1 Integral Basics

5.3 Average Value

Integrals are pretty much an infinite amount of Riemann sums. If f is integrable on $[a, b]$, then its average value (or mean) on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.

5.5 U-Substitution

Since du can be substituted for $(\frac{du}{dx})dx$, u-substitution is possible.
Ex. Find $\int (x^3 + x)^5(3x^2 + 1)dx$.

$$\begin{aligned}
\int_a^b (x^3 + x)^5(3x^2 + 1)dx \\
\int_a^b (u)^5(3x^2 + 1)dx \\
u = x^3 + x \\
\frac{du}{dx} = 3x^2 + 1 \\
\int_{a^3+a}^{b^3+b} (u)^5(\frac{du}{dx})dx \\
\int_{a^3+a}^{b^3+b} (u)^5 du \\
\int_{a^3+a}^{b^3+b} (u)^5 du \\
\frac{(b^3 + b)^6}{6} - \frac{(a^3 + a)^6}{6}
\end{aligned} \tag{1}$$

5.6 Area Between Curves

If $f(x) \geq g(x)$ for $[a, b]$, then the area between both curves from a to b is equal to $A = \int_a^b [f(x) - g(x)]dx$.

6.1 Volume Using Integrals: Disk and Washer Methods

The volume of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is equal to $\int_a^b A(x)dx$.
The volume of a disk created by rotating a function $R(x)$ around $y = 0$ (the x-axis) for the same bounds is equal to $\int_a^b \pi[R(x)]^2 dx$.
The volume of a disk created by rotating a function $R(y)$ around $x = 3$ for the same bounds is equal to $\int_{R((a)}^{R(b)} \pi[R(y) - 3]^2 dy$.
The reason 3 is subtracted from $R(y)$ is because the axis of rotation is closer to the area being rotated, so the radius of rotation will be less than if $x = 0$.
The volume of a washer created by rotating the space between $R(x)$ and $r(x)$ around $y = 1$ is equal to $\int_a^b \pi([R(x) - 1]^2 - [r(x) - 1]^2)dx$. The reason a 1 is subtracted is because the axis of rotation is closer to the area being rotated, so the radius of rotation will be less than if $y = 0$.
The volume of a washer created by rotating the space between R(y) and r(y) around x=0 (the y-axis) is equal to $\int_{R(a)}^{R(b)} \pi([R(y)]^2 - [r(y)]^2)dx$.

6.2 Volumes Using Integrals: Shell Method

Summarize Shell Method

2 Integral Applications and Sections

8.2 Integration by Parts

1. Pick a u to derive
2. Pick a dv to integrate
3. $\int u * dv = u * v - \int v * du$

8.3 Trigonometric Integrals

$sincos$ integals $\rightarrow sin^2(x) + cos^2(x) = 1$
$\int sin(x)cos^n(x)dx$
1. Case 1: m is odd $\rightarrow u = cosx$
- Make $sin^2(x) = (1 - cos^2(x))$
2. Case 2: n is odd $\rightarrow u = sinx$
- Make $cos^2(x) = (1 - sin^2(x))$
3. Case 3: m and n are even
- Use the following identities...
$sin^2(x) = \frac{1-cos2x}{2}$ and $cos^2(x) = \frac{1+cos2x}{2}$

Eliminating Square Roots
- Use following identities..

$$1.sin^2x + cos^2x = 1$$
$$2.cos2x = cos^2x - sin^2x$$
$$3.tan^2x + 1 = sec^2x$$
$$4.sin2x = 2sinxcosx$$
$$5.sinx^2 = \frac{1 - cos2x}{2}$$
$$6.cos^2x = \frac{1 + cos2x}{2}$$

Products of sin and cos

$$- sin(mx)sin(nx) = \frac{1}{2}(cos(m-n)x - cos(m+n)x)$$
$$- sin(mx)cos(nx) = \frac{1}{2}(sin(m-n)x + sin(m+n)x)$$
$$- cos(mx)cos(nx) = \frac{1}{2}(cos(m-n)x + cos(m+n)x)$$

8.4 Trigonometric Substitutions

1.

8.5 Partial Fraction Decomposition

*Degree

Density Problems

- 1-dimensional

3 Sequences, Series, and Tests

8.8

10.1

10.2

10.3

10.4

4 Sequences, Series, and Tests cont.

10.5

10.6

10.7

10.8

10.9

5 Parametric Curves

10.10

6.3 Arc Length

Pythagorean's Theoreom can be applied to find the length of a segment $f(x)$. If ds is equal to a single straight segment in $f(x)$, then dx is equal to the horizontal length and dy is equal to its vertical length.

$$\begin{aligned} (ds)^2 &= (dx)^2 + (dy)^2 \\ \sqrt{(ds)^2} &= \sqrt{(dx)^2 + (dy)^2} \\ ds = dx&\sqrt{(dx)^2/(dx)^2 + (dy)^2/(dx)^2} \\ ds &= \sqrt{1 + (dy/dx)^2}dx \end{aligned} \tag{2}$$

By taking the integral of this, you can get the total length of the segment.

$$s = \int_a^b \sqrt{1 + (f(x))^2}\, dx \tag{3}$$

6.4

11.1

11.2

6 Polar Coordinates

11.3

11.4

11.5

Complex Numbers

Standard form $= a + bi$. a is real portion, bi is imaginary. To graph: 3 units to the right, imaginary is 4 so 4 units up. To calculate absolute value, $/absa + bi = /sqrta^2 + b^2$
For -5 + 12i:
Graph 5 units to the left, and up 12 units.

$|-5+12i| = \sqrt{(-5)^2+12^2}$
$|-5+12i| = \sqrt{(25+144}$
$|-5+12i| = \sqrt{169}$
$|-5+12i| = 13$

An example of how a square root of negative looks: $\sqrt{-80} = i * \sqrt{16} * \sqrt{5} = 4i\sqrt{5}$

$$\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned} \tag{4}$$

Idea: Associate polar coordinates to complex numbers
$z = x + iy$
$r = |z| = \sqrt{x^2+y^2}$
if $x \geq 0$, $\theta = arctan\frac{y}{x}$
if $x \leq 0$, $\theta = arctan\frac{y}{x} + \pi$
if $x = 0, \theta = \pm\frac{\pi}{2}$
Find polar coordinates of $z = -2\sqrt{2} + 2\sqrt{2}i$
$r = |z| = \sqrt{(-2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$
Since $x \leq 0$, $\theta = arctan\frac{y}{x} + \pi = \frac{3\pi}{4}$
Polar Coordinates = $(4cos\frac{3\pi}{4}, 4sin\frac{3\pi}{4})$