1 Integral Basics

5.3 Average Value

Integrals are pretty much an infinite amount of Riemann sums. If f is integrable on $[a, b]$, then its average value (or mean) on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.

5.5 U-Substitution

Since du can be substituted for $(\frac{du}{dx})dx$, u-substitution is possible.
Ex. Find $\int(x^3+x)^5(3x^2+1)dx$.

$$\begin{aligned}
\int_a^b (x^3+x)^5(3x^2+1)dx \\
\int_a^b (u)^5(3x^2+1)dx \\
u = x^3+x \\
\frac{du}{dx} = 3x^2+1 \\
\int_{a^3+a}^{b^3+b} (u)^5(\frac{du}{dx})dx \\
\int_{a^3+a}^{b^3+b} (u)^5 du \\
\int_{a^3+a}^{b^3+b} (u)^5 du \\
\frac{(b^3+b)^6}{6} - \frac{(a^3+a)^6}{6}
\end{aligned} \tag{1}$$

5.6 Area Between Curves

If $f(x) \geq g(x)$ for $[a, b]$, then the area between both curves from a to b is equal to $A = \int_a^b [f(x) - g(x)]dx$.

6.1 Volume Using Integrals: Disk and Washer Methods

The volume of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is equal to $\int_a^b A(x)dx$.
The volume of a disk created by rotating a function $R(x)$ around $y = 0$ (the x-axis) for the same bounds is equal to $\int_a^b \pi[R(x)]^2dx$.
The volume of a disk created by rotating a function $R(y)$ around $x = 3$ for the same bounds is equal to $\int_{R((a)}^{R(b)} \pi[R(y)-3]^2dy$. The reason 3 is subtracted from $R(y)$ is because the axis of rotation is closer to the area being rotated, so the radius of rotation will be less than if $x = 0$.
The volume of a washer created by rotating the space between $R(x)$ and $r(x)$ around $y = 1$ is equal to $\int_a^b \pi([R(x)-1]^2 - [r(x)-1]^2)dx$. The reason a 1 is subtracted is because the axis of rotation is closer to the area being rotated, so the radius of rotation will be less than if $y = 0$.
The volume of a washer created by rotating the space between R(y) and r(y) around x=0 (the y-axis) is equal to $\int_{R(a)}^{R(b)} \pi([R(y)]^2 - [r(y)]^2)dx$.

6.2 Volumes Using Integrals: Shell Method

$V = 2\pi\int_a bR(x)h(x)dx$ Draw a rectangle parallel to the axis of rotation. The change in the height of this rectangle between a and b is modeled by $h(x)$. The change in the distance of the rectangle from the axis of rotation is modeled by $R(x)$. Basically integrating an area $R(x) * h(x)$ over a certain interval times 2π.
When using the area between two curves, $h(x) = f(x) - g(x)$ where $f(x) > g(x)$ for [a, b].

2 Integral Applications and Sections

8.2 Integration by Parts

1. Pick a u to derive
2. Pick a dv to integrate
3. $\int u * dv = u * v - \int v * du$

8.3 Trigonometric Integrals

$sincos$ integals $\rightarrow sin^2(x) + cos^2(x) = 1$
$\int sin(x)cos^n(x)dx$
1. Case 1: m is odd $\rightarrow u = cosx$
- Make $sin^2(x) = (1 - cos^2(x))$
2. Case 2: n is odd $\rightarrow u = sinx$
- Make $cos^2(x) = (1 - sin^2(x))$
3. Case 3: m and n are even
- Use the following identities...
$sin^2(x) = \frac{1-cos2x}{2}$ and $cos^2(x) = \frac{1+cos2x}{2}$

Eliminating Square Roots
- Use following identities..

$$1.sin^2x + cos^2x = 1$$
$$2.cos2x = cos^2x - sin^2x$$
$$3.tan^2x + 1 = sec^2x$$
$$4.sin2x = 2sinxcosx$$
$$5.sinx^2 = \frac{1 - cos2x}{2}$$
$$6.cos^2x = \frac{1 + cos2x}{2}$$

Products of sin and cos

$$-sin(mx)sin(nx) = \frac{1}{2}(cos(m-n)x - cos(m+n)x)$$
$$-sin(mx)cos(nx) = \frac{1}{2}(sin(m-n)x + sin(m+n)x)$$
$$-cos(mx)cos(nx) = \frac{1}{2}(cos(m-n)x + cos(m+n)x)$$

8.4 Trigonometric Substitutions

$1.Case \quad 1: a^2 + x^2$
$\rightarrow x = atan\theta$
$\rightarrow x = asec^2\theta d\theta$
$\rightarrow a^2 + x^2 = a^2 + a^2tan^2\theta = a^2(1 + tan^2\theta) = a^2sec^2\theta$
$2.Case \quad 2: a^2 - x^2$
$\rightarrow x = asin\theta$
$\rightarrow dx = acos\theta d\theta$
$\rightarrow a^2 - x^2 = a^2 - a^2sin^2\theta = a^2(1 - sin^2\theta) = a^2cos^2\theta$
$3.Case \quad 3: x^2 - a^2$
$\rightarrow x = asec\theta$
$\rightarrow dx = asec\theta tan\theta d\theta$
$\rightarrow x^2 - a^2 = a^2sec^2\theta - a^2 = a^2(sec^2\theta - 1) = a^2tan^2\theta$

8.5 Partial Fraction Decomposition

*Degree on top must be less than degree on bottom, or long divide

- If there is a linear factor $\rightarrow \frac{A}{x+1} + \frac{B}{(x+1)^2}$
- If there is a quadratic factor $\rightarrow \frac{Ax+B}{x^2+bx+c} + \frac{Cx+D}{(x^2+bx+c)^2}$
- Set $\frac{f(x)}{g(x)}$ equal to the sum of its partial fractions by multiplying
- Equate coefficients of powers of x to solve for variables

$$\frac{-x+3}{x^2-9x+20}$$
$$\frac{-x+3}{(x-4)(x-5)}$$
$$\frac{-x+3}{(x-4)(x-5)} = \frac{A}{x-4} + \frac{B}{x-5}$$
$$\frac{-x+3}{(x-4)(x-5)} * (x-4)(x-5) = \frac{A}{x-4}(x-4)(x-5) + \frac{B}{x-5}(x-4)(x-5)$$
$$-x+3 = A(x-5) + B(x-4)$$
$$x = 5 : -2 = A(0) + B(1) \rightarrow So, B = -2$$
$$x = 4 : -1 = A(-1) + B(0) \rightarrow So, A = 1$$
$$\frac{-x+3}{x^2-9x+20} = \frac{1}{x-4} - \frac{2}{x-5}$$

Density Problems

- 1-dimensional $\rightarrow$ need length or mass (δx meters or grams)

- 2-dimensional $\rightarrow$ need area of a section (rectangle bh where h is δx) (or other shape)

- 3-dimensional $\rightarrow$ need volume of a section (cylinder $= \pi r^2 h$ where h is δy / other shape)

3 Sequences, Series, and Tests

8.8

$$Test$$

10.1

10.2

10.3

10.4

4 Sequences, Series, and Tests cont.

10.5

10.6

10.7

10.8

10.9

5 Parametric Curves

10.10

6.3 Arc Length

Pythagorean's Theoreom can be applied to find the length of a segment $f(x)$. If ds is equal to a single straight segment in $f(x)$, then dx is equal to the horizontal length and dy is equal to its vertical length.

$$\begin{aligned} (ds)^2 &= (dx)^2 + (dy)^2 \\ \sqrt{(ds)^2} &= \sqrt{(dx)^2 + (dy)^2} \\ ds &= dx\sqrt{(dx)^2/(dx)^2 + (dy)^2/(dx)^2} \\ ds &= \sqrt{1 + (dy/dx)^2}dx \end{aligned} \tag{2}$$

By taking the integral of this, you can get the total length of the segment.

$$s = \int_a^b \sqrt{1 + (f(x))^2}\, dx \tag{3}$$

6.4

11.1

11.2

6 Polar Coordinates

11.3

11.4

11.5

Complex Numbers

Standard form $= a + bi$. a is real portion, bi is imaginary. To graph: 3 units to the right, imaginary is 4 so 4 units up. To calculate absolute value, $/absa + bi = /sqrta^2 + b^2$
For -5 + 12i:
Graph 5 units to the left, and up 12 units.
$|-5 + 12i| = \sqrt{(-5)^2 + 12^2}$
$|-5 + 12i| = \sqrt{(25 + 144}$
$|-5 + 12i| = \sqrt{169}$
$|-5 + 12i| = 13$

An example of how a square root of negative looks: $\sqrt{-80} = i * \sqrt{16} * \sqrt{5} = 4i\sqrt{5}$

$$\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned} \tag{4}$$

Idea: Associate polar coordinates to complex numbers
$z = x + iy$
$r = |z| = \sqrt{x^2 + y^2}$
if $x \geq 0$, $\theta = arctan\frac{y}{x}$
if $x \leq 0$, $\theta = arctan\frac{y}{x} + \pi$
if $x = 0, \theta = \pm\frac{\pi}{2}$
Find polar coordinates of $z = -2\sqrt{2} + 2\sqrt{2}i$
$r = |z| = \sqrt{(-2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$
Since $x \leq 0$, $\theta = arctan\frac{y}{x} + \pi = \frac{3\pi}{4}$
Polar Coordinates $= (4cos\frac{3\pi}{4}, 4sin\frac{3\pi}{4})$