CS 205 Homework 1

Spring 2021

1. Let p, q, and r be the propositions

   - p: You have the flu
   - q: You miss the final exam
   - r: You pass the course

   Express each of these as an English sentence:

   (a) $p \to q$
   (b) $(p \to \neg r) \vee (q \to \neg r)$
   (c) $(p \wedge q) \vee (\neg q \wedge r)$

2. State the converse, inverse, and contrapositive for each of the following:

   (a) If it snows tonight, then I will stay home.
   (b) I go to the beach whenever it's a sunny summer day.
   (c) When I stay up late, it's necessary that I sleep until noon.

3. Show the following:

   (a) $p \leftrightarrow q$ is equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$
   (b) $(p \to r) \wedge (q \to r)$ is equivalent to $(p \vee q) \to r$

4. Find an expression that's equivalent to $p \vee q$ but uses only the $\neg$ and $\wedge$ operators. Prove that these are equivalent.

5. Prove that $p \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ is a tautology.

6. Find a satisfying assignment if one exists for the following, or if not, prove that it's a contradiction:

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (\neg r \vee \neg p) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$$

7. What is the negation of the statement "if you take every quiz, you get a cookie"?

8. Given these predicates:

   - $C(x)$ means x has a cat
   - $D(x)$ means x has a dog
   - $F(x)$ means x has a ferret

   Write the following as predicate logic expressions:

   (a) A student in class has a cat, a dog, and a ferret

   (b) All students in the class have a cat, a dog, or a ferret

   (c) Some student in class has a cat and a ferret, but not a dog

   (d) No student in class has a cat, a dog, and a ferret

   (e) For each of the three kinds of animal, there is a student in class who has this kind of animal as a pet.

9. Determine the truth value of these expressions, assuming the domain is the real numbers.

   (a) $\exists x(x^3 = -1)$

   (b) $\exists x(x^4 < x^2)$

   (c) $\forall x((-x)^2 = x^2)$

   (d) $\forall x(2x > x)$

10. What is the negation of each of the following?

    (a) $\forall x(\exists y(P(x) \rightarrow Q(y)))$

    (b) $\exists y(P(y) \vee \exists x(R(x) \wedge R(y)))$