CS 205 Homework 4

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1. **Show that for any** $n \in N$**,** $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$**.**
   Proving $n \to n+1$:
   Assume for $k = n$: $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2 = \frac{n(n+1)(2n+1)}{6}$.
   Then for $k = n+1$: $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$.
   Since both contain $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2$, this can be substituted for $\frac{n(n+1)(2n+1)}{6}$.
   Therefore, $\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$.
   Expanding these results in: $\frac{2n^3+9n^2+13n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$. ■

2. **Show that for any** $n \in N$ **where** $n \geq 1$**,** $\prod_{k=1}^{n}(1 + \frac{1}{k}) = n+1$**.**
   Proving $n \to n+1$:
   Assume for $k = n$: $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n}) = (n+1)$.
   Then for $k = n+1$: $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n}) * (1 + \frac{1}{n+1}) = (n+2)$
   Since both contain $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n})$, this can be substituted for $(n+1)$.
   Therefore, $n + 1 * (1 + \frac{1}{n+1}) = (n+2)$.
   Expanding the left side results in $(n+2)$ and $n+2 = n+2$. ■