CS 205 Homework 2

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1. Prove or disprove: Every odd number is the difference of two squares.
   Disproof: Using 1, the lowest squares are $1^2 = 1$ and $2^2 = 4$ which already have a difference of 3. Each successive square beyond these has a large and larger difference in between. Therefore, there does not exist two squares whose difference is equal to 1.

2. Prove or disprove: If $x > 3$ and $y < 2$, then $x^2 - 2y > 5$.
   Proof: The lowest possible value of x is 4 so the lowest possible value of the first term is 16 or $(4)^2$. The largest possible value of y of 1, so the largest possible value of the second term is 2 or $(2) * 1$. Under these conditions, the left side is equal to 15, which is greater than 5. As x continues to rise and y continues to decline, this value will continue to rise, but the lowest possible value that can be found is 15.

3. Prove or disprove: For all real numbers $x > 0$, there exists a real number y such that $x = y(y + 1)$.
   Proof: $x = y(y + 1)$ can be rerranged as $y^2 + y - x$. Since x is a constant, this is in the form of a quadratic equation. The quadratic equation $x^2 + x + C$ (the base of the aforementioned one) has at least one real zero as long as $C \leq 0.25$, so for the original equation, since the constant x is on the opposite side, as long as x is greater than -0.25 there will be a real number for y that satisfies the equation. Since the original restrictions are $x > 0$, and this shows that $0 > -0.25$, the original statement is true.

4. Suppose $2y + 3x = 3y - 4x$, and x and y are not both zero. Prove that $y \neq 0$.
   Disproof: If $y = 0$, the equation would be reduced to $3x = -4x$, which could be true if $x = 0$ but both numbers are not allowed to equal zero. $\lightning$