

CS 205 Homework 3

Keith Lehman, kpl56@scarletmail.rutgers.edu

March 5, 2021

1. Prove the following distributive law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
If the left side were a set, it would contain all of A , as well as the intersection of B and C . So for the set L that is the left side, $A \subseteq L$, and $(B \cap C) \subseteq L$.
For the set R that is the right side, the union of everything in $A + B$ and everything in $A + C$ would contain everything in A since everything in A is also in A . It would also contain all elements at the intersection of B and C . So, like L , $A \subseteq R$, and $(B \cap C) \subseteq R$. Since both sides have the exact same components, they must be equal.
2. Suppose $A \subseteq B$ and $C \subseteq D$. Show that $A \times C \subseteq B \times D$.
Since A is a subset of B and C is a subset of D , $\forall x(x \in A \implies x \in B) \wedge \forall y(y \in C \implies y \in D)$. Therefore, any products that can be created between A and C will also be products that can be created between B and D . Therefore, $(A \times C \subseteq B \times D)$.
3. Prove that $A - (B \cap C) = (A - B) \cup (A - C)$.
The left side is A minus any elements of B that are also in C . The right side is two operations, $A - B$ and $A - C$. After doing both of these and creating a set of the union of both, what will remain is A minus the elements of B that are also in C because for example, for $x \in A \wedge x \in B \wedge x \notin C$, x will be subtracted from A in $A - B$ but not in $A - C$, so $(A - B) \cup (A - C)$ will still contain x because it will not be removed from A in $A - C$. Only elements that are in both B and C will not be included, alike the left side.
4. An ordered pair (a, b) can be defined as the set $\{a, \{a, b\}\}$. Show that $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. For an ordered pair to be equal to a, a, b , by the definition of an ordered pair, the first term of the ordered pair must be equal to a and the second must be equal to b . There are no exceptions to this. Therefore, for $(a, b) = (c, d)$, c must equal a and d must equal b .
5. Let $f : R \rightarrow R$ be defined by $f(x) = 4x^3 - 2$.
 - (a) Is f injective?
 f is injective since it maps to each member of the codomain once and never more than once.
 - (b) Is f surjective?
 f is surjective since it maps to each member of the codomain once.
6. Let $S = P(R)$. Let $f : R \rightarrow S$ be defined by $f(x) = \{y \in R : y^2 < x\}$.
 - (a) Is f injective?
 f is not injective because for any given value of x , there are multiple values of y that can satisfy the equation.
 - (b) Is f surjective? f is not surjective. While all members of the reals are mapped to, the empty set is not and cannot be mapped to with the given function.
7. Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$.
 - (a) Prove that if f is surjective and g is not injective, $g \circ f$ is not injective.
While every element of B is mapped to by an element of A , since g is not injective, there are elements of C that are mapped to more than once by values of B . Therefore, for $g \circ f : A \rightarrow B \rightarrow C$, there will be multiple elements of A that will map to the same element in C , making $g \circ f$ not injective.
 - (b) Prove that if f is not surjective and g is injective, $g \circ f$ is not surjective.
While there are no elements of C mapped to more than once by elements of B , not all elements of B are mapped to by elements of A . Therefore, since each element in B maps to a unique element in C , but not every element in B is mapped to by an element of A , there will be elements of C not mapped to by elements of A , making $g \circ f$ not surjective.
8. Suppose $f : B \rightarrow C$, $g : A \rightarrow B$, and $h : A \rightarrow B$. If f is injective, prove that if $f \circ g = f \circ h$, then $g = h$.