CS 205 Homework 5

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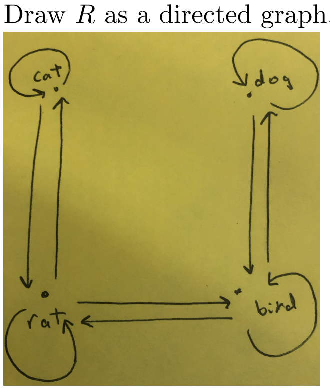
1. Given an arbitrary relation R, suppose we compute two new relations. Prove $R_1 = R_2$ for all R.
   For R_1 to be the reflixive closure of the transitive closure of R, it first must have every element map to the next element and every succeeding element. For it to be reflexive, each element mentioned must also be related to itself. For R_2 to be the transitive closure of the reflexive closure of R, it first must have every element related to itself, then, for each element, it must relate to the following element, as well as every other following element. Under these conditions, both R_1 and R_2 will continue the exact same elements.

2. Let $A = \{\text{cat, dog, bird, rat}\}$ and R be a relation on A defined by $\{(x, y) : x \text{ and } y \text{ have at least one letter in common}\}$.

   (a) Draw R as a directed graph.

   (b) Is R reflexive, symmetric, and/or transitive?
   R is reflexive as every element has every letter in common with itself. R is also symmetric because if one element has a letter in common with another element, that other element also has that same letter in common with the first. R is NOT transitive. While element 1 may have a letter in common with element 2 and element 2 might have a letter in common with element 3, these could be different letters meaning that element 1 may not be related to element 3, making it NOT transitive.

3. Given a relation R on a set A, prove that if R is transitive, then so is R^{-1}.
   For R to be transitive on set A, substituting indexes for elements, R must contain $\{(0, 1), (1, 2), (2, 3)..(n, n+1), (0, n+1)\}$. Therefore, we know that R^{-1} must contain $\{(n+1, n)..(3, 2), (2, 1), (1, 0), (n+1, 0)\}$. While in a different order, the first element is still related to the last, making the entire relation transitive.

4. Suppose R and S are symmetric relations on a set A. Prove that $R \circ S$ is symmetric iff $R \circ S = S \circ R$.