CS 205 Homework 4

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1. **Show that for any $n \in N$, $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.**
   Base Case: $0^2 = \frac{0(0+1)(0+1)}{6} = 0$
   Proving $n \rightarrow n+1$:
   Assume for $k = n$: $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2 = \frac{n(n+1)(2n+1)}{6}$.
   Then for $k = n+1$: $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$.
   Since both contain $0^2 + 1^2 + 2^2 + 3^2 + 4^2 + .. + n^2$, this can be substituted for $\frac{n(n+1)(2n+1)}{6}$.
   Therefore, $\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$.
   Expanding these results in: $\frac{2n^3+9n^2+13n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$. $\blacksquare$

2. **Show that for any $n \in N$ where $n \geq 1$, $\prod_{k=1}^{n}(1 + \frac{1}{k}) = n + 1$.**
   Base Case: $1 + \frac{1}{1} = 1 + 1 = 2$
   Proving $n \rightarrow n+1$:
   Assume for $k = n$: $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n}) = (n+1)$.
   Then for $k = n+1$: $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n}) * (1 + \frac{1}{n+1}) = (n+2)$
   Since both contain $(1+1) * (1 + \frac{1}{2}) * (1 + \frac{1}{3}) * .. * (1 + \frac{1}{n})$, this can be substituted for $(n+1)$.
   Therefore, $n + 1 * (1 + \frac{1}{n+1}) = (n+2)$.
   Expanding the left side results in $(n+2)$ and $n+2 = n+2$. $\blacksquare$

3. Find and prove a closed-form formula for the sum of row k of Pascal's triangle.
   Formula: 2^k.
   Proof: Pascal's triangle is not only a fun mathematical diagram but also shows the coefficients for terms in binomial expansion.
   $(a+b)^0 = 1$
   $(a+b)^1 = 1a + 1b$
   $(a+b)^2 = 1a^2 + 2ab + 1b^2$
   $(a+b)^k = 1a^k + ka^{(k-1)}b + .. + kab^{(k-1)} + 1b^k$
   Setting both a and b to 1 would allow us to calculate the sum at each level. However, replacing both with 1 also simplifies the expansion to $(a+b)^k = (2)^k$ which can then be used as the formula for the sum of a given row.

4. Find and prove a closed-form formula for the nth pentagonal number.