CS 205 Homework 1

Keith Lehman, kpl56@scarletmail.rutgers.edu

February 7, 2021

1. Express propositions as English sentences:

   (a) You miss the final exam unless you don't have the flu.

   (b) You will not pass the course if you have the flu or miss the final.

   (c) You have the flu and miss the final exam or you pass the course and attend the final exam.

2. State the converse, imverse, and contrapositive of English sentences:

   (a) Converse: If I stay home, it will snow tonight.
   Inverse: If it doesn't snow tonight, I won't stay home.
   Contrapositive: If I don't stay home, it won't snow tonight.

   (b) Converse: If I go the beach, then it is a sunny summer day.
   Inverse: If it isn't a sunny summer day, then I won't go the beach.
   Contrapositive: If I don't go to the beach, then it isn't a sunny summer day.

   (c) Converse: When I sleep until noon, I absolutely must have stayed up late the previous night.
   Inverse: When I don't stay up late, I don't sleep until noon.
   Contrapositive: When I don't sleep until noon, then I must have not stayed up late the previous night.

3. Prove some statements:

   (a) $p \iff q$ is equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$:

p	q	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \iff q$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	F	T	T	T

   (b) $(p \to r) \wedge (q \to r)$ is equivalent to $(p \vee q) \to r$:

p	q	r	$(p \to r) \wedge (q \to r)$	$(p \vee q) \to r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

4. Find an expression equivalent to $p \vee q$ using only $\neg$ and $\wedge$:

   (a) Answer: $\neg(\neg p \wedge \neg q)$. Proof:

p	q	$p \vee q$	$\neg(\neg p \wedge \neg q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

5. Prove that $p \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ is a tautology.

(a)

p	q	$p \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

6. Find a satisfying assignment if one exists for the following, or if not, prove that it's a contradiction:
$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (\neg r \vee \neg p) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$.
Starting from the left, either p is true, q is false, or both.
Test: If p is true, r must be false to make the third epxression true. If r is false, then q must be false in order for the fifth expression to be true. Therefore, $p = \text{T}$, $q = \text{F}$, $r = \text{F}$.

7. What is the negation of the statement "if you take every quiz, you get a cookie"?
Answer: "Despite taking every quiz, you did not receive a cookie".

8. (a) $\exists x\, C(x) \wedge D(x) \wedge F(x)$
   (b) $\forall x\, C(x) \vee D(x) \vee F(x)$
   (c) $\exists x\, C(x) \wedge -D(x) \wedge F(x)$
   (d) $\forall x\, C(x) \oplus D(x) \oplus F(x)$
   (e) $\exists xyz\, C(x) \wedge D(y) \wedge F(z)$

9. Determine truth values of expressions:
   (a) True, $x = -1$
   (b) True, $x = \frac{1}{2}$
   (c) True, works with all real numbers
   (d) False, fails with all negative numbers

10. (a) $\exists x(\neg\forall y(P(x) \to Q(y)))$
    (b) $\neg\forall y(P(y) \vee \neg\forall x(R(x) \wedge R(y)))$