HW2

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1 Problem 1: "Monetary policy rules and macroeconomic stability: evidence and some theory" Clarida, Gali and Gertler (2000)

1.1 (a)

Both r_t and ϵ_t contain $\pi_{t,k}$ and $X_{t,q}$ by definition. Therefore it will not be the case that $\mathbb{E}[\epsilon_t|y_t] \neq 0$ where $y_t = \bar{\alpha} + \beta\pi_{t,k} + \gamma X_{t,q}$. So, we will have an endogeniety problem if we simply use NLS.

1.2 (b)

Let's choose some $z_t \in \Omega_t$

$$\mathbb{E}(z_t, \epsilon_t) = \mathbb{E}[\mathbb{E}(z_t, \epsilon_t|\Omega_t)] = \mathbb{E}[z_t\mathbb{E}(\epsilon_t|\Omega_t)] = 0 \text{ Because: } \mathbb{E}(\epsilon_t|\Omega_t) = 0 \text{ (see 1.3(c) below)}$$

So we can use any $z_t \in \Omega_t$ as an exogenous valid instrument under this setup.

1.3 (c)

First we note: $\mathbb{E}[\epsilon_t] = \mathbb{E}[\nu_t + \mu_t] = \mathbb{E}[\nu_t] + \mathbb{E}[\mu_t] = 0 + \mathbb{E}[\mu_t]$

We want to show: $\mathbb{E}[\mu_t, \mu_{t-1}] = 0$

$$\mathbb{E}[\mu_t, \mu_{t-1}] = \mathbb{E}[\mathbb{E}(\mu_t, \mu_{t-1}|\Omega_t)] =$$

$$\mathbb{E}[\mathbb{E}(\mu_t, -(1-\rho)[(\pi_{t-1,1} - \mathbb{E}(\pi_{t-1,1}|\Omega_{t-1}) + \gamma(X_{t-1,1} - \mathbb{E}(X_{t-1,1}|\Omega_{t-1})|\Omega_t)]] =$$

$$\mathbb{E}[-(1-\rho)[(\pi_{t-1,1} - \mathbb{E}(\pi_{t-1,1}|\Omega_{t-1}) + \gamma(X_{t-1,1} - \mathbb{E}(X_{t-1,1}|\Omega_{t-1})]E(\mu_t|\Omega_t)]$$

Because $-(1-\rho)[(\pi_{t-1,1} - \mathbb{E}(\pi_{t-1,1}|\Omega_{t-1}) + \gamma(X_{t-1,1} - \mathbb{E}(X_{t-1,1}|\Omega_{t-1})]$ is known at time t we can take it outside the conditional expectation. Furthermore:

$$\mathbb{E}[-(1-\rho)[(\pi_{t-1,1} - \mathbb{E}(\pi_{t-1,1}|\Omega_{t-1}) + \gamma(X_{t-1,1} - \mathbb{E}(X_{t-1,1}|\Omega_{t-1})]E(\mu_t|\Omega_t)] = 0$$

Because:

$$\mathbb{E}[\mu_t|\Omega_t] = \mathbb{E}[-(1-\rho)[(\pi_{t,1} - \mathbb{E}(\pi_{t,1}|\Omega_t) + \gamma(X_{t,1} - \mathbb{E}(X_{t,1}|\Omega_t)] =$$

$$-(1-\rho)[(\mathbb{E}(\pi_{t,1}|\Omega_t) - \mathbb{E}(\pi_{t,1}|\Omega_t) + \gamma(\mathbb{E}(X_{t,1}|\Omega_t) - \mathbb{E}(X_{t,1}|\Omega_t)] = 0$$

Therefore ϵ_t is serially uncorrelated.

Here are their original results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$	p
Pre-Volcker	4.24	.83	.27	.68	.834
	(1.09)	(.07)	(.08)	(.05)	
Volcker-Greenspan	3.58	2.15	.93	.79	.316
	(.50)	(.40)	(.42)	(.04)	

Here are my results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$	p
Pre-Volcker	4.75	.92	1.04	.63	.000
	(.22)	(.08)	(.22)	(.07)	
Volcker-Greenspan	3.51	2.65	.46	.84	.000
	(.15)	(.47)	(.31)	(.04)	

My point estimates are pretty close for $\hat{\beta}$, $\hat{\pi^*}$ and $\hat{\rho}$ but not $\hat{\gamma}$. Estimates of $\hat{\gamma}$ are sensitive to the choice of the output gap measure, and if we've made different choices than the authors this could be the source of this difference in these point estimates of $\hat{\gamma}$. Obviously, one major difference is in the p-values associated with the test of the over-identifying restrictions. The authors fail to reject the null, and thus find evidence in support of their restrictions, whereas we are strongly rejecting the null.

1.4 (d)

Here are my NLS results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$
Pre-Volcker	4.47	1.08	1.085	.71
	(.30)	(.10)	(.29)	(.06)
Volcker-Greenspan	3.44	1.95	1.19	.85
	(.50)	(.53)	(.88)	(.06)

Here are the results of my Hausman tests:

	$J - Stat$	$p - value$
Pre-Volcker	2.16	.706
Volcker-Greenspan	18.65	.000

In the Pre-Volcker estimation, I do not reject the null hypothesis that there is no difference between our parameter estimates using NLS and two-stage GMM. Given how close the point estimates and standard errors are, this is not surprising. In the Volcker-Greenspan estimation, we do reject the null though. We expected our results using NLS to suffer from the endogeneity issues described in part (a), so the results of the Pre-Volcker estimation are puzzling, whereas the results from the Volcker-Greenspan estimation seem to confirm these issues.

1.5 (e)

Here are my results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$	p
Pre-Volcker	4.79	1.03	.97	.64	.008
	(.23)	(.11)	(.26)	(.08)	
Volcker-Greenspan	3.37	2.64	.43	.84	.001
	(.23)	(.66)	(.53)	(.07)	

The results are not very different from the results in (c).

1.6 (f)

Here are my results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$	p
$k = 4, q = 1$					
Pre-Volcker	19.21	.21	1.04	.79	.000
	(1.09)	(.07)	(.08)	(.05)	
Volcker-Greenspan	-118.79	3.72	-76.93	1.00	.081
	(3.49e+03)	(479.78)	(1.16e+04)	(.03)	
$k = 4, q = 2$					
Pre-Volcker	19.32	.21	1.13	.80	.000
	(1.78)	(.03)	(.39)	(.06)	
Volcker-Greenspan	-119.86	2.40	-48.59	1.00	.091
	(3.52e+03)	(185.55)	(4.81e+04)	(.03)	

Obviously, these are incredibly different from what the authors found under these alternative horizon specifications, and in some cases flat out impossible (-118.79 for inflation for example).

I was unable to determine the cause. First, the original paper gives very little information about how the authors produced their results. I tried altering several aspects of my code to uncover the root of the problem. I first suspected that my thetainit values were the culprit, but even when I altered them my results didn't change. I then suspected my optimal weighting matrix, and possibly the correction for the serial correlation using Newey-West. Even when I tinkered with these, I still didn't achieve anything close to the original table. I finally noticed that the thetaGMM1 results were incredibly off as well. I tried altering my second-stage to start with the thetinit values set to the results in table 4 of the original paper. This also didn't substantially alter the results. This has led me to believe that it is perhaps something related to fminunc, and how it is being utilized in the code since I am getting these wildly incorrect results for NLS, and both the first and second stages of my GMM.

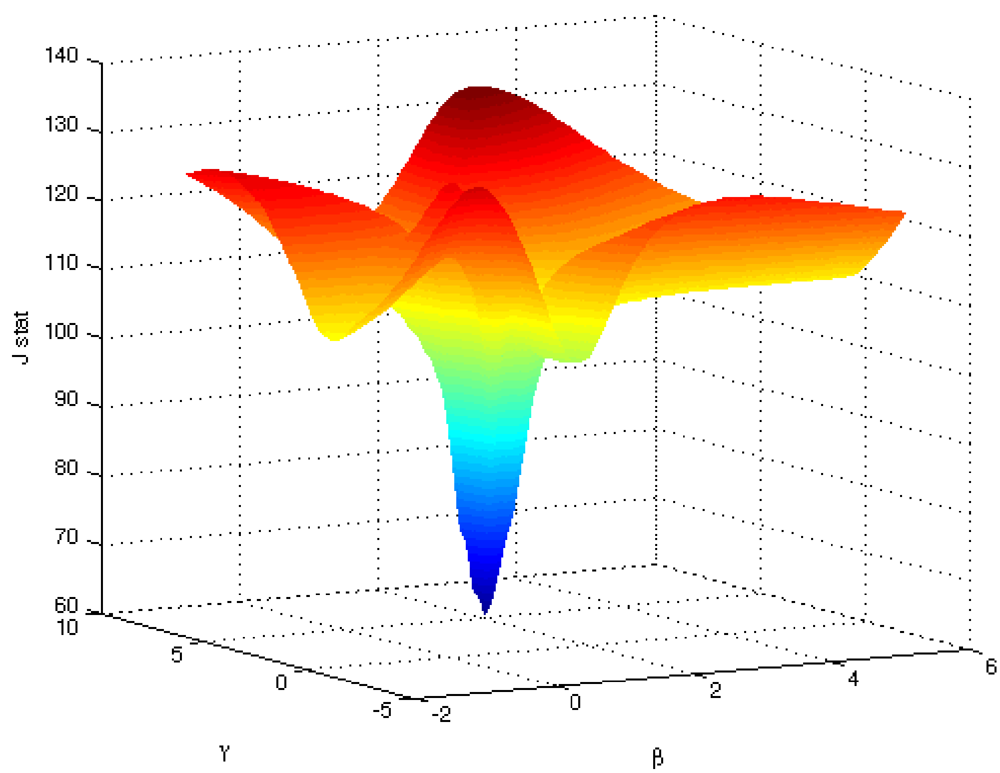
1.7 (g)

Here are my results:

	$\hat{\pi^*}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\rho}$	p
Pre-Volcker	4.75	.92	1.04	.63	.000
	(.22)	(.08)	(.22)	(.07)	
Volcker-Greenspan	3.51	2.65	.46	.84	.000
	(.15)	(.47)	(.31)	(.04)	
Post-CGG	2.39	1.72	.43	.92	.000
	(.36)	(1.40)	(.12)	(.04)	

While, the point estimate of $\hat{\beta}$ seems to be some evidence for this post-CGG structural break around 2000, the large standard error associated with the estimate doesn't allow us to say that it is different from the Volcker-Greenspan estimate. The estimates of $\hat{\gamma}$ are very close and don't support the existence of this structural break. Note: I did change the subsample 3 to starting at 2000Q3, as opposed to 1997Q1 as it was initially set. This change didn't alter my results.

1.8 (h)



2 Problem 2: "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models" Hansen and Singleton (1982)

2.1 Replicating Table 1.

Here are the results from the errata to the original paper:

NLAG	$\hat{\alpha}$	$\hat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\hat{SE}(\hat{\beta})$	χ^2	DF	Prob
1	-1.2028	.7789	.9976	.0027	1.457	1	.7726
2	-.5761	.7067	.9975	.0027	5.819	3	.8792
4	-.6565	.6896	.9978	.0027	7.923	7	.6606
6	-.9638	.6425	.9985	.0027	10.522	11	.5159

Here are my results:

NLAG	$\hat{\alpha}$	$\hat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\hat{SE}(\hat{\beta})$	χ^2	DF	Prob
1	-1	.6363	1.0010	.0030	1.3614	1	.7567
2	-.7460	.6226	1.0010	.0029	3.7692	3	.7125
4	-.8040	.6686	1.0010	.0027	7.9063	7	.6591
6	-1	.5982	1.0020	.0025	9.3617	11	.4155

My results, while fairly close are different in a few ways. First, I never find an estimate of α larger than one. Second, they never obtain estimates of β larger than one, whereas this is the case for all of my estimates (although my standard errors don't preclude the estimates from being below one). Obviously, a discount factor greater than one is not consistent with theory. Beyond these two, the differences are fairly minor. My SE's are different, especially for α when $NLAG = 1, 2$. Everything else is pretty close, and while my p-values are slightly different, they are consistent with not rejecting the null hypothesis that the over identifying restrictions are valid for $NLAG = 1, 2, 4, 6$.

There are several reasons why my results differ from the original paper. First, we've used the SP500 as our measure for stock returns. This is different from what the authors used, and additionally we are only using stock prices to measure returns, whereas they use both prices and dividends. Secondly, we've used a method to obtain a Heteroskedasticity and Autocorrelation Consistent Covariance Estimate (Newey West) that was not developed until after this paper was originally published. Finally, I believe that our numerical minimization technique (grid search in matlab) is likely different from the technique they used to obtain their results.

2.2 Updating Table 1.

Here are my results for 1979:01-2013:04:

NLAG	$\hat{\alpha}$	$\hat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\hat{SE}(\hat{\beta})$	χ^2	DF	Prob
1	-.5	.9387	.9950	.0024	1.6941	1	.8069
2	-.5	.7592	.9940	.0021	5.4042	3	.8555
4	-.5	.7284	.9930	.0021	6.9115	7	.5618
6	-.5	.6975	.9920	.0020	10.1913	11	.4867

These results are fairly close to my old results, except for two major differences. First, here my point estimates for $\hat{\beta}$ are all less than one, and the standard errors make it a far more convincing case that the discount factor is less than one. This is a result more in line with what we might expect from theory. The other big difference is my estimates for $\hat{\alpha}$ don't vary with the number of lags, although with the associated standard errors I can't make a case for them being different from my estimates in 2.1. In fact these standard errors for $\hat{\alpha}$ are all larger in this second estimation. Other than the obvious differences in the actual data, I don't see any underlying reason why these estimates would be different. I don't see anything about the two time periods that would lead me to believe that discount factors and risk aversion would have changed between these periods. These results seem to confirm this.

3 Appendix: m-files

3.1 Question 1 main file

3.2 NLSobjmain file

3.3 Question 2 main file

3.4 GMMoptW file

3.5 GMMcovmat main file

3.1

```
clear;
clc;




% Robert Ackerman
% Homework 2
% Problem 1
% October 15, 2013

% Original Paper: R. Clarida, J. Gali, & M. Gertler (2000).
% Monetary Policy Rules and Macroeconomic Stability: Evidence
% and Some Theory. Quarterly Journal of Economics, 115, 147-180.

%% Step 1: Preliminary Settings
% choose subsamples
subsample = 1;  % 1 for Pre-Volcker (60Q1-79Q2; 1-78; #78)
                % 2 for Volcker-Greenspan (79Q3-96Q4; 79-148; #70)
                % 3 for Post-CGG (97Q1-2012Q2; 149-210; #62)
% choose horizons
k = 1;   % horizon for inflation (must be 1 or 4)
q = 1;   % horizon for output gap (must be 1 or 2)

% choose initial parameter values
thetainit1 = [0.83; 0.27; 0.68; 4.24];  % Pre-Volcker
thetainit2 = [2.15; 0.93; 0.79; 3.58];  % Volcker-Greenspan
thetainit3 = [2.15; 0.93; 0.79; 3.58];  % Post-CGG

% # of lags of instruments
nlag = 4;

% set fminunc options for NLS
optionsNLS = optimset('Display','iter','GradObj','on','LargeScale','on');
% set fminunc options for first-stage GMM
optionsGMM1 = optimset('Display','iter','LargeScale','off');
% set fminunc options for second-stage GMM
optionsGMM2 = optimset('Display','iter', 'GradObj', 'on','LargeScale','on');

% Preparation for 95% confidence region
betavec1  = (-2:0.1:6)';    % grid for beta in subsample 1
gammavec1 = (-2:0.1:6)';   % grid for gamma in subsample 1
betavec2  = (-2:0.1:6)';    % grid for beta in subsample 2
gammavec2 = (-2:0.1:6)';   % grid for gamma in subsample 2
betavec3  = (-2:0.1:6)';    % grid for beta in subsample 3
gammavec3 = (-2:0.1:6)';   % grid for gamma in subsample 3

%% Step 2: Data Management
load HW2Q1b.txt;   % 210-by-9 data matrix
data = HW2Q1b;

% HW2Q1b contains
% [r, r(-1), pi1, pi4, X1, X2, Commodity price growth, M2 growth, Spread]
% from 1960Q1 - 2012Q2
FF = data(:,1);     FF_lag = data(:,2);  pi1 = data(:,3);
pi4 = data(:,4);    X1 = data(:,5);      X2 = data(:,6);
COM = data(:,7);    M2 = data(:,8);      SPR = data(:,9);

% choose inflation measure (k must be 1 or 4)
if k == 1
    inflation = pi1;
elseif k == 4
    inflation = pi4;
else
    error('Invalid k');
end;
```

```
% choose output gap measure (q must be 1 or 2)
if q == 1
    X = X1;
elseif q == 2
    X = X2;
else
    error('Invalid q');
end;

% Based on CGG's theory, we do not have to use Newey-West HAC estimator when k = q = 1.
% We do have to use it otherwise.
if (k == 1) && (q == 1)
    lambda = 0;     % no HAC
else
    lambda = 'NW' ;  % HAC with Newey and West's (1994) automatic lag selection.
end;

T = size(FF, 1);   % entire sample size

% collect all variables in model
data_in_model = [FF, inflation, X, FF_lag];

%% Step 3: Nonlinear Least Squares (NLS)
% choose subsample automatically
if subsample == 1   % Pre-Volcker (60Q1-79Q2; 1-78)
    ssind = 1:78;
elseif subsample == 2   % Volcker-Greenspan (79Q3-96Q4; 79-148)
    ssind = 79:148;
elseif subsample == 3   % Post-CGG (97Q1-2012Q2; 149-210)
    ssind = 163:T;
end;

data_NLS = data_in_model(ssind,:);   % instruments not needed for NLS

% choose initial parameter values automatically
if subsample == 1
    thetainit = thetainit1;
elseif subsample == 2
    thetainit = thetainit2;
elseif subsample == 3
    thetainit = thetainit3;
end;

% get NLS estimator
theta_NLS = fminunc('NLSobj', thetainit, optionsNLS, data_NLS);

% get asymptotic covariance matrix for theta_NLS
[~, ~, ~, cov_NLS] = NLSobj(theta_NLS, data_NLS);
se_NLS = sqrt(diag(cov_NLS));

% save results
table_NLS = [theta_NLS, se_NLS];

%% Step 4: Two-Stage Generalized Method of Moments (GMM)
r = 6 * nlag;         % # of instruments
nobs = T - nlag;      % effective sample size

% construct instruments (nobs x r)
% First nlag columns have lags of FF_lag, second nlag columns have lags of
% pi1, ..., last nlag columns have lags of SPR.
Z = zeros(nobs, r);
for i = 1:nlag
     first = nlag + 1 - i;
     last = T - i;
     tind = first:last;
     lagind = i:nlag:(5 * nlag + i);
     Z(:, lagind) = [FF_lag(tind), pi1(tind), X1(tind), COM(tind), M2(tind), SPR(tind)];
```

```
end;

% collect variables in model
% choose subsample automatically
% discard the first nlag observations
data_GMM = data_in_model((nlag + 1):T, :);
if subsample == 1
    ssind = 1:(78-nlag);
elseif subsample == 2
    ssind = (79-nlag):(148-nlag);
elseif subsample == 3
    ssind = (149-nlag):(T-nlag);
end;
data_GMM = data_GMM(ssind,:);
Z = Z(ssind,:);

% 1st-stage estimation: Take W = identity
W = eye(r);
theta_GMM1= fminunc('GMMobjCGG', thetainit, optionsGMM1, data_GMM, Z, W);

% get optimal weighting matrix
W = GMMoptWCGG(theta_GMM1, data_GMM, Z, lambda);

% 2nd-stage estimation:: Take optimal W
% The second output of fminunc is the minimized objective function.
theta_GMM2= fminunc('GMMobjCGG', theta_GMM1, optionsGMM2, data_GMM, Z, W);

% get asymptotic covariance matrix for theta_GMM2
cov_GMM = GMMcovmatCGG(theta_GMM2, data_GMM, Z, W);
[Q,~,~,~]=GMMobjCGG(theta_GMM2, data_GMM,Z,W);
se_GMM = sqrt(diag(cov_GMM));      % standard error
J = nobs * Q;              % J statistic
prob = 1 - chi2cdf(J, r);  % p-value.

% save results
table_GMM = [theta_GMM2, se_GMM];

%% Step 5: Statistical Inference
% % Hausman test
nparam=size(theta_NLS,1);
COV=([cov_GMM-cov_NLS]\eye(nparam));
H=(theta_NLS-theta_GMM2)'*COV*(theta_NLS-theta_GMM2);
probH = 1 - chi2cdf(H, nparam);  % p-value.


% 95% confidence region wrt. (beta, gamma)
% choose possible beta's and gamma's according to subsamples
if subsample == 1
    betavec = betavec1; gammavec = gammavec1;
elseif subsample == 2
    betavec = betavec2; gammavec = gammavec2;
elseif subsample == 3
    betavec = betavec3; gammavec = gammavec3;
end;

nb = size(betavec,1);    % # of beta's tried
ng = size(gammavec,1);   % # of gamma's tried

% compute J statistic for each pair of (beta, gamma)
% fix rho and pistar at their GMM estimates
Jmat = zeros(nb,ng);
for i = 1:nb
     for j = 1:ng
          beta = betavec(i);
          gamma = gammavec(j);
          theta = [beta; gamma; theta_GMM2(3:4,1)];
          W = GMMoptWCGG(theta, data_GMM, Z, lambda);
          Qtemp = GMMobjCGG(theta, data_GMM, Z, W);
```

```
        Jmat(i,j) = nobs * Qtemp;
    end;
end;

betamat = kron(betavec, ones(1,ng));     % linescale for beta
gammamat = kron(gammavec', ones(nb,1));  % linescale for gamma

figure(1)  % plot 3D figure of all J statistics
surf(betamat, gammamat, Jmat, 'EdgeColor','none', 'FaceColor','interp',
'FaceLighting','phong');
xlabel('\beta','FontSize',11); ylabel('\gamma','FontSize',11);
zlabel('J stat','FontSize',11);
set(gca, 'FontSize', 11);

figure(2)  % plot contour of all J statistics
contour(betamat, gammamat, Jmat, 'ShowText', 'on');
xlabel('\beta','FontSize',11); ylabel('\gamma','FontSize',11);
```

3.2

```
function [SSR, dSSR, hess, cov] = NLSobj(theta, data_NLS)
% PURPOSE: compute (the average of) the sum of squared residuals,
%          its first derivatives wrt. theta, hessian, and the asymptotic
%          covariance matrix
%-------------------------------------------------------------------
% USAGE: [SSR, dSSR, hess, cov] = NLSobj(theta, data_NLS)
% where: theta     = parameters (nparam x 1)
%        data_NLS  = data matrix (T x nvar)
%        nparam    = # of parameters
%        nvar      = # of variables
%-------------------------------------------------------------------
% RETURNS: SSR   = sum of squared residuals / T (scalar)
%          dSSR  = gradient (nparam x 1)
%          hess  = Hessian (nparam x nparam)
%          cov   = asymptotic covariance matrix for theta_hat (nparam x nparam)
%-------------------------------------------------------------------
% Reference: R. Clarida, J. Gali, & M. Gertler (2000).
%            Monetary Policy Rules and Macroeconomic Stability: Evidence
%            and Some Theory. Quarterly Journal of Economics, 115, 147-180.
% ------------------------------------------------------------------
% Written by
% Updated by Robert Ackerman, UNC Chapel Hill.
% Oct. 15, 2013.
% ------------------------------------------------------------------

% parameters
beta = theta(1);
gamma = theta(2);
rho = theta(3);
pistar = theta(4);

nobs = size(data_NLS,1);
nparam = size(theta,1);

% get each series
FF = data_NLS(:,1);
pi1 = data_NLS(:,2);
X1 = data_NLS(:,3);
FF_lag = data_NLS(:,4);

% get epsilon
alpha_ = mean(FF, 1) - beta * pistar;
y = alpha_ + beta * pi1 + gamma * X1;
eps = FF - (1 - rho) * y - rho * FF_lag;

% get individual moment vector and average moment vector
g = eps;
g_bar = mean(g, 1)';
%W = diag(eps'*eps);
% NLS objective function
Q = (1/nobs)*(g'*g);

% derivative of y wrt theta
dy = zeros(nobs, nparam);
dy(:,1) = -pistar + pi1;
dy(:,2) = X1;
dy(:,4) = -beta;

% derivative of epsilon wrt theta
deps = -(1-rho) * dy;
deps(:,3) = y - FF_lag;

% derivative of g wrt theta
dg = deps;
dg_bar = mean(dg, 1);

% derivative of Q wrt theta
```

```
dQ = (2/nobs) * (dg' * g);
SSR=Q;
dSSR=dQ;

hess = zeros (nparam, nparam, nobs);
hess(1,3,:) = -pistar + pi1;
hess(1,4,:) = 1-rho;
hess(2,3,:) = X1;
hess(3,1,:) = -pistar + pi1;
hess(3,2,:) = X1;
hess(3,4,:) = -beta;
hess(4,1,:) = 1-rho;
hess(4,3,:) = -beta;

%Get Omegahat and compute cov
Omegahat=var(eps);
cov = ((deps'*deps)\eye(4))*deps'*Omegahat*deps*((deps'*deps)\eye(4));
```

3.3

```
% Robert Ackerman
% Homework 2
% Problem 2
% October 15, 2013

% L. P. Hansen and K. J. Singleton (1982). Generalized Instrumental
% Variables Estimation of Nonlinear Rational Expectations Models,
% Econometrica, 50, 1269-1286.

% See also: GMMobj.m, GMMcovmat.m, GMMoptW.m, NWHAC.m.
%%
clc;
clear;
load 'HW2Q2b.txt';
%Set range of sample 1=Feb1959 239=Dec1978 651=Apr2013
first = 1;
last = 239;
%set instrument lags
nlag = 1;
%Use NWHAC.m to get Newy West 1987 HAC estimator
lambda = 'NW';

% grid for alpha beta first stage est (note: picking range based on what we
% think is the likely alpha and beta)
avec1 = (-1:0.01:0)';
bvec1 = (0.9:0.01:1.1)';

% grid for alpha beta second stage est (note: increment is smaller here
avec2 = (-1:0.001:-0.5)';
bvec2 = (0.9:0.001:1.1)';
%% PT2 Grid search
X = HW2Q2b(first:last, :);   % get x1 and x2
T = size(X,1);                  % sample size
na1 = size(avec1,1);            % # of alpha's tried in 1st stage
nb1 = size(bvec1,1);            % # of beta's tried in 1st stage
na2 = size(avec2,1);            % # of alpha's tried in 2nd stage
nb2 = size(bvec2,1);            % # of beta's tried in 2nd stage

% 1st stage
Q = Inf;                        % start with infinite obj. fnc. and update
for i = 1:na1      % loop wrt. alpha, CRRA parameter
     alpha = avec1(i);
     for j = 1:nb1  % loop wrt. beta, discount factor
          beta = bvec1(j);
          theta = [alpha; beta];
          Q_temp = GMMobj(X, theta, nlag); % 1st stage with identity weighting matrix

          if Q_temp < Q      % if new theta improves Q, then save it.
                             % Otherwise keep the previous Q and theta.
              Q = Q_temp;
              GMME = [alpha; beta];
          end;
     end;
end;

W = GMMOptW(X, GMME, nlag, lambda);
%%
% Second stage using the optimal weighting matrix
Q2 = zeros(na2, nb2);  % save all candidate values to draw a 3D figure

% 2nd stage
Q = Inf;                        % start with infinite obj. fnc. and update
```

```
for i = 1:na2      % loop wrt. alpha, CRRA parameter
     alpha = avec2(i);
     for j = 1:nb2  % loop wrt. beta, discount factor
          beta = bvec2(j);
          theta = [alpha; beta];
          Q_temp = GMMobj(X, theta, nlag, W); % 2nd stage with optimal weighting matrix
          Q2(i,j)=Q_temp;
          if Q_temp < Q      % if new theta improves Q, then save it.
                             % Otherwise keep the previous Q and theta.
              Q = Q_temp;
              GMME = [alpha; beta];
          end;
     end;
end;


%% Step 3: Statistical Inference
cov = GMMcovmat(X, GMME, nlag, W);
se = sqrt(diag(cov));   % standard error
J = T * Q;              % J statistic
dof = 2 * nlag - 1;     % degrees of freedom
                        % # of moment restrictions = 2*nlag + 1
                        % # of parameters = 2
prob = chi2cdf(J, dof);  % p-value

% make Table 1 in p.1282
table = [nlag; GMME(1); se(1); GMME(2); se(2); J; dof; prob];

% plot objective function for each pair of (alpha, beta)
amat = kron(avec2, ones(1,nb2));   % linescale for alpha
bmat = kron(bvec2', ones(na2,1));  % linescale for beta
figure(1)  % plot 3D figure
surf(amat, bmat, Q2, 'EdgeColor','none', 'FaceColor','interp', 'FaceLighting','phong');
xlabel('\alpha','FontSize',11); ylabel('\beta','FontSize',11);
zlabel('Q','FontSize',11);
set(gca, 'FontSize', 11);

% clear original data
 clear HW2Q2b
```

3.4

```
function OptW=GMMOptW(X, GMME, nlag, lambda)
% PURPOSE: compute the optimal GMM weighting matrix for second-stage of two-stage GMM
%-------------------------------------------------------------------
% USAGE: OptW = GMMOptW(X, GMME, nlag, lambda)
% where: X    = data matrix (T x 2)
%                 X(:,1) is x1, while X(:,2) is x2.
%        GMME = GMM estimates from first-stage
%      lambda = tuning parameter for Newey and West's (1987) HAC
%                 'NW' for Newey and West's (1994) automatic selection
%-------------------------------------------------------------------
% RETURNS: OptW   = the optimal GMM weighting matrix for second-stage of two-stage GMM
%-------------------------------------------------------------------
% References:  L. P. Hansen and K. J. Singleton (1982). Generalized Instrumental
% Variables Estimation of Nonlinear Rational Expectations Models,
% Econometrica, 50, 1269-1286.
% ------------------------------------------------------------------
% Written by
% Updated by Robert Ackerman, UNC Chapel Hill.
% Oct. 14, 2013.
% Also see NWHAC.m and GMMobj.m
% -------------------------------------------------------------------

[Q Z]=GMMobj(X, GMME, nlag);
Sigma=NWHAC(Z,lambda);
OptW=inv(Sigma);
end
```

3.5

```
function cov=GMMcovmat(X, GMME, nlag, OptW)
% PURPOSE: compute the asymptotic covariance matrix of two-stage GMM
% estimates
%---------------------------------------------------------------------
% USAGE: cov = GMMcovmat(X, GMME, nlag, OptW)
% where: X   = data matrix (T x 2)
%                  X(:,1) is x1, while X(:,2) is x2.
%       GMME = GMM estimates from second-stage
%     OptW = Optimal weighting matrix
%---------------------------------------------------------------------
% RETURNS: cov   = the asymptotic covariance matrix of two-stage GMM
% estimates
%---------------------------------------------------------------------
% References:  L. P. Hansen and K. J. Singleton (1982). Generalized Instrumental
% Variables Estimation of Nonlinear Rational Expectations Models,
% Econometrica, 50, 1269-1286.
% --------------------------------------------------------------------
% Written by
% Updated by Robert Ackerman, UNC Chapel Hill.
% Oct. 14, 2013.
% Also see NWHAC.m, GMMOptW.m and GMMobj.m
%
T = size(X,1);     % sample size
[Q, f, fderiv] = GMMobj(X, GMME, nlag, OptW, 1);
G=mean(fderiv,3);
cov=1/T*inv(G'*OptW*G);
----------------------------------------------------------------
```