**Problem-1:**

This exercise is based on a famous paper: "Monetary policy rules and macroeconomic stability: evidence and some theory" by Richard Clarida, Jordi Gali, Mark Gertler published in The Quarterly Journal of Economics, Vol. 115, No. 1 (2000), pp. 147-180. Please read at least from the Introduction to the end of Section 3 of the article before answering the questions. The basic story is as follows:

*Background:*

The goal here is to investigate how the Federal Reserve responds to expected inflation and output while setting the target Federal Funds rate (FFR). Let us postulate the following model:

$$r_t^* = \alpha + \beta\left(E[\pi_{t,k}|\Omega_t] - \pi^*\right) + \gamma E[X_{t,q}|\Omega_t] \tag{1}$$

where $r_t^*$ is the target FFR in period $t$; $\pi_{t,k}$ is the percent change in price level between periods $t$ and $t+k$ expressed in annual rate; $pi^*$ is the Federal Reserve's target of inflation; $X_{t,q}$ is a measure of average output gap between period $t$ and $t+q$; $\Omega_t$ is collection of all information available when the interest rate is set in period $t$; and $\alpha$ is the desired nominal rate when inflation and output are at target. Note that this gives the implied rule for (ex-ante) real rate target $(rr_t^*)$ as:

$$rr_t^* = \tilde{\alpha} + (\beta - 1)\left(E[\pi_{t,k}|\Omega_t] - \pi^*\right) + \gamma E[X_{t,q}|\Omega_t] \tag{2}$$

where $\tilde{\alpha} = \alpha - \pi^*$. $\beta > 1$ is considered a stabilizing policy whereas $\beta < 1$ is destabilizing. Same for $\gamma >$ or $< 0$. Be sure to keep this interpretations in mind while explaining your results.

In practice the Federal Reserve cannot, however, react fully to expected inflation and output gap for various reasons. Instead it is realistic to assume that while the target FFR is given by (1), there is some smoothing involved in the reaction to the actual FFR $r_t$. In particular, assume that there is partial adjustment of FFR to eliminate a fraction $(1-\rho)$ of the gap between last year FFR and the current target level, i.e.,

$$r_t = (1-\rho)r_t^* + \rho r_{t-1} + v_t \tag{3}$$

where $v_t$ is some unanticipated (at time $t$) money demand shock. We will assume $v_t$ is serially uncorrelated over time. Therefore, combining (1) and (3) we obtain:

$$r_t \quad = \quad (1-\rho)\left[\alpha + \beta\left(E[\pi_{t,k}|\Omega_t] - \pi^*\right) + \gamma E[X_{t,q}|\Omega_t]\right] + \rho r_{t-1} + v_t.$$

Unfortunately you or I do not know $E[\pi_{t,k}|\Omega_t]$ or $E[X_{t,q}|\Omega_t]$. So to estimate the above model a natural strategy will be to replace them by the actual quantities: $\pi_{t,k}$ and $X_{t,q}$ respectively. This gives:

$$r_t = (1-\rho)\left[\bar{\alpha} + \beta\pi_{t,k} + \gamma X_{t,q}\right] + \rho r_{t-1} + \epsilon_t, \tag{4}$$

where $\bar{\alpha} := \alpha - \beta\pi^*$, $\epsilon_t := v_t + u_t$ and $u_t := -(1-\rho)\left[(\pi_{t,k} - E[\pi_{t,k}|\Omega_t]) + \gamma\,(X_{t,q} - E[X_{t,q}|\Omega_t])\right]$ is a function of two the prediction errors. Our goal is to estimate the parameters in (4). This will give us some idea on how the Federal Reserve reacts. The original paper clearly showed a sharp distinction in reaction in the pre and post Volcker era.

We will use the same data as in the original paper. Please read from the beginning of Section 3 to that of Section 3.1 to learn more about the data source so that you can download it from UNC library (Odum Institute Archive Dataverse). Please make sure to follow the appropriate citation requirement for the data when you turn in your homework.

*Questions:* Answer the following:

(a) Explain the endogeneity issues that will arise if you want to estimate (4) by nonlinear least squares.

(b) So you will need instrumental variables. Fortunately any element belonging to $\Omega_t$ can be used as an exogenous (valid) instrument under this setup. Why is that the case?

(c) First take $k = q = 1$, i.e., one period ahead forecast. This is the baseline model (Section 3.1 and Table - 1) in the original paper. Explain why the error $\epsilon_t$ is serially uncorrelated under this setup. Now replicate the top panel (i.e., only excluding the test for structural break part) of Table - 1 of the paper.

(d) Under the setup of problem 1(c) estimate the parameters of (4) by nonlinear least squares. Perform a Hausman test under the assumption that the setup of problem 1(c) is correct.

(e) Instead of using 4 lags of the variables as instruments, use 2 lags only. So now you have 12 instead of 24 instruments. Lesser number of instruments can help to reduce finite-sample bias of GMM estimators. Redo problem 1(c).

(f) $k = q = 1$ is typically an unrealistic target horizon. We did it so that you can work out the problem with serially uncorrelated error. However, in general $u_t$ is $MA(\max(k, q) - 1)$. This type of serial correlation is easy to deal with because all we will require is to account for this serial correlation when computing the optimal weighting matrix. Using the Newey-West kernel estimator for the asymptotic variance of the average moment restrictions, replicate Table - 2 of the original paper. Again, feel free to exclude the structural break part.

(g) Redo problem 1(c) by including more recent data. Some people claim there was a structural break around 2000. So consider 3 sets of periods with the last one starting from quarter 3 of 2000. Comment on your results.

(h) Based on the setup of problem 1(c) construct joint weak-identification-robust confidence regions for $(\beta, \gamma)$ for the two periods. See the famous paper by Stock and Wright (2000, Econometrica) for reference beyond my lecture notes.