# Problem 2, Homework 2

Due on October 14, 2013

## 1 Motivation

This extra problem is based on L. P. Hansen and K. J. Singleton (1982), "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models", Econometrica, Vol. 50, pp. 1269-1286. This paper is one of the earliest and most well-known economic applications of nonlinear GMM. They start with the basic consumer problem (i.e. maximizing utility given budget constraint) and derive the Euler equation as the first order condition. Using the Euler equation as a moment restriction, they estimate the constant relative risk aversion (CRRA) parameter $\alpha$ and the discount factor $\beta$.

Our goal is to replicate a part of their empirical results step by step (see Table 1 in p.1282). This exercise will improve your understanding of the GMM theory, data-handling skills, and Matlab coding skills on numerical optimizations.

## 2 Questions

**(a) Data collection.** Download four monthly series from January 1959 through April 2013:

1. S&P 500 prices (SP500). For convenience of data collection, use the closing price of the first date of each month.
2. total population: all ages including armed forces overseas (POP)
3. real personal consumption expenditures: nondurable goods, chain-type quantity index (CND)
4. personal consumption expenditures: nondurable goods, implicit price deflator (CNDP)

SP500 can be found at Yahoo! Finance. POP, CND, and CNDP can be found at FRED, but CNDP is available only quarterly. We thus need to interpolate it into monthly series. While there are many ways of interpolation, we employ a simple one such that the monthly growth rate is kept constant between two adjacent quarters. See **qtom_int.m** for further details.

**(b) Data management.** Using these four monthly series, calculate $\{x_{1t+1}\}$ and $\{x_{2t+1}\}$ in p.1281, where

$$x_{1t+1} = \frac{P_{1t+1} + D_{1t+1}}{P_{1t}}$$

and

$$x_{2t+1} = \frac{C_{t+1}}{C_t}$$

based on the original notation. The best approximation we can do is:

$$x_{1t+1} = \frac{p_{t+1}}{p_t}$$

and

$$x_{2t+1} = \frac{c_{t+1}}{c_t},$$

where $p_t$ is SP500 divided by CNDP and $c_t$ is CND divided by POP. Export these two series into Matlab.

**(c) Replicating Table 1.** Taking the sample period 1959:02 - 1978:12, replicate the last four rows of Table 1 (recall that SP500 is a value-weighted index). Make sure to take the two-step procedure with the first step involving the identity weighting matrix and the second involving the optimal weighting matrix.

Comment on (1) how your results and theirs are different and (2) where those differences are potentially coming from.

Based on your own results, interpret (i) the magnitude and statistical significance of $\hat{\alpha}$ and $\hat{\beta}$ and (ii) the J-statistic.

**(d) Updating Table 1.** Taking the sample period 1979:01 - 2013:04, update the last four rows of Table 1. Comment on (1) how your updated results and your old results are different and (2) where those differences are potentially coming from.