Problem-1: [Reading: Section 9 (pages 2215-2226) "Large Sample Estimation and Hypothesis Testing", W. Newey and D. McFadden, Handbook of Econometrics, 1994.]

Let $Z_1, \ldots, Z_n$ be i.i.d. copies of a random variable Z whose distribution satisfies the moment restrictions:

$$E[g(Z, \beta)] = 0 \text{ for } \beta \in \mathcal{B} \text{ if and only if } \beta = \beta^0.$$

where $g : \text{Support}(Z) \times \mathcal{B} \mapsto \mathbb{R}^k$ and $\mathcal{B}$ is a compact subset of $\mathbb{R}^p$ and $\beta^0 \in \text{interior}(\mathcal{B})$.

We want to test the null hypothesis $H_0 : a(\beta^0) = 0$ against a local alternative $H_l : a(\beta^0) = \frac{\delta}{\sqrt{n}}$ and a fixed alternative $H_f : a(\beta^0) \neq 0$ where $a : \mathcal{B} \mapsto \mathbb{R}^r$ and $r \leq p$.

(i) Construct the Wald, LM and LR test for this purpose. In this chapter the authors use the term DM for LR.

(ii) Show that the three tests are asymptotically equivalent under H_0 and H_l under standard GMM assumptions.

(iii) Show that the three tests are consistent under H_f.

Please make sure that you explain each step of the proofs for the above results.

Problem-2: Suppose that we are interested in two null hypotheses: $H_0^\beta : \beta = 1$ and $H_0^\gamma : \gamma = 0$ in the context of Problem-1(c) of HW-2. Test these two null hypotheses separately using Wald, LM and LR tests. Perform the tests for both sample periods.