LPS ECON-103-601, Fall 2014
HW Due 09.10.14 Name: ______________________

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

How many unique, four digit pin code numbers are there? Each digit in the pin code must be chosen from the range 0-9 (inclusive).

Each digit (0-9) is unique, so any pin code we construct from these digits will be unique. By the Fundamental Counting Principle, the total number of ways to arrange 10 numbers in 4 slots with replacement is

$$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10^4$$

Q2

Hilary is stacking eight books on a shelf. How many ways she arrange the books if three books are identical and the other five are distinct from all others?

We want to know the number of distinct permutations of the books. Only one subset of books has a multiplicity of 3, so

$$\frac{8!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

Q3

Kevin is preparing for battle and can make one battle kit that contains exactly one suit of armor, one helmet, and two weapons. There are four distinct suits of armor, three distinct helmets, and five distinct weapons. Given his choices, how many distinguishable battle kits can Kevin make?

By the Fundamental Counting Principle, the total number of ways the kit can be made is

$$\underline{{}_4C_1} \cdot \underline{{}_3C_1} \cdot \underline{{}_5C_2} = 120$$

Q4

Fifteen identical boxes are being sent to five distinct people. How many different ways can the boxes get distributed if it is possible for people to get no boxes (e.g. all the boxes get lost)?

Conceptually, all boxes being lost is the same as all boxes going to a sixth person. With this insight and identical boxes, the problem is equivalent to putting 5 identical dividers between six different "people." There are 20 positions to place the dividers in, because there are 15 boxes and 5 dividers to be placed. There number of ways to assign 20 positions to five dividers is $_{20}C_5$.

Q5

A committee consisting of two monks and three knights is being formed from a group of six monks and five knights; every monk or knight is a distinguishable person. How many distinct committees are possible if one monk and one knight refuse to serve together?

The total number of possible committees (by the Fundamental Counting Principle) is $\underline{_6C_2} \cdot \underline{_5C_1} = 150$. Fixing the stubborn monk and stubborn knight on the committee, the number of invalid committees is $\underline{_5C_1} \cdot \underline{_4C_2} = 30$. Hence, the number of committees that can be formed is $150 - 30 = 120$.

Q6

A robot named Karel is trapped on a Cartesian grid. Karel travels in steps where each step either moves Karel to (x + 1, y) or (x, y + 1), where (x, y) is Karels location before making the step. If Karel is located at (1, 1), how many distinct paths can Karel take to get to (6, 8)? (Two paths are distinct if the sequences of coordinates that Karel travels to contain at least one different coordinate pair.)

In order to reach (6,8) from (1,1), Karel must take 5 steps right and 7 steps up (because the robot cannot move backwards). Choosing when to go up fixes when to go right. The number of ways to assign the steps where to go up is $_{12}C_7 = 242$, because there are 12 different steps and Karel must go up 7 times.

Q7

HongVan is doing a photo shoot at South Beach with ten puppies. She starts by taking a photo of herself. Next, she takes a photo of each dog individually. Then she takes photos of every possible pair of dogs, every possible group of three dogs, every possible group of four dogs, and continues taking photos in this manner until she has taken a photo of all ten dogs together. Including the photo she took of herself, how many photos did HongVan take?

This problem is adding up the combinations to obtain the total number of photos, so it is just the sum of the elements in the 10^{th} row of Pascal's Triangle.

$$ {}_{10}C_0 + {}_{10}C_1 + {}_{10}C_2 + ... + {}_{10}C_{10} = (1+1)^{10} = 2^{10} \tag{1} $$

Q8

How many four-digit numbers have a digit that repeats at least 3 times?

Case I: 4 digits repeats

Numbers do not start with zero, so there are 9 possible numbers with 4 digits repeating.

Case IIa: 3 digit repeat (leading digit is repeating)

There are 9 choses for the leading digit, because it cannot be zero. The final three places represent three positions, and we need to assign two of the those positions to the number in the leading digit's place. There are ${}_3C_2 = 3$ ways to choose these positions. There are 9 digits left to choose the final digit, and its place is fixed, since we already chose were to place the repeating digit. By the Fundamental Counting Principle, there are $\underline{9} \cdot \underline{3} \cdot \underline{9} = 243$ possible numbers.

Case IIb: 3 digit repeat (leading digit is not repeating)

There are 9 possible digits for the first digit, because it cannot be zero. The digits that follow are identical and there are 9 possible digits left to choose from. So by the Fundamental Counting Principle, there are $\underline{9} \cdot \underline{9} = 81$ possible numbers.

So in total there $9 + 243 + 9 = 333$ numbers with digits that repeat at least 3 times.

Bonus

Sandy is throwing 88 darts at 9 different dartboards. Every dart she throws hits and sticks to one of the dartboards and each dart is colored exclusively red, yellow, or blue. After Sandy throws all the darts, combinatorics guarantees that one of the dartboards will have at least n darts of the same color. What is the maximum value of n?

Consider the worst case scenario where Sandy throws the darts uniformly (worst case because otherwise there will be a dartboard with more than 10 darts for sure). We are ensured that one dartboard will have at least 10 darts

for sure, because $88/9 = 9r.7$. Again, consider the worst case where the color distribution of the darts is uniform. Then we are guarenteed a dartboard will have 4 of the same color for sure because $10/3 = 3r.1$. This concept is known as the Pigeonhole Principle.