Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

For Q1-Q3, suppose $X_i \sim i.i.d.N(\mu, \sigma^2)$

Q1

Prove $\frac{\bar{X}-\mu}{\sigma} \sim N(0,1)$.

Q2

Prove $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$. You may use the fact that $\bar{X}$ and S^2 are independent.

Q3

Prove $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$.

Q4

Suppose S^1 and S^2 are the sample variances of independent random sample of size n_1 and n_2 for normal populations (i.e. normal random variables) with variances σ_1^2 and σ_2^2. Prove $\sigma_2^2 S_1^2 / \sigma_1^2 S_2^2 \sim F_{n_1-1, n_2-1}$.