Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

For this homework, you are given the data $\{x_i, z_i, y_i\}_{i=1}^m$.

Q1

Suppose the true model is $y_i = z_i\gamma + \varepsilon_i$ where $\mathbb{E}[\varepsilon_i] = 0$ and $\varepsilon \sim i.i.d.$ over i. Derive the OLS estimator for γ (be sure to write out the OLS minimisation problem, write out the FOC, verify the SOC, and use the FOC to find $\hat{\gamma}_{OLS}$).

Q2

Show that $\hat{\gamma}_{OLS}$ is unbiased.

Q3

Now derive $Var(\hat{\gamma}_{OLS})$ and determine whether $\hat{\gamma}_{OLS}$ is consistent (i.e. $Var(\hat{\gamma}_{OLS}) \to 0$ as $n \to \infty$).

Q4

Now suppose we were wrong and the true model is actually $y_i = z_i\gamma + x_i\beta + \varepsilon_i$ where $\mathbb{E}[\varepsilon_i] = 0$. Show that $\hat{\gamma}_{OLS}$ may be biased and derive an expression for the potential bias.

Q5

Under what condition will $\hat{\gamma}_{OLS}$ be unbiased?