

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

Let $X_1 \sim \text{Bernoulli}(1/2)$ independently of $X_2 \sim \text{Bernoulli}(2/3)$ and define $Y = 2X_1$ and $Z = X_2 - X_1$.

- (a) Express the joint PMF of Y and Z in tabular form.
- (b) Find the marginal PMF of Y .
- (c) Find the conditional PMF of Z ?
- (d) Calculate $\text{Cov}(Y, Z)$. Are Y and Z independent? What is the intuition?

Q2

Let X be a continuous random variable CDF $F(x) = \log_c(x)$ where $c \in \mathbb{R}_{++}$.

- (a) What is the support of X ?
- (b) What is the $P(a \leq x \leq b)$ where $a < e$ and $b > e$?
- (c) Calculate $\mathbb{E}[X]$.
- (d) Calculate $\mathbb{E}[X^2]$.