Sequence and Series Review

Definition A *sequence* is an ordered collection of numbers $\{a_1,\ a_2,\ ...,\ a_n, ...\}$, which may be finite or infinite.

Example Arithmetic Sequence
$a_n\ =\ a_{n-1}\ +\ d,\ d \in \mathbb{R} \Rightarrow a_n\ =\ a_1\ +\ (n-1)d$

Example Geometric Sequence
$a_n\ =\ r \cdot a_{n-1},\ r \in \mathbb{R} \Rightarrow a_n\ =\ a_1 \cdot r^{n-1}$

Example Harmonic Sequence
$a_n\ = \frac{1}{n},\ n \in \mathbb{N}$

Definition A *series*, S_n, is the sum of a sequence, which may also be finite or infinite.

Example Finite Arithmetic Series
$S_n\ = \sum\limits_{n=1}^{N} a_n = \frac{n(a_1+a_n)}{2}$

Example Finite Geometric Sequence
$S_n\ = \sum\limits_{n=1}^{N}\ a_1 r^{n-1} = a_1 \frac{1-r^N}{1-r}$

Example Infinite Geometric Sequence
$S_n\ = \sum\limits_{n=1}^{\infty}\ a_1 r^{n-1} = \frac{a_1}{1-r},\ |r| < 1$

Example Riemann Sum
$R_k\ = \sum\limits_{k=1}^{N} f(x_k^*) \cdot \Delta x_k$ where x_n^* is an arbitrary point in the k^{th} subinterval of a closed interval $[a, b]$. Recall that $\int\limits_a^b f(x)\mathrm{d}x = \lim\limits_{\max \Delta x_k \to 0} R_k$

Notation Sometimes we use the shorthand $\sum\limits_n$ instead of $\sum\limits_{n=1}^{N}$ and sometimes we drop the subscript entirely when it is well understood like $\sum x_n$. You should always be clear as to what the series is when there are multiple subscripts or indices.

Remark The summation passes through *affine transformations.* A transformation is *affine* if preserves collinearity and relative distances. An affine transformation of x_n in $\mathbb{R}$ is $a \cdot x_n\ + b$ where $a,\ b \in \mathbb{R}$. In other words,

$$\sum_n (a \cdot x_n + b)\ =\ a \left(\sum_n x_n \right) +\ b$$