

Prereqs, Counting, Combinations and Permutations

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

Suppose we observe the sequence $\{x_n, y_n\}_{n=1}^N$ and know that $y_n = b \cdot x_n + e_n$. Using all of the data available and one assumption of your choice on e_n .

a) Determine your b .

b) Is b unique for your assumption? Justify your answer.

Q2

Let $f(a) = \sum_{n=1}^N (x_n - a)^2 + (y - a)^2$. Find the minimum and maximum value of f .

Q3

Consider the game of musical chairs with 8 players and 7 chairs. When the music stops, some players are sitting and some are standing. An outcome is the elimination of certain players in this first round.

- a) How many possible outcomes does this scenario have once the music stops?
- b) How many possible outcomes exists if there are only 5 chairs instead of 7?
- c) How many more outcomes exists if there are only 5 chairs but now an outcome is who lands in a particular chair rather than just any chair?

Q4

Let $\{a_n\}_{n=1}^N$ be a non-zero sequence. Are the following statements TRUE or FALSE? Justify your answers.

- a) $\left(\sum_n a_n\right)^m = \left(\sum_n a_n^m\right)$
- b) $\left(\sum_n a_n b_n\right) / \left(\sum_n a_n^2\right) = \sum_n \frac{a_n b_n}{\sum_n a_n^2}$

Q5

Using the binomial formula, expand $(x + y)^4$ (your coefficients should be integers).

Q6

a) Find the value of c such that $\int_a^b \int_0^x f(cx, y) dx dy = 1$ where $f(x, y) = x^2 y^3$.

a) Find $\frac{\partial f}{\partial x}$ of $\int_a^b \int_0^x f(x, y) dx dy$ where $f(x, y) = 2x^2 y^3$.

c) Find the Hessian of $f(x, y)$ (i.e. $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$) where $f(x, y) = \cos(x^2 \sqrt{y})$. Is $f(x, y)$ convex, concave and/or both? Justify your answer.