

### Prereqs, Counting, Combinations and Permutations

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

#### Q1

Suppose we observe the sequence  $\{x_n, y_n\}_{n=1}^N$  and know that  $y_n = b \cdot x_n + e_n$  but cannot observe  $\{e_n\}_{n=1}^N$ . Using all of the data available and one assumption of your choice on  $e_n$ .

a) Determine your  $b$ .

b) Is  $b$  unique for your assumption? Justify your answer.

#### Q2

Let  $f(a) = \sum_{n=1}^N (x_n - a)^2 + (y - a)^2$ . Find the minimum and maximum value of  $f$ .

### Q3

Consider the game of musical chairs with 8 players and 7 chairs. When the music stops, some players are sitting and some are standing. An outcome is the elimination of certain players in this first round.

- a) How many possible outcomes does this scenario have once the music stops?
- b) How many possible outcomes exists if there are only 5 chairs instead of 7?
- c) How many more outcomes exists if there are only 5 chairs but now an outcome is who lands in a particular chair rather than just any chair?

### Q4

Let  $\{a_n\}_{n=1}^N$  be a non-zero sequence. Are the following statements TRUE or FALSE? Justify your answers.

- a)  $\left(\sum_n a_n\right)^m = \left(\sum_n a_n^m\right)$
- b)  $\left(\sum_n a_n b_n\right) / \left(\sum_n a_n^2\right) = \sum_n \frac{a_n b_n}{\sum_n a_n^2}$

### Q5

Using the binomial formula, expand  $(x + y)^4$  (your coefficients should be integers).

### Q6

a) Find a value of  $c$  such that  $\int_a^b \int_0^x f(cx, y) dx dy = 1$  where  $f(x, y) = x^2 y^3$ .

a) Find  $\frac{\partial f}{\partial x}$  of  $\int_a^b \int_0^x f(x, y) dx dy$  where  $f(x, y) = 2x^2 y^3$ .

c) Find the Hessian of  $f(x, y)$  (i.e.  $\frac{\partial^2 f}{\partial x^2}$ ,  $\frac{\partial^2 f}{\partial y^2}$ ,  $\frac{\partial^2 f}{\partial x \partial y}$ ) where  $f(x, y) = \cos(x^2 \sqrt{y})$ . Is  $f(x, y)$  convex, concave and/or both? Justify your answer.