Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

Let $\hat{\theta}$ be the statistic $\frac{\bar{X}}{c}$ and $\mathbb{E}[X] = \rho$ and $Var(X) = \phi^2$ where $c \in \mathbb{R} \setminus \{0\}$.

(a) What is $\mathbb{E}[\hat{\theta}]$?

(b) What is $Var(\hat{\theta})$?

(c) Find the values of c such that $Var(\hat{\theta}) < Var(\bar{X})$.

Q2

Suppose the CDF of x is $F(x) = \log x, x \in [1, e]$ and the random variable $Y = \sqrt{X}$. Find the PDF of Y called $g(y)$.