Prereqs, Counting, Combinations and Permutations

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

Suppose we observe the sequence $\{x_n,\ y_n\}_{n=1}^{N}$ and know that $y_n = b \cdot x_n + e_n$ but cannot observe $\{e_n\}_{n=1}^{N}$. Using all of the data available and one assumption of your choice on e_n.

a) Determine your b.

b) Is b unique for your assumption? Justify your answer.

Q2

Let $f(a) = \sum_{n=1}^{N}(x_n - a)^2 + (y - a)^2$. Find the maximum and maximum value of f.

Q3

Consider the game of musical chairs with 8 players and 7 chairs. When the music stops, some players are sitting and some are standing. An outcome is the elimination of certain players in this first round.

a) How many possible outcomes does this scenario have once the music stops?

b) How many possible outcomes exists if there are only 5 chairs instead of 7?

c) How many more outcomes exists if there are only 5 chairs but now an outcome is who lands in a particular chair rather than just any chair?

Q4

Let $\{a_n\}_{n=1}^{N}$ be a non-zero sequence. Are the following statements TRUE or FALSE? Justify your answers.

a) $\left(\sum_n a_n \right)^m = \left(\sum_n a_n^m \right)$

b) $\left(\sum_n a_n b_n \right) / \left(\sum_n a_n^2 \right) = \sum_n \frac{a_n b_n}{\sum_n a_n^2}$

Q5

Using the binomial formula, expand $(x + y)^4$ (your coefficients should be integers).

Q6

a) Find the value of c such that $\int_a^b \int_0^x f(cx, y)\mathrm{d}x\mathrm{d}y = 1$ where $f(x, y) = x^2 y^3$.

a) Find $\frac{\partial f}{\partial x}$ of $\int_a^b \int_0^x f(x, y)\mathrm{d}x\mathrm{d}y$ where $f(x, y) = 2x^2 y^3$.

c) Find the Hessian of $f(x, y)$ (i.e. $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$) where $f(x, y) = \cos(x^2 \sqrt{y})$. Is $f(x, y)$ convex, concave and/or both? Justify your answer.