

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Suppose $X_1, \dots, X_n \sim i.i.d.N(\mu, \sigma^2)$

Q1

Derive the likelihood function $L(\mu, \sigma^2; X)$.

Q2

Derive the maximum likelihood estimator for μ . Is it unbiased? Is it consistent? Justify your answers.

Q3

Derive the maximum likelihood estimator for σ^2 . Is it unbiased? Justify your answer.

Q4

Construct a 95% confidence interval for σ^2 using $\hat{\sigma}_{MLE}^2$.

Q5

Derive a method of moments estimator for μ and σ^2 . How do the MME estimators for μ and σ^2 compare to the MLE estimators?