

Show all work clearly and in order. Circle or box your final answer but points will be awarded based on a correct solution. A solution should always justify the steps taken and explain the assumptions needed to reach a final answer (e.g. how do you know you are not dividing by zero in the last step?).

Q1

Prove by induction that

$$P(E_1 \cup E_2 \cup \dots \cup E_n) \leq \sum_{i=1}^n P(E_i)$$

for any finite sequence of events E_1, E_2, \dots and E_n .

Q2

Prove $P(A \cap B^c) = P(A) - P(A \cap B)$.

Q3

Prove $P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A \cap B)$

Q4

Ten fair coins are dropped on the floor. What is the probability that at least two of them show heads?

Q5

A fair coin is flipped ten times. What is the probability that heads comes up at least once?

Q6

Three integers are picked randomly from the range 1–20, inclusive. What is the probability that the value of the second number lies exclusively in between the values of the first and third number?

Q7

David is dealt a hand consisting of five cards from a Salty Card Deck. A Salty Card Deck has fifty-four cards: a numberless silver card, a numberless golden card, and a standard deck of fifty-two playing cards. What is the probability that David gets dealt two pairs?

Q8

John is mailing letters to n friends, all of whom have different addresses. He has his n letters and n envelopes already addressed to his friends, but, in a fit of whimsy, John decides to randomly assign each letter to an envelope. Each envelope receives exactly one letter. Richard and Diane are two of John's friends who are sent letters. What is the probability that Richard or Diane (but not both) receive the correct letter in the envelope addressed to him or her?