Resource-Constrained Project Scheduling Problem with Alternative Subgraphs (RCPSP-AS)

1 Problem Definition

A project is represented as a directed acyclic graph $G = (N, A)$ of non-preemptive activities with fixed durations and precedence constraints. A set of renewable resources with limited capacities constrains which activities can run simultaneously. RCPSP-AS extends the standard RCPSP by introducing alternative subgraphs, each with a principal activity p_l whose immediate successors represent mutually exclusive branches—exactly one branch must be selected from each subgraph. The set of non-branching arcs is $A_{\text{prop}} = A \setminus \{(p_l, j) \in A : l \in L\}$. Activity 0 is the source and activity n is the sink.

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\begin{aligned}
\min \quad & S_n + d_n && && (1)\\
\text{s.t.} \quad & x_0 = 1 && && (2)\\
& x_i \le x_j && \forall (i,j) \in A_{\text{prop}} && (3)\\
& \sum_{j\colon (p_l, j) \in A} x_j = x_{p_l} && \forall l \in L && (4)\\
& S_i + d_i \le S_j + M(2 - x_i - x_j) && \forall (i,j) \in A && (5)\\
& \sum_{\substack{i \in N: \\ S_i \le t < S_i + d_i}} r_{i,k} \cdot x_i \le R_k && \forall k \in R,\ \forall t \in \mathcal{T} && (6)\\
& x_i \in \{0,1\} && \forall i \in N && (7)
\end{aligned}
$$

where M is a sufficiently large constant and $\mathcal{T} = \{0, 1, \ldots, T_{\max}\}$ is the planning horizon.

Symbol	**Description**
$N = \{0, \ldots, n\}$	Set of activities (topologically ordered)
$A \subseteq N \times N$	Set of precedence arcs
A_{prop}	Non-branching arcs
L	Set of alternative subgraphs
R	Set of renewable resources
d_i	Duration of activity i
$r_{i,k}$	Demand of activity i for resource k
R_k	Capacity of resource k
p_l	Principal activity of subgraph l
$x_i \in \{0,1\}$	1 if activity i is selected, 0 otherwise
$S_i \in \mathbb{Z}_{\ge 0}$	Start time of activity i

1. Minimize the makespan (completion time of the sink activity).
2. The source activity is always selected.
3. If activity i is selected and (i, j) is a non-branching arc, then j must be selected.

4. Exactly one branch is selected from each alternative subgraph iff its principal activity is selected.
5. If both activities i and j are selected and $(i, j) \in A$, then i must finish before j starts.
6. The total resource demand at any time must not exceed capacity.
7. Decision variables: $x_i \in \{0, 1\}$ (activity selection) and $S_i \in \mathbb{Z}_{\geq 0}$ (start times).

2 References

Servranckx, T., & Vanhoucke, M. (2019). A tabu search procedure for the resource-constrained project scheduling problem with alternative subgraphs. *European Journal of Operational Research*, 273(3), 841–860.