In The Name Of God,
The Merciful.

" Consideration of Ellipse Equation around at Polar Coordinate System "

Keywords: Ellipse,Equation,Around,Scope

1-Overview:

Consideration at polar system may be very simplest that Decart coordinate system because the integral is very easy to solve. First we illustrate and describe the polar system equation and solved the integral and consider the scope of answer.

2-Polar equation:

The *polar* equation of an ellipse is shown at the below. The θ in this equation should not be confused with the parameter θ in the parametric equation. In celestial mechanics, the θ in the polar equation is called the *true anomaly* (sometimes denoted by w), while the parameter is called the *eccentric anomaly* (sometimes denoted by E). The two constants in the polar equation are the *semi-latus rectum* p and the *eccentricity* e. The origin is a *focus* F of the ellipse. There is a second focus F' symmetrically located on the axis. The point P at which r is a minimum is called *perihelion* in an orbit about the sun, while A is the *aphelion*. Hence, $2a = p/(1 + e) + p/(1 - e) = 2p/(1 - e^2)$, relating p to the semi-major axis a. ***p is, of course, the radius when θ = 90°.***

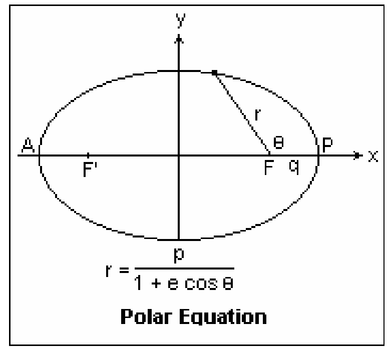

Figure 1-Polar Equation Of Ellipse

Now let c be the distance from the center to either of the foci. Then $c = a - p/(1 + e) = ep/(1 - e^2) = ea$. This is the clearest definition of the eccentricity: $e = c/a$. ***We see that e < 1, and that e = 0 gives a circle.*** These things are illustrated in the diagram at the right. Note especially the right triangle with legs b and c, and hypotenuse a. From this triangle, we can prove that $b = a\sqrt{(1 - e^2)}$, which we claimed above. Below the ellipse is shown the canonical equation of an ellipse, which includes the lengths a and b. ***Two parameters are necessary to specify an ellipse, either a, b or p, e for example***

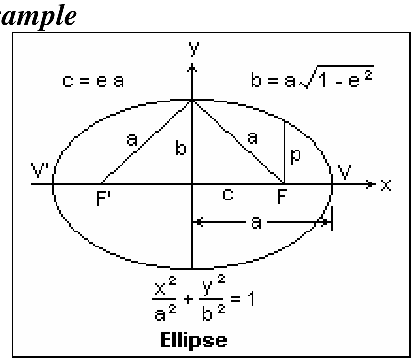

Figure 2-The ellispe description

3-The around equation finding:

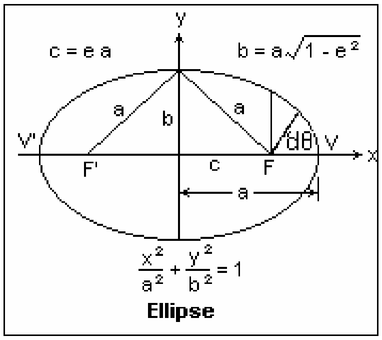

Figure 3-The Elemans

Suppose we draw a line as radius from F point to ellipse around and the Angle differential be very small. We know that the relation between the length element ($d\tau$) and other elements can be written as:

$$\sin(d\theta) = \frac{d\tau}{r}$$

And for being small of differential angle the angelic function can be ignored and can be written as:

$$d\theta = \frac{d\tau}{r}$$

So:

$$\tau = 4 * \int_{0}^{\pi - ArcTan(\frac{\sqrt{1-e^2}}{e})} r d\theta$$

That:

$$r = \frac{p}{1 + e\cos(\theta)}$$

And:

$$\tau = 4 * \int_{0}^{\pi - ArcTan(\frac{\sqrt{1-e^2}}{e})} \frac{p}{1 + e\cos(\theta)} d\theta$$

Solving Above integral leads to:

$$\tau = \frac{8 * p}{\sqrt{1-e^2}} ArcTan\left[\sqrt{\frac{1-e}{1+e}} Tan(\theta/2)\right] From_{0}^{\pi - ArcTn(\frac{\sqrt{1-e^2}}{e})}$$

That:

$$e = \sqrt{1 - \frac{p}{a}}$$

And p is radius when $\theta = \frac{\pi}{2}$.

4-Finding p base on a,b,c from Figure 3

Form Ellipse equation at Decart coordinate system we can calculate p that is equal:

$$p = b\sqrt{1 - \frac{c^2}{a^2}} \quad \text{or} \quad p = \frac{b^2}{a}$$

That with replacing in second integral theory scope relation should be satisfied below:

$$a \succ b\sqrt{1 - \frac{c^2}{a^2}}$$

Reference:

http://mysite.du.edu/~jcalvert/math/ellipse.htm