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FINITE ELEMENT TORQUE CALCULATION IN ELECTRICAL MACHINES WHILE CONSIDERING THE MOUVEMENT

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Abstract: In this paper different methods are presented for the calculation of torque as a function of rotation angle in an electrical machine. These methods are integrated in a calculation code by using the finite element method. The movement is taken into account by means of the technique of the Moving Band, constituted by quadrilateral finite elements in the airgap. The torque is calculated during the displacement of the moving part by using the following methods: Maxwell stress tensor, Coenergie derivation, Coulomb's virtual work, Arkkio's method and the Magnetizing current method. The results obtained by the different methods are compared with experimental data and allow to deduce practical informations concerning the advantages and limitations of each method.

INTRODUCTION

Actually the design of electrical machines involves increasingly the calculation codes of the electromagnetic fields based on the finite element method. The knowledge of the torque variations in terms of the rotation angle of an electrical machine is very important for the designer. In fact, this knowledge allows him to evaluate the performances and operating qualities of the machine which is to be designed. The determination of the torque variations as a function of position, involves two coupled techniques:

a) Movement included methods.
b) The torque calculation method which is the best adapted to the movement included method.

This paper studies this coupled problem, examines the different techniques of torque calculation and provides comparison elements for choosing the best method.

Considering the movement during the field calculations

The techniques to take into account the movement can be separated in two categories:

1) The airgap is not discretized; in this case, the airgap can be modeled by the boundary-element method [1] or by using analytical solutions [2].

2) The airgap is discretized; here, the airgap is subdivided into meshes and the rotation can take place by means of a layer of finite elements placed in the airgap. This layer can occupy all the airgap region or just a part of it. This technique is known as the Moving band method [3]. At each step of the moving part displacement, the elements of the Moving band are distorted and when the distorsion is large enough, then the airgap is remeshed. In this way, the number of domain nodes can increase, but by using a dynamic allocation of the periodicity or anti-periodicity conditions, the size of the corresponding matrices is not increased.

As refered to the computation time, the Moving band method is much faster than the methods in which the airgap is not discretized. For this reason we have chosen this method for our study. By considering the importance of an adequate representation of the airgap in which the Moving band is placed, we have chosen quadrilateral elements for its composition (fig.1).

While the rotor displacement is equal to the Moving band discretization step, the integration of these quadrilateral elements is carried out by dividing them into four triangles, as illustrated in figure 2. However, if the displacement is equal to a fraction of the discretization step, this division in the two directions can yield a pair of triangles more flattened than the other pair, as shown in figure 3. In this case, between all the elements which compose the Moving band, we look for the triangles that can be derived from the quadrilateral element, and for the integration we will retain the division into two triangles with the largest quality factor [4].

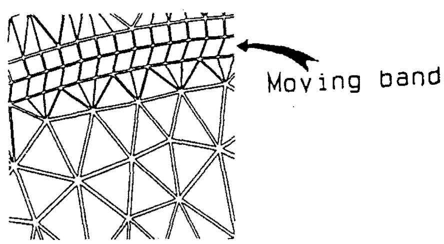

Figure 1: Airgap and Moving band.

Methods of Torque calculation

Several methods have been presented in the literature for torque calculation:

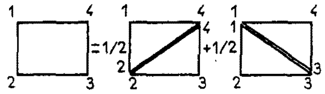

Figure 2: Division of quadrilateral element when the displacement step is equal to the discretization step.

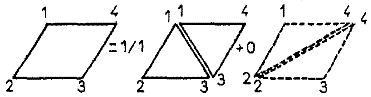

Figure 3: Division of quadrilateral element when the displacement step is different from the discretization step.

1) <u>Maxwell stress tensor:</u> the force applied to one part of the magnetic circuit can be obtained by integrating the well-known Maxwell stress tensor along a surface Γ, placed in the air and enclosing this part. In the case of electrical machines, this surface is normally placed in the airgap and the torque can be calculated by the following relationship:

$$T=L\int_{\Gamma}\{\mathbf{r}\times[(1/\mu_0)(\mathbf{B}.\mathbf{n})\mathbf{B}-(1/2\mu_0)\mathbf{B}^2\mathbf{n}]\}d\Gamma \qquad (1)$$

where L is the length, $\mathbf{B}$ is the induction vector in the elements and $\mathbf{r}$ is the lever arm, i.e. the vector which connects the origin to the midpoint of segment dΓ. In order to have a better accuracy, dΓ is drawn by connecting the midpoints of the triangular element edges [5]. When the quadrilateral elements are considered, the average torque is obtained by dividing the elements according the two directions (fig.1).

2) <u>Arkkio's method:</u> this is a variant of the Maxwell stress tensor and consists in integrating the torque given by (1) in the whole volume of the airgap comprised between the layers of radii r_r and r_s. This method has been presented by Arkkio [6], with the following expression for the torque:

$$T=\{L/[\mu_0(r_s-r_r)]\}\int_S rB_rB_\phi dS \qquad (2)$$

in which L is the length, B_r and B_ϕ denote the radial and tangential inductions in the elements of surface S, comprised between radii r_r and r_s, and dS is the surface of one element.

3) <u>Method of magnetic coenergy derivation:</u> the torque can be calculated by deriving the magnetic coenergy W, by maintaing the current constant:

$$T=LdW/d\alpha=d\{\int_V\int_0^H \mathbf{B}.d\mathbf{H}\ dV\}/d\alpha \qquad (3)$$

In the numerical modeling, this derivation is approximated by the difference between two successive calculations, that is:

$$T=L(W_{\alpha+\delta\alpha}-W_\alpha)/\delta_\alpha \qquad (4)$$

where L is the length and δ_α represents the displacement.

4) <u>Coulomb's Virtual Work:</u> based on the principle of virtual works [7], gives the following expression for the torque:

$$T=\int_\Omega L[-B^tG^{-1}(dG/d\Phi)H + \int_0^H BdH|G|^{-1}(d|G|/d\Phi)]d\Omega \qquad (5)$$

where the integration is carried out over the elements situated between the fixed and moving parts, having undergone a virtual deformation. In equation (5), L is the length, G denotes the jacobian matrix, dG/dΦ is its derivative representing the element deformation during the displacement dΦ, |G| is the determinant of G and (d|G|/dΦ) is the derivative of the determinant, representing the variation of the element volume during the displacement dΦ.

5) <u>Magnetizing Current Method:</u> this method is based on the calculation of the magnetizing current and the flux density over the element edges which constitute the boundary between the iron or permanent magnet and the air [8]. Here the torque can be determined by the following expression:

$$T=(L/\mu_0)\int_{\Gamma_c}\{\mathbf{r}\times[(B_{t1}^2-B_{t2}^2)\mathbf{n} - (B_{t1}B_{t2}-B_{n2}^2)\mathbf{t}]\}d\Gamma_c \qquad (6)$$

where L is the machine length, Γ_c denotes all the interface between the iron or permanent magnet and the air, and $d\Gamma_c$ is the length of the element edge located at the boundary. The vector $\mathbf{r}$ is the lever arm, that is the vector which connects the origin to the midpoint of $d\Gamma_c$. B_{ti} and B_{ni} denote respectively, tangential and normal flux densities with respect to $d\Gamma_c$. The subscript 1 refers to the iron or permanent magnet, while 2 corresponds to the air.

OBTAINED RESULTS AND DISCUSSION

In order to compare the above mentioned methods, two permanent magnet machine structures have been considered:

1) The first structure (fig.4) is a motor with three pole pairs and has, in the region under study, three blocs of permanent magnet, magnetized in parallel with the center magnet axis. For the purpose of numerical calculations, the airgap mesh is composed of two layers of quadrilateral elements, one of which being the Moving band. The angular discretization step is taken equal to 1.25 degrees.

We present two sets of simulation results for the static torque obtained with two series phases, supplied by a constant DC current: when the rotation step is equal to the discretization step and when it is equal to on half of the discretization step.

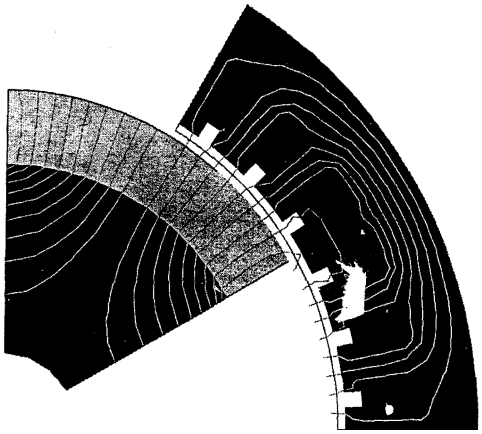

Figure 4: First machine

a) Rotation step equal to 1.25 degrees: the corresponding simulation results are illustrated in figure 5.

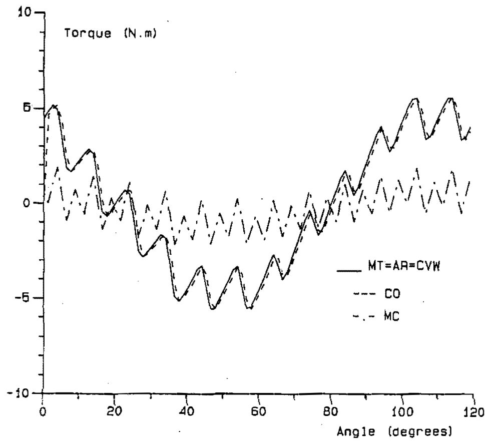

Figure 5: Static torque obtained by:
Maxwell stress tensor (MT)
Coenergie Derivation (CO)
Arkkio's method (AR)
Coulomb's Virtual Work (CVW)
Magnetizing Current (MC)

It can be observed that the obtained results with different methods are very close, except for the Magnetizing current method. In order to understand the reasons of this discrepancy, we have carried out a specific study on a simple structure composed of a permanent magnet and an iron part, as shown in figure 6a. The attraction force on the iron part has been calculated by the Maxwell stress tensor and also by the Magnetizing current method which yields the surface distribution of forces shown in figure 6b. Different mesh densities have been used, allowing to observe that the Magnetizing current method is very sensitive to the mesh size. Here, we need a large number of elements in order to obtain results close to those given by the Maxwell stress tensor, the latter being less sensitive to the mesh density. This is considered as a drawback if the structure is an electrical machine in which the number of elements is usually large.

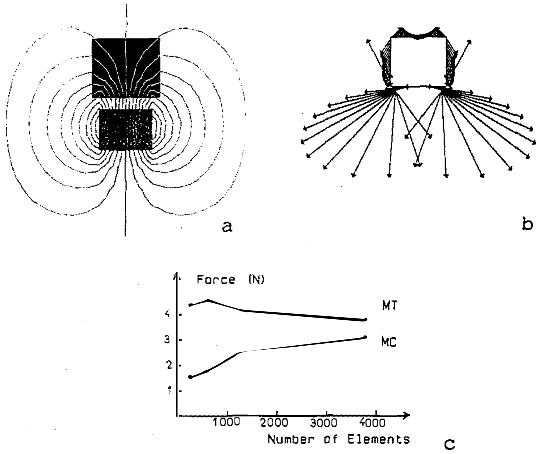

Figure 6: a) Permanent magnet and iron piece.
b) Surface distribution of forces obtained by the Magnetizing current method (MC)
c) Global force, determined by the Maxwell stress tensor (MT) and Magnetizing current method (MC) in terms of number of elements.

b) Rotation step is equal to 0.75 degrees: figure 7 shows the results obtained for this rotation step, which is the half of the Moving band discretization step.

It can be seen that because of the elements distortion in the Moving band, the Coenergie derivation, which is integrated along the whole structure , and Arkkio's method which is integrated along the airgap, present some numerical fluctuations. These oscillations do not appear for the Maxwell stress tensor and the Coulomb's virtual work methods since they are calculated in the second layer of quadrilateral elements located in the air gap and consequently, are not distorted during the movement.

2) The second motor (fig.8) presents radial magnets in the stator and the currents are placed in the rotor.

Figure 9 illustrates the simulation and experimental results obtained for the static torque with two series phases supplied by a constant DC current, and with a rotation step equal to the discretization step.

It can be seen, once more all the methods yield relatively close results for a rotation step equal to the discretization step, here taken equal to one degree. Figure 9 shows that the results obtained by the methods of Maxwell stress tensor and Coulomb's virtual work are equal. In fact, it can be demonstrated that when first-order triangular finite elements are used, then (5) gives an expression analog to (1) in which the lever arm represents the vector connecting the origin to the vertex of the element. In this way, when the size of elements used in the torque calculation is small then this lever arm is practically the same in the two expressions.

A good agreement is observed in the comparison between the theoretical results and experimental data. This fact validates the accuracy of the Maxwell stress tensor method.

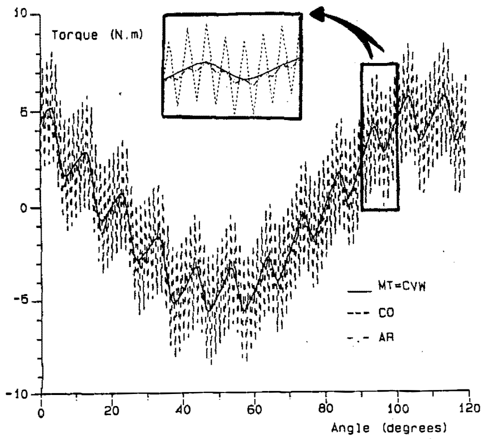

Figure 7: Static torque obtained by:
Maxwell stress tensor (MT)
Coulomb's virtual work (CVW)
Coenergie derivation (CO)
Arkkio's method (AR)

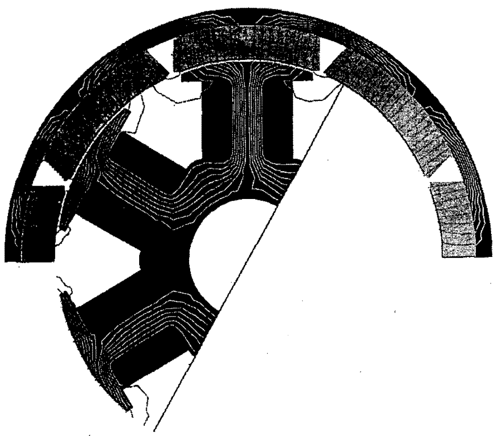

Figure 8: Second machine

CONCLUSION

In this work, we have presented a comparative study of different methods for torque calculation in electrical motors by taking into account the movement. The rotation is done by means of a Moving band, constituted by quadrilateral finite elements. All the methods of torque calculation studied in this work, yield very close results if the rotation step is taken equal to the discretization step along the Moving band. An exception is observed, however, concerning the Magnetizing current method, in which large mesh densities are necessary in order to obtain good results. Meanwhile, if the rotation imposes a rotation step different from the discretization step, then the Coenergy derivation and Arkkio's method present unwanted numerical oscillations. In order to overcome this problem, an efficient

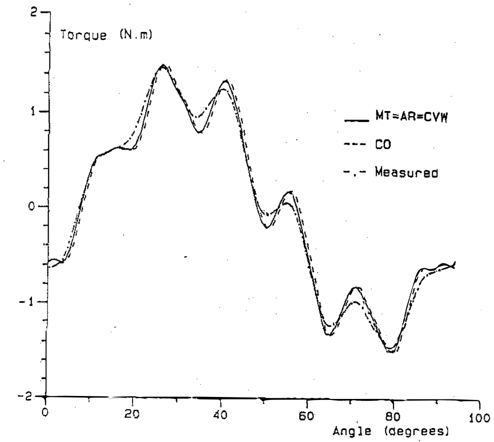

Figure 9: Static torque calculated by:
Maxwell stress tensor (MT)
Coenergie derivation (CO)
Arkkio's method (AR)
Coulomb's virtual work (CVW)

Measured data

solution consists in using the Maxwell strees tensor, integrated over a layer of quadrilateral elements in the airgap and outside the Moving band.

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