The result is

$$H_x(x,y) = \frac{I}{\pi g}\left(\tan^{-1}\frac{x+g/2}{y} - \tan^{-1}\frac{x-g/2}{y}\right) - \frac{I}{2\pi}\frac{y}{x^2+y^2} + \frac{I}{4\pi}\frac{p+g/2}{x^2+y^2}\left[\frac{x^2-y^2}{x^2+y^2}\left(\tan^{-1}\frac{x+p+g/2}{y} - \tan^{-1}\frac{x-p-g/2}{y} - \pi\right) + \frac{xy}{x^2+y^2} \cdot \ln\frac{(x+p+g/2)^2+y^2}{(x-p-g/2)^2+y^2}\right]. \tag{7}$$

This function is plotted in Fig. 2 such that it is directly comparable with certain curves in Figs. 4 and 6 of PSH.

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[3] See, for instance, J. D. Jackson, *Classical Electrodynamics.* New York: Wiley, 1962, pp. 18–19.

Simple Equations for the Magnetization and Reluctivity Curves of Steel

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Abstract—The normal B-H curves of several types of steel are shown to be approximated by the equation

$$H = (k_1\, e^{k_2 B^2} + k_3)\, B. \tag{1}$$

The reluctivity (H/B) and its derivative with respect to B are given by simple equations that are especially useful in the finite element method of analyzing steel magnetic circuits.

Digital computer calculation of the performance of devices such as transformers, motors, and generators requires use of an equation for the B-H curve of the steel. A recent letter [1] proposed a five term equation for the normal (ac) B-H curves of various steels and cited some of the several equations proposed earlier by others. The earlier equations were found to be valid only over certain ranges of B. The five-term equation has five constants that must be evaluated by a trial and error search using five potentiometers on an analog computer.

Equation (1) proposed here for H as a function of B contains three constants that are easily calculated from the B-H data points of a particular type of steel. Fig. 1 contains three sets of points relating normal B and H, which were obtained experimentally with three common types of steel [2]. The figure also contains curves drawn using (1). Each curve closely fits a set of the experimental data points.

The values k_1, k_2, and k_3 of (1) for a particular type of steel are determined by constraining (1) to pass through three of the experimental data points. The equation fits all points well when the three constrained points include one point slightly above the origin, one point slightly below the knee of the curve, and one point slightly above the knee of the curve. The point slightly above the origin is used to give the initial slope of the curve, which is assumed linear near the origin. For example, for cold rolled steel the initial reluctivity is found from the data points near the origin to equal approximately 400 m/H. The two points of constraint near the knee are chosen for cold rolled steel as ($B = 1.30$, $H = 709$) and ($B = 1.65$, $H = 2953$). Then different values of k_1 are tried until a solution is obtained with $k_1 = 3.8$ as follows. At $B = 0$, (1) gives $400 = 3.8 + k_3$. Thus $k_3 = 396.2$. At $B = 1.30$, $709/1.30 = 3.8e^{(1.30)^2 k_2} + (396.2)$. Thus $k_2 = 2.17$. Check at $B = 1.65$, $2953/1.65 \simeq 396.2 + 3.8e^{2.17(1.65)^2}$.

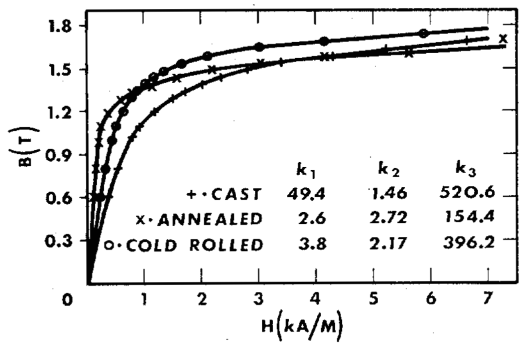

Fig. 1. Comparison between experimental normal B-H data points of three types of steel and the solid curves given by (1).

It should be noted that (1) always gives $H = 0$ at $B = 0$, which agrees with experimental normal B-H curves. The previous five-term equation does not necessarily give $H = 0$ at $B = 0$ [1].

Reluctivity ν is defined as the ratio (H/B). An equation for the reluctivity of steel is needed in the finite element method of analyzing magnetic circuits that contain steel. Recently a highly efficient method of finite element analysis was developed which requires an equation for ν as a function of B^2 and an equation for the derivative $d\nu/d(B^2)$ as a function of B^2. The function used for the derivative must be the derivative of the function used for the reluctivity and must be continuously differentiable [3].

From (1) simple formulas are obtained for steel reluctivity and its derivative:

$$\nu = k_1 e^{k_2 B^2} + k_3 \tag{2}$$

$$\frac{d\nu}{d(B^2)} = k_1\, k_2\, e^{k_2 B^2}. \tag{3}$$

Both (2) and (3) are continuously differentiable and are very easily programmed in finite element computer programs.

Also from (1) a simple equation for the magnetic energy density in steel is obtained:

$$w_m = \int_0^B H\, dB = \frac{k_1}{2k_2}\,(e^{k_2 B^2} - 1) + \frac{1}{2}\, k_3 B^2. \tag{4}$$

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Comments on "Magnetic Domain Wall Dislocation Analogy and Intrinsic Coercivity"

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In a recent paper[1] Weiner has published theoretical results for the intrinsic coercive field of narrow domain walls. The author uses the method applied by him previously for the calculation of the Peierls stress of dislocations in the Frenkel–Kontorova model. According to Weiner's calculations for domain walls not too narrow the coercive

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