COMPUTATION OF ELECTROMAGNETIC FORCES FROM FINITE ELEMENT FIELD SOLUTIONS

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ABSTRACT

Designers of electromechanical devices often need to predict magnetic forces from finite element solutions with good accuracy. This paper considers two common methods of force computation, namely the Maxwell stress tensor (MST) and the Coulomb virtual work (CVW) methods. These methods require the definition of either an integration path (in the MST method) or a sheared layer (in the CVW method), and results have been found to depend on the path or layer selected. This difficulty is associated with local errors in the computed field, which can translate into significant force errors. To overcome the problem, this paper investigates a particular implementation based on the CVW method. In this implementation, the predicted force is computed from several layers of distorted elements of the free space region surrounding a body under force thereby increasing the number of elements contributing to the force computation.
Two practical examples are considered. One is a linear case, consisting of two infinitely long parallel conductors of rectangular cross section for which an analytical result is possible and the other is a severely non-linear case for which the axial force in an axially symmetric actuator has been measured experimentally. The improved accuracy of the new implementation compared with the MST method and the CVW method with a single sheared layer is demonstrated.

INTRODUCTION

The operation of many electromechanical and electromagnetic devices depends upon the forces that act upon either current-carrying conductors or magnetised parts. In such devices, therefore, the calculation of forces is a subject of great importance. In most engineering applications, it is generally required to know both the total forces exerted on movable parts and the distribution of forces within the material of the device. This paper is concerned only with the calculation of total forces. In finite element based magnetostatics, the two most popular force formulations are the Maxwell stress tensor (MST) and the Coulomb virtual work (CVW) methods. The application of the MST method in the finite element context necessitates only a single field solution and requires the integration of a simple force density expression over any closed surface surrounding the part on which the force is needed. Accuracy problems associated with the MST method are well documented, particularly with regard to the dependency of the computed force on the type of elements crossed by the integration path, the placement of the path inside the elements, and its location within a finite element mesh [1,2]. A number of recent papers on techniques to improve the accuracy and the reliability of the MST method have been published, and alternative methods of force calculation have been suggested. Amongst these methods was the CVW method sometimes known as the local Jacobian derivatives method [3,4]. The CVW method is based on the principle of conservation of energy and the principle of virtual displacement. The global force acting on the moving part of a device is evaluated by direct, closed form differentiation of energy or coenergy associated with the free space region between the movable and the fixed parts of the device. In a finite element field solution, this method requires the selection of one or several sheared layers of free space elements surrounding the moving part.
The original intention in the derivation of the CVW method was to overcome the problems associated with the MST method. It was initially claimed that the CVW approach has all the advantages of the MST method without its drawbacks [3]. This claim was later partially refuted by McFee and Lowther [2] who showed that the CVW method when used with one element thick sheared layer was prone to the same serious accuracy and consistency problems suffered by the MST method. In fact from a theoretical point of view, the two methods were found to be equivalent. The force computed by the CVW with a single element thick sheared layer is exactly the same as the one computed by the MST method with its integration path passing by the midpoints of each two sides of triangular elements of the sheared layer [5].
Basically, the problems of accuracy and reliability of the MST, and the CVW method with a single element thick sheared layer are due to errors in the computed field: these in turn are due to discretisation (particularly important for problems with sharp corners) and failure to match the continuity of the tangential component of the magnetic field intensity at the interfaces of elements. In addition, whilst the field obtained as a result of minimisation of the total coenergy of the problem will be a good solution overall, there may still

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be significant local errors. Therefore, inaccuracies in the computed force are attributed to the fact that both methods are only related to flux densities of the elements of the sheared layer and the elements crossed by the integration path. There are at least five schemes to improve the reliability of the MST and CVW methods. All of them are based on estimating and /or reducing errors in the computed field as briefly explained in reference [6]. Thus neither the MST nor the CVW method can be used with confidence especially by a non expert user of finite elements. In an attempt to overcome the above difficulties, a particular implementation of the CVW method which uses several layers of free space elements surrounding the body, thereby increasing the number of elements contributing to the force computation is examined. Finally, two examples are presented to illustrate the problems of force computation. Also the merit of the particular implementation of the CVW method will be demonstrated and discussed.

MAXWELL STRESS TENSOR METHOD

The Maxwell stress approach computes local stress at all points of a bounding surface, then sums the local stresses by means of a surface integral to find the global magnetic force. Thus if the total field surrounding a body is known, then the force exerted on the body in terms of the field alone can be expressed by [5]

$$F = \int_S \left(\mu_0 \overline{H}(\overline{n}.\overline{H}) - \frac{\mu_0}{2}(\overline{H}.\overline{H})\overline{n}\, dS \right) \quad (1)$$

where n is the unit outward normal to the surface S, and S is any closed surface surrounding the body. The components of the force density normal and tangential to the surface S are given by, respectively:

$$f_n = \frac{\mu_0}{2}(H_n^2 - H_t^2) \quad (2)$$

$$f_t = \mu_0 H_n H_t \quad (3)$$

where H_t and H_n are the tangential and normal components of the field, respectively.

The advantages of the MST method when used in the finite element context are that (a) in principle, any surface can be chosen provided that it crosses free space and surrounds the body in question, (b) the field is only computed once, and (c) in principle, local force densities on surfaces can be derived.

The disadvantages are that (a) in practice, the computed global force is dependent on the order of element used and the path selected, i.e. the position of the elements crossed and the path through each element, and (b) definition of the surface of integration becomes difficult in three dimensional problems.

COULOMB VIRTUAL WORK METHOD

The CVW method is based upon the law of conservation of energy and the principle of virtual displacement. The global magnetic force acting on the moving part of an electromechanical device is calculated as the derivative of the energy/coenergy of the system with respect to the virtual displacement ∂q while the magnetic flux /current is held constant.

Mathematically, the global magnetic force in the q direction is given by [3,4]:-

$$F_q = -\frac{1}{2\mu_0}\sum_e \left\{ \Omega_e \frac{\partial B^2}{\partial q} + B^2 \frac{\partial \Omega_e}{\partial q} \right\} \quad (4)$$

where the summation is over all the finite elements in a sheath of air surrounding the part on which the force is to be calculated. Ω_e is the finite element volume, and B is the flux density in the element obtained from a finite element field solution by the relation B=curl A. To evaluate the derivatives involved in the equation of the force, the finite element mesh of the device is divided into three regions, the movable region, the fixed region, and the intermediate region. The movable part of the device which undergoes the virtual displacement and the fixed region are not affected geometrically by the virtual displacement. The intermediate, or the sheared, region is the part of free space between the movable and the fixed parts that is selected to absorb the virtual displacement of the movable region.

The derivatives $\partial B^2/\partial q$ and $\partial \Omega_e/\partial q$ require the knowledge of the co-ordinate derivatives of the nodes in each of the three regions. Numerical values are given to these nodal derivatives by assigning a factor between 0 and 1 to each node of the mesh. Nodes of the movable region are assigned a virtual displacement of 1. Nodes which are not unaffected by motion such as those attached to the fixed part are assigned a factor of 0. Intermediate factors between 0 and 1 may be assigned to nodes in the free space surrounding the movable part in such way that the virtual displacement is absorbed by the finite elements in the air region.

The force algorithm in the finite element field computation involves the following steps:-

1. The nodal values of the magnetic vector potential are computed in the area of definition of the problem using the classical finite element method.

2. Sheared layers of free space elements surrounding the movable part of the device are identified at the time of mesh generation by giving them separate labels. As illustrated in Fig.1 each sheared layer is of one element thick and can be of any shape.

It will be assumed that irrespective of layer thickness, the virtual displacement is uniformly distributed over all the sheared layers between 0 value for the nodes belonging to the inner boundary of the fixed part and 1 for the nodes of the outer boundary of the movable region so that the nodal derivatives are assigned as follows:-

(i) d_i= 1.0 for the outer nodes of the movable region ,

(ii) $$d_i = \frac{n-m}{n} \quad (5)$$

for the outer nodes of the m sheared layer

(iii) d_i=0.0 for the inner nodes of the fixed region.

3. The contributions of sheared elements of each sheared layer to the global magnetic force may be obtained by application of Equation (4) giving:-

$$F_q = -\left[\sum_{p=1}\sum_{e=1}\frac{1}{2\mu_0}\cdot\left\{\frac{\partial B^2}{\partial q}\Omega_e + B^2\frac{\partial \Omega_e}{\partial q}\right\}\right] \quad (6)$$

The first summation is over the total number of sheared layers, while the second is over the total number of sheared elements.

Alternatively (and this was the method adopted), the CVW method using several sheared layers of free space surrounding the moving part may be implemented in the finite element method as follows: First, from a set of sheared layers select a single one element thick sheared layer of free space and assign nodal derivatives of 0 and 1 to the outer and inner nodes of this layer, respectively. Second, compute the global force due the single sheared layer considered. Finally, compute the global magnetic force by taking an average value of the forces computed considering various layers separately.

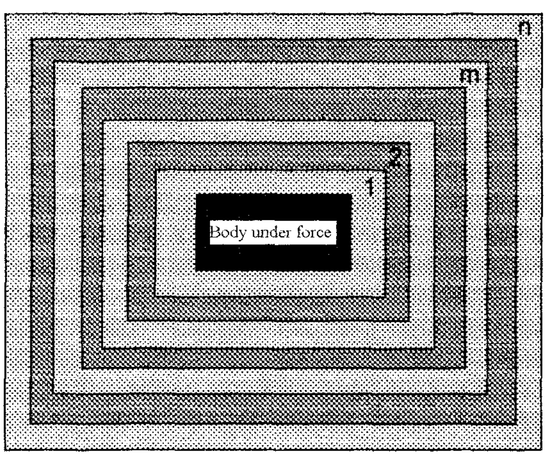


Figure 1: Body under force surrounded by sheared layers

EXAMPLE 1

The two dimensional trial system considered consisted of two parallel current-carrying conductors of square section in free space (Fig.2). An analytical solution is available for the force between them, so permitting a check on the results of the MST and CVW methods of force calculation. The distance between the centres of conductors is 1m, the dimension of each conductor is 5x5mm. The conductors carry equal and opposite currents of 1 Ampere.

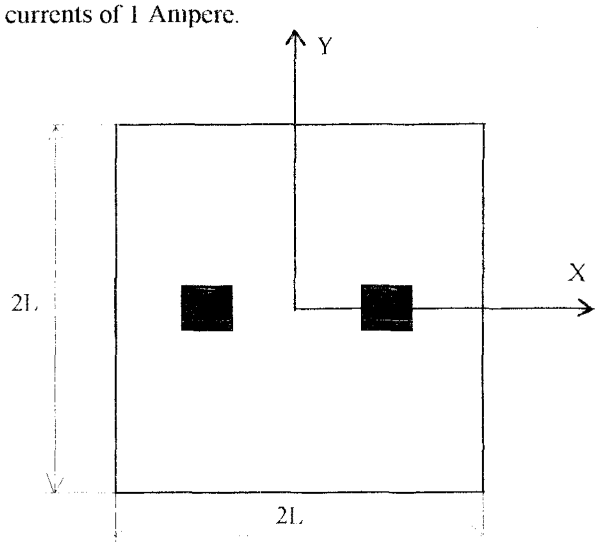


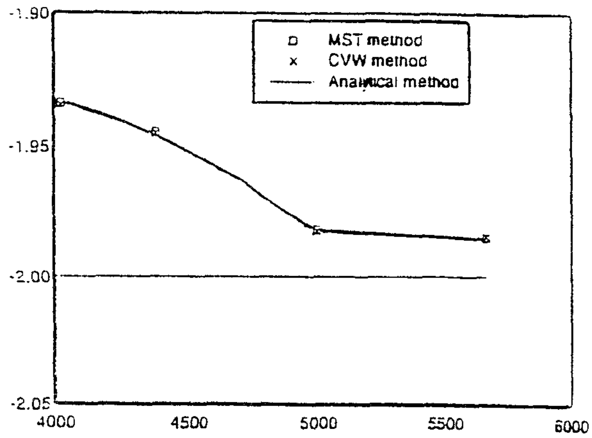
Figure 2: Outline of two parallel current-carrying conductors.

The forces computed by both the MST and the CVW methods can change with the mesh discretisation and the boundary of the mesh so that several meshes were prepared to investigate these problems. To investigate the effect of mesh discretisation, sheared layer (integration path) number 3 was used to evaluate the convergence of the computed forces with the numbers of nodes. This result is shown in Fig.3, where it can be seen that the computed force by both methods converges to its analytical value as the number of nodes is increased. Agreement within 1% is obtained in this case with a finite element mesh containing 5680 nodes. To investigate the effect of the boundary, the computed force is plotted against the distance L from the conductor to its remote boundary (Fig.4) . The remote boundary of L=10m used in the earlier calculations was found to be adequate.

Having determined satisfactory discretisation and boundary positions, several sheared layers of free space elements surrounding the conductor were selected for the CVW method. Integration paths within each sheared layer were also selected for the MST method.

Two different cases were considered for the MST method: in the first case, the integration path is selected arbitrarily and randomly within the sheared layer, in the second, it is selected to pass through the centroid of elements of the sheared layer (centroid path).

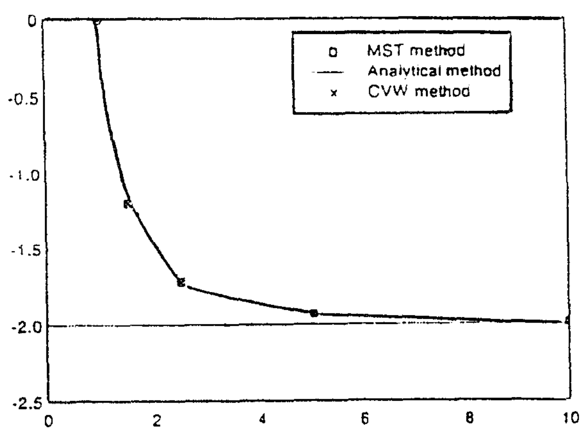
Force per unit length [10^{-7}N/m]



Number of nodes

Figure 3: Computed force versus number of nodes in mesh.

Force per unit length [10^{-7}N/m]



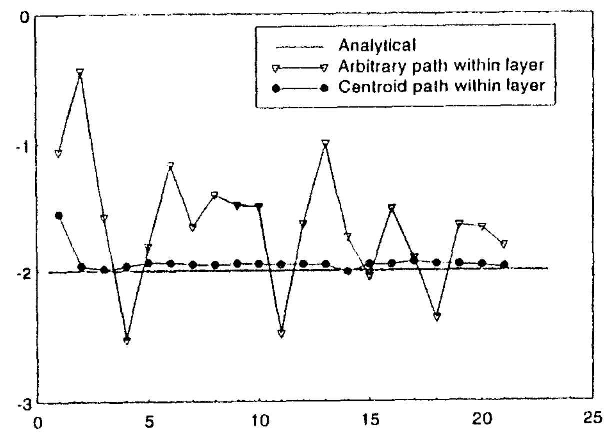
Distance L[m]

Figure 4: Computed force versus distance L to remote boundary.

The computed force by the MST method was plotted against the position of the integration path for the two cases considered. as shown in Fig.5. The computed force using a random integration path is very inaccurate and highly dependent on the position of the path within the mesh. The force computed using centroid paths, though still dependent upon the elements chosen, yields better results than the case of the random paths. The accuracy of the computed force may be improved by taking an average value of forces computed from several centroid paths.

The force computed by the CVW method against the layer position is plotted in Fig.6. A comparison between the results of Fig.6 and Fig.5 indicates that when the integration path is the centroid path within the sheared layer, the computed force by the MST method agrees closely with the CVW method for that layer. From Figs. 5 and 6, it can be noticed that the computed force by both the MST and the CVW methods is multivalued i.e., its value is not the same when various integration paths and sheared layers were used. This unreliability in the computed force was alleviated by the implementation of the CVW method over several layers as illustrated in Fig. 6.

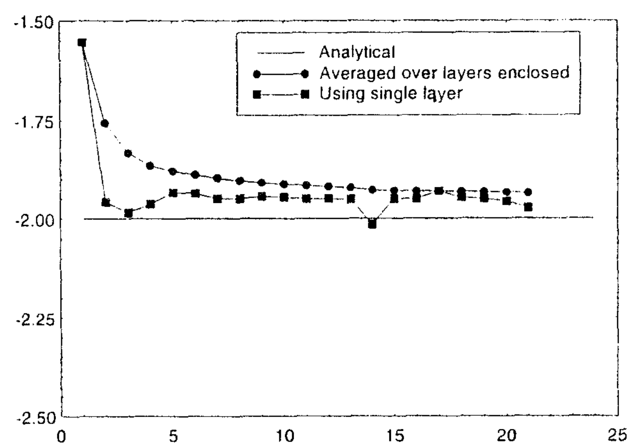
Force per unit length [10^{-7} N/m]



Layer position (see Fig.1)

Figure 5: Computed force from MST method versus position of path within layer.

Force per unit length [10^{-7} N/m]



Layer position

Figure 6: Computed force from CVW method versus position and number of sheared layer

EXAMPLE 2

The subject of this trial is an improved version of the experimental iron-cored actuator described in a previous publication [7]. It is axisymmetric with the radial section as shown in Fig. 7. The design is such as to give considerable local saturation at the higher currents. The actuator is composed of three different parts: the fixed part, the moving part, and the coil winding. The fixed part is made of mild steel and is in the form of a cylinder with a base, a lid, and a central pillar around which was wound a coil of 1450 turns. The moving part consists of a cylindrical liner of nyloil which carries a mild steel cylinder which is acted on by electromagnetic forces: the nyloil allows almost friction-free displacement of the cylinder along the central pillar. An aluminium extension is added to the central pillar in order to preserve concentricity of the actuator and to improve stability during the motion of the moving cylinder. An aluminium collar surrounds the bottom end of the central pillar in order to support the moving cylinder. A hole exists on the axis of the pillar in order to hang weights from the moving cylinder.

A direct current in the coil produces a magnetic field which creates an upward or downward force on the moving cylinder. the direction of the force depending upon the position of the cylinder.

Force versus current characteristics for various positions were obtained as follows:-

i) the position of the cylinder was fixed by non magnetic washers. and measured by an LVDT.

ii) By means of a known mass, a known gravitational force was applied to the cylinder.

iii) First demagnetisation of the actuator was carried out to remove any residual magnetism by subjecting the actuator to a sequence of hysteresis cycles of decreasing amplitude (the demagnetisation state B=H=0, is approached by a series of decreasing cycles). Then the coil current was increased slowly until the balance between the magnetic force and the gravitational force was obtained. The cylinder was allowed to move only by 0.016 mm to simplify its magnetic history. and then the current was reduced until the initial reading of the transducer was obtained. This last operation was controlled by the help of an oscilloscope displaying the transducer voltage.

The results were found to be closely repeatable.

In the same manner as example 1, several meshes with different number of nodes and boundaries were constructed in order to investigate the convergence of the computed force. Increasing the number of nodes beyond 9000 and taking the remote boundary to about eight times the outer radius of the actuator gave

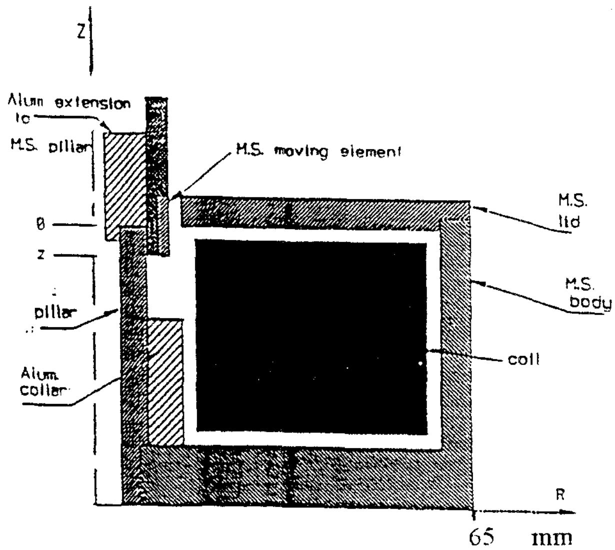


Figure 7: Geometry of the actuator.

negligible change in the computed forces.

The CVW method was applied to eight concentric sheared layers in the airgap around the cylinder, while the MST was applied to contours within each sheared layer and passing through the centroid of each of its elements. Since the force computed by the CVW method is almost equal to that of the MST when the integration contour is passing through the centroid of elements of the sheared layer, therefore only the computed force by the CVW method will be considered and compared to its experimental value.

Figure 8 and 9 illustrate the comparison between the computed force in terms of the applied current for two positions. The computed forces were obtained by the CVW method in two different ways: the first considered only the first layer of elements surrounding the moving cylinder, and the second averaged the results over the remaining seven layers of elements.

A comparison between the computed and the measured results shows that a good agreement is obtained when seven sheared layers of elements are used in the computation of the force (discrepancies are between 0.5% and 6.5%). However, when the force is computed with only the first sheared layer, the comparison with measurement showed discrepancies between 9.5% and 15.5%. This large departure between the measured and the computed forces may be attributed to the large errors in the computed fields of elements near or adjacent to the sharp edges of the moving cylinder (this effect accounts for the behaviour of the curves of Fig.6).

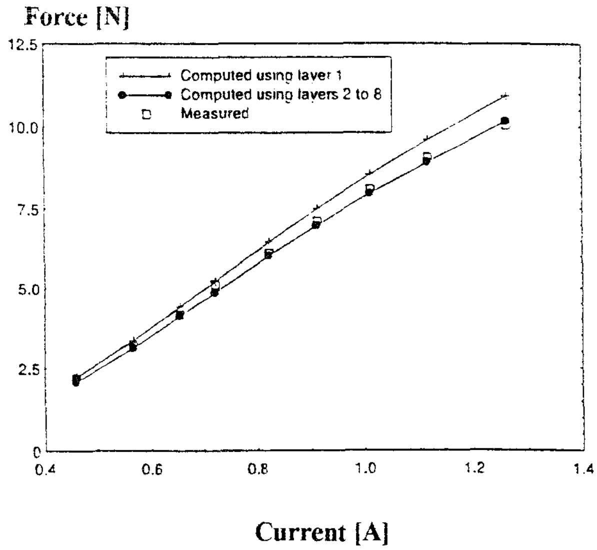


Figure 8: Comparison between computed and measured force for z=0.04mm.

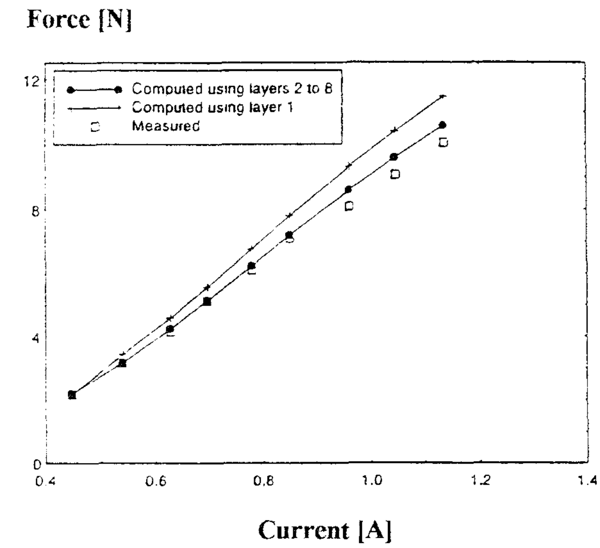


Figure 9: Comparison between computed and measured force for z=1.05mm.

CONCLUSION

The paper has highlighted the difficulties associated with both the MST method, and the CVW method with single element thick sheared layer. The use of the MST method with an arbitrary integration path within a layer was found to be very inaccurate and unreliable, When the integration path is the centroid path within a layer, the results obtained by the MST method were much better than for an arbitrary path, and were found to be very close to those of the CVW method with a one element thick sheared layer. Although the CVW method with one element thick sheared layer has solved some of the problems of the MST method, it is still subject to reliability problems. The computed force is multivalued i.e. its computed value is not the same when various sheared layers are used, and there is no sure way to tell which is most accurate. This problem was alleviated by implementing the CVW method with as many sheared layers as possible. The merit of this implementation lies in the fact that it yields a unique and accurate value for the magnetic force for the non-expert user.

Whilst there is no increase in accuracy in predicting global forces using multi-layer CVW compared with a multiple path MST case, the problem of path definition is eased. The advantage is expected to be much greater in the full 3-D case.

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