

The concept of *type theory* is vital in the understanding of mathematical logic and theoretical computer science. It focuses on the question of “what exactly is a proof”. The importance of a type is based on clarity and absence of wrong definitions. A formalism is a pure, rigorous presentation in which we need to manipulate symbols and their meaning. We can observe a resemblance between a formalism and principles from Plato's philosophy. Both describe pure, ideal thinking and the existence of the objects described will not lead to contradictions. Classical logic, developed in the Greek antiquity comprises the law of excluded middle.

$$\wedge I \quad \frac{A \quad B}{A \wedge B}$$

This formula is called conjunction-introduction. It says that if we know A and we know B, then we can conclude that $A \wedge B$ is true

$$\wedge E \quad \frac{A \wedge B}{A} \quad \frac{A \wedge B}{B}$$

It is called conjunction elimination. It says that if we know that $A \wedge B$ is true then we may state that both A and B are true.

$$\vee I \quad \frac{A}{A \vee B} \quad \frac{B}{A \vee B}$$

This rule is called disjunction-introduction. If we know A to be true, then we may conclude that $A \vee B$ is true. Similarly, the idea applies if we know that B is true.

$$\vee E \quad \frac{\begin{array}{cc} [A] & [B] \\ \mathcal{D} & \mathcal{D}' \\ C & C \end{array}}{C}$$

This rule is called disjunction-elimination. It states that if we know A or B and if we can derive from the proposition A a proposition C, and if we can derive from B a proposition C, then we can state that C is true.

$$\rightarrow I \quad \frac{\begin{array}{c} [A] \\ \mathcal{D} \\ B \end{array}}{A \rightarrow B}$$

Implication introduction. If we can derive from proposition A the proposition B, then we state that A implies B.

$$\rightarrow E \quad \frac{A \quad A \rightarrow B}{B}$$

Implication elimination (modus ponens)- If we know that A implies B and we also know that A is true, then we can derive B.

$$\perp E \quad \frac{\perp}{A}$$

False-hood elimination- if we can derive False, then any proposition is true.

$$\forall I \quad \frac{\begin{array}{c} \mathcal{D} \\ A(x) \end{array}}{\forall x A(x)}$$

For-all quantification introduction: if we have to prove that for all x, A(x) is true, and we have a derivation that for all instances of x the predicate A is satisfied, then the goal is achieved.

$$\forall E \quad \frac{\forall x A(x)}{A(t)}$$

Forall-elimination. If we know that forall x , $A(x)$ is true, then we substitute x for the term t and $A(t)$ is true.

Existential –introduction: If we know that $A(t)$ is true then we substitute this term t with x in the conclusion and we can deduce that there exists at least one x that satisfies the property.

$$\exists I \quad \frac{A(t)}{\exists x A(x)}$$

$$\exists E \quad \frac{\exists x A(x) \quad \begin{array}{c} [A(x)] \\ \mathcal{D} \\ C \end{array}}{C}$$

Existential elimination states that if we know that there exists an x such that $A(x)$ is satisfied, then if we know that there exists a derivation of C from $A(x)$, then we can state that C is true.

Calculus of inductive constructions.