ADVANCED BAYESIAN MODELING

Direct Simulation

Review

When Bayesian tools are not exactly computable, they may be approximated based on simulated parameter values

$$\theta^1, \quad \theta^2, \quad \ldots, \quad \theta^S$$

usually sampled from the posterior.

We first consider methods that use <u>independent</u> sampling.

(Note: NOT related to sampling distribution of data!)

Standard Distributions

Many software packages have functions for independent random sampling from standard distributions (normal, uniform, beta, ...).

Standard distributions arise often in simulation from

- The prior distribution
- The posterior distribution when using a (fully) conjugate prior
- Posterior conditional distributions when using partially conjugate priors

R functions for independent sampling from standard distributions are usually named starting with `r` and have the number of samples `S` as the first argument.

For example:

`rnorm(S, mu, sigma)`: vector of `S` independent samples from $\mathrm{N}(\mu, \sigma^2)$

`rbeta(S, alpha, beta)`: vector of `S` independent samples from $\mathrm{Beta}(\alpha, \beta)$

Note: Arguments `mu`, `sigma`, ... may be vectors (of length `S`).

Warning: Check R help to verify parameterization – may be different than in BDA3.

Remark: Basic features of standard distributions often have explicit formulas (mean, variance, mode, ...) or pre-programmed functions (quantiles, distribution function).

Use these instead of simulation whenever possible.

For example, in R,

`qbeta(0.9, alpha, beta)` gives 0.9 (lower) quantile of $\mathrm{Beta}(\alpha, \beta)$

Grid Sampling

Suppose we can evaluate unnormalized continuous posterior density

$$q(\theta) \quad \propto \quad p(\theta \mid y)$$

on a fixed grid of θ values

$$\theta^1, \quad \theta^2, \quad \ldots, \quad \theta^g$$

Then an *approximate* posterior sample is obtained by discretely sampling the grid points with probabilities proportional to

$$q(\theta^1), \quad q(\theta^2), \quad \ldots, \quad q(\theta^g)$$

If θ is scalar, there is a convenient R function:

```
sample(grid.points, S, replace=TRUE, prob=q(grid.points))
```

This takes `S` independent samples from vector `grid.points`, weighted by values of vectorized function `q()`.

(Using `sample` for higher dimensions is more complicated.)

Remarks:

- Practically limited to low dimensions (1 or 2).
- Requires advance knowledge of good range for grid.
- Deterministic numeric integration can sometimes be used instead.

Conditional Sampling

Assume an independent posterior sample

$$\theta^1_{-j}, \quad \theta^2_{-j}, \quad \dots, \quad \theta^S_{-j}$$

of $\theta_{-j} \; = \;$ the θ vector without its jth component.

Then a posterior sample of jth component θ_j is obtained by independently sampling θ_j^s from conditional posterior density $p(\theta_j \mid \theta^s_{-j}, y)$.

Convenient if θ_j has partially conjugate prior.

Another application: Sampling from a posterior predictive distribution.