STAT 578: Advanced Bayesian Modeling

Week 4 – Lesson 2

Normal Hierarchical Model in R/JAGS

Fall 2019

JAGS Models for 2016 Polls

2016 Polls Example

$$y_j = \text{Clinton lead (percentage points) in poll } j$$

$$\sigma_j = \text{half margin of error of } y_j$$

$$j = 1, \ldots, 7$$

Regard σ_js as fixed and known.

Recall data in file polls2016.txt:

```
# 2016 U.S. presidential election race between H. Clinton and D. Trump
# National poll results for two-way race, conducted November 3 and later
# y = percentage of Clinton lead, ME = margin of error

poll        y    ME
YouGov      4    1.7
Bloomberg   3    3.5
ABCWaPo     3    2.5
Fox         4    2.5
IBD         1    3.1
Monmouth    6    3.6
NBCWSJ      5    2.73
```

Hierarchical Models

Model similar to one proposed in BDA3, Sec. 5.4:

$$y_j \mid \theta_j \sim \mathrm{N}(\theta_j, \sigma_j^2) \qquad j = 1, \ldots, 7$$

$$\theta_j \mid \mu, \tau \sim \mathrm{N}(\mu, \tau^2) \qquad j = 1, \ldots, 7$$

$$\mu \sim \text{flat on } (-\infty, \infty)$$

$$\tau \sim \text{flat on } (0, \infty)$$

Since JAGS doesn't allow improper priors, approximate with

$$y_j \mid \theta_j \; \sim \; \mathrm{N}(\theta_j, \sigma_j^2) \qquad j = 1, \dots, 7$$

$$\theta_j \mid \mu, \tau \; \sim \; \mathrm{N}(\mu, \tau^2) \qquad j = 1, \dots, 7$$

$$\mu \; \sim \; \mathrm{U}(-1000, 1000)$$

$$\tau \; \sim \; \mathrm{U}(0, 1000)$$

(Hyperpriors wide enough? Need data to decide.)

JAGS model (polls20161.bug):

```
model {

  for (j in 1:length(y)) {
    y[j] ~ dnorm(theta[j], 1/sigma[j]^2)
    theta[j] ~ dnorm(mu, 1/tau^2)
  }

  mu ~ dunif(-1000,1000)
  tau ~ dunif(0,1000)

}
```

Note:

▶ dnorm parameterizes the normal distribution using the precision, not the variance.

▶ JAGS allows mathematical expressions as arguments to distributions.

Alternative Model

Another possibility:

$$y_j \mid \theta_j \sim \mathrm{N}(\theta_j, \sigma_j^2) \qquad j = 1, \ldots, 7$$

$$\theta_j \mid \mu, \tau^2 \sim \mathrm{N}(\mu, \tau^2) \qquad j = 1, \ldots, 7$$

$$\mu \sim \mathrm{N}(0, 1000^2)$$

$$\tau^2 \sim \mathrm{Inv}\text{-}\chi^2(1, 1)$$

Motivation: *Partial* conjugacy (later)

(Hyperpriors diffuse enough? Need data to decide.)

From BDA3, Table A.1:

$$\text{Inv-}\chi^2(1,1) \;=\; \text{Inv-gamma}(1/2, 1/2)$$

Thus, equivalently,

$$1/\tau^2 \;\sim\; \text{Gamma}(1/2, 1/2)$$

(Recall: Precision parameter)

Needed because JAGS does not have inverse chi-square or inverse gamma

JAGS model (polls20162.bug):

```
model {

  for (j in 1:length(y)) {
    y[j] ~ dnorm(theta[j], 1/sigma[j]^2)
    theta[j] ~ dnorm(mu, tausqinv)
  }

  mu ~ dnorm(0, 0.000001)
  tausqinv ~ dgamma(0.5, 0.5)

  tau <- sqrt(1/tausqinv)

}
```

Note:

Choosing to let JAGS compute tau, but could just as well let R compute it later

(Advantage of using JAGS to compute it: Automatic monitoring and inference)

See JAGS manual for list of mathematical functions.