Group Coursework Submission Form

Specialist Masters Programme

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DATA TRANSFORMATION AND ORDER OF INTEGRATION OF THE VARIABLES

For this report we were provided daily time series data of 4 foreign exchange rates of US \$, JAPANESE YEN, UK £ and SWISS FRANC to EURO € over a 20-year period (01/09/2003 to 01/09/2023). To investigate for order of integration, we aggregate the series quarterly and log transform the series to obtain lp1q, lp2q, lp3q, and lp4q[1] .

From the level plots, lp1q, and lp3q show an upward sloping trend compared to lp4q which shows a downward sloping trend, but lp2q possesses no trend at all, note all series show non-stationarity at their levels. Based on the detrended residual plots of lp1q, lp2q, lp3q, and lp4q, we noticed lp1q, lp2q, and lp3q are rescaled plots of the original plots (i.e., difference stationary), unlike lp4q which shows a new pattern compared to its original plot, (i.e., trend stationarity).

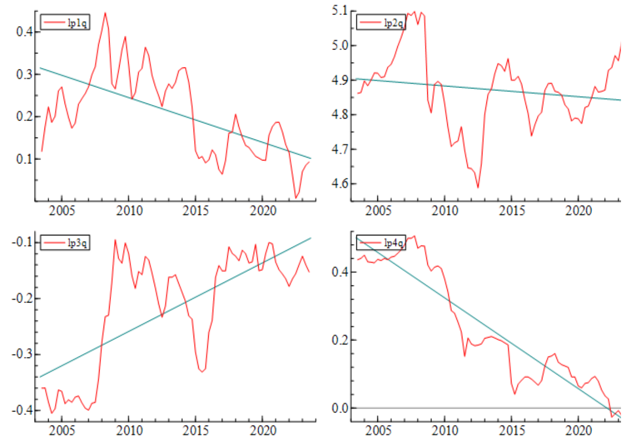

Figure 1: Plots of Level Series

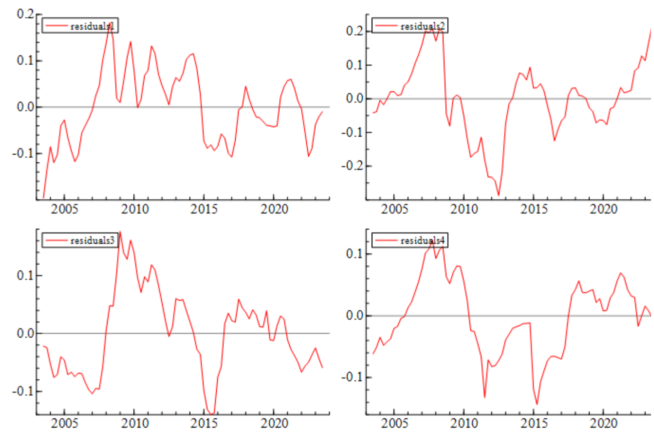

Figure 2: Plots of Residual Plots

But these are all based on eyeballing, so we need to further investigate this, by applying the ADF test. For the lp1q, lp2q and lp3q, we run an ADF[2] test for an equation containing both trend and a constant as seen in Ex 1.1, where $\propto$ *is the constant and* βt *is the trend, with our hypothesis below,*

$$\Delta Y_t = \propto \ + \beta t + \ \theta Y_{t-1} + \varepsilon_t \tag{1.1}$$

$$H_0 : \theta = 0, Y_t \sim I(1) \ a\ unit\ root \tag{1.2}$$

$$H_1 : \theta \ \neq 0 \ , Y_t \sim I(0) \ , possesses\ stationarity \tag{1.3}$$

Moreover, for further investigation into their first differences, we found the quarterly log transform the differences of Dlp1q, Dlp2q, Dlp3q, Dlp4q. Best t-values were selected based on the lowest AIC[3]. In conclusion, DLp1q, DLp2q, Dlp3q and DLp4q are all of I (0), thus possess weak stationarity and their level forms are I (1) as they all possess a stochastic trend and are difference stationary.

MULTIVARIATE COINTEGRATION ANALYSIS

Cointegration is the phenomenon where nonstationary processes can have linear combinations that are stationary. (Granger, 1983).From the ADF test, we know lp1q, lp2q, lp3q and lp4q are all I (1) - non-stationarity. Hence, we can run an ADF test on the residuals of each pair, to investigate convergence. Based on this all pairs converge except Lp4q regressed on lp1q. Table 2 below, shows if the Regressand and regressors are integrated and converge with each, this method can be used to obtain the pairs of cointegrated movements based on the non-stationary variables.

As stated previously, lp1q, lp2q, lp3q and lp4q are all integrated of order 1, we can now run a multivariate cointegration analysis using VAR and VECM approach. To identify our number of cointegrating pairs, we ran an unrestricted VAR on our 4 series, Dlp1q, Dlp2q, Dlp3q, Dlp4q from an order of 0 to 4, as we were dealing with quarterly series.

[1] lp1q=ln (US \$ TO EURO), lp2q=ln(JAPANESE YEN TO EURO), lp3q=ln(UK £ TO EURO), and lp4q=ln(SWISS FRANC TO EURO).
[2] ADF – Augmented Dickey fuller
[3] AIC-Akaike information Criteria

Regressand	Regressor	Period: 2003(1) - 2023(3)			Integrated	Convergence
		T-prob from OLS	ADF statistic[4] of Residual	General restrictions Test		
Lp1q	Lp2q	0.2223	-2.274*	1.7822e+05[0] **	Yes	Yes
	Lp3q	0.2534	-2.627**	1576.2 [0] **	Yes	Yes
	Lp4q	0.0000	-3.629**	158.50 [0] **	Yes	Yes
Lp2q	Lp1q	0.2223	-2.123*	1.4292e+05[0] **	Yes	Yes
	Lp3q	0.0000	-2.430*	2.0745e+05[0] **	Yes	Yes
	Lp4q	0.0014	-2.035*	1.5824e+05[0] **	Yes	Yes
Lp3q	Lp1q	0.2534	-2.354*	1507.9 [0.0000] **	Yes	Yes
	Lp2q	0.0000	-2.391*	2.4799e+05[0] **	Yes	Yes
	Lp4q	0.0000	-3.038**	1115.0 [0] **	Yes	Yes
Lp4q	Lp1q	0.0000	-2.332*	4.3958 [0.1110]	Yes	No
	Lp2q	0.0014	0.1999	69236. [0] **	No	Yes
	Lp3q	0.0000	-1.364	1115.0 [0] **	No	Yes

Table 1: Integration and Convergence of Regressand and Regressors

We observe all our VAR models show misspecification due to normality and their function form (i.e., RESET23 test). However, as we are selecting based on BIC/AIC and misspecification, we choose a VAR (0) and VAR (2) as our best selections, as VAR (0) has the least BIC due to it being more parsimonious and having the least number of parameters. Also, we choose VAR (2), as our 2nd best, as it has the lowest AIC, with more parameters, and the least misspecification, only in normality.

VAR	AIC	BIC/SC
0	-28.1383	-28.0165
1	-28.2223	-27.6136
2	-28.3019	-27.2142
3	-28.0063	-26.4235
4	-27.8586	-25.7733

Table 2:VAR(p) results of BIC and AIC, we choose the lowest BIC and AIC model, as this tells us its more parsimonious. To fix any misspecification, we add dummies as exogenous variables, whiles timeseries are endogenous.

We then run an I (1) cointegrating analysis to find the cointegrating pairs of our best models VAR (0) and VAR (2), with hypothesis,

$$H_0^1: at\ maximum\ k-1\ vectors\ of\ cointegration$$
$$H_1^1: k\ vectors\ of\ cointegration$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$H_0^k: at\ maximum\ 0\ vectors\ of\ cointegration$$
$$H_1^k: 1\ vectorof\ cointegration$$

Our results show that the specified VAR of order 0 and 2, have no cointegrating pairs, based on the trace test and eigen values for the returns (first difference level), but have 3 or more cointegrating pairs on the price level.

Richards explains the evidence that cointegration does not generally exist between return series is hardly surprising, given that basic models of asset pricing would preclude cointegration, which is synonymous to our results as we had no cointegrating pairs of returns at a VAR (0) to VAR (4).

Richards further argues that apparent findings of cointegration in other studies may often be due to the use of asymptotic, rather than small-sample, critical values. In fact, economic theory suggests that cointegration is unlikely to be observed in efficient markets. To further investigate this issue, the paper examined the statistical basis for the rejection by Kasa (1992) of the null hypothesis of no cointegration between different markets. Simulation evidence suggests that the finding of such a strong cointegrating relationship is due to a failure to adjust asymptotic critical values to take account of the small number of degrees of freedom that remain in the Johansen (1988) multivariate estimation procedure.

Setting asymptotic critical values based on no statistical test may be misleading in small samples as setting thresholds for the number of pairs of cointegrating factors, might not be accurate for a small sample of data. Let's

[4] * Means at 5% significance level of -1.94 and ** means at 1% significance level of -2.59

say we say we decide to reject no cointegration at the 5% significance level based on instincts, on a sample of 100 observations, there's a high chance we are inaccurate due to the small sample. Gregory (1994) shows that the size of the Johansen tests is significantly worse in cases of small samples and a high number of explanatory variables than the size of other tests of cointegration. This applies to our results, as we had a small sample, and our results may be affected by this issue. In particular, as the VAR sample size (T) falls, the critical values should be adjusted upwards (or the test statistics downwards). In our case, the T dropped from 80 to 76, as our VAR order increased. Also, Kasa's tests imply, that serial correlation is not present in the low-order VARs, so mean reversion would not appear to be a major factor, which can be shown in our misspecification results of no serial correlation. The addition of extra lags to 'remove' the nonnormality of residuals would be inappropriate if changes in stock prices are fundamentally fat-tailed or otherwise nonnormal. In cases of such extremely small samples, asymptotic results are unlikely to hold, requiring researchers to examine the robustness of their results through Monte Carlo analysis or some similar technique. Based on this our results may not be accurate due to this asymptotic behaviour of our critical value.

GARCH AND MGARCH MODELS

In this section, we determined the appropriate GARCH and MGARCH models for the daily foreign exchange rates. To do this, we obtained the daily log returns of the series, lp1, lp2, lp3 and lp4, which showed all series are I (1) as well as the mean-reverting series - the first difference of the log returns, Dlp1, Dlp2, Dlp3 and Dlp4 as I (0). To determine if our differenced series need volatility modelling, we run the F-test to test for ARCH effects. Based on H_0 of no ARCH effects, all our **p-values < 0.05**, all our series showed presence of ARCH effects in the squared first log differenced series, thus we can estimate the volatility of the returns GARCH models.

To estimate the univariate GARCH estimation, we first fitted ARMA (0,0) – GARCH (1,1) onto the differenced series, using a normal distribution and t-distribution as comparison of distributions.
Based on misspecification even though the BIC was slightly higher, the ARMA (0,0) – GARCH (1,1) **[Normal]** was a better fit compared to the ARMA (0,0) – GARCH (1,1) **[Student]**, as the Information criteria were lower, and all variables showed presence of convergence to the normal distribution, and the model was well specified except none possessed normality due to the presence of outliers.

Compared to the t-distribution, the ARMA (0,0) – GARCH (1,1), which showed misspecification in the squared-residuals of the differenced log returns, indicating the presence of serial correlation. Even though the t-distribution GARCH models had lower Information criteria, we chose the normally distributed GARCH due to it being well specified, $\alpha + \beta < 1$(stationarity), no serial correlation and ARCH effects. Moreover, all univariate GARCH models had statistically significant conditional variance, and alpha and beta apart from the conditional mean, and no normality of residuals as only misspecification.

In modelling Multivariate GARCH (MGARCH) model, one needs to make sure that the specifications are,
- Flexible to capture the dynamics of the conditional variances and covariance.
- Model specifications should be parsimonious.
- Positive definiteness of covariance matrices.
- Estimated parameters should have direct interpretation.

However, the number of parameters in the MGARCH model tend to increase rapidly as the time series variables increases, thus we adopted various models to identify the best MGARCH. We applied the variance targeting approach to reduce dimensionality of our parameters using G@RCH9 in OxMetrics.

The MGARCH models are estimated using ML[5] or QML[6] approaches, where we check if the timeseries show convergence to a distribution, information criteria[7] and specification in terms of,
- Normality of residuals
- No serial correlation of residuals
- No serial correlation of squared residuals

To find the appropriate MGARCH fit of all 4-time series (i.e., m) we split, the model approaches into the number of steps of estimating the conditional variance-covariance matrix of residuals H_t from the VAR model and type of combination of GARCH,
- **Direct univariate combination of GARCH and 1-step estimation of H_t**
    - ***Scalar -BEKK*** – restricted case of the VECH model, where we estimated a ARMA (0,0)-Scalar BEKK (1,1) using the variance targeting model. To choose between the 2 model we checked the BIC (-31.89 to -32.9)

[5] ML – Maximum Likelihood
[6] QML – Quasi maximum likelihood
[7] Information Criteria may be BIC or AIC depending on preference.

and picked the more parsimonious model with the lowest BIC which was the t-distributed model over the Normal model, as it was better specified. However, the parameters were more than the Normally distributed model by one, but this is not an issue as its, due to the t-degree of freedom.

$$H_t = C'C + \sum_{i=1}^{p} A_i{}' \, \Xi_{t-i} \, \Xi_{t-i}{}' A_i + \sum_{j=1}^{q} B_j' H_{t-j} B_j \tag{3.1}$$

- o ***Diagonal BEKK*** – special case of the DVEC – the variance targeting approach produced erroneous values for the normal distribution, thus we chose the t-distribution DBEKK model, as it had more accurate results, except the model had non-normal residuals and serial correlation in the squared residuals of Dlp3q up to the 20th lag.

- **Nonlinear combination of GARCH and 2-step estimation of H_t,** as such they decompose the variance matrix into the conditional volatilities and conditional correlations.
  - o ***Constant Conditional Correlation (CCC) model***: we first fit the 4 univariate GARCH model as explained previously to obtain $h_{ii,t}$ thus populating the matrix D_t. Secondly, estimating the correlation matrix R, using 3.3. CCC model is based on constant correlation.

Based on this, we estimated CCC model on the t-distribution and Normal distribution. The Normal-distribution CCC model is better suited to our data, due to it BIC, it is well specified, has statistically significant variance and coefficients, no serial correlation in residuals as well as absence of ARCH effect apart from the fact we have non-normality.

$$H_t = D_t R D_t = \left(\rho_{ij}\sqrt{h_{ii,t} h_{jj,t}}\right) \tag{3.2}$$

$$\overline{\rho_{ij}} = \left(\sum_{t=1}^{n} \frac{\varepsilon_{i,t}}{\sqrt{\widehat{h_{i,t}}}} \frac{\varepsilon_{j,t}}{\sqrt{\widehat{h_{j,t}}}}\right) \Bigg/ \left(\sqrt{\sum_{t=1}^{n}\left(\varepsilon_{i,t}/\sqrt{\widehat{h_{i,t}}}\right)^2 \sum_{t=1}^{n}\left(\varepsilon_{j,t}/\sqrt{\widehat{h_{j,t}}}\right)^2}\right) \tag{3.3}$$

We further ran the Constant correlation of Tse and Engle and Sheppard test of a null of constant correlation, which were both rejected, indicating we have no constant correlation and must run the DCC model.

- o ***Dynamic Conditional Correlation (DCC):*** the DCC is akin to the CCC except, it is time dependent and has dynamic correlations, which allows it to outperform the CCC model. Mostly best to split data samples to get a better view of model, but we will use full sample to view the whole outlook.

$$H_t = D_t R_t D_t \tag{3.4}$$

  - ▪ DCC Engle: we compared the T-distribution model to the Normally fitted model, they were both well specified, but the t-distribution fitted the DCC model better with a BIC of -32.69 compared to -32.02, even though we had more parameters.
  - ▪ DCC Tse and Tsui had similar results to the DCC Engle, except that the BIC's were slightly higher of -31.98 for Normal and -32.65 for Student t distribution. Moreover, the conditional correlations are the same, with the m estimated as 4.

			Number of Parameters		
Model	**Formula**	**Distribution**	**Non-variance targeting**	**Variance targeting**	**BIC**
BEKK	ARMA(0,0)-BEKK(1,1)	Student T	***17***	7	-32.90
DBEKK	ARMA(0,0)-DBEKK(1,1)	Student T	***22***	Error[8]	*-32.01*
CCC	ARMA(0,0)-CCC(1,1)	Normal/GARCH	23	0	-31.84
DCC	ARMA(0,0)-DCCE(1,1)	Student T/GARCH	***25***	25	-32.69

Table 3: Chosen Models based on BIC and Other Factors.

There is no single best MGARCH model, as this depends on the time series, reason for MGARCH, and other specifications such as computation speed. MGARCH modelling is important as it allows us to combine all time series as one GARCH model and capture their relationships between volatilities of the different assets such as their correlations, covariance, and any spillover effects as seen previously. Compared to univariate GARCH, where each series has its own volatility model. However, financial practitioners might prefer an MGARCH as it offers more insight into portfolio risk drivers, such as correlations in markets, which was shown by the DCC models conditional correlations plots, or their covariances, thus helping with risk allocation, hedging and

[8] Error- due to Oxmetrics not being able to compute output

diversification. In finance, DCC and CCC models are more relevant as they seem more efficient in estimation procedure and make better estimates, based on this we will pick the ARMA (0,0)- DCC (1,1), even though it has a lot of parameters, it estimates the conditional covariance and correlations, based on realistic assumptions unlike the CCC which assumes all correlations are constant and not capturing negative correlations through the period.

TESTING FOR CONTAGION

4A. Forbes and Rigobon (2002)

Contagion is defined as the significant increase in cross-markets linkage after a shock to one country or to a group of countries. While there are several established methods of contagion testing currently established in the literature including the usage of an ARCH or GARCH framework or by examining changes in the cointegrating factor, Forbes and Rigobon's paper narrows in on the most straightforward approach in contagion testing – examining changes in the cross-market correlation coefficients. This method is as intuitive as it seems to suggest: an increase in cross-market correlations during turmoil as compared to that that of during stability could strongly suggest the possibility of contagion between two markets. This is illustrated plainly through a simple linear model:

$$y_t = a + Bx_t + \varepsilon_t \tag{4.1}$$

where any statistically significant change in the B could imply a significant change in the relationship between x_t and y_t as well.

However, a critical analysis of this particular methodology by Forbes and Rigobon uncovers a key flaw in such a comparison where the independence between two markets could be misconstrued as contagion. This is primarily due to the presence of heteroskedasticity which biases the tests for contagion based on cross-market correlations where the increase in volatility between the periods of stability and crisis contributes to the increase of in cross-market correlations. Referring to the linear model, this means that the volatility of the error term ε_t changes between x_t and y_t from the stable to the crisis period which violates the assumption of homoskedasticity. As such, this effectively makes the interpretation of any significant change in B limited as it implies that the observed increase in correlations is driven by the transmission mechanisms from existing cross-market linkages and not through new cross-market linkages that sprung from the crisis.

To circumvent this problem, the Forbes and Rigobon paper differentiates the ideas between the conditional and unconditional correlation coefficients and relates the two numbers through the factor δ which measures the relative increase in the variance of the variable x_t. The relationship between ρ and ρ^* is:

$$\rho = \frac{\rho^*}{\sqrt{(1 + \delta(1 - (p^*)^2))}} \tag{4.2}$$

where ρ is the unconditional correlation coefficient and ρ^* is the conditional correlation coefficient which is conditioned on the variance of x_t.

By comparing the values of the unconditional correlation coefficient during the crisis period and the entire period through a t-test, we can be more confident in attributing any statistically significant rise in the unconditional correlation coefficient to new cross-market linkages that emerged during the crisis and establish contagion between the two variables.

In the context of natural logarithm of the differenced data on weekly foreign exchange rates, the aforementioned method can be applied using a VAR model to test for contagion from US$ to Euro to the: (A) Japanese Yen to Euro, (B) UK £ to Euro and (C) Swiss Franc to Euro. The unconditional correlation coefficients are compared through a t-test at the 95% confidence level where the test statistic at the critical level is 1.65. The crisis to test for contagion would be the COVID-19 crisis delimited from 1/11/2019 to 31/12/2022. The stable period begins after the 2008 financial crisis from 1/1/2010 to 31/10/2019.

Critical Value:	**Unconditional Correlation Coefficients**			**Test Statistic**	**Contagion**
t-value: 1.65	Stable	Crisis	Full		
(A) Japanese Yen to Euro	0.5896	0.4919	0.5115	1.5987	No
(B) UK £ to Euro	0.4295	0.021	0.0222	20.7913	Yes
(C) Swiss Franc to Euro	0.2469	0.3668	0.3836	3.0406	Yes

Table 4: Unconditional Correlation, Volatility and Test Statistic of (A), (B) and (C) with respect to the US$ to Euro

From Table 6, there appears to be no contagion from US$ to Euro to the (A) Japanese Yen to Euro as the test statistic at 1.5986942 is below the critical value of 1.65 of the 95% confidence level. However, there appears to be contagion from the US$ to Euro to (B) UK £ to Euro and (C) Swiss Franc to Euro and the test statistics of (B) and (C) are 20.791283 and 3.0406132 respectively. As they are both above the critical value of 1.65, there appears to be contagion at the 95% confidence level. Unconditional Correlation, Volatility and Test Statistic of (A), (B) and (C) with respect to the US$ to Euro

Reviewing the data and implementation of the methodology, further improvements and adjustments could be made to heighten our certainty of our conclusions above. During the construction of the VAR model, it should be noted that the VAR models failed to pass the normality test. This could be due to the outliers that still exist even with the differenced weekly data. As such, further differencing could be made for the data to be stationary for a better VAR model.

4B. Frey, Martin, and Tang (2010)

The contagion test described in the paper by Frey, Martin, and Tang (2010) centres on detecting significant shifts in the co-skewness between asset returns across pre-crisis and crisis periods. Co-skewness reflects how one asset's returns are asymmetrically related to the squared returns of another, and understanding this relationship is crucial for identifying contagion effects. The paper outlines two variants of the test determined using either returns or squared returns in computing co-skewness. These tests, labelled CS1 and CS2, are designed to ascertain if the onset of a crisis influences the co-movement of asset returns. Specifically, CS1 evaluates the change in co-skewness involving one asset's returns and the other's squared returns, and CS2 addresses the inverse relationship. The computation of co-skewness involves the average of the product of deviations from the mean returns, each standardized by their respective standard deviations.

In the exploration of financial contagion effects, two methodological approaches were employed: direct analysis using actual return data and indirect analysis utilizing the residuals from a Vector Autoregression (VAR) model. These methods serve distinct purposes and offer different insights into the dynamics of financial markets during periods of crisis.

Direct analysis offers simplicity and an intuitive grasp of market volatility, making it straightforward to implement. However, it struggles to differentiate between correlations from market fundamentals and genuine contagion effects. On the other hand, indirect analysis through VAR model residuals allows for a detailed understanding of contagion by accounting for market interdependencies and isolating effects beyond fundamentals. This approach, while offering a more refined analysis, introduces complexity in model specification and dependency on parameter selection, which can influence outcomes.
Upon applying both methods to analyse the contagion effect, it was observed that the statistical outcomes differed between the two approaches, highlighting the distinct analytical frameworks they employ. However, the conclusion regarding whether to accept or reject the hypothesis of contagion remained consistent across both methodologies. This consistency suggests that, despite the differing statistical magnitudes of each approach, they converge on a similar conclusion regarding the fundamental question of contagion. It underscores the utility of employing multiple analytical perspectives to make our conclusions stronger.

Critical value: χ_1^2= 3.8415 @ 95%	**CS1**		**CS2**	
	Return data	Residuals	Return data	Residuals
Dlp2w	0.1459	0.27888	0.2038	0.1089
Dlp3w	1.0470	1.5229	11.7012	16.3483
Dlp4w	28.3841	39.7636	0.0921	0.0848

Table 5: Result Statistics for Contagion Tests based on Changes in Co-skewness

Table 7 presents the results of contagion tests using both return data and residuals for variables Dlp2w, Dlp3w, and Dlp4w, evaluated against a critical value of 3.8414588. Contagion test results reveal no significant evidence of contagion from the US Dollar to Euro rate to the Japanese Yen to Euro rate across both CS1 and CS2 tests. However, for the UK £ to Euro rate, while the CS1 test indicates no contagion, the CS2 test shows a significant contagion effect in the residuals. In contrast, the Swiss Franc to Euro rate demonstrates a strong contagion effect in the CS1 test with both returns and residuals, significantly exceeding the critical threshold.

In financial contagion analysis, both correlation and co-skewness tests serve distinct purposes. Correlation-based methods measure how closely the returns of two assets align. Changes in correlations during a crisis can be an intuitive indicator of contagion effects. However, correlations do not sensitively reflect changes in volatility and may not capture extreme events or asymmetric risks well. On the other hand, co-skewness -based methods assess the asymmetric co-movements in asset returns. These methods are particularly useful for capturing changes in relationships during extreme market movements. Co-skewness can provide information that correlation alone does not capture, especially regarding how tail risks in asset returns change during a crisis. While both papers have proposed important improvements to methodology of contagion testing, the method proposed by Frey, Martin and Tang provides a more sophisticated approach as it attempts to account for more underlying mechanisms such as market liquidity that drive the contagion on top of the market-specific or global factors that may influence correlations between two markets during a crisis proposed by Forbes.

ALTERNATIVE MEASURES OF VOLATILITY

The use of alternative volatility measures is an important consideration when working with high-frequency data. Existing literature suggests there are significant benefits to using alternative dispersion measures in forecasting via stochastic volatility models. It is found by Blair, Poon, and Taylor (2001) that the VIX index is a coherent measure of volatility as evidenced through robust ARCH results and high degree of correlation between the VIX implied volatility and the realised volatility measured during the same periods. Andersen and Benzoni (2008) extend the discussion to a more generalised framework by developing concepts around the relation between conditional return variance against and their quadratic variation and realised volatility.

The study conducted by Blair, Poon, and Taylor (2001) demonstrates that using the VIX index to forecast volatility yields more accurate results compared to any conditional variance estimation method and choice of time series frequency. The VIX index naturally contains a greater amount of information pertaining to leverage effects and market perception by virtue of the VIX being a measure of implied volatility in the aggregated options market. It is also found that measures of realised volatility generally outperform the traditional counterparts, especially as a leading indicator of future volatility.

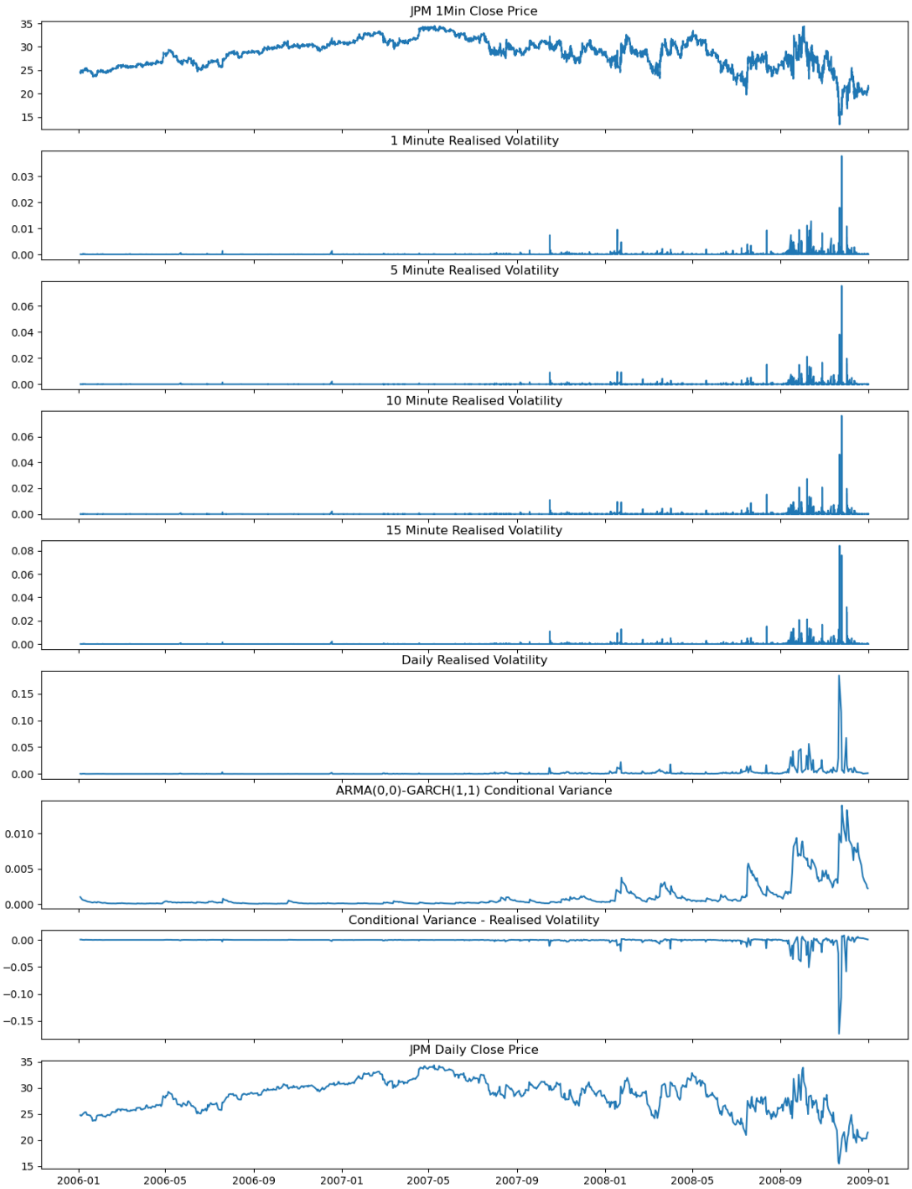

Figure 3: JPM Prices, Realised Volatilities, and ARMA(0,0)-GARCH(1,1) Daily Conditional Variance

The intuition behind realised volatility computed from high-frequency returns is that the data is closer to a continuous time representation. Intraday frequencies for long time series are considerably more granular than traditional approaches and will capture shorter-term dynamics of the data. Naturally, there are limitations in the fact that such high frequency will potentially also capture unwanted elements such as, microstructure noise, herding effects, or other idiosyncratic disruptions to the information content of the data.

In an intraday frequency setting, one possible measure of realised volatility is that of the squared return. In this study, all returns have been computed via the differencing of log prices to maintain their additive properties. Subsequently, the squared market returns can be summed to compute the desired frequency of realised volatility.

$$\sigma_n^2 = \sum_{i=1}^{m} r_i^2 \tag{5.1}$$

Namely, an n period realised volatility can be computed by the summation of all m period squared returns within the n period. It is purported that measuring volatility at this frequency will increase the information content extracted compared to lower-frequency traditional data collection methods. This technique of computing realised volatility is possible by virtue of the assumption that returns follow the dynamic of a random walk process with no drift. Reiteratively, this method is only valid given that returns are the first difference of log levels.

Using this method to compute realised volatility, an experimental application has been conducted to evaluate the quality of this measure. Intraday data obtained from AlphaVantage was first assessed for its quality in terms of data completeness and accuracy. While it was possible to download more than twenty years at a time, it was found that many securities had issues around missing data. A sample of JPMorgan intraday minute-frequency data between 2006 and 2009 was found to be of the highest completeness of the samples inspected. However, an interesting feature which is not present in this sample is that of jumps in the data. A feature which was present in other data samples which were not studied due to large amounts of missing data.

The prices were first interpolated at minute frequency using the linear method $y = \frac{y_0(x_1-x)+y_1(x-x_0)}{x_1-x_0}$ to ensure the completeness and synchronicity of data. While this application was strictly univariate, the interpolation is still considered due to a surplus of dispersion created by the series being treated as properly discretised without accounting for missing points.

The logs of these levels are then differenced to compute one-minute returns for the series. Squaring these returns then yields the realised volatility for that miunte. The additive properties of log returns and the associated no-drift random walk assumptions allow us to compute realised volatility for any period via summation. The five, ten, fifteen minute, and daily realised volatilities have been computed and presented in figure 3. It is also presented in figure 3 that the realised volatilities can potentially model short bursts of high volatility relating to abnormal price movements. Furthermore, the realised volatility seems to lead extreme short-term abnormal returns. For instance, on the 17th of July, 2008, the sharp increase in the level is not percieved greatly by the realised volatility measure. However, the highly volatile period starting in September 2008 seems to be predicted by the realised volatility measure by virture of the leading increase in dispersion. The $ARMA(p,q) - GARCH(p,q)$ model – aginst which the realised volatility measure is asessed – exhibits similar properties with one notable distiction. The realised volatility is especially persistent leading up to the market carsh with its trough on the 21st of November 2008. The overall magnitude of variation is similar between the models. However, there is a considerably higher degree of persistence in the conditional volatility measured by the $ARMA(0,0) - GARCH(1,1)$ model. This information can be useful in modelling short-term extreme shocks in the series being analysed. In that respect, the model might have potential applications in modelling jumps for stochastic volatility models.

In order to gauge whether our realised volatility is similar to the conditional variances of the daily returns, we performed a $GARCH$ test – which estimated a $ARMA(0,0) - GARCH\ (1,1)$ for both the student's t and normal distributions – that was well specified in terms of no serial correlations in residuals, statistical significance of estimated parameters, and with no stationarity as $\alpha + \beta \nleq 1$ and non-normality of residuals. Based on the BIC, we chose the student t-distribution (-4.98) over the normal (-4.92).

Variable	Coefficient	Std. Error
Cst(M)	0.000821	0.00052731
Cst(V) x 10^4	0.067645	0.03452
ARCH(Alpha1)	0.173502	0.041712
GARCH(Beta1)	0.832814	0.036735

Table 6: Coefficients and Standard Errors of the ARMA(0,0)-GARCH(1,1) Conditional Variance Estimation

Additional accompanying materials for the entire report can be found here