Brute-forcing Monte Carlo Simulation

Radu Briciu
BSc Finance (Hons)
University of Westminster

City, University of London
Faculty of Finance

February 1, 2025

Overview

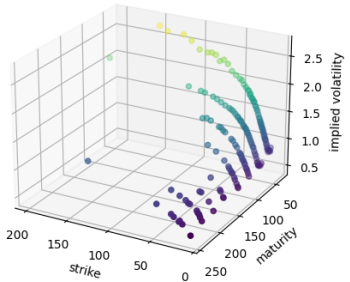
In this session we explore a proposed method for estimating nonlinear stochastic functions in the context of path dependent financial derivatives

1. Financial Derivatives

2. Remedy

Financial Derivatives

What are financial derivatives?



Financial Derivatives

We can begin thinking about the problem by advancing on the illustrious concept of the Myron Black and Fischer Scholes pricing model for plain European call options.

$$C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2) \tag{1}$$

$$\tag{2}$$

where

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \quad \text{and} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{4}$$

Financial Derivatives

Problems with the classical Black-Scholes model (as presented):

- Constant volatility
- Arbitrage free
- Instantaneous returns
- No dividends
- Cannot price complex financial derivatives

The Heston (1993) Model

- Geometric Brownian Motion
- Arbitrary correlation between asset volatility and return

$$dS_t = \left(r - \frac{v_t}{2}\right) dt + \sqrt{v_t}\left(\rho dW_t + \sqrt{1-\rho^2} dB_t\right) \tag{5}$$

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t} dW_t \tag{6}$$

1. v_0 represents the initial variance,
2. θ is the long-run variance,
3. ρ is the correlation between the log-price process and its volatility,
4. κ is the mean reversion of the variance to θ,
5. η is the volatility of the variance process, and
6. B_t, W_t are continuous random walks.

Applications

Calibration of the Heston (1993) model permits usage of more sophisticated pricing algorithms in determining a correct asset price. We are able to leverage path-dependent dynamics of the underlying price to create more sophisticated financial derivative products. One such example is the *Asian* option:

$$C_t^{\text{Asian}} = e^{-r(T-t)} \times \frac{1}{m} \sum_{i=1}^{m} (S_T^{\text{avg}} - K)^+ \tag{7}$$

$$\tag{8}$$

where S_T^{avg} is the average price of the underlying spot price

References

[1] Radu Briciu. "Estimating non-linear stochastic functions from generated structured data using multi-layer perceptron models with applications to pricing path-dependent financial derivatives". In: (Jan. 2025). DOI: 10.2139/ssrn.5104328. URL: https://papers.ssrn.com/abstract=5104328 (visited on 01/21/2025).

The End