Laboratory Project for a search course in artificial intelligence - with Prof. Ariel Felner

IDA* vs. A*

8 Puzzle Game

Radwan Ganem 322509951

radwan@post.bgu.ac.il 054-7236777

Introduction

8 Puzzle Game: The 8 puzzle is a sliding puzzle. It has 8 square tiles numbered 1 to 8 in a frame that is 3 tile positions high and 3 tile positions wide, with one unoccupied position. Tiles in the same row or column of the open position can be moved by sliding them horizontally or vertically, respectively. The goal of the puzzle is to place the tiles in numerical order (from left to right, top to bottom).

For Example:

Initial State

1	2	3
8		4
7	6	5

Goal State

	1	2	3
>	4	5	6
	7	8	

More Explanation in this link:

https://www.almabetter.com/bytes/tutorials/artificial-intelligence/8-puzzle-problem-in-ai

You Can also try and play this game here: https://sliding.toys/mystic-square/8-puzzle/

Algorithms

1. A* is a graph traversal and pathfinding algorithm, which is used in many fields of computer science due to its completeness, optimality, and optimal efficiency. Given a weighted graph, a source node and a goal node, the algorithm finds the shortest path (with respect to the given

weights) from source to goal. Peter Hart, Nils Nilsson and Bertram Raphael of Stanford Reasearch Institute published the algorithm in 1968. A* has better performance with the help of heuristics. it can be also seen as an extension of Dijkstra's algorithms.

A* is an informed search algorithm, or a best-first search, meaning that it is formulated in terms of weighted graphs: starting from a specific starting node of a graph, it aims to find a path to the given goal node having the smallest cost (least distance travelled, shortest time, etc.). It does this by maintaining a tree of paths originating at the start node and extending those paths one edge at a time until the goal node is reached.

At each iteration of its main loop, A* needs to determine which of its paths to extend. It does so based on the cost of the path and an estimate of the cost required to extend the path all the way to the goal. Specifically, A* selects the path that minimizes f(n) = g(n) + h(n), where n is the next node on the path, g(n) is the cost of the path from the start node to n, and h(n) is a heuristic function that estimates the cost of the cheapest path from n to the goal. The heuristic function is problem-specific. If the heuristic function is admissible – meaning that it never overestimates the actual cost to get to the goal – A* is guaranteed to return a least-cost path from start to goal.

Algorithm Complexity: let b – average branching factor, d – depth of graph from initial state to goal state

Time Complexity: O(bd)

Space Complexity: O(bd)

Completeness: Yes

Optimality: Yes

Source: https://en.wikipedia.org/wiki/A* search algorithm

2. Iterative Deepening A* / IDA* is a graph traversal and path search algorithm that can find the shortest path between a designated start node and any member of a set of goal nodes in a weighted graph. It is a variant of iterative deepening depth-first search that borrows the idea to use a heuristic function to conservatively estimate the remaining cost to get to the goal from the A* search algorithm. Since it is a depth-first search algorithm, its memory usage is lower than in A*, but unlike ordinary iterative deepening search, it concentrates on exploring the most promising nodes and thus does not go to the same depth everywhere in the search tree. Unlike A*, IDA* does not utilize dynamic programming and therefore often ends up exploring the same nodes many times. While the standard iterative deepening depth-first search uses search depth as the cutoff for each iteration, the IDA* uses the more informative f(n) = g(n)+h(n), where g(n) is the cost to travel from the root to node n and h(n)) is a problem-specific heuristic estimate of the cost to travel from n to the goal. The algorithm was first described by Richard Korf in 1985. Iterative-deepening-A* works as follows: at each iteration, perform a depth-first search, cutting off a branch when its total cost f(n) = g(n) +h(n), exceeds a given threshold. This threshold starts at the estimate of the cost at the initial state, and increases for each iteration of the algorithm. At each iteration, the threshold used for the next iteration is the minimum cost of all values that exceeded the current threshold. Like A*, IDA* is guaranteed to find the shortest path leading from the given start node to any goal node in the problem graph, if the heuristic function h is admissible, that is $h(n) \le h*(n)$ for all nodes n, where h* is the true cost of

the shortest path from n to the nearest goal (the "perfect heuristic").

IDA* is beneficial when the problem is memory constrained. A* search

keeps a large queue of unexplored nodes that can quickly fill up memory.

By contrast, because IDA* does not remember any node except the ones

on the current path, it requires an amount of memory that is only linear in

the length of the solution that it constructs. Its time complexity is

analyzed by Korf et al. under the assumption that the heuristic cost

estimate h is consistent, meaning that $h(n) \le \cos(n, n') + h(n')$ for all

nodes n and all neighbors n' of n; they conclude that compared to a brute-

force tree search over an exponential-sized problem, IDA* achieves a

smaller search depth (by a constant factor), but not a smaller branching

factor.

Algorithm Complexity: let b – average branching factor, d – depth of graph

from initial state to goal state

Time Complexity: O(b^d)

Space Complexity: O(b * d)

Completeness: Yes

Optimality: Yes

Source: https://en.wikipedia.org/wiki/Iterative_deepening_A*

- Misplaced Heuristic: The number of tiles that are not in their goal position. Except the empty tile or zero tile.
- 4. Manhattan Distance: The Manhattan distance heuristic is used not only for its simplicity but also for its ability to estimate the number of moves required to bring a given puzzle state to the solution state. Manhattan distance is simply computed by the sum of the distances of each tile from where it should belong. or the sum of the absolute values of the horizontal and vertical distances of the tiles from their goal positions.
 Except the empty tile or zero tile.

The Purpose of the experiment

in this experiment I will test the hypothesis: on 8 puzzle game IDA* is more memory efficient than A* and A* is faster than IDA* and Heuristic Manhattan distance is better than misplaced. I will test this by comparing the number of vertices, vertices explored, time taken, number of duplicate vertices detected/undetected, and vertices saved in memory through the whole search.

Description of the experiment

Code overview: full code is at the end of the document it include a classes:

Game_8_Puzzle, A_Star and IDA_Star and functions for executing and plotting graphs. Also full code with results can be found here:

https://colab.research.google.com/drive/1c8Z37rOuRrfo8YOi0kYFRDmUjfZiMcQ C?usp=sharing

Experimental Setup

Runs: Each experiment was run on at least 10 initial puzzle states generated randomly to ensure accuracy and few initial state that include easy, medium and hard levels of initial puzzle.

Environment: The experiment were conducted on Google Colab notebook, Python 3 Google Compute Engine Backend, Ram: 12.7 GB.

The 8 puzzle problem was solved using both A* and IDA* algorithms.

Important Note: Not all 8 puzzles are solvable, I will only use solvable puzzles

For an initial state, 8 puzzle is not possible to solve an instance of 8 puzzle if

number of invertions is odd in the initial state input.

Source: https://www.geeksforgeeks.org/check-instance-8-puzzle-solvable/

Results

Figure 1:

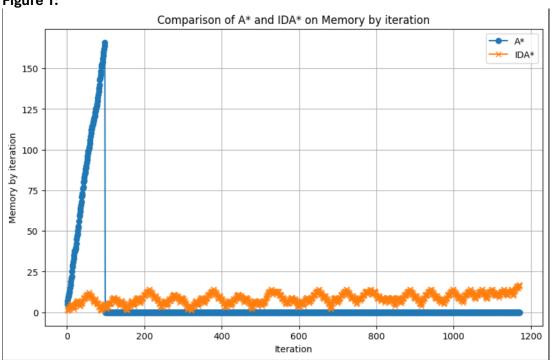


Figure 2:

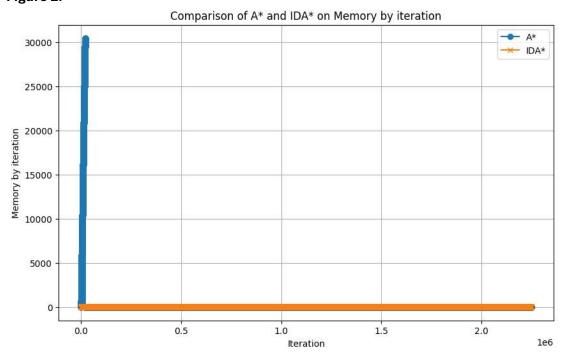


Figure 1 showcase the **behavior of memory usage** by iteration for both algorithms A* and IDA* with Manhattan distance on initial input: [[1,3,5], [7,0,8], [2,6,4]] that it's optimal solution is 16, it can be shown that A* took a lot less iterations to find optimal solution and used a lot more memory than IDA*, Figure 2 is the same but with more

extreme initial input: [[6,4,7],[8,5,0],[3,2,1]] that takes the most moves to solve optimally with 31 move this initial input is taken from here:

https://www.cs.princeton.edu/courses/archive/spring19/cos226/assignments/8puzzle/c
hecklist.php, it can be shown from the two figures that A* is faster and using a lot more
memory as it keeps running through the iterations than IDA* on their path to find the
optimal solution for a given initial input.

Figure 3.a:

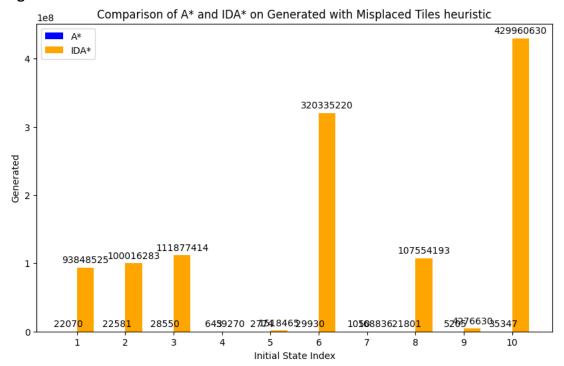


Figure 3.b:

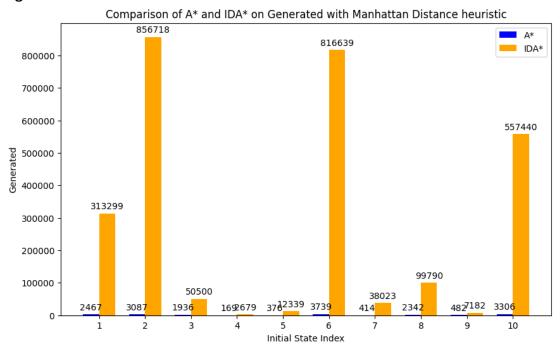


Figure 3.a show the number of Generated Vertices on given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 3.b show the same but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states puzzle. it can be seen that with A* expand less vertices than IDA* and using Manhattan Distance both algorithms expand less vertices for all initial states.

Figure 4.a:

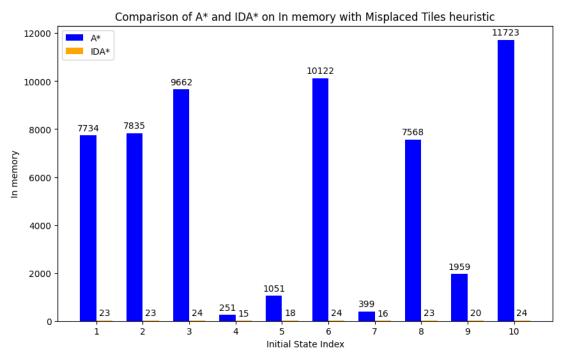


Figure 4.b:

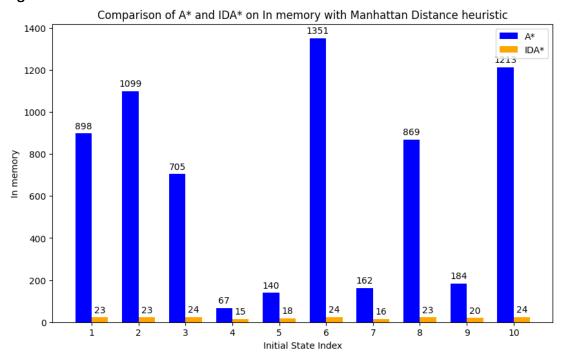


Figure 4.a show the number of in memory (number of states saved until reaching the optimal solution) Vertices on given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 4.b show the same but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states

puzzle. it can be seen that with A* use more memory than IDA* ,and using

Manhattan Distance A* used less memory than using Misplaced and IDA* memory

usage stayed similar for both heuristics.

Figure 5.a:

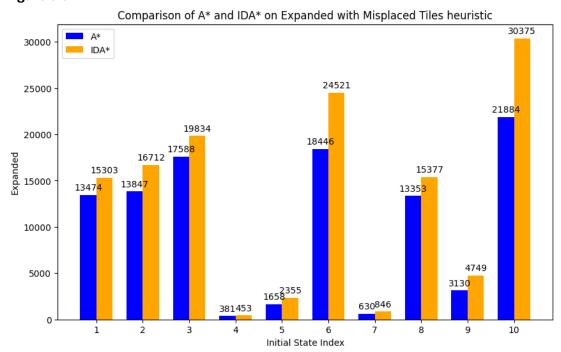


Figure 5.b:

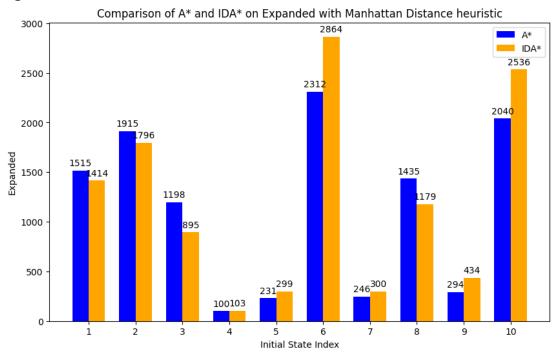


Figure 5.a show the number of Expanded Vertices on given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 5.b show the same

but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states puzzle. it can be seen that with A* Expanded less vertices than IDA* with Misplaced ,and using Manhattan Distance A* and IDA* both expanded less than using Misplaced and for initial states 1, 2, 3, 8 A* expanded more than IDA* and for initial states 4,5,6,7,9,10 IDA* expanded more than A*.

Figure 6.a:

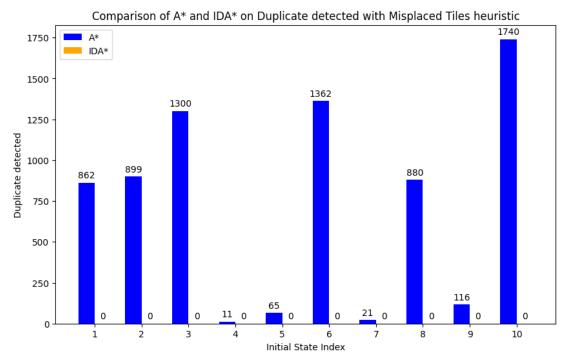


Figure 6.b:

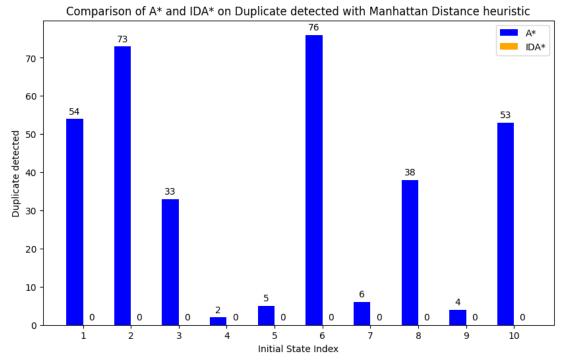


Figure 6.a show the number of duplicate Vertices detected and ignored on given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 6.b show the same but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states puzzle. it can be seen that IDA* doesn't detect

any duplicates using both heuristics and A* detected on both heuristic, and using Manhattan Distance A* detected less vertices (encountered less duplicates).

Figure 7.a:

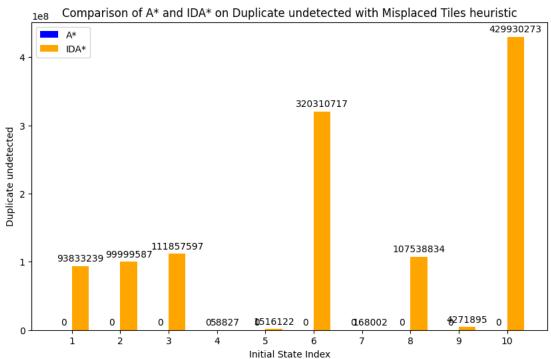


Figure 7.b:

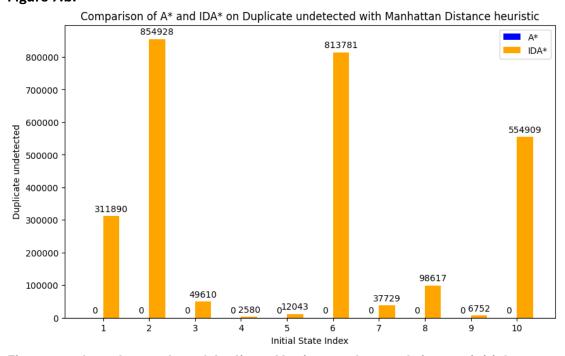


Figure 7.a show the number of duplicate Vertices undetected given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 7.b show

the same but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states puzzle. it can be seen that IDA* doesn't detect any duplicates using both heuristics and IDA* undetected less vertices using Manhattan distance, A* doesn't encounter any duplicate nodes that are undetected by the algorithm with both heuristics.

Figure 8.a:

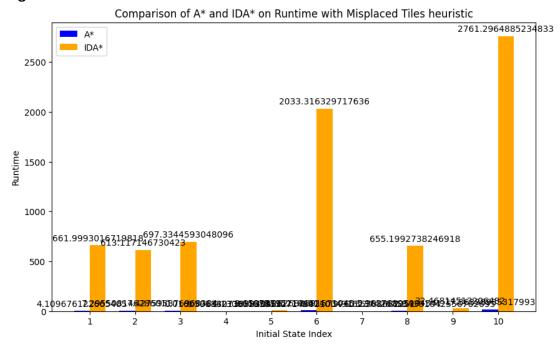


Figure 8.b:

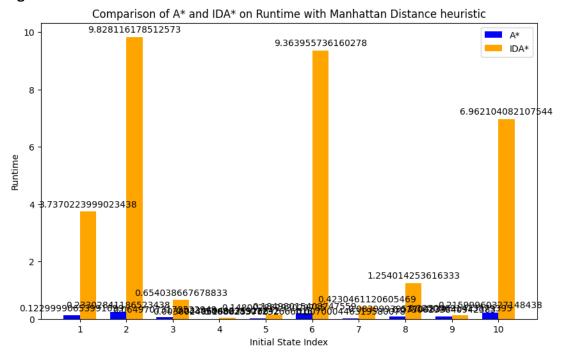
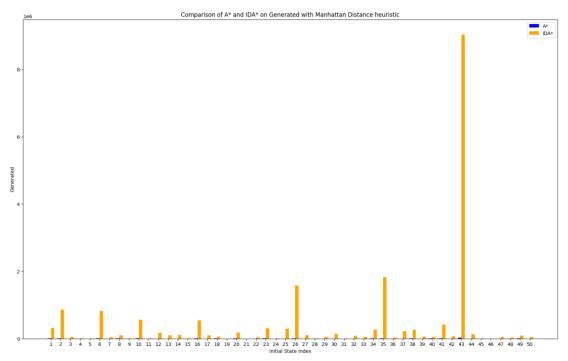


Figure 8.a show the runtime in ms on given 10 initial states with the algorithms A* and IDA* using the misplaced heuristic, and Figure 8.b show the same but with Manhattan Distance Heuristic, both graphs showcase the same 10 initial states puzzle. it can be seen that A* is faster than IDA* using both heuristics and that both A* and IDA* are faster using Manhattan Distance than using Misplaced.

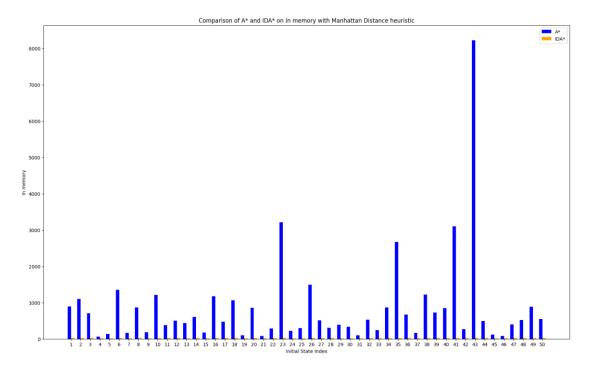
To ensure accuracy the following figures showcase IDA* vs A* on 50 different initial states with Manhattan distance heuristic

Figure 9:



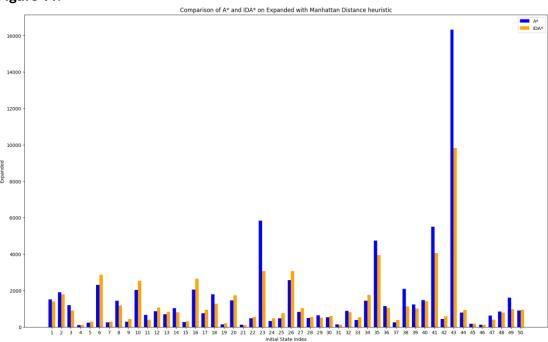
It is shown that IDA* generates more than A* with Manhattan Distance.

Figure 10:



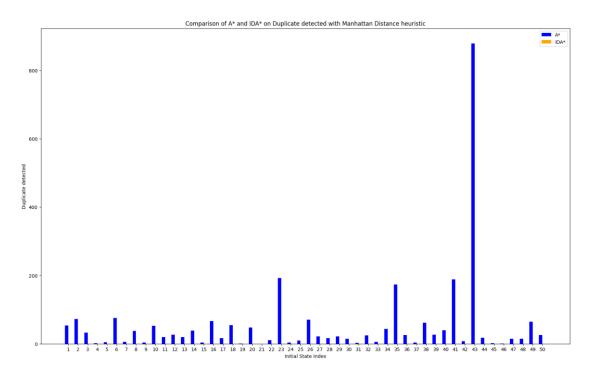
It is shown that A* uses more memory than IDA* with Manhattan Distance.

Figure 11:



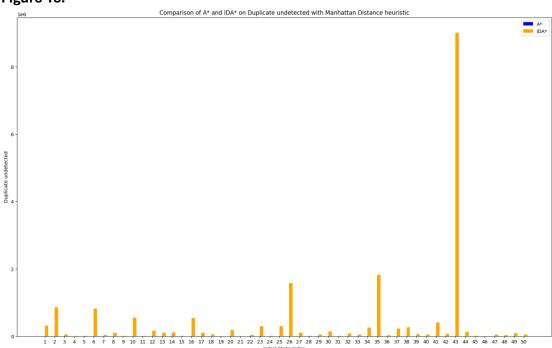
It is shown that A* expand more vertices than IDA* on initial input indexes 1,2,3,8,11,14,18,21,23,29,31,32,35,36,38,39,40,41,43,45,47,48,49. And the rest IDA* expanded more than A*.

Figure 12:



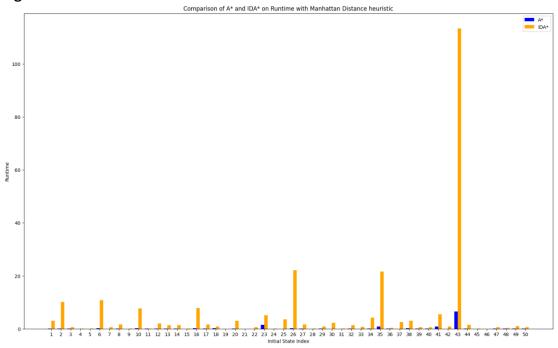
It is shown that A* detected duplicate and IDA* didn't detect any at all.

Figure 13:



It is shown that A* didn't encounter any undetected vertices at all and IDA* did.

Figure 14:



It is shown that IDA* took more time to find the solution in ms compared to A*.

Conclusions

After evaluating and comparing A* vs. IDA* with misplaced and Manhattan distance heuristics for solving 8 puzzle game. I can conclude from the results and graphs that my hypothesis was correct. Manhattan distance is more efficient than Misplaced (if we look at the number of states it explores and runtime before arriving at a solution), and The most efficient runtime algorithm is A* with Manhattan distance heuristic and the most efficient memory algorithm is IDA* with Manhattan distance heuristic. Also I avoided unnecessary runtime errors by checking first the 8 – puzzle initial state is solvable. Also A* outperforms IDA* because A* have open and closed lists so that a given state is not explored multiple times through the graph whereas in IDA* if there are multiple paths to a given state in the graph it will explore those states again and again.

References

A* - https://en.wikipedia.org/wiki/A*_search_algorithm

IDA* - https://en.wikipedia.org/wiki/Iterative_deepening_A*

8 puzzle solvable - https://www.geeksforgeeks.org/check-instance-8-puzzle-

solvable/

8 puzzle – https://www.almabetter.com/bytes/tutorials/artificial-intelligence/8-

puzzle-problem-in-ai

8 puzzle online game – https://sliding.toys/mystic-square/8-puzzle/

Examples of 8 puzzle initial states and their optimal solution –

https://www.cs.princeton.edu/courses/archive/spring19/cos226/assignments/8

puzzle/checklist.php

Appendex

Initial states Inputs: (this file was generated using the program)



- 1	\mathbf{r}	n	1 11
- 1			
		r	u

3 4 1

3 4 6

0 4 8

8 1 3

8 4 2

0 4 8

3 4 2

8 4 7

0 1 8

2 1 0

6 1 4

4 1 8

8 4 1

8 1 3

4 3 1

3 2 6

5 4 8

3 4 5

5 3 4

3 4 5

3 5 1

3 1 0

4 1 8

3 1 6

7 1 0

Output values for A* and IDA* with Misplaced, Manhatten Distance: (this file

was generated using the program)



Initial_	Algo	Heuri	Generated	Vertices_I	Expanded	Duplicate	Duplicate_	Run_
State	rithm	stic	_Vertices	n_Memory	_Vertices	_Detected	Undetected	Time
673		Mispl						
1 4 8		aced						4.10
502	A*	Tiles	22070	7734	13474	862	0	9676
673		Mispl						
1 4 8		aced						661.
502	IDA*	Tiles	93848525	23	15303	0	93833239	9993
		Manh						
673		attan						
1 4 8		Dista						0.12
5 0 2	A*	nce	2467	898	1515	54	0	3
		Manh						
673		attan						
1 4 8		Dista						3.73
5 0 2	IDA*	nce	313299	23	1414	0	311890	7022
2 1 6		Mispl						
074		aced						7.29
3 5 8	A*	Tiles	22581	7835	13847	899	0	5508
2 1 6		Mispl						
074		aced						613.
3 5 8	IDA*	Tiles	1E+08	23	16712	0	99999587	1171

		Manh						
216		attan						
074		Dista						0.23
358	A*	nce	3087	1099	1915	73	0	3028
		Manh						
216		attan						
074		Dista						9.82
358	IDA*	nce	856718	23	1796	0	854928	8116
6 1 7		Mispl						
234		aced						7.42
850	A*	Tiles	28550	9662	17588	1300	0	7595
617		Mispl						
234		aced						697.
850	IDA*	Tiles	1.12E+08	24	19834	0	1.12E+08	3345
		Manh						
6 1 7		attan						
234		Dista						0.06
850	A*	nce	1936	705	1198	33	0	4971
		Manh						
6 1 7		attan						
234		Dista						0.65
850	IDA*	nce	50500	24	895	0	49610	4039
3 4 1		Mispl						
025		aced						0.01
786	A*	Tiles	643	251	381	11	0	6039

3 4 1		Mispl						
025		aced						0.50
786	IDA*	Tiles	59270	15	453	0	58827	6842
		Manh						
3 4 1		attan						
025		Dista						0.00
786	A*	nce	169	67	100	2	0	3002
		Manh						
3 4 1		attan						
025		Dista						0.03
786	IDA*	nce	2679	15	103	0	2580	3969
152		Mispl						
807		aced						0.13
3 4 6	A*	Tiles	2774	1051	1658	65	0	0012
152		Mispl						
8 0 7		aced						9.65
3 4 6	IDA*	Tiles	1518465	18	2355	0	1516122	8778
		Manh						
152		attan						
807		Dista						0.00
3 4 6	A*	nce	376	140	231	5	0	8029
		Manh						
152		attan						
807		Dista						0.14
3 4 6	IDA*	nce	12339	18	299	0	12043	8003

0 4 8		Mispl						
126		aced						8.26
5 3 7	A*	Tiles	29930	10122	18446	1362	0	2683
0 4 8		Mispl						
126		aced						2033
5 3 7	IDA*	Tiles	3.2E+08	24	24521	0	3.2E+08	.316
		Manh						
0 4 8		attan						
126		Dista						0.18
5 3 7	A*	nce	3739	1351	2312	76	0	498
		Manh						
0 4 8		attan						
126		Dista						9.36
5 3 7	IDA*	nce	816639	24	2864	0	813781	3956
7 1 3		Mispl						
452		aced						0.02
086	A*	Tiles	1050	399	630	21	0	1005
713		Mispl						
452		aced						0.97
086	IDA*	Tiles	168836	16	846	0	168002	6055
		Manh						
7 1 3		attan						
4 5 2		Dista						0.00
086	A*	nce	414	162	246	6	0	7
713								
4 5 2		Manh						0.42
086	IDA*	attan	38023	16	300	0	37729	3046

		Dista						
		nce						
6 3 1		Mispl						
470		aced						5.96
582	A*	Tiles	21801	7568	13353	880	0	3769
6 3 1		Mispl						
470		aced						655.
582	IDA*	Tiles	1.08E+08	23	15377	0	1.08E+08	1993
		Manh						
6 3 1		attan						
470		Dista						0.08
582	A*	nce	2342	869	1435	38	0	3999
		Manh						
6 3 1		attan						
470		Dista						1.25
582	IDA*	nce	99790	23	1179	0	98617	4014
0 5 4		Mispl						
712		aced						0.34
386	A*	Tiles	5205	1959	3130	116	0	301
054		Mispl						
712		aced						32.4
386	IDA*	Tiles	4276630	20	4749	0	4271895	6815
		Manh						
0 5 4		attan						
712		Dista						0.07
386	A*	nce	482	184	294	4	0	3002

		Manh						
0 5 4		attan						
712		Dista						0.12
386	IDA*	nce	7182	20	434	0	6752	5007
6 3 4		Mispl						
107		aced						17.3
285	A*	Tiles	35347	11723	21884	1740	0	6399
6 3 4		Mispl						
107		aced						2761
285	IDA*	Tiles	4.3E+08	24	30375	0	4.3E+08	.296
		Manh						
6 3 4		attan						
107		Dista						0.21
285	A*	nce	3306	1213	2040	53	0	6
		Manh						
6 3 4		attan						
107		Dista						6.96
285	IDA*	nce	557440	24	2536	0	554909	2104

Output values for A* vs IDA* with Manhattan Distance: (this file was generated using the program)



Initial_	Algor	Heuri	Generated	Vertices_In	Expanded	Duplicate_	Duplicate_U	Run_
State	ithm	stic	_Vertices	_Memory	_Vertices	Detected	ndetected	Time
		Manh						
673		attan						
1 4 8		Dista						0.082
502	A*	nce	2467	898	1515	54	0	007
		Manh						
673		attan						
1 4 8		Dista						3.042
502	IDA*	nce	313299	23	1414	0	311890	271
		Manh						
2 1 6		attan						
074		Dista						0.113
3 5 8	A*	nce	3087	1099	1915	73	0	003
		Manh						
2 1 6		attan						
074		Dista						10.18
3 5 8	IDA*	nce	856718	23	1796	0	854928	695
		Manh						
6 1 7		attan						
2 3 4		Dista						0.079
8 5 0	A*	nce	1936	705	1198	33	0	034

	1			I	ı	ı		
		Manh						
6 1 7		attan						
2 3 4		Dista						0.617
8 5 0	IDA*	nce	50500	24	895	0	49610	011
		Manh						
3 4 1		attan						
0 2 5		Dista						0.003
786	A*	nce	169	67	100	2	0	006
		Manh						
3 4 1		attan						
0 2 5		Dista						0.031
786	IDA*	nce	2679	15	103	0	2580	991
		Manh						
152		attan						
807		Dista						0.006
3 4 6	A*	nce	376	140	231	5	0	997
		Manh						
152		attan						
807		Dista						0.143
3 4 6	IDA*	nce	12339	18	299	0	12043	007
		Manh						
0 4 8		attan						
126		Dista						0.240
5 3 7	A*	nce	3739	1351	2312	76	0	003
0 4 8								
126		Manh						10.77
5 3 7	IDA*	attan	816639	24	2864	0	813781	711
	l		<u> </u>	I	l	l .	<u> </u>	

		Dista						
		บเรเส						
		nce						
		Manh						
7 1 3		attan						
4 5 2		Dista						0.010
086	A*	nce	414	162	246	6	0	003
		Manh						
7 1 3		attan						
4 5 2		Dista						0.593
086	IDA*	nce	38023	16	300	0	37729	037
		Manh						
6 3 1		attan						
470		Dista						0.115
582	A*	nce	2342	869	1435	38	0	973
		Manh						
6 3 1		attan						
470		Dista						1.583
582	IDA*	nce	99790	23	1179	0	98617	049
		Manh						
0 5 4		attan						
712		Dista						0.012
386	A*	nce	482	184	294	4	0	001
		Manh						
0 5 4		attan						
712		Dista						0.132
386	IDA*	nce	7182	20	434	0	6752	999
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		Manh						
6 3 4		attan						
107		Dista						0.217
285	A*	nce	3306	1213	2040	53	0	005
		Manh						
6 3 4		attan						
107		Dista						7.656
285	IDA*	nce	557440	24	2536	0	554909	117
		Manh						
6 4 5		attan						
207		Dista						0.030
8 1 3	A*	nce	1057	379	658	20	0	002
		Manh						
6 4 5		attan						
207		Dista						0.075
8 1 3	IDA*	nce	6116	22	398	0	5721	97
		Manh						
5 7 3		attan						
8 4 2		Dista						0.041
106	A*	nce	1393	506	860	27	0	029
		Manh						
573		attan						
8 4 2		Dista						2.036
106	IDA*	nce	167080	21	1063	0	166022	995
6 2 5								
7 1 3		Manh						0.097
0 4 8	A*	attan	1153	436	697	20	0	033
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	1		1	1		4	I	1
		Dista						
		nce						
		Manh						
6 2 5		attan						
7 1 3		Dista						1.355
0 4 8	IDA*	nce	95781	20	828	0	94958	014
		Manh						
086		attan						
3 4 2		Dista						0.062
175	A*	nce	1684	609	1036	39	0	004
		Manh						
086		attan						
3 4 2		Dista						1.432
175	IDA*	nce	111518	22	812	0	110711	019
		Manh						
285		attan						
174		Dista						0.009
0 3 6	A*	nce	459	177	278	4	0	979
		Manh						
285		attan						
174		Dista						0.080
0 3 6	IDA*	nce	6416	20	306	0	6114	02
		Manh						
362		attan						
8 4 7		Dista						0.162
150	A*	nce	3295	1177	2051	67	0	969
L	i	L	i	l	L	i	L	

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		Manh						
362		attan						
8 4 7		Dista						7.809
150	IDA*	nce	543132	24	2649	0	540489	096
		Manh						
0 1 8		attan						
753		Dista						0.051
264	A*	nce	1244	471	756	17	0	999
		Manh						
0 1 8		attan						
753		Dista						1.696
264	IDA*	nce	103763	22	938	0	102830	017
		Manh						
8 7 1		attan						
503		Dista						0.235
264	A*	nce	2921	1064	1802	55	0	005
		Manh						
8 7 1		attan						
503		Dista						0.851
264	IDA*	nce	56347	24	1284	0	55067	009
		Manh						
485		attan						
2 1 0		Dista						
763	A*	nce	241	96	144	1	0	0.005
485								
210		Manh						0.026
763	IDA*	attan	1790	17	197	0	1596	139
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		Dista						
		nce						
		Manh						
0 3 5		attan						
782		Dista						0.103
614	A*	nce	2361	856	1457	48	0	867
		Manh						
0 3 5		attan						
782		Dista						3.095
614	IDA*	nce	183460	24	1739	0	181726	037
		Manh						
652		attan						
4 1 8		Dista						0.006
307	A*	nce	211	84	127	0	0	027
		Manh						
652		attan						
418		Dista						0.008
307	IDA*	nce	530	19	107	0	426	972
		Manh						
5 1 7		attan						
683		Dista						0.027
0 4 2	A*	nce	775	286	478	11	0	004
		Manh						
5 1 7		attan						
683		Dista						0.627
0 4 2	IDA*	nce	38942	22	555	0	38391	005

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		Manh						
6 1 7		attan						
8 5 4		Dista						1.497
230	A*	nce	9250	3216	5841	193	0	045
		Manh						
6 1 7		attan						
8 5 4		Dista						5.095
230	IDA*	nce	300488	28	3076	0	297418	663
		Manh						
0 6 4		attan						
127		Dista						0.023
583	A*	nce	554	219	331	4	0	072
		Manh						
0 6 4		attan						
127		Dista						0.100
583	IDA*	nce	7156	20	475	0	6685	36
		Manh						
503		attan						
186		Dista						0.020
2 4 7	A*	nce	787	301	476	10	0	943
		Manh						
503		attan						
186		Dista						3.535
2 4 7	IDA*	nce	292666	19	749	0	291922	346
3 5 1								
078		Manh						0.225
264	A*	attan	4140	1493	2576	71	0	633
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		Dista						
		nce						
		Manh						
3 5 1		attan						
078		Dista						22.07
264	IDA*	nce	1579351	25	3084	0	1576272	477
		Manh						
3 0 2		attan						
8 4 1		Dista						0.057
765	A*	nce	1355	509	824	22	0	999
		Manh						
3 0 2		attan						
8 4 1		Dista						1.672
765	IDA*	nce	98347	21	1043	0	97309	647
		Manh						
8 1 3		attan						
670		Dista						0.023
524	A*	nce	821	308	496	17	0	998
		Manh						
8 1 3		attan						
670		Dista						0.152
5 2 4	IDA*	nce	8965	21	546	0	8423	616
		Manh						
260		attan						
8 4 3		Dista						0.104
175	A*	nce	1056	394	640	22	0	641
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		Manh						
260		attan						
8 4 3		Dista						0.833
175	IDA*	nce	50531	20	507	0	50029	173
		Manh						
4 3 1		attan						
0 2 5		Dista						0.025
876	A*	nce	882	330	537	15	0	997
		Manh						
4 3 1		attan						
0 2 5		Dista						2.326
876	IDA*	nce	142433	19	604	0	141835	23
		Manh						
160		attan						
7 4 2		Dista						0.006
8 3 5	A*	nce	254	101	150	3	0	003
		Manh						
160		attan						
7 4 2		Dista						0.046
8 3 5	IDA*	nce	2815	16	126	0	2692	999
		Manh						
3 2 8		attan						
605		Dista						0.054
174	A*	nce	1430	528	877	25	0	004
3 2 8								
605		Manh						1.324
174	IDA*	attan	79028	22	803	0	78230	016
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		Dista						
		nce						
		Manh						
182		attan						
0 6 4		Dista						0.016
573	A*	nce	624	241	377	6	0	996
		Manh						
182		attan						
0 6 4		Dista						0.799
5 7 3	IDA*	nce	51095	19	529	0	50571	04
		Manh						
1 4 8		attan						
750		Dista						0.109
3 2 6	A*	nce	2353	863	1446	44	0	124
		Manh						
1 4 8		attan						
750		Dista						4.194
3 2 6	IDA*	nce	259050	23	1758	0	257298	927
		Manh						
467		attan						
802		Dista						0.862
1 3 5	A*	nce	7589	2667	4748	174	0	05
		Manh						
467		attan						
802		Dista						21.57
1 3 5	IDA*	nce	1825301	26	3954	0	1821352	325

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5 4 8 Dista Color Color <td< td=""><td></td><td></td><td>Manh</td><td></td><td></td><td></td><td></td><td></td><td></td></td<>			Manh						
3 6 2 A* nce 1832 668 1138 26 0 967 Manh 071 Attan 2015ta 0.331 0.331 0.331 0.331 0.331 0.331 0.331 0.331 0.331 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007	071		attan						
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5 4 8 Dista 0.331 3 6 2 IDA* nce 28745 24 1050 0 27699 054 Manh attan 0.007 0 27699 054 1 2 0 attan 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.007 0.00			Manh						
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1 2 0 attan Dista 0.007 8 7 5 A* nce 414 161 249 4 0 984 1 2 0 Manh attan 2.534 2.534 2.534 2.534 2.534 376 0 218525 03 1 6 7 nce 218895 16 376 0 218525 03 8 0 2 Dista 0.162 0.162 0.162 0.162 0.162 3 4 5 A* nce 263531 24 1133 0 262402 002 6 7 0 Manh Manh 0.130 0.130 0.130	362	IDA*	nce	28745	24	1050	0	27699	054
4 6 3 Dista nce A* nce 414 161 249 4 0 984 Manh attan Manh attan 2.534 8 7 5 IDA* nce 218895 16 376 0 218525 03 Manh attan Manh attan 0.162 3 4 5 A* nce 3370 1223 2085 62 0 002 Manh attan Manh attan 0 262402 002 16 7 Atan nce 263531 24 1133 0 262402 002 6 7 0 Manh Manh 0 0.130			Manh						
8 7 5 A* nce 414 161 249 4 0 984 1 2 0 attan Amath A	120		attan						
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4 6 3 Dista 2.534 8 7 5 IDA* nce 218895 16 376 0 218525 03 Manh Attan 0 0 218525 03 B 0 2 Dista 0.162 0 0.162 A 5 A* nce 3370 1223 2085 62 0 002 Manh attan 0 2.995 0 2.995 0 0 262402 002 6 7 0 0 0 0 0.130 0 0.130 0 0.130			Manh						
8 7 5 IDA* nce 218895 16 376 0 218525 03 Manh attan 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	120		attan						
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8 0 2 Dista 0.162 3 4 5 A* nce 3370 1223 2085 62 0 002 Manh Mattan 0.162 0.162 0 002 002 8 0 2 Dista 0.130 0.130 262402 002 6 7 0 Manh 0.130 0.130			Manh						
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1 6 7 attan 8 0 2 Dista 3 4 5 IDA* nce 263531 6 7 0 Manh 5 3 4 Manh 1 6 7 0 0.130	3 4 5	A*	nce	3370	1223	2085	62	0	002
8 0 2 Dista 2.995 3 4 5 IDA* nce 263531 24 1133 0 262402 002 6 7 0 Manh 0.130			Manh						
3 4 5 IDA* nce 263531 24 1133 0 262402 002 6 7 0 Manh Manh 0.130 0.130	167		attan						
6 7 0	802		Dista						2.995
5 3 4 Manh 0.130	3 4 5	IDA*	nce	263531	24	1133	0	262402	002
	670								
1 2 8 A* attan 1993 731 1235 27 0 085	5 3 4		Manh						0.130
	128	A*	attan	1993	731	1235	27	0	085

		Dista						
		nce						
		Manh						
670		attan						
5 3 4		Dista						0.635
128	IDA*	nce	54912	24	1019	0	53898	211
		Manh						
607		attan						
3 4 5		Dista						0.091
8 1 2	A*	nce	2370	845	1485	40	0	002
		Manh						
607		attan						
3 4 5		Dista						0.588
8 1 2	IDA*	nce	50397	25	1426	0	48975	006
		Manh						
604		attan						
281		Dista						0.898
5 3 7	A*	nce	8798	3103	5506	189	0	01
		Manh						
6 0 4		attan						
281		Dista						5.557
5 3 7	IDA*	nce	415664	27	4057	0	411612	198
		Manh						
6 1 2		attan						
7 5 4		Dista						0.015
8 3 0	A*	nce	718	273	437	8	0	998

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		Manh						
6 1 2		attan						
7 5 4		Dista						0.836
830	IDA*	nce	72981	20	582	0	72404	015
		Manh						
086		attan						
7 2 4		Dista						6.538
3 5 1	A*	nce	25432	8225	16328	879	0	996
		Manh						
086		attan						
7 2 4		Dista						113.3
3 5 1	IDA*	nce	9025262	30	9823	0	9015447	925
		Manh						
3 1 0		attan						
568		Dista						0.036
427	A*	nce	1303	490	795	18	0	800
		Manh						
3 1 0		attan						
5 6 8		Dista						1.491
427	IDA*	nce	127462	22	938	0	126530	417
		Manh						
120		attan						
3 8 5		Dista						0.010
476	A*	nce	292	115	175	2	0	003
120								
385		Manh						0.067
476	IDA*	attan	4395	16	173	0	4227	999
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	1	1 =	1	1	1	I	1	,
		Dista						
		nce						
		Manh						
5 2 4		attan						
170		Dista						0.004
6 3 8	A*	nce	215	85	129	1	0	001
		Manh						
5 2 4		attan						
170		Dista						
6 3 8	IDA*	nce	551	19	119	0	435	0.008
		Manh						
3 0 2		attan						
4 1 8		Dista						0.029
657	A*	nce	1039	397	627	15	0	001
		Manh						
3 0 2		attan						
4 1 8		Dista						0.568
657	IDA*	nce	45542	21	412	0	45135	039
		Manh						
5 2 8		attan						
3 1 6		Dista						0.039
0 4 7	A*	nce	1379	522	842	15	0	967
		Manh						
528		attan						
3 1 6		Dista						0.368
0 4 7	IDA*	nce	31407	22	796	0	30616	038
	l	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

	l							
		Manh						
8 2 5		attan						
6 4 3		Dista						0.100
7 1 0	A*	nce	2569	888	1616	65	0	001
		Manh						
8 2 5		attan						
6 4 3		Dista						1.036
7 1 0	IDA*	nce	90461	24	977	0	89491	015
		Manh						
174		attan						
253		Dista						0.099
068	A*	nce	1474	548	900	26	0	993
		Manh						
174		attan						
253		Dista						0.597
068	IDA*	nce	52703	22	936	0	51773	289

Full Code with the results can be found in this link:

https://colab.research.google.com/drive/1c8Z37rOuRrfo8YOi0kYFRDmUjfZiMcQC?usp

=sharing

Full Code text:

```
from cmath import inf
from collections import deque
import random
import time
class Game_8_Puzzle:
 def init (self, puzzle):
  # puzzle: a list of lists of 8 puzzle matrix, initial node
  self.puzzle = puzzle
  self.goal = [[1, 2, 3], [4, 5, 6], [7, 8, 0]]
 def __str__(self):
  s = ""
  for i in range(3):
  for j in range(3):
    s += str(self.puzzle[i][j]) + " "
  s += "\n"
  #s += "\n"
  return s
 def move(self, x1, y1, x2, y2):
  # swap two tiles on the puzzle
  new_puzzle = [list(ias) for ias in self.puzzle]
  tmp = new_puzzle[x1][y1]
  new_puzzle[x1][y1] = new_puzzle[x2][y2]
  new_puzzle[x2][y2] = tmp
  return new_puzzle
 def get_position_i_j(self, num, puzzle=None):
  # find number and return the i, j of the position
  if not puzzle:
    puzzle = self.puzzle
  for i in range(3):
    for j in range(3):
       if puzzle[i][j] == num:
         return i, j
def options(self):
```

```
# find all possible moves and return all options
    options = []
    x, y = self.get_position_i_j(0)
    if x > 0: # up
      options.append(Game_8_Puzzle(self.move(x, y, x - 1, y)))
    if x < 2: # down
      options.append(Game_8_Puzzle(self.move(x, y, x + 1, y)))
    if y > 0: # left
      options.append(Game_8_Puzzle(self.move(x, y, x, y - 1)))
    if y < 2: # right
      options.append(Game_8_Puzzle(self.move(x, y, x, y + 1)))
    return options
def MD(self):
    # heurustic function manhattan distance
    h = 0
    for x in range(3):
     for y in range(3):
        if self.puzzle[x][y] != 0:
          i, j = self.get_position_i_j(self.puzzle[x][y], self.goal)
          h += abs(x - i) + abs(y - j)
    return h
def M(self):
    # heurustic function misplaced tiles
    h = 0
    for x in range(3):
     for y in range(3):
        if self.puzzle[x][y] != 0 and self.puzzle[x][y] != self.goal[x][y]:
    return h
def NO(self):
    # heurustic function that return always 0 in case of A* then it behave exactly
like Uniform Cost Search
    h = 0
    return h
defis_valid_puzzle_and_can_be_solved(self):
    # return true if puzzle is valid and solvable as explaned above
    cnt = 0
    puzzle_as_1d_list = [num for i in self.puzzle for num in i if not(num == 0)]
    for x in range(len(puzzle_as_1d_list)):
     for y in range(x + 1, len(puzzle as 1d list)):
        if puzzle_as_1d_list[x] > puzzle_as_1d_list[y]:
          cnt += 1
    return cnt % 2 == 0
```

```
class A_Star:
 def __init__(self, initial_node, h):
 self.puzzle = initial node
 self.h = h # heuristic function
 def run(self, get_memory_used = False):
 # array of memory usage: will contain len(closed set) + len(priority queue)
 memory used = []
 # each item in the priority queue is (heuristic value, puzzle) it will help getting
the min path by its heuristic value faster
 priority_queue = [(self.h(self.puzzle), [self.puzzle])] # open_set
 closed set = set() # expanded nodes
 # i will also calculate:
 number of expanded vertices = 0
 number_of_duplicate_vertices_detected = 0
 number_of_generated_vertices = 0
 while priority queue:
  # get lowest heurestic
  index = priority_queue.index(min(priority_queue, key=lambda x: x[0]))
  min_h, puzzle_path = priority_queue.pop(index)
  min_puzzle = puzzle_path[-1]
  if min_puzzle.puzzle == min_puzzle.goal:
   break
  if str(min_puzzle.puzzle) in closed_set:
   number_of_duplicate_vertices_detected += 1
   continue
  for option in min_puzzle.options():
   if str(option.puzzle) in closed_set:
     continue
   tmp = puzzle_path+[option]
   priority_queue.append((len(puzzle_path)-1 + self.h(option), tmp))
   number_of_generated_vertices += 1
  closed_set.add(str(min_puzzle.puzzle))
  if get_memory_used:
   memory_used.append(len(closed_set) + len(priority_queue))
  number_of_expanded_vertices += 1
 #number_of_vertices = len(closed_set)
 # print("len(open_list): "+str(len(priority_queue)))
 # print("len(closed_set): "+str(number_of_vertices))
 # print("num of expanded: "+str(number_of_expanded_vertices))
 number_of_vertices_in_queue = len(priority_queue)
 number_of_duplicate_vertices_undetected = 0
```

```
return puzzle_path, number_of_generated_vertices,
number_of_vertices_in_queue, number_of_expanded_vertices,
number of duplicate vertices detected,
number_of_duplicate_vertices_undetected, memory_used
def print_path_to_sol(path):
for x in path:
 print(x.__str__())
def play(game, print_boo=True, get_memory_used=False):
 assert game.puzzle.is_valid_puzzle_and_can_be_solved() == True
 start = time.time()
 path, num1, num2, num3, num4, num5, arr = game.run(get_memory_used)
 end = time.time()
total_time = end - start
 if print boo:
 print("solution: ")
 print("num of moves: "+str(len(path)-1))
 print_path_to_sol(path)
 print("number of generated vertices: "+str(num1))
 print("number of vertices in (queue for A*/stack for IDA*): "+str(num2))
 print("number of expanded vertices: "+str(num3))
 print("amount of time(ms): "+str(total_time))
 print("number of duplicate vertices detected: "+str(num4))
 print("number of duplicate vertices undetected: "+ str(num5))
 if get_memory_used:
  print("memory used array:")
  print(arr)
 if get_memory_used:
 return arr
return num1, num2, num3, num4, num5, total_time
class IDA_Star():
 def __init__(self, initial_node, h):
 self.puzzle = initial node
 self.h = h # heuristic function
 self.expanded_nodes = set() # used this only to count undectected duplicates
 def run(self, get_memory_used=False):
 number_of_generated_vertices, number_of_vertices_in_stack,
number_of_expanded_vertices, number_of_duplicate_vertices_detected,
number of duplicate vertices undetected = 0, 0, 0, 0, 0
 # memory used array: will have the size of the path at each time
 memory_used = [1] # starting with 1 (only initial state)
 # here
 def search(path, g, bound):
```

```
nonlocal number_of_generated_vertices, number_of_vertices_in_stack,
number_of_expanded_vertices, number_of_duplicate_vertices_detected,
number_of_duplicate_vertices_undetected, memory_used
  node = path[-1]
  if(str(node.puzzle) in self.expanded_nodes):
   number_of_duplicate_vertices_undetected += 1
  else:
   self.expanded nodes.add(str(node.puzzle))
  f = g + self.h(node)
  if f > bound:
   return f
  if str(node.puzzle) == str(node.goal):
   return True
  min = inf
  for option in node.options():
    number of generated vertices += 1
    if option not in path:
      number_of_vertices_in_stack += 1
      path.append(option)
      if get_memory_used:
       memory_used.append(len(path))
      t = search(path, g + 1, bound)
      if t == True:
       return True
      if t < min:
       min = t
      path.pop()
      if get_memory_used:
       memory_used.append(len(path))
      number_of_vertices_in_stack -= 1
    else:
      number_of_duplicate_vertices_detected += 1
  return min
 bound = self.h(self.puzzle)
 path = [self.puzzle]
 while 1:
  t = search(path, 0, bound)
  if t == True:
   return path, number_of_generated_vertices, number_of_vertices_in_stack,
len(self.expanded_nodes), number_of_duplicate_vertices_detected,
number of duplicate vertices undetected, memory used
  if t == inf:
   return False
  bound = t
```

```
import matplotlib.pyplot as plt
import numpy as np
genrated_initial_states_number = 50
def generate_random_board():
 boo = True
 while boo:
  #Generate a random 8-puzzle board
  board = list(range(9))
  random.shuffle(board)
  board = [board[i:i+3] for i in range(0, 9, 3)]
  board = Game_8_Puzzle(board)
  boo = not board.is_valid_puzzle_and_can_be_solved()
 return board
def generate_initial_states(n):
 #Generate n initial states for the 8-puzzle game.
 initial_states = []
 dups = set()
 while len(initial_states) < n:
   board = generate_random_board()
   if str(board) not in dups:
    initial_states.append(board)
    dups.add(str(initial_states))
 return initial_states
# example of random generated puzzle game
print(generate_random_board().__str__())
initial_states = [puzzle]
def run_experiment_MD_show_memory(initial_states): # run experiment with no
heuristic
 results = {
   "A*": {"memory_by_iteration": []},
   "IDA*": {"memory_by_iteration": []}
 }
 for state in initial_states:
   for algo in ["A*", "IDA*"]:
     game = None
     if algo == "A*":
      game = A_Star(state, Game_8_Puzzle.MD)
     elif algo == "IDA*":
      game = IDA_Star(state, Game_8_Puzzle.MD)
```

```
generated, in_memory, expanded, duplicate_detected,
duplicate_undetected, runtime = 0, 0, 0, 0, 0, 0
      mem = play(game,True, True) # replace this later to false so no printing will
be involved
     results[algo]["memory_by_iteration"] = mem
 #return results
 metrics = ["memory_by_iteration"]
 num_states = max(len(results["A*"]["memory_by_iteration"]),
len(results["IDA*"]["memory_by_iteration"]))
 delta = abs(len(results["IDA*"]["memory_by_iteration"]) -
len(results["A*"]["memory_by_iteration"]))
 if len(results["A*"]["memory_by_iteration"]) <
len(results["IDA*"]["memory_by_iteration"]):
   results["A*"]["memory_by_iteration"] += [0] * delta
 elif len(results["A*"]["memory_by_iteration"]) >
len(results["IDA*"]["memory_by_iteration"]):
   results["IDA*"]["memory_by_iteration"] += [0] * delta
 x = np.arange(1, num_states + 1)
 for metric in metrics:
   plt.figure(figsize=(10, 6))
   plt.plot(x, results["A*"][metric], label="A*", marker='o')
   plt.plot(x, results["IDA*"][metric], label="IDA*", marker='x')
   plt.xlabel("Iteration")
   plt.ylabel(metric.replace("_", " ").capitalize())
   plt.title(f"Comparison of A* and IDA* on {metric.replace('_', ' ').capitalize()}")
   plt.legend()
   plt.grid(True)
   plt.show()
run_experiment_MD_show_memory(initial_states)
initial_states = generate_initial_states(1)
def run_experiment_MD_show_memory(initial_states): # run experiment with no
heuristic
 results = {
   "A*": {"memory_by_iteration": []},
   "IDA*": {"memory_by_iteration": []}
 }
 for state in initial_states:
   for algo in ["A*", "IDA*"]:
     game = None
     if algo == "A*":
      game = A_Star(state, Game_8_Puzzle.MD)
      elif algo == "IDA*":
      game = IDA_Star(state, Game_8_Puzzle.MD)
```

```
generated, in_memory, expanded, duplicate_detected,
duplicate_undetected, runtime = 0, 0, 0, 0, 0, 0
     mem = play(game,True, True) # replace this later to false so no printing will
be involved
     results[algo]["memory by iteration"] = mem
 #return results
 metrics = ["memory_by_iteration"]
 num_states = max(len(results["A*"]["memory_by_iteration"]),
len(results["IDA*"]["memory_by_iteration"]))
 delta = abs(len(results["IDA*"]["memory_by_iteration"]) -
len(results["A*"]["memory_by_iteration"]))
 if len(results["A*"]["memory_by_iteration"]) <
len(results["IDA*"]["memory_by_iteration"]):
   results["A*"]["memory_by_iteration"] += [0] * delta
 elif len(results["A*"]["memory_by_iteration"]) >
len(results["IDA*"]["memory_by_iteration"]):
   results["IDA*"]["memory_by_iteration"] += [0] * delta
 x = np.arange(1, num_states + 1)
 for metric in metrics:
   plt.figure(figsize=(10, 6))
   plt.plot(x, results["A*"][metric], label="A*", marker='o')
   plt.plot(x, results["IDA*"][metric], label="IDA*", marker='x')
   plt.xlabel("Iteration")
   plt.ylabel(metric.replace("_", " ").capitalize())
   plt.title(f"Comparison of A* and IDA* on {metric.replace('_', ' ').capitalize()}")
   plt.legend()
   plt.grid(True)
   plt.show()
run_experiment_MD_show_memory(initial_states)
initial_states = generate_initial_states(genrated_initial_states_number)
import openpyxl
import pandas as pd
def write_rows_to_excel(file_name, sheet_name, rows):
 # Create a new workbook and select the active worksheet
 workbook = openpyxl.Workbook()
 sheet = workbook.active
 sheet.title = sheet_name
 # Write the rows to the worksheet
 for row in rows:
   sheet.append(row)
 # Save the workbook to a file
 workbook.save(file name)
```

```
rows = [["input"]]
for x in initial_states:
  rows.append([x.__str__()])
print(rows)
write_rows_to_excel("inputs.xlsx", "Sheet1", rows)
initial_states = []
inputs_num = 10
# reading the inputs
df = pd.read_excel("inputs.xlsx", sheet_name='Sheet1', usecols="A:A",
nrows=inputs_num, skiprows=0, index_col=0)
#print(df)
for x in df.index:
 tmp = x.split('\n')[:3]
 new_input = []
 for t in tmp:
   tmp2 = t.split(' ')[:3]
   list_of_ints = [int(item) for item in tmp2]
   new_input.append(list_of_ints)
 initial_states.append(Game_8_Puzzle(new_input))
 #print(new_input)
#print(initial_states)
def run_experiment_M(initial_states): # run experiment with no heuristic
 results = {
   "A*": {"generated": [], "in_memory": [], "expanded": [], "duplicate_detected":
[], "duplicate_undetected": [], "runtime": []},
   "IDA*": {"generated": [], "in_memory": [], "expanded": [],
"duplicate_detected": [], "duplicate_undetected": [], "runtime": []}
 }
 for state in initial_states:
   for algo in ["A*", "IDA*"]:
     game = None
     if algo == "A*":
      game = A_Star(state, Game_8_Puzzle.M)
     elif algo == "IDA*":
      game = IDA_Star(state, Game_8_Puzzle.M)
     generated, in_memory, expanded, duplicate_detected,
duplicate_undetected, runtime = play(game,False)
     results[algo]["generated"].append(generated)
     results[algo]["in_memory"].append(in_memory)
     results[algo]["expanded"].append(expanded)
     results[algo]["duplicate_detected"].append(duplicate_detected)
     results[algo]["duplicate_undetected"].append(duplicate_undetected)
```

```
results[algo]["runtime"].append(runtime)
 return results
def run experiment MD(initial states): # run experiment with no heuristic
 results = {
   "A*": {"generated": [], "in_memory": [], "expanded": [], "duplicate_detected":
[], "duplicate_undetected": [], "runtime": []},
   "IDA*": {"generated": [], "in_memory": [], "expanded": [],
"duplicate_detected": [], "duplicate_undetected": [], "runtime": []}
 }
 for state in initial_states:
   for algo in ["A*", "IDA*"]:
     game = None
     if algo == "A*":
      game = A Star(state, Game 8 Puzzle.MD)
     elif algo == "IDA*":
      game = IDA_Star(state, Game_8_Puzzle.MD)
     generated, in_memory, expanded, duplicate_detected,
duplicate_undetected, runtime = play(game,False)
     results[algo]["generated"].append(generated)
     results[algo]["in_memory"].append(in_memory)
     results[algo]["expanded"].append(expanded)
     results[algo]["duplicate_detected"].append(duplicate_detected)
     results[algo]["duplicate_undetected"].append(duplicate_undetected)
     results[algo]["runtime"].append(runtime)
 return results
def plot results(results, h):
 metrics = ["generated", "in_memory", "expanded", "duplicate_detected",
"duplicate_undetected", "runtime"]
 num_states = len(initial_states)
 x = np.arange(1, num_states + 1) # Label initial states from 1 to num_states
 for metric in metrics:
   a_star_values = results["A*"][metric]
   ida_star_values = results["IDA*"][metric]
   bar width = 0.35 # Width of the bars
   fig, ax = plt.subplots(figsize=(10, 6))
   bar1 = ax.bar(x - bar_width/2, a_star_values, bar_width, label='A*',
color='blue')
```

bar2 = ax.bar(x + bar_width/2, ida_star_values, bar_width, label='IDA*',

color='orange')

```
ax.set_xlabel('Initial State Index')
   ax.set_ylabel(metric.replace("_", " ").capitalize())
   ax.set_title(f'Comparison of A* and IDA* on {metric.replace("_", "
").capitalize()} with {h} heuristic')
   ax.set_xticks(x)
   ax.legend()
   # Optional: Add labels above the bars
   def add_labels(bars):
     for bar in bars:
       height = bar.get_height()
       ax.annotate(f'{height}',
             xy=(bar.get_x() + bar.get_width() / 2, height),
             xytext=(0, 3), #3 points vertical offset
             textcoords="offset points",
             ha='center', va='bottom')
   add_labels(bar1)
   add_labels(bar2)
   plt.show()
for i in range(len(initial_states)):
print("initial state #"+str(i+1)+" :")
 print(initial_states[i])
print("----")
# Run the experiment
results_M = run_experiment_M(initial_states)
# Plot the results
plot_results(results_M, "Misplaced Tiles")
# Run the experiment
results_MD = run_experiment_MD(initial_states)
# Plot the results
plot_results(results_MD, "Manhattan Distance")
rows = [["Initial_State", "Algorithm", "Heuristic", "Generated_Vertices",
"Vertices_In_Memory", "Expanded_Vertices", "Duplicate_Detected",
"Duplicate_Undetected", "Run_Time"]]
# ["generated", "in_memory", "expanded", "duplicate_detected",
"duplicate_undetected", "runtime"]
for i in range(len(initial_states)):
```

```
rows.append([initial_states[i].__str__(), "A*", "Misplaced Tiles",
results_M["A*"]["generated"][i], results_M["A*"]["in_memory"][i],
results_M["A*"]["expanded"][i], results_M["A*"]["duplicate_detected"][i],
results_M["A*"]["duplicate_undetected"][i], results_M["A*"]["runtime"][i]])
rows.append([initial_states[i].__str__(), "IDA*", "Misplaced Tiles",
results_M["IDA*"]["generated"][i], results_M["IDA*"]["in_memory"][i],
results_M["IDA*"]["expanded"][i], results_M["IDA*"]["duplicate_detected"][i],
results_M["IDA*"]["duplicate_undetected"][i], results_M["IDA*"]["runtime"][i]])
rows.append([initial_states[i].__str__(), "A*", "Manhattan Distance",
results_MD["A*"]["generated"][i], results_MD["A*"]["in_memory"][i],
results_MD["A*"]["expanded"][i], results_MD["A*"]["duplicate_detected"][i],
results_MD["A*"]["duplicate_undetected"][i], results_MD["A*"]["runtime"][i]])
rows.append([initial_states[i].__str__(), "IDA*", "Manhattan Distance",
results_MD["IDA*"]["generated"][i], results_MD["IDA*"]["in_memory"][i],
results_MD["IDA*"]["expanded"][i], results_MD["IDA*"]["duplicate_detected"][i],
results_MD["IDA*"]["duplicate_undetected"][i], results_MD["IDA*"]["runtime"][i]])
# results[algo]["generated"].append(generated)
# results[algo]["in_memory"].append(in_memory)
# results[algo]["expanded"].append(expanded)
# results[algo]["duplicate_detected"].append(duplicate_detected)
# results[algo]["duplicate_undetected"].append(duplicate_undetected)
# results[algo]["runtime"].append(runtime)
print(rows)
write_rows_to_excel("data_M_vs_MD.xlsx", "Sheet1", rows)
initial_states = []
inputs_num = 50
# reading the inputs
df = pd.read_excel("inputs.xlsx", sheet_name='Sheet1', usecols="A:A",
nrows=inputs_num, skiprows=0, index_col=0)
#print(df)
for x in df.index:
 tmp = x.split('\n')[:3]
 new_input = []
 for t in tmp:
   tmp2 = t.split(' ')[:3]
   list_of_ints = [int(item) for item in tmp2]
   new_input.append(list_of_ints)
 initial_states.append(Game_8_Puzzle(new_input))
 #print(new_input)
#print(initial_states)
# Run the experiment
```

```
results_MD = run_experiment_MD(initial_states)
# Plot the results
plot_results(results_MD, "Manhattan Distance")
def plot_results_without_labels(results, h):
 metrics = ["generated", "in_memory", "expanded", "duplicate_detected",
"duplicate_undetected", "runtime"]
 num_states = len(initial_states)
 x = np.arange(1, num_states + 1) # Label initial states from 1 to num_states
 for metric in metrics:
   a_star_values = results["A*"][metric]
   ida_star_values = results["IDA*"][metric]
   bar_width = 0.35 # Width of the bars
   fig, ax = plt.subplots(figsize=(20, 12))
   bar1 = ax.bar(x - bar_width/2, a_star_values, bar_width, label='A*',
color='blue')
   bar2 = ax.bar(x + bar_width/2, ida_star_values, bar_width, label='IDA*',
color='orange')
   ax.set_xlabel('Initial State Index')
   ax.set_ylabel(metric.replace("_", " ").capitalize())
   ax.set_title(f'Comparison of A* and IDA* on {metric.replace("_", "
").capitalize()} with {h} heuristic')
   ax.set_xticks(x)
   ax.legend()
   ## Optional: Add labels above the bars
   # def add labels(bars):
   # for bar in bars:
   #
         height = bar.get_height()
   #
         ax.annotate(f'{height}',
               xy=(bar.get_x() + bar.get_width() / 2, height),
   #
               xytext=(0, 3), # 3 points vertical offset
   #
               textcoords="offset points",
               ha='center', va='bottom')
   # add labels(bar1)
   # add_labels(bar2)
   plt.show()
plot_results_without_labels(results_MD, "Manhattan Distance")
```

```
rows = [["Initial_State", "Algorithm", "Heuristic", "Generated_Vertices",
"Vertices_In_Memory", "Expanded_Vertices", "Duplicate_Detected",
"Duplicate_Undetected", "Run_Time"]]
# ["generated", "in_memory", "expanded", "duplicate_detected",
"duplicate_undetected", "runtime"]
for i in range(len(initial_states)):
rows.append([initial_states[i].__str__(), "A*", "Manhattan Distance",
results_MD["A*"]["generated"][i], results_MD["A*"]["in_memory"][i],
results_MD["A*"]["expanded"][i], results_MD["A*"]["duplicate_detected"][i],
results_MD["A*"]["duplicate_undetected"][i], results_MD["A*"]["runtime"][i]])
rows.append([initial_states[i].__str__(), "IDA*", "Manhattan Distance",
results_MD["IDA*"]["generated"][i], results_MD["IDA*"]["in_memory"][i],
results_MD["IDA*"]["expanded"][i], results_MD["IDA*"]["duplicate_detected"][i],
results_MD["IDA*"]["duplicate_undetected"][i], results_MD["IDA*"]["runtime"][i]])
# results[algo]["generated"].append(generated)
# results[algo]["in_memory"].append(in_memory)
# results[algo]["expanded"].append(expanded)
# results[algo]["duplicate_detected"].append(duplicate_detected)
# results[algo]["duplicate_undetected"].append(duplicate_undetected)
# results[algo]["runtime"].append(runtime)
print(rows)
write_rows_to_excel("data_MD.xlsx", "Sheet1", rows)
```