Radzimiński Jan #35

Numerical Methods (ENUME 2019) – Project
Assignment A: Solving linear algebraic equations

1. Design a procedure for generation of the following matrices:

$$
\mathbf{A}_{N,x} = \begin{bmatrix}
x^2 & \frac{2x}{3} & -\frac{2x}{3} & \frac{2x}{3} & \cdots & \frac{2x}{(-1)^{N-1}\cdot 3} & \frac{2x}{(-1)^{N}\cdot 3} \\
\frac{2x}{3} & \frac{8}{9} & -\frac{8}{9} & \frac{8}{9} & \cdots & \frac{8}{(-1)^{N-1}\cdot 9} & \frac{8}{(-1)^{N}\cdot 9} \\
-\frac{2x}{3} & -\frac{8}{9} & \frac{12}{9} & -\frac{12}{9} & \cdots & \frac{12}{(-1)^{N-4}\cdot 9} & \frac{12}{(-1)^{N-3}\cdot 9} \\
\frac{2x}{3} & \frac{8}{9} & -\frac{12}{9} & \frac{16}{9} & \cdots & \frac{16}{(-1)^{N-5}\cdot 9} & \frac{16}{(-1)^{N-4}\cdot 9} \\
-\frac{2x}{3} & -\frac{8}{9} & \frac{12}{9} & -\frac{16}{9} & \cdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & & \frac{(N-1)\cdot 4}{9} & -\frac{(N-1)\cdot 4}{9} \\
\frac{2x}{(-1)^{N}\cdot 3} & \frac{8}{(-1)^{N}\cdot 9} & \frac{12}{(-1)^{N-3}\cdot 9} & \frac{16}{(-1)^{N-4}\cdot 9} & \cdots & -\frac{(N-1)\cdot 4}{9} & \frac{N\cdot 4}{9}
\end{bmatrix}
$$

2. For each matrix $\mathbf{A}_{N,x}$, generated for $N \in \{3, 10, 20\}$ and $x = \ln(\alpha)$:
   - determine the smallest positive value α_N of α which yields $\det(\mathbf{A}_{N,x}) = 0$;
   - draw the dependence of $\det(\mathbf{A}_{N,x})$ on α for $\alpha \in [\alpha_N - 0.01, \alpha_N + 0.01]$;
   - draw the dependence of $\mathrm{cond}(\mathbf{A}_{N,x})$ on α for $\alpha \in [\alpha_N - 0.01, \alpha_N + 0.01]$.
3. Design a procedure for inverting the matrix $\mathbf{A}_{N,x}$ according to the scheme presented on the lecture slide #3-16 – in two versions: (a) based on the LU factorisation, (b) based on the LLT factorisation. Check the correctness of this procedure using several low-dimensional positive definite matrices.
4. Apply the above procedure for finding the estimates $\hat{\mathbf{A}}_{N,x}^{-1}$ of the matrices $\mathbf{A}_{N,x}$ generated for $N \in \{3, 10, 20\}$ and $x = \frac{2^k}{300}$ with $k \in \{0, 1, 2, \ldots, 21\}$.
5. For each estimate $\hat{\mathbf{A}}_{N,x}^{-1}$ determine the following indicators of its uncertainty:

   $\delta_2 = \left\| \mathbf{A}_{N,x} \cdot \hat{\mathbf{A}}_{N,x}^{-1} - \mathbf{I}_N \right\|_2$ (the root-mean-square error)

   $\delta_\infty = \left\| \mathbf{A}_{N,x} \cdot \hat{\mathbf{A}}_{N,x}^{-1} - \mathbf{I}_N \right\|_\infty$ (the maximum error)

   Compute the norms of the matrices according to the formulae presented on the lecture slide #1-15 (compare the norms obtained in this way with the corresponding norms computed by means of the operator ***norm*** implemented in MATLAB). Compare the estimates $\hat{\mathbf{A}}_{N,x}^{-1}$, obtained by means of the procedure defined in Section 3, with the estimates obtained by means of the operator of matrix inversion ***inv*** implemented in MATLAB. Draw the dependence of δ_2 and δ_∞ on x.