Assignment

Question 1

Question 1

dual-core cpu & disk   60 minutes.

a) Service demand of c1, c2 and disk

$D(j) = V(j)\,S(j).$

$D(j) = U(j) / X(0)$

$U(j) = B(j) / T$

Busy time / Throughput

$X(0) = \frac{C(0)}{T}$ num request

$D(j) = B(j) / C(0).$

$c1 = 2828 / 1347 = 2.0995$

$c2 = 1728 / 1347 = 1.283.$

$disk = 2665 / 1347 = 1.98.$

b). Bottle neck analysis, asymptotic bound on system throughput   30 interaktive users, think time job =15sec.

$$X(0) \le \min\left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{k} D_i}\right]$$

$$D(c1) > D(disk) > D(c2)$$

$$\cancel{\frac{1}{1.283}} \quad \frac{1}{2.0995} = 0.4763$$

$$\frac{N}{D(c1) + D(c2) + D(disk) + ThinkTime}$$

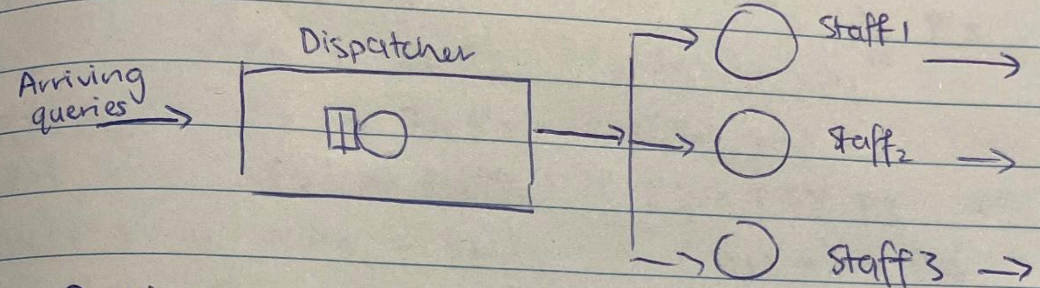
$$= \frac{30}{2.0995 + 1.283 + 1.98 + 15} = 1.4733$$

$X(0) \le 0.4763$ (job/s).

Question 2

Question 2.

call centre, 3 Staff, queue hold 2 calls at dispatcher, no queueing facility at staff end

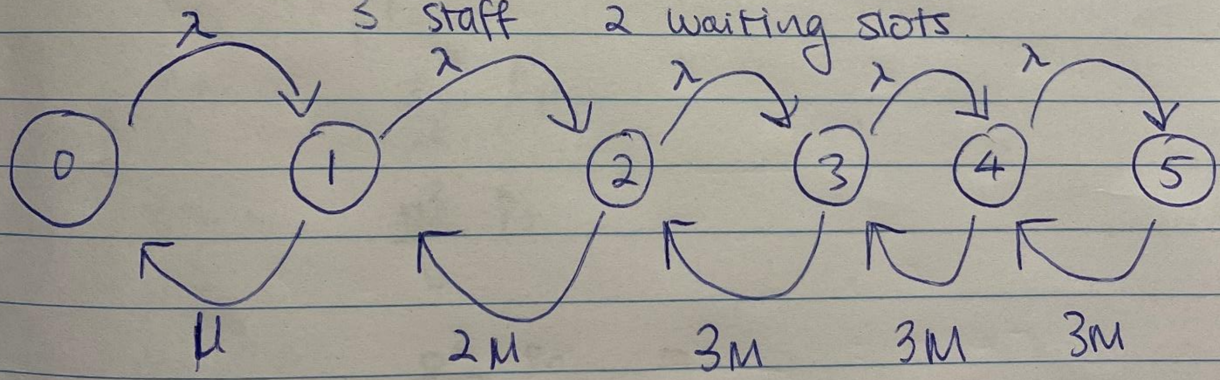


Receives 12.7 queries per hour.

Each staff complete on avg 4.1 queries per hr.

a). Continuous time Markov chain, definition of states & transition rates between states.

3 staff 2 waiting slots.



state 0 = 0 job, idle.

state 1 = 1 jobs in the system. (1 staff busy)

state 2 = 2 jobs in the system. (2 staff busy)

state 3 = 3 jobs in the system. (3 staff busy)

state 4 = 3 jobs ~~in the system~~ busy, 1 waiting.

state 5 = 3 jobs ~~in the system~~ busy, 2 waiting

Question 2.

b). Balance equation for markov chain

P_k = Prob k jobs in system.

State 0 $= \lambda P_0 = M P_1 \rightarrow P_1 = \frac{\lambda}{u} P_0$

state 1 $= \lambda P_0 + 2MP_2 = (M+\lambda)P_1$

State 2 $= \lambda P_1 + 3MP_3 = (2M+\lambda)P_2$

State 3 $= \lambda P_2 + 3MP_4 = (3M+\lambda)P_3$

state 4 $= \lambda P_3 + 3MP_5 = (3M+\lambda)P_4$

state 5 $= \lambda P_4 = 3MP_5$

c) Expressions for the steady state probabilities

let $p = \lambda / M$

$P_1 = pP_0$

$P_2 = \frac{p^2}{2} P_0$

$P_3 = \frac{p^3}{6} P_0$

$P_4 = \frac{p^4}{18} P_0$

$P_5 = \frac{p^5}{54} P_0$

$P_1 + P_2 + P_3 + P_4 + P_5 = 1$

$\left(p + \frac{p^2}{2} + \frac{p^3}{6} + \frac{p^4}{18} + \frac{p^5}{54}\right) P_0 = 1$

$P_0 = \frac{1}{(1+p + \frac{p^2}{2} + \frac{p^3}{6} + \frac{p^4}{18} + \frac{p^5}{54})}$

Question 2

) Find the probability an arriving query is rejected.

rejected = 3 busy staff & 2 in queue

= State 5

$\lambda = 12.7$ $\quad \mu = 4.1$

$p = 12.7/4.1 = 3.097561$

$$P_0 = \frac{1}{\left(1 + 3.0975 + \frac{3.0975^2}{2} + \frac{3.0975^3}{6} + \frac{3.0975^4}{18} + \frac{3.0975^5}{54}\right)}$$

$P_0 = \cancel{0.43}\ \cancel{0.043}\ 0.04124$

$P(5) = P_0 \times p^5 \times \frac{1}{54}$

$= \cancel{0.043}\ 0.04124 \times 3.0975^5 \times \frac{1}{54}$

$P(5) = \cancel{0.227}\ 0.2178$

Probability of rejection = 0.2178.

Question 2

e) Mean waiting time for accepted query in queue.

Little's law → $N = XR$.

$$N = \sum_{K=0}^{5} K \times P(k)$$

$= 0 \times P(0) + 1 \times P(1) + 2 \times P(2) + 3 \times P(3) + 4 \times P(4) + 5 \times P(5)$.

$\rho = 3.0975$

$P(0) = 0.043$

$P(1) = 3.0975 \times 0.043 = 0.1332$

$P(2) = \frac{3.0975^2}{2} \times 0.043 = 0.2063$

$P(3) = \frac{3.0975^3}{6} \times 0.043 = 0.213$

$P(4) = \frac{3.0975^4}{18} \times 0.043 = 0.2199$

$P(5) = \frac{3.0975^5}{54} \times 0.043 = 0.227$

$N = 0 + 0.1332 + 2 \times 0.2063 + 3 \times 0.213 + 4 \times 0.2199 + 5 \times 0.227$

$N = 3.1994$

$X = (1 - P(5)) \times \lambda$

$X = (1 - 0.227) \times 12.7 = 9.8171$

$R = N / X = 0.3259 = 1173.24$ sec

$W = R - S = 1173.24 - 878.0488 = 295.1912$

Question 3

1

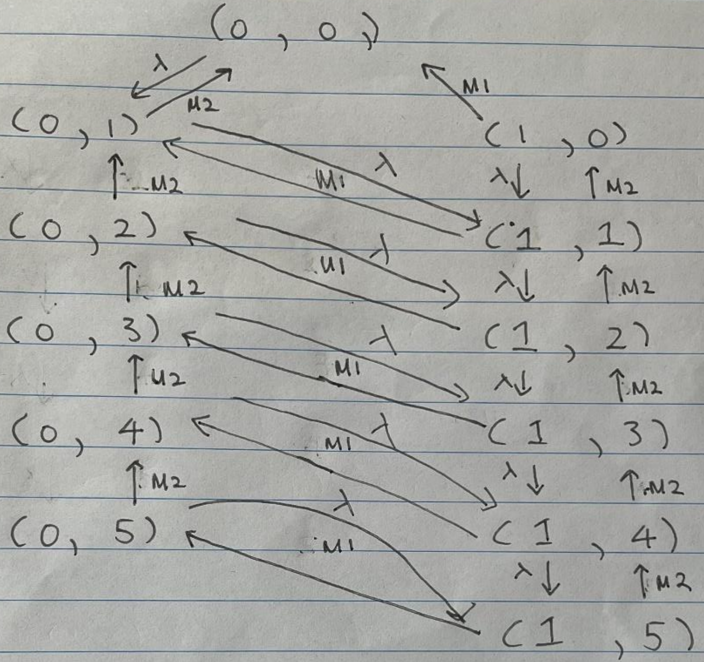
a). $n_1 = 1$, $n_2 = 5$

$\mu_1 = 0.5/s$, $\mu_2 = 0.7/s$, $\lambda = 1/s$

Let us use 2-tuple (x, y) where x is j^1 and $y = j^2$.

$\mu_1 = 1/0.5 = 2$ $\quad \mu_2 = 1/0.7 = 1.428$

$\lambda = 1/1 = 1$



Equations are:

1 $P(0,0)\lambda = P(0,1)\mu_2 + P(1,0)\mu_1$

2 $P(0,1)(\lambda+\mu_2) = P(0,0)\lambda + P(1,1)\mu_1 + P(0,2)\mu_2$

3 $P(0,2)(\lambda+\mu_2) = P(0,3)\mu_2 + P(1,2)\mu_1$

4 $P(0,3)(\lambda+\mu_2) = P(0,4)\mu_2 + P(1,3)\mu_1$

5 $P(0,4)(\lambda+\mu_2) = P(0,5)\mu_2 + P(1,4)\mu_1$

6 $P(0,5)(\mu_2+\lambda) = P(1,5)\mu_1$

7 $P(1,0)(\mu_1+\lambda) = P(1,1)\mu_2$

8 $P(1,1)(\lambda+\mu_1+\mu_2) = P(1,2)\mu_2 + P(1,0)\lambda + P(0,1)\lambda$

9 $P(1,2)(\lambda+\mu_1+\mu_2) = P(1,3)\mu_2 + P(1,1)\lambda + P(0,2)\lambda$

10 $P(1,3)(\lambda+\mu_1+\mu_2) = P(1,4)\mu_2 + P(1,2)\lambda + P(0,3)\lambda$

11 $P(1,4)(\lambda+\mu_1+\mu_2) = P(1,5)\mu_2 + P(1,3)\lambda + P(0,4)\lambda$

12 $P(1,5)(\mu_1+\mu_2) = P(1,4)\lambda + P(0,5)\lambda$

....../....../......

a) i). $P(0,0) + P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(0,5)$
$+ P(1,0) + P(1,1) + P(1,2) + P(1,3) + P(1,4) + P(1,5)$
$= 1$

ii). Rejected = S_1 is full, S_2 is full.
Used matlab! $\rightarrow P(1,5). = 0.0013$

iii) Mean Response time of S_1

$$\text{Mean Response time} = \frac{\text{Mean number of jobs}}{\text{Throughput of } S_1}$$

Mean number of jobs in S_1 = $\sum_{k=0}^{K} K \times P_K$ (K is num of jobs in S_1).

We know that:

$P(0,0) = 0.4960$ $P(0,1) = 0.2911$ $P(0,2) = 0.0299$
$P(0,3) = 0.0103$ $P(0,4) = 0.0035$ $P(0,5) = 0.0011$
$P(1,0) = 0.0401$ $P(1,1) = 0.0841$ $P(1,2) = 0.0290$
$P(1,3) = 0.0100$ $P(1,4) = 0.0035$ $P(1,5) = 0.0013.$

Mean jobs = $1 \times (0.0401 + 0.0841 + 0.0290 + 0.01 + 0.0035 + 0.0013).$

$= 0.168$

Throughput = Utilisation $\times$ Service rate
$(0.168) \times 2 = 0.336.$

Mean response time = $0.168 / 0.336 = 0.5s.$

Using Untitled.m in Matlab we add the values we get from balancing the equation to a matrix, we can obtain the probability for each state for n1 = 1.

Editor - Untitled.m

Untitled.m

```
A=[ 1,-1/0.7,0,0,0,0,-2,0,0,0,0,0;
    -1,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0;
    0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0;
    0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0;
    0,0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0;
    0,0,0,0,0,1+(1/0.7),0,0,0,0,0,-2;
    0,0,0,0,0,0,3,-1/0.7,0,0,0,0;
    0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0;
    0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0;
    0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0;
    0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7;
    0,0,0,0,0,-1,0,0,0,0,-1,2+(1/0.7);
    1,1,1,1,1,1,1,1,1,1,1,1];
b=[0 0 0 0 0 0 0 0 0 0 0 0 1]';
x = A\b;
```

Editor - Untitled.m

x | A

12x1 double

	1	2	3	4	5
1	0.4960				
2	0.2911				
3	0.0299				
4	0.0103				
5	0.0035				
6	0.0011				
7	0.0401				
8	0.0841				
9	0.0290				
10	0.0100				
11	0.0035				
12	0.0013				
13					
14					
15					
16					
17					
18					

iv). Mean Response time of server farm = $S_1 + S_2$.

(3)

Mean jobs in S_2 = $1 \times (0.2911 + 0.0841) +$
$2 \times (0.0299 + 0.029) + 3 \times (0.0103 + 0.01)$
$+ 4 \times (0.0035 + 0.0035) + 5 \times (0.0011 + 0.0013)$
$= 0.5939$

Throughput = Utilisation × Service rate

$\cancel{0.66257}$ $0.4638 \times 1/0.7 = 0.66257$

Mean response time of S_2 = $0.5939 / 0.66257 = 0.89636$ sec

Mean Response time of Server farm = $0.89636 + 0.5$
$= 1.39636$ sec

b) Response time of Server for $n_1 = 3$..

= Jobs / throughput of system.

Jobs = Probability of all state − P(0,0)

= using excel we get = 0.496362

Throughput of system = Throughput of S_1 + S_2

S_1 = 0.176676 × 2 = 0.353352

S_2 = (0.319686, + 0.11760 + 0.014003 + 0.00131) × $\frac{1}{0.7}$ = 0.646571

Total Throughput = 0.99992343

Response time = 0.4964 s.

For n_1 = 2

Jobs = 1 − 0.4960 = 0.504.

Throughput of system 1 : 0.33697

Throughput of S_2 : 0.464 × $\frac{1}{0.7}$ = 0.662931

Response time = 0.50405 s.

Using the balance.py to get a matrix according to n1, we can compute that into MATLAB, which gives the probability for each state, which can then be used to find the response time. Once we get the column with the probabilities, we can copy that into Excel to get the sum of the selected rows to calculate the throughput for the systems.

Below is the matrix and probabilities for n1 = 2

Editor - Untitled.m

Untitled.m

```
A=[ 1,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0;
    -1,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0;
    0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0;
    0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0;
    0,0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0;
    0,0,0,0,0,1+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0;
    0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0;
    0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0;
    0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0;
    0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0;
    0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0;
    0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),0,0,0,0,0,-2;
    0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7;
    0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,2+(1/0.7);
    1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
b=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]';
x = A\b;
s=sum(A);
```

	1
1	0.4960
2	0.2911
3	0.0299
4	0.0103
5	0.0034
6	9.6729e-04
7	0.0401
8	0.0841
9	0.0290
10	0.0100
11	0.0035
12	0.0012
13	2.8312e-06
14	5.9455e-06
15	1.6449e-05
16	4.6831e-05
17	1.3366e-04
18	3.8157e-04
19	

Below is the matrix for n1 = 3 and the probabilities for each state.

Editor - Untitled.m

Untitled.m

```
A=[ 1,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    -1,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,1+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0;
    0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0;
    0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0;
    0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0;
    0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0;
    0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0;
    0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0;
    0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0;
    0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0;
    0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,2+(1/0.7);
    1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
b=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]';
x = A\b;
s=sum(A);
```

	1	2
1	0.5043	
2	0.2929	
3	0.0236	
4	2.6902e-04	
5	7.4997e-05	
6	1.6756e-05	
7	0.0429	
8	0.0867	
9	0.0284	
10	2.7309e-04	
11	7.9100e-05	
12	2.0346e-05	
13	0.0024	
14	0.0038	
15	0.0076	
16	4.1370e-04	
17	1.2312e-04	
18	3.6674e-05	
19	8.6512e-04	
20	0.0018	
21	0.0024	
22	6.9156e-04	
23	2.0687e-04	
24	7.1035e-05	
25		
26		
27		
28		

Response time for ~~se~~ $n_1 = 4$.

Jobs $= 1 - 0.503652 = 0.496348$

Throughput of $S_1 = 0.176722 \times 2 = 0.353444$

Throughput of $S_2 = (0.319626 + 0.117565 + 0.013979 + 0.00129 + 0.000113) \times \frac{1}{0.7}$

$= 0.64653\,3$

Total throughput $= 1.14288$

Response time $= 0.43429$ s.

Response time for $n_1 = 5$

Jobs $= 1 - 0.50365 = 0.49635$

Throughput of $S_1 = 0.17673 \times 2 = 0.35346$.

Throughput of $S_2 = (0.31962 + 0.11756 + 0.01398 + 0.00129 + 0.000112 + 0.00000931) \times \frac{1}{0.7} = 0.64653$

Total: 1.14288

Response time: 0.43435.

c). when $n_1 = 4$, it gives the smallest response time for the server farm.

Below is the matrix for n1 = 4 and the probabilities for each state.

Editor - Untitled.m

```matlab
A=[ 1,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    -1,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,1+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,2+(1/0.7);
    1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
b=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]';
x = A\b;
s=sum(A);
```

Variables - s

	1	2
1	0.5037	
2	0.2934	
3	0.0249	
4	0.0012	
5	5.5375e-05	
6	2.5122e-06	
7	0.0422	
8	0.0866	
9	0.0294	
10	0.0015	
11	6.5446e-05	
12	3.0505e-06	
13	0.0015	
14	0.0030	
15	0.0083	
16	0.0026	
17	1.1505e-04	
18	5.4986e-06	
19	4.5483e-05	
20	9.3712e-05	
21	2.5488e-04	
22	7.1407e-04	
23	2.1810e-04	
24	1.0650e-05	
25	1.2874e-06	
26	2.7035e-06	
27	7.4798e-06	
28	2.1295e-05	
29	6.0778e-05	
30	2.0833e-05	
31		

Below is the matrix for n1 = 5 and the probabilities for each state.

```
Editor - Untitled.m
Untitled.m
A=[ 1,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    -1,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,1+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,1+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,3+(1/0.7),0,0,0,0,0,-2,0,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0,-2,0,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0,0,-2,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,3+(1/0.7),0,0,0,0,0,-2;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,3,-1/0.7,0,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7,0;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,3+(1/0.7),-1/0.7;
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,-1,2+(1/0.7);
    1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
b=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]';
x = A\b;
```

I
0.50365
0.29341
0.02493
0.00123
5.49E-05
2.26E-06
0.04225
0.08665
0.0294
0.00145
6.51E-05
2.74E-06
0.00148
0.00304
0.00826
0.00255
0.00011
4.94E-06
4.57E-05
9.39E-05
0.00026
0.00071
0.00022
9.56E-06
1.40E-06
2.88E-06
7.80E-06
2.18E-05
6.11E-05
1.87E-05
4.51E-08
9.47E-08
2.62E-07
7.46E-07
2.13E-06
6.08E-06