Raeeca Narimani
Professor Gupta
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Bifurcation Diagrams From Cubic Iterators

Chaos theory is a current area of study in mathematics and chaotic systems are found in many scientific fields. Bifurcation diagrams are among the simplest forms of chaotic systems. The most famous bifurcation diagrams is Feigenbaum's diagram. This diagram is produced using the quadratic iterator function:

$$X_{n+1} = aX_{n+1}(1 - X_n)\ ,\ \ n = 1, 2, \ldots$$

We have several questions. What factors cause bifurcation? Where does the first split of a bifurcation diagram occur? At what point does the nth split occur? Does there exist a point at which the bifurcation diagram stops branching, and if so where? We change the iterator function's changes from a quadratic form to a cubic, and analyze all these questions. The intersection points of a line $y = A_1x + B_1$ and $y = Ax^3 + Bx^2 + Cx + D$, where A and $A_1$are non-zero values, are the fixed points. It has been determined whether a fixed point X_0 is attractive by observing whether the iteration's approach is towards X_0. The study of attractive or repulsive points is important since bifurcation only occurs around an attractive fixed point. However, not all the attractive points implement the occurrence of bifurcation. In fact, from studying quadratics, as well as the cubics, it was concluded that the derivative of a polynomial, in relevance with the slope of the intersecting straight line, gives a clue of where bifurcation happens. It was figured that in a cubic iterator, bifurcation happens if and only if the cubic function has local maximum and minimum. Also by inductive reasoning, we may hypothesize that bifurcation occurs with polynomials that within finite intervals, have a form similar to a quadratic function. Interestingly, the bifurcation diagram from a cubic iterator, has a pattern very similar to a quadratic's, which supports the hypothesis.

Works Cited

Peitgen, Heinz-Otto, H. Jürgens, and Dietmar Saupe. Chaos and Fractals: New Frontiers of Science. New York: Springer-Verlag, 1992. Print.