CIVENG 3C03 Assignment 1

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Question 1

Using the graphic method of solution, find the optimal solution for the following LP model:

Minimize:

$$Z = 2200 - 7X_1 - 2X_2$$

Subject to:

$$X_1 \leq 100$$
$$3X_1 + 5X_2 \leq 900$$
$$4X_1 + 2X_2 \leq 600$$
$$X_1, X_2 \geq 0$$

The graphical solution to this problem is found in Figure 1. The optimal value for Z is approximately 184.857, when $X_1 = 85.714$ and $X_2 = 128.571$.

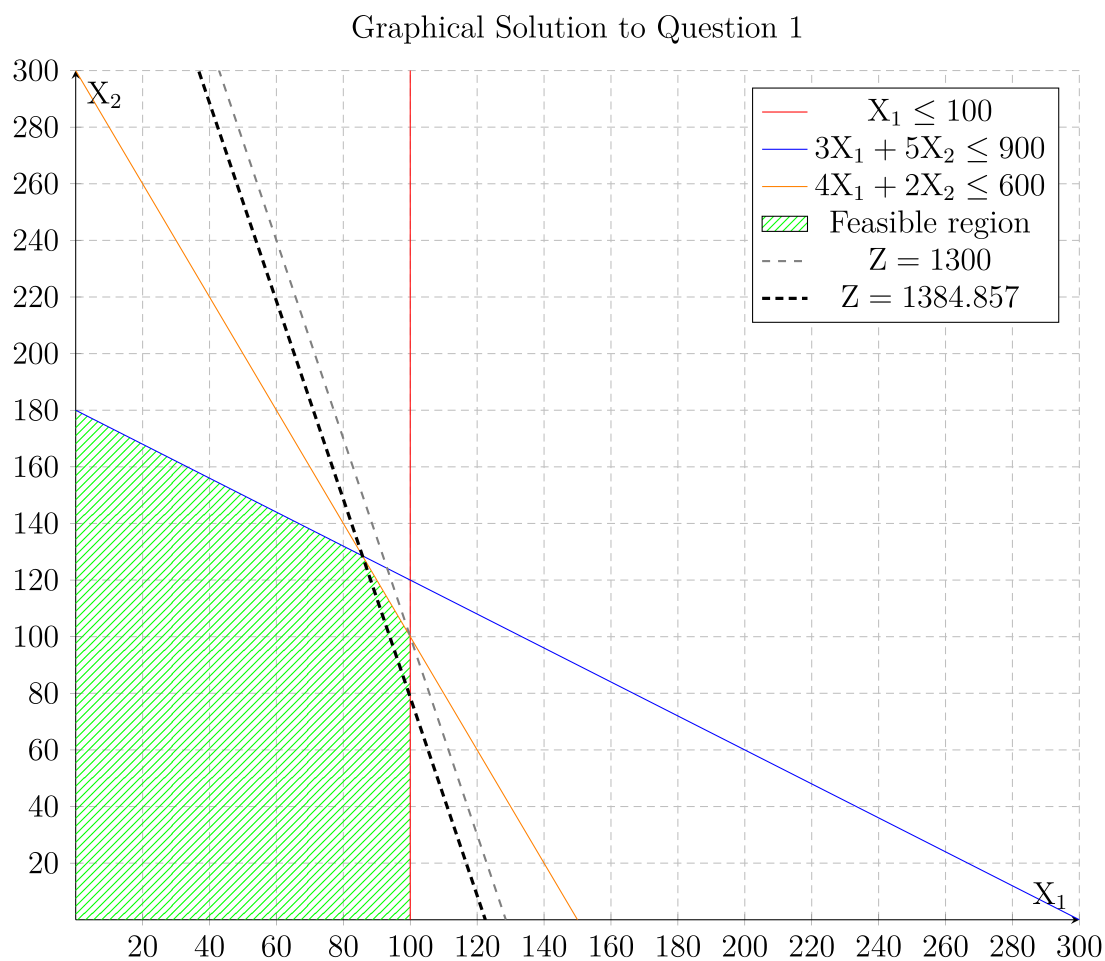

Figure 1: Question 1: Graphical Solution

Question 2

The operating cost per week is $5400 - 15X_1 - 20X_2 - 10X_3$, and the chemical cost per week is $3.47X_1 + 4.235X_2 + 2.725X_3$, therefore the total cost per week is the operating cost per week plus the chemical cost per week.

LP Model

Total cost (\$/per week) = $5400 - (15 - 2.5 \cdot 0.98 - 2 \cdot 0.51)X_1$ - $(20 - 2.5 \cdot 0.98 - 3.5 \cdot 0.51)X_2$ - $(10 - 2 \cdot 0.98 - 1.5 \cdot 0.51)X_3$
Total cost (\$/per week) = $5400 - 11.53X_1$ - $15.765X_2$ - $7.275X_3$

Minimize: Z= $5400 - 11.53X_1$ - $15.765X_2$ - $7.275X_3$ (Objective Function)

Subject to:
$6X_1 + 8X_2 + 4X_3 \leq 1000$ (Total hour constraint)
$2.5X_1 + 2.5X_2 + 2X_3 \leq 380$ (C_1 constraint)
$2X_1 + 3.5X_2 + 1.5X_3 \leq 400$ (C_2 constraint)
$X_1 \leq 50$ (Available X_1 constraint)
$X_1, X_2, X_3 \geq 0$

The canonical form of the model is expressed as follows:

Minimize: Z= $5400 - 11.53X_1$ - $15.765X_2$ - $7.275X_3$

Subject to:
$6X_1 + 8X_2 + 4X_3 + S_1 = 1000$
$2.5X_1 + 2.5X_2 + 2X_3 + S_2 = 380$
$2X_1 + 3.5X_2 + 1.5X_3 + S_3 = 400$
$X_1 + S_4 = 50$
$X_1, X_2, X_3, S_1, S_2, S_3, S_4 \geq 0$

1. The Simplex Tableau for this problem can be seen in Table 1. The solution was found in three iterations of the Simplex method. The optimal weekly production of the three paint types is $[X_1 = 50, X_2 = 75, X_3 = 25]$. The corresponding production cost of the optimal production policy is $Z = 5400 - (11.53 \cdot 50) - (15.765 \cdot 75) - (7.275 \cdot 25) = 3459.25$.

2. Since $S_1, S_3, S_4 = 0$, the binding constraints are:
   $6X_1 + 8X_2 + 4X_3 \leq 1000$
   $2X_1 + 3.5X_2 + 1.5X_3 \leq 400$
   $X_1 \leq 50$

   The non-binding constraint is:
   $2.5X_1 + 2.5X_2 + 2X_3 \leq 380$

Table 1: Question 2: Simplex Tableau

C_i	Baisc Variables	C_j	-11.53	-15.77	-7.28	0	0	0	0	b_i/a_{ij}
		b_i	X_1	X_2	X_3	S_1	S_2	S_3	S_4	
0	S_1	1000	6	8	4	1	0	0	0	125
0	S_2	380	2.5	2.5	2	0	1	0	0	152
0	S_3	400	2	3.5	1.5	0	0	1	0	114.29
0	S_4	50.00	1	0	0	0	0	0	1	∞
		ΔZ_j	-11.53	-15.77	-7.28					
0	S_1	85.71	1.43	0.00	0.57	1.00	0.00	-2.29	0.00	60
0	S_2	94.29	1.07	0.00	0.93	0.00	1.00	-0.71	0.00	88
-15.765	X_2	114.29	0.57	1.00	0.43	0.00	0.00	0.29	0.00	200
0	S_4	50.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	50
		ΔZ_j	-2.52		-0.52			4.50		
0	S_1	14.29	0	0	0.57	1	0	-2.29	-1.43	25
0	S_2	40.71	0	0	0.93	0	1	-0.71	-1.07	43.85
-15.765	X_2	85.71	0	1	0.43	0	0	0.29	-0.57	200
-11.53	X_1	50	1	0	0	0	0	0	1	∞
		ΔZ_j			-0.52			4.5	2.52	
-7.275	X_3	25	0	0	1	1.75	0	-4	-2.5	
0	S_2	17.5	0	0	0	-1.63	1	3	1.25	
-15.765	X_2	75	0	1	0	-0.75	0	2	0.5	
-11.53	X_1	50	1	0	0	0	0	0	1	
		ΔZ_j				0.91			1.23	

Question 3

1. From the Simplex Tableau provided, we can determine that the canonical form for the Linear Programming problem is:

   Minimize:

   $$\mathrm{C} = 250 - 3\mathrm{X}_1 - 8\mathrm{X}_2$$

   Subject to:

   $$\begin{gathered} 4\mathrm{X}_1 + \mathrm{X}_2 + \mathrm{S}_1 = 40 \\ 3\mathrm{X}_1 + 5\mathrm{X}_2 + \mathrm{S}_2 = 75 \\ \mathrm{X}_2 + \mathrm{S}_3 = 13 \\ \mathrm{X}_1, \mathrm{X}_2, \mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3 \geq 0 \end{gathered}$$

   This means the LP model used to represent the LP problem is:

   Minimize:

   $$\mathrm{C} = 250 - 3\mathrm{X}_1 - 8\mathrm{X}_2$$

   Subject to:

   $$\begin{gathered} 4\mathrm{X}_1 + \mathrm{X}_2 \leq 40 \\ 3\mathrm{X}_1 + 5\mathrm{X}_2 \leq 75 \\ \mathrm{X}_2 \leq 13 \\ \mathrm{X}_1, \mathrm{X}_2 \geq 0 \end{gathered}$$

2. The optimal production policy for the $[\mathrm{X}_1 = 3.33, \mathrm{X}_2 = 13]$. The corresponding cost of the optimal production policy is $\mathrm{C} = 250 - (3 * 3.33) - (13 * 8) = 136$.

3. Since $\mathrm{S}_2, \mathrm{S}_3 = 0$, the binding constraints are:

   $$\begin{gathered} 3\mathrm{X}_1 + 5\mathrm{X}_2 \leq 75 \\ \mathrm{X}_2 \leq 13 \end{gathered}$$