CIVENG 3C03 Assignment 2

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October 21, 2022

Question 1

If the number of bronze, silver, and gold passes issued are X_1, X_2, and X_3, then the profit margins for the park can be represented as:

$$\text{Max: } Z = X_1 + 3X_2 + 10X_3$$

The passes are also subject to the following constraints:
$X_1 + X_2 + 3.5X_3 \leq 22000$ (administrative working units constraint)
$X_1 + X_2 + X_3 \leq 10000$ (maximum number of passes)
$X_1 \geq 1200$ (minimum number of bronze tickets)
X_1, X_2, $X_3 \geq 0$ (non-negativity constraint)

From the objective function and the constraints, we can represent the problem in canonical form:

Minimize:

$$Z^* = -X_1 - 3X_2 - 10X_3 + MA_1$$

Subject to: Subject to:

$$\begin{gathered} X_1 + X_2 + 3.5X_3 + S_1 = 22000 \\ X_1 + X_2 + X_3 + S_2 = 10000 \\ X_1 - E_1 + A_1 = 1200 \\ X_1, X_2, X_3, S_1, S_2, E_1, A_1 \geq 0 \end{gathered}$$

The Simplex Tableau for this problem can be seen in Table 1. The solution was found in three iterations of the Simplex method. The optimal number of each pass type the park should issue to maximize its profit is $[X_1 = 1200, X_2 = 4000, X_3 = 4800]$. The corresponding total profit margin cost of the optimal pass policy is $Z = 1200 + (3 \cdot 4000) + (10 \cdot 4800) =$ \$61200.

Since $S_1, S_2, E_1, A_1 = 0$, all constraints are binding:

$$\begin{gathered} X_1 + X_2 + 3.5X_3 \leq 22000 \\ X_1 + X_2 + X_3 \leq 10000 \\ X_1 \geq 1200 \end{gathered}$$

Question 2

The solution to Question 1 was redone using Excel solver, producing the same solution as the Simplex method using in question 1 as seen in Figure 1. The solution is [X_1 = 1200, X_2 = 4000, X_3 = 4800] for a total profit margin of \$61200. The Excel spreadsheet is included with the assignment submission as `question2.xlsx`.

Table 1: Question 1: Simplex Tableau

C_i	Baisc Variables	C_j	-1	-3	-10	0	0	0	M	b_i/a_{ij}
		b_i	X_1	X_2	X_3	S_1	S_2	E_1	A_1	
0	S_1	22000	1	1	3.5	1	0	0	0	22000
0	S_2	10000	1	1	1	0	1	0	0	10000
M	A_1	1200	1	0	0	0	0	-1	1	1200
		ΔZ_j	-1-M	-3	-10			M		
0	S_1	20800	0	1	3.5	1	0	1	-1	5942.86
0	S_2	8800	0	1	1	0	1	1	-1	8800
-1	X_1	1200	1	0	0	0	0	-1	1	∞
		ΔZ_j		-3	-10			-1	M+1	
-10	X_3	5942.86	0	0.29	1	0.29	0	0.29	-0.29	20800
0	S_2	2857.14	0	0.71	0	-0.29	1	0.71	-0.71	4000
-1	X_1	1200	1	0	0	0	0	-1	1	∞
		ΔZ_j		-0.14		2.86		1.86	M-1.86	
-10	X_3	4800	0	0	1	0.4	-0.4	0	0	
-3	X_2	4000	0	1	0	-0.4	1.4	1	-1	
-1	X_1	1200	1	0	0	0	0	-1	1	
		ΔZ_j				2.8	0.2	2	M-2	

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Variables		X1	X2	X3					
3		Values		1200	4000	4800					
4		Cost Cofficients		-1	-3	-10			-61200		
5											
6		Struct Coeff		1	1	3.5		#1	22000	<=	22000
7				1	1	1		#2	10000	<=	10000
8				1	0	0		#5	1200	>=	1200

Figure 1: Excel Solver Solution

Question 3

To determine if the current production policy can be continued, we check the sensitivity of the optimal solution due to the changes in each of the cost coefficients. To determine this, we must first calculate the ΔZ_j values for the non-basic variables as done in Table 2.

Table 2: Question 3: ΔZ_j calculation

Baisc Variables	C_j	-2	-2.5	-5	0	0	0	0
	b_i	X_1	X_2	X_3	S_1	S_2	S_4	S_4
X_1	17.71	1	0	0	1.14	0	-0.29	-2.29
S_2	225.54	0	0	0	-2.57	1	0.54	4.54
X_2	12.34	0	1	0	-0.57	0	0.34	0.34
X_3	25	0	0	1	0	0	0	1
	ΔZ_j				0.855		0.27	1.27

We can use this information to determine the acceptable range of each cost/profit coefficient.

For C^1, the allowable range is $1.25 \leq C^1 \leq 2.55$.

$$\text{Max } C^1 = -2 + \min \begin{cases} \frac{0.855}{1.14} = 0.75 \\ \text{ignore because } a_{ij} < 0 \\ \text{ignore because } a_{ij} < 0 \end{cases}$$
$$= -2 + 0.75 = -1.25$$
$$\text{Min } C^1 = -2 + \max \begin{cases} \text{ignore because } a_{ij} > 0 \\ \frac{0.27}{-0.29} = -0.93 \\ \frac{1.27}{-2.29} = -0.55 \end{cases}$$
$$= -2 - 0.55 = -2.55$$

For C^2, the allowable range is $1.71 \leq C^2 \leq 4$.

$$\text{Max } C^2 = -2.5 + \min \begin{cases} \text{ignore because } a_{ij} < 0 \\ \frac{0.27}{0.34} = 0.79 \\ \frac{1.27}{0.34} = 3.74 \end{cases}$$
$$= -2.5 + 0.79 = -1.71$$
$$\text{Min } C^2 = -2.5 + \max \begin{cases} \frac{0.855}{-0.57} = -1.5 \\ \text{ignore because } a_{ij} > 0 \\ \text{ignore because } a_{ij} > 0 \end{cases}$$
$$= -2.5 - 1.5 = -4$$

For C^3, the allowable range is $3.73 \leq C^3 \leq \infty$.

$$\text{Max } C^3 = -5 + \min \begin{cases} \frac{0.855}{0} = \infty \\ \frac{0.27}{0} = \infty \\ \frac{1.27}{1} = 1.27 \end{cases}$$

$$= -5 + 1.27 = -3.73$$

$$\text{Min } C^3 = -5 + \max \begin{cases} \frac{0.855}{0} = \infty \\ \frac{0.27}{0} = \infty \\ \text{ignore because } a_{ij} > 0 \end{cases}$$

$$= -5 - \infty = -\infty$$

With the acceptable ranges for profit coefficients calculated, we can determine if the profit coefficients can be changed without changing the optimal solution. We can see that these new unit profit coefficients all fall within the acceptable range for each cost coefficient:

$$1.25 \leq 1.60 \leq 2.55$$
$$1.71 \leq 1.90 \leq 4$$
$$3.73 \leq 3.75 \leq \infty$$

Therefore, the company does not need to change its production policy and can continue with its current production policy.