

Assignment 1
Due: September 30, 2022 – 11:59 PM
Late submission is not permitted

Question 1

Using the graphical method of solution, find the optimal solution for the following LP model:

Minimize $Z = 2200 - 7X_1 - 2X_2$

Subject to:

$$X_1 \leq 100$$

$$3X_1 + 5X_2 \leq 900$$

$$4X_1 + 2X_2 \leq 600$$

$$X_1 \geq 0, X_2 \geq 0$$

Question 2

A painting company produces three types of paint (P_1 , P_2 , and P_3). Each paint type requires a different combination of two different chemical compositions (C_1 and C_2), as shown in Table 1. The typical working hours needed to produce one cubic meter of each type are also shown in Table 1. The maximum number of working hours available to the company is 1000 hours/week. Due to some technical issues, the production of the first paint type (P_1) cannot exceed 50 m³ per week. The weekly operating cost required to produce the paint is defined according to the following equation:

$$\text{Operating cost (\$/week)} = 5400 - 15X_1 - 20X_2 - 10X_3$$

Where X_1 , X_2 , and X_3 are the number of units produced from the three types of paint each week, respectively. The unit cost and the maximum available supply of each chemical composition are presented in Table 2. You are asked to:

- 1) Determine the optimal weekly production of the three paint types and the corresponding production cost using the Simplex method.
- 2) Identify the binding and the non-binding constraints.

Table 1

Paint Type	Chemical Composition Combination (in units) per m ³ of paint		Working hours per m ³ of paint
	C ₁	C ₂	
P ₁	2.5	2	6
P ₂	2.5	3.5	8
P ₃	2	1.5	4

Table 2

Chemical composition	Cost (\$ / unit)	Supply (units/week)
C ₁	0.98	380
C ₂	0.51	400

Question 3

ABC company is interested in minimizing its production cost (C), which is expressed according to the following equation:

$$C = 250 - 3X_1 - 8X_2$$

Where X_1 and X_2 are the quantities produced of two different products manufactured by the company. The company set up a mathematical model to obtain the optimal production strategy that minimizes production cost. The Simplex algorithm was utilized to obtain the optimal production strategy. The Simplex tableau is presented in the table below.

- 1) Retrieve the LP model used to represent the LP problem addressed in this question
- 2) What is the optimal production policy of the company?
- 3) What are the binding constraints of this LP problem?

C _i	Basic Variables	C _j	-3	-8	0	0	0	b _j /a _{ij}
		b _i	X ₁	X ₂	S ₁	S ₂	S ₃	
0	S ₁	40	4	1	1	0	0	40
0	S ₂	75	3	5	0	1	0	15
0	S ₃	13	0	1	0	0	1	13
		ΔZ_j	-3	-8				
0	S ₁	27	4	0	1	0	-1	6.75
0	S ₂	10	3	0	0	1	-5	3.33
-8	X ₂	13	0	1	0	0	1	∞
		ΔZ_j	-3				8	
0	S ₁	13.67	0	0	1	-1.33	5.67	
-3	X ₁	3.33	1	0	0	0.33	-1.67	
-8	X ₂	13	0	1	0	0	1	
		ΔZ_j				1	3	