CIVENG 3C03 Assignment 3

Raeed Hassan
hassam41

November 11, 2022

Question 1

The optimal solution to the provided LP solution was found using the Microsoft Excel Solver. The LP model setup in Excel can be seen in Figure 1. The optimal solution is found using the Excel Solver with Simplex LP Solving Method. The optimal solution is $[X_1 = 0,\ X_2 = 11.5,\ X_3 = 14.8,\ X_4 = 10.4]$ with Max Z = 889.3, as seen in Figure 2.

The acceptable range for the cost coefficients are:

$$
\begin{gathered}
-\infty \leq C_1 \leq 29.1 \\
6.8 \leq C_2 \leq 86.4 \\
33.17 \leq C_3 \leq \infty \\
-\infty \leq C_4 \leq 32.5
\end{gathered}
$$

The acceptable range for the stipulations are:

$$
\begin{gathered}
153.33 \leq B_1 \leq 230 \\
50.01 \leq B_2 \leq \infty \\
0 \leq B_3 \leq 35.33 \\
33 \leq B_4 \leq 56.57
\end{gathered}
$$

The Excel spreadsheet is included with the assignment submission as `question1.xlsx`.

Variables	X1	X2	X3	X4					
Values	0	0	0	0					
Cost Coefficients	25	15	40	12			0		
Struct Coeff	3	4	4	7		#1	0	<=	178
	4.2	1.5	0	3.15			0	<=	112
	1	2	0	0			0	>=	23
	0	0	1	3			0	>=	46

Figure 1: Excel Solver Model

Objective Cell (Max)

Cell	Name	Original Value	Final Value
J4	Cost Coefficients	0	889.3

Variable Cells

Cell	Name	Original Value	Final Value	Integer
D3	Values X1	0	0	Contin
E3	Values X2	0	11.5	Contin
F3	Values X3	0	14.8	Contin
G3	Values X4	0	10.4	Contin

Figure 2: Excel Solver Solution

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	Values X1	0	-4.1	25	4.1	1E+30
E3	Values X2	11.5	0	15	71.4	8.2
F3	Values X3	14.8	0	40	1E+30	6.83333333
G3	Values X4	10.4	0	12	20.5	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
J6	#1	178	21.6	178	52	24.6666667
J7		50.01	0	112	1E+30	61.99
J8		23	-35.7	23	12.3333333	23
J9		46	-46.4	46	10.5714286	13

Figure 3: Excel Solver Sensitivity

Question 2

The unbalanced transportation problem was solved using the MODI Method, with all iterations shown in Figures 4–6. The optimal solution can be seen in Figure 6.

(a) The optimal allocation is shown in Table 1. As the demand exceeds supply, there is a shortage of 30m^3 for both locations B and C that must be fulfilled by external sources.

Table 1: Optimal Solution

Route	Allocation
I-A	45
I-B	75
II-C	120
II-D	135

(b) The transportation cost corresponding to the solution obtained in part (a) is shown in Table 2.

Table 2: Transportation Cost

Route	Allocation	Unit Cost	Total Cost
I-A	45	9	405
I-B	75	10	750
II-C	120	6	720
II-D	135	3	405
Total Cost			2280

Figure 4: Initial Iteration

		Location				
		A	B	C	D	
Quarry	Ui \ Vj	9	10	8	5	Available
I	0	[9] 45	[10] 75	[15]	[11]	120
II	-2	[12]	[8] 30	[6] 150	[3] 75	255
III	-5	[0]	[0]	[0]	[0] 60	60
Demand		45	105	150	135	435 \ 435

Non-basic Cell	Improvement Index
I-C	7
I-D	6
II-A	5
III-A	-4
III-B	-5
III-C	-3

Figure 5: First Iteration

		Location				
		A	B	C	D	
Quarry	Ui \ Vj	9	10	13	10	Available
I	0	[9] 45	[10] 75	[15]	[11]	120
II	-7	[12]	[8]	[6] 150	[3] 105	255
III	-10	[0]	[0] 30	[0]	[0] 30	60
Demand		45	105	150	135	435 \ 435

Non-basic Cell	Improvement Index
I-C	2
I-D	1
II-A	10
III-A	1
II-B	5
III-C	-3

Figure 6: Second Iteration

		Location				
		A	B	C	D	
Quarry	Ui \ Vj	9	10	10	7	Available
I	0	[9] 45	[10] 75	[15]	[11]	120
II	-4	[12]	[8]	[6] 120	[3] 135	255
III	-10	[0]	[0] 30	[0] 30	[0]	60
Demand		45	105	150	135	435 \ 435

Non-basic Cell	Improvement Index
I-C	5
I-D	4
II-A	7
III-A	1
II-B	2
III-D	3

Question 3

The develop a plan for returning the four vehicles to the four local offices, we can identify the problem as an Assignment Problem and solve the problem using the Hungarian method. As the distances are not provided between each drop-off location and office, Google Maps was used to get these distances. The shortest distance for each route was used and rounded down to be an integer. The setup of the Hungarian method and initial two steps are shown in Figure 7. After the first two steps, optimality is not reached, therefore we find an improved solution in Figure 8 which we discover is an optimal solution. The optimal solution for returning the four vehicles to the four local offices is shown in Table 3.

Figure 7: Hungarian Method Setup

Drop-off Location	Office Location			
	Hamilton	Square One	Toronto	Kitchener
YYZ	64	16	27	88
BUF	118	153	172	184
YXU	119	161	182	94
Milton GO	38	28	52	57

Drop-off Location	Office Location			
	Hamilton	Square One	Toronto	Kitchener
YYZ	26	0	0	31
BUF	80	137	145	127
YXU	81	145	155	37
Milton GO	0	12	25	0

Drop-off Location	Office Location			
	Hamilton	Square One	Toronto	Kitchener
YYZ	26	0	0	31
BUF	0	57	65	47
YXU	44	108	118	0
Milton GO	0	12	25	0

Figure 8: Hungarian Method Optimal Solution

Drop-off Location	Office Location			
	Hamilton	Square One	Toronto	Kitchener
YYZ	38	0	0	43
BUF	0	45	53	47
YXU	44	96	106	0
Milton GO	0	0	13	0

Table 3: Optimal Solution

Drop-off Location	Return Location
YYZ	Downtown Toronto
BUF	Hamilton
YXU	Kitchener
Milton GO	Square One