Laboratory 5: Design the Sallen-Key Low-Pass Filter

ELEC ENG 2CJ4: Circuits and Systems
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1 Objective

Active filters are widely used in analog circuits, e.g., in power, communication and control systems. In this experiment, we will study a useful second-order Butterworth filter — the Sallen-Key low-pass filter.

2 Euqipment

The following equipments are used in this laboratory:

- DC voltage source with positive and negative output($\pm$9V); Oscilloscope; Function signal generator
- Op-Amp LM358
- Resistors: $10\Omega \times 1$, $10k\Omega \times 3$, $25k\Omega \times 1$, $100k\Omega \times 1$
- Capacitors: $10nF(103) \times 2$
- Please download LTSpice to your machines using the link: https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html

3 Introduction to Sallen-Key Low-pass Filters

The Sallen-Key topology was introduced by R. P. Sallen and E. L. Key of MIT Lincoln Laboratory in 1955[1].

3.1 The Transfer Function

Consider the circuit in Fig. 1. To simplify our analysis, we assume that the Op-Amp is ideal and it works in the linear region. Hence,

$$V_p = V_n. \tag{1}$$

[1] https://en.wikipedia.org/wiki/Sallen%E2%80%93Key_topology

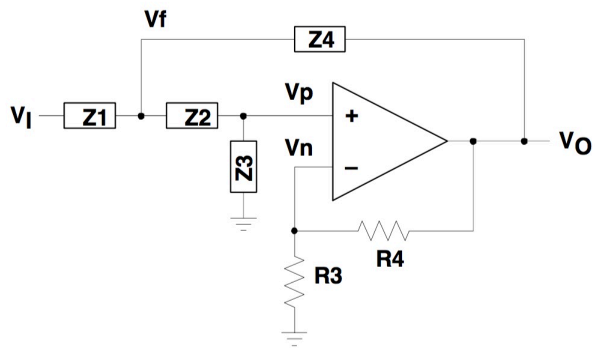

Figure 1: A generic Sallen-Key filter topology Karki (1999)

Also note that

$$V_p = V_f \frac{Z_3}{Z_2 + Z_3}, \tag{2a}$$

$$V_n = V_o \frac{R3}{R_3 + R_4}. \tag{2b}$$

It follows by the KCL that

$$\frac{V_i - V_f}{Z_1} = \frac{V_f - V_o}{Z_4} + \frac{V_f}{Z_2 + Z_3}. \tag{3}$$

Combing the above equations yields

$$\frac{V_o}{V_i} = \frac{K}{\frac{Z_1 Z_2}{Z_3 Z_4} + \frac{Z_1}{Z_3} + \frac{Z_2}{Z_3} + \frac{Z_1(1-K)}{Z_4} + 1} \tag{4a}$$

$$= \frac{K Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_4 + Z_1 Z_3 (1-K) + Z_3 Z_4}, \tag{4b}$$

where $K = 1 + \frac{R_4}{R_3}$.

3.2 Low-Pass Sallen-Key Circuit

Consider the filter in Fig. 2. We let $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = \frac{1}{sC_1}$ and $Z_4 = \frac{1}{sC_2}$. The transfer function for this low-pass filter is given by

$$H(s) = \frac{K}{R_1 R_2 C_1 C_2 s^2 + s\big(R_1 C_1 + R_2 C_1 + R_1 C_2 (1-K)\big) + 1}. \tag{5}$$

It can be verified that $f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$ and $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1 + R_1 C_2 (1-K)}$. Setting $R_1 = R_2 = R$, $C_1 = C_2 = C$, and $K \approx 1$ gives $f_c = \frac{1}{2\pi RC}$ and $Q = 1/2$.

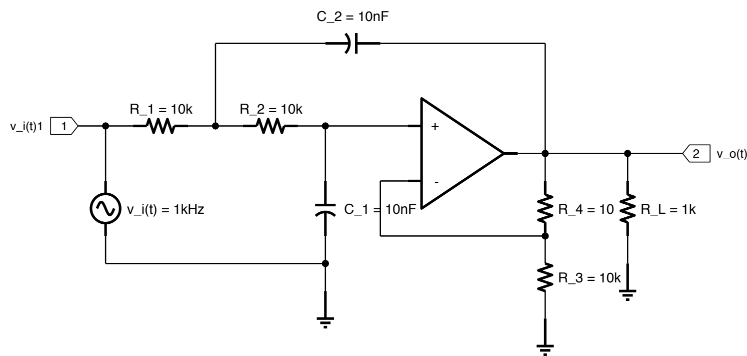

Figure 2: Low-pass Sallen-Key circuit

4 Experiment

4.1 Preparation:

a. Review the connection diagram of LM358.

b. Calculate the cutoff frequency of the filter in Fig. 2. Use Matlab to plot the transfer function (**Hint: a part of the code is provided in page 5**). Change the value of R_1 to $25K$, $100K$ and repeat your calculation.

c. Design a low pass filter with cutoff frequency approximately $10kHz$ and a gain of $2(6dB)$ in the passband. Give the values of R_1, R_2, C_1, C_2 and try to maximize Q for the elements you have chosen. Also use Matlab to plot the frequency response.

4.2 Experiment:

a. Build the low-pass filter on LTSpice (**procedures are provided in page 6 of this lab manual**) according to Fig. 2.

b. Sketch the waveform of $v_o(t)$, where the input $v_i(t)$ is chosen to be sinusoid, square, and triangular with frequency 1kHz and 5kHz.

c. Sweep the input signal frequency and try to plot transfer function with $v_i(t)$ chosen to be sinusoid. Hint: You can change the input signal frequency and then measure the input and output waveforms, respectively. To save your time, you can first use a large step size and then use a small step size to add more points to your plot. The input signal should be as large as possible in the permitted range to improve the accuracy. Compare the measured results with the theoretical analysis obtained using Matlab. If they do not agree with each other, can you explain why?

d. *Remove R_L and then sweep the input signal from low frequency (e.g., $10Hz$) to high frequency(e.g, $1MHz$), and observe the output signal.

5 Results and Conclusions

Your report should include the following things:

a. Derive the transfer function of the low-pass Sallen-Key circuit. Calculate the cutoff frequency and the Q-factor of the circuit.

b. Use Matlab to plot the transfer function.

c. Compare the theoretical analysis with the measured results. Explain the discrepancies (if any).

d. You must submit your report in PDF format, in addition to LTSpice files and Matlab m-files.

References

Karki, J. (1999). Analysis of the Sallen Key architecture. Technical report, Texas Instrucments, Dallas, Texas, USA.

Transfer function plot in Matlab:

To plot H(s) in Matlab, we replace s by jw, where w=2*pi*f, and draw magnitude of H(s) versus a sweep of freqquncy.

A part of Matlab code that can be employed to plot the transfer function on Matlab is as follow:

```
close all
clear
clc
K   = 1;
R1 = 10e3;
R2 = 10e3;
C1 = 10e-9;
C2 = 10e-9;
fc = 1/(2*pi*sqrt(R1*R2*C1*C2);
% cut off frequency
f   =  fmin:stepsize: fmax;
% f = fc/10:fc/100:10*fc;
 % fmin is the minimum frequency which should be less than or equal to 0.1fc;
 %step size a suitable step size should be chosen according to fmin and fmax,
 % fmax is the maximum frequency which should be greater than or equal to10fc.
w  =  2*pi*f;
 s = 1j*w;
Hs  =   K./(R1*R2*C1*C2*s.^2+(R1*C1+R2*C2+R1*C2*(1-K))*s+1);
semilogx( f,20*log10( abs(Hs) ) ) % semilogarithmic plot
% plot magnitude of H(jw) in dB versus log10(f)
or
semilogx( f,abs(Hs)) % plot magnitude of H(jw) versus log10(f)
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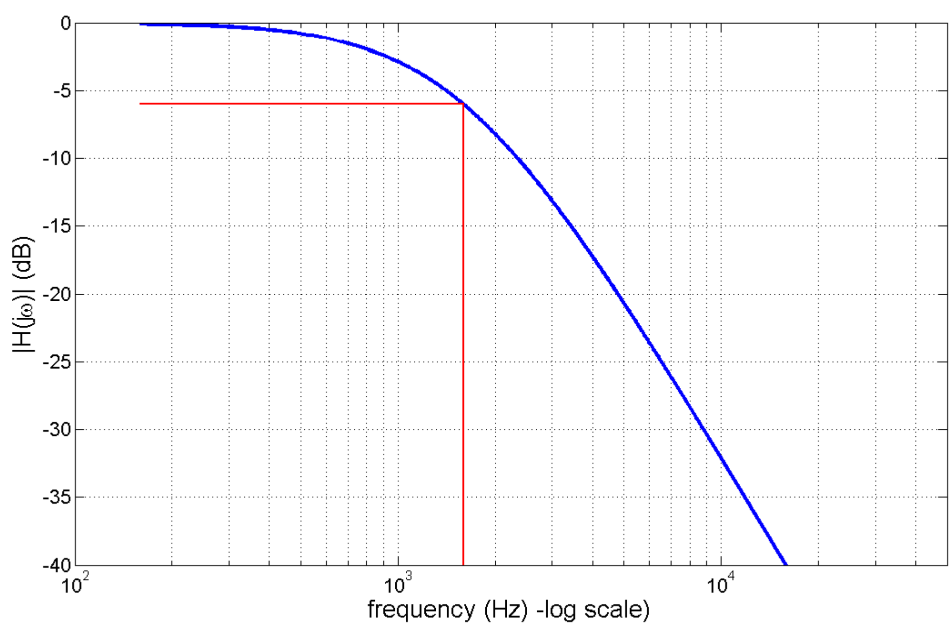

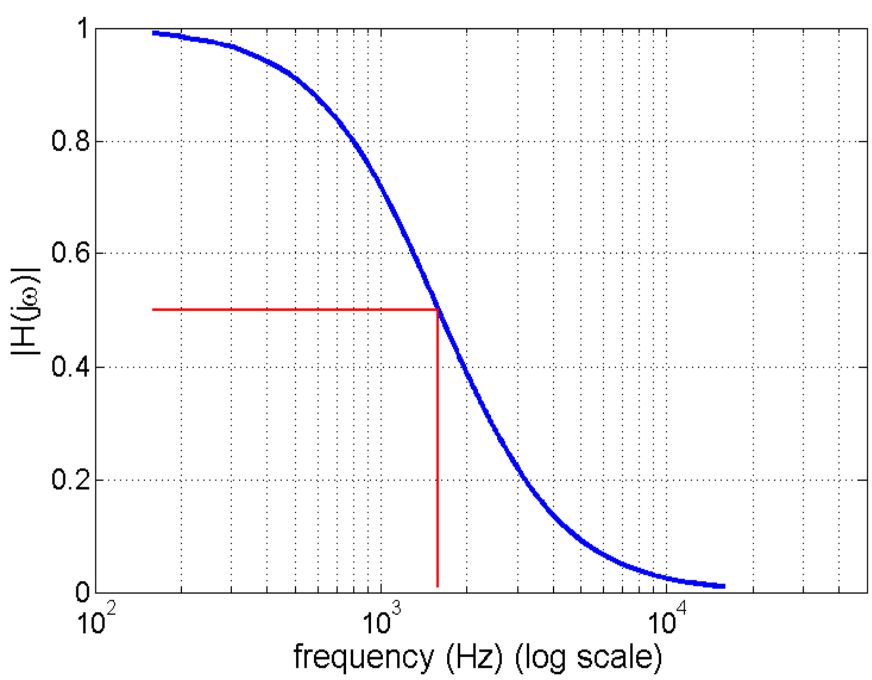

Figure 3: Transfer function plot

LTSpice Circuit Setup procedures:

a. Open LTSpice.

b. Select File > New > New Schematic.

c. Under the New Schematic Page, Right click > Draft > Component. Build the circuit layout as shown in Fig. 4, Each component can be placed by searching each component: OP37(Opamps), Res(Resister), Cap(Capacitor), voltage(DC Voltage Source), Signal(AC Voltage Source). Ground can be added by right clicking the wire > Label Net > GND, and click "Ok" (attention: OP37's +IN and -IN are in the opposite position of those in Fig. 4).

d. Select and right-click a component and enter the new value for the component.

e. Right click > Draft > wires. Connect two components with the selected wires.

f. Right click >Edit Simulation Cmd. Set stop time 10ms, click "OK". This command sets the simulation period up to 10ms.

g. Square input voltage source: Right click > Draft >signal> Right click>Functions: PULSE, V_{on}(V)=1, T_{on}=0.5/frequency, T_{period}=1/frequency.

h. Triangular input voltage source: Right click > Draft >signal> Right click>Functions: PULSE, V_{on}(V)=1, T_{rise}=0.5/frequency, T_{fall}=0.5/frequency, T_{on}=1n, T_{period}=1/frequency.

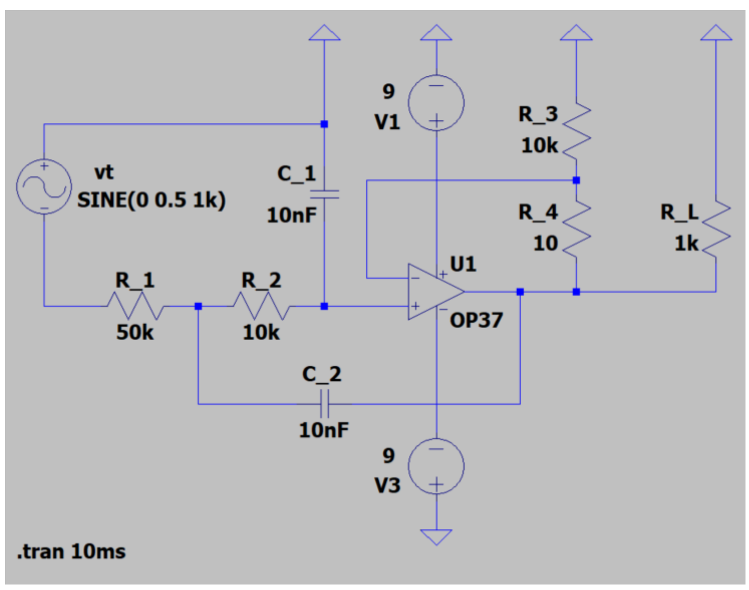

Figure 4: Layout of the Low-pass Sallen-Key circuit

Run the simulation:

a. Right click > Run. This opens a simulation window.

b. Click the wire which you want to observe, and view voltage waveforms of the selected wire on the simulation window.