ELECENG 2FL3 ASSIGNMENT 2

Raeed Hassan
McMaster University

Variation #00:

$$A_1(A_1x, A_1y, A_1z) = (0.40, 0.70, 0.00)$$
$$A_2(A_2x, A_2y, A_2z) = (0.30, 0.10, 0.10)$$
$$P(x, y, z) = (2.80, 3.70, 4.60)$$

2 Problem Statement

2.1 Rectangle Coordinates

a) the dot product $A_1 \cdot A_2$

$$\begin{aligned} A_1 \cdot A_2 &= A_1\hat{x} \cdot A_2\hat{x} + A_1\hat{y} + A_2\hat{y} + A_1\hat{z} + A_2\hat{z} \\ &= (0.40 \cdot 0.30) + (0.70 \cdot 0.10) + (0.00 \cdot 0.10) \\ &= 0.12 + 0.07 + 0.00 \\ &= 0.19 \end{aligned}$$

b) the projection of A_1 onto A_2

$$\begin{aligned} proj_{A_2} A_1 &= \frac{A_1 \cdot A_2}{A_2 \cdot A_2} A_2 \\ &= \frac{0.19}{(0.30)^2 + (0.10)^2 + (0.10)^2}(0.30\hat{x} + 0.10\hat{y} + 0.10\hat{z}) \\ &= 0.5182\hat{x} + 0.1727\hat{y} + 0.1727\hat{z} \end{aligned}$$

c) the angle between A_1 and A_2

$$\begin{aligned} \cos\theta &= \frac{A_1 \cdot A_2}{\|A_1\|\|A_2\|} \\ \theta &= \cos^{-1}\left(\frac{A_1 \cdot A_2}{\|A_1\|\|A_2\|}\right) \\ &= \cos^{-1}\left(\frac{0.19}{\sqrt{(0.40)^2 + (0.70)^2 + (0.00)^2}\sqrt{(0.30)^2 + (0.10)^2 + (0.10)^2}}\right) \\ &= 0.7805 \text{ rad} \end{aligned}$$

d) the cross product $A_1 \times A_2$

$$\begin{aligned} A_1 \times A_2 &= A_1\hat{x} \times A_2\hat{y} + A_1\hat{x} \times A_2\hat{z} + A_1\hat{y} \times A_2\hat{x} + A_1\hat{y} \times A_2\hat{z} \\ &\quad + A_1\hat{z} \times A_2\hat{x} + A_1\hat{z} \times A_2\hat{y} \\ &= 0.40\hat{x} \times 0.10\hat{y} + 0.40\hat{x} \times 0.10\hat{z} + 0.70\hat{y} \times 0.30\hat{x} \\ &\quad + 0.70\hat{y} \times 0.10\hat{z} + 0.00\hat{z} \times 0.30\hat{x} + 0.00\hat{z} \times 0.10\hat{y} \\ &= 0.04\hat{z} - 0.04\hat{y} - 0.21\hat{z} + 0.07\hat{x} + 0 + 0 \\ &= 0.07\hat{x} - 0.04\hat{y} - 0.17\hat{z} \end{aligned}$$

e) the distance from the origin to the line defined by A_1 at P

$$
\begin{aligned}
\overrightarrow{A_1O} &= \overrightarrow{OP} + \overrightarrow{A_1P} \\
&= (2.80 - 0.40)\hat{x} + (3.70 - 0.70)\hat{y} + (4.60 - 0.00)\hat{z} \\
&= 2.40\hat{x} + 3.00\hat{y} + 4.60\hat{z} \\
\overrightarrow{A_1P} &= -A_1 \\
&= -0.40\hat{x} - 0.70\hat{y} \\
\overrightarrow{d} &= \overrightarrow{A_1O} \times \frac{\overrightarrow{A_1P}}{|\overrightarrow{A1_P}|} \\
&= 2.40\hat{x} + 3.00\hat{y} + 4.60\hat{z} \times \frac{-0.40\hat{x} - 0.70\hat{y}}{|-0.40\hat{x} - 0.70\hat{y}|} \\
d &= |\overrightarrow{d}| \\
&= 4.6384 \text{ units}
\end{aligned}
$$

f) the distance from the origin to the plane defined by A_1 and A_2 at P

$$
\begin{aligned}
D &= |\overrightarrow{A_1O} \cdot \hat{a}_n| \\
\hat{a}_n &= \frac{A_1 \times A_2}{|A_1 \times A_2|} \\
&= |2.40\hat{x} + 3.00\hat{y} + 4.60\hat{z} \cdot \frac{0.07\hat{x} - 0.04\hat{y} - 0.17\hat{z}}{|0.07\hat{x} - 0.04\hat{y} - 0.17\hat{z}|}| \\
&= 3.9012 \text{ units}
\end{aligned}
$$

2.2 Cyclindrical Coordinates

Transform the rectangular coordinates of P into cylindrical ones.

$$P(x, y, z) = (2.80, 3.70, 4.60)$$

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2} & \Phi &= \tan^{-1}(y/x) & z &= z \\
&= \sqrt{2.80^2 + 3.70^2} & &= \tan^{-1}(3.70/2.80) & &= 4.60 \\
&= 4.64 & &= 0.9230 \text{ rad} & &
\end{aligned}
$$

$$P(r, \Phi, z) = 4.64\hat{r} + 0.923\hat{\Phi} + 4.60\hat{z}$$

Transform A_1 and A_2 into cylindrical-component form.

$$
\begin{aligned}
A_1(A_1x, A_1y, A_1z) &= (0.40, 0.70, 0.00) \\
r &= \sqrt{x^2 + y^2} \\
&= \sqrt{0.40^2 + 0.70^2} \\
&= 0.8062 \\
\Phi &= \tan^{-1}(y/x) \\
&= \tan^{-1}(0.70/0.40) \\
&= 1.0517 \text{ rad} \\
z &= z \\
&= 0.00
\end{aligned}
$$

$$
\begin{aligned}
A_2(A_2x, A_2y, A_2z) &= (0.30, 0.10, 0.10) \\
r &= \sqrt{x^2 + y^2} \\
&= \sqrt{0.30^2 + 0.10^2} \\
&= 0.3162 \\
\Phi &= \tan^{-1}(y/x) \\
&= \tan^{-1}(0.10/0.30) \\
&= 0.3218 \text{ rad} \\
z &= z \\
&= 0.10
\end{aligned}
$$

$$
\begin{aligned}
A_1(A_1r, A_1\Phi, A_1z) &= 0.8062\hat{r} + 1.0517\hat{\Phi} \\
A_2(A_2r, A_2\Phi, A_2z) &= 0.3162\hat{r} + 0.3218\hat{\Phi} + 0.10\hat{z}
\end{aligned}
$$

Finally, find the dot product between the so transformed vectors. Is it the same as the dot product obtained in the rectangular coordinate system?

$$\begin{aligned} A_1 \cdot A_2 &= A_1\hat{r} \cdot A_2\hat{r} \cdot \cos(A_1\hat{\Phi} - A_2\hat{\Phi}) + A_1\hat{z} \cdot A_2\hat{z} \\ &= (0.8062 \cdot 0.3162 \cdot \cos(1.0517 - 0.3218)) + (0.00 \cdot 0.10) \\ &= 0.19 \end{aligned}$$

The dot product obtained in the rectangular coordinate system is the same dot product that was calculated in the rectangle coordinate system.

2.3 Spherical Coordinates

Transform the rectangular coordinates of P into spherical ones.

$$P(x, y, z) = (2.80, 3.70, 4.60)$$

$$\begin{aligned} R &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{2.80^2 + 3.70^2 + 4.60^2} \\ &= \sqrt{42.69} \\ &= 6.5338 \end{aligned} \qquad \begin{aligned} \theta &= \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{2.80^2 + 3.70^2}}{4.60}\right) \\ &= 0.7897 \text{ rad} \end{aligned} \qquad \begin{aligned} \Phi &= \tan^{-1}(y/x) \\ &= \tan^{-1}(3.70/2.80) \\ &= 0.9230 \text{ rad} \end{aligned}$$

$$P(R, \theta, \Phi) = 6.5338\hat{R} + 0.7897\hat{\theta} + 0.923\hat{\Phi}$$

Transform A_1 and A_2 into spherical-component form.

$$\begin{aligned} A_1(A_1x, A_1y, A_1z) &= (0.40, 0.70, 0.00) \\ R &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{0.40^2 + 0.70^2 + 0.00^2} \\ &= 0.8062 \\ \theta &= \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{0.40^2 + 0.70^2}}{0.00}\right) \\ &= \pi/2 \text{ rad} \\ \Phi &= \tan^{-1}(y/x) \\ &= \tan^{-1}(0.70/0.40) \\ &= 1.0517 \text{ rad} \end{aligned}$$

$$\begin{aligned} A_2(A_2x, A_2y, A_2z) &= (0.30, 0.10, 0.10) \\ R &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{0.30^2 + 0.10^2 + 0.10^2} \\ &= 0.3317 \\ \theta &= \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{0.30^2 + 0.10^2}}{0.10}\right) \\ &= 1.2645 \text{ rad} \\ \Phi &= \tan^{-1}(y/x) \\ &= \tan^{-1}(0.10/0.30) \\ &= 0.3218 \text{ rad} \end{aligned}$$

$$A_1(R, \theta, \Phi) = 0.8062\hat{R} + \frac{\pi}{2}\hat{\theta} + 1.0517\hat{\Phi}$$

$$A_2(R, \theta, \Phi) = 0.3317\hat{R} + 1.2645\hat{\theta} + 0.3218\hat{\Phi}$$

Find the cross product between the so transformed A_1 and A_2.

$$\begin{aligned} A_1 \times A_2 &= A_1\hat{R} \times A_2\hat{\theta} + A_1\hat{R} \times A_2\hat{\Phi} + A_1\hat{\theta} \times A_2\hat{R} + A_1\hat{\theta} \times A_2\hat{\Phi} \\ &= 0.1882\hat{R} + 2.6988\hat{\theta} - 0.5192\hat{\Phi} \end{aligned}$$

Transform the so found cross product into rectangular component form.

$$(A_1 \times A_2)_{RCS} = 0.07\hat{x} - 0.04\hat{y} - 0.17\hat{z}$$

The cross product obtained in the rectangular coordinate system is the same cross product that was calculated in the spherical coordiante system.