ELECENG 2FL3 ASSIGNMENT 6

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2 Problem Statement

A finite uniform linear charge $\rho L = 4$ nC/m lies in the $z = 0$ plane. For an example plot, see Figure 3.4 on p. 20 of the MATLAB® Manual. The coordinates of the two ends of the line charge, A and B, are given in Table 1. Find $\mathbf{E}$ at $(0, 0, 0)$.

3 Variation

#	A(m)	B(m)
0	5, 0, 0	0, 5, 0

Solution

$$
\begin{aligned}
E_{\parallel} & \\
E_{\perp} & \\
\vec{D_p} &= \hat{a_L} \times (\vec{AP} \times \hat{a_L}) \\
\vec{AP} &= -5\hat{a_x} \\
\hat{a_L} &= \frac{\vec{AB}}{|\vec{AB}|} = \frac{-5\hat{a_x} + 5\hat{a_y}}{\sqrt{50}} = \frac{-\hat{a_x} + \hat{a_y}}{\sqrt{2}} \\
\vec{AP} \times \hat{a_L} &= (-5\hat{a_x}) \times \frac{-\hat{a_x} + \hat{a_y}}{\sqrt{2}} = \frac{-5\hat{a_z}}{\sqrt{2}} \\
\vec{D_p} = \hat{a_L} \times (\vec{AP} \times \hat{a_L}) &= \frac{-\hat{a_x} + \hat{a_y}}{\sqrt{2}} \times \frac{-5\hat{a_z}}{\sqrt{2}} \\
&= \frac{-5\hat{a_y} - 5\hat{a_x}}{2} \\
\hat{a_\perp} &= \frac{\vec{D_p}}{|\vec{D_p}|} = \frac{-\hat{a_x} - \hat{a_y}}{\sqrt{2}} \\
\theta_1 &= 45^\circ, \theta_2 = 135^\circ \\
E_{\parallel} &= \frac{\rho_l}{4\pi\epsilon_0} \cdot \frac{1}{\pi} \cdot (\sin 135^\circ - \sin 45^\circ) = 0 \\
E_{\perp} &= \frac{\rho_l}{4\pi\epsilon_0} \cdot \frac{1}{\pi} \cdot (\cos 45^\circ - \cos 135^\circ) \\
&= \frac{\sqrt{2} \times \rho_l}{4\pi\epsilon_0} \cdot \frac{1}{r} \\
\vec{E}(0,0,0) &= \frac{\sqrt{2}\rho_l}{4\pi\epsilon_0 r}\hat{a_\perp} \\
r = |\vec{D_p}|, \hat{a_\perp} &= \frac{\vec{D_p}}{|\vec{D_p}|} \\
\vec{E}(0,0,0) &= \frac{\sqrt{2}\rho_l}{4\pi\epsilon_0} \cdot \frac{\vec{D_p}}{|\vec{D_p}|^2} \\
&= 9 \times 10^9 \cdot \sqrt{2} \cdot 4 \times 10^{-9} \cdot (\frac{-\hat{a_x} - \hat{a_y}}{5}) \\
\boxed{\vec{E}(0,0,0) = \frac{36\sqrt{2}}{5}(-\hat{a_x} - \hat{a_y})\ ,\ V/m}
\end{aligned}
$$