ELECENG 2FL3 ASSIGNMENT 5

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Analytical Problem

a) The reflection coefficient Γ at the load is equal to $0.0112 - 0.1051j$.

$$\begin{aligned}
\Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} \\
Z_L &= R_s + j\omega L_s + \frac{1}{j\omega C_s} \\
&= 50 + j \cdot 2 \cdot \pi \cdot 3 \cdot 10^9 \cdot 1.0 \cdot 10^{-9} + \frac{1}{j \cdot 2 \cdot \pi \cdot 3 \cdot 10^9 \cdot 1.8 \cdot 10^{-12}} \\
&= 50 - 10.6236j \\
\Gamma &= \frac{(50 - 10.6236j) - 50}{(50 - 10.6236j) + 50} \\
&= \frac{-10.6236j}{100 - 10.6236j} \\
\Gamma &= 0.0112 - 0.1051j
\end{aligned}$$

b) The SWR in the line is equal to 1.2362.

$$\begin{aligned}
SWR &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \\
&= \frac{1 + |0.0112 - 0.1051j|}{1 - |0.0112 - 0.1051j|} \\
&= 1.2362
\end{aligned}$$

c) The input impedance Z_{in} is equal to $40.4543 + 0.4404j$.

$$\begin{aligned}
Z_{in} &= Z_0 \cdot \left[\frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}\right] \\
&= 50 \cdot \left[\frac{(50 - 10.6236j) + j50 \tan(\frac{2\pi \cdot 3 \cdot 10^9}{299792458} \cdot 0.2136)}{50 + j(50 - 10.6236j) \tan(\frac{2\pi \cdot 3 \cdot 10^9}{299792458} \cdot 0.2136)}\right] \\
&= 40.4543 + 0.4404j
\end{aligned}$$