ELECENG 3CL4 Lab 2 Pre-lab

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February 15, 2021

1 Description of Laboratory Equipment

Pre-lab Question 1

The step response of the model $G(s)$ is in Equation 1. The derivation of the step response is shown below.

$$\begin{aligned}
\mathscr{L}^{-1}\left\{G(s)\frac{1}{s}\right\} &= \mathscr{L}^{-1}\left\{\frac{A}{s(s\tau_m+1)}\frac{1}{s}\right\} \\
&= A\ \mathscr{L}^{-1}\left\{\frac{1}{s^2(s\tau_m+1)}\right\} \\
&= A\ \mathscr{L}^{-1}\left\{\frac{{\tau_m}^2}{s\tau_m+1} - \frac{\tau_m}{s} + \frac{1}{s^2}\right\} \\
&= A\left[\mathscr{L}^{-1}\left\{\frac{{\tau_m}^2}{s\tau_m+1}\right\} - \mathscr{L}^{-1}\left\{\frac{\tau_m}{s}\right\} + \mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\}\right] \\
&= A\left[\tau_m e^{-\frac{t}{\tau_m}} - \tau_m u(t) + t\right] \qquad (1)
\end{aligned}$$

Pre-lab Question 2

The step response is not bounded because the term t is not bounded.

2 Closed Loop System Identification

Pre-lab Question 3

(i) The output signal of the model can be expressed as:

$$\begin{aligned}
Y(s) = \frac{G_c(s)G(s)}{1+H(s)G_c(s)G(s)}R(s) + \frac{G(s)}{1+H(s)G_c(s)G(s)}T_d(s) \\
+ \frac{H(s)G_c(s)G(s)}{1+H(s)G_c(s)G(s)}N(s)
\end{aligned}$$

If we neglect the effects of the disturbance $T_d(s)$ and the noise $N(s)$, we are left with:

$$Y(s) = \frac{G_c(s)G(s)}{1+H(s)G_c(s)G(s)}R(s)$$

When $H(s) = 1$ and we let $\Theta(s) = Y(s)$, we can find the transfer function $T(s)$ in Equation 2 (idk why this isn't working), which is equal to Equation 4a from the lab document.

$$\begin{aligned}
T(s) &= \frac{\Theta(s)}{R(s)} \\
&= \frac{\frac{G_c(s)G(s)}{1+H(s)G_c(s)G(s)}R(s)}{R(s)} \\
&= \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \qquad (2)
\end{aligned}$$

(ii) When we substitute $G(s) = \frac{A}{s(s\tau_m+1)}$ and $G_c(s) = K$ into Equation 2, and then substituting $\sqrt{\frac{KA}{\tau_m}} = \omega_n$ and $\frac{1}{2\omega_n\tau_m}$ into the result, we can derive Equation 3 which is equal to Equation 4b from the lab document:

$$\begin{aligned}
T(s) &= \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \\
&= \frac{\frac{A}{s(s\tau_m+1)}K}{1+\frac{A}{s(s\tau_m+1)}K} \\
&= \frac{KA}{s(s\tau_m+1)(1+\frac{A}{s(s\tau_m+1)}K)} \\
&= \frac{KA}{s(s\tau_m+1+\frac{KA}{s})} \\
&= \frac{KA}{s^2\tau_m+s+KA} \\
&= \frac{KA/\tau_m}{s^2+(1/\tau_m)s+KA/\tau_m} \\
&= \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad (3)
\end{aligned}$$

Pre-lab Question 4

When given ζ and ω_n, we can derive Equation 4 for τ_m:

$$\begin{aligned}
\zeta &= \frac{1}{2\omega_n\tau_m} \\
\tau_m &= \frac{1}{2\omega_n\zeta} \qquad (4)
\end{aligned}$$

By substituting Equation 4 into the equation for ω_n, we can derive Equation 5 for A:

$$\begin{aligned}
\omega_n &= \sqrt{\frac{KA}{\tau_m}} \\
\omega_n^2 &= \frac{KA}{\tau_m} \\
A &= \frac{\omega_n^2\tau_m}{K} \qquad (5) \\
A &= \frac{\omega_n/2\zeta}{K}
\end{aligned}$$

2.1 Closed-loop System Identification from the Step Response

Pre-lab Question 5

The percent overshoot is determined at the peak time, T_p, of the step response. For the provided step response, this peak time occurs the first time $d\theta_{\text{step}}(t)/dt = 0$, which occurs when $\omega_n\sqrt{1-\zeta^2}t = \pi$. Therefore, the peak time T_p is equal to $\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$. At the peak time, the peak value of the step response is $1+\exp\left(-\zeta\pi/\sqrt{1-\zeta^2}\right)$. Therefore, the percent overshoot can be defined as P.O. $= 100\frac{(1+\exp^{-(\zeta\pi/\sqrt{1-\zeta^2})})-1}{1} = 100\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$.

Pre-lab Question 6
As discussed in the solution to Pre-lab Question 5, the peak time occurs the first time $d\theta_{\text{step}}(t)/dt = 0$, which occurs when $\omega_n\sqrt{1-\zeta^2}t = \pi$. Therefore, the peak time T_p is equal to $\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$.

Pre-lab Question 7
From the equation for P.O., you can calculate ζ as:

$$\begin{aligned}
P.O. &= 100\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)\\
\frac{P.O.}{100} &= \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)\\
\ln\left(\frac{P.O.}{100}\right) &= \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\\
\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2} &= \frac{\zeta^2}{1-\zeta^2}\\
\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2} &= 1-\frac{1}{1+\zeta^2}\\
1-\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2} &= \frac{1}{1+\zeta^2}\\
1+\zeta^2 &= \frac{1}{\left(1-\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2}\right)}\\
\zeta^2 &= \frac{1}{\left(1-\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2}\right)}-1\\
\zeta &= \sqrt{\frac{1}{\left(1-\frac{\left(\ln\left(\frac{P.O.}{100}\right)\right)^2}{\pi^2}\right)}-1} \qquad (6)
\end{aligned}$$

ok elts try something thats probably a bit easier

$$\begin{aligned}
T_p &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}\\
\omega_n &= \frac{\pi}{T_p i\sqrt{1-\zeta^2}} \qquad (7)
\end{aligned}$$

2.2 Closed-Loop System Identification using the Frequency Response

Pre-lab Question 8

Pre-lab Question 9

Pre-lab Question 10