ELEC ENG 3CL4: Introduction to Control Systems Lab 5: Phase Lead-Lag Compensator Design Using Root Locus (Pre-Lab Only)

Tim Davidson davidson@mcmaster.ca

Objective

To design a phase lead-lag compensator for a marginally-stable servomotor using the root locus method, and to examine the frequency domain properties of the design.

Assessment

This laboratory is conducted in groups of no more than two students. The students are required to attend their assigned lab section. The assessment of this lab will occur based on your pre-lab design report, in-lab design and experiment activities, and a final lab report. Each group is required to submit their pre-lab report electronically through the course webpage by 12:01pm on the day of the lab. Pre-labs submitted after 12:01pm but before 2:30pm will be subject to a penalty of 50%. No marks will be awarded to pre-labs submitted after 2:30pm. You will earn a maximum of 100 marks from Lab 5 activities. Lab 5 will contribute to a maximum of 25% of your total lab grade for this course. The components of the assessment are:

- Pre-lab root locus based phase lead-lag control design and evaluation **(30 marks)**;
- In-lab design and experiment activities **(45 marks)**;
- Final laboratory report **(25 marks)**.

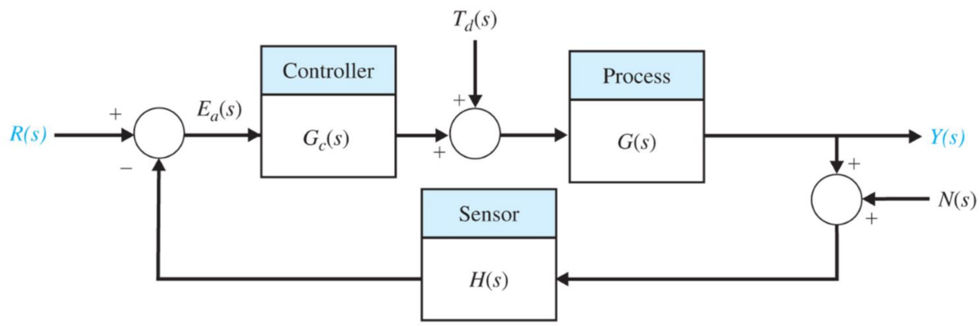

Figure 1: A feedback control system. We will focus on the case in which $H(s) = 1$. (Figure 10.1 of Dorf and Bishop, *Modern Control Systems*, 11th edition, Prentice Hall, 2008.)

1 Pre-lab Design Exercise

Consider the closed loop feedback control system in Fig. 1 in the case in which $H(s) = 1$ and $G(s) = \frac{4.7}{s(s+3.2)}$. In the pre-lab exercise of Lab 4, you designed a phase-lead compensator of the form $G_{c,\text{lead}}(s) = K_c \frac{(s+z_{\text{lead}})}{(s+p_{\text{lead}})}$, for which $p_{\text{lead}} > z_{\text{lead}}$ so that for a unit step input the percentage overshoot of the output $y(t)$ was 30% and its 2% settling time was 0.75 second. In this pre-lab exercise, you will modify your controller from the pre-lab of Lab 4 to satisfy the following additional design requirement:

- The steady-state error due to a step disturbance $T_d(s) = \frac{A}{s}$ is one fifth of what it was when the lead compensator was used.

You should ensure that in achieving this additional requirement you do not (significantly) violate the percentage overshoot and settling time requirements.

To achieves this objective, you can augment your phase-lead controller from the pre-lab of Lab 4 by adding a phase-lag compensator and by doing so form a lead-lag compensator:

$$G_{c,\text{lead-lag}}(s) = \frac{(s+z_{\text{lag}})}{(s+p_{\text{lag}})} G_{c,\text{lead}}(s) = K_c \frac{(s+z_{\text{lag}})}{(s+p_{\text{lag}})} \frac{(s+z_{\text{lead}})}{(s+p_{\text{lead}})} \tag{1}$$

Here $G_{c,\text{lead}}(s)$ is the phase-lead controller from the pre-lab exercise of Lab 4, and $\frac{(s+z_{\text{lag}})}{(s+p_{\text{lag}})}$, with $z_{\text{lag}} > p_{\text{lag}}$ is an additional lag compensator to be designed. The phase lead component provides for the transient response requirements whereas the phase lag part helps achieve the disturbance rejection objective.

To this end,

1. For the closed loop system in Fig. 1 with $H(s) = 1$ derive the transfer function from the disturbance to the output, $\frac{Y(s)}{T_d(s)}$. **(3 marks)**

2. For the case in which $G(s) = \frac{4.7}{s(s+3.2)}$, derive the steady-state value of the output in the absence of any input, when the disturbance is a step function of size A. That is, derive an expression for the steady state value of $y(t)$ when $r(t) = 0$ and $t_d(t) = Au(t)$. This expression will be a function of an aspect of the (generic) controller transfer function, $G_c(s)$. **(4 marks)**

3. For the case of a lead compensator, write the expression that you derived in Item 2 as a function of K_c, z_{lead}, and p_{lead}. **(3 marks)**

4. For the case of a lead-lag compensator, write the expression that you derived in Item 2 as a function of K_c, z_{lead}, p_{lead}, z_{lag}, and p_{lag}. **(3 marks)**

5. Retrieve the parameters of the phase lead compensator, K_c, z_{lead}, and p_{lead} from the pre-lab exercise of Lab 4. **(3 marks)**

6. Calculate the steady-state value of the step disturbance error when this controller is used. **(3 marks)**

7. What relationship between z_{lag} and p_{lag} will ensure that when the lead-lag controller is used the steady-state response due to a step disturbance will be five times smaller than the response when the lead controller is used. **(4 marks)**

8. From the pre-lab exercise for Lab 4, retrieve the positions of the closed-loop poles of the lead compensated loop.

9. How can we choose z_{lag} and p_{lag} so that the relationship in Item 7 is preserved and the transient performance of the closed loop is not changed very much? **(4 marks)**

10. Select values for z_{lag} and p_{lag} that you think will be appropriate. (*Hint:* Consider making an argument for why choosing $z_{\text{lag}} = 0.2$ is a reasonable choice.) **(3 marks)**

11. Now verify your design using the following sequence of Matlab operations. There are no marks for this verification, but you may find it quite helpful. (The Matlab script `Lab5_prelab.m` might simplify this stage.)

    (a) In Matlab, type `help tf` and build Matlab transfer functions models for $G(s)$, $G_{c,\text{lead}}(s)$ and your designed $G_{c,\text{lead-lag}}(s)$.

    (b) Type `help series`, and build Matlab transfer functions models for the open loop transfer functions $L_{\text{lead}}(s) = G_{c,\text{lead}}(s)G(s)$ and $L_{\text{lead-lag}}(s) = G_{c,\text{lead-lag}}(s)G(s)$

    (c) Type `help rlocus` and plot, perhaps on the same graph, the root loci of the lead compensated and lead-lag compensated open loop transfer functions. (Type `help hold` for information on how to plot multiple curves on a single graph.)

    (d) Type `help feedback` and build Matlab transfer function models for the input-output transfer functions for both the lead compensated and lead-lag compensated cases.

    (e) Type `help pole` and use that function to plot the closed-loop poles of the lead-compensated closed loop and the lead-lag compensated closed loop on the root locus plots.

    (f) Recall that the zeros of the closed-loop transfer function from input to output are the same as the zeros of the open loop transfer function.

    (g) Type `help step` and use that function to plot the input-output step responses of the lead-compensated closed loop and the lead-lag compensated closed loop. Comment on the relationships between these responses. Are the design criteria related to the transient response satisfied? Describe how your answers could have been predicted from the closed-loop pole and zero positions.

    (h) Type `help lsim` and use that function to plot the response of each system to a unit ramp input. Make sure that the length of the ramp signal is at least four times the time constant associated with the slowest pole that you identified in Item 11e. (You may wish to review item n) from the pre-lab for Lab 4 before doing this step.)

    (i) Comment on any differences you observe. Pay particular attention to the steady state.

    (j) Calculate the velocity error constant of the lead-compensated open loop and the lag-compensated open loop. Do these constants predict any of your observations.

    (k) Type `help bode` and use that function to draw the Bode diagram of the open loop transfer functions $L_{\text{lead}}(s) = G_{c,\text{lead}}(s)G(s)$ and $L_{\text{lead-lag}}(s) = G_{c,\text{lead-lag}}(s)G(s)$. Do these diagrams predict any of your observations?

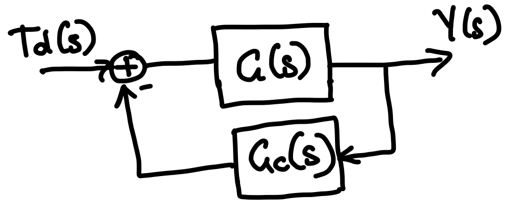

Figure 2: A equivalent diagram for Fig. 1 when $R(s) = 0$ and $N(s) = 0$.

(l) From those diagrams, read off the cross-over frequency, ω_c, for each open loop. The cross-over frequency is the frequency at which $|L(j\omega_c)| = 1$. Once you have determined the cross-over frequency, read off the phase margin, in degrees, which is $180° + \angle L(j\omega_c)$.

(m) Observe that in the absence of an input signal and the noise signal, we can redraw the closed-loop in Fig. 1 in the form in Fig. 2.

(n) Use the `feedback` command to build a Matlab model for the transfer function from disturbances to output for both the lead-compensated loop and the lead-lag compensated loop.

(o) Use the `step` command to display the step-disturbance responses for those two closed loop. Comment on the results in relation to the design objectives and the closed-loop pole positions.