ELECENG 3CL4 Lab 3 Pre-lab

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1 Proportional Control of DC Motor

Pre-Lab Question 1 The closed loop transfer function $T(s)$ is:

$$\begin{aligned} T(s) &= \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \\ &= \frac{k_pG(s)}{1+k_pG(s)} \end{aligned}$$

We find the characteristic equation of the closed loop transfer function $T(s)$ below:

$$\begin{aligned} 0 &= 1 + k_pG(s) \\ 0 &= 1 + \frac{k_pA}{s(s\tau_m+1)} \\ 0 &= 1 + \frac{k_pA}{s^2\tau_m + s} \\ -1 &= \frac{k_pA}{s^2\tau_m + s} \\ -s^2\tau_m - s &= k_pA \\ 0 &= s^2\tau_m + s + k_pA \end{aligned}$$

The closed-loop poles of the system is found by determining the poles of the characteristic equation, which is done in Equation 1.

$$\begin{aligned} p_{1,2} &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ a &= \tau_m,\ \ b = 1,\ \ c = k_pA \\ &= \frac{-1 \pm \sqrt{1-4\tau_m k_pA}}{2\tau_m} \\ p_{1,2} &= -\frac{1}{2\tau_m} \pm \frac{1}{2\tau_m}\sqrt{1-4k_pA\tau_m} \end{aligned} \tag{1}$$

Pre-Lab Question 2

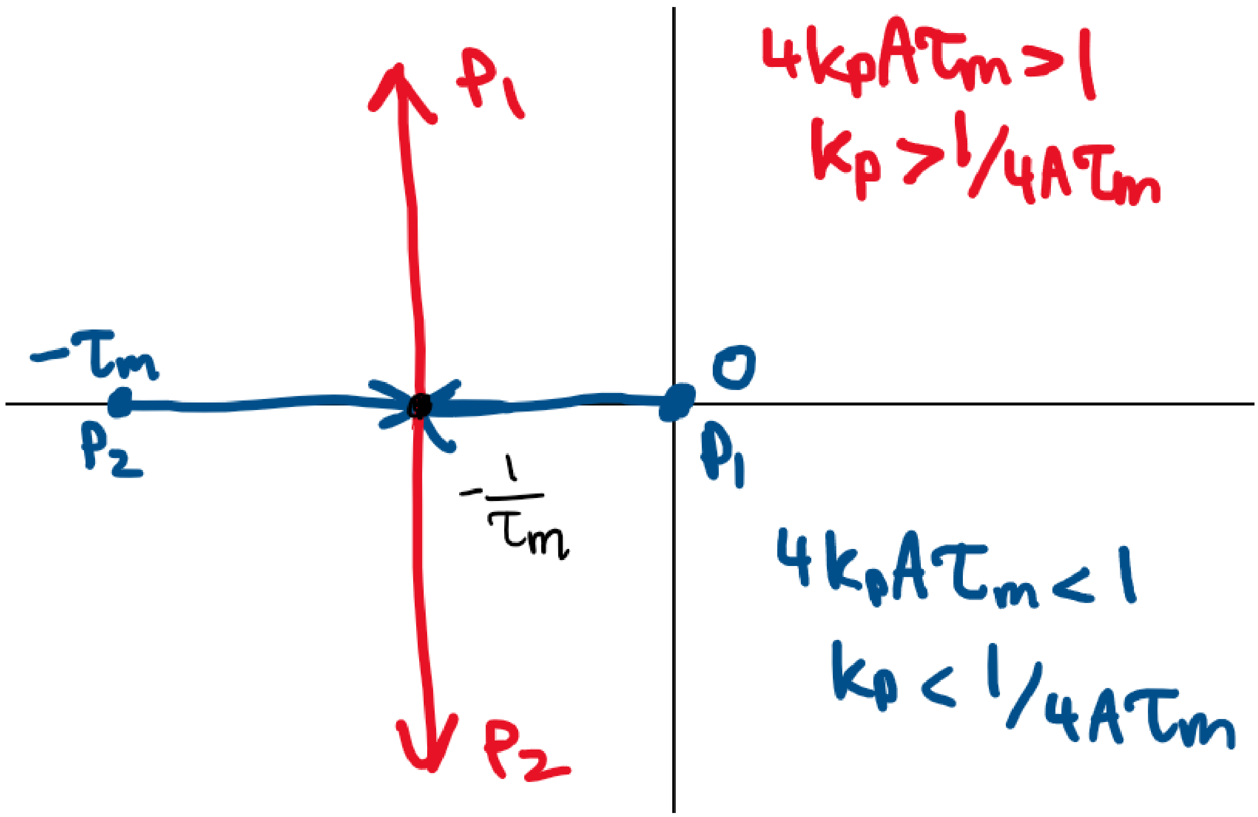

Figure 1: q2

Pre-Lab Question 3

$$
\begin{aligned}
T(s) &= \frac{k_p G(s)}{1 + k_p G(s)} \\
&= \frac{\frac{k_p A}{s^2\tau_m + s}}{1 + \frac{k_p A}{s^2\tau_m + s}} \\
&= \frac{k_p A}{s^2\tau_m + s + k_p A} \\
&= \frac{\frac{k_p A}{\tau_m}}{s^2 + \frac{s}{\tau_m} + \frac{k_p A}{\tau_m}} \\
F_2(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\
\omega_n^2 &= \frac{k_p A}{\tau_m} \\
\omega_n &= \sqrt{\frac{k_p A}{\tau_m}} \\
2\zeta\omega_n s &= \frac{s}{\tau_m} \\
\zeta\omega_n &= \frac{1}{2\tau_m} \\
\zeta &= \frac{1}{2\tau_m\omega_n} \\
\zeta &= \frac{1}{2\tau_m\sqrt{\frac{k_p A}{\tau_m}}} \\
\zeta &= \frac{1}{2\sqrt{k_p A \tau_m}}
\end{aligned}
$$

Pre-Lab Question 4

$$
\begin{aligned}
\zeta &= \frac{1}{2\sqrt{k_p A \tau_m}} \\
&= \frac{1}{2\sqrt{\frac{A\tau_m}{4A\tau_m}}} \\
&= \frac{1}{2\sqrt{\frac{1}{4}}} \\
&= \frac{1}{2(0.5)}\zeta \qquad = 1
\end{aligned}
$$

c'est critically damped

Pre-Lab Question 5

$$
\begin{aligned}
p_{1,2} &= -\frac{1}{2\tau_m} \pm \frac{1}{2\tau_m}\sqrt{1-4k_pA\tau_m} \\
&= \frac{1}{2\tau_m}\left(-1 \pm \sqrt{1-4k_pA\tau_m}\right)
\end{aligned}
$$

2 Trade-offs in Proportional Control of a Servomotor: Theoretical Insight

Pre-Lab Question 6

$$
\begin{aligned}
T_s &\approx \frac{4}{\zeta\omega_n} \\
&\approx \frac{4}{\frac{\omega_n}{2\omega_n\tau_m}} \\
&\approx \frac{4}{\frac{1}{2\tau_m}} \\
&\approx 8\tau_m \\
P.O. &= 100\exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \\
&= 100\exp\left(-\frac{\frac{\pi}{2\omega_n\tau_m}}{\sqrt{1-\left(\frac{1}{2\omega_n\tau_m}\right)^2}}\right) \\
&= 100\exp\left(-\frac{\frac{\pi}{2\omega_n\tau_m}}{\sqrt{1-\frac{1}{4\omega_n^2\tau_m^2}}}\right) \\
&= 100\exp\left(-\frac{\pi}{2\omega_n\tau_m\sqrt{1-\frac{1}{4\omega_n^2\tau_m^2}}}\right) \\
&= 100\exp\left(-\frac{\pi}{2\omega_n\tau_m\sqrt{\frac{4\omega_n^2\tau_m^2-1}{4\omega_n^2\tau_m^2}}}\right) \\
&= 100\exp\left(-\frac{\pi}{2\omega_n\tau_m\frac{\sqrt{4\omega_n^2\tau_m^2-1}}{\sqrt{4\omega_n^2\tau_m^2}}}\right) \\
&= 100\exp\left(-\frac{\pi}{\sqrt{4\omega_n^2\tau_m^2-1}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 100\exp\left(-\frac{\pi}{\sqrt{4\frac{k_pA}{\tau_m}\tau_m^2 - 1}}\right) \\
&= 100\exp\left(-\frac{\pi}{\sqrt{4k_pA\tau_m - 1}}\right) \\
T_{r1} &\approx \frac{2.16\zeta + 0.6}{\omega_n} \\
&\approx \frac{\frac{2.16}{2\omega_n\tau_m} + 0.6}{\omega_n} \\
&\approx \frac{\frac{2.16+1.2\omega_n\tau_m}{2\omega_n\tau_m}}{\omega_n} \\
&\approx \frac{2.16 + 1.2\omega_n\tau_m}{2\omega_n^2\tau_m} \\
&\approx \frac{2.16 + 1.2\sqrt{\frac{k_pA}{\tau_m}}\tau_m}{2\frac{k_pA}{\tau_m}\tau_m} \\
&\approx \frac{2.16 + 1.2\sqrt{k_pA\tau_m}}{2k_pA}
\end{aligned}
$$

Pre-Lab Question 7 T_s does not change with k_p. P.O. increases as k_p increases and it approaches a horizontal asymptote of 100%. T_{r1} decreases as k_p decreases and approaches the horizontal asymptote of 0.

Pre-Lab Question 8 The 2% settling time will not change no matter what we do for k_p.

Pre-Lab Question 9

Pre-Lab Question 10

Pre-Lab Question 11

6 Proportional Controller with Velocity Feedback

Pre-Lab Question 12 If we use block diagram transforms we can transform the block diagram to Figure(ref figure here). From the block diagram, we can see that the total

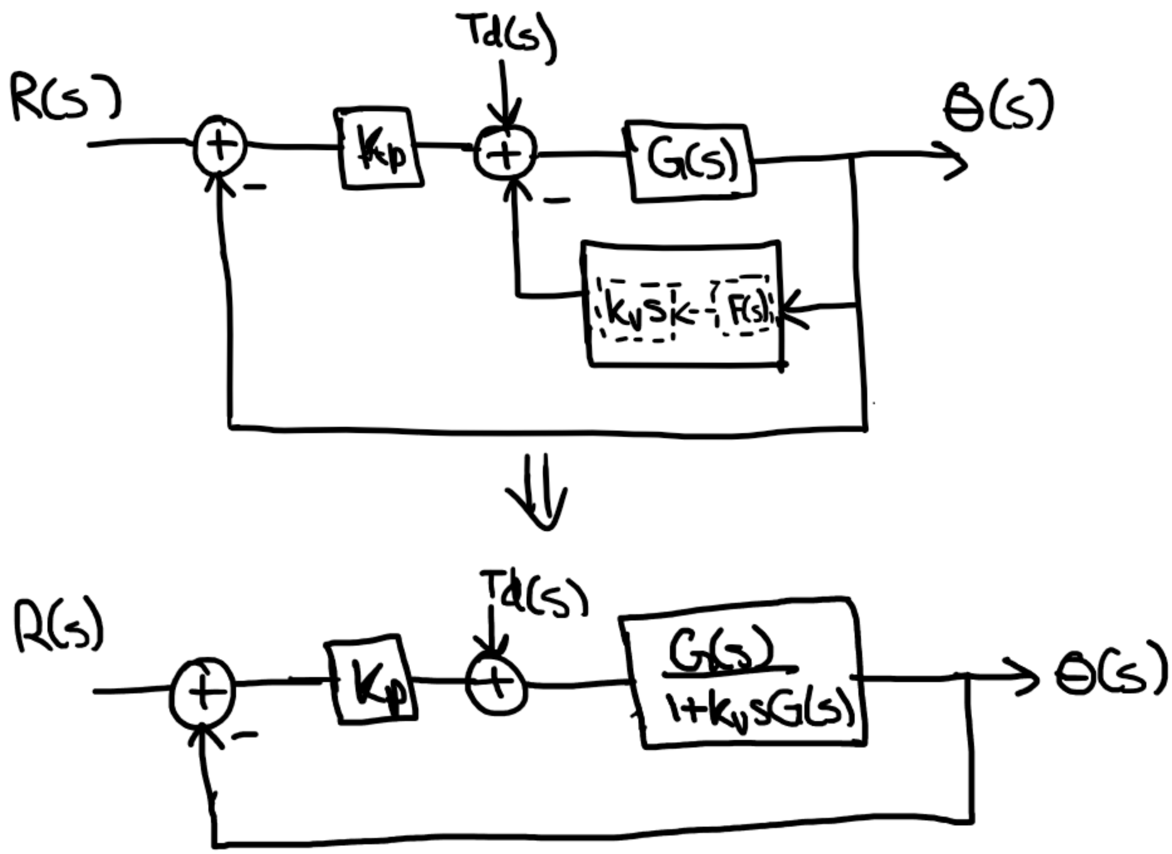

Figure 2: q12

output response when $F(s) \approx 1$ can be written as:

$$
\begin{aligned}
\Theta(s) &= \frac{k_p G(s)}{1 + k_v s G(s) + k_p G(s)} R(s) + \frac{G(s)}{1 + k_v s G(s) + k_p G(s)} T_d(s) \\
&= \frac{\frac{k_p A}{s(s\tau_m+1)}}{1 + s\frac{k_v A}{s(s\tau_m+1)} + \frac{k_p A}{s(s\tau_m+1)}} R(s) + \frac{\frac{A}{s(s\tau_m+1)}}{1 + s\frac{k_v A}{s(s\tau_m+1)} + \frac{k_p A}{s(s\tau_m+1)}} T_d(s) \\
&= \frac{k_p A}{s(s\tau_m + 1) + k_v A s + kpA} R(s) + \frac{A}{s(s\tau_m + 1) + k_v A s + kpA} T_d(s) \\
&= \frac{\frac{k_p A}{\tau_m}}{s^2 + \frac{1+k_v A}{\tau_m} s + \frac{k_p A}{\tau_m}} R(s) + \frac{\frac{A}{\tau_m}}{s^2 + \frac{1+k_v A}{\tau_m} s + \frac{k_p A}{\tau_m}} T_d(s)
\end{aligned} \tag{2}
$$

Pre-Lab Question 13

$$
\begin{aligned}
e_{ss} &= \lim_{s \to 0} s \frac{A}{s(s\tau_m + 1) + k_v A s + k_p A} T_d(s) \\
&= \lim_{s \to 0} s \frac{A}{s(s\tau_m + 1) + k_v A s + k_p A} \frac{\tau_d}{s} \\
&= \lim_{s \to 0} \frac{A\tau_d}{s(s\tau_m + 1) + k_v A s + k_p A} \\
&= \frac{A\tau_d}{k_p A + \lim_{s \to 0} s(s\tau_m + 1) + k_v A s} \\
&= \frac{A\tau_d}{k_p A} \\
&= \frac{\tau_d}{k_p}
\end{aligned} \tag{3}
$$

Pre-Lab Question 14 The closed-loop transfer function can be written as:

$$
\begin{aligned}
T(s) &= \frac{k_p G(s)}{1 + k_v s G(s) + k_p G(s)} \\
&= \frac{\frac{k_p A}{s(s\tau_m+1)}}{1 + \frac{k_v A}{s(s\tau_m+1)} s + \frac{k_p A}{s(s\tau_m+1)}} \\
&= \frac{k_p A}{s^2 \tau_m + (1 + k_v A)s + k_p A} \\
&= \frac{\frac{k_p A}{\tau_m}}{s^2 + \frac{1+k_v A}{\tau_m} s + \frac{k_p A}{\tau_m}}
\end{aligned}
$$

We can find ζ by writing the closed-loop transfer function in the form of a standard secondorder system.

$$F_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{k_p A}{\tau_m}$$

$$\omega_n = \sqrt{\frac{k_p A}{\tau_m}}$$

$$2\zeta\omega_n s = \frac{1 + k_v A}{\tau_m} s$$

$$\zeta\omega_n = \frac{1 + k_v A}{2\tau_m}$$

$$\zeta = \frac{1 + k_v v A}{2\tau_m \omega_n}$$

$$\zeta = \frac{1 + k_v A}{2\tau_m \sqrt{\frac{k_p A}{\tau_m}}}$$

$$\zeta = \frac{1 + k_v A}{2\sqrt{k_p A \tau_m}}$$

Pre-Lab Question 15 Increasing k_p will decrease ζ, which will lead to a decrease in the rise time and an increase in the maximum overshoot, while decreasing k_p will increase the rise time and decrease the maximum overshoot. Increasing k_v will increase ζ, which will lead to an increase in the rise time and a decrease in the maximum overshoot, while decreasing k_v will decrease the rise time and increase the maximum overshoot. The steady-state error to a constant disturbance is inversely proportional to k_p.