ELECENG 3CL4 Lab 5 Pre-lab

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April 4, 2021

1. The output of the model can be expressed as:

$$Y(s) = \frac{G_c(s)G(s)}{1 + H(s)G_c(s)G(s)}R(s) + \frac{G(s)}{1 + H(s)G_c(s)G(s)}T_d(s) + \frac{H(s)G_c(s)G(s)}{1 + H(s)G_c(s)G(s)}N(s) \quad (1)$$

From Equation 1, we can derive $\frac{Y(s)}{T_d(s)}$ in Equation 2 when $R(s) = 0$ and $H(s) = 1$.

$$\begin{aligned} Y(s) &= \frac{G(s)}{1 + H(s)G_c(s)G(s)}T_d(s) \\ \frac{Y(s)}{T_d(s)} &= \frac{G(s)}{1 + H(s)G_c(s)G(s)} \\ &= \frac{G(s)}{1 + G_c(s)G(s)} \end{aligned} \quad (2)$$

2. The steady-state value of the output with a step disturbance of size A and the absence of any input can be derived through the final value theorem as:

$$\begin{aligned} e_{ss} &= \lim_{s \to 0} s \frac{G(s)}{1 + G_c(s)G(s)} \frac{A}{s} \\ &= \lim_{s \to 0} \frac{AG(s)}{1 + G_c(s)G(s)} \\ &= \lim_{s \to 0} \frac{A\frac{4.7}{s(s+3.2)}}{1 + G_c(s)\frac{4.7}{s(s+3.2)}} \\ &= \lim_{s \to 0} \frac{4.7A}{s(s+3.2) + 4.7G_c(s)} \\ &= 4.7A \lim_{s \to 0} \frac{1}{s(s+3.2) + 4.7G_c(s)} \\ &= 4.7A \frac{1}{\lim_{s \to 0}(s(s+1)) + \lim_{s \to 0}(4.7G_c(s))} \\ &= 4.7A \frac{1}{0 + 4.7 \lim_{s \to 0}(G_c(s))} \\ &= \frac{4.7A}{4.7 \lim_{s \to 0}(G_c(s))} \\ &= \frac{A}{\lim_{s \to 0} G_c(s)} \end{aligned} \quad (3)$$

3.
$$
\begin{aligned}
&= \frac{A}{\lim_{s\to 0} G_c(s)} \\
&= \frac{A}{\lim_{s\to 0}\left(K_c \frac{(s+z_{lead})}{(s+p_{lead})}\right)} \\
&= \frac{A}{K_c\left(\frac{z_{lead}}{p_{lead}}\right)} \\
&= \frac{A}{K_c}\left(\frac{p_{lead}}{z_{lead}}\right)
\end{aligned} \tag{4}
$$

4.
$$
\begin{aligned}
&= \frac{A}{\lim_{s\to 0} G_c(s)} \\
&= \frac{A}{\lim_{s\to 0}\left(K_c \frac{(s+z_{lead})}{(s+p_{lead})}\frac{(s+z_{lag})}{(s+p_{lag})}\right)} \\
&= \frac{A}{K_c\left(\frac{z_{lead}}{p_{lead}}\right)\left(\frac{z_{lag}}{p_{lag}}\right)} \\
&= \frac{A}{K_c}\left(\frac{p_{lead}}{z_{lead}}\right)\left(\frac{p_{lag}}{z_{lag}}\right)
\end{aligned} \tag{5}
$$

5. $K_c = 62.1$, $z_{lead} = 6$, $p_{lead} = 15.51$

6.

7.

8. poles at $-6 \pm 15j$

9. We can choose values of z_{lag} and p_{lag} that are near the origin. This would not affect the transient performance of the closed loop very much as the transient responses of the zero and pole would decay very quickly.

10.