ELECENG 3CL4 Lab 3 Report

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Member Contributions

Both group members contributed an even amount to both the exercises and the report. Both members went through the exercises together and contributed to all sections of the report.

Objective

To design a proportional controller and a proportional controller with velocity feedback for the DC motor servomechanism, and explore trade-offs involved in the selection of the controller parameters and their impact on transient and steady-state responses of the control system.

3 Experiment: Qualitative Trade-offs in Rise-time, Steady-State Error, Overshoot, and Settling Time for a Proportional Controller

In order to measure the peak overshoot (with respect to the final value) and steady state error, we measured the delta between the first peak and the steady state value using the peak finder tool for the peak and a cursor for the steady state value. The settling time measurement was performed by moving a cursor to the point at which the last oscillation was ending and the plot showed a straight line after. The time value shown on the cursor table was then taken as the settling time. The rise time was measured using a cursor as the time from 0 to the first instance of when the output reached the steady state value. We tried the "Bilevel Measurements" tool, but we found it to be inaccurate.

k_p	0.5	1	2	3	4	5
peak overshoot (%)	0.7	38.4	54.7	65.7	75.3	82.4
steady-state error (deg)	26.89	8.09	5.10	3.69	2.64	1.93
settling time (sec)	0.336	0.469	0.618	0.771	0.881	1.067
rise time (sec)	0.336	0.125	0.100	0.076	0.064	0.058

Table 1: Measurements for Experiment 1

ok

4 Experiment: Proportional Controller Design

To determine the values for k_p that would meet the design specifications (produce underdamped with maximum overshoot of 50%, 55%, 60%, and 65%), we used the expression from the lab document to estimate the percent overshoot of an underdamped system. The expression was rearranged to solve for k_p in Equation 1. The calculations to determine the values that would meet the specifications were done in MATLAB, with the MATLAB code shown in Listing 1. The values of A and τ_m were the average of their determined values from time-domain and frequency-domain analysis in lab 2.

$$\begin{aligned}
P.O. &= 100 \exp\left(-\frac{\pi}{\sqrt{4k_pA\tau_m - 1}}\right) \\
\ln\left(\frac{P.O.}{100}\right) &= -\frac{\pi}{\sqrt{4k_pA\tau_m - 1}} \\
\sqrt{4k_pA\tau_m - 1} &= \frac{-\pi}{\ln\left(\frac{P.O.}{100}\right)} \\
4k_pA\tau_m - 1 &= \left(\frac{-\pi}{\ln\left(\frac{P.O.}{100}\right)}\right)^2 \\
4k_pA\tau_m &= \left(\frac{-\pi}{\ln\left(\frac{P.O.}{100}\right)}\right)^2 + 1 \\
k_p &= \left(\left(\frac{-\pi}{\ln\left(\frac{P.O.}{100}\right)}\right)^2 + 1\right) \; / \; 4A\tau_m \qquad (1)
\end{aligned}$$

Listing 1: Calculating k_p to meet specifications

```
%% Experiment 2
tau_m = (0.133 + 0.155)/2; % average of lab 2 determined values
A = (25.877 + 30.303)/2; % average of lab 2 determined values

po = [50 55 60 65]; % percent overshoot specifications
kp = ((-pi./log(po./100)).^2 + 1) ./ (4 * A * tau_m)
```

k_p	1.3314	1.7685	2.3395	3.3489
theoretical peak overshoot (%)	50	55	60	65
peak overshoot (%)	41.0	50.4	59.8	69.9
theoretical steady-state error (deg)	7.51	5.65	4.17	2.99
steady-state error (deg)	6.68	7.03	3.34	2.11
settling time (sec)	0.53	0.51	0.58	0.70

Table 2: Measurements for Experiment 2

6 Joint Design of Proportional Controller and Velocity Feedback

There are two design requirements to consider. We know that the magnitude of the steady-state error can be given by the expression $|e_{ss}| = \frac{\tau_d}{k_p}$. If the steady-state error at $k_p = 1$ is 10 degrees, to reduce the steady-state by a factor of 5 (to 2 degrees), we need to increase k_p by a factor of 5. Therefore the design requirement for the magnitude of the steady-state error can be met by any $k_p > 5$.

$$\zeta = -\frac{\ln\left(\frac{P.O.}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{P.O.}{100}\right)}} \tag{2}$$

rearrange expression for ζ and k_v

$$\zeta = \frac{1 + k_v A}{2\sqrt{k_p A \tau_m}}$$
$$k_v = \frac{2\zeta\sqrt{k_p A \tau_m} - 1}{A} \tag{3}$$

Listing 2: Calculating k_v to meet specifications

```
%% Experiment 3
po = 10;
zeta = -log(po/100)/sqrt(pi^2 + log(po/100)^2);
kp = [5 6 7];
kv = (zeta*2*sqrt(kp*A*tau_m)-1)/A
```