# ELECENG 3TQ3 Assignment 4

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1. Given Gaussian random variables $X$ and $Y$ with zero means and covariance matrix $\left[\begin{smallmatrix} 4 & 1 \\ 1 & 9 \end{smallmatrix}\right]$ find random variables $U = aX + bY$ and $V = cX + dY$ so that $U$ and $V$ are uncorrelated. Hint: there are multiple solutions for this problem you just need to pick one.
   From the covariance matrix, we can determine that:
   $\operatorname{cov}(x, y) = \operatorname{cov}(y, x) = 1$, $\operatorname{Var}[X] = \sigma_X^2 = 4$, $\operatorname{Var}[Y] = \sigma_Y^2 = 9$

$$\begin{aligned}
\operatorname{cov}(u, v) &= \operatorname{cov}(ax + by, cx + dy) \\
&= ac \operatorname{cov}(x, x) + ad \operatorname{cov}(x, y) + bc \operatorname{cov}(y, x) + bd \operatorname{cov}(y, y) \\
&= ac \operatorname{Var}[X] + ad(1) + bc(1) + bd \operatorname{Var}[Y] \\
\operatorname{cov}(u, v) &= 4ac + ad + bc + 9bd \\
0 &= 4ac + ad + bc + 9bd
\end{aligned}$$

   This equation has multiple solutions. A possible solution is $a = 1$, $b = 1$, $c = 1$, and $d = -0.5$.

2. Assume that the number of trucks entering weigh station per day is distributed as a Poisson random variable with $\lambda$=50 and that the weights of each truck are iid Gaussian distributed with mean 7500 kg and standard deviation = 500 kg and independent of the number of trucks entering the stop. Let M be total weight measured per day in the weigh station. Find the expected value and variance of M. Hint: you can either use moment generating functions or central limit theorem.
   Let $N$ = Poisson random variable for number of trucks entering weigh station
   Let $X$ = Gaussian random variable for weights of each truck

$$\begin{aligned}
\phi_M(s) &= \phi_N(\ln \phi_X(s)) \\
\phi_X(s) &= e^{s\mu + s^2\sigma^2/2} = e^{7500s + 125000s^2} \\
\phi_N(s) &= e^{\lambda(e^s - 1)} = e^{50(e^s - 1)} \\
\phi_M(s) &= e^{50(e^{\ln(e^{7500s+125000s^2})} - 1)} \\
\phi_M(s) &= e^{50(e^{(7500s+125000s^2)} - 1)} \\
\phi_M'(s) &= 50e^{50\left(e^{7500s+125000s^2} - 1\right) + 125000s^2 + 7500s}\,(250000s + 7500) \\
\mathrm{E}[M] &= \phi_M'(0) = \mathrm{E}[N]\mathrm{E}[X] = 50 \cdot 7500 = 375000 \text{ kg} \\
\operatorname{Var}[M] &= \mathrm{E}[N]\operatorname{Var}[X] + \operatorname{Var}[N](\mathrm{E}[X])^2 = 50 \cdot 500^2 + 50(7500)^2 = 2.825 \times 10^9 \text{ kg}
\end{aligned}$$