Assignment 2 – ELEC ENG 3TQ3
Due Date Nov 1st 8:30 a.m.

1. (5 points) Let us consider breaking a chocolate bar of length L randomly into two pieces of lengths L1 and L2 such that L=L1+L2. Find
   a) Probability that L1 > L2
   b) Expected value of L1
   c) Expected value of min(L1,L2)

   Note: Students are allowed to use Matlab to validate their answers. However EXACT answers are expected.

2. (5 points) John and Susan are going out on a first date. They agree to meet at Nathan Phillips Square at 8:00 p.m. Since it is their first date John decides that he will be their early in order to impress Susan and he plans to arrive at 7:45. Due to randomness in TTC performance the difference between his intended arrival time and actual arrival time is Gaussian distributed with mean 0 and variance 25. Susan decides to be fashionably late and she plans to arrive at 8:10 p.m. She decides to take a cab and hence the difference between her arrival time and intended arrival time is Gaussian distributed with mean 0 and variance 9.
   a) Find probability that John arrives before 8 p.m.
   b) Find probability that John arrives after 8:10 p.m.
   c) Using MATLAB find the probability that join arrives before Susan. Hint: using randn command in matlab you can generate two independent Gaussian random variables.
      X=5*randn; generates random variable with mean 0 and variance 25 and Y=3*randn; generates random variable with mean 0 and variance 9. Using these two commands you can generate large number of experiments and count number of times John arrives before Susan. Note: answers derived using probability theory will also be accepted.

3. (5 points) Consider a Gaussian random variable with mean 4 and unknown variance σ^2. Find the variance so that $P[1 \leq X \leq 3]$ is maximized.

4. (5 points) Assume that the life cycle of the light bulb is a function of the power. Let the expected life be exponentially distributed variable such that 60W bulb has expected life of 2000 hours and 120W has expected life of 1200 hours. Consider a long hallway in which we have one 60W bulb and one 120W bulb replaced at the same time. Also assume that their lifetimes are independent. a) Find the probability that after 1 month we will have at least one functioning bulb in the hallway. b) Find the probability that we will not have to change bulbs for at least 1 year.

5. (5 points) The starship Enterprise arrives at newly discovered planet Haldurian. The scientists find that there are 3 different genders on Haldurian with different height distributions. The height of gender A is Gaussian distributed with mean 6ft and 4 inches and variance 36. The height of gender B is Gaussian distributed with mean 7ft and 10 inches and variance 16. The height of gender C is Gaussian distributed with mean 5 ft and 10 inches and variance 25. Your data also indicated that 70% of Haldurian population is gender A, 20% population is gender B and, 10% population is gender C. Find

   a) Probability that randomly chose Haldurian is taller than 8 feet.
   b) Probability that no Haldurian is taller than 9ft if the population of Haldurian is one billion.

6. (5 points) Consider two random variables X and Y with join pdf such that

$$f_{X,Y}(x,y) = \begin{cases} cx^2y & when\ 0 \le x \le y \le 2 \\ 0 \end{cases}$$

   a) Find c.
   b) Find marginal distributions of X and Y.