ELECENG 3TR4 Lab 4: Random Processes

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Numerical Experiment #1: Evaluation of Autocorrelation and Power Spectral Density

i) The theoretical autocorrelation function of the output is derived in Equation 1.

$$
\begin{aligned}
y(n) &= h(n) * w(n) \\
h(n) &= 2B\,\mathrm{sinc}(2Bn) \\
&= 2 \cdot 250 sinc(2 \cdot 250n) \\
R_y(m) &= E\left[y(n) \cdot y(n+m)\right] \\
&= \sum_k \sum_j h(k)h(j)E[w(n-k)w(n+m-j)] \\
&= \sum_k h(k)h(k+m)\sigma_w^2 \\
&= \sum_k (500\,\mathrm{sinc}(500k))(500\,\mathrm{sinc}(500(k+m)))\sigma_w^2 \\
&= \sum_k 250000\,\mathrm{sinc}(500k)\,\mathrm{sinc}(500(k+m))\sigma_w^2
\end{aligned} \tag{1}
$$

The PSD is just the Fourier transform of the autocorrelation function. The PSD is derived in Equation 2, where k is the gain of the noise.

$$
\begin{aligned}
S_y(f) &= k\left|H(f)\right|^2 \\
&= k\,\mathrm{rect}(\frac{f}{2 \cdot 250}) \\
&= k\,\mathrm{rect}(\frac{f}{500})
\end{aligned} \tag{2}
$$

The theoretical results from Equation 1 and Equation 2 can be compared to the results of the numerical experiment in Figure 1. We can see that both the autocorrelation and PSD match between the theoretical results and the numerical results. Both autocorrelation functions result in sinc functions with the same frequency. Similarly, both PSDs produce a rect function with a bandwidth of 250 Hz, although is noise in the passband of the MATLAB PSD, as the signal used for the MATLAB plot does not use an ideal white noise signal.

ii) We can see the impact of increasing the maxlag to 200 in Figure 2 and increasing the maxlag to 500 in Figure 3. We can see that there is more information in the PSD as we increase the maxlag, as doing so increases the frequency resolution of our PSD.

iii) We can estimate the bandwidth of the filter using the autocorrelation plot. We can see that in all three plots of the autocorrelation function, the zeros of the sinc function are spaced 0.002 seconds apart. This means the sinc function repeats peaks and troughs every 0.004 seconds. We can determine the bandwidth of the filter to be $\frac{1}{0.004\text{ s}}$ or 250 Hz.

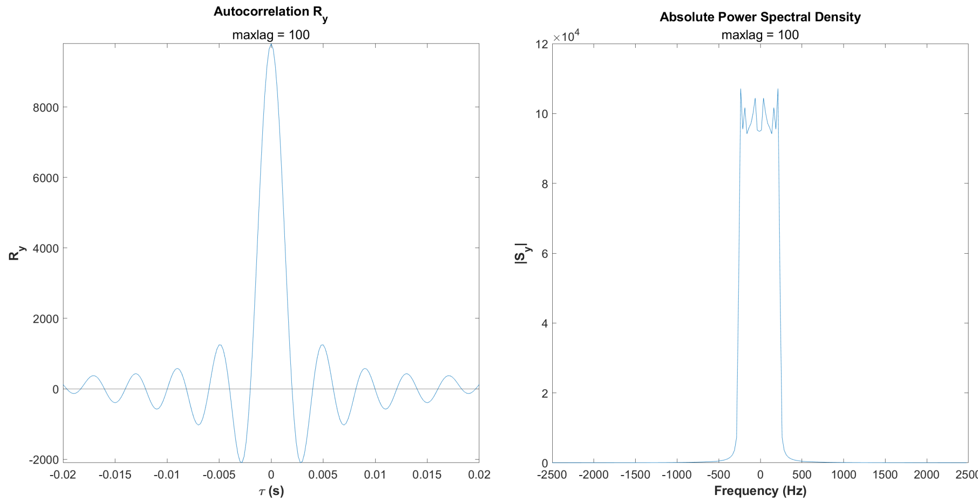

Figure 1: Autocorrelation and PSD for maxlag = 100

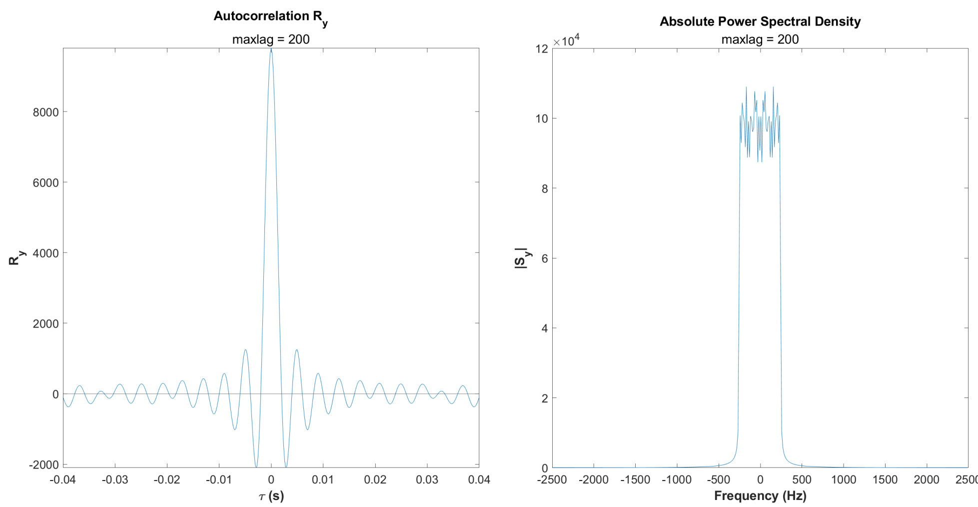

Figure 2: Autocorrelation and PSD for maxlag = 200

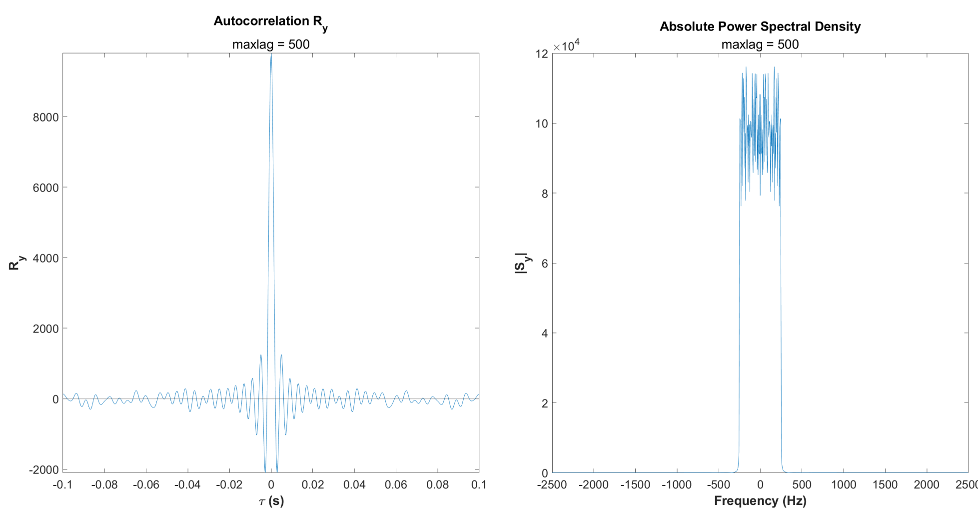

Figure 3: Autocorrelation and PSD for maxlag = 500

Numerical Experiment #2: A sinusoid buried in noise

Autocorrelation function.

$$\begin{aligned}
y(t) &= A\sin(2\pi f_c t + \theta) + w(t) \\
R_y(\tau) &= E\left\{y(t)y(t+\tau)\right\} \\
&= E\left\{\left[A\sin(\underbrace{2\pi f_c t + \theta}_{x}) + w(t)\right]\cdot\left[A\sin(\underbrace{2\pi f_c (t+\tau) + \theta}_{y}) + w(t+\tau)\right]\right\} \\
&= E\left\{A^2\sin(x)\sin(y) + A\sin(x)\cdot w(t+\tau) + w(t)\cdot A\sin(y) + w(t)\cdot w(t+\tau)\right\} \\
&= \frac{A^2}{2}\sin(2\pi f_c\tau) + 0 + 0 + \frac{N_0}{2}\delta(\tau) \\
&= \frac{A^2}{2}\sin(2\pi f_c\tau) + \frac{N_0}{2}\delta(\tau)
\end{aligned}$$

PSD is just the Fourier transform of the autocorrelation function.

$$\begin{aligned}
R_y(\tau) &= \frac{A^2}{2}\sin(2\pi f_c\tau) + \frac{N_0}{2}\delta(\tau) \\
S_y(f) &= \left|F\left\{\frac{A^2}{2}\sin(2\pi f_c\tau)\right\} + F\left\{\frac{N_0}{2}\delta(\tau)\right\}\right| \\
&= \left|i\cdot\frac{A^2}{4}\left[\delta(f-f_c) - \delta(f+f_c)\right]\right| + \frac{N_0}{2} \\
&= \frac{A^2}{4}\left[\delta(f-f_c) - \delta(f+f_c)\right] + \frac{N_0}{2}
\end{aligned}$$

Numerical Experiment #3: Delay Estimation

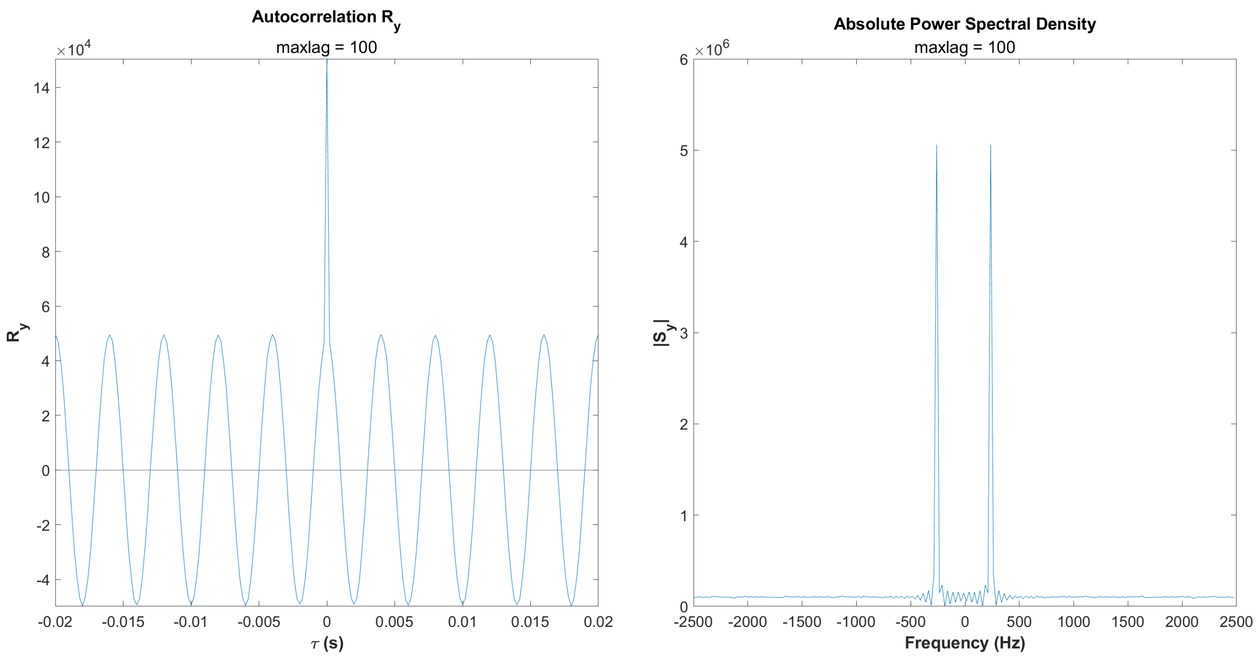

Figure 4: Autocorrelation and PSD for maxlag = 100

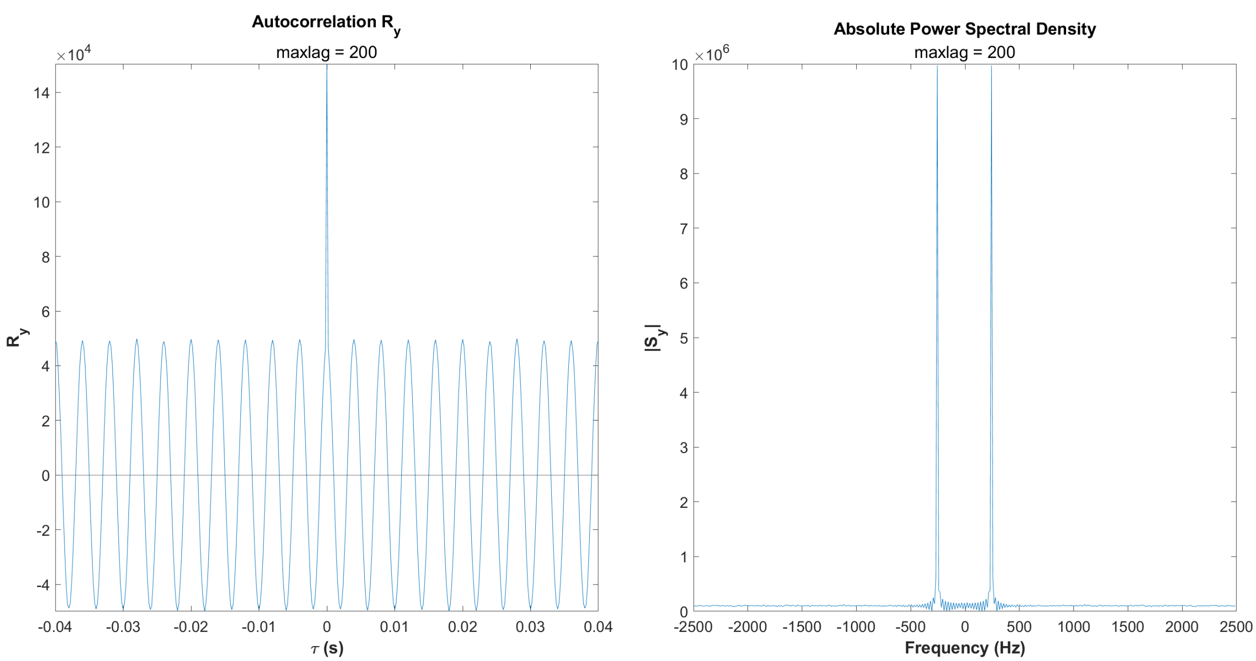

Figure 5: Autocorrelation and PSD for maxlag = 200

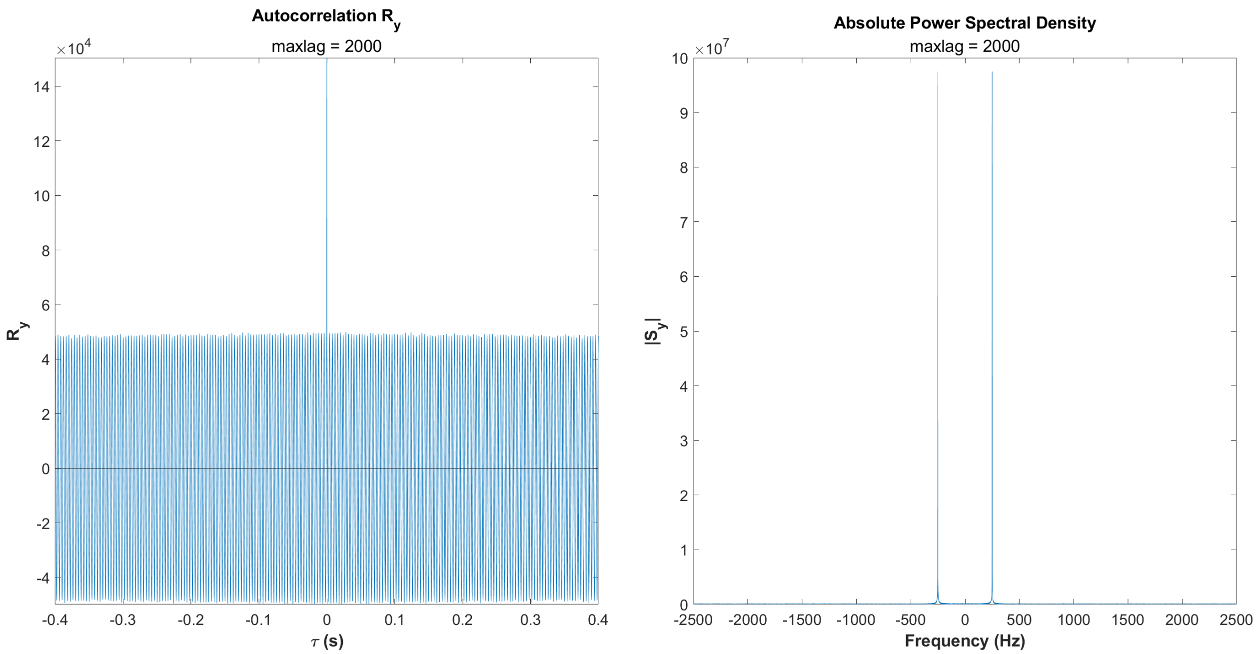

Figure 6: Autocorrelation and PSD for maxlag = 2000

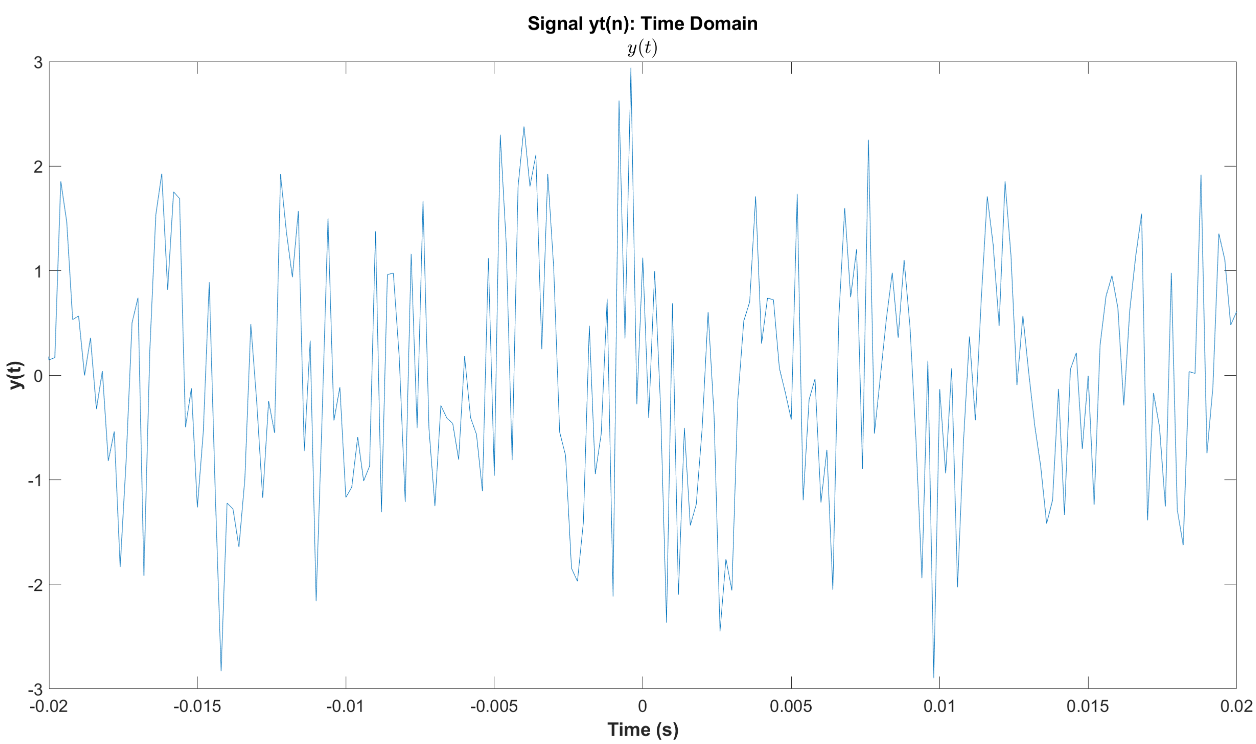

Figure 7: Time Domain

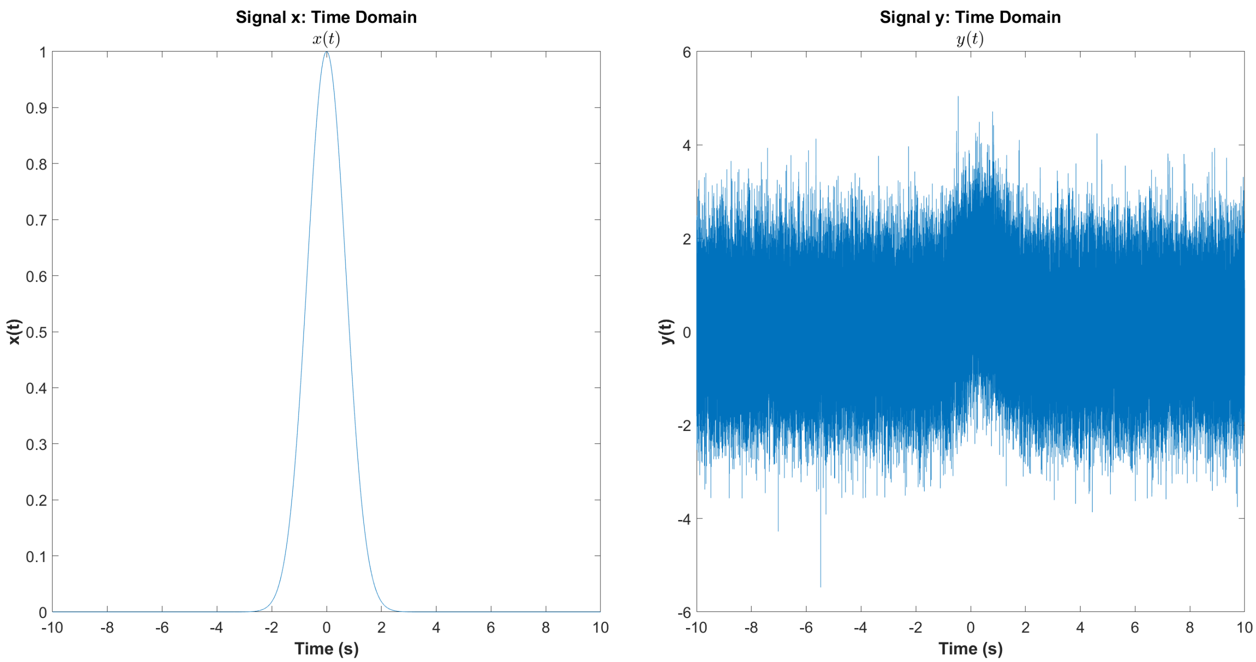

Figure 8: Time Domain

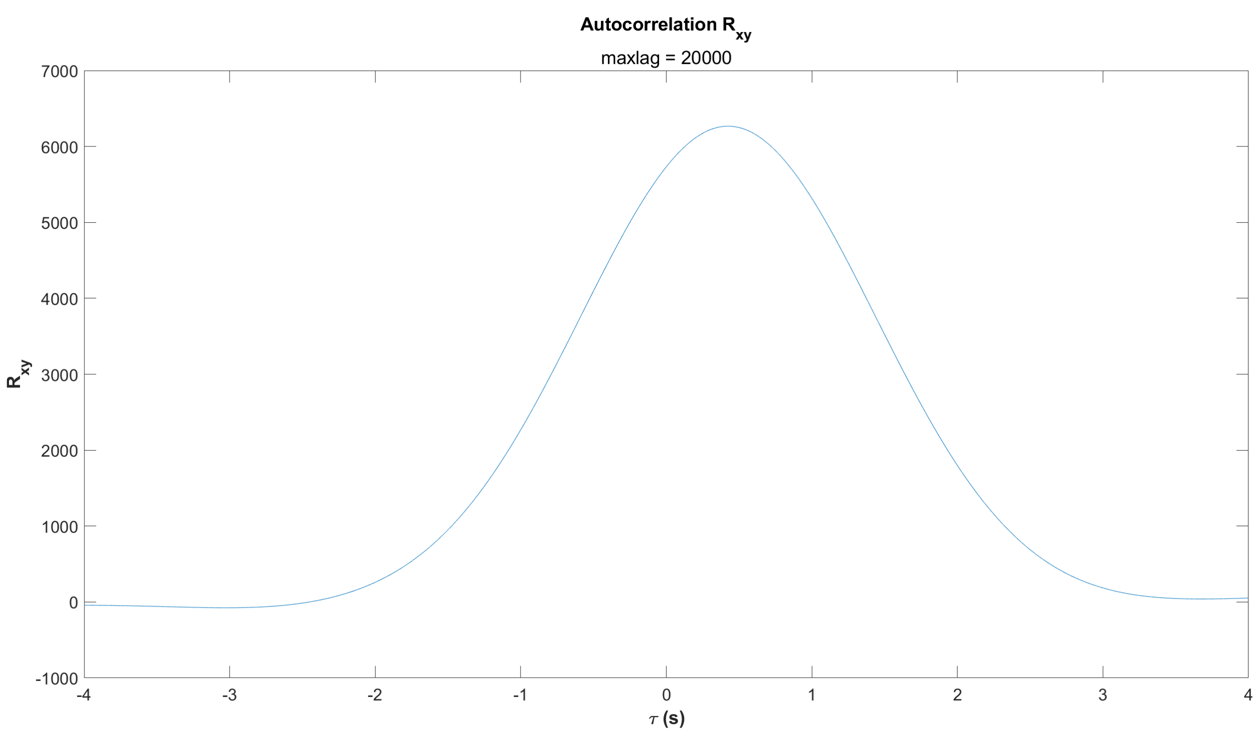

Figure 9: Autocorrelation