EE3TR4 Lab 1: Fourier Analysis

Please work individually or in pairs. ***Please note that each group is expected to work independently.*** Please be familiar with the guidelines on academic integrity, summarized on the course outline. The late penalty is 10% per day.

Numerical Experiment: Filter Analysis of a Periodic Signal

The output of a function generator is a square wave with 50% duty cycle and the period of the wave is 0.1 ms. You may assume that the amplitude of the square wave is 1 Volt. Design a second order Butterworth filter so that any harmonics in the output is at least 23 dB below the fundamental sinusoid at 10 kHz, and the loss of the fundamental sinusoid due to filtering should be less than 2 dB. The purpose of this experiment is to generate a high amplitude sinusoidal signal out of a square wave (to have a high quality sinusoid, the amplitudes of harmonics should be sufficiently low). You should choose the cutoff frequency of the filter (i.e. 3 dB bandwidth of the filter) such that the above design constraints are met. Note that the cutoff frequency is not unique and there could be a range of cutoff frequencies over which the design constraints could be met. Plot the time domain signal, amplitude spectrum and phase spectrum at the filter input and output. You may assume that the peak of the filter transfer function (H(0)) is unity. Plot the transfer function of the filter as a function of frequency. Show analytically that the filter you have designed meet the design constraints. Discuss the impact of changing the cutoff frequency on design constraints (For example, what happens if the cutoff frequency is too large? Can you satisfy the design constraint that the any harmonic in the output is at least 25 dB below the fundamental sinusoid?). Find the range of cutoff frequencies over which the design constraints can be met. If the design constraints cannot be met using the second order filter, consider using a higher order filter. Tutorial 1 deals with a similar problem. We expect a report that is clear, articulate and accurate.

Hints and Suggestions:

1. You can calculate the output response $y(t)$ of the filter in at least two different ways--Fourier series representation and discrete Fourier transform. We have $Y(nf_0) = X(nf_0)H(nf_0)$, where f_0 is the fundamental frequency of the periodic input signal.
   Fourier series representation: Multiply the Fourier coefficients of the input (C_n) by $H(nf_0)$ to get the Fourier coefficients of the output. The filter frequency response $H(f)$ is available from the supplementary material for this lab. Do the weighted sum of the complex sinusoids (i.e. Fourier series) to find the output signal in time domain. Note that the transfer functions of the filters shown in the supplementary material are normalized to 1 radian/sec. To achieve a cutoff frequency of ω_c radians/sec., replace ω with the quantity $\frac{\omega}{\omega_c}$. Follow the approach of Tutorial 1 or in the matlab code "Triangular_filtering.m".
2. *Discrete Fourier Transform*: The spectrum $X(f)$ is known, since it can be calculated from the Fourier transform of the input signal. Use the matlab built in function, fft() to compute the Fourier transform. We have $Y(f) = X(f)H(f)$. The corresponding time-domain signal $y(t)$ can be found by inverting the Fourier transform quantity $Y(f)$, as we discussed in class. Use the matlab built in function, ifft() to compute the inverse Fourier transform. Assemble

a vector in Matlab of values of $H(j2\pi f)$ according to the format of the Discrete Fourier Transform. Make sure the sampling frequency f_s is large enough so that $\left|H\left(\frac{f_s}{2}\right)\right|$ is small (e.g. 0.01) in comparison to $|H(0)|$ to avoid aliasing error in frequency.

3. Make sure your cutoff frequency is a reasonable choice, relative to the frequency of your input signal. You don't want the cutoff frequency of the filter to be so large that few frequency components of the input are removed by the filter.

Lab 1 Report

1. Description of the design of a butterworth filter
   Simulation details – input wave, filter and output. Provide design parameters. Show your analytical work and explain how this choice of parameters meet the design constraints. Provide figures corresponding to time and frequency domain (amplitude and phase spectrum) representation and explain the figures. If the phase is zero for all the frequencies, you may simply state this in the report and no need to plot the phase spectrum. See the matlab code 'triangular_filtering.m' in class (also posted on avenue). Provide the figure showing the absolute transfer function of the filter. Provide explanation on the possible range of cutoff frequencies. Discuss the impact of changing the cutoff frequency on design constraints.
2. Attach the matlab code.