1.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

first we use a base case of n = 1 to see if it is true

$$1^2 = \frac{1 \cdot (1+1)\,(2 \cdot\ 1+1)}{6}$$

which is true because$1^2 = 1$ then we assume that $n = m$ is true

$$\sum_{i=1}^{m} i^2 = \frac{m\,(m+1)\,(2m+1)}{6}$$

and now we need to prove that $n = m + 1$ is true

$$\sum_{i=1}^{m+1} i^2 = \frac{(m+1)\,((m+1)+1)\,(2\,(m+1)+1)}{6}$$

now we need to prove the truth of this expression

$$\sum_{i=1}^{m+1} i^2 = (m+1)^2 + \sum_{k=1}^{m} k^2$$

and using the induction method gets us

$$(m+1)^2 + \frac{m\,(m+1)\,(2m+1)}{6} = \frac{(m+1)\,(m+1+1)\,(2\,(m+1)+1)}{6}$$

$$(m+1)^2 \cdot 6 + \frac{m\,(m+1)\,(2m+1)}{6} \cdot 6 = \frac{(m+1)\,(m+1+1)\,(2\,(m+1)+1)}{6} \cdot 6$$

$$6\,(m+1)^2 + m\,(m+1)\,(2m+1) = (m+1)\,(m+2)\,(2\,(m+1)+1)$$

$$2m^3{+}9m^2{+}13m{-}\big(2m^3 + 9m^2 + 13m\big) = 2m^3{+}9m^2{+}13m{-}\big(2m^3 + 9m^2 + 13m\big)$$

$$0 = 0$$

after some simplifying and extending of the equality, we get that right term equels the left term which proves the sum of all n values.

2.

$$\sum_{j=1}^{n} (2j-1) = n^2$$

first we use a base case of n = 1 to see if it is true

$$(2 \cdot\ 1 - 1) = 1^2$$

$$1 = 1$$

which is true assume that $n = m$ is true

$$\sum_{j=1}^{m} (2j - 1) = m^2$$

and then we prove for $n = m + 1$ is true

$$\sum_{j=1}^{m+1} (2j - 1) = (m + 1)^2$$

now we need to prove the truth of this expression

$$\sum_{j=1}^{m+1} (2j - 1) = (2 (m + 1) - 1) + \sum_{j=1}^{m} (2j - 1)$$

$$2 (m + 1) - 1 + m^2 = (m + 1)^2$$

$$m^2 + 2m + 1 = m^2 + 2m + 1$$

$$0 = 0$$

after some simplifying and extending of the equality, we get that right term equels the left term which proves the sum of all n values.