Project III: Confidence Intervals

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1 Results

Table 1 below shows the results of the experiments on given functions.

Function	Validity	Confidence Level	Asymptotically Valid
1	✓	0.0001	✓
2	×	-	-
3	✓	0.05	✓
4	×	-	-
5	✓	0.25	×
6	×	0	-
7	✓	0.15	×
8	×	-	-
9	✓	0.01	✓
10	✓	0.10	×

Table 1: Results of Confidence Intervals Experiments

2 Background

This project aims to experiment with given ten functions that allegedly compute confidence intervals. We identify the validity and confidence levels for given functions in the experiments. Formally, given an independent identically distributed data X_1, X_2, ..., X_N, a confidence interval is a pair of random variables A_N, B_N, such that

$$Pr[A_N \leqslant \theta \leqslant B_N] = 1 - \alpha$$

Here, θ is a property of the distribution generating X_i (e.g., the mean), and α is the confidence level (e.g., 0.05). We will also see asymptotic confidence intervals that are a set of random variables A_N, B_N, such that

$$\lim_{n\to\infty} Pr[A_N \leqslant \theta \leqslant B_N] = 1 - \alpha$$

3 Methodology

The confidence interval functions are tested with various samples of size $N \in \{10, 100, 1000, 10000\}$) from Ber(0.01), Ber(0.5), and Uni(0.3, 0.7). The confidence intervals are reported for only the valid functions.

A valid confidence interval has to satisfy all the distributions. Whenever it's returning confidence intervals that completely capture the true mean for some distribution and completely miss the true mean for others, it's deemed invalid. As a heuristic, if the gap is more than 0.5 between the minimum and maximum missed rates for large values of N (1000, 10000), we consider the method generating the confidence interval as invalid.

And, some functions meet the definition of valid asymptotic functions (only valid for higher values of N). So, these intervals fail to trap the true parameter for small N, but only fail a fraction α of the time for larger N. The test we used to identify asymptotic confidence interval is given a valid confidence interval, the missed rates are close for large values of N irrespective of the distribution and the values are different for smaller values of N, it's asymptotic else not.