ALGEBRA

Factoring

1. $x^2 - a^2 = (x + a)(x - a)$

2. $x^2 + 2ax + a^2 = (x + a)^2$

3. $x^2 - 2ax + a^2 = (x - a)^2$

4. $x^2 + (a + b)x + ab = (x + a)(x + b)$

5. $x^3 + 3ax^2 + 3a^2 x + a^3 = (x + a)^3$

6. $x^3 - 3ax^2 + 3a^2 x - a^3 = (x - a)^3$

7. $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

8. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

Properties of Exponent

1. $a^n a^m = a^{n+m}$

2. $(a^n)^m = a^{nm}$

3. $(ab)^n = a^n b^n$

4. $a^0 = 1,\ a \neq 0$

5. $\dfrac{a^n}{a^m} = a^{n-m}$

6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

7. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

8. $a^{-n} = \dfrac{1}{a^n}$

Properties of Radicals

1. $\sqrt[n]{a} = a^{\frac{1}{n}}$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
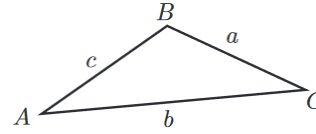4. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Trigonometric Identities

1. $Tan\,\theta = Sin\,\theta\,/\,Cos\,\theta$
2. $Cot\,\theta = Cos\,\theta\,/\,Sin\,\theta$
3. $Csc\,\theta = 1/\,Sin\,\theta$
4. $Sec\,\theta = 1/\,Cos\,\theta$
5. $Cot\,\theta = 1/\,Tan\,\theta$
6. $Sin^2\,\theta + Cos^2\,\theta = 1$
7. $1 + Tan^2\,\theta = Sec^2\,\theta$
8. $1 + Cot^2\,\theta = Csc^2\,\theta$
9. $Sin\,2\theta = 2Sin\theta\,Cos\theta$
10. $Cos\,2\theta = Cos^2\,\theta - Sin^2\,\theta$
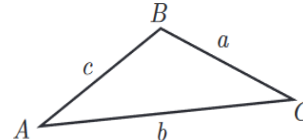11. $Cos\,2\theta = 2Cos^2\,\theta - 1$
12. $Cos\,2\theta = 1 - Sin^2\,\theta$

Sine Law

$$\frac{Sin\,A}{a} = \frac{Sin\,B}{b} = \frac{Sin\,C}{c}$$

Cosine Law
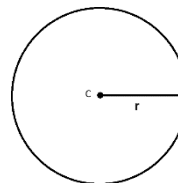

$$a^2 = b^2 + c^2 - 2bcCosA$$

$$b^2 = a^2 + c^2 - 2acCosB$$

$$c^2 = a^2 + b^2 - 2abCosC$$

ANALYTIC GEOMETRY

Conic sections

Circle

General equation:

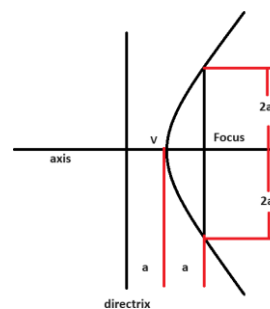$$Ax^2 + By^2 + Dx + Ey + F = 0$$

Standard equation with center at origin and radius r:

$$x^2 + y^2 = r^2$$

Standard equation with center at (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

Parabola

Where,

V –vertex Focus

Latus rectum = 4a

Distance from focus to vertex = a Distance from vertex to directrix = a

Standard equation:

Vertex at origin V(0, 0)

1. $y^2 = 4ax,\ parabola\ opening\ to\ the\ right$

2. $y^2 = -4ax,\ parabola\ opening\ to\ the\ left$

3. $x^2 = 4ay,\ parabola\ opening\ upward$

4. $x^2 = -4ay,\ parabola\ opening\ downward$

Vertex at h, k. V(h, k)
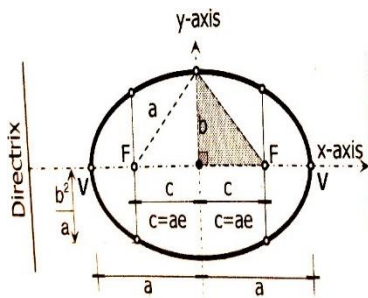
1. $(y - k)^2 = 4a(x - h),\ opening\ to\ the\ right$

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2. $(y - k)^2 = -4a(x - h)$, *opening to the left*
3. $(x - h)^2 = 4a(y - k)$, *opening upward*
4. $(x - h)^2 = -4a(y - k)$, *opening downward*

Ellipse

Where,
a = semi-major axis
2a = major axis
b = semi-minor axis
2b = minor axis
c = focal distance
V = vertices
F = foci

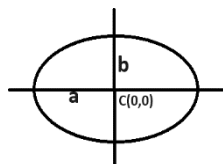**General equation of an ellipse**

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Standard equation:

Center at origin, C(0, 0)

Major Axis Horizontal

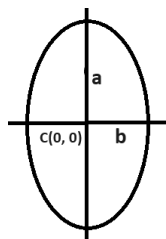1. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Where: a = semi-major axis
b = semi-minor axis

Major Axis Vertical

2. $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$
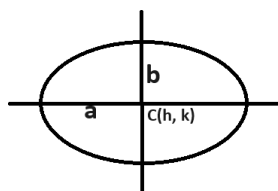

Where: a = semi-major axis
b = semi-minor axis

Center at C(h, k)

Major Axis horizontal

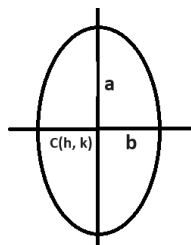3. $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
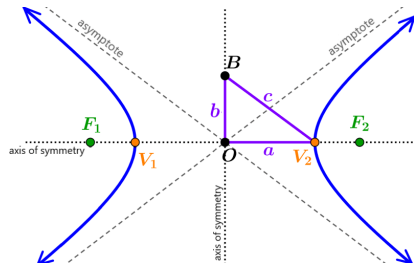

Where: a = semi-major axis
b = semi-minor axis

Major Axis vertical

4. $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

Where: a = semi-major axis
b = semi-minor axis

Hyperbola

General equation of the hyperbola

$$Ax^2 - Cy^2 + Dx + Ey + F = 0$$

Standard equation

Center at origin, C(0, 0)

Transverse axis horizontal

3. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Transverse axis vertical

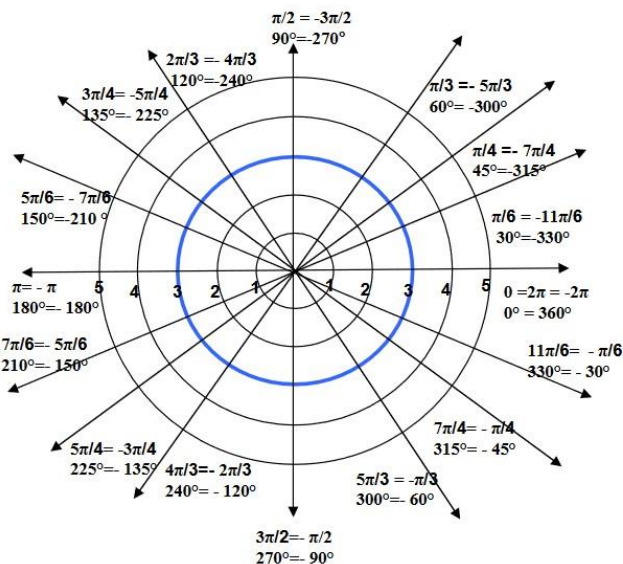4. $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$

Center at C(h,k)

Transverse axis horizontal

5. $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
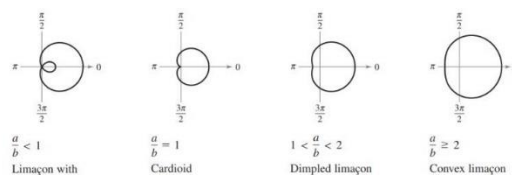
Transverse axis vertical

6. $\dfrac{(x-h)^2}{b^2} - \dfrac{(y-k)^2}{a^2} = 1$
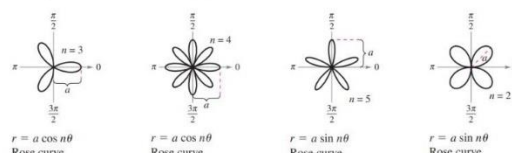
Graphs of polar equations
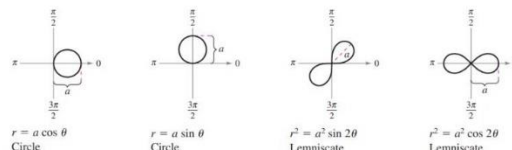
PRACTICE PROBLEMS

Algebra

1. Find the value/s of b. $8(2b^2 + 4)^{2/3} = 32$

   Ans. $b = \pm\sqrt{2}$

2. Get the factor of the polynomial by synthetic division. $x^4 - x^3 - 7x^2 + x + 6$

   Ans. $(x + 1)(x - 1)(x + 2)(x - 3)$

3. Solve the system of linear equation
   $$7x - 8y = -12$$
   $$-4x + 2y = 3$$

   Ans. $x = 0$, $y = 3/2$

4. Solve the inequality $-14 < -7(3x + 2) < 1$

   Ans. $(-5/7, 0)$

5. Perform the operation.

   $$\left(\frac{x^2-49}{2x^2-3x-5}\right)\left(\frac{x^2+7x+6}{x^2-x-42}\right)$$

   Ans. $\frac{x+7}{2x-5}$

Trigonometry

6. A triangle has the following, sides a = 8 cm, b = 11 cm, and angle C = 37 degrees. Find the value of side c.

   Ans. $c = 6.67cm$

7. A hot air balloon that is 4280 ft above the ground measures an *angle of depression* of 24degrees from its horizontal line of sight to the base of a house on the ground. Assuming the ground is flat, how far away along the ground is the house from the balloon?

   Ans. $9{,}613\ ft$

8. What is the value/s of x in the interval $0° \leq x \leq 180°$ for $Tan3x = -1$

   Ans. $x = 45°, 105°, 165°$

9. The hypotenuse of a right triangle is 34m. What is the lengths of the two legs if one leg is 14m longer than the other.

   Ans. $16m\ and\ 30m$

Analytic Geometry

10. *What is the general equation of a circle with center at* $(-3, 5)$ *and diameter of* 20.

    Ans. $x^2 + y^2 + 6x - 10y = 66$

11. Determine the standard equation of the ellipse.

    $$x^2 + 3y^2 + 8x - 6y = -7$$

    Ans. $\frac{(x+4)^2}{12} + \frac{(y-1)^2}{4} = 1$

12. Determine the coordinate of the vertex of the parabola with an equation of $y = 4x^2 - 4x + 1$

    Ans. $(1/2, 0)$

13. What do you call the polar equation $r = 5cos\theta$

    Ans. *a circle*

14. *Determine the radius and center of the circle with an equation of* $x^2 + y^2 + 14x - 8y = -56$

    Ans. $r = 3, c(-7, 4)$

15. *Sketch and describe the curve,* $r = 4\ sin\theta$

    Ans. *It is a circle with radius of* 2.

16. *Sketch the curve,* $r = 7 \sin 2\theta$

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