Empirical Analysis of Greedy Algorithms for Online Graph Coloring

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Abstract – This study aims to evaluate and compare the performance of the greedy algorithms, FirstFit and CBIP, to effectively color online graphs. The empirical study involves generating multiple random k-colorable graphs of various sizes and calculating the competitive ratios of the algorithms, where k = {2, 3, 4} for FirstFit and k = {2} for CBIP. It is observed that the average competitive ratio for FirstFit gradually increases with an increase in k and lies between 1 and 2 for studied k values. In contrast, the average competitive ratio for the CBIP algorithm is closer to 1 for k = 2, indicating better performance than the FirstFit algorithm. The results for both algorithms were recorded for the average competitive ratios on the same test scenarios, based on a ratio of 1.35 between the number of vertices n and instances of graphs N, which concludes that CBIP outperforms FirstFit in all the cases. These results indicate a good efficiency of using these algorithms for solving graph coloring problems.
Keywords – Online Graph, Graph Coloring, First Fit, CBIP

I. Introduction and Problem Description

In Algorithm Design Techniques, graphs play a vital role in solving complex problems, which are often vital to humanity. One such fundamental problem is Graph coloring, which is helpful in applications like scheduling and map coloring. In this study, we must understand the concept of an online graph. It is a graph where vertices and edges of the graph appear in an online fashion [1]. This means that the vertices are received one at a time, and the information is then processed about the vertex and its neighbours currently existing in the graph [2]. Here, the algorithm correctly colors the received vertices $v_1, v_2, \ldots v_n$, belonging to the online graph G and the vertex v_i is colored after considering the subgraph G_{sub} with vertices $v_1, v_2, \ldots v_{i-1}$. The minimum number of colors required to color a graph G, such that no two vertices having an edge between them, is the chromatic number $\chi(G)$ of the graph G. Thus, $\chi(G)$ shows the best performance of an algorithm on the online graph G [3]. In a graph coloring problem, Algorithm A tries to color the vertices of a graph and the number of colors used should be minimum. Once the vertex is colored, we do not change the color vertex again as we are considering the greedy versions of the graph coloring algorithms.

We are considering two greedy algorithms for online graph coloring for this study - the FirstFit Algorithm and the CBIP algorithm. In the First Fit algorithm, we color each vertex with the least available color, provided that its neighbouring vertices do not use the color. The CBIP algorithm partitions the current partial graph into k-partitions such that all vertices in each partition have a single color and do not have any neighbours among themselves, then colors the incoming vertex with the least available color not assigned by any vertex in its complimentary partition sets.

For an Algorithm to run, we need to provide it with a graph that is k-colorable, with k=2,3,4 for the FirstFit Algorithm and k=2 for the CBIP Algorithm. For this, we are using a graph generation method through which we can create these k-colorable graphs. Initially, we create n vertices, then we assign each vertex randomly to k mutually exclusive subsets, and edges are created between vertices across the subsets based on a random probability. This process is repeated to generate random k-colorable graphs that can be used to perform empirical analysis on the FirstFit and CBIP algorithms.

This study's primary goal is to perform an empirical study on the FirstFit and CBIP algorithms. For this, we must understand the concept of competitive ratio as it is the comparing factor between the two graph coloring algorithms in the project. It can be defined as follows:

$$\rho(A, G) = \frac{No. of\ colors\ used\ by\ the\ algorithm\ A}{\chi(\mathrm{G})}$$

Here, $\rho(A, G)$ refers to the competitive ratio of the algorithm applied to the online graph G and $\chi(\mathrm{G})$ refers to the chromatic number of the graph [1].

Our main goal is to apply the FirstFit and CBIP algorithms on different colorable graphs to perform an empirical study on the competitive ratios. This report is further organized as follows. In section II, we perform a literary review of the works done by leading researchers and discuss their works. Further, in section III, we try to understand the concepts of FirstFit and CBIP algorithms and discuss the implementation of the algorithms, followed by the results of the empirical study performed in section IV. We conclude the report with a conclusion and summary of the project in section V.

II. Literature Review

Li, Y., et al., 2020 [2] investigated the problem of online graph coloring with the additional constraint that the graph is restricted to k-connected components. They tested the performance of the FirstFit and CBIP algorithms on various graphs with k-connected components and observed how the competitiveness of these algorithms is impacted by an infinite sequence of input under this condition.

Kierstead, H.A. 1998 [5] proposed three findings concerning online-graph coloring. The outcomes were obtained for k-chromatic graphs, which are graph classes where the chromatic number is limited by the clique number, and for the First Fit algorithm's performance on interval graphs. The limitations were expressed using the chromatic number, clique number, and independence number. The study offers theorems and proofs illustrating the effectiveness of online-graph coloring issues in various situations.

Kierstead, H.A. et al., 1992 [6] investigated the use of First-Fit for coloring interval graphs and aimed to obtain more precise bounds for such graphs. The study involved proving various theorems expressed in terms of little ω asymptotic notation, with a focus on identifying the conditions that an interval graph must satisfy for a given chromatic number, clique size, proper vertex coloring, and walls. Overall, the study provides valuable insights into the efficient use of First-Fit for coloring interval graphs.

Zarrabi-Zadeh, H. 2007 [11] conducted research on the issue of coloring co-interval graphs online. This refers to a situation in which an algorithm has to color a sequence of intervals presented one at a time on the natural line while minimizing the number of colors used. The research focused on determining the competitive ratio for co-interval graphs using the First-Fit algorithm, establishing an upper bound of 2. The study also explored the application of randomised algorithms to optimize scheduling, partitioning, or resource allocation models that are akin to graph coloring problems.

Now, we proceed to the Algorithm Implementation in Section III, where we discuss the different algorithms which we will be implementing.

III. Algorithm Implementation

Graph coloring algorithms such as FirstFit and CBIP requires an online graph to be provided as an input. An online graph G(V, E) is such that a new vertex is provided to the Algorithm in every iteration. In i^{th} iteration, a new vertex v_i arrives, [2] and is colored by the Algorithm based on the colors already assigned to the neighbourhood of the vertex i, $N(v)$ where $N(v) = \{ i \in V,\ i \sim v\} \, \forall \, v \in V$ [6].

For generating these types of k-colorable graphs, we can use the following intuition as suggested [7]. Let us consider a method for generating graphs with chromatic number χ. Here, $\chi \in \mathbb{N}$ denotes the minimum number of colors that can be used to color the graph. We first partition the vertex set *V(G)* of a graph G into mutually exclusive subsets, assigning each vertex v to a set with a random probability of assignment, say, V_1, V_2, V_3 …. up to V_k, where $V_{1....k} \subset V$ and $V_1 \cap V_2 \cap ... V_k = \phi$. We then introduce edges between vertices across the mutually exclusive subsets with a random probability, also ensuring that no two vertices in the same set share an edge. Every edge (u, v) satisfies the condition that $u, v \notin V_i \, \forall \, V_i \subset V$. This results in a graph G with the chromatic number χ(G), which is also a k-colorable graph by definition. This technique can be used repeatedly to create diverse k-chromatic graphs, resulting in random k-colorable graphs.

Elaborating on the above, let k = number of colors that can be used to color the graph, n = the number of vertices of the graph p1 = probability of assigning a vertex to a disjoint vertex set, and p_i = probability of choosing an edge between the two vertices belonging to mutually exclusive subsets $\forall \; 0 < p_1,\ p_i < 1$ [7]. Let us consider the vertex set V = { v_1, v_2, v_3 … v_n }, which is partitioned into k disjoint mutually exclusive subsets, say, V_1, V_2, V_3 …. up to V_k, where $V_{1....k} \subset V$. Now, we add an edge between vertices (v_i,v_j) $\forall \; 1 \leq i < j \leq n$ with a probability $p_i \, \forall \, i, j \notin$

$V_t \; \forall \; V_t \subset V$ for any number $t \leq k$ and $t \in \mathbb{N}$. This leads to the generation of k-colorable graphs with different characteristics.

Since we are using these graphs to apply our algorithms, we will briefly discuss the implementation of the FirstFit and CBIP algorithms. *FirstFit* can be considered a greedy algorithm [2,3,5,6] in the sense that when the Algorithm receives a new vertex v_i, it checks for a least available color $c \in \mathbb{N}$ that can be used to color the vertex v_i that is not present in the neighbourhood of the vertex v_i.

We perform this iteratively until all vertices have been colored and this gives us the total number of unique colors used by the *FirstFit* Algorithm on graph G. In the FirstFit graph coloring algorithm, we pass an input graph G generated by the graph generator implemented in the project. The Algorithm iterates over the vertices $v \in V(G)$ in an online fashion, and tries to color each vertex with the least available c ∈ ℕ in color set C. Here, C denotes a key-value map of colors for graph G. Finally, we return C that contains key as vertex v_i and value as color $c(v_i)$. This can be well understood by the pseudocode of this Algorithm 1 below.

Algorithm 1 *FirstFit graph coloring algorithm.*

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procedure *FirstFit (G)*

$G_{cur} \longleftarrow \phi$

$C \quad \longleftarrow \phi$

for each $v \in V(G)$*:*

vertex v arrives to G_{cur} *with* $N(v) \in G_{cur}$

$C(v) \longleftarrow \min(c), s.t.\; c \in \mathbb{N} \; and \; c \notin C(v_x), v_x \in N(v)$

return *C*

In the second Algorithm, we study the *CBIP algorithm*. Similar to *FirstFit Algorithm*, when a vertex v_i arrives, i.e., a vertex is received by the Algorithm, it first finds the entire connected components of the vertex v_i, say CC, of the graph currently received. We will partition the connected components into two independent sets, say S_1 and S_2, as we apply this Algorithm to 2-colorable graphs which are bipartite. In bipartite graphs, there are two mutually exclusive subsets where every node in one of the sets may or may not have an edge over some other node in the complimentary set and does not have an edge to any node in the same set. A bipartite graph can also be formally defined as a graph $G_{bipartite}(V,E)$ if $\exists \, S_1, \; S_2 \; where, S_1 \cap S_2 = \emptyset, S_1 \cup S_2 = V \; s.t. \forall \, (x,y) \in E, (x \in S_1 \wedge y \in S_2) \vee (y \in S_1 \wedge x \in S_2)$. In our case, we perform a modified BFS similar to a level order traversal algorithm, starting at vertex v_i to find the connected components on the partial graph using queues. Here, v_i belongs to the first level and all its neighbors belong to the second level, further their neighbours go to the third level and so on. Each odd level goes to the S_1, and each even partition goes to S_2. Since, $v_i \in S_1$, the color of v_i is assigned the minimum available color in the set S_2. Thus, we find the number of unique colors assigned by the *CBIP algorithm* on Graph G [2,9]. The pseudocode for this Algorithm 2 is as follows:

Algorithm 2 *CBIP graph coloring algorithm.*

procedure *CBIP*

$G_{cur} \longleftarrow \phi$

$C \quad \longleftarrow \phi$

for $v \in V(G)$:

vertex v arrives to G_{cur} *with* $N(v) \in G_{cur}$

$CC \longleftarrow connected\ component\ of\ v_i\ in\ G\ using\ BFS$

$Partition\ v \in CC\ into\ S_{1,}S_2\ based\ on\ alternate\ iteration\ of\ BFS\ \forall\ (v_{s1}, v_{s2}) \in E$

$C(v) \longleftarrow \min(c), s.t.\ c \in \mathbb{N}\ and\ c \notin C(v_x), v_x \in S_2$

return *C*

We will use *FirstFit* and *CBIP* graph coloring algorithms to calculate the average competitive ratios on a predefined number of graphs N. This can be further analysed by varying the number of vertices on the randomly generated k-colorable graphs for the algorithms. We find the ratio of number of colors used by the greedy Algorithm to the chromatic number χ(G) to find the competitive ratio $\rho(A, G)$. This refers to the competitive ratio of the online graph algorithm applied to graph G and χ(G) refers to the chromatic number of the graph [1]. We can find the $avg(\rho(A, G))$ over N graphs to compare the different online algorithms on k-colorable graphs. In Section IV, these experiments are carried out, and the results are summarized.

IV. EXPERIMENTAL RESULTS

This section is focused on the experimental results found by applying the coloring graph algorithms on k-colorable graphs. Our main goal is to find the average competitive ratios of k = {2,3,4} colorable graphs using FirstFit and k = 2 colorable graphs for the CBIP algorithm.

A. *FirstFit Algorithm*

The FirstFit Algorithm experiments were carried out on k = {2,3,4} colorable graphs generated using a k-colorable graph generator. For our experiment, we fixed the number of graphs (N) = 100. Varying the number of vertices in each of the graphs gives us the average competitive ratio $\rho(A, G)$. We can perform an empirical study on the same, and the result of the study is shown in the below table.

Vertices	50	100	200	400	600	800	1000	1200	1400	1600
Avg Comp Ratio	1.14	1.19	1.15	1.16	1.145	1.18	1.17	1.22	1.08	1.135

Table 1. Average Competitive Ratios for different no of vertices for k = 2, N = 100 using First Fit Algorithm

Here, in Table 1, we see that the Average Competitive Ratios using the different number of vertices tend to lie between 1 and 1.5, which is an excellent competitive ratio. There is no direct proportion of Average Competitive Ratio to Vertices in the graph, so there is no guarantee that the Average Competitive Ratio will increase with an increasing number of vertices to color. This can easily be observed by the graph plotted in Fig. 1.

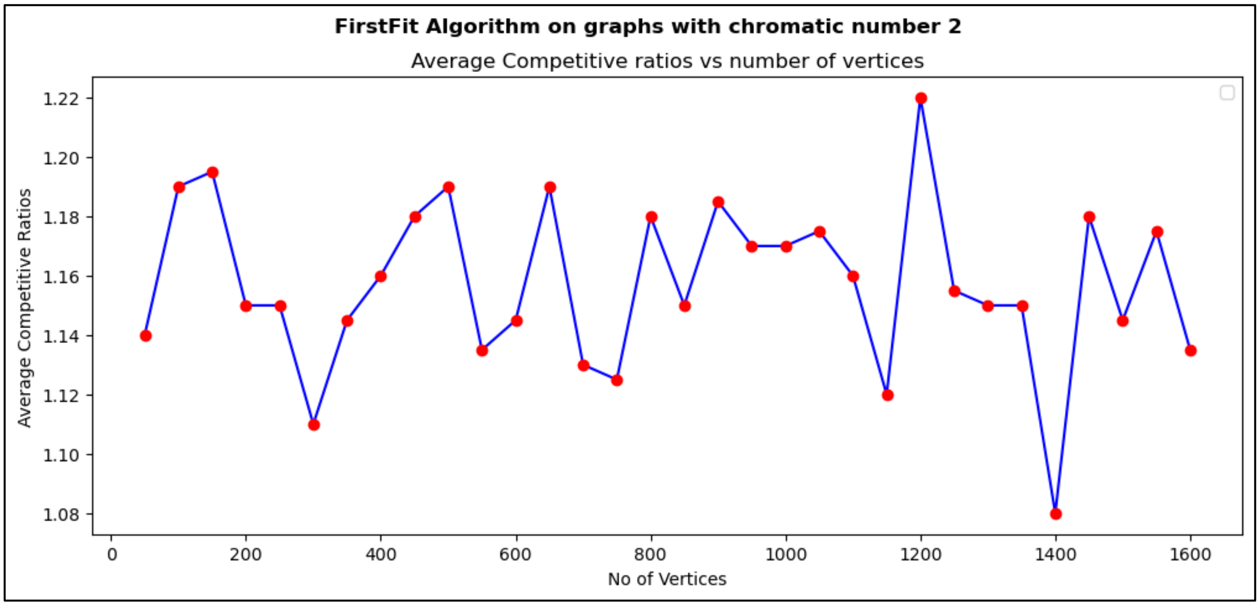

Fig. 1. Average Competitive Ratios on 2-colorable graphs using First Fit Algorithm

We can now try to do some analysis on the data points; we can try to see if there is some simple function which can fit the data in the plot. On preliminary analysis, we see that simple functions are not able to fit the data, so we simply plot the data as mentioned in the requirement of the project.

Similarly, we can perform the experiment on the 3-colorable graph with First Fit, where k = 3 and the numbers of graph generated N = 100. We perform the empirical study based on the average competitive ratios of increasing numbers v, as shown below in Table 2.

Vertices (v)	50	100	200	400	600	800	1000	1200	1400	1600
Avg Comp Ratio	1.337	1.353	1.37	1.29	1.28	1.353	1.283	1.363	1.353	1.28

Table. 2. Average Competitive Ratios for different no of vertices for k = 3, N = 100 using First Fit Algorithm

Compared to k = 2, we see that the average competitive ratios have increased slightly on 3-colorable graphs when applying First Fit Algorithm, which can also be seen from the plot graph in Fig. 3. below.

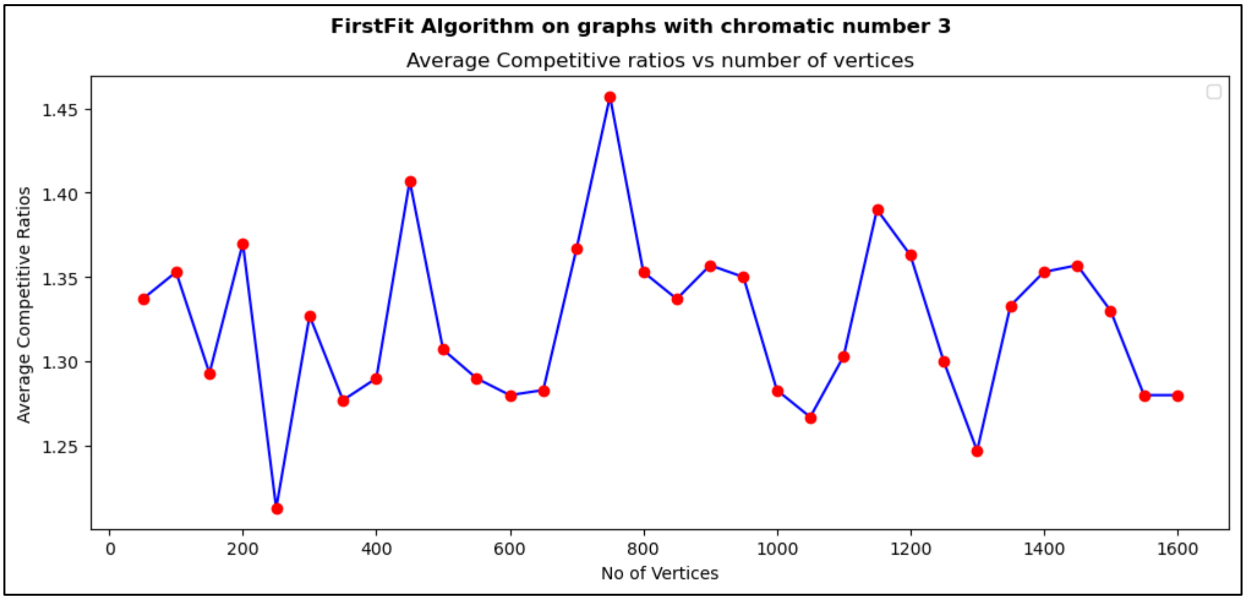

Fig. 2 Average Competitive Ratios on 3-colorable graphs using First Fit Algorithm

Finally, we can perform the experiment on 4-colorable graphs with First Fit, where k = 4 or the graph is 4-colorable, and the numbers of graphs generated N = 100. We also do this iteratively with an increment of 50 vertices each time and plot the graph. This graph can be seen in Fig. 4., where we notice the competitive ratios have increased quite significantly. We perform the empirical study based on the average competitive ratios of increasing numbers of v, as shown below in Table 3.

Vertices (v)	50	100	200	400	600	800	1000	1200	1400	1600
Avg Comp Ratio	1.51	1.515	1.502	1.49	1.495	1.49	1.387	1.558	1.438	1.442

Table. 3. Average Competitive Ratios for different no of vertices for k = 4, N = 100 using First Fit Algorithm

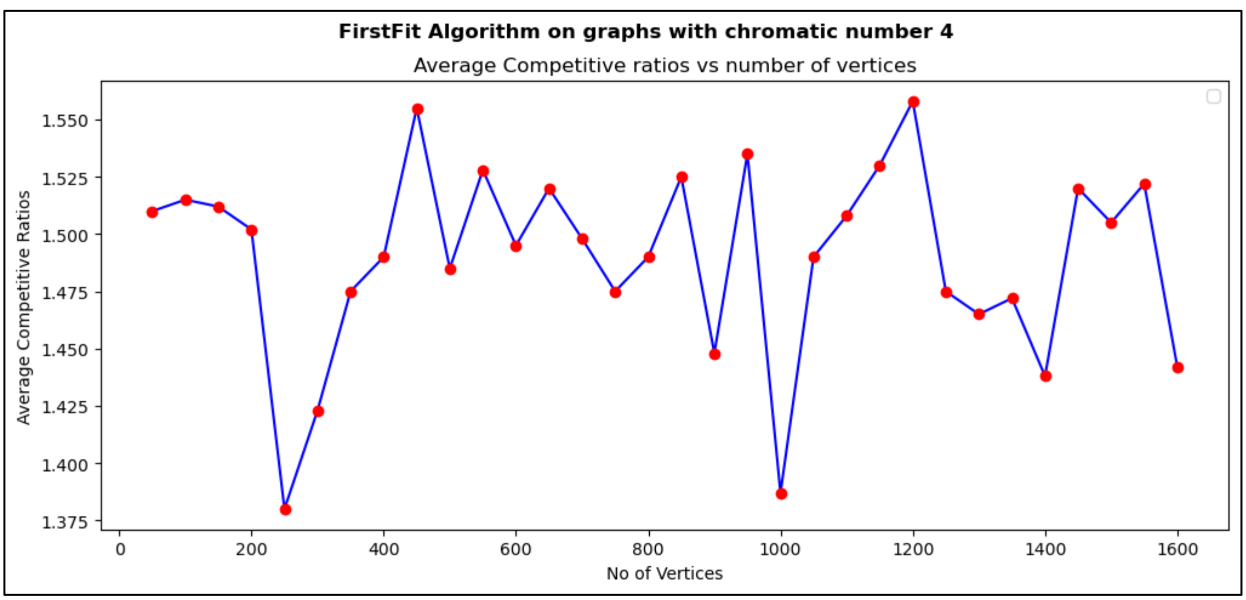

Fig. 3 Average Competitive Ratios on 4-colorable graphs using First Fit Algorithm

After applying First Fit for both 3 and 4 colorable graphs, there is no such plot which is able to fit the data perfectly, so we plot the graph similarly as done for 2 colorable graphs in Fig 3.

B. CBIP Algorithm

For the results on the CBIP algorithm, the Algorithm was run on different bipartite graphs, i.e., the chromatic number of the graphs is 2 or the graph is two colorable. To find the average competitive ratios, we used the number of graphs N = 100 and ran the Algorithm in increments of 50 vertices each time. The empirical study is represented by Table 3., which shows some of the results of the experiment as shown below.

Vertices (v)	50	100	200	400	600	800	1000	1200	1400	1600
Avg Comp Ratio	1.14	1.16	1.10	1.17	1.17	1.15	1.15	1.13	1.11	1.13

Table 4. Average Competitive Ratios for different no of vertices for k = 2, N = 100 using CBIP Algorithm

Table 4., shows that when k=2 with varying numbers of vertices, the Average Competitive Ratios for the CBIP algorithm fall between 1 and 1.25, indicating better performance than the FirstFit algorithm. This trend remains consistent even as the number of vertices increases, as demonstrated in Fig. 4, where the competitive ratio does not fluctuate.

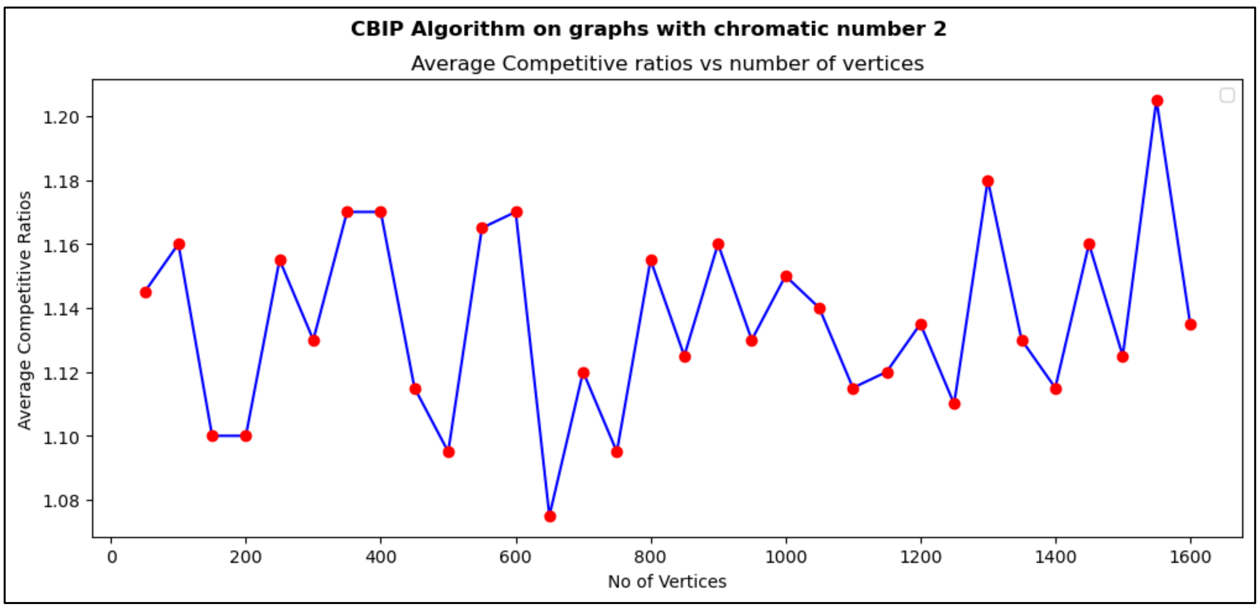

Fig. 4 Average Competitive Ratios on 2-colorable graphs using CBIP Algorithm

C. First Fit Algorithm vs CBIP Algorithm

In order to perform a comparative analysis between CBIP and FirstFit algorithms for 2-colorable graphs, we use an increasing number of graphs as the vertices increases to improve precision of our results as suggested. We have kept a ratio of 1.35 between vertices count and number of graphs the algorithm is run on, for example, when v=1600 N=2160. In this experiment, the First Fit and CBIP algorithms are run on the same graphs in order to provide a correct analysis of the average competitive ratios.

As observed in Table 4., the average competitive ratios of First Fit algorithms lie between 1.119 and 1.185 and the total average of the ratios is found to be 1.159. In the case of CBIP algorithm on the same data, the average competitive ratios are seen to lie between 1.104 and 1.160. The total average of the competitive ratios over different test cases is found to be 1.140 which is better than First Fit algorithm.

Vertices (v)	50	100	200	400	600	800	1000	1200	1400	1600
Avg Comp Ratio (First Fit)	1.119	1.141	1.181	1.136	1.177	1.159	1.151	1.157	1.18	1.165
Avg Comp Ratio (CBIP)	1.104	1.107	1.156	1.119	1.16	1.143	1.134	1.136	1.156	1.148

*Table. 4. Average Competitive Ratios for different no of vertices for k = 2, N = 1.35 * v for both algorithms*

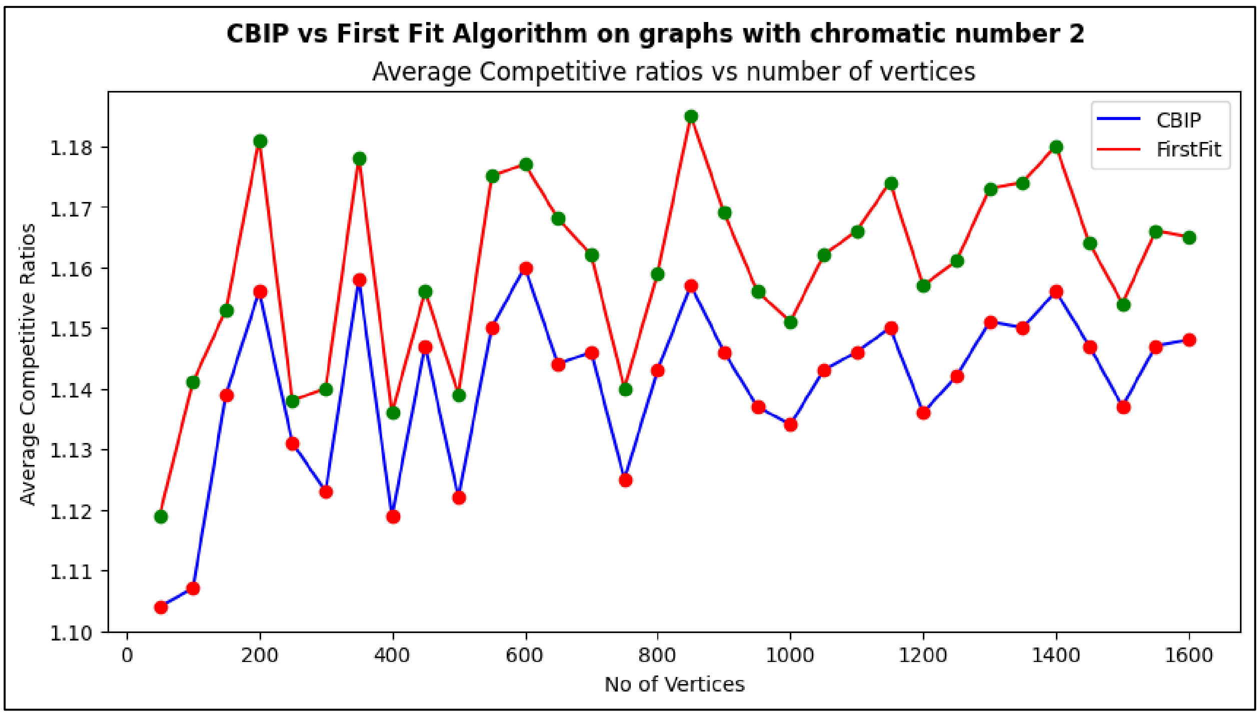

Fig. 5 Average Competitive Ratios on 2-colorable graphs using FirstFit vs CBIP Algorithm

From Fig. 5., we see that CBIP is always performing better than First Fit for all cases. The data does not seem to fit any particular polynomial function, so we plot the data to make our observations.

D. Result Observation

After proper analysis of the First Fit and CBIP Algorithms, we see that both of the algorithms perform pretty efficiently on the varying graphs provided to the Algorithm. The graph generator provides a large set of k-colorable graphs, which can be used by the graph coloring algorithms for analysis. In the First Fit algorithm, the average competitive ratios were seen to be between 1.08 and 1.25 for 2-colorable graphs. The total average competitive ratios come out to be 1.15 with the First Fit Algorithm for k=2 for k-colorable graphs. In the same way, for k= 3, we see that the average competitive ratios were between 1.21 and 1.45, while the total averages of the average competitive ratios come out to be 1.32 for 3-colorable graphs. Finally, we try to do the same analysis for 4-colorable graphs, where the average competitive ratios fall between 1.38 and 1.55 while the total average of the average competitive ratios is 1.48, which is a pretty significant increase from the value for 2-colorable graphs. We observe that when we increase the k in the k-colorable graphs, the total average competitive ratios are seen to increase for First Fit Algorithm. Thus, we can state that $k \propto$ average competitive ratio, which is true because as the complexity of the graphs increases and more colors get used than required. Regardless, the average competitive ratios are found to be near-optimal in most cases.

In the case of the CBIP algorithm, since it is only applicable to bipartite graphs, we only use 2-colorable graphs for our analysis . In the case of using the CBIP algorithm, we see that the average competitive ratios lie between 1.075 and 1.20. Overall, the total average of average competitive ratios is found to be 1.13 . One crucial factor to notice is that CBIP cannot be used to solve k-colorable graphs with $k \geq 3$. In our implementation, we have used two independent sets to store the vertices using connected components. When $k \geq 3$, we have to consider k-partitions for which vertices in each partition do not have an edge to any other vertex in the same partition. This becomes a 3-coloring recurring problem, and that falls under the class of NP-hard problems. There is no efficient solution to this problem which can be used to solve it in polynomial time. The solution proposed by us will also run in exponential time. Thus, CBIP Algorithm is feasible to be used for bipartite graphs or 2-colorable graphs.

Even though both the algorithms, at first glance, appear to be inefficient, but our empirical study clears the misconceptions, and we see that the algorithms result to a near-optimal coloring for online graphs in most cases. When comparing the First Fit algorithm with the CBIP algorithm for k=2, we see that the total average of the average competitive ratios is always higher for First Fit Algorithm having a value 1.159 in comparison to CBIP Algorithm having a value of 1.140. For our experiment a ratio of 1.35 between vertices and graphs generated was selected to provide a verifiable method for our conclusion.

We can verify the correct coloring visually by producing a visualization of the experiments run on First Fit and CBIP graph coloring algorithms. In Fig. 6., we see that the First Fit algorithm has colored the graph using 5 colors for 4-colorable graph having 13 vertices.

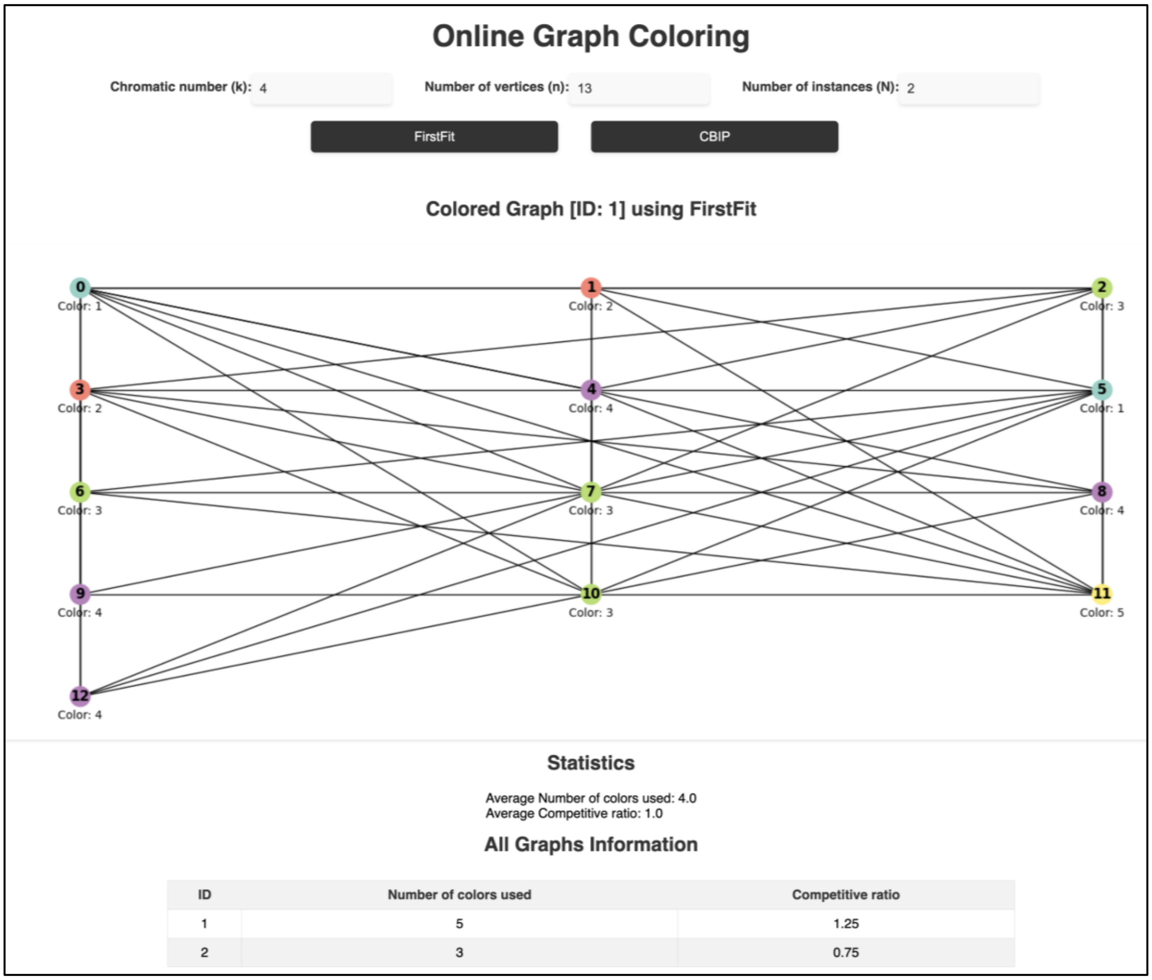

Fig. 6. Visualization of FirstFit Algorithm on 4-colorable graph

In Fig. 7., we see the visualization of CBIP graph coloring algorithm with 13 vertices on a bipartite graph. Here, we see that the CBIP has used 4 colors which brings the competitive ratio to 2.

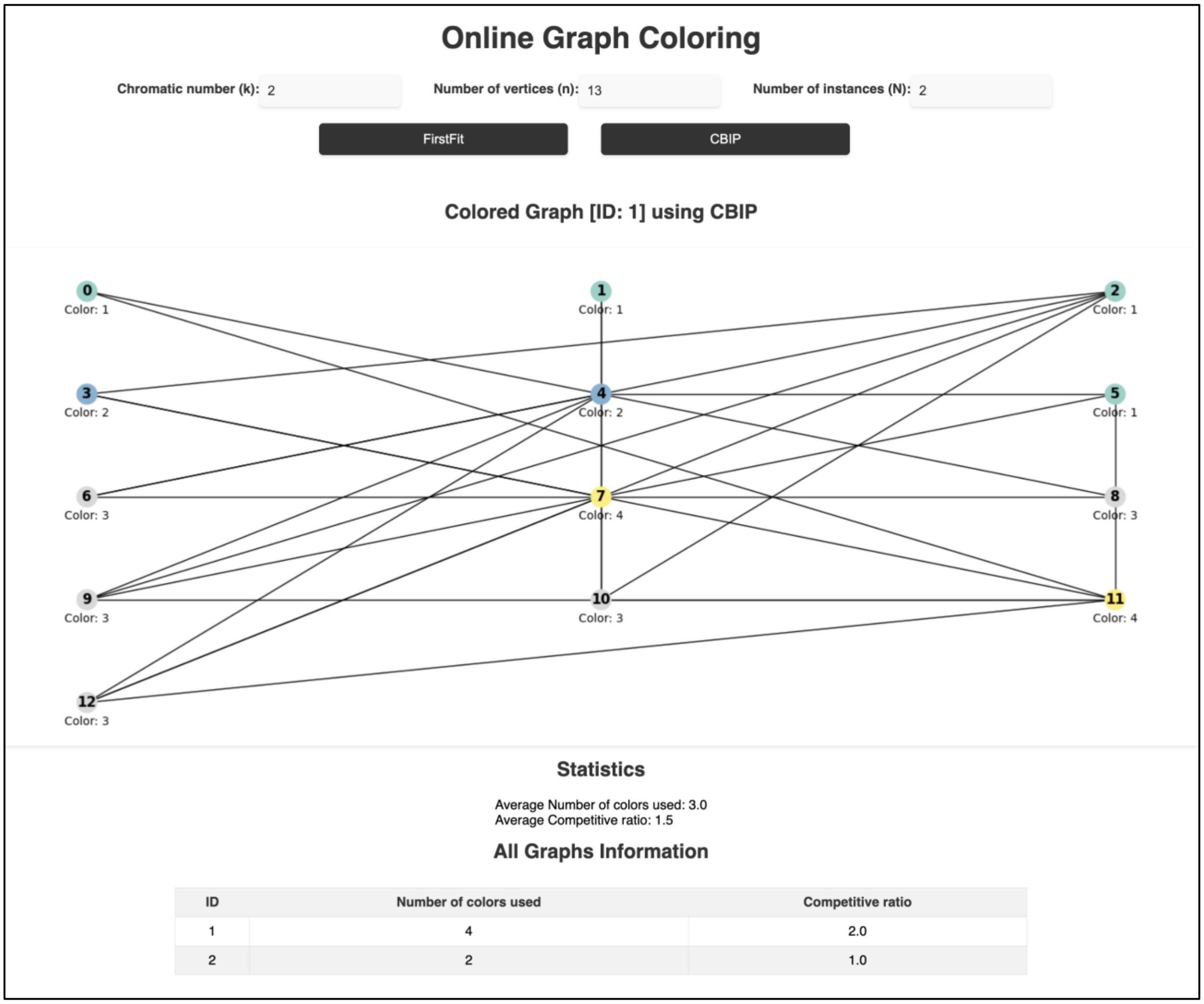

Fig. 7 Visualization of CBIP Algorithm on 2-colorable graph

In section V, we will provide the conclusions of our findings for the competitive ratios for the First Fit and CBIP graph coloring algorithms. Further, we will discuss the future scope of works that can be done relating to our empirical study.

V. Conclusion

To conclude, the analysis of the First Fit and CBIP algorithms for online graph coloring problems shows that both algorithms perform efficiently on varying graphs. The average competitive ratios for First Fit were observed to be between 1.08 and 1.55 for 2, 3, and 4-colorable graphs, with an increasing competitive ratio as the value of k increases. On the other hand, the CBIP algorithm, which is only applicable to bipartite or 2-colorable graphs, showed average competitive ratios ranging between 1.075 and 1.20, with the overall average of average competitive ratios of 1.13, which is close to being optimal.

When we directly compare First Fit and CBIP graph coloring algorithms using the same test graphs, we find that CBIP algorithm always performs better than First Fit algorithm in all test cases as seen in Fig. 5. We can finally conclude that CBIP algorithm is better than First Fit algorithm in terms of optimality in the coloring problem.

There is future scope to understand if there is any correlation between k and the average competitive ratio values for the First Fit Algorithm by performing empirical studies for many values of k for k-colorable graphs. Also, there is scope to see if there is a relation between the number of graphs and vertices used on First Fit and the CBIP algorithm in large random graphs with varying colorable graphs.

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VII. Live Visualization Demo

The visualization for our project is hosted online and can be accessed through the following link:

http://gurpreetnanda.pythonanywhere.com/

VIII. TEAMWORK DETAILS

The below Table refers to the teamwork distribution done for the successful completion of the project in the requirement for the COMP 6651 course.

Item	**Primary Contributor**	**Secondary Contributor(s)**
Report	Himangshu Shekhar Baruah	Ali Affan, Gurpreet Singh Nanda
First Fit Algorithm	Ali Affan	Gurpreet Singh Nanda
CBIP Algorithm	Gurpreet Singh Nanda	Himangshu Shekhar Baruah, Ali Affan
Graph Generation	Gurpreet Singh Nanda	Himangshu Shekhar Baruah
Graph Plotting	Ali Affan	Gurpreet Singh Nanda
Empirical Analysis	Himangshu Shekhar Baruah	Ali Affan
Interactive Display	Gurpreet Singh Nanda	Himangshu Shekhar Baruah, Ali Affan