Homework2-INF552-Rahul Ethiraj

1.) Time Series Classification

a) Importing data

b) Train test split data :

x_train_df
x_test_df

c)

(i) Feature Extraction:

Minimum, maximum, mean, standard deviation, skewness, Fourier transforms of a range, periodicity, serial correlation, chaos, nonlinearity of time series are various methods of time series classification.

(ii) Time-domain features:

	min1	max1	mean1	median1	std1	1st quart1	3rd quart1	min2	max2	mean2	median2	std2	1st quart2	3rd quart2	min3	max3	mean3	median
0	37.25	45.00	40.624792	40.500	1.476967	39.2500	42.0000	0.0	1.30	0.358604	0.430	0.322605	0.0000	0.5000	4.00	29.50	19.040937	19.25
1	38.00	45.67	42.812812	42.500	1.435550	42.0000	43.6700	0.0	1.22	0.372438	0.470	0.289158	0.0000	0.5000	2.00	29.50	20.096979	21.00
2	35.00	47.40	43.954500	44.330	1.558835	43.0000	45.0000	0.0	1.70	0.426250	0.470	0.338690	0.0000	0.5000	6.50	29.75	22.122354	23.00
3	33.00	47.75	42.179813	43.500	3.670666	39.1500	45.0000	0.0	3.00	0.696042	0.500	0.630860	0.0000	1.1200	8.50	30.00	22.183625	23.00
4	33.00	45.75	41.678063	41.750	2.243490	41.3300	42.7500	0.0	2.83	0.535979	0.500	0.405469	0.4300	0.7100	3.00	28.25	19.006562	19.12
5	37.00	48.00	43.454958	43.250	1.386098	42.5000	45.0000	0.0	1.58	0.378083	0.470	0.315566	0.0000	0.5000	5.75	27.00	15.793333	15.00
6	36.25	48.00	43.969125	44.500	1.618364	43.3100	44.6700	0.0	1.50	0.413125	0.470	0.263111	0.4300	0.5000	1.50	26.33	15.868021	16.25
7	12.75	51.00	24.562958	24.250	3.737514	23.1875	26.5000	0.0	6.87	0.590833	0.430	0.837408	0.0000	0.7100	0.00	25.33	19.121333	20.25
8	0.00	42.75	27.464604	28.000	3.583582	25.5000	30.0000	0.0	7.76	0.449708	0.430	0.767197	0.0000	0.5000	7.50	35.00	20.842542	20.75

(iii) Bootsrap confidence interval:

Features	Standard Deviation	Bootstrapped 90% confidence intervals
min	11.4378	[5.35032197 7.01170455]
max	14.8498	[17.6605303 19.76914773]
mean	13.8775	[11.32430434 13.30879596]
median	13.9591	[11.35162879 13.35388258]
std	1.56337	[2.08876688 2.31493206]
first_quart	13.4966	[9.95552557 11.86963068]
third_quart	14.6016	[12.74616004 14.84235322]

(iv)

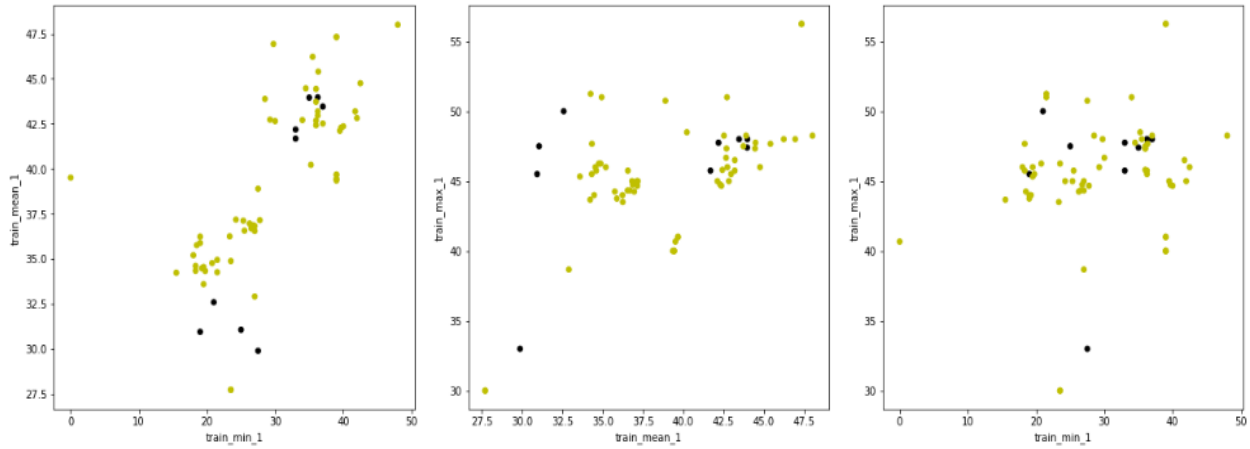
The time-domain feature ‘max’ contributed more to the target variable (classification index: bending vs others). I chose min, max and mean as the time-domain features and moved forward.

(d) Binary Classification Using Logistic Regression

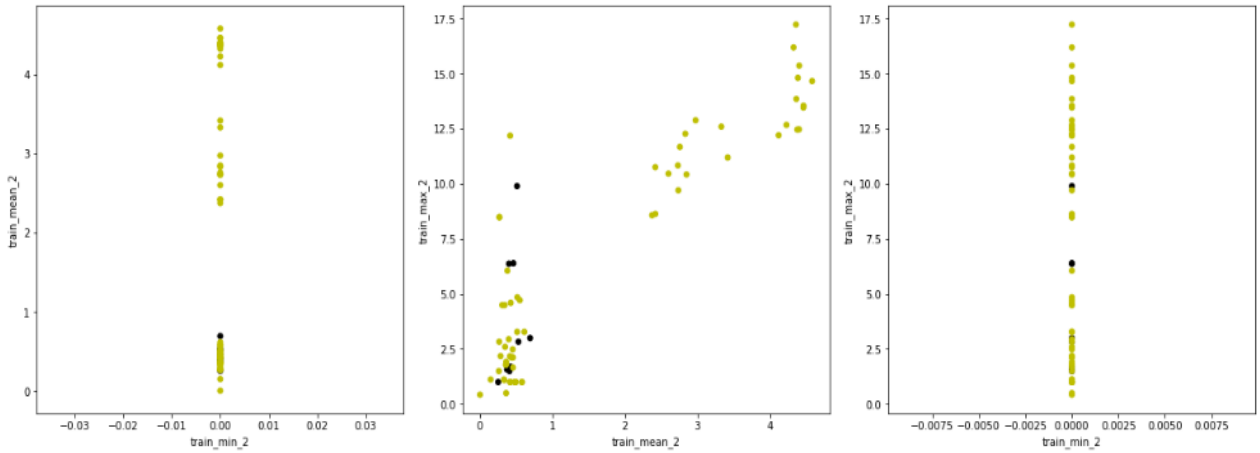
(i) Scatter plots of the features:

Note : Black represents bending and yellow others

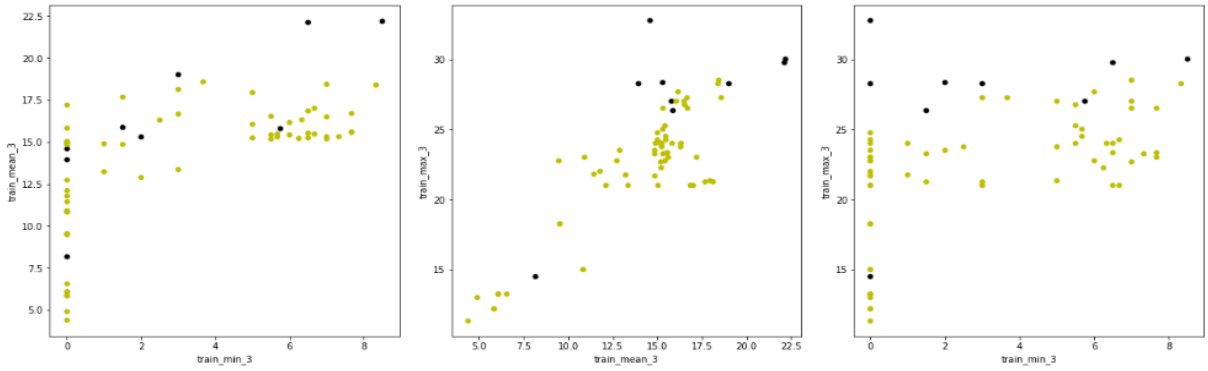
Scatter plot for Time series : 1



Scatter plot for Time series : 2



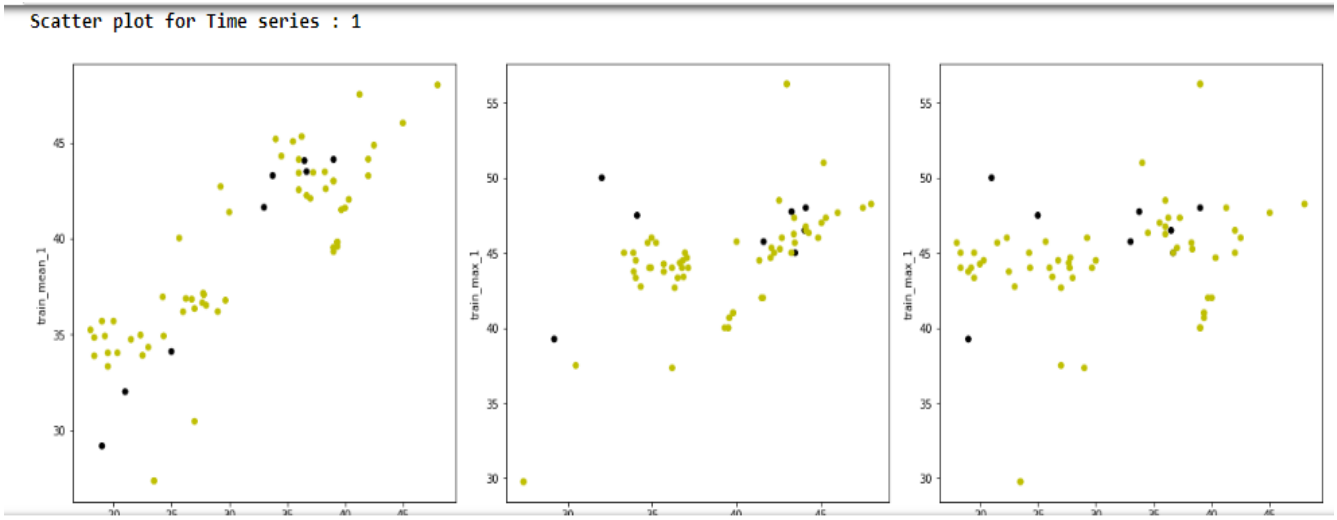
Scatter plot for Time series : 3

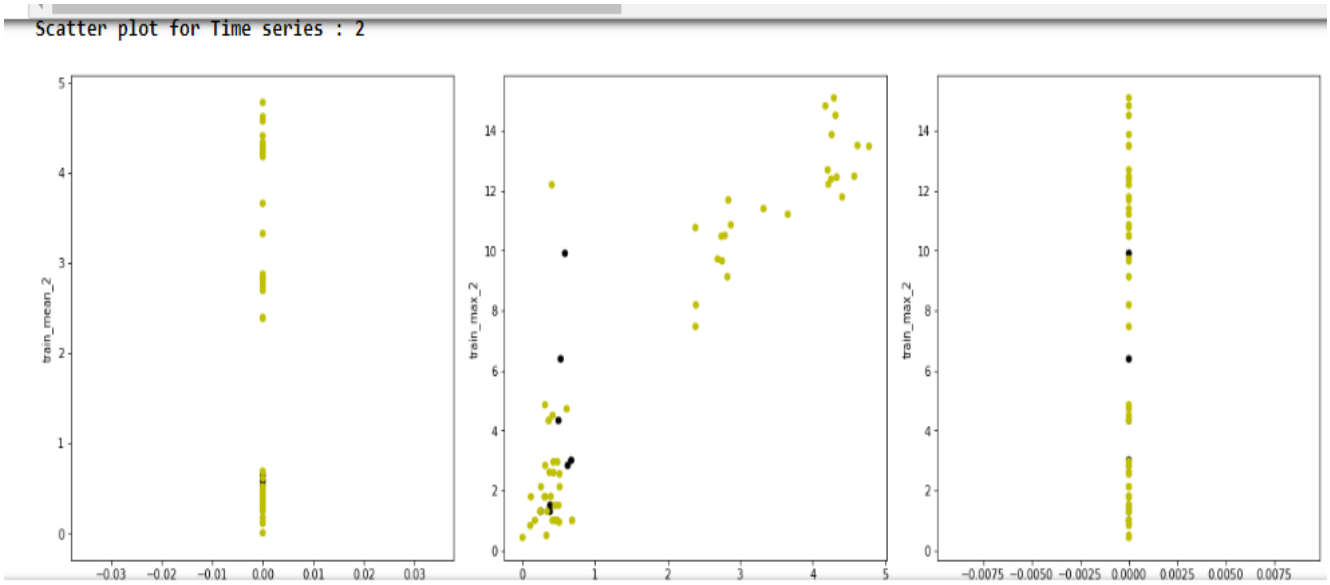


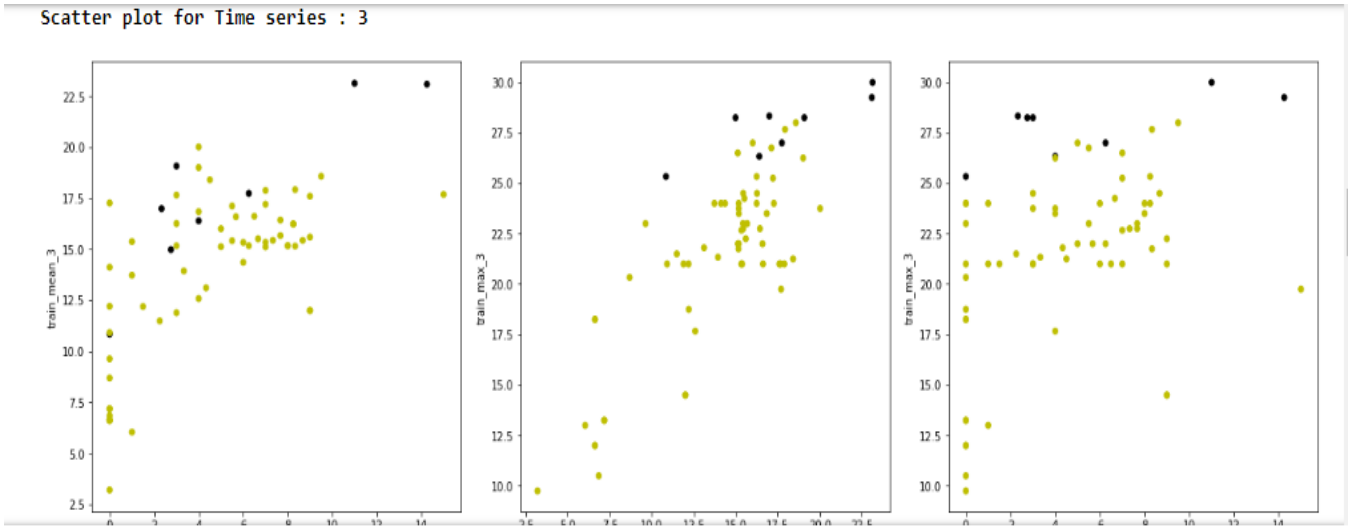
(ii)

Dividing the train dataset into two parts, L=2

L=1:

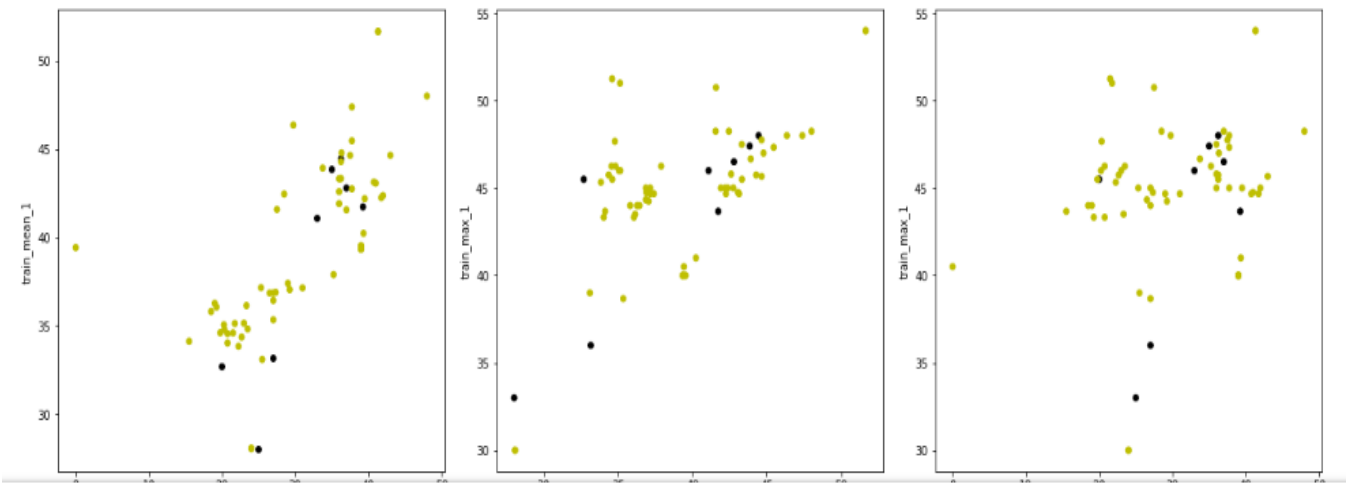




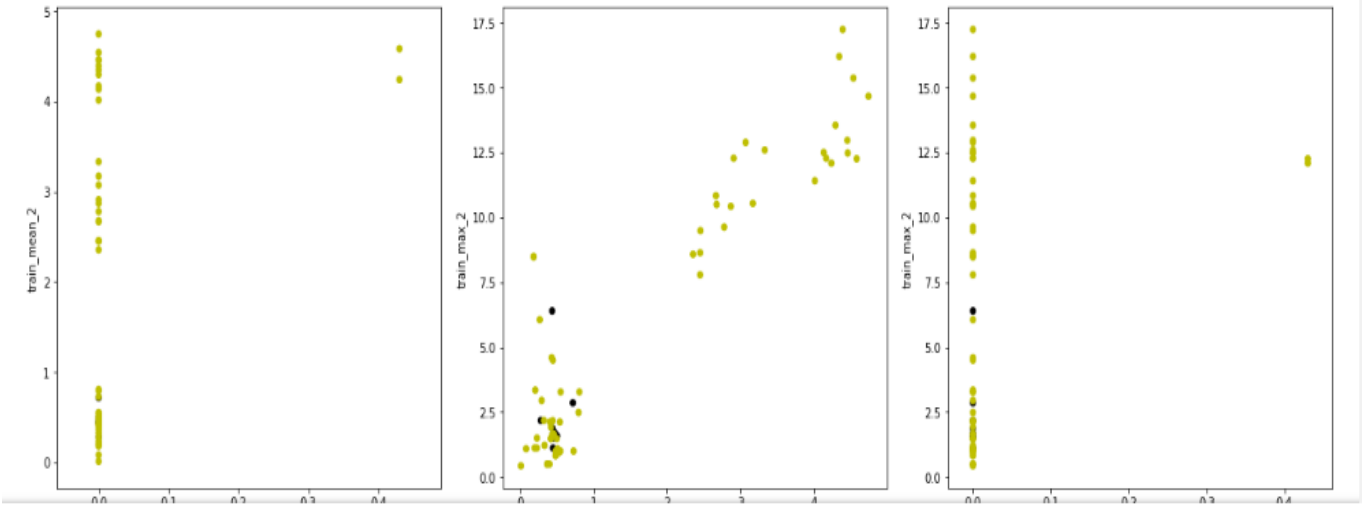


L=2:

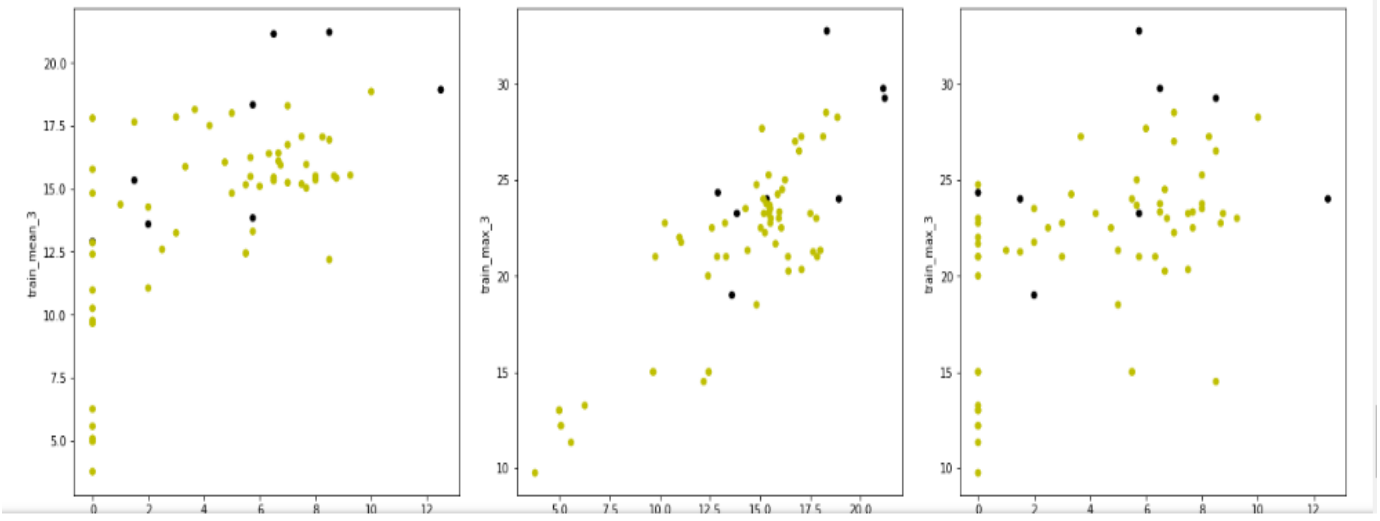
Scatter plot for Time series : 1



Scatter plot for Time series : 2



Scatter plot for Time series : 3



Splitting the data is recommended here for this dataset as that modifies the features treating the whole data as two new datasets, which can lead to better training of the model.

This is evident from the following the results using the model.score() function.

(iii) Breaking each time series at different sizes

For l = {1,2, 3,…,20} time series.

IN L= 1
In part: 1
(69, 43)
Starting : 0
Ending : 480
Fold: 0, Score: 1.0

p values for X_train_K :

[8.50600808e-01 7.21466227e-01 3.38076083e-01 6.97191503e-01
1.06793874e-01 4.72624481e-01 7.46107796e-01 nan
2.57166794e-02 1.11614701e-01 4.28838813e-02 2.25191771e-01
2.71620102e-02 9.19249113e-03 3.94769109e-01 1.66112842e-02
2.13167491e-04 1.74284208e-02 1.09156965e-01 4.36611947e-02
7.79064203e-03 nan 1.24483775e-01 6.09470586e-01
6.09981461e-02 6.97956688e-01 1.23017963e-01 8.60829278e-02
6.34846753e-31 1.12626733e-10 1.18576815e-12 4.49172603e-10
4.04163452e-01 7.90099117e-12 7.39335177e-09 8.02265410e-01
6.83797960e-02 1.69673886e-01 5.90469738e-02 4.33482249e-01
5.86908338e-02 3.56414347e-02]
Fold: 1, Score: 1.0

p values for X_train_K :

[5.61643838e-01 8.93587320e-01 8.92314287e-01 7.43788750e-01
6.74439422e-01 8.71427760e-01 8.07071376e-01 nan
2.55660879e-02 1.01210198e-03 3.55548048e-02 1.37643358e-01
5.19466052e-02 1.02614038e-02 4.37909670e-01 5.86476646e-02
2.36426176e-02 6.18554711e-02 5.57474143e-01 7.49382105e-02
5.64079836e-02 nan 1.16856313e-01 6.35934313e-01
7.41741457e-02 6.03420558e-01 1.42802306e-01 7.73013447e-02
4.63281722e-47 5.70187249e-13 1.84516780e-08 9.14592233e-13
4.67350566e-01 2.55382756e-16 5.44996491e-10 8.02265410e-01
4.26656681e-02 1.02602109e-01 4.56038526e-02 2.81732824e-01
4.19379352e-02 2.55628787e-02]
Fold: 2, Score: 1.0

p values for X_train_K :

[8.89681752e-01 2.62979509e-01 8.78639428e-01 2.87454990e-01

4.25141950e-01 1.63043371e-01 1.92581005e-01 nan
6.27842250e-03 1.93137654e-03 1.00210185e-02 1.05265970e-01
1.15897675e-02 1.60158515e-03 2.84971175e-01 1.68961762e-01
1.33325428e-02 1.31703387e-01 1.12272400e-01 2.73614921e-01
8.40849746e-02 nan 4.74204717e-02 5.17158856e-01
3.02225347e-02 4.77759842e-01 3.02362456e-02 2.62503441e-02
1.27310593e-22 6.52854551e-08 1.36949592e-09 2.95815730e-07
3.30813319e-01 1.70667955e-08 1.56687498e-06 8.02265410e-01
4.11077377e-02 1.70806940e-01 3.80312888e-02 3.35961277e-01
3.85617885e-02 2.02754272e-02]
Fold: 3, Score: 1.0

p values for X_train_K :

[7.34863443e-01 5.67661637e-01 9.83285393e-01 5.44475162e-01
8.72003636e-01 6.36480762e-01 4.43187857e-01 nan
1.36136740e-02 3.07795012e-04 2.54393247e-02 1.07052713e-01
3.72490423e-02 3.32238148e-03 8.13566634e-01 1.09774392e-01
7.48756134e-03 1.12548215e-01 2.27890851e-01 1.99114876e-01
5.99090105e-02 nan 7.48459875e-02 9.61116321e-01
3.98339988e-02 5.59814442e-01 4.55118019e-02 4.81204514e-02
7.21386930e-29 1.91031023e-08 7.12069027e-10 9.88873224e-08
8.10441535e-01 4.57362630e-10 2.28507947e-06 8.02265410e-01
4.64123648e-02 1.95497077e-02 4.97780064e-02 2.72383878e-01
5.63427206e-02 2.37107143e-02]
Fold: 4, Score: 1.0

p values for X_train_K :

[9.15594845e-01 3.31906285e-01 9.55786346e-01 3.41852073e-01
3.62091804e-01 2.21721444e-01 2.68085243e-01 nan
7.67888706e-03 3.94198665e-03 1.30223749e-02 1.07127471e-01
1.96734941e-02 2.02241726e-03 2.62024422e-01 3.01977043e-01
1.00999486e-02 3.29513157e-01 9.37273484e-02 5.41602127e-01
1.40035490e-01 nan 6.62121987e-02 9.76627494e-01
3.17478736e-02 6.04854512e-01 5.33885342e-02 3.60199754e-02
9.22256044e-21 7.70861092e-08 5.64703737e-10 3.00298952e-07
4.12664527e-01 2.09612864e-08 1.80436965e-06 nan
3.58441417e-02 1.73869241e-01 3.33648574e-02 3.02241446e-01
3.94896862e-02 1.39319173e-02]
Final Score for L= 1 : 1.0
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IN L= 2
In part: 1
(69, 43)

Starting : 0
Ending : 240
Fold: 0, Score: 1.0

p values for X_train_K :

[9.06655449e-01 7.57295333e-01 8.15301500e-01 4.49291632e-01
 3.21144067e-01 5.64848252e-01 8.74419349e-01   nan
 1.80750537e-02 9.67663108e-03 2.67035095e-02 1.82260970e-01
 1.43092575e-02 6.73220066e-03 1.43677531e-01 3.55619290e-02
 1.07130992e-02 3.52254271e-02 3.17449719e-01 5.79419803e-02
 3.13377119e-02 8.02265410e-01 1.05101218e-01 6.68637084e-01
 6.91098107e-02 6.04872028e-01 1.10683351e-01 7.00443255e-02
 9.66173621e-23 1.73267538e-11 1.37374694e-10 5.93428862e-11
 2.99685078e-01 1.53922297e-11 5.06072199e-11 8.02265410e-01
 7.50205675e-02 2.24114643e-01 8.32587970e-02 4.20447944e-01
 5.03444919e-02 6.26987373e-02]
Fold: 1, Score: 1.0

p values for X_train_K :

[9.86657167e-01 8.58220294e-01 9.50331243e-01 6.89307646e-01
 5.17882873e-01 8.25661343e-01 8.64501020e-01   nan
 1.25480467e-02 5.87772636e-04 1.74292877e-02 1.41841654e-01
 1.70421842e-02 5.89467242e-03 4.19463298e-01 4.08601824e-02
 1.03791721e-02 3.74529998e-02 5.45700068e-02 2.33799739e-01
 4.53790852e-03 7.23232351e-01 9.80476880e-02 4.02145605e-01
 7.02862000e-02 5.78336566e-01 9.36779324e-02 7.54987237e-02
 1.51014206e-29 2.13243076e-12 5.57132492e-11 5.30130391e-12
 7.58139278e-01 8.65305772e-14 5.99588527e-11 6.59642350e-01
 4.32752947e-02 1.48011691e-02 5.75710990e-02 2.86738179e-01
 4.29580270e-02 2.75684179e-02]
Fold: 2, Score: 1.0

p values for X_train_K :

[4.80384028e-01 4.71307035e-01 8.19991983e-01 2.70398931e-01
 1.07889138e-01 2.94186324e-01 9.75182776e-01   nan
 2.23791852e-02 1.13160231e-02 2.66120303e-02 2.32005394e-01
 2.20447569e-02 9.96301129e-03 4.78938789e-01 2.83231881e-01
 1.65555215e-02 3.11843872e-01 2.92881863e-02 9.99173227e-01
 6.26438792e-02 7.23232351e-01 1.71147783e-01 9.65549418e-01
 9.97710605e-02 8.36210936e-01 1.86416338e-01 1.28830523e-01
 1.24468347e-24 6.03023138e-11 6.01518774e-09 2.06829775e-10
 7.44998285e-01 1.98384754e-12 3.12384170e-09 6.59642350e-01
 9.04160448e-02 4.35196938e-01 9.57611407e-02 5.22603901e-01
 6.28019457e-02 7.07344521e-02]

Fold: 3, Score: 1.0

p values for X_train_K :

[7.33954838e-01 8.83695615e-01 4.01517983e-01 4.35416400e-01
 4.31364431e-02 5.41428551e-01 5.12632886e-01 nan
 1.22780177e-02 3.08690688e-02 1.72115119e-02 1.73965165e-01
 1.73370226e-02 3.70702432e-03 8.73385349e-01 6.67017047e-02
 9.16523666e-04 6.78167718e-02 8.56655114e-03 4.95957192e-01
 5.45129683e-03 7.23232351e-01 1.17217294e-01 8.87904994e-01
 6.56532371e-02 7.69961326e-01 7.33391612e-02 8.31280771e-02
 5.86133697e-17 1.33024143e-10 5.20442922e-12 2.57676463e-10
 1.04374938e-01 3.17852763e-10 1.90801928e-10 7.17139501e-01
 7.14819460e-02 3.36357729e-01 5.75205842e-02 5.28408090e-01
 4.39723958e-02 4.38575479e-02]
Fold: 4, Score: 1.0

p values for X_train_K :

[3.87892400e-01 5.46515791e-01 9.65591501e-01 2.52743601e-01
 6.87198191e-02 3.10192901e-01 8.63355523e-01 nan
 1.06113296e-02 1.07231732e-02 1.96442551e-02 1.59790263e-01
 1.12401858e-02 3.13525797e-03 8.44538179e-01 4.94189721e-02
 3.03935708e-03 4.86483144e-02 2.94200089e-02 2.84114595e-01
 6.04356087e-03 7.88925395e-01 6.03372583e-02 2.86584636e-01
 3.50074438e-02 5.89098356e-01 5.10128067e-02 4.20061585e-02
 8.31493402e-29 1.02953317e-12 1.39799316e-11 3.48062001e-12
 2.17797482e-01 1.31071506e-12 2.09250607e-12 6.37772926e-01
 4.31024966e-02 1.70367150e-01 4.53232265e-02 4.11178434e-01
 2.45193686e-02 2.81385612e-02]

IN L= 2
In part: 2
(69, 43)
Starting : 240
Ending : 480
Fold: 0, Score: 0.9285714285714286

p values for X_train_K :

[7.57585286e-01 1.30282872e-01 9.90416137e-02 8.59149788e-02
 7.51591963e-01 1.10978404e-01 1.73947427e-01 8.02265410e-01
 1.95275342e-02 3.24596874e-04 2.17968216e-02 1.49460407e-01
 3.81699108e-02 1.22271932e-02 2.58654925e-01 1.99231044e-01
 1.06480872e-01 1.50053899e-01 5.64155042e-01 3.13239831e-01
 1.52466478e-01 nan 1.02366977e-01 4.43913241e-01
 6.20688367e-02 5.49901860e-01 1.33614630e-01 6.28051421e-02

4.63666725e-12 7.67540850e-06 1.22039626e-04 2.34281630e-05
9.45016542e-01 1.07082899e-06 3.75754067e-05 6.59642350e-01
6.03501766e-02 1.95371670e-02 6.77705910e-02 2.67995312e-01
1.10845581e-01 2.77637582e-02]
Fold: 1, Score: 1.0

p values for X_train_K :

[3.75546669e-01 5.14963366e-01 1.33039786e-01 4.68825669e-01
3.98762090e-02 7.32858644e-01 2.84615797e-01 8.02265410e-01
7.45417350e-03 1.04715855e-06 1.02276367e-02 5.42148984e-02
3.36217996e-02 3.24913916e-03 1.65887071e-01 4.95866031e-01
4.51204709e-01 3.50706323e-01 6.37362752e-01 6.31356463e-01
3.96398383e-01 8.02265410e-01 3.91117568e-02 3.96070973e-02
2.69754866e-02 3.62654992e-01 4.49844537e-02 1.69770768e-02
2.48175822e-16 2.03441369e-06 3.71060791e-03 2.39334470e-06
3.02910975e-01 4.62438545e-08 5.03062944e-05 6.12419504e-01
2.11737224e-02 2.09180482e-03 1.90699940e-02 1.53354448e-01
6.62930905e-02 9.04107895e-03]
Fold: 2, Score: 0.9285714285714286

p values for X_train_K :

[6.64957224e-01 5.30372019e-01 3.59196965e-01 3.51054067e-01
4.10011005e-01 5.86822723e-01 4.97558279e-01 7.23232351e-01
1.40529402e-02 1.29654482e-04 2.85713612e-02 8.28116673e-02
7.93733808e-02 3.37010858e-03 4.27930710e-01 1.53760575e-01
1.24465148e-01 1.50788550e-01 9.30804305e-01 1.20781503e-01
1.75733205e-01 8.02265410e-01 5.44238353e-02 2.76959019e-01
4.03118338e-02 4.09009564e-01 5.18471041e-02 2.88096295e-02
1.48491187e-21 5.78701221e-07 1.29915468e-04 1.77853816e-06
4.89384414e-01 1.39493028e-08 7.91220469e-06 7.23232351e-01
4.50092915e-02 2.47744922e-03 4.27573941e-02 2.13479071e-01
1.06234406e-01 2.17752928e-02]
Fold: 3, Score: 0.9285714285714286

p values for X_train_K :

[9.17280241e-01 4.72857013e-01 3.67459924e-01 3.19902245e-01
8.15567024e-01 4.38994998e-01 5.67380541e-01 7.23232351e-01
1.23088747e-02 1.04727915e-04 1.91750283e-02 1.07449698e-01
3.79841780e-02 5.87491519e-03 2.93044855e-01 2.65816421e-02
1.35795717e-02 2.26966764e-02 4.02987128e-01 3.45149589e-02
1.68907673e-02 8.02265410e-01 7.57617648e-02 3.41352066e-01
4.12185183e-02 4.98684270e-01 9.23819596e-02 3.96907114e-02
4.15752921e-23 2.49563718e-08 1.26925366e-06 1.39322735e-07
9.21658503e-01 2.10628080e-09 1.68511797e-07 6.59642350e-01

3.51012906e-02 5.98980110e-03 2.80085392e-02 2.55999638e-01
5.31129428e-02 1.65165563e-02]
Fold: 4, Score: 0.9230769230769231

p values for X_train_K :

[7.43084557e-01 7.27702970e-01 4.80362188e-01 5.45620319e-01
5.75010937e-01 7.16394172e-01 7.52237102e-01 7.04989460e-01
1.19323507e-02 1.45762030e-05 1.44226506e-02 8.89040600e-02
4.99342933e-02 5.01723805e-03 2.99269301e-01 5.64586012e-02
2.71675509e-02 4.66859795e-02 5.63581870e-01 9.72811710e-02
3.37680016e-02 7.88925395e-01 5.50478571e-02 2.04560286e-01
3.57539536e-02 4.10414355e-01 5.16605651e-02 3.00740908e-02
1.07529015e-26 4.40713376e-10 1.48479790e-06 4.27250018e-09
4.79194561e-01 2.78354679e-12 2.57379348e-08 5.88092328e-01
2.09241397e-02 1.54025009e-03 2.09775032e-02 1.69423982e-01
4.57883959e-02 9.90754586e-03]
Final Score for L= 2 : 0.9417582417582417

Similarly, all the values for L=3,4…,20 were observed, by iterating the loops for different sizes.

Final Score for L= 1 : 1.0
Final Score for L= 2 : 0.9417582417582417
Final Score for L= 3 : 0.9417582417582417
Final Score for L= 4 : 0.9571428571428573
Final Score for L= 5 : 0.9560439560439562
Final Score for L= 6 : 0.9428571428571428
Final Score for L= 7 : 0.9571428571428571
Final Score for L= 8 : 0.9571428571428571
Final Score for L= 9 : 0.9417582417582417
Final Score for L= 10 : 0.9417582417582417
Final Score for L= 11 : 0.956043956043956
Final Score for L= 12 : 0.9571428571428571
Final Score for L= 13 : 0.956043956043956
Final Score for L= 14 : 0.9417582417582417
Final Score for L= 15 : 0.9417582417582417
Final Score for L= 16 : 0.9703296703296704
Final Score for L= 17 : 0.9417582417582417
Final Score for L= 18 : 0.9417582417582417
Final Score for L= 19 : 0.956043956043956
Final Score for L= 20 : 0.9714285714285715

Best L : 1
Pruned number of features : 5
Error inverse : 1.0
Features list : [False False False False False False True False False False False False
False False False False False True False False False False False
False False False False False True True False False True False False
False False False False False False]

For each value of L the parts of data passed to the model were different, which resulted in different set of features to be pruned each time by the model.

I used Recursive Feature Elimination method for pruning the full 42 features.

Cross-validation can be done in two different ways: the wrong way and the right way. The only difference is that in the former, we perform the variable selection before cross validation using all the samples. In the latter, we perform the variable selection within a K-fold cross validation loop each and every time.

Re-fitting the model using pruned set of features by RFE

Regression Coef:

```
 [[-0.30272987 -0.22798283  0.78625013  0.42167481 -0.51325141]]
```

p values for x_train :

```
[3.90271388e-01 2.84100194e-01 5.46927780e-08 8.13227556e-06
 2.18370678e-08]
```

Final Score for L = 1 : 0.927536231884058

```
Printing all feature p-values :
==============================================================================
Printing for third_quart1
                            OLS Regression Results
==============================================================================
Dep. Variable:                 target   R-squared:                       0.023
Model:                            OLS   Adj. R-squared:                  0.008
Method:                 Least Squares   F-statistic:                     1.581
Date:                Mon, 02 Jul 2018   Prob (F-statistic):              0.213
Time:                        20:17:48   Log-Likelihood:                -22.008
No. Observations:                  69   AIC:                             48.02
Df Residuals:                      67   BIC:                             52.48
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept          0.6059      0.380      1.593      0.116      -0.153       1.365
I(third_quart1)   -0.0119      0.009     -1.257      0.213      -0.031       0.007
==============================================================================
```

```
Omnibus:                       39.023   Durbin-Watson:
0.134
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
76.067
Skew:                           2.150   Prob(JB):                     3
.04e-17
Kurtosis:                       5.822   Cond. No.
377.
==============================================================================
=======

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.


==============================================================================
```
Printing for median3
```
                            OLS Regression Results
==============================================================================
=======
Dep. Variable:                 target   R-squared:
0.011
Model:                            OLS   Adj. R-squared:
-0.004
Method:                 Least Squares   F-statistic:
0.7142
Date:                Mon, 02 Jul 2018   Prob (F-statistic):
0.401
Time:                        20:17:48   Log-Likelihood:
-22.447
No. Observations:                  69   AIC:
48.89
Df Residuals:                      67   BIC:
53.36
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
=======
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
-------
Intercept      0.0269      0.129      0.208      0.836      -0.231
0.285
I(median3)     0.0073      0.009      0.845      0.401      -0.010
0.025
==============================================================================
=======
Omnibus:                       39.247   Durbin-Watson:
0.145
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
76.573
Skew:                           2.169   Prob(JB):                     2
.36e-17
Kurtosis:                       5.795   Cond. No.
47.2
```

```
==============================================================================
=======

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.


==============================================================================
```
Printing for max5
```
                            OLS Regression Results
==============================================================================
=======
Dep. Variable:                  target   R-squared:
0.206
Model:                             OLS   Adj. R-squared:
0.194
Method:                  Least Squares   F-statistic:
17.34
Date:                 Mon, 02 Jul 2018   Prob (F-statistic):           9
.14e-05
Time:                         20:17:48   Log-Likelihood:
-14.873
No. Observations:                  69   AIC:
33.75
Df Residuals:                      67   BIC:
38.21
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
=======
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
-------
Intercept     -0.4415      0.142     -3.106      0.003      -0.725
-0.158
I(max5)        0.0291      0.007      4.164      0.000       0.015
0.043
==============================================================================
=======
Omnibus:                       30.715   Durbin-Watson:
0.467
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
49.943
Skew:                           1.773   Prob(JB):                     1
.43e-11
Kurtosis:                       5.190   Cond. No.
79.2
==============================================================================
=======

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.


==============================================================================
```

Printing for first_quart5

OLS Regression Results

Dep. Variable:	target	R-squared:	0.236
Model:	OLS	Adj. R-squared:	0.224
Method:	Least Squares	F-statistic:	20.68
Date:	Mon, 02 Jul 2018	Prob (F-statistic):	2.34e-05
Time:	20:17:48	Log-Likelihood:	-13.533
No. Observations:	69	AIC:	31.07
Df Residuals:	67	BIC:	35.53
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-0.3143	0.104	-3.016	0.004	-0.522	-0.106
I(first_quart5)	0.0312	0.007	4.547	0.000	0.018	0.045

Omnibus:	42.084	Durbin-Watson:	0.594
Prob(Omnibus):	0.000	Jarque-Bera (JB):	98.497
Skew:	2.120	Prob(JB):	4.09e-22
Kurtosis:	7.034	Cond. No.	44.1

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Printing for mean5

OLS Regression Results

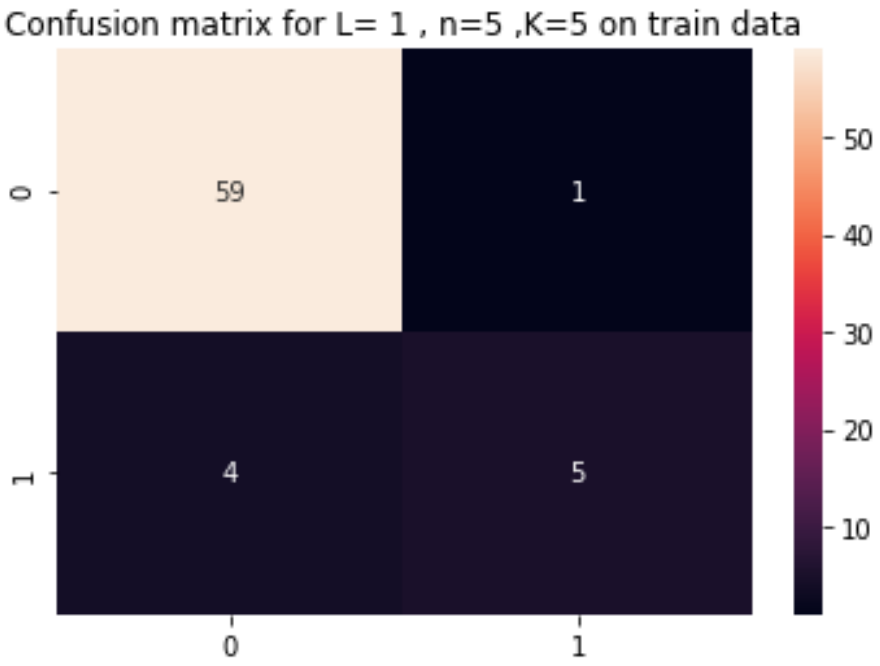
Dep. Variable:	target	R-squared:	0.253
Model:	OLS	Adj. R-squared:	0.241

```
Method:                 Least Squares   F-statistic:
22.64
Date:                Mon, 02 Jul 2018   Prob (F-statistic):           1
.08e-05
Time:                        20:17:48   Log-Likelihood:
-12.767
No. Observations:                  69   AIC:
29.53
Df Residuals:                      67   BIC:
34.00
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
=======
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
-------
Intercept     -0.3839      0.114     -3.374      0.001      -0.611
-0.157
I(mean5)       0.0329      0.007      4.759      0.000       0.019
0.047
==============================================================================
=======
Omnibus:                       39.290   Durbin-Watson:
0.636
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
84.668
Skew:                           2.019   Prob(JB):                     4
.12e-19
Kurtosis:                       6.626   Cond. No.
52.9
==============================================================================
=======

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

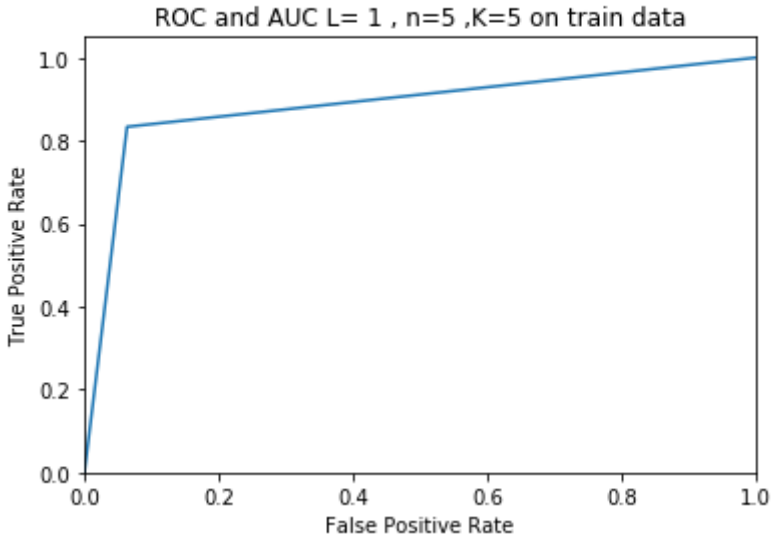
iv) Regression coefficients and p-values for L=1, Number of features=5 using 5-fold CV:

```
Regression Coef:
 [[-0.30272987 -0.22798283  0.78625013  0.42167481 -0.51325141]]

p values for x_train :
[3.90271388e-01 2.84100194e-01 5.46927780e-08 8.13227556e-06
 2.18370678e-08]

Final Score for L = 1 : 0.927536231884058
```

Confusion matrix, ROC & AUC:

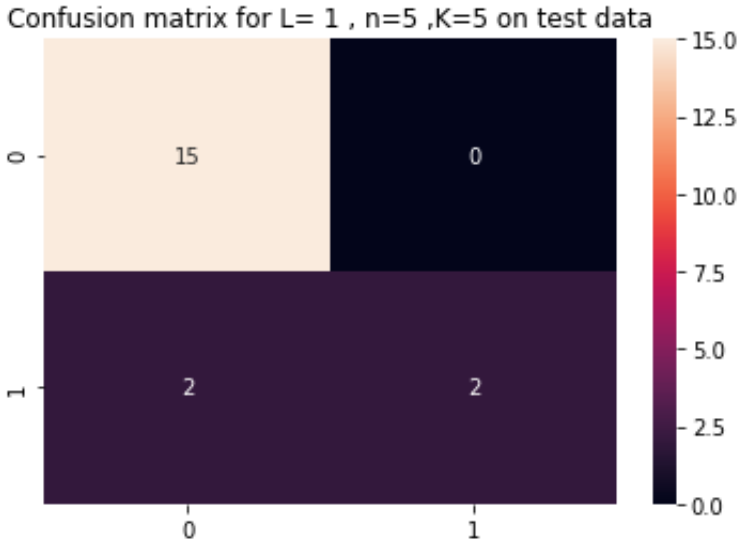



```
AUC :  0.8849206349206349
```

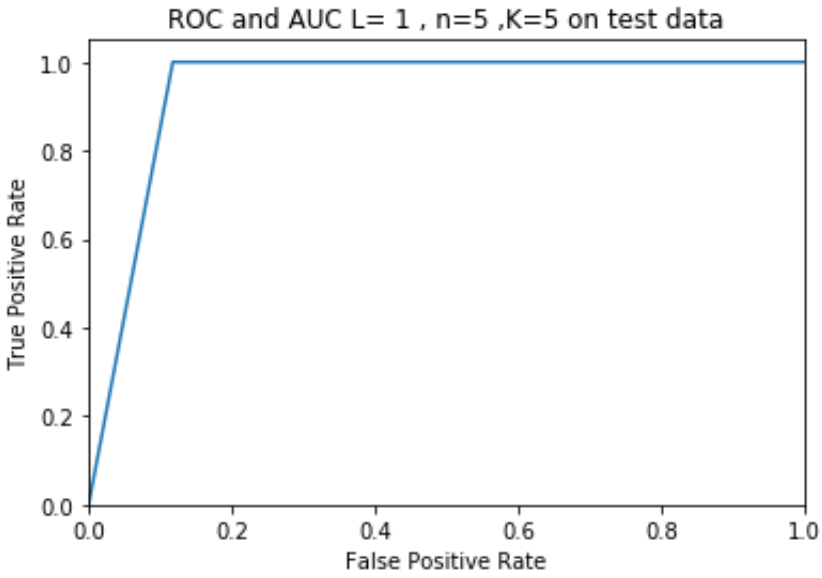


v)

```
Train data final training score for L= 1 : 0.927536231884058
Test  data final testing  score for L= 1 : 0.8947368421052632
<class 'numpy.ndarray'>
```




```
AUC score :  0.9411764705882353
```



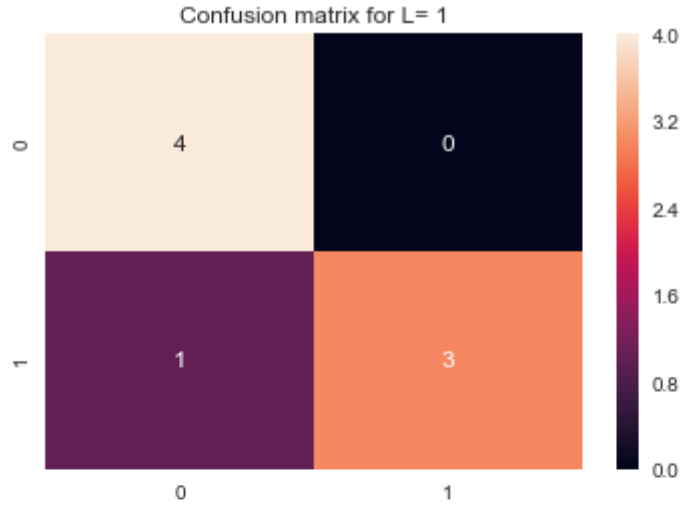
Test score is very much closer to the train score, which implies that the model has performed well on the testing data.

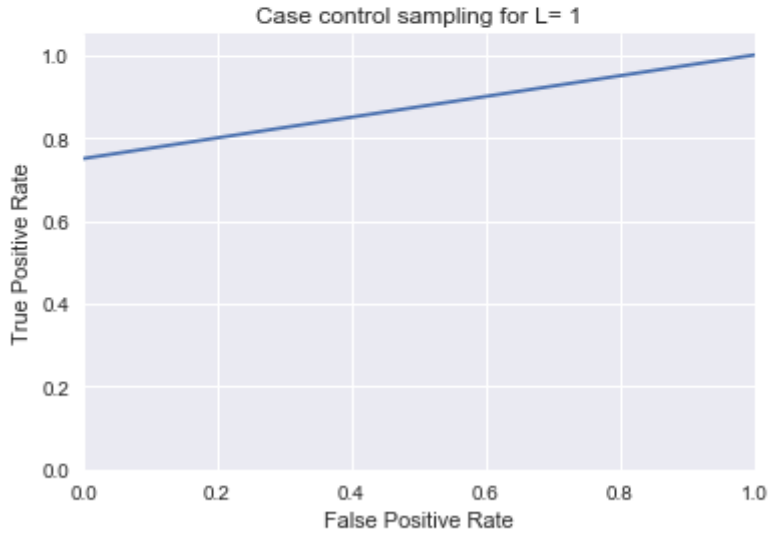
vi)

For training:
We had 5 misclassifications in total.
An AUC score of 1 is a result of 0% overlapping degree, and a score of about 0.89 signifies there is an overlap in the data.
Classes seem to be not well-separated to cause instability in calculating logistic regression parameters.

For testing:
We have 2 misclassifications.
An AUC score of 1 is a result of 0% overlapping degree, and a score of about 0.94 signifies there is an overlap in the data.
Classes seem to be not well-separated to cause instability in calculating logistic regression parameters.

vii) Case control sampling:






```
AUC              :  0.875
Error for L= 1   :  0.125
```

There exists imbalance in the data and therefore under-sampled the majority class(0).
Both the classes are now well-balanced (1:1 ratio)

(e) Binary classification using L1-penalized logistic regression:

The code tries 10 values of C and cross validates on them as well as on the train set. Following are the results:

```
IN L= 1
In part: 1
Starting : 0
Ending :  480
Fold: 0, Score: 0.9285714285714286
```
Regression coef-values :
```
[[ 0.          0.          0.          0.          0.         -0.272753
73
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   8.6146879   0.          0.          0.          0.          0.
   0.          0.          0.          0.         -2.94195997  0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 1, Score: 0.9285714285714286
```
Regression coef-values :
```
[[ 0.         -3.18615869  0.          0.          0.          0.
  -0.15209024  0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
  -0.11191496  0.          0.          0.          2.25442505  0.
   5.77754826  0.          0.          0.          0.          0.
   0.          0.         -3.47295174  0.          0.          0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 2, Score: 0.8571428571428571
```
Regression coef-values :
```
[[-5.17993223 -1.23371418  0.          0.          0.59100634 -7.258080
31
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          3.85567301  0.
   0.          0.          0.          0.          0.          0.
  -0.06565019  0.         -5.69097591  0.         24.63381078  0.
   6.3476103   0.          0.          0.          0.          0.
   0.          0.          0.          0.         -9.98324396  0.
]]
C :  [166.81005372]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 3, Score: 1.0
```
Regression coef-values :
```
[[ 0.         -3.65947558  0.          0.          0.          0.
  -0.97927771  0.          0.         -4.37417825  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
  -2.49172622  0.          0.          0.         10.65660604  0.
  14.82580117  0.         -1.00592686  0.          0.          0.
```

```
   0.          0.          0.         -0.93694531  0.          0.
]]
C :  [21.5443469]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 4, Score: 1.0
```
Regression coef-values :
```
[[ -4.69688181   0.           0.           0.           0.
    0.         -10.05972797   0.           0.          -2.54805489
    0.           0.           0.           0.           0.
    0.           0.          -0.11028776   0.           0.
   -1.70744181   0.           0.           4.63845158  -1.50871159
    0.           0.           0.          23.71731575   0.
   18.22184272   0.           0.           0.           0.
    0.           0.           0.           0.           0.
   -9.12127452   0.        ]]
C :  [166.81005372]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
```
Final Score for L= 1 : [0.9428571428571428]
```
==========================================================================
==========================================================================

IN L= 2
In part: 1
Starting : 0
Ending :  240
Fold: 0, Score: 0.9285714285714286
```
Regression coef-values :
```
[[ 0.          0.          0.         -3.2463717   0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          2.33742012  0.
   5.13832456  0.          0.          0.          0.          0.
   0.          0.          0.          0.         -4.23608703  0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 1, Score: 0.9285714285714286
```
Regression coef-values :
```
[[ 0.          0.          0.         -2.39728467  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          2.28244987  0.
   5.61525796  0.          0.          0.          0.          0.
   0.          0.          0.          0.         -3.39501999  0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 2, Score: 1.0
```

Regression coef-values :

```
[[ 0.          0.          0.         -6.28830876  0.          0.
   0.          0.          0.         -1.944261    0.          0.
   0.          0.          0.          0.          0.          0.
   0.         -0.74317906  0.          0.          0.          0.082060
9
   0.          0.          0.          0.         13.7467722   0.
  12.0990617   0.          0.          0.          0.          0.
   0.          0.          0.          0.         -6.0920041   0.
]]
C :  [21.5443469]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 3, Score: 1.0
```

Regression coef-values :

```
[[ 0.          0.          0.         -0.69137297  0.         -3.294860
98
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.         -2.66942252  0.          0.          0.          0.
  -4.10828134  0.          0.          0.         11.2863585   0.
  15.12561329  0.          0.          0.          0.          0.
   0.          0.          0.          0.         -2.84244273  0.
]]
C :  [21.5443469]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 4, Score: 0.9230769230769231
```

Regression coef-values :

```
[[ 0.          0.          0.         -1.91279038  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.3881247   0.
   7.95487675  0.          0.          0.          0.          0.
   0.          0.          0.          0.         -3.59724743  0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]

IN L= 2
In part: 2
Starting : 240
Ending :  480
Fold: 0, Score: 0.8571428571428571
```

Regression coef-values :

```
[[ -4.15478151  -4.25909276   0.           0.         -22.50674674
    0.         -20.97308507   0.           0.           0.
    0.         -11.53023299   0.          -1.25970788  -6.65882722
    3.24874939  17.62642168   0.          -9.84307051   1.83283607
    0.           0.           0.          -6.88174669  -9.73233496
   23.58106466   0.           0.           3.19927128   5.85744322
   12.45289477   2.16007223 -23.8365537    1.12149946   3.00484062
    0.           0.           0.         -11.06729863   0.
```

```
   36.95331207 -22.56911476]]
C :  [1291.54966501]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 1, Score: 0.9285714285714286
Regression coef-values :
[[-0.18691306 -0.34875786  0.         -2.51811923  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          2.30999279  0.
   3.19029898  0.          0.          1.86844195  0.          0.
   0.          0.         -0.34368656  0.          0.         -4.024897
07]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 2, Score: 0.8571428571428571
Regression coef-values :
[[  0.           0.           0.          -0.16472102   0.
    0.         -14.40755269   0.           0.           0.
    0.           0.           0.          -5.94692698   1.75875836
    0.           1.36447388   0.          -9.17500526   0.
    0.           0.           0.           0.          -0.14940421
    0.          -1.55173381   0.           0.           0.
   16.17977678   0.           0.           2.34297872   0.
    0.           0.           0.           0.          -1.13876694
    0.          -0.8494328 ]]
C :  [21.5443469]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 3, Score: 1.0
Regression coef-values :
[[  0.           0.           0.           0.          -2.64319536
    0.         -12.86437522   0.           0.           0.
    0.           0.           0.           0.          -2.69649671
    0.          13.10929323   0.          -6.43492774   0.
    0.           0.           0.           0.          -3.38680311
    0.           0.           0.           0.           0.
   13.58650063   0.          -5.19641778   0.           0.
    0.           0.           0.          -7.55897776   0.
    0.           0.        ]]
C :  [21.5443469]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
Fold: 4, Score: 0.8461538461538461
Regression coef-values :
[[ 0.          0.          0.         -5.51950478  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          1.27924777  0.
   0.          0.          0.          0.          0.          0.
  -5.28792832  0.          0.          0.          6.13185612  0.
   0.28815944  0.          0.          0.          0.          0.
```

```
   0.          0.          0.          0.          0.          0.
]]
C :  [2.7825594]
CS_ :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e-02
 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02
 1.29154967e+03 1.00000000e+04]
```
Final Score for L= 2 : [0.9428571428571428, 0.8978021978021978]

==========

==========

Similarly, results were obtained for other values of L.

Final Score for L= 1 : [0.9428571428571428]
Final Score for L= 2 : [0.8978021978021978]
Final Score for L= 3 : [0.9274725274725275]
Final Score for L= 4 : [0.856043956043956]
Final Score for L= 5 : [0.9142857142857143]
Final Score for L= 6 : [0.8703296703296702]
Final Score for L= 7 : [0.8846153846153847]
Final Score for L= 8 : [0.8846153846153847]
Final Score for L= 9 : [0.9142857142857143]
Final Score for L= 10 : [0.8846153846153847]
Final Score for L= 11 : [0.8703296703296705]
Final Score for L= 12 : [0.9285714285714285]
Final Score for L= 13 : [0.8549450549450549]
Final Score for L= 14 : [0.8857142857142858]
Final Score for L= 15 : [0.9]
Final Score for L= 16 : [0.8989010989010989]
Final Score for L= 17 : [0.856043956043956]
Final Score for L= 18 : [0.8714285714285716]
Final Score for L= 19 : [0.8989010989010989]
Final Score for L= 20 : [0.8857142857142858]

Training errors:

With p-values:

Final Score for L=1: **0.927536231884058**

With L-1 penalization:

Final Score for L=1: **0.9428571428571428**

Best L : **1**

Regression coef-values :

```
[[-3.41374304e+00  0.00000000e+00  0.00000000e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00 -1.06322415e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00 -1.11427729e-03  0.00000000e+0
0
```

```
   0.00000000e+00  0.00000000e+00  2.21379049e+00 -4.34208948e+0
0

   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00  1.27603509e+01  0.00000000e+0
0

   0.00000000e+00  9.79191014e+00  0.00000000e+00  0.00000000e+0
0

   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+0
0

  -9.91925872e+00  0.00000000e+00]]

C :  [21.5443469]

C_s :  [1.00000000e-04 7.74263683e-04 5.99484250e-03 4.64158883e
-02

 3.59381366e-01 2.78255940e+00 2.15443469e+01 1.66810054e+02

 1.29154967e+03 1.00000000e+04]
```

The L-1 penalization is slightly better in the accuracy and is easier to implement because of readymade libraries.

(f)

(i) Multi-class classification (The realistic case):

```
Final Score for L= 1 : 0.9056372549019608
Final Score for L= 2 : 0.8487745098039217
Final Score for L= 3 : 0.7862745098039216
Final Score for L= 4 : 0.7571078431372549
Final Score for L= 5 : 0.732107843137255
Final Score for L= 6 : 0.7571078431372549
Final Score for L= 7 : 0.795343137254902
Final Score for L= 8 : 0.7848039215686274
Final Score for L= 9 : 0.757843137254902
Final Score for L= 10 : 0.7654411764705882
Final Score for L= 11 : 0.7688725490196078
Final Score for L= 12 : 0.7938725490196078
Final Score for L= 13 : 0.8041666666666668
Final Score for L= 14 : 0.7661764705882353
Final Score for L= 15 : 0.7446078431372549
Final Score for L= 16 : 0.7571078431372549
Final Score for L= 17 : 0.7661764705882353
Final Score for L= 18 : 0.6946078431372549
Final Score for L= 19 : 0.721078431372549
```

```
Final Score for L= 20 : 0.7370098039215687
```

Best L : 1

Error inverse : 0.9056372549019608

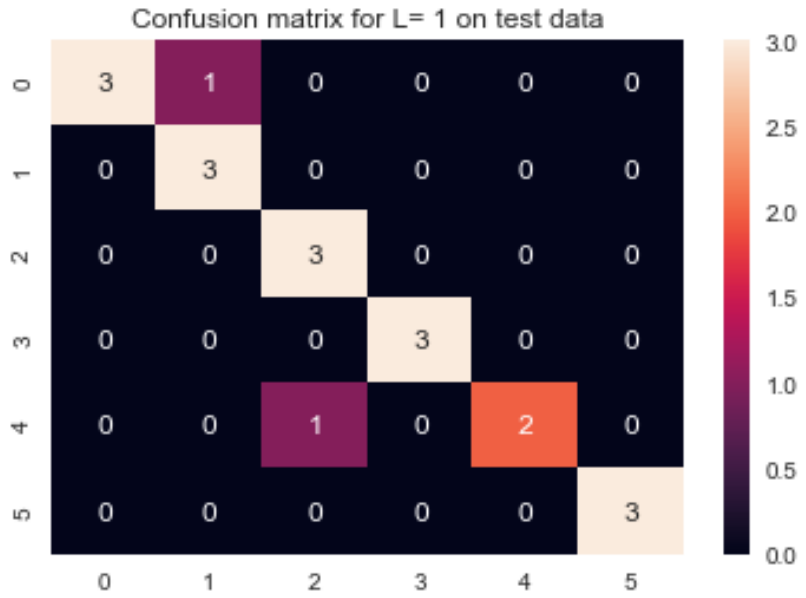
Regression coef-values :

```
[[ 0.          0.          0.          0.          0.         -1.052550
69
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          2.89333516  0.          0.          1.46603467  0.
   2.9138691   0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
]
 [ 0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.         -5.34272818  0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
]
 [ 0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.         -0.04881261  0.          0.          0.
   2.5877929   0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
]
 [ 0.          0.          0.          1.98994936  0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
]
 [ 0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          3.23716979  0.
   4.2369885   0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
]]
```

Train data final training score for L= 1 : 0.9056372549019608

```
Test  data final testing  score for L= 1 : 0.8947368421052632
```

Test score is very much closer to the train score, which implies that the model has performed well on the testing data.



ii) Naïve Bayes with Gaussian prior:

```
Final Score for L= 1 : 0.9132352941176471
Final Score for L= 2 : 0.8563725490196077
Final Score for L= 3 : 0.7669117647058823
Final Score for L= 4 : 0.7377450980392156
Final Score for L= 5 : 0.758578431372549
Final Score for L= 6 : 0.7223039215686274
Final Score for L= 7 : 0.7397058823529411
Final Score for L= 8 : 0.6529411764705882
Final Score for L= 9 : 0.695343137254902
Final Score for L= 10 : 0.7017156862745099
Final Score for L= 11 : 0.6813725490196079
Final Score for L= 12 : 0.7071078431372548
Final Score for L= 13 : 0.6794117647058824
Final Score for L= 14 : 0.6502450980392156
Final Score for L= 15 : 0.6884803921568627
Final Score for L= 16 : 0.7036764705882353
Final Score for L= 17 : 0.6419117647058823
Final Score for L= 18 : 0.6115196078431373
Final Score for L= 19 : 0.5995098039215687
Final Score for L= 20 : 0.6024509803921569
```

Best L : 1
Error inverse : 0.9132352941176471

```
Train data final training score for L= 1 : 0.9132352941176471
Test  data final testing  score for L= 1 : 0.8947368421052632
```

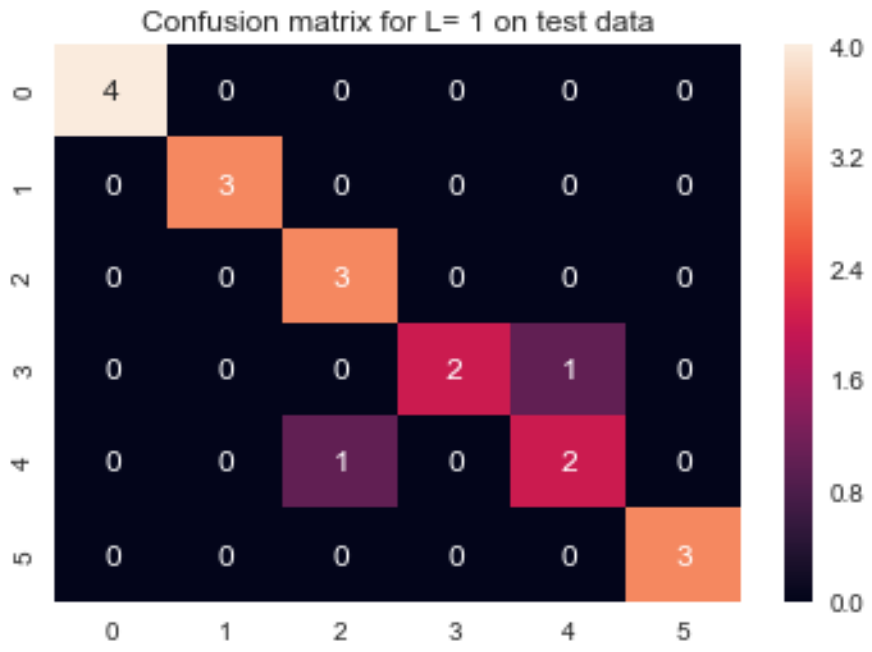
Test score is very much closer to the train score, which implies that the model has performed well on the testing data.

Naïve Bayes with Multinomial prior:

Final Score for L= 1 : 0.7134803921568629
Final Score for L= 2 : 0.6980392156862745
Final Score for L= 3 : 0.6911764705882354
Final Score for L= 4 : 0.6294117647058823
Final Score for L= 5 : 0.6676470588235295
Final Score for L= 6 : 0.6642156862745099
Final Score for L= 7 : 0.5995098039215686
Final Score for L= 8 : 0.6127450980392156
Final Score for L= 9 : 0.5634803921568629
Final Score for L= 10 : 0.5482843137254902
Final Score for L= 11 : 0.6044117647058823
Final Score for L= 12 : 0.5987745098039216
Final Score for L= 13 : 0.6051470588235295
Final Score for L= 14 : 0.5468137254901961
Final Score for L= 15 : 0.5835784313725491
Final Score for L= 16 : 0.5218137254901961
Final Score for L= 17 : 0.6392156862745098
Final Score for L= 18 : 0.535049019607843
Final Score for L= 19 : 0.6169117647058824
Final Score for L= 20 : 0.6002450980392157

Best L : 1
Error inverse : 0.7169117647058824

Train data final training score for L= 1 : 0.7134803921568629
Test data final testing score for L= 1 : 0.736842105263158



iii)

Naïve Bayes with Gaussian prior performed best with almost a 91% accuracy of predictions.

This method is better for multi-class classification in this problem.

2.) ISLR 3.7.4

Question 2 : (ISLR 3.7.4)

a.) It is difficult to say, we need more information for this. I would expect the polynomial regression to have a lower training RSS than the linear regression because it could make a tighter fit against data that matched with a wider irreducible error (Var ϵ). Train RSS for linear model are expected to be lower than the cubic regressor.

b.) We need more information on the test set. I would expect the polynomial regression to have a higher test RSS as the overfit from training would have more error than the linear regression.

c.) Polynomial regression has lower train RSS than the linear fit because of higher flexibility : no matter what the underlying true relationship is the more flexible model will closer follow points and reduce train RSS.

d.) There is more information required to tell which will be lower. We do not know how far away from linear the true nature of the model is. If its closer to linear than cubic, the LR test RSS could be lower. Or, if it is closer to cubic than linear, the cubic regression test RSS could be lower. Unless the model is clearly specified it is difficult to say which test RSS is lower.

3.) ISLR 4.7.3 & 4.) ISLR 4.7.7:

Question 3: (ISLR 4.7.3)

$$X \in N(\mu_k, \sigma_k)$$

Bayes Theorem:

$$p(x_k) = \frac{\pi_k f(x_k)}{\sum_{k'=1}^{k} \pi_{k'} f(x_{k'})} \quad \text{— (1)}$$

For normal distribution,

$$f(x_k) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{(x-\mu_k)^2}{2\sigma^2}} \quad \text{— (2)}$$

$$p(x_k) = \frac{\frac{\pi_k}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(x-\mu_k)^2}{2\sigma^2}}}{\sum_{k'=1}^{k} \frac{\pi_{k'}}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu_{k'})^2}{2\sigma^2}}} \quad \text{— From (1) \& (2)}$$

To maximize this we need large value of k,

Taking ln on both sides,

$$\ln(p(x_k)) = \ln\left[\frac{\pi_k}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{(x-\mu_k)^2}{2\sigma^2}}\right] - \ln\left[\sum_{k'=1}^{k} \frac{\pi_{k'}}{\sqrt{2\pi\sigma_{k'}^2}} \times e^{-\frac{(x-\mu_{k'})^2}{2\sigma^2}}\right]$$

This term will be a constant for all k's. Thus it does not influence k; and thus we can ignore it.

Thus we get,

$$\ln(p(x_k)) = \ln \pi_k + \ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x-\mu_k)^2}{2\sigma^2}$$

Again this term $\frac{1}{\sqrt{2\pi}}$ doesn't contribute to k.

So, we ignore it.

$$\delta(x_k) = \ln \pi_k - \ln \sigma_k - \frac{1}{2}\left(\frac{x^2}{\sigma_k^2} - \frac{2\mu_k x}{\sigma_k} + \frac{\mu_k^2}{\sigma_k^2}\right)$$

$$\delta(x_k) = ax^2 + bx + c$$

where

$$a = \frac{-1}{2\sigma_k^2}$$

$$b = \frac{-2\mu_k}{\sigma_k}$$

$$c = \ln \pi_k - \ln \sigma_k - \frac{1}{2}\frac{\mu_k^2}{\sigma_k^2}$$

It is of the form Quadratic equation.

Thus, Naive Bayes classifier here is quadratic for $X \sim N(\mu_k, \sigma_k)$.

Question 4: (ISLR 4.7.7)

Here $k = 2$ (Yes or No)

Let $k=1$ signifies Yes

$k=2$ signifies No

$\bar{X}$ for $k=1$ $= 10$, $\mu_1 = 10$

$\mu_2 = 0$

$\hat{\sigma}_2 = 36$, $x = 4$

$\pi_1 = \frac{80}{100}$, $\pi_2 = \frac{20}{100}$

$$P(x_k) = \frac{\pi_k \cdot f(x_k)}{\sum_{l=1}^{k} \pi_l \cdot f(x_l)}$$

So, $f_k(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_k)^2}{2\sigma^2}}$

Thus plugging in the values, we get

$$P(k=1 \mid x=4) = \frac{0.8 \times \frac{1}{\sqrt{2\pi \cdot 36}} e^{-\frac{(4-10)^2}{2\times 36}}}{0.8 \times \frac{1}{\sqrt{2\pi \times 36}} \times e^{-\frac{(4-10)^2}{2\times 36}} + 0.2 \times \frac{1}{\sqrt{2\pi \times 36}} \times e^{-\frac{(4-0)^2}{2\times 36}}}$$

$$= \frac{0.8 \times 0.6065}{0.8 \times 0.6065 + 0.2 \times 0.801} = 0.7519$$

$\Rightarrow$ $P(k=1 \mid x=4) = 0.7519$ is the probability.