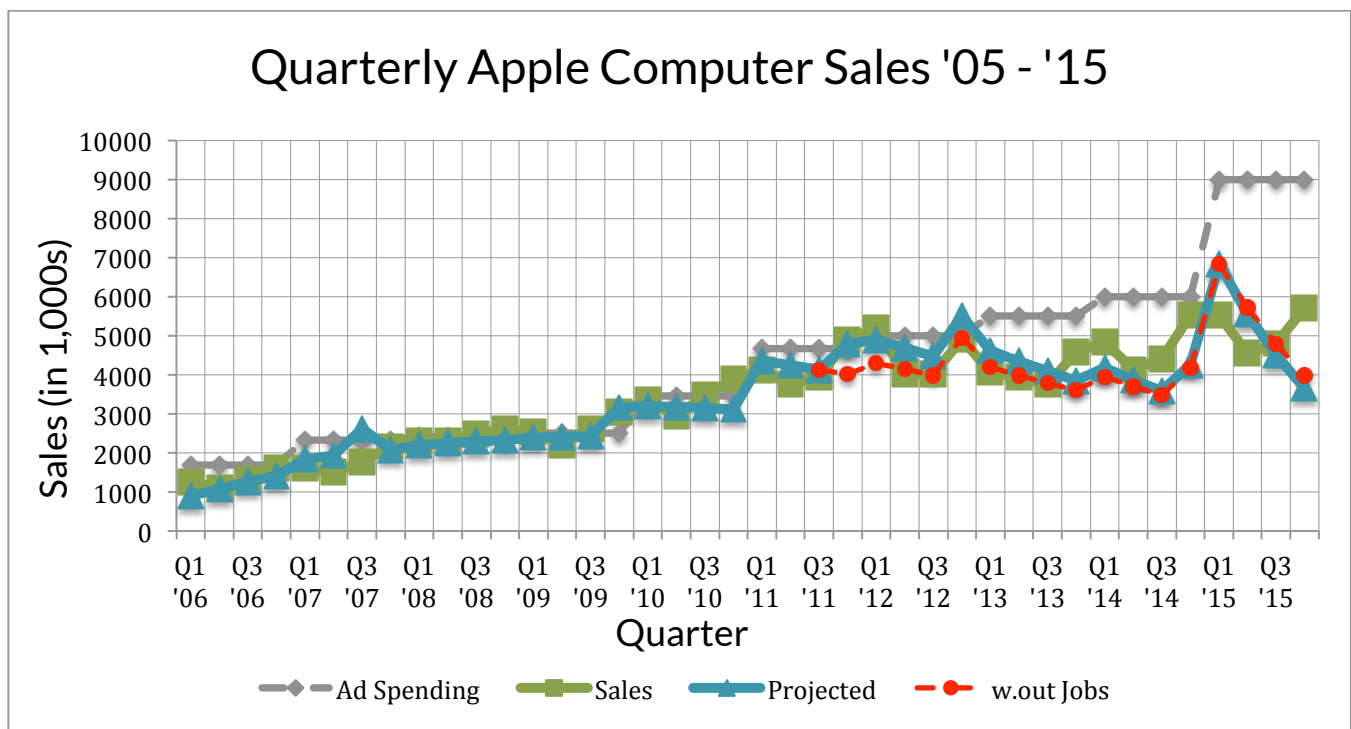


Projecting Apple Mac Sales

Traditional regression methods aren't sufficient for predicting Apple Mac Sales. This analysis presents a probability model that estimates quarterly Mac sales (including both iMac desktops and MacBook laptops) in 2005 – 2015 with a MAPE of .126. The best fitting model is a Weibull that incorporates the covariates of Apple advertisement expenditures, releases of new iMac models, and the impact of Steve Jobs' death.



lambda	0.00282	gamma	0.203
b_steve	0.181	delta	13.574
b_iMac	0.264	BIC	1083259
c	1.343	MAPE	0.1264
b_adv	1.480	LL	-541618

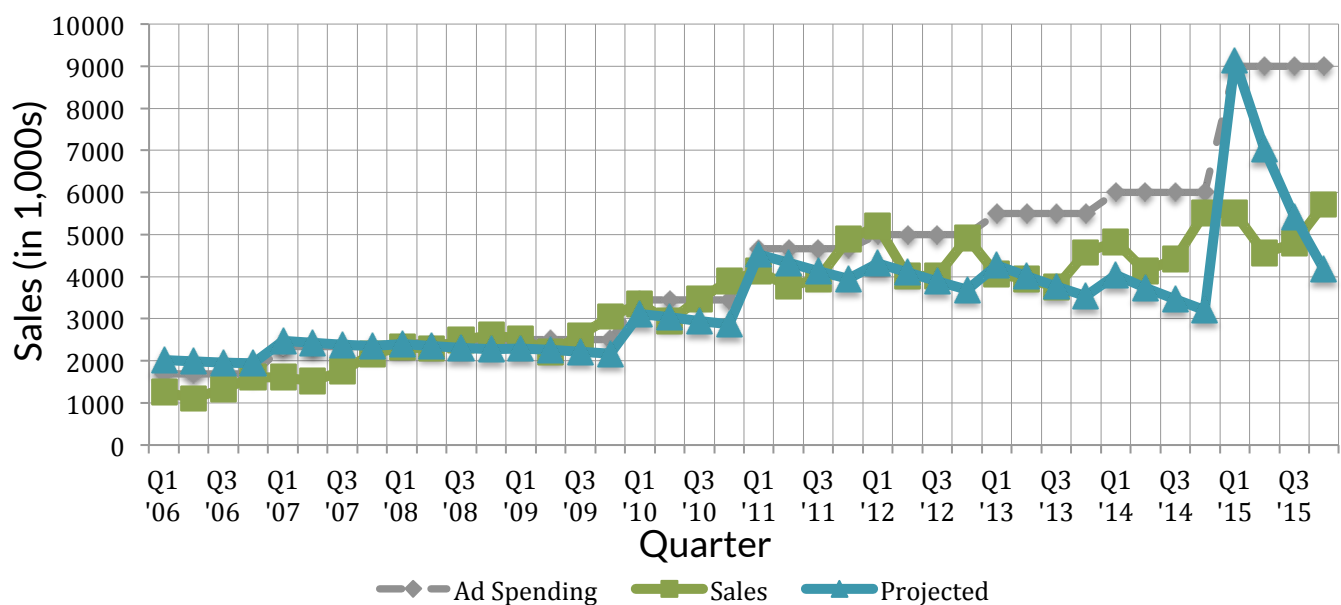
As predicted, advertisement expenditure is the strongest indicator for sales and new iMac releases are also somewhat indicative. Surprisingly, the death of Steve Jobs has a positive correlation with the probability model; however this positive

correlation seems only to be a temporary one, as described later in this analysis. If Apple disclosed their 2016 advertising budget and any planned iMac launches, we could quite accurately predict what their Mac sales figures will be using this model.

Benchmark

All models in this analysis were built in R with the `optim()` function (source code in Appendix 1) and also double-checked in Excel using the GRG Nonlinear Solver. The simplest probability model that can capture some of this behavior is an Exponential with only the advertisement expense covariate. With a MAPE of .22, this model can arguably outperform a traditional linear or exponential regression, although the model's performance in the most recent quarters is not impressive.

Exponential + Advertising Covariate



lambda	0.0068	BIC	1088719
b_adv	2.0278	MAPE	0.21846
LL	-544348		

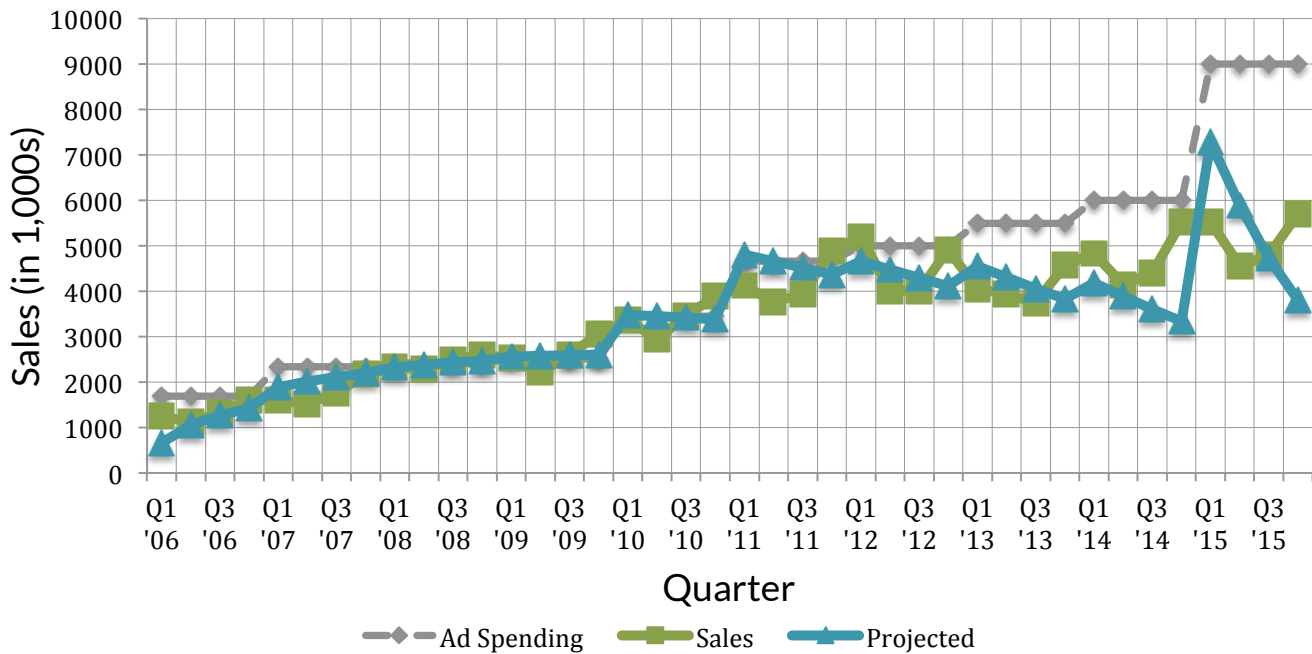
An attempt to incorporate heterogeneity into this model by building an Exponential Gamma does not succeed at improving the model's fit (as seen in appendix 2A.) This could possibly be explained by the fact that Apple builds expensive high quality products and offers convenient customer service, making it unlikely that any customer can or will purchase Apple products very frequently. Alternatively, it can be explained by remarkable customer brand loyalty: almost all customers are repeat customers or recent converts that will become repeat customers. More data about customer demographics could help us deduce a rigorous explanation.

Another potential point of interest is the inherent disparity between businesses and individuals purchasing Apple computers. It is possible that latent classes can reveal some of the heterogeneity that a Exponential Gamma couldn't capture. However, a 2-segment model (shown in Appendix 2B) also does not yield any substantial improvement over the simple Exponential shown above.

Duration Dependence

A more fruitful improvement on the Exponential turns out to be a Weibull, which incorporates the effect of duration dependence on purchase likelihood. With a duration dependence of $c = 1.4$, the Weibull improves MAPE by .08 and log likelihood by 2000.

Weibull + Cov



lambda	0.00262	BIC	1084658
c	1.376	MAPE	0.1374
b_adv	1.5395	LL	-542317

This effect shouldn't be surprising because of the inherently finite and relatively predictable lifespans of Apple products (i.e. a consumer is highly unlikely to purchase 2 Macs within one year, but as time goes on, the probability that the user will need to replace an old computer increases.)

Covariates

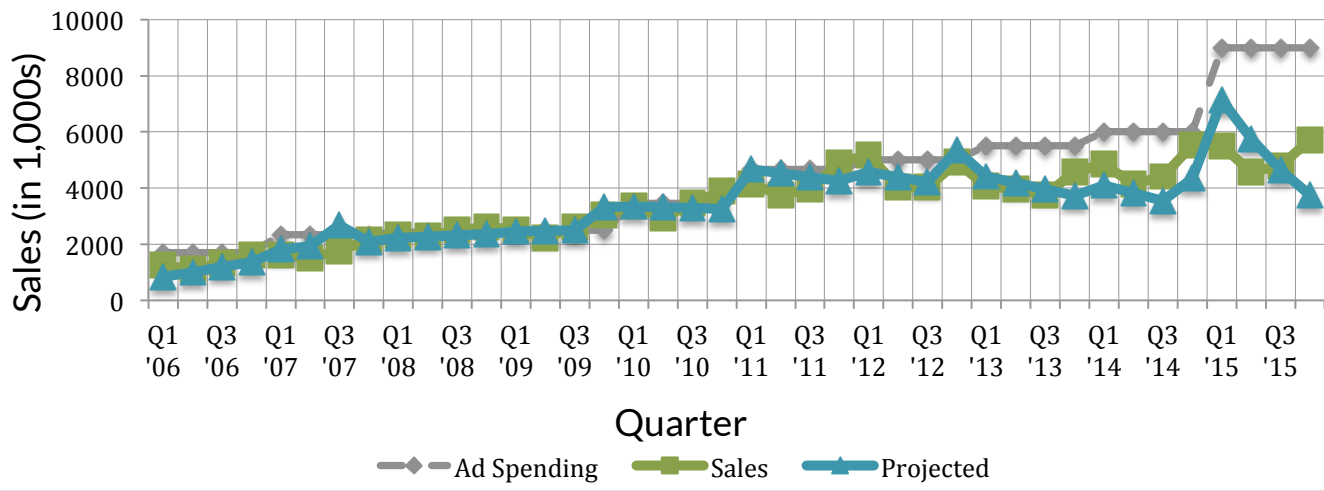
Advertisement expenditure is the baseline covariate because it has the strongest and most logical connection to Mac sales. However, many other covariates that could possibly impact the sales of Macs were also analyzed, including the impact of various Apple product launches as well as the death of Steve Jobs (October 5th, 2011.)

Product Launches

Among product launches that could possibly impact total Mac sales, this analysis tests the announcement impact for each of the three major computer types: iMac, MacBook Pro, and MacBook Air. It also examines the impact of new iPhone releases on Mac sales. Fascinatingly, of these 4 products – only the release of new iMac models has any noticeable impact on overall Mac sales. (It's worth noting that this does *not* show that new Pro/Air/iPhone model releases have *no* impact on Pro/Air sales, but rather that the impact of new Pro/Air/iPhone model releases is indecipherable when it comes to Mac computer sales as a whole.)

Even the co-variation with new iMac releases isn't particularly strong, with a coefficient of 0.29, a decrease in LL by 500, and a MAPE that is only .004 lower than the previous model. It shouldn't be surprising that new product releases are substantially less indicative of sales than advertisement (after all, almost nobody can tell the difference between a 2012 and a 2013 MacBook Pro.)

Weibull + 2 Cov (Ads, iMac)



lambda	0.00248	MAPE	0.1336
b_iMac	0.2894	BIC	1083548
b_adv	1.5682		
c	1.375	LL	-541762

Why is it that iMac was somewhat predictive while Pro, Air, and iPhone were not at all (as shown in Appendix 3)? Most likely, the impact of Pro, Air, and iPhone releases is so negligible because they get released very often, with respective averages of 1.9, 0.8, and 0.9 models per year being released. In comparison, there was only a 50% chance that a new iMac got released in any given year. Interestingly, the only other covariate that showed any correlation whatsoever was the Air (with $b_{air} = .009$), thus adding another tiny bit of evidence towards this explanation.

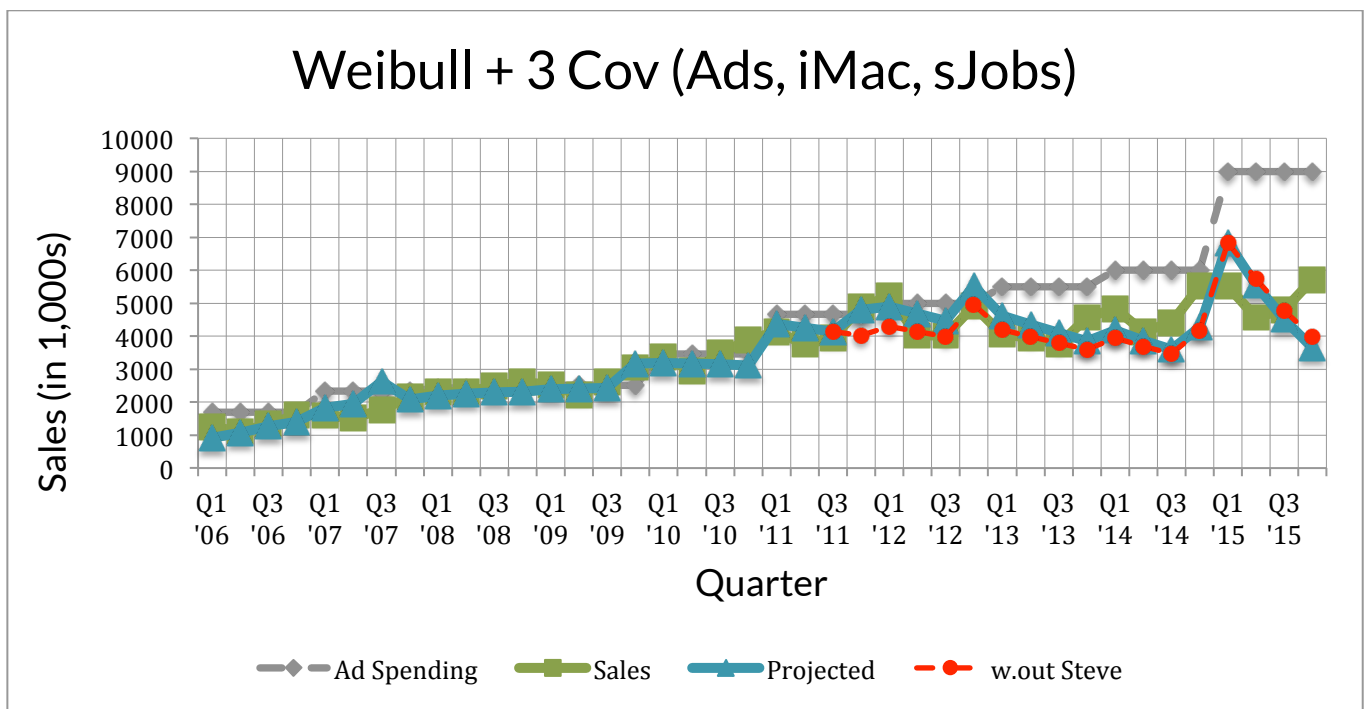
The Legacy of Steve Jobs

Steve Jobs, Founder and CEO of Apple, is responsible for ushering in 3 revolutionary products that permanently altered their industries and ushered in new ones. Under his leadership, Apple brought the personal computer to the masses, put 500+ songs in everyone's pocket, and built an unprecedentedly popular mobile Internet communications device. This analysis provides intriguing insights about how his death impacts the future of Apple.

Steve's death didn't only impact Apple (and Mac sales) the quarter he died, but rather it left a lasting impact on the trajectory of Apple. This analysis captures that effect using a the technique that Ryan Dismukes employed in his 9/11 paper. The covariate b_steve , is captured with the following formula:

$$\beta_{Steve} [1 - \gamma(1 - e^{-\delta|\tau-T|})]$$

Where γ is the normal sales level (before Steve's death), δ is the rate of change for sales going back to their new post-Steve level, and T is the quarter that he passed away (Q4 2011.) In blue is the model; in red is the model without incorporating this covariate:

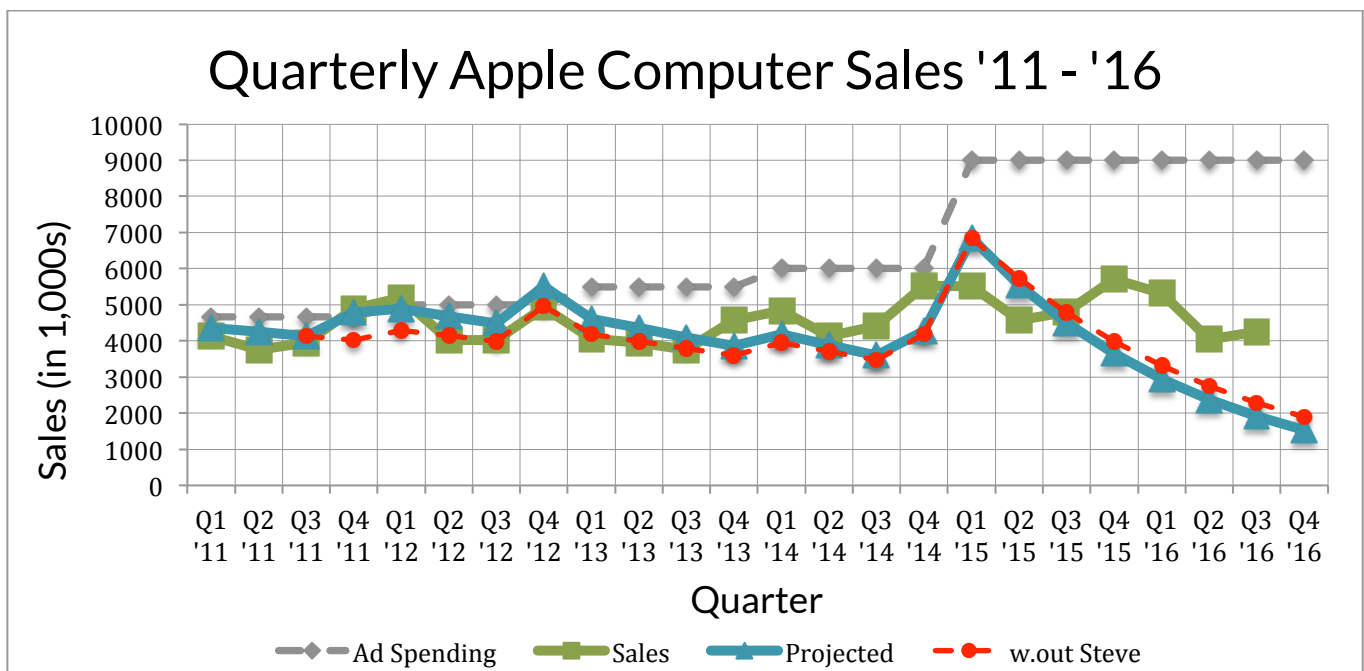


lambda	0.00282	gamma	0.203
b_steve	0.181	delta	13.574
b_iMac	0.264	BIC	1083259
c	1.343	MAPE	0.126
b_adv	1.480	LL	135709

My hypothesis that Steve Job's death would begin the gradual decline of Apple does not appear to be validated, especially with b_steve being positive. It is quite surprising that projected sales for all of 2011-2014 are higher than they would have

been without Steve's death. A plausible explanation for this is that his death and its publicity acted as a "free" marketing boost for Apple, i.e. the media was writing about Jobs and Apple all over, thus increasing customer's propensity to purchase.

Although the impact is noticeably positive over 2011-2014, it seems like in Q1 2015 it actually starts being slightly negative. The following graph shows the W + 3Cov projection vs. actual sales in 2016, under the assumption that Apple's advertisement budget is the same as last year (actual figures have not yet been disclosed.)



It's noteworthy that the model was not trained on 2016 datapoints (except to get the cumulative sales-to-date number, N.) This figure presents the prior argument with more clarity: it is evident that Q1 '15 is a sort of turning point where the impact of Jobs's death flips from positive to negative. While Steve's death provided a temporary sales boost via increased publicity – this evidence suggests that Jobs provided substantial long-term value that is likely to diminish without him. Conducting this analysis in a wide range of 2016 advertisement expenditures still yields the same result: that Steve's death is negatively impacting sales (as shown in Appendix 4.)

Conclusion

The final model (Weibull + 3 Cov) provides stunning accuracy when compared to traditional regression methods. With a MAPE of .126, it is able to predict the effectiveness of Apple's advertisements, the impact of a new iMac model release, and the future of a Steve-less Apple. With the use of internally available data, Apple could employ this as a reliable forecast for Mac-related revenue and even test the effectiveness of marketing campaigns or product launches.

It is intriguing but not surprising to observe how products that are constantly being updated and re-released (like iPhone or Macbook Pro) bring little impact to the sales of Apple computers, while products that are released less frequently (iMac, Macbook Air) tend to have more impact on overall sales figures. In some sense it validates Steve Jobs' marketing philosophy that product launches need to be really big, bold, and suspenseful to bring impact.

Most fascinating is the insight gained on Steve Jobs' legacy. Earlier in the company's lifespan, Jobs was ousted from CEO – and later asked to return – because of his monumental yet controversial influence on the company. It has been surprising to not notice decline in Apple over the last 5 years since his death (as both stock price and mac sales have stayed at relatively similar levels.) However, this Weibull + 3 Cov model gives a great under-the-hood explanation of this mystery: Over the first 3.5 years since his death, the lack of Steve's value added was more-or-less compensated by some sort of publicity boost. Soon the lack of Jobs' leadership became a more powerful force than the hype surrounding his death – resulting in a bleak future for Apple, where sales are projected to fall under any advertisement scenario. It will be interesting to see if Tim Cook proves this analysis wrong.

Appendix 1: Model Mechanics in R

#Total number of sales is known N = 150,452#

```
iData <- read.csv("Dropbox/iData.csv")
```

```
imatr <- iData[,1:4]
```

###E + Cov###

```
eCov <- function(x) { ## function to optimize
```

```
  x1 <- x[1] #lambda
```

```
  x2 <- x[2] # beta
```

```
  imatr$exb=exp(imatr$adv..bn.*x2)
```

```
  imatr$A=cumsum(imatr$exb)
```

```
  imatr$Pt<-1-exp(-x1*imatr$A)
```

```
  imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
```

```
  imatr$dP[1]=imatr$Pt[1]
```

```
  imatr$LL=imatr$sales*log(imatr$dP)
```

```
    -sum(imatr$LL, (150452-sum(imatr[,3]))*log(1-imatr$Pt[40]))
```

```
}
```

```
initial_guess=c(.005, 2)
```

```
optim(initial_guess, eCov)
```

```
plot (imatr[,1], imatr$dP * 150452, type="l", col="blue", xlab="quarter", ylab="sales", pch=16, cex=1.2,  
lwd=2)
```

```
lines (imatr[,1], imatr[,3], pch=16, cex=1.2, lwd=2)
```

###W + Cov###

```
wCov <- function(x) { ## function to optimize
```

```
  x1 <- x[1] #lambda
```

```
  x2 <- x[2] # beta
```

```
  x3 <- x[3] # c
```

```
  imatr$exb=exp(imatr$adv..bn.*x2)
```

```
  imatr$A=cumsum((((imatr$t^x3) - (c(0, imatr$t[1:39]))^x3)*imatr$exb)
```

```
  imatr$Pt<-1-exp(-x1*imatr$A)
```

```
  imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
```

```
  imatr$dP[1]=imatr$Pt[1]
```

```
  imatr$LL=imatr$sales*log(imatr$dP)
```

```
  -sum(imatr$LL, (150452-sum(imatr[,3]))*log(1-imatr$Pt[40]))
```

```
}
```

```
> initial_guess=c(.001, 1.5, 1.5)
```

```
> optim(initial_guess, wCov)
```

```
plot (imatr[,1], imatr$dP * 150452, type="l", col="blue", xlab="quarter", ylab="sales", pch=16, cex=1.2,  
lwd=2)
```

```
lines (imatr[,1], imatr[,3], pch=16, cex=1.2, lwd=2)
```

###EG###

```
EG <- function(x) { ## function to optimize
```

```
  x1 <- x[1] #r
```

```
  x2 <- x[2] #alpha
```

```
  x3 <- x[3] #b
```

```
  imatr$exb=exp(imatr$adv..bn.*x3)
```

```
  imatr$A=cumsum(imatr$exb)
```

```

imatr$Pt<-1-((x2/(x2+imatr$A)))^x1
imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
imatr$dP[1]=imatr$Pt[1]
imatr$LL=imatr$sales*log(imatr$dP)
-sum(imatr$LL, (150452-sum(imatr[,3]))*log(1-imatr$Pt[40]))
}

```

```

initial_guess=c(300, 50000, 1.5)
optim(initial_guess, EG)

```

```

x1 <- optim(initial_guess, EG)$par[1] #r
x2 <- optim(initial_guess, EG)$par[2] #alpha
x3 <- optim(initial_guess, EG)$par[3] #b
imatr$exb=exp(imatr$adv.bn.*x3)
imatr$A=cumsum(imatr$exb)
imatr$Pt<-1-((x2/(x2+imatr$A)))^x1
imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
imatr$dP[1]=imatr$Pt[1]
plot (imatr[,1], imatr$dP * 150452, type="l", col="blue", xlab="quarter", ylab="sales", pch=16, cex=1.2,
lwd=2)
lines (imatr[,1], imatr[,3], pch=16, cex=1.2, lwd=2)

```

###W + iMac + Adv###

```

w2Cov <- function(x) { ## function to optimize
  x1 <- x[1] #lambda
  x2 <- x[2] # beta
  x3 <- x[3] # c
  x4 <- x[4] #b_iMac
  imatr$exb=exp(imatr$adv.bn.*x2 + imatr[,10]*x4)
  imatr$A=cumsum(((imatr$t^x3) - (c(0, imatr$t[1:39]))^x3)*imatr$exb)
  imatr$Pt<-1-exp(-x1*imatr$A)
  imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
  imatr$dP[1]=imatr$Pt[1]
  imatr$LL=imatr$sales*log(imatr$dP)
  -sum(imatr$LL, (150452-sum(imatr[,3]))*log(1-imatr$Pt[40]))
}

```

```

initial_guess=c(0.5, 1.5, 1.5, 0.5)
optim(initial_guess, w2Cov)

```

```

x1 <- optim(initial_guess, w2Cov)$par[1] #lamda
x2 <- optim(initial_guess, w2Cov)$par[2] #b_ads
x3 <- optim(initial_guess, w2Cov)$par[3] #c
x4 <- optim(initial_guess, w2Cov)$par[4] #b_iMac
imatr$exb=exp(imatr$adv.bn.*x2 + imatr[,10]*x4)
imatr$A=cumsum(((imatr$t^x3) - (c(0, imatr$t[1:39]))^x3)*imatr$exb)
imatr$Pt<-1-exp(-x1*imatr$A)
imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
imatr$dP[1]=imatr$Pt[1]
imatr$LL=imatr$sales*log(imatr$dP)
plot (imatr[,1], imatr$dP * 150452, type="l", col="blue", xlab="quarter", ylab="sales", pch=16, cex=1.2,
lwd=2)

```

```
lines (imatr[,1], imatr[,3], pch=16, cex=1.2, lwd=2)
```

###W + iMac + Adv + Steve###

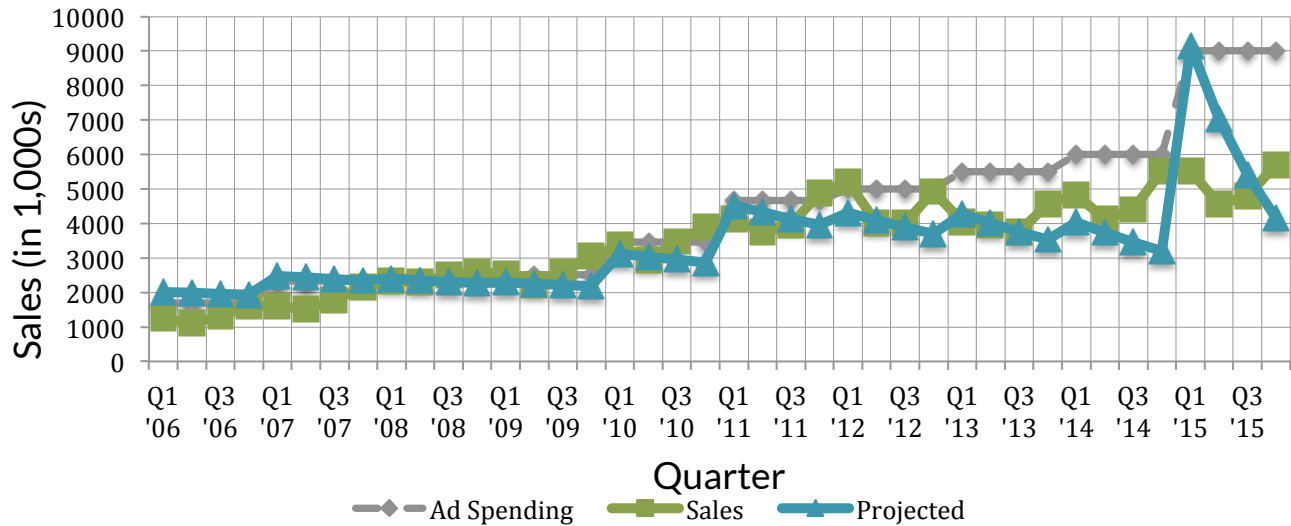
```
w3Cov <- function(x) { ## function to optimize
x1 <- x[1] #lambda
x2 <- x[2] # beta
x3 <- x[3] # c
x4 <- x[4] #b_iMac
x5 <- x[5] #b_steve
x6 <- x[6] #gamma
x7 <- x[7] #delta
imatr$sJobs <- c(array(0, c(1,23)), 1-x6*(1-exp(-x7*abs(imatr$t-24)))[24:40])
imatr$exb=exp(imatr$adv.bn.*x2 + imatr$iMac*x4 + imatr$sJobs*x5)
imatr$A=cumsum(((imatr$t^x3) - (c(0, imatr$t[1:39]))^x3)*imatr$exb)
imatr$Pt<-1-exp(-x1*imatr$A)
imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
imatr$dP[1]=imatr$Pt[1]
imatr$LL=imatr$sales*log(imatr$dP)
-sum(imatr$LL, (150452-sum(imatr[,3]))*log(1-imatr$Pt[40]))
}
```

```
initial_guess=c(0.005, 1.5, 1.5, 0.5, 0.5, 0.2, 14)
optim(initial_guess, w3Cov)
```

```
x1 <- optim(initial_guess, w3Cov)$par[1] #lamda
x2 <- optim(initial_guess, w3Cov)$par[2] #b_ads
x3 <- optim(initial_guess, w3Cov)$par[3] #c
x4 <- optim(initial_guess, w3Cov)$par[4] #b_iMac
x5 <- optim(initial_guess, w3Cov)$par[5] #b_steve
x6 <- optim(initial_guess, w3Cov)$par[6] #gamma
x7 <- optim(initial_guess, w3Cov)$par[7] #delta
imatr$sJobs <- c(array(0, c(1,23)), 1-x6*(1-exp(-x7*abs(imatr$t-24)))[24:40])
imatr$exb=exp(imatr$adv.bn.*x2 + imatr$iMac*x4 + imatr$sJobs*x5)
imatr$A=cumsum(((imatr$t^x3) - (c(0, imatr$t[1:39]))^x3)*imatr$exb)
imatr$Pt<-1-exp(-x1*imatr$A)
imatr$dP[2:nrow(imatr)]=diff(imatr$Pt)
imatr$dP[1]=imatr$Pt[1]
imatr$LL=imatr$sales*log(imatr$dP)
plot (imatr$t, imatr$dP * 150452, type="l", col="blue", xlab="quarter", ylab="sales", pch=16, cex=1.2,
lwd=2)
lines (imatr$t, imatr$sales, pch=16, cex=1.2, lwd=2)
```

Appendix 2A: Exponential Gamma

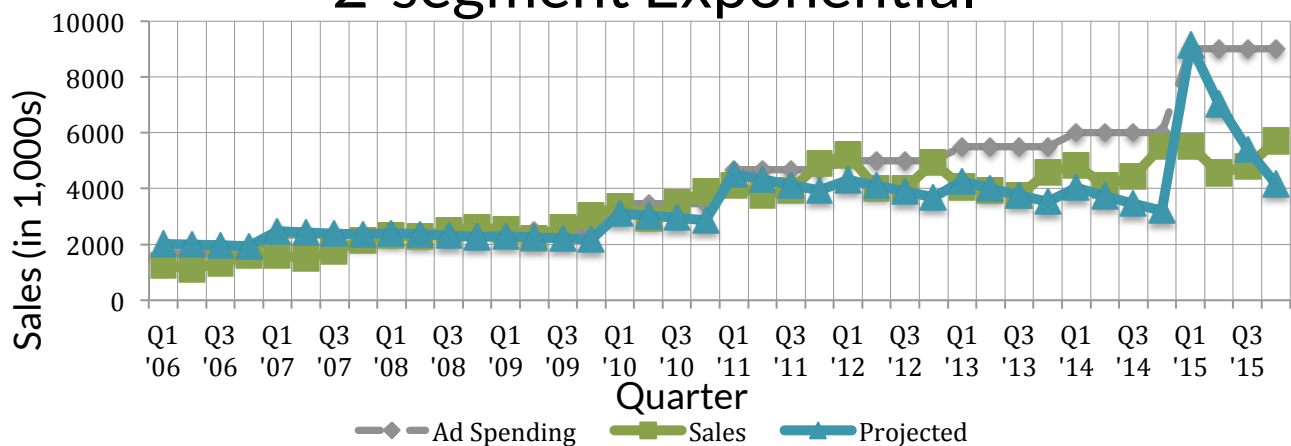
Exponential Gamma



r	36004	BIC	1088718
alpha	5297748	MAPE	0.218
b_rev	2.0278	LL	-544348

Appendix 2B: 2-segment Exponential

2-segment Exponential



lambda	0.00679,	0.00679			
b_adv	2.0278	BIC	1088720	Pi1	0.395
LL	-544348	MAPE	0.2185	Pi2	0.605

Appendix 3: Macbook Pro, Macbook Air, and iPhone Covariates

Weibull + Macbook Pro

c	b_MBP	b_iMac	b_rev	LL	MAPE	BIC
1.466	0	0.1932	0.7133	-540861	0.1094	1081744

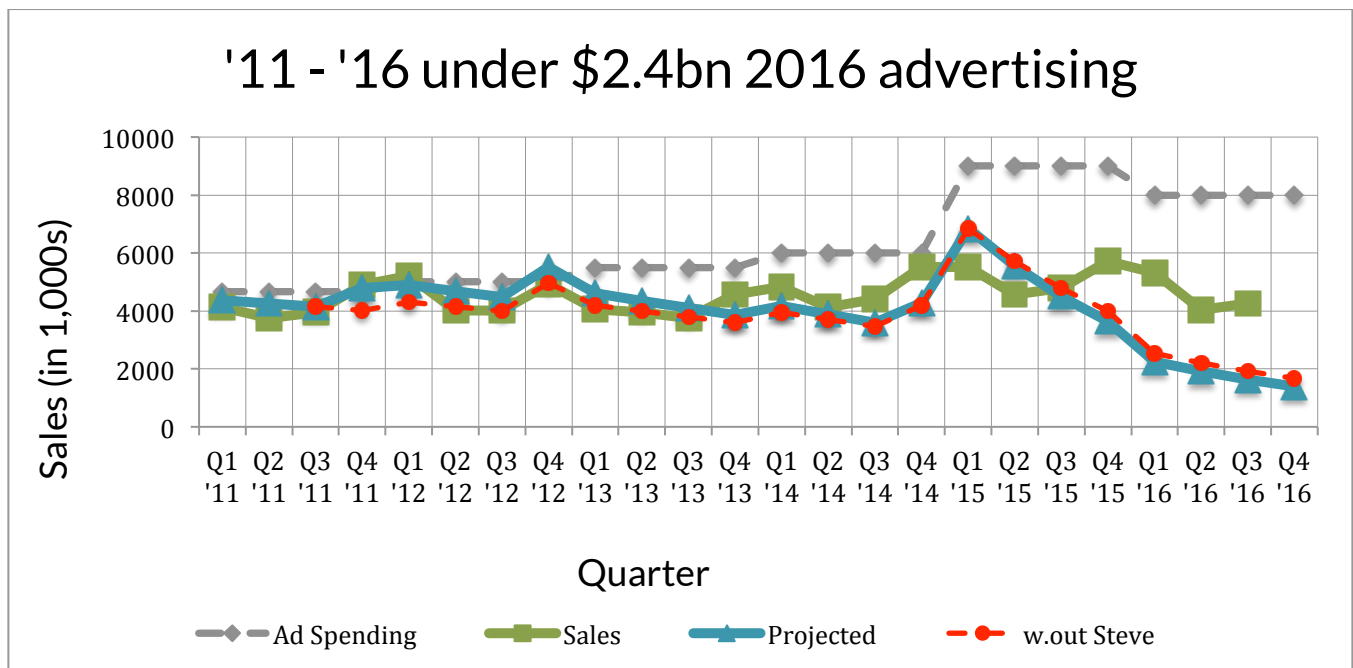
Weibull + Macbook Air

c	lambda	b_steve	b_iMac	b_air	b_rev	LL	MAPE	BIC
1.466	0.00308	1.241	0.195	0.00889	0.708	-540859	0.1096	1081743

Weibull + iPhone

lambda	b_steve	b_iPh	b_iMac	c	b_rev	LL	BIC	MAPE
0.00308	1.2314	0	0.193	1.466	0.714	-540860	1081744	0.109

Appendix 4: Impact of S.J. under various 2016 ad expenses



'11 - '16 under \$1.6bn 2016 advertising

