AoL SC

1.)

(a) Linear Interpolasi

even Years

If $X_m = 1982$ $y_m = ?$

$\Delta y_A = y_m - y_1$ $\Delta x_A = x_m - x_1$

$\Delta y_B = y_2 - y_1$ $\Delta x_B = x_2 - x_1$

$$\frac{\Delta y_A}{\Delta y_B} = \frac{\Delta x_A}{\Delta x_B} \Rightarrow \frac{y_m - 14.1999}{14.2411 - 14.1999} = \frac{1982 - 1981}{1983 - 1981}$$

$$= \frac{y_m - 14.1999}{0.0412} = \frac{1}{2} \Rightarrow y_m = 14.1999 + \frac{0.0412}{2}$$

$= 14.2205 \rightarrow$ Temp In 1982

$X_m = 1984$ $y_m = ?$

$$\frac{y_m - 14.2411}{14.0342 - 14.2411} = \frac{1984 - 1983}{1985 - 1983} \Rightarrow y_m = 14.2411 - \frac{0.2069}{2}$$

$= 14.1376 \rightarrow$ Temp In 1984

$X_m = 1986$ $y_m = ?$

$$\frac{y_m - 14.0342}{14.2696 - 14.0342} = \frac{1}{2} \Rightarrow y_m = 14.0342 + \frac{0.2354}{2}$$

$= 14.1519 \rightarrow$ Temp In 1986

$X_m = 1988 \quad y_m = ?$

$$\frac{y_m - 14.2696}{14.197 - 14.2696} = \frac{1}{2} \Rightarrow y_m = 14.2696 - \frac{0.0726}{2}$$

$$= 14.2333 \rightarrow \text{Temp In } 1988$$

$X_m = 1990 \quad y_m = ?$

$$\frac{y_m - 14.197}{14.3055 - 14.197} = \frac{1}{2} \Rightarrow y_m = 14.197 + \frac{0.1085}{2}$$

$$= 14.2512 \rightarrow \text{Temp In } 1990$$

$X_m = 1992 \quad y_m = ?$

$$\frac{y_m - 14.3055}{14.1853 - 14.3055} = \frac{1}{2} \Rightarrow y_m = 14.3055 - \frac{0.1202}{2}$$

$$= 14.2454 \rightarrow \text{Temp In } 1992$$

$X_m = 1994 \quad y_m = ?$

$$\frac{y_m - 14.1853}{14.3577 - 14.1853} = \frac{1}{2} \Rightarrow y_m = 14.1853 + \frac{0.1724}{2}$$

$$= 14.2715 \rightarrow \text{Temp In } 1994$$

$X_m = 1996 \quad y_m = ?$

$$\frac{y_m - 14.3577}{14.4187 - 14.3577} = \frac{1}{2} \Rightarrow y_m = 14.3577 + \frac{0.061}{2}$$

$$= 14.3882 \rightarrow \text{Temp In } 1996$$

$X_m = 1998 \quad y_m = ?$

$$\frac{y_m - 14.4187}{14.3438 - 14.4187} = \frac{1}{2} \Rightarrow y_m = 14.4187 - \frac{0.0749}{2}$$

$$= 14.3812 \rightarrow \text{Temp In } 1998$$

Interpolasi linear menghitung value diantara dua (2) data dengan adanya garis lurus diantaranya.

Sedangkan, Interpolasi kuadratik menghitung value menggunakan fungsi kuadrat yang mana memiliki tiga (3) data.

terakhir, Interpolasi kubik memiliki perbedaan di perhitungan value yang menggunakan fungsi polinomial kubik dan memiliki 4 data.

Dalam proses perhitungan tersebut saya menggunakan Interpolasi linear.

(b) least square Regression

x	y	xy	x^2
1981	14.1999	28130,0019	3924361
1983	14.2411	28240,1013	3932289
1985	14.0342	27857,887	3940225
1987	14.2696	28.353,6952	3948169
1989	14.197	28237,833	3956121
1991	14.3055	28482,2505	3964081
1993	14.1853	28271,3029	3972049
1995	14.3577	28643,6115	3980025
1997	14.4187	28794,1439	3988009
1999	14.3438	28673,2562	3996001
19900	142.5528	283684,0834	39601330

Σ

$$\boxed{y = mx + b} \quad n = 10$$

$$m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$= \frac{10(283684,0834) - (2836800,72)}{10(39601330) - (396.010.000)}$$

$$= \frac{40,114}{3300}$$

$$\boxed{m = 0,01215}\ldots$$

$$b = \frac{\Sigma y - m \Sigma x}{n} = \frac{142,5528 - (0,0121\ldots)(19900)}{10}$$

$$\boxed{b = -9,93467\ldots}$$

$$\boxed{y = mx + b} \Rightarrow \boxed{y = 0,01215x - 9,93}$$

Even Years :

$x = 1982 \rightarrow y = (0,01215)(1982) - 9,93 = 14.1513$
$x = 1984 \rightarrow y = (0,01215)(1984) - 9,93 = 14.1756$
$x = 1986 \rightarrow y = (0,01215)(1986) - 9,93 = 14.1999$
$x = 1988 \rightarrow y = (0,01215)(1988) - 9,93 = 14.2242$
$x = 1990 \rightarrow y = (0,01215)(1990) - 9,93 = 14,2485$
$x = 1992 \rightarrow y = (0,01215)(1992) - 9,93 = 14,2728$
$x = 1994 \rightarrow y = (0,01215)(1994) - 9,93 = 14,2971$
$x = 1996 \rightarrow y = (0,01215)(1996) - 9,93 = 14.3214$
$x = 1998 \rightarrow y = (0,01215)(1998) - 9,93 = 14.3437$

(c)

Linear InterPolasi

Year	Temp
1982	14,2205
1984	14,1376
1986	14,1519
1988	14,2333
1990	14,2512
1992	14,2454
1994	14,2715
1996	14,3882
1998	14,3812

Regression

Year	temp
1982	14,1513
1984	14,1756
1986	14,1999
1988	14,2242
1990	14,2485
1992	14,2728
1994	14,2971
1996	14,3214
1998	14,3437

Dari Perhitungan InterPolasi dan Regresi yang saya lakukan, hasil dari InterPolasi memberikan hasil yang lebih akurat dan masuk akal dibanding dengan Regresi. Hal ini disebabkan karena Regresi hanya memberikan Prediksi.

AoL SC

2.) (a) 1

$F(x) = \sin x \quad F(0) = 0$

$F'(x) = \cos x \quad F'(0) = 1$

$F''(x) = -\sin x \quad F''(0) = 0$

$F'''(x) = -\cos x \quad F'''(0) = -1$

$F^{(4)}(x) = \sin x \quad F^{(4)}(0) = 0$

$$\sin x = 0 + \frac{1}{1!}(x-0) + \frac{0}{2!}(x-0)^2 + \left(-\frac{1}{3!}\right)(x-0)^3$$

$$+ \frac{0}{4!}(x-0)^4$$

$$\sin x = (x-0) - \frac{1}{3!}(x-0)^3$$

$$= x - \frac{1}{3!}(x)^3$$

2

$F(x) = \cos x \quad F(0) = 1$

$F'(x) = -\sin x \quad F'(0) = 0$

$F''(x) = -\cos x \quad F''(0) = -1$

$F'''(x) = \sin(x) \quad F'''(0) = 0$

$F^{(4)}(x) = \cos x \quad F^{(4)}(0) = 1$

$$\cos x = 1 + \frac{0}{1!}(x-0) + \left(-\frac{1}{2!}\right)(x-0)^2 + \frac{0}{3!}(x-0)^3 + \frac{1}{4!}(x-0)^4$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

3

$f(x) = \sin(x)\cos(x)$ $\quad F(0) = 0$

$F'(x) = \cos(2x)$ $\quad F'(0) = 1$

$F''(x) = -2\sin(2x)$ $\quad F''(0) = 0$

$F'''(x) = -4\cos(2x)$ $\quad F'''(0) = -4$

$F^4(x) = 8\sin(2x)$ $\quad F^4(0) = 0$

$$\sin(x)\cos(x) = 0 + \frac{1}{1!}(x-0) + \frac{0}{2!}(x-0)^2 + \left(-\frac{4}{3!}\right)(x-0)^3 + \frac{0}{4!}(x-0)^4 + \ldots$$

$$= (x-0) - \frac{4}{3!}(x-0)^3$$

$$= x - \frac{2}{3}(x)^3$$

(b) When $x = \pi/2$

$$\sin x = x - \frac{1}{3!}x^3$$

$$= \frac{\pi}{2} - \frac{1}{3!}\left(\frac{\pi}{2}\right)^3 = 0.9248$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$= 1 - \frac{(\pi/2)^2}{2!} + \frac{(\pi/2)^4}{4!} = 0.0199$$

$$\sin(x)\cos(x) = x - \frac{2}{3}(x)^3$$

$$= \frac{\pi}{2} - \frac{2}{3}\left(\frac{\pi}{2}\right)^3 = -1.0130$$

less error terjadi
Pada fungsi cos x

(c.) $x = \frac{\pi}{4}$

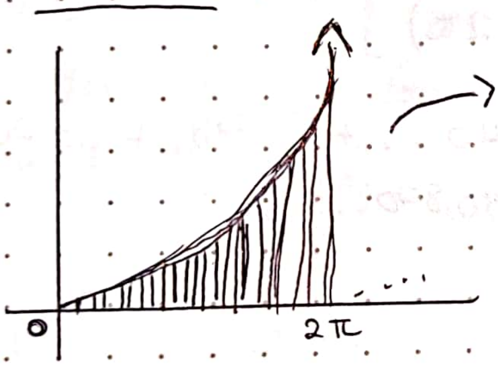
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} = 1 - \frac{(\pi/4)^2}{2!} + \frac{(\pi/4)^4}{4!} = 0.9452$$

3.) (a) $F(x) = x^3 - 0.3x^2 - 8.56x + 8.448$

$$\int_0^{2\pi} F(x)\,dx = \frac{x^4}{4} - \frac{0.3x^3}{3} - \frac{8.56x^2}{2} + 8.448x \Big]_0^{2\pi}$$

$$= \frac{1}{4}x^4 - 0.1x^3 - 4,28x^2 + 8.448x \Big]_0^{2\pi}$$

Riemann



Jumlah Kotak = 20 = n

$$\Delta x = \frac{b-a}{n} = \frac{2\pi - 0}{20} = \frac{\pi}{10}$$

left Riemann : $0, \frac{\pi}{10}, \frac{\pi}{5}, \frac{3\pi}{10}, \frac{2\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{5}, \frac{7\pi}{10},$

$\frac{4\pi}{5}, \frac{9\pi}{5}, \pi, \frac{11}{10}\pi, \frac{6\pi}{5}, \frac{13\pi}{10}, \frac{7\pi}{5},$

$\frac{3\pi}{2}, \frac{8\pi}{5}, \frac{17}{10}\pi, \frac{9\pi}{5}, \frac{19}{10}\pi, 2\pi$

$$A_L = \Delta x \left[F(0) + F(\tfrac{\pi}{10}) + F(\tfrac{\pi}{5}) + \dots + F(\tfrac{9\pi}{5}) + F(\tfrac{19\pi}{10}) \right]$$

$$= \frac{\pi}{10} \left[8.448 + 5.70 + 3.199\ldots + \ldots + 131.277\ldots + 159.336\ldots \right]$$

$$= 221.23.23 = 221$$

Trapezoid

$$T_n = \frac{\Delta x}{2}\left[f(0) + 2f\left(\frac{\pi}{10}\right) + 2f\left(\frac{\pi}{5}\right) + \ldots + 2f\left(\frac{9\pi}{5}\right) + 2f\left(\frac{19\pi}{10}\right) + f(2\pi)\right]$$

$$= \frac{\frac{\pi}{10}}{2}\left[8.448 + 11.520\ldots + 6.398\ldots + \ldots + 262.555\ldots + 318.672\ldots + 190.870\ldots\right]$$

$$= 249.8871 = \underline{\underline{250}}$$

Simpson's

$$S_n = \frac{\Delta x}{3}\left[f(0) + 4\left(f\left(\frac{\pi}{10}\right)\right) + 2f\left(\frac{\pi}{5}\right) + \ldots + 2f\left(\frac{9\pi}{5}\right) + 4f\left(\frac{19}{10}\pi\right) + f(2\pi)\right]$$

$$= \frac{\pi/10}{3}\left[8.448 + 23{,}040\ldots + 6{,}398\ldots + \ldots + 262{,}555\ldots + 637{,}345\ldots + 190{,}870\ldots\right]$$

$$= 248{,}9441 = \underline{\underline{248}}$$

(b) Perbedaan dari ketiga metode tersebut terletak pada rumusnya yang mana

rumus riemann : $\Delta x\left[f(x_0) + f(x_1) + f(x_2) \ldots f(x_{n-2}) + f(x_{n+1})\right]$

rumus trapezoid : $\frac{\Delta x}{2}\left[f(x_0) + f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$

rumus Simpson's : $\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

(b) Jika dibandingkan dengan menghitung langsung Integralnya:

$$\frac{1}{4}x^4 - 0.1x^3 - 4{,}28x^2 + 8.448x \Big]_0^{2\pi}$$

$$= \left[\frac{1}{4}(2\pi)^4 - 0.1(2\pi)^3 - 4{,}28(2\pi)^2 + 8.448(2\pi)\right] -$$

$$[0]$$

$$= 388{,}84 - 24{,}76 - 168{,}79 + 53{,}05 - 0$$

$$= 248.34 = \underline{\underline{248}}$$

Menghitung Integralnya secara langsung lebih mudah dibanding metode-metode tersebut. Hanya metode Simpsons yang mendekati/sama dgn hasil aslinya.

(c) $f(x) = x^3 - 0.3x^2 - 8.56x + 8.448$

$f'(x) = 3x^2 - 0.6x - 8{,}56$

$f''(x) = 6x - 0{,}6$

x	-1.1	-0.3	0.8	1.9
f(x)	15,180	10,962	1,920	-2,040

$$f(x) = 15{,}180 \Rightarrow \frac{(x-x_0)\cdot(x-x_2)\cdot(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x)$$

$$= \frac{(x+0{,}3)(x-0{,}8)(x-1{,}9)}{(-1{,}1+0{,}3)(-1{,}1-0{,}8)(-1{,}1-1{,}9)} \times 15{,}180$$

$$= 1{,}2171x^3$$

$$f(x) = 10{,}962 \Rightarrow \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x)$$

$$= \frac{(x+1{,}1)(x-0{,}8)(x-1{,}9)}{(-0{,}3+1{,}1)(-0{,}3-0{,}8)(-0{,}3-1{,}9)} \times 10{,}962$$

$$= 0{,}8211\,x^2$$

$$f(x) = 1{,}920 \Rightarrow \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x)$$

$$= \frac{(x+1.1)(x+0.3)(x-1.9)}{(0.8+1.1)(0.8+0.3)(0.8+1.9)} \times 1{,}920$$

$$= 8{,}4059\,x$$

$$f(x) = -2{,}040 \Rightarrow \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x)$$

$$= \frac{(x+1.1)(x+0.3)(x-0{,}5)}{(1{,}9+1{,}1)(1{,}9+0.3)(1{,}9-0{,}8)} \times (-2{,}040)$$

$$= 8{,}547$$

$$f(x) = 1{,}2171\,x^3 - 0{,}8211\,x^2 - 8{,}4059\,x + 8{,}547$$

$$f'(x) = (3)(1{,}2171)\,x^2 - (2)(0{,}8211)\,x - 8{,}4059$$

$$f''(x) = (2)(3)(1{,}2171)\,x - (2)(0{,}8211)$$

$x = 0$

$f'(0) = -8{,}4059$

$f''(0) = -2 \cdot 0{,}8211 = -1{,}6422$

(d) $f'(x) = 3x^2 - 0{,}6x - 8{,}56$

$f'(0) = -8{,}56$

$f'(x) = (3)(1{,}2171)x^2 - (2)(0{,}8211)x - 8{,}4059$

$f'(0) = -8{,}4059$

$$\text{Error} \Rightarrow \left| \frac{-8{,}56 + 8{,}4059}{-8{,}56} \right| \times 100 = \left| -\frac{0{,}1541}{-8{,}56} \right| \times 100$$

$$= |-1{,}8| = 1{,}8\,\%$$

$f''(x) = 6x - 0{,}6$ $\quad$ $f''(x) = (2)(3)(1{,}2171)x - (2)(0{,}8211)$

$f''(0) = -0{,}6$

$f''(0) = -1{,}6422$

$$\text{Error} \Rightarrow \left| \frac{-0{,}6 + 1{,}6422}{-0{,}6} \right| \times 100 = \left| \frac{1{,}0422}{-0{,}6} \right| \times 100$$

$$= 173\,\%$$

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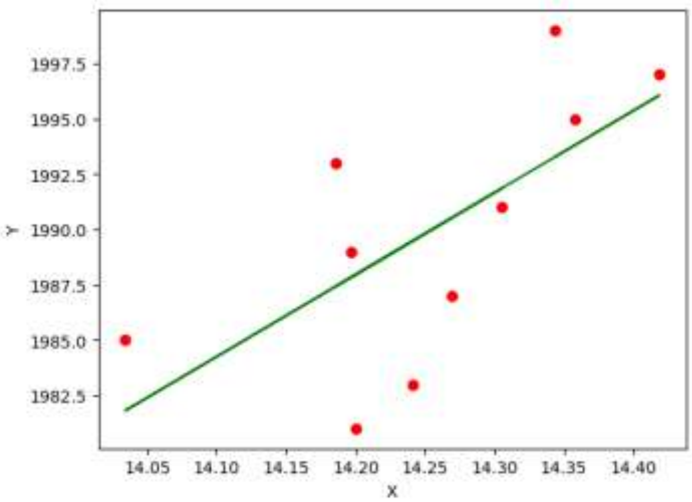
JAWABAN AOL SCIENTIFIC COMPUTING

1.

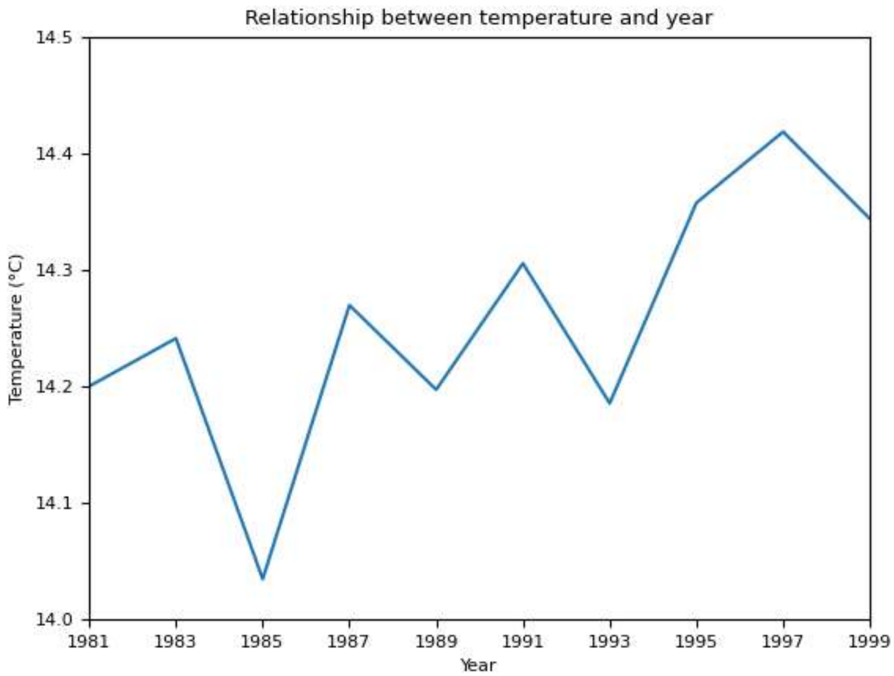
b) Least square regression python code :

```
import numpy as np
import matplotlib.pyplot as plt

x = [14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055,
14.1853, 14.3577, 14.4187, 14.3438]
y = [1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997,
1999]

x = np.array(x)
y = np.array(y)

A = np.vstack([x, np.ones(len(x))]).T
B = np.vstack([y, np.ones(len(y))]).T

y = y[:, np.newaxis]

alpha = np.dot(np.dot(np.linalg.inv(np.dot(A.T, A)), A.T), y)

plt.plot(x,y, "ro")
plt.plot(x, alpha[0]*x + alpha[1], "g")
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



d) Plot python code :

```
import matplotlib.pyplot as plt

year = [1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 
1999]

temp = [14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 
14.1853, 14.3577, 14.4187, 14.3438]

years = ['1981', '1983', '1985', '1987', '1989', '1991', 
'1993', '1995', '1997', '1999']

plt.plot(year, temp)

plt.ylim(14.0 , 14.5)
plt.xlim(1981 , 1999)

plt.title("Relationship between temperature and year")
plt.ylabel('Temperature (°C)')
plt.xlabel('Year')

plt.xticks(year, years)
plt.rcParams.update({'font.size': 8, 'font.serif':'Arial'})
```



2.

a) Taylor python code :

```
import math
from math import sin, cos

c = 0
v = 4

def sin_series(x, n):
    sin_series = 0

    for i in range(n):
        sin_term = (-1)**i * x**(2*i+1) /
math.factorial(2*i+1)
        sin_series += sin_term

    return sin_series

def cos_series(x, n):
    cos_series = 0

    for i in range(n):
        cos_term = (-1)**i * x**(2*i) /
math.factorial(2*i)
        cos_series += cos_term

    return cos_series

def sin_cos_product_series(x, n):
    sin_cos_series = 0

    for i in range(n):
        term = (-1)**i * x**(2*i+1) /
math.factorial(2*i+1)
        sin_cos_series += term

    return sin_cos_series
```

3.

a) Riemann, Trapezoid & Simpson python code :

```
import numpy as np

def f(x):
    return x**3 - 0.3*x**2 - 8.56*x + 8.448

phi = 3.14

b = 2 * phi
a = 0

n = 20

width = (b - a) / (n - 1)

x = np.linspace(a, b, n)

y = f(x)

# Left Riemann
left = width * sum(y[:n-1])
print("Left Riemann: ", left)

# Right Riemann
right = width * sum(y[1:])
print("Right Riemann: ", right)

# Mid Riemann
left_x = x[:n - 1]
right_x = x[1:]

xMid = (left_x + right_x) / 2
yMid = f(xMid)

mid = width * sum(yMid)
print("Mid Riemann: ", mid)

# Trapezoid
trapezoid = 1/2 * width * sum(y[:n - 1] + y[1:])
print("Trapezoid: ", trapezoid)

# Simpson
```

```python
dx = (b-a)/n
x = np.linspace(a,b,n+1)
y = f(x)
simpson = dx/3 * np.sum(y[0:-1:2] + 4*y[1::2] + y[2::2])
print("Simpson: ", simpson)
```

```
Output:
Left Riemann:  219.287541423158
Right Riemann:  279.47137010526325
Mid Riemann:  247.81520427789488
Trapezoid:  249.37945576421063
Simpson:  248.33662144
```