R Notebook

```r
#packages
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 4.2.3
```

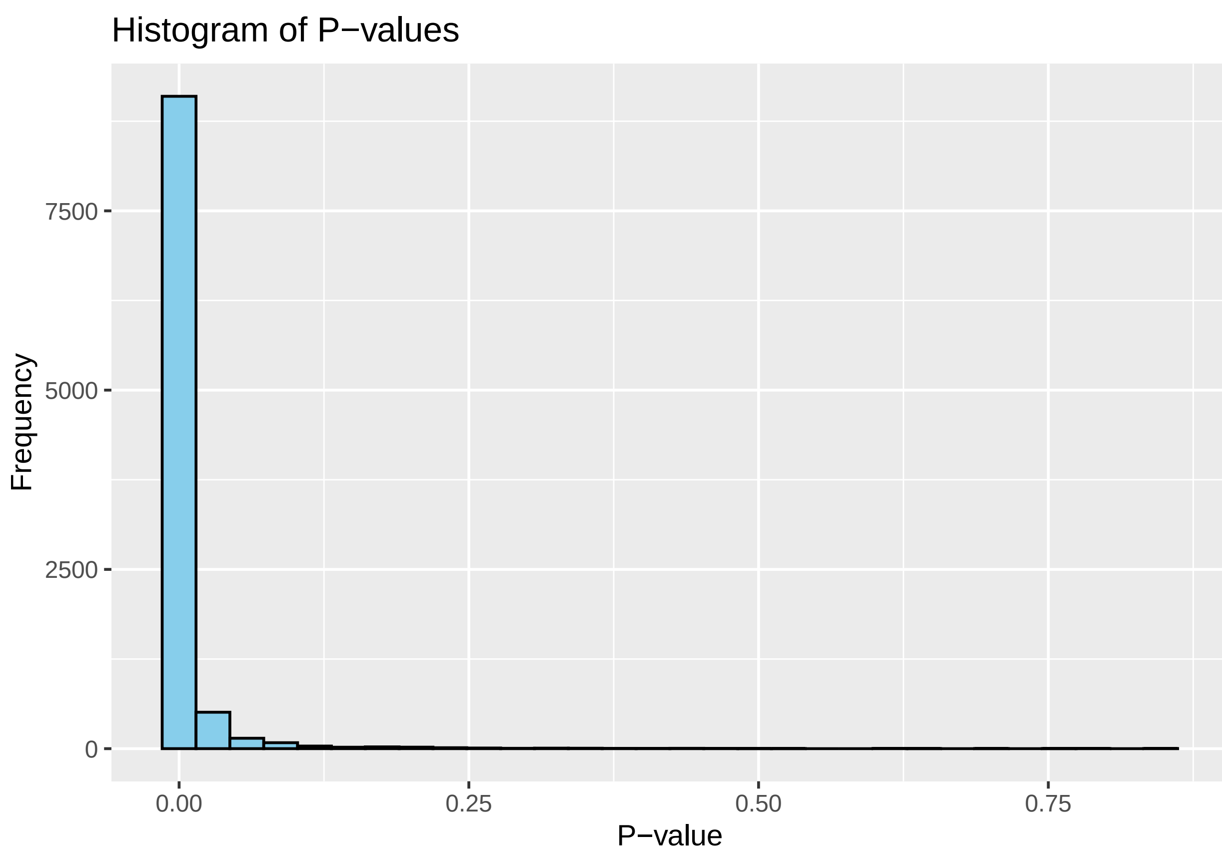
```r
library(knitr)
```

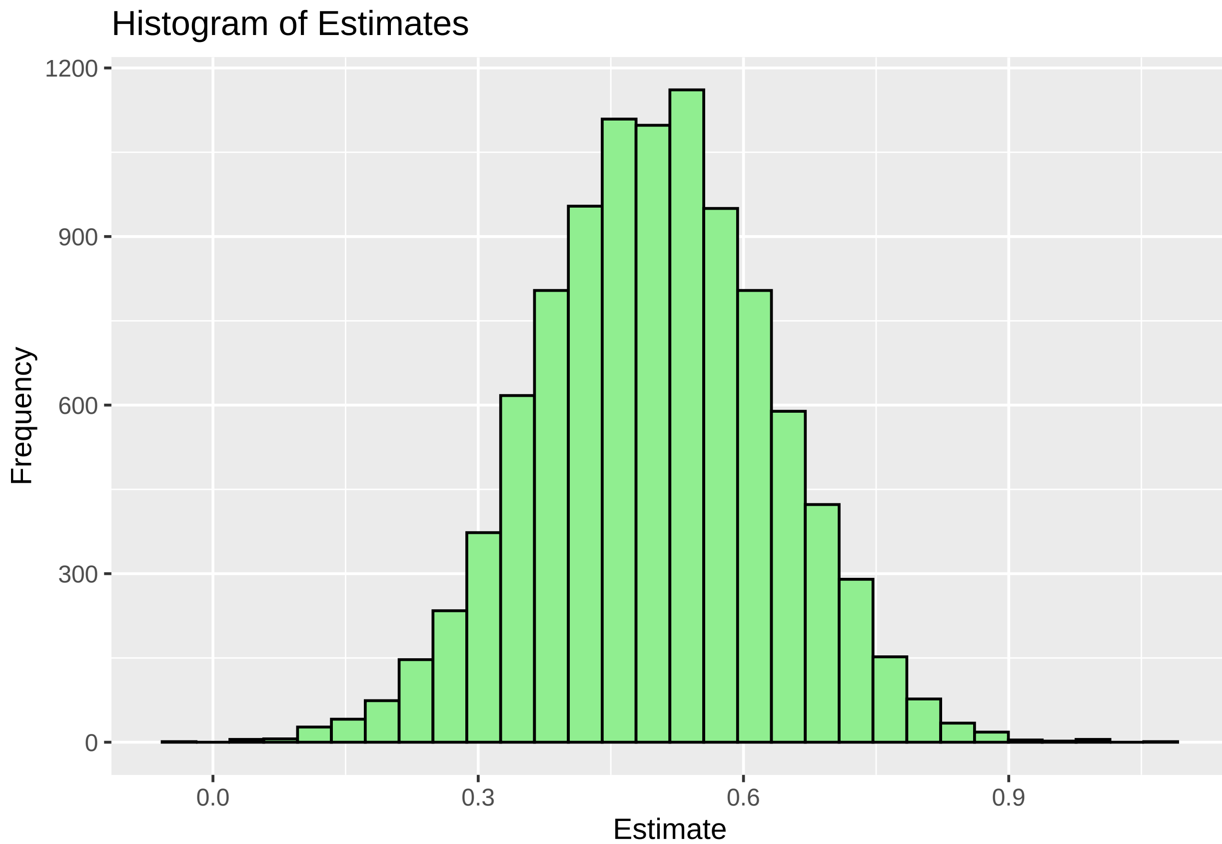
```r
#define function
set.seed(123)

simulate_t_tests <- function(n_sims, n1, n2, true_diff) {
  p_values <- numeric(n_sims)
  test_stats <- numeric(n_sims)
  estimates <- numeric(n_sims)
  rejections <- logical(n_sims)

  for (i in 1:n_sims) {
    x1 <- rnorm(n1, mean = 0, sd = 1)
    x2 <- rnorm(n2, mean = 0.5, sd = 3)

    test_result <- t.test(x1, x2)

    p_values[i] <- test_result$p.value
    test_stats[i] <- test_result$statistic
    estimates[i] <- test_result$estimate[2] - test_result$estimate[1]
    rejections[i] <- test_result$p.value < 0.05
  }

  simulation_data <- data.frame(
    p_values = p_values,
    test_stats = test_stats,
    estimates = estimates,
    rejections = rejections,
    bias = estimates - true_diff
  )

  mean_stats <- data.frame(
    mean_p_value = mean(simulation_data$p_values),
    mean_test_stat = mean(simulation_data$test_stats),
    mean_estimate = mean(simulation_data$estimates),
    power = mean(simulation_data$rejections),
    mean_bias = mean(simulation_data$bias)
  )

  print(mean_stats)
```

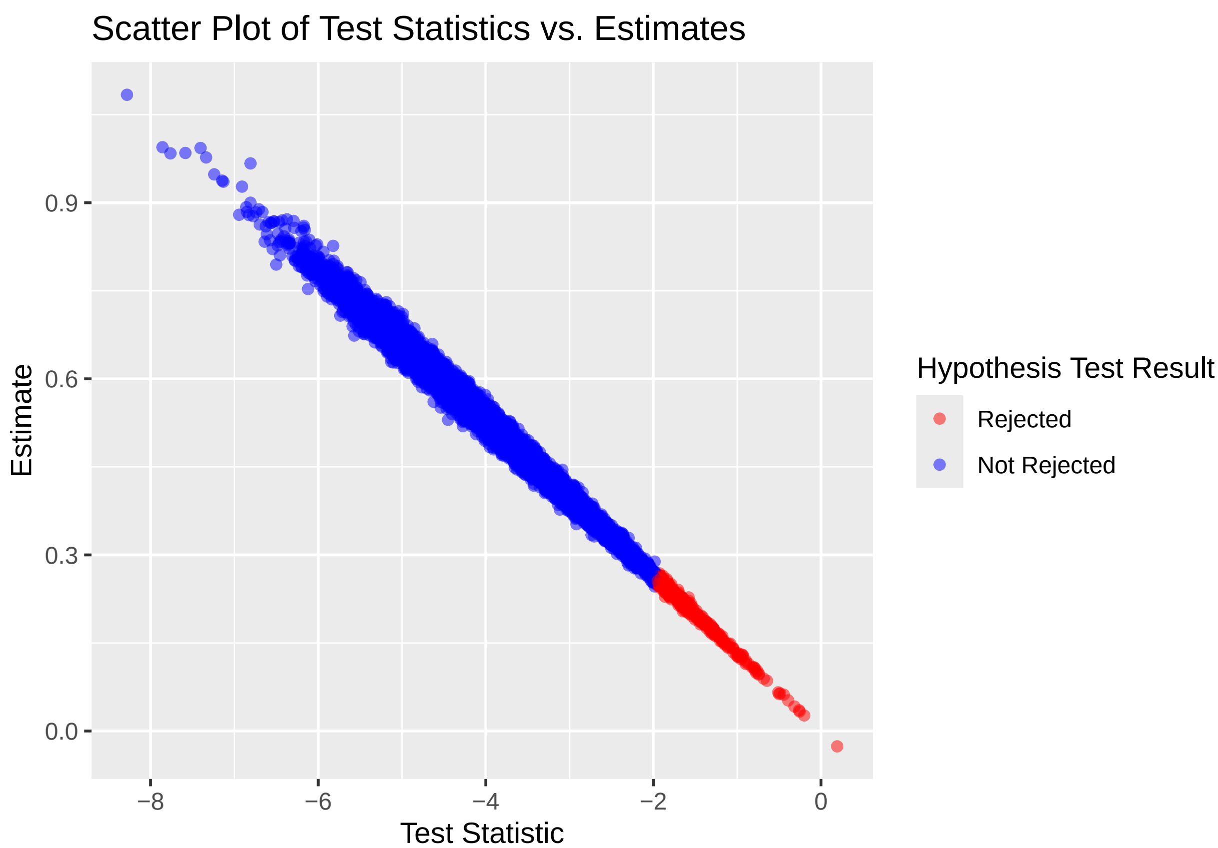
```r
  p_value_plot <- ggplot(simulation_data, aes(x = p_values)) +
    geom_histogram(bins = 30, fill = "skyblue", color = "black") +
    labs(title = "Histogram of P-values", x = "P-value", y = "Frequency")

  estimate_plot <- ggplot(simulation_data, aes(x = estimates)) +
    geom_histogram(bins = 30, fill = "lightgreen", color = "black") +
    labs(title = "Histogram of Estimates", x = "Estimate", y = "Frequency")

  scatter_plot <- ggplot(simulation_data, aes(x = test_stats, y = estimates, color = rejections)) +
    geom_point(alpha = 0.5) +
    scale_color_manual(values = c("red", "blue"), labels = c("Rejected", "Not Rejected")) +
    labs(title = "Scatter Plot of Test Statistics vs. Estimates",
         x = "Test Statistic", y = "Estimate", color = "Hypothesis Test Result")

  print(p_value_plot)
  print(estimate_plot)
  print(scatter_plot)
}
```

```r
#simulation 1, n_sims = 10000, n1 = 400, n2 = 600, true_diff
simulate_t_tests(n_sims = 10000, n1 = 400, n2 = 600, true_diff = 0.5)
```

```
##   mean_p_value mean_test_stat mean_estimate  power     mean_bias
## 1  0.008017772      -3.777353     0.4992039 0.9636 -0.0007960896
```

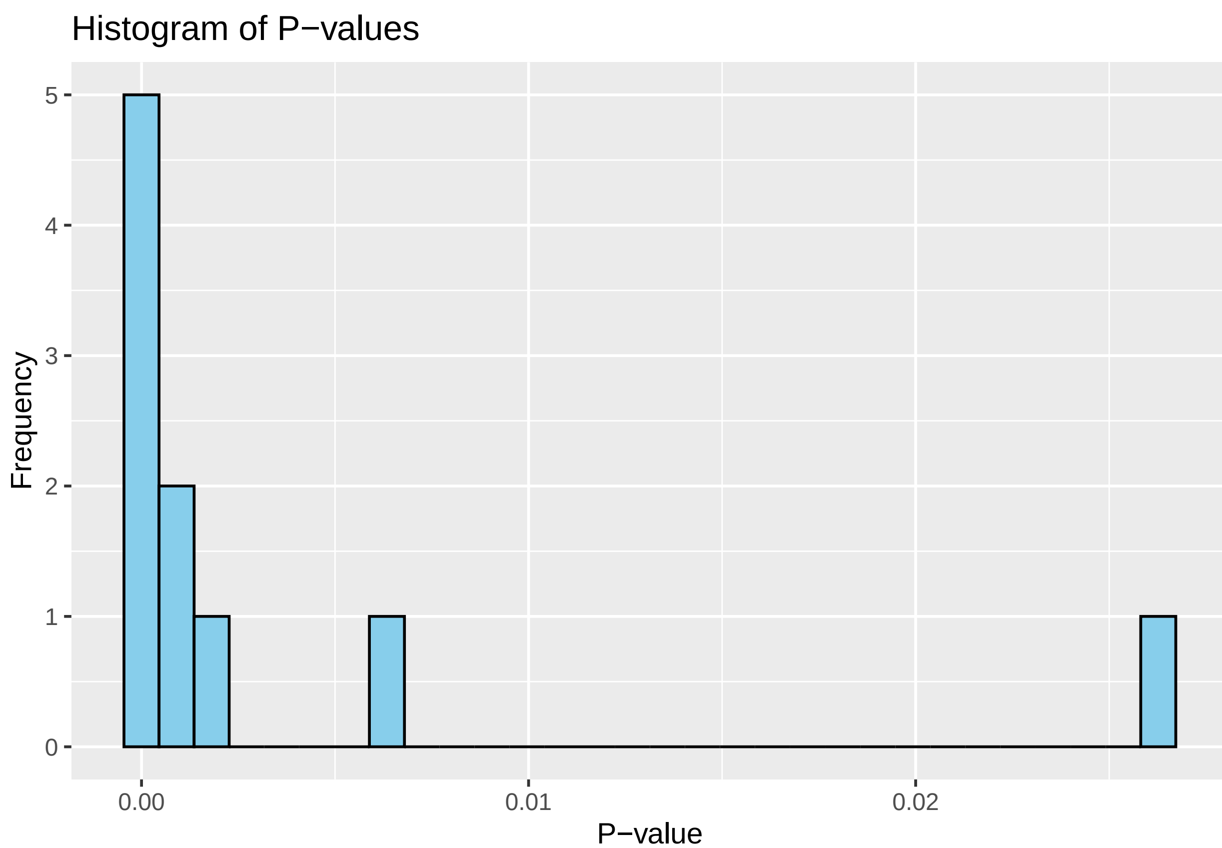


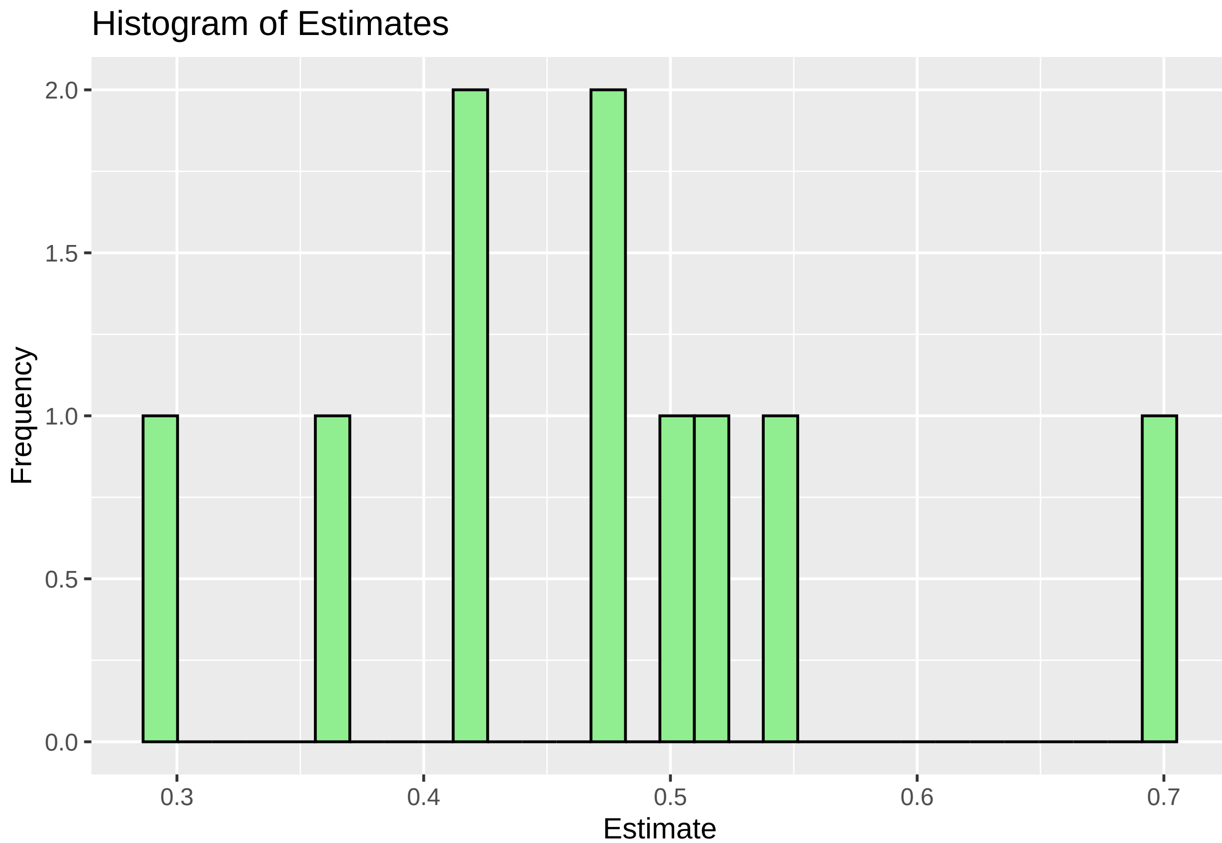

Histogram of Estimates
1200
900
600
300
0
Frequency
0.0
0.3
0.6
0.9
Estimate

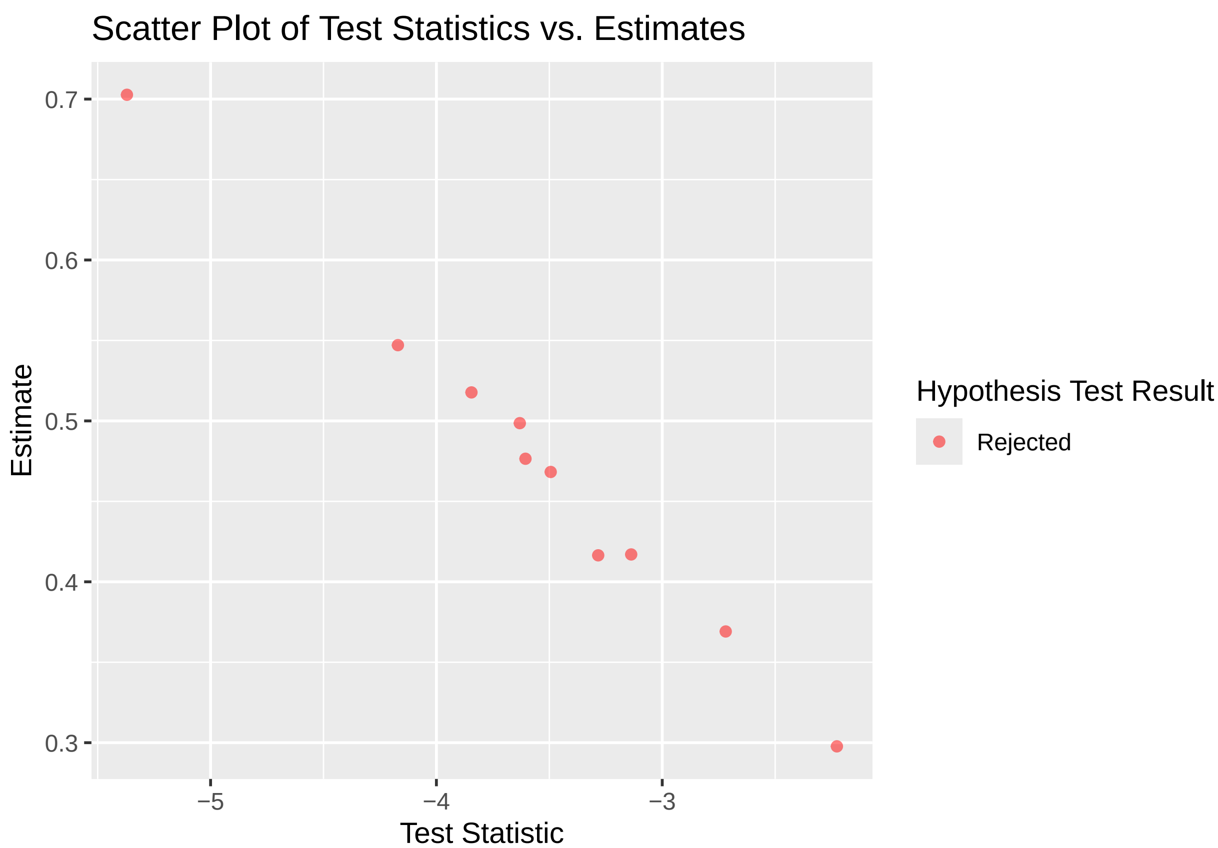



```
#simulation 2, n_sims = 10, n1 = 400, n2 = 600, true_diff = 0.5
simulate_t_tests(n_sims = 10, n1 = 400, n2 = 600, true_diff = 0.5)
```

```
##   mean_p_value mean_test_stat mean_estimate power   mean_bias
## 1  0.003710012      -3.548266     0.4711115     1 -0.02888847
```

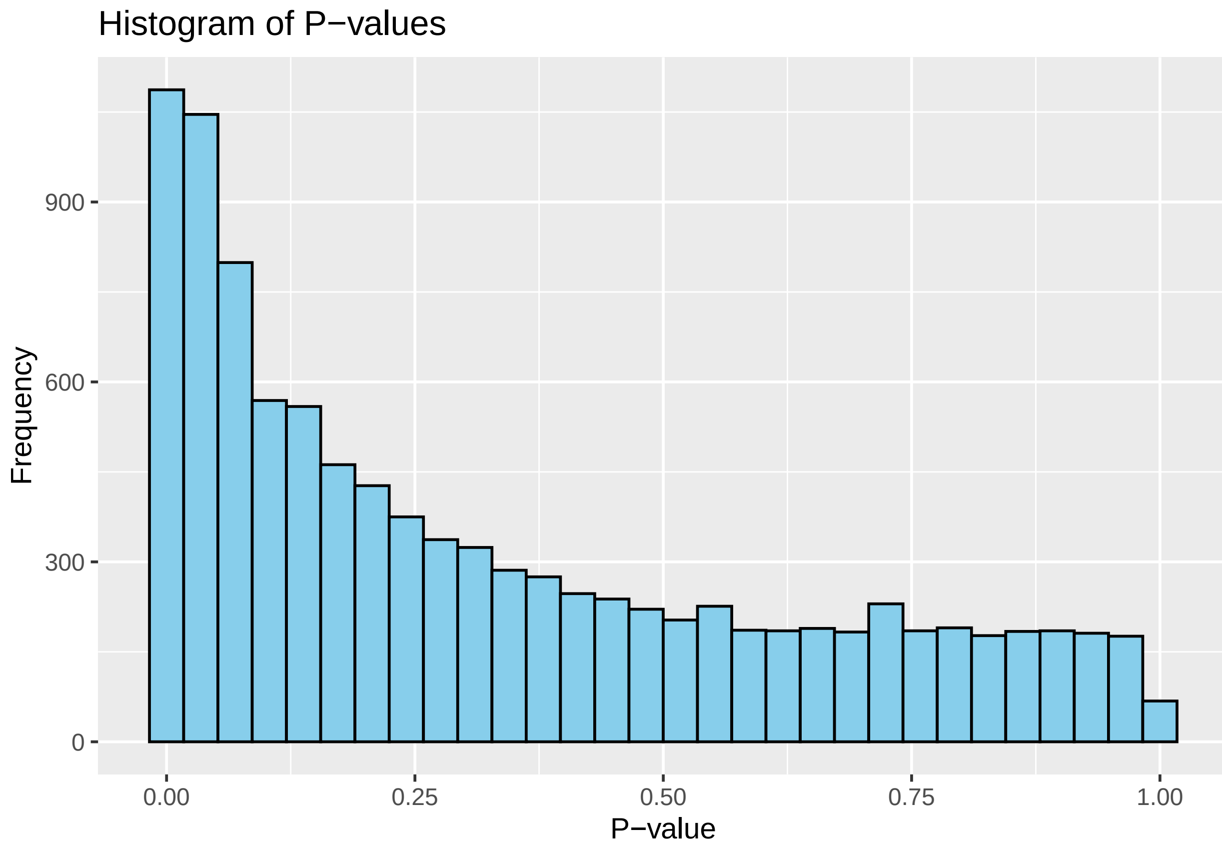

Histogram of P−values
Frequency
5
4
3
2
1
0
0.00
0.01
0.02
P−value


Histogram of Estimates
Frequency
2.0
1.5
1.0
0.5
0.0
0.3
0.4
0.5
0.6
0.7
Estimate

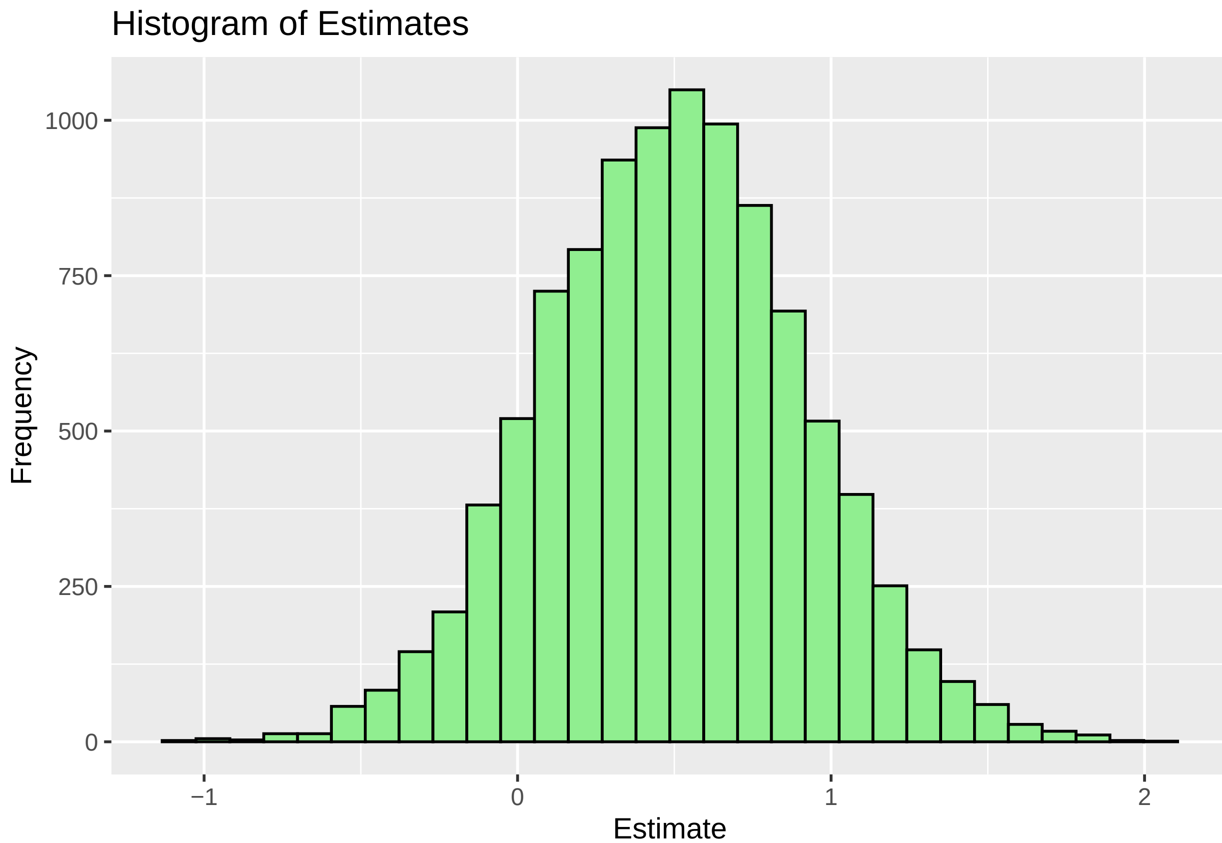


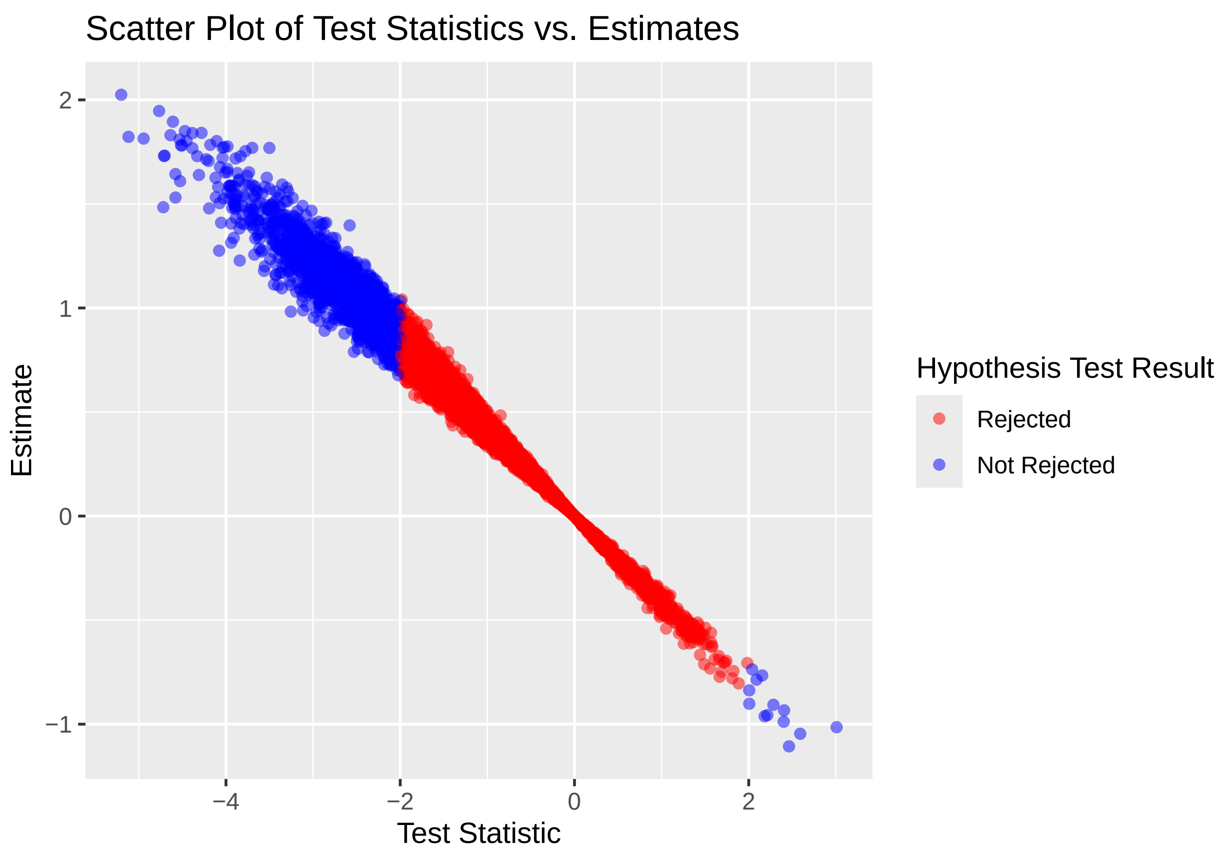

```r
#simulation 3, n_sims = 10000, n1 = 40, n2 = 60, true_diff = 0.5
simulate_t_tests(n_sims = 10000, n1 = 40, n2 = 60, true_diff = 0.5)
```

```
##   mean_p_value mean_test_stat mean_estimate  power    mean_bias
## 1    0.3238261      -1.196644     0.4961934 0.2092 -0.003806602
```



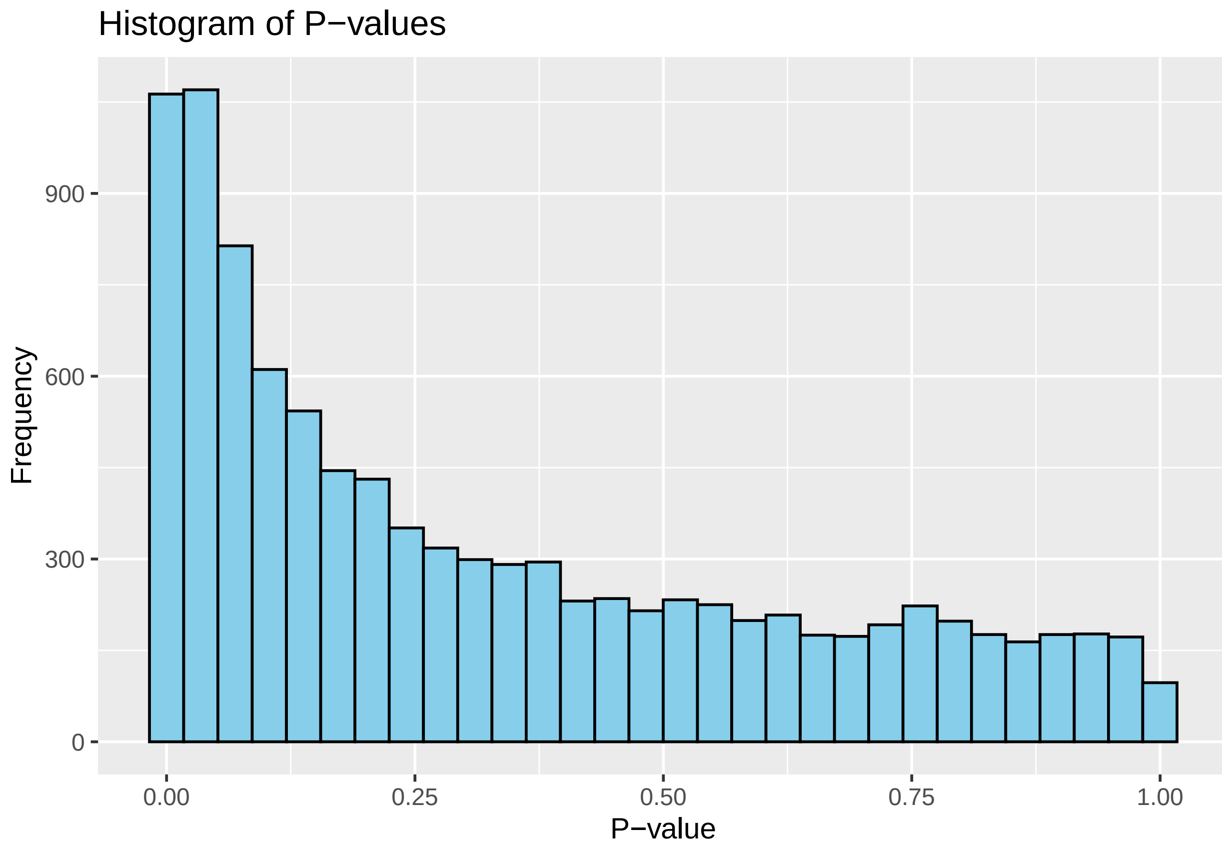
Histogram of P−values
Frequency
900
600
300
0
0.00
0.25
0.50
0.75
1.00
P−value

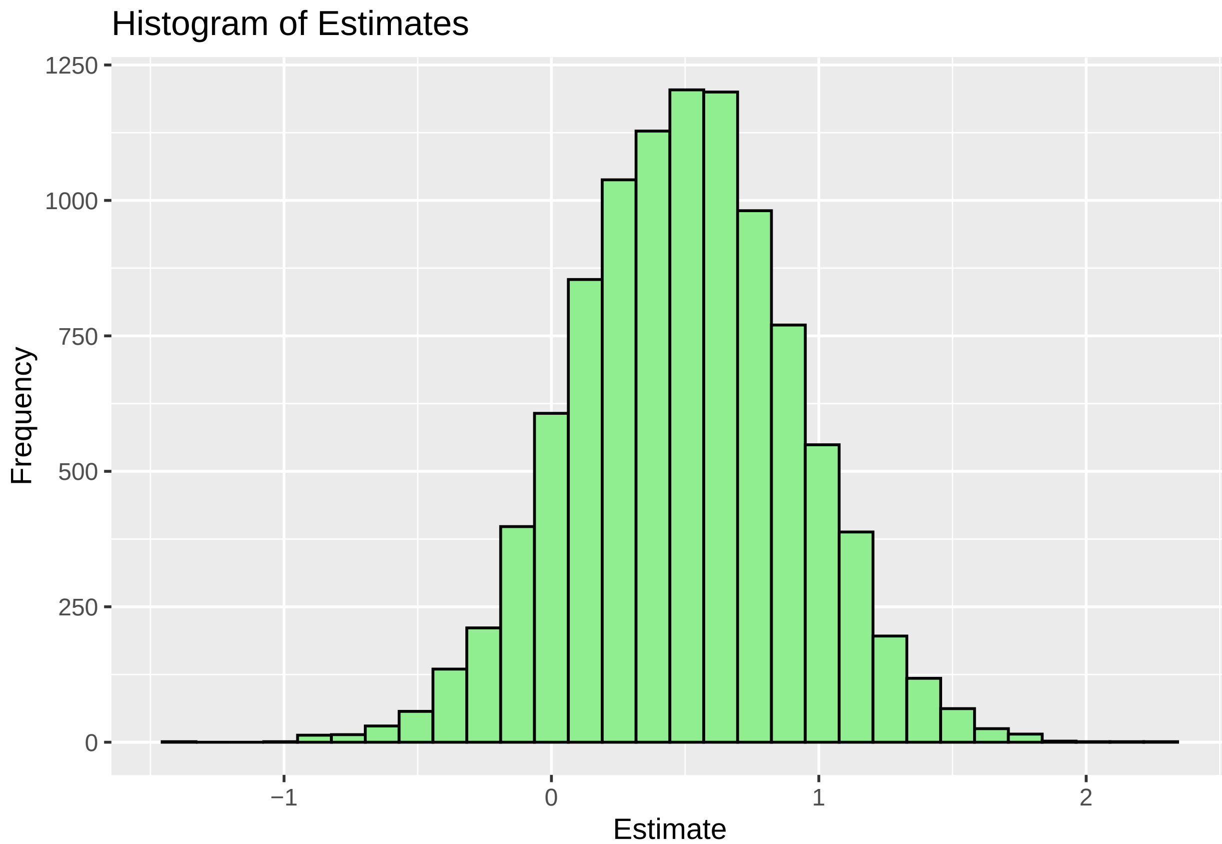

Histogram of Estimates
1000
750
500
250
0
Frequency
−1
0
1
2
Estimate

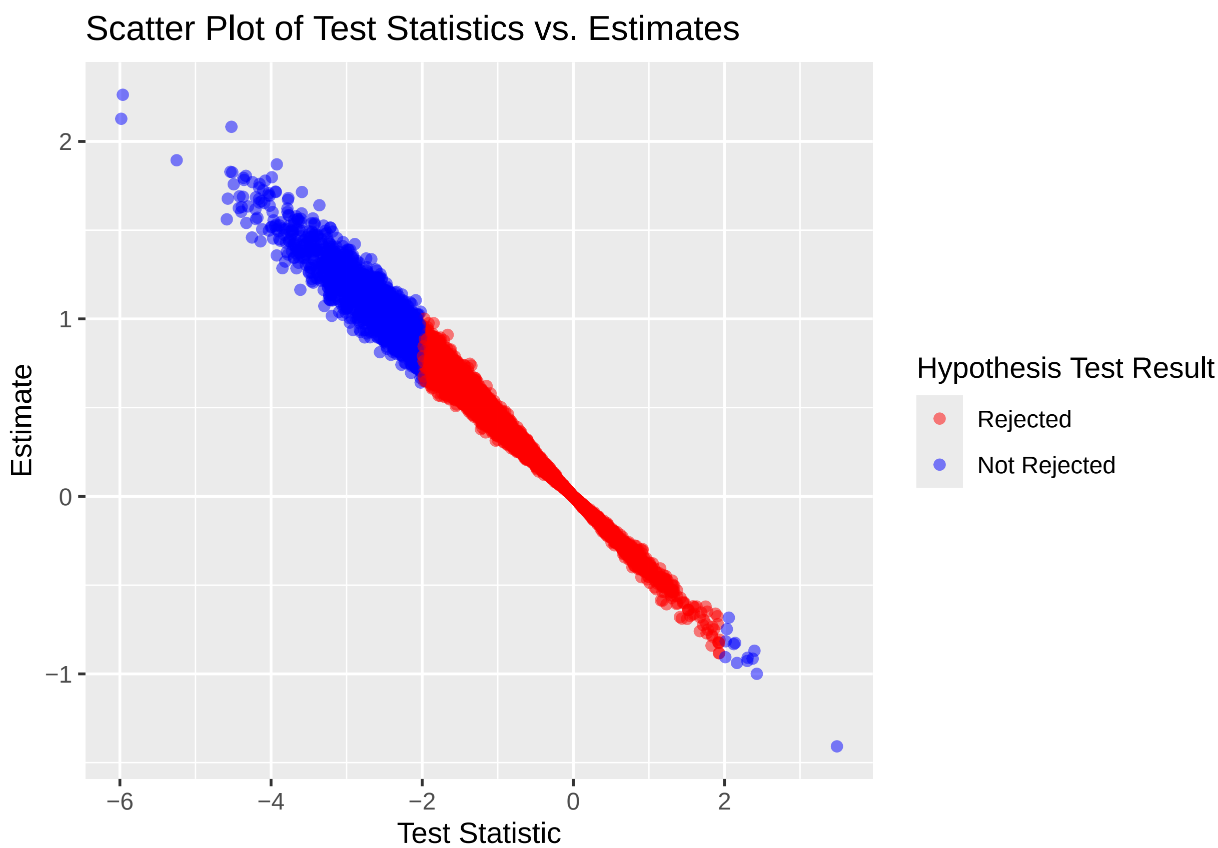



```r
#simulation 4, n_sims = 10, n1 = 40, n2 = 60, true_diff = 0.5
simulate_t_tests(n_sims = 10000, n1 = 40, n2 = 60, true_diff = 0.5)
```

```
##   mean_p_value mean_test_stat mean_estimate  power    mean_bias
## 1     0.324488      -1.195792     0.4954334 0.2093 -0.004566562
```


Histogram of P−values
Frequency
900
600
300
0
0.00
0.25
0.50
0.75
1.00
P−value


Histogram of Estimates
1250
1000
750
500
250
0
Frequency
−1
0
1
2
Estimate



Analysis:

Decreasing Sample Size: Reducing the sample size increased the variance of the estimator. This change reduced the power of the test (the ability to correctly reject the null hypothesis when it is false), and it also increased the bias (underhitting to more extent).

Increasing Sample Size: Increasing the sample size enhanced the precision of the estimator, and increased the power of the test. There are more cases when the p-value is very small, indicating a higher likelihood of rejecting the null hypothesis when it is indeed false. Replicating with Different n_sims:

Changing the number of simulations (n_sims) impacted the estimates of power and bias. The power increased and the bias was very small.