

Aplikasi Komputer

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Kelas : Matematika E

EMT untuk Perhitungan Aljabar

Pada notebook ini Anda belajar menggunakan EMT untuk melakukan berbagai perhitungan terkait dengan materi atau topik dalam Aljabar. Kegiatan yang harus Anda lakukan adalah sebagai berikut:

- Membaca secara cermat dan teliti notebook ini;
- Menerjemahkan teks bahasa Inggris ke bahasa Indonesia;
- Mencoba contoh-contoh perhitungan (perintah EMT) dengan cara meng ENTER setiap perintah EMT yang ada (pindahkan kursor ke baris perintah)
- Jika perlu Anda dapat memodifikasi perintah yang ada dan memberikan keterangan/penjelasan tambahan terkait hasilnya.
- Menyisipkan baris-baris perintah baru untuk mengerjakan soal-soal Aljabar dari file PDF yang saya berikan;
- Memberi catatan hasilnya.
- Jika perlu tuliskan soalnya pada teks notebook (menggunakan format LaTeX).
- Gunakan tampilan hasil semua perhitungan yang eksak atau simbolik dengan format LaTeX. (Seperti contoh-contoh pada notebook ini.)

Contoh pertama

Menyederhanakan bentuk aljabar:

$$6x^{-3}y^5 \times -7x^2y^{-9}$$

```
>$&6*x^(-3)*y^5*-7*x^2*y^(-9)
```

Menjabarkan:

$$(6x^{-3} + y^5)(-7x^2 - y^{-9})$$

```
>$&showev('expand((6*x^(-3)+y^5)*(-7*x^2-y^(-9))))
```

Baris Perintah

Baris perintah Euler terdiri dari satu atau beberapa perintah Euler diikuti dengan titik koma ";" atau koma ",". Titik koma mencegah pencetakan hasil. Koma setelah perintah terakhir dapat dihilangkan.

Baris perintah berikut hanya akan mencetak hasil ekspresi, bukan tugas atau perintah format.

```
>r:=2; h:=4; pi*r^2*h/3
```

16.7551608191

Perintah harus dipisahkan dengan yang kosong. Baris perintah berikut mencetak dua hasilnya.

```
>pi*2*r*h, %+2*pi*r*h // Ingat tanda % menyatakan hasil perhitungan terakhir sebelumnya
```

```
50.2654824574  
100.530964915
```

Baris perintah dieksekusi dalam urutan yang ditekan pengguna kembali. Jadi Anda mendapatkan nilai baru setiap kali Anda menjalankan baris kedua.

```
>x := 1;  
>x := cos(x) // nilai cosinus (x dalam radian)
```

```
0.540302305868
```

```
>x := cos(x)
```

```
0.857553215846
```

Jika dua garis terhubung dengan "..." kedua garis akan selalu dieksekusi secara bersamaan.

```
>x := 1.5; ...  
>x := (x+2/x)/2, x := (x+2/x)/2, x := (x+2/x)/2,
```

```
1.41666666667  
1.41421568627  
1.41421356237
```

Ini juga merupakan cara yang baik untuk menyebarkan perintah panjang pada dua atau lebih baris. Anda dapat menekan Ctrl+Return untuk membagi garis menjadi dua pada posisi kursor saat ini, atau Ctrl+Back untuk menggabungkan garis.

Untuk melipat semua multi-garis tekan Ctrl + L. Kemudian garis-garis berikutnya hanya akan terlihat, jika salah satunya memiliki fokus. Untuk melipat satu multi-baris, mulailah baris pertama dengan "%+".

```
>%+ x=4+5; ...
```

Garis yang dimulai dengan %% tidak akan terlihat sama sekali.

Euler mendukung loop di baris perintah, selama mereka masuk ke dalam satu baris atau multi-baris. Dalam program, pembatasan ini tidak berlaku, tentu saja. Untuk informasi lebih lanjut lihat pengantar berikut.

```
>x=1; for i=1 to 5; x := (x+2/x)/2, end; // menghitung akar 2
```

```
1.5  
1.41666666667  
1.41421568627  
1.41421356237  
1.41421356237
```

Tidak apa-apa untuk menggunakan multi-line. Pastikan baris diakhiri dengan "...".

```
>x := 1.5; // comments go here before the ...  
>repeat xnew:=(x+2/x)/2; until xnew~=x; ...  
>  x := xnew; ...  
>end; ...  
>x,
```

```
1.41421356237
```

Struktur bersyarat juga berfungsi.

```
>if E^pi>pi^E; then "Thought so!", endif;
```

```
Thought so!
```

Saat Anda menjalankan perintah, kursor dapat berada di posisi mana pun di baris perintah. Anda dapat kembali ke perintah sebelumnya atau melompat ke perintah berikutnya dengan tombol panah. Atau Anda dapat mengklik ke bagian komentar di atas perintah untuk menuju ke perintah.

Saat Anda menggerakkan kursor di sepanjang garis, pasangan tanda kurung atau kurung buka dan tutup akan disorot. Juga, perhatikan baris status. Setelah kurung buka fungsi `sqrt()`, baris status akan menampilkan teks bantuan untuk fungsi tersebut. Jalankan perintah dengan tombol kembali.

```
>sqrt(sin(10°)/cos(20°))
```

```
0.429875017772
```

Untuk melihat bantuan untuk perintah terbaru, buka jendela bantuan dengan F1. Di sana, Anda dapat memasukkan teks untuk dicari. Pada baris kosong, bantuan untuk jendela bantuan akan ditampilkan. Anda dapat menekan escape untuk menghapus garis, atau untuk menutup jendela bantuan.

Anda dapat mengklik dua kali pada perintah apa pun untuk membuka bantuan untuk perintah ini. Coba klik dua kali perintah `exp` di bawah ini di baris perintah.

```
>exp(log(2.5))
```

2.5

Anda dapat menyalin dan menempel di Euler ke. Gunakan Ctrl-C dan Ctrl-V untuk ini. Untuk menandai teks, seret mouse atau gunakan shift bersama dengan tombol kursor apa pun. Selain itu, Anda dapat menyalin tanda kurung yang disorot.

Euler tahu fungsi matematika biasa. Seperti yang Anda lihat di atas, fungsi trigonometri bekerja dalam radian atau derajat. Untuk mengonversi ke derajat, tambahkan simbol derajat (dengan tombol F7) ke nilainya, atau gunakan fungsi rad(x). Fungsi akar kuadrat disebut kuadrat dalam Euler. Tentu saja, $x^{1/2}$ juga dimungkinkan.

Untuk menyetel variabel, gunakan "=" atau ":=". Demi kejelasan, pengantar ini menggunakan bentuk yang terakhir. Spasi tidak masalah. Tapi ruang antara perintah diharapkan.

Beberapa perintah dalam satu baris dipisahkan dengan ";" atau ";". Titik koma menekan output dari perintah. Di akhir baris perintah ";" diasumsikan, jika ";" hilang.

```
>g:=9.81; t:=2.5; 1/2*g*t^2
```

30.65625

EMT menggunakan sintaks pemrograman untuk ekspresi. Memasuki

$$e^2 \cdot \left(\frac{1}{3 + 4 \log(0.6)} + \frac{1}{7} \right)$$

Anda harus mengatur tanda kurung yang benar dan menggunakan / untuk pecahan. Perhatikan tanda kurung yang disorot untuk bantuan. Perhatikan bahwa konstanta Euler e diberi nama E dalam EMT.

```
>E^2*(1/(3+4*log(0.6))+1/7)
```

8.77908249441

Untuk menghitung ekspresi rumit seperti

$$\left(\frac{\frac{1}{7} + \frac{1}{8} + 2}{\frac{1}{3} + \frac{1}{2}} \right)^2 \pi$$

Anda harus memasukkannya dalam bentuk baris.

```
>((1/7 + 1/8 + 2) / (1/3 + 1/2))^2 * pi
```

```
23.2671801626
```

Letakkan tanda kurung dengan hati-hati di sekitar sub-ekspresi yang perlu dihitung terlebih dahulu. EMT membantu Anda dengan menyorot ekspresi bahwa braket penutup selesai. Anda juga harus memasukkan nama "pi" untuk huruf Yunani pi.

Hasil dari perhitungan ini adalah bilangan floating point. Secara default dicetak dengan akurasi sekitar 12 digit. Di baris perintah berikut, kita juga belajar bagaimana kita bisa merujuk ke hasil sebelumnya dalam baris yang sama.

```
>1/3+1/7, fraction %
```

```
0.47619047619  
10/21
```

Perintah Euler dapat berupa ekspresi atau perintah primitif. Ekspresi terbuat dari operator dan fungsi. Jika perlu, itu harus berisi tanda kurung untuk memaksa urutan eksekusi yang benar. Jika ragu, memasang braket adalah ide yang bagus. Perhatikan bahwa EMT menunjukkan tanda kurung buka dan tutup saat mengedit baris perintah.

```
>(cos(pi/4)+1)^3*(sin(pi/4)+1)^2
```

14.4978445072

Operator numerik Euler meliputi

- + unary atau operator plus
- unary atau operator minus
- *, /
- . produk matriks
- a^b daya untuk positif a atau bilangan bulat b (a**b juga berfungsi)
- n! operator faktorial

dan masih banyak lagi.

Berikut adalah beberapa fungsi yang mungkin Anda butuhkan. Ada banyak lagi.

sin,cos,tan,atan,asin,acos,rad,deg
log,exp,log10,sqrt,logbase
bin,logbin,logfac,mod,lantai,ceil,bulat,abs,tanda
conj,re,im,arg,conj,nyata,kompleks
beta,betai,gamma,complexgamma,ellrf,ellf,ellrd,elle
bitand, bitor, bitxor, bitnot

Beberapa perintah memiliki alias, mis. Untuk log.

```
>ln(E^2), arctan(tan(0.5))
```

```
2  
0.5
```

```
>sin(30°)
```

```
0.5
```

Pastikan untuk menggunakan tanda kurung (kurung bulat), setiap kali ada keraguan tentang urutan eksekusi! Berikut ini tidak sama dengan $(2^3)^4$, yang merupakan default untuk 2^3^4 di EMT (beberapa sistem numerik melakukannya dengan cara lain).

```
>2^3^4, (2^3)^4, 2^(3^4)
```

```
2.41785163923e+24  
4096  
2.41785163923e+24
```

Bilangan Asli

Tipe data utama dalam Euler adalah bilangan real. Real direpresentasikan dalam format IEEE dengan akurasi sekitar 16 digit desimal.

```
>longest 1/3
```

0.3333333333333333

Representasi ganda internal membutuhkan 8 byte.

```
>printdual(1/3)
```

[illegible]

```
>printhex(1/3)
```

$$5.5555555555554 \times 16^{-1}$$

Sebuah string dalam Euler didefinisikan dengan "...".

```
>"A string can contain anything."
```

```
A string can contain anything.
```

String dapat digabungkan dengan `|` atau dengan `+`. Ini juga berfungsi dengan angka, yang dikonversi menjadi string dalam kasus itu.

```
>"The area of the circle with radius " + 2 + " cm is " + pi*4 + " cm^2."
```

```
The area of the circle with radius 2 cm is 12.5663706144 cm^2.
```

Fungsi `print` juga mengonversi angka menjadi string. Ini dapat mengambil sejumlah digit dan sejumlah tempat (0 untuk keluaran padat), dan secara optimal satu unit.

```
>"Golden Ratio : " + print((1+sqrt(5))/2,5,0)
```

```
Golden Ratio : 1.61803
```

Ada string khusus tidak ada, yang tidak dicetak. Itu dikembalikan oleh beberapa fungsi, ketika hasilnya tidak masalah. (Ini dikembalikan secara otomatis, jika fungsi tidak memiliki pernyataan pengembalian.)

```
>none
```

Untuk mengonversi string menjadi angka, cukup evaluasi saja. Ini juga berfungsi untuk ekspresi (lihat di bawah).

```
>"1234.5"()
```

```
1234.5
```

Untuk mendefinisikan vektor string, gunakan notasi vektor [...].

```
>v:=["affe","charlie","bravo"]
```

```
affe  
charlie  
bravo
```

Vektor string kosong dilambangkan dengan `[none]`. Vektor string dapat digabungkan.

```
>w:=[none]; w|v|v
```

```
affe  
charlie  
bravo  
affe  
charlie  
bravo
```

String dapat berisi karakter Unicode. Secara internal, string ini berisi kode UTF-8. Untuk menghasilkan string seperti itu, gunakan `u"..."` dan salah satu entitas HTML.

String Unicode dapat digabungkan seperti string lainnya.

```
>u"&alpha; = " + 45 + u"&deg; " // pdfLaTeX mungkin gagal menampilkan secara benar
```

```
= 45°
```

I

Dalam komentar, entitas yang sama seperti `alpha;`, `beta;` dll dapat digunakan. Ini mungkin alternatif cepat untuk Lateks. (Lebih detail di komentar di bawah).

Ada beberapa fungsi untuk membuat atau menganalisis string unicode. Fungsi `strtochar()` akan mengenali string Unicode, dan menerjemahkannya dengan benar.

```
>v=strtochar(u"&Auml; is a German letter")
```

```
[196, 32, 105, 115, 32, 97, 32, 71, 101, 114, 109, 97, 110,  
32, 108, 101, 116, 116, 101, 114]
```

Hasilnya adalah vektor angka Unicode. Fungsi kebalikannya adalah `chartoutf()`.

```
>v[1]=strtochar(u"&Uuml;")[1]; chartoutf(v)
```

```
Ü is a German letter
```

Fungsi `utf()` dapat menerjemahkan string dengan entitas dalam variabel menjadi string Unicode.

```
>s="We have &alpha;=&beta;."; utf(s) // pdfLaTeX mungkin gagal menampilkan secara benar
```

```
We have =.
```

Dimungkinkan juga untuk menggunakan entitas numerik.

```
>u"&#196;hnliches"
```

Ähnliches

Nilai Boolean

Nilai Boolean direpresentasikan dengan 1=true atau 0=false dalam Euler. String dapat dibandingkan, seperti halnya angka.

```
>2<1, "apel"<"banana"
```

0
1

"dan" adalah operator "&&" dan "atau" adalah operator "||", seperti dalam bahasa C. (Kata-kata "dan" dan "atau" hanya dapat digunakan dalam kondisi untuk "jika".)

```
>2<E && E<3
```

1

Operator Boolean mematuhi aturan bahasa matriks.

```
>(1:10)>5, nonzeros(%)
```

```
[0, 0, 0, 0, 0, 1, 1, 1, 1, 1]  
[6, 7, 8, 9, 10]
```

Anda dapat menggunakan fungsi bukan nol() untuk mengekstrak elemen tertentu dari vektor. Dalam contoh, kami menggunakan isprima bersyarat(n).

```
>N=2|3:2:99 // N berisi elemen 2 dan bilangan2 ganjil dari 3 s.d. 99
```

```
[2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29,  
31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57,  
59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85,  
87, 89, 91, 93, 95, 97, 99]
```

```
>N[nonzeros(isprime(N))] //pilih anggota2 N yang prima
```

```
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,  
53, 59, 61, 67, 71, 73, 79, 83, 89, 97]
```

Format Keluaran

Format output default EMT mencetak 12 digit. Untuk memastikan bahwa kami melihat default, kami mengatur ulang format.

```
>defformat; pi
```

```
3.14159265359
```

Secara internal, EMT menggunakan standar IEEE untuk bilangan ganda dengan sekitar 16 digit desimal. Untuk melihat jumlah digit penuh, gunakan perintah "format terpanjang", atau kita gunakan operator "terpanjang" untuk menampilkan hasil dalam format terpanjang.

```
>longest pi
```

```
3.141592653589793
```

Berikut adalah representasi heksadesimal internal dari bilangan ganda.

```
>printhex(pi)
```

```
3.243F6A8885A30*16^0
```

Format output dapat diubah secara permanen dengan perintah format.

```
>format(12,5); 1/3, pi, sin(1)
```

```
0.33333  
3.14159  
0.84147
```

Standarnya adalah format (12).

```
>format(12); 1/3
```

```
0.333333333333
```

Fungsi seperti "shortestformat", "shortformat", "longformat" bekerja untuk vektor dengan cara berikut.

```
>shortestformat; random(3,8)
```

```
0.66    0.2    0.89    0.28    0.53    0.31    0.44    0.3  
0.28    0.88    0.27    0.7    0.22    0.45    0.31    0.91  
0.19    0.46    0.095    0.6    0.43    0.73    0.47    0.32
```

Format default untuk skalar adalah format (12). Tapi ini bisa diubah.

```
>setscalarformat(5); pi
```

3.1416

Fungsi "format terpanjang" mengatur format skalar juga.

```
>longestformat; pi
```

3.141592653589793

Untuk referensi, berikut adalah daftar format output yang paling penting.

```
format terpendek format pendek format panjang, format terpanjang  
format(panjang,digit) format baik(panjang)  
fracformat (panjang)  
mengubah bentuk
```

Akurasi internal EMT adalah sekitar 16 tempat desimal, yang merupakan standar IEEE. Angka disimpan dalam format internal ini.

Tetapi format output EMT dapat diatur dengan cara yang fleksibel.

```
>longestformat; pi,
```

```
3.141592653589793
```

```
>format(10,5); pi
```

```
3.14159
```

Standarnya adalah `defformat()`.

```
>defformat; // default
```

Ada operator pendek yang hanya mencetak satu nilai. Operator ”terpanjang” akan mencetak semua digit angka yang valid.

```
>longest pi^2/2
```

```
4.934802200544679
```

Ada juga operator pendek untuk mencetak hasil dalam format pecahan. Kami sudah menggunakannya di atas.

```
>fraction 1+1/2+1/3+1/4
```

25/12

Karena format internal menggunakan cara biner untuk menyimpan angka, nilai 0,1 tidak akan direpresentasikan dengan tepat. Kesalahan bertambah sedikit, seperti yang Anda lihat dalam perhitungan berikut.

```
>longest 0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1-1
```

-1.110223024625157e-16

Tetapi dengan "format panjang" default Anda tidak akan melihat ini. Untuk kenyamanan, output dari angka yang sangat kecil adalah 0.

```
>0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1-1
```

0

String atau nama dapat digunakan untuk menyimpan ekspresi matematika, yang dapat dievaluasi oleh EMT. Untuk ini, gunakan tanda kurung setelah ekspresi. Jika Anda bermaksud menggunakan string sebagai ekspresi, gunakan konvensi untuk menamakannya "fx" atau "fxy" dll. Ekspresi lebih diutamakan daripada fungsi.

Variabel global dapat digunakan dalam evaluasi.

```
>r:=2; fx:="pi*r^2"; longest fx()
```

```
12.56637061435917
```

Parameter ditetapkan ke x, y, dan z dalam urutan itu. Parameter tambahan dapat ditambahkan menggunakan parameter yang ditetapkan.

```
>fx:="a*sin(x)^2"; fx(5,a=-1)
```

```
-0.919535764538
```

Perhatikan bahwa ekspresi akan selalu menggunakan variabel global, bahkan jika ada variabel dalam fungsi dengan nama yang sama. (Jika tidak, evaluasi ekspresi dalam fungsi dapat memberikan hasil yang sangat membingungkan bagi pengguna yang memanggil fungsi tersebut.)

```
>at:=4; function f(expr,x,at) := expr(x); ...  
>f("at*x^2",3,5) // computes 4*3^2 not 5*3^2
```

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Jika Anda ingin menggunakan nilai lain untuk "at" daripada nilai global, Anda perlu menambahkan "at=value".

```
>at:=4; function f(expr,x,a) := expr(x,at=a); ...  
>f("at*x^2",3,5)
```

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Untuk referensi, kami berkomentar bahwa koleksi panggilan (dibahas di tempat lain) dapat berisi ekspresi. Jadi kita bisa membuat contoh di atas sebagai berikut.

```
>at:=4; function f(expr,x) := expr(x); ...  
>f({{"at*x^2",at=5}},3)
```

45

Ekspresi dalam x sering digunakan seperti fungsi.

Perhatikan bahwa mendefinisikan fungsi dengan nama yang sama seperti ekspresi simbolik global menghapus variabel ini untuk menghindari kebingungan antara ekspresi simbolik dan fungsi.

```
>f &= 5*x;  
>function f(x) := 6*x;  
>f(2)
```

12

Dengan cara konvensi, ekspresi simbolik atau numerik harus diberi nama fx, fxy dll. Skema penamaan ini tidak boleh digunakan untuk fungsi.

```
>fx &= diff(x^x,x); $&fx
```

Bentuk khusus dari ekspresi memungkinkan variabel apa pun sebagai parameter tanpa nama untuk evaluasi ekspresi, bukan hanya "x", "y" dll. Untuk ini, mulai ekspresi dengan "@(variabel) ...".

```
>"@(a,b) a^2+b^2", %(4,5)
```

```
@(a,b) a^2+b^2
```

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Ini memungkinkan untuk memanipulasi ekspresi dalam variabel lain untuk fungsi EMT yang membutuhkan ekspresi dalam "x".

Cara paling dasar untuk mendefinisikan fungsi sederhana adalah dengan menyimpan rumusnya dalam ekspresi simbolis atau numerik. Jika variabel utama adalah x, ekspresi dapat dievaluasi seperti fungsi.

Seperti yang Anda lihat dalam contoh berikut, variabel global terlihat selama evaluasi.

```
>fx &= x^3-a*x; ...  
>a=1.2; fx(0.5)
```

-0.475

Semua variabel lain dalam ekspresi dapat ditentukan dalam evaluasi menggunakan parameter yang ditetapkan.

```
>fx(0.5,a=1.1)
```

-0.425

Sebuah ekspresi tidak perlu simbolis. Ini diperlukan, jika ekspresi berisi fungsi, yang hanya diketahui di kernel numerik, bukan di Maxima.

Matematika Simbolik

EMT melakukan matematika simbolis dengan bantuan Maxima. Untuk detailnya, mulailah dengan tutorial berikut, atau telusuri referensi untuk Maxima. Para ahli di Maxima harus mencatat bahwa ada perbedaan sintaks antara sintaks asli Maxima dan sintaks default ekspresi simbolik di EMT.

Matematika simbolik terintegrasi dengan mulus ke dalam Euler dengan &. Ekspresi apa pun yang dimulai dengan & adalah ekspresi simbolis. Itu dievaluasi dan dicetak oleh Maxima.

Pertama-tama, Maxima memiliki aritmatika "tak terbatas" yang dapat menangani angka yang sangat besar.

```
>&44!
```

Dengan cara ini, Anda dapat menghitung hasil yang besar dengan tepat. Mari kita hitung

$$C(44, 10) = \frac{44!}{34! \cdot 10!}$$

```
>& 44!/(34!*10!) // nilai C(44,10)
```

Tentu saja, Maxima memiliki fungsi yang lebih efisien untuk ini (seperti halnya bagian numerik dari EMT).

```
>$binomial(44,10) //menghitung C(44,10) menggunakan fungsi binomial()
```

Untuk mempelajari lebih lanjut tentang fungsi tertentu klik dua kali di atasnya. Misalnya, coba klik dua kali pada "&binomial" di baris perintah sebelumnya. Ini membuka dokumentasi Maxima seperti yang disediakan oleh penulis program itu.

Anda akan belajar bahwa yang berikut ini juga berfungsi.

$$C(x,3) = \frac{x!}{(x-3)!3!} = \frac{(x-2)(x-1)x}{6}$$

```
>$binomial(x,3) // C(x,3)
```

Jika Anda ingin mengganti x dengan nilai tertentu, gunakan "dengan".

```
>$&binomial(x,3) with x=10 // substitusi x=10 ke C(x,3)
```

Dengan begitu Anda dapat menggunakan solusi persamaan dalam persamaan lain.

Ekspresi simbolik dicetak oleh Maxima dalam bentuk 2D. Alasan untuk ini adalah bendera simbolis khusus dalam string.

Seperti yang akan Anda lihat pada contoh sebelumnya dan berikut, jika Anda telah menginstal LaTeX, Anda dapat mencetak ekspresi simbolis dengan Lateks. Jika tidak, perintah berikut akan mengeluarkan pesan kesalahan.

Untuk mencetak ekspresi simbolis dengan LaTeX, gunakan \$ di depan & (atau Anda dapat menghilangkan &) sebelum perintah. Jangan menjalankan perintah Maxima dengan \$, jika Anda tidak menginstal LaTeX.

```
>$ (3+x)/(x^2+1)
```

Ekspresi simbolik diuraikan oleh Euler. Jika Anda membutuhkan sintaks yang kompleks dalam satu ekspresi, Anda dapat menyertakan ekspresi dalam "...". Untuk menggunakan lebih dari ekspresi sederhana adalah mungkin, tetapi sangat tidak disarankan.

```
>&"v := 5; v^2"
```

Untuk kelengkapan, kami menyatakan bahwa ekspresi simbolik dapat digunakan dalam program, tetapi perlu diapit dalam tanda kutip. Selain itu, jauh lebih efektif untuk memanggil Maxima pada waktu kompilasi jika memungkinkan.

```
>$&expand((1+x)^4), $&factor(diff(%,x)) // diff: turunan, factor: faktor
```

Sekali lagi, % mengacu pada hasil sebelumnya.

Untuk mempermudah, kami menyimpan solusi ke variabel simbolik. Variabel simbolik didefinisikan dengan "&=".

```
>fx &= (x+1)/(x^4+1); $&fx
```

Ekspresi simbolik dapat digunakan dalam ekspresi simbolik lainnya.

```
>$&factor(diff(fx,x))
```


Masukan langsung dari perintah Maxima juga tersedia. Mulai baris perintah dengan "::<". Sintaks Maxima disesuaikan dengan sintaks EMT (disebut "mode kompatibilitas").

```
>&factor(20!)
```

```
2432902008176640000
```

```
>::: factor(10!)
```

```
8 4 2  
2 3 5 7
```

```
>:: factor(20!)
```

```
18 8 4 2  
2 3 5 7 11 13 17 19
```

Jika Anda ahli dalam Maxima, Anda mungkin ingin menggunakan sintaks asli Maxima. Anda dapat melakukannya dengan "::<".

```
>::: av:g$ av^2;
```

$$g^2$$

```
>fx &= x^3*exp(x), $fx
```

$$x^3 E$$

Variabel tersebut dapat digunakan dalam ekspresi simbolik lainnya. Perhatikan, bahwa dalam perintah berikut sisi kanan &= dievaluasi sebelum penugasan ke Fx.

```
>&(fx with x=5), $%, &float(%)
```

$$125 \cdot 5$$

18551.64488782208

```
>fx(5)
```

18551.6448878

Untuk evaluasi ekspresi dengan nilai variabel tertentu, Anda dapat menggunakan operator "with".

Baris perintah berikut juga menunjukkan bahwa Maxima dapat mengevaluasi ekspresi secara numerik dengan float().

```
>&(fx with x=10)-(fx with x=5), &float(%)
```

$$1000 \cdot 10^5 - 125 \cdot 5$$

2.20079141499189e+7

```
>$factor(diff(fx,x,2))
```

Untuk mendapatkan kode Lateks untuk ekspresi, Anda dapat menggunakan perintah tex.

```
>tex(fx)
```

```
x^3\,e^{\,x}
```

Ekspresi simbolik dapat dievaluasi seperti ekspresi numerik.

```
>fx(0.5)
```

```
0.206090158838
```

Dalam ekspresi simbolis, ini tidak berfungsi, karena Maxima tidak mendukungnya. Sebagai gantinya, gunakan sintaks "with" (bentuk yang lebih bagus dari perintah at(...) dari Maxima).

```
>$&fx with x=1/2
```

Penugasan juga bisa bersifat simbolis.

```
>$&fx with x=1+t
```

Perintah solve memecahkan ekspresi simbolik untuk variabel di Maxima. Hasilnya adalah vektor solusi.

```
>$&solve(x^2+x=4,x)
```

Bandingkan dengan perintah numerik "selesaikan" di Euler, yang membutuhkan nilai awal, dan secara opsional nilai target.

```
>solve("x^2+x",1,y=4)
```

1.56155281281

Nilai numerik dari solusi simbolik dapat dihitung dengan evaluasi hasil simbolis. Euler akan membaca tugas x= dll. Jika Anda tidak memerlukan hasil numerik untuk perhitungan lebih lanjut, Anda juga dapat membiarkan Maxima menemukan nilai numerik.

```
>sol &= solve(x^2+2*x=4,x); $sol, sol(), $float(sol)
```

```
[-3.23607, 1.23607]
```

Untuk mendapatkan solusi simbolis tertentu, seseorang dapat menggunakan "with" dan index.

```
>$solve(x^2+x=1,x), x2 &= x with %[2]; $x2
```

Untuk menyelesaikan sistem persamaan, gunakan vektor persamaan. Hasilnya adalah vektor solusi.

```
>sol &= solve([x+y=3,x^2+y^2=5],[x,y]); $sol, $x*y with sol[1]
```

Ekspresi simbolis dapat memiliki bendera, yang menunjukkan perlakuan khusus di Maxima. Beberapa flag dapat digunakan sebagai perintah juga, yang lain tidak. Bendera ditambahkan dengan "|" (bentuk yang lebih bagus dari "ev(...,flags)")

```
>$ diff((x^3-1)/(x+1),x) //turunan bentuk pecahan  
>$ diff((x^3-1)/(x+1),x) | ratsimp //menyederhanakan pecahan  
>$factor(%)
```

Fungsi

Dalam EMT, fungsi adalah program yang didefinisikan dengan perintah "fungsi". Ini bisa berupa fungsi satu baris atau fungsi multibaris.

Fungsi satu baris dapat berupa numerik atau simbolis. Fungsi satu baris numerik didefinisikan oleh ":=".

```
>function f(x) := x*sqrt(x^2+1)
```

Untuk gambaran umum, kami menunjukkan semua kemungkinan definisi untuk fungsi satu baris. Suatu fungsi dapat dievaluasi sama seperti fungsi Euler bawaan lainnya.

```
>f(2)
```

4.472135955

Fungsi ini akan bekerja untuk vektor juga, dengan mematuhi bahasa matriks Euler, karena ekspresi yang digunakan dalam fungsi divektorkan.

```
>f(0:0.1:1)
```

[0, 0.100499, 0.203961, 0.313209, 0.430813, 0.559017, 0.699714,
0.854459, 1.0245, 1.21083, 1.41421]

Fungsi dapat diplot. Alih-alih ekspresi, kita hanya perlu memberikan nama fungsi.
Berbeda dengan ekspresi simbolik atau numerik, nama fungsi harus diberikan dalam string.

```
>solve("f",1,y=1)
```

0.786151377757

Secara default, jika Anda perlu menimpa fungsi bawaan, Anda harus menambahkan kata kunci "menimpa". Menimpa fungsi bawaan berbahaya dan dapat menyebabkan masalah untuk fungsi lain tergantung pada fungsi tersebut.

Anda masih dapat memanggil fungsi bawaan sebagai "...", jika itu adalah fungsi di inti Euler.

```
>function overwrite sin (x) := _sin(x°) // redine sine in degrees  
>sin(45)
```

0.707106781187

Lebih baik kita menghapus redefinisi dosa ini.

```
>forget sin; sin(pi/4)
```

0.707106781187

Fungsi numerik dapat memiliki parameter default.

```
>function f(x,a=1) := a*x^2
```

Menghilangkan parameter ini menggunakan nilai default.

```
>f(4)
```

16

Menyetelnya akan menimpa nilai default.

```
>f(4,5)
```

80

Parameter yang ditetapkan menyimpannya juga. Ini digunakan oleh banyak fungsi Euler seperti plot2d, plot3d.

```
>f(4,a=1)
```

16

Jika suatu variabel bukan parameter, itu harus global. Fungsi satu baris dapat melihat variabel global.

```
>function f(x) := a*x^2  
>a=6; f(2)
```

24

Tetapi parameter yang ditetapkan menimpa nilai global.

Jika argumen tidak ada dalam daftar parameter yang telah ditentukan sebelumnya, argumen tersebut harus dideklarasikan dengan ":=".

```
>f(2,a:=5)
```

20

Fungsi simbolis didefinisikan dengan "&=". Mereka didefinisikan dalam Euler dan Maxima, dan bekerja di kedua dunia. Ekspresi yang mendefinisikan dijalankan melalui Maxima sebelum definisi.

```
>function g(x) &= x^3-x*exp(-x); $&g(x)
```

Fungsi simbolik dapat digunakan dalam ekspresi simbolik.

```
>$&diff(g(x),x), $&% with x=4/3
```

Mereka juga dapat digunakan dalam ekspresi numerik. Tentu saja, ini hanya akan berfungsi jika EMT dapat menginterpretasikan semua yang ada di dalam fungsi tersebut.

```
>g(5+g(1))
```

178.635099908

Mereka dapat digunakan untuk mendefinisikan fungsi atau ekspresi simbolik lainnya.

```
>function G(x) &= factor(integrate(g(x),x)); $$G(c) // integrate: mengintegralkan  
>solve(&g(x),0.5)
```

0.703467422498

Berikut ini juga berfungsi, karena Euler menggunakan ekspresi simbolis dalam fungsi g, jika tidak menemukan variabel simbolik g, dan jika ada fungsi simbolis g.

```
>solve(&g,0.5)
```

0.703467422498

```
>function P(x,n) &= (2*x-1)^n; $$P(x,n)  
>function Q(x,n) &= (x+2)^n; $$Q(x,n)  
>$$P(x,4), $$expand(%)  
>P(3,4)
```

625

```

>$P(x,4)+ Q(x,3), $expand(%)
>$P(x,4)-Q(x,3), $expand(%) , $factor(%)
>$P(x,4)*Q(x,3), $expand(%) , $factor(%)
>$P(x,4)/Q(x,1), $expand(%) , $factor(%)
>function f(x) &= x^3-x; $f(x)

```

Dengan &= fungsinya simbolis, dan dapat digunakan dalam ekspresi simbolik lainnya.

```

>$integrate(f(x),x)

```

Dengan := fungsinya numerik. Contoh yang baik adalah integral tak tentu seperti

$$f(x) = \int_1^x t^t dt,$$

yang tidak dapat dinilai secara simbolis.

Jika kita mendefinisikan kembali fungsi dengan kata kunci "peta" dapat digunakan untuk vektor x. Secara internal, fungsi dipanggil untuk semua nilai x satu kali, dan hasilnya disimpan dalam vektor.

```

>function map f(x) := integrate("x^x",1,x)
>f(0:0.5:2)

```

```

[-0.783431, -0.410816, 0, 0.676863, 2.05045]

```

Fungsi dapat memiliki nilai default untuk parameter.

```
>function mylog (x,base=10) := ln(x)/ln(base);
```

Sekarang fungsi dapat dipanggil dengan atau tanpa parameter "basis".

```
>mylog(100), mylog(2^6.7,2)
```

```
2  
6.7
```

Selain itu, dimungkinkan untuk menggunakan parameter yang ditetapkan.

```
>mylog(E^2,base=E)
```

```
2
```

Seringkali, kita ingin menggunakan fungsi untuk vektor di satu tempat, dan untuk elemen individual di tempat lain. Ini dimungkinkan dengan parameter vektor.

```
>function f([a,b]) &= a^2+b^2-a*b+b; $&f(a,b), $&f(x,y)
```

Fungsi simbolik seperti itu dapat digunakan untuk variabel simbolik.

Tetapi fungsinya juga dapat digunakan untuk vektor numerik.

```
>v=[3,4]; f(v)
```

17

Ada juga fungsi simbolis murni, yang tidak dapat digunakan secara numerik.

```
>function lapl(expr,x,y) &&= diff(expr,x,2)+diff(expr,y,2)//turunan parsial kedua
```

```
diff(expr, y, 2) + diff(expr, x, 2)
```

```
>${realpart}((x+I*y)^4), ${lapl}(% ,x,y)
```

Tetapi tentu saja, mereka dapat digunakan dalam ekspresi simbolik atau dalam definisi fungsi simbolik.

```
>=-function f(x,y) &= factor(lapl((x+y^2)^5,x,y)); $&f(x,y)
```

Syntax error in expression, or unfinished expression!

Error in:

```
=-function f(x,y) &= factor(lapl((x+y^2)^5,x,y)); $&f(x,y) ...  
^
```

Untuk meringkas

- &= mendefinisikan fungsi simbolis,
- := mendefinisikan fungsi numerik,
- &&= mendefinisikan fungsi simbolis murni.

Memecahkan Ekspresi

Ekspresi dapat diselesaikan secara numerik dan simbolis.

Untuk menyelesaikan ekspresi sederhana dari satu variabel, kita dapat menggunakan fungsi `solve()`. Perlu nilai awal untuk memulai pencarian. Secara internal, `solve()` menggunakan metode secant.

```
>solve("x^2-2",1)
```

```
1.41421356237
```

Ini juga berfungsi untuk ekspresi simbolis. Ambil fungsi berikut.

```
>$&solve(x^2=2,x)  
>$&solve(x^2-2,x)  
>$&solve(a*x^2+b*x+c=0,x)  
>$&solve([a*x+b*y=c,d*x+e*y=f],[x,y])  
>px &= 4*x^8+x^7-x^4-x; $&px
```

Sekarang kita mencari titik, di mana polinomialnya adalah 2. Dalam `solve()`, nilai target default $y=0$ dapat diubah dengan variabel yang ditetapkan. Kami menggunakan $y=2$ dan memeriksa dengan mengevaluasi polinomial pada hasil sebelumnya.

```
>solve(px,1,y=2), px(%)
```

```
0.966715594851  
2
```

Memecahkan ekspresi simbolis dalam bentuk simbolis mengembalikan daftar solusi. Kami menggunakan pemecah simbolik `solve()` yang disediakan oleh Maxima.

```
>sol &= solve(x^2-x-1,x); $&sol
```

Cara termudah untuk mendapatkan nilai numerik adalah dengan mengevaluasi solusi secara numerik seperti ekspresi.

```
>longest sol()
```

```
-0.6180339887498949      1.618033988749895
```

Untuk menggunakan solusi secara simbolis dalam ekspresi lain, cara termudah adalah "dengan".

```
> $x^2 with sol[1], $expand(x^2-x-1 with sol[2])
```

Memecahkan sistem persamaan secara simbolis dapat dilakukan dengan vektor persamaan dan solver simbolis solve(). Jawabannya adalah daftar daftar persamaan.

```
> $solve([x+y=2,x^3+2*y+x=4],[x,y])
```

Fungsi f() dapat melihat variabel global. Namun seringkali kita ingin menggunakan parameter lokal.

lateks: $a^x - x^a = 0.1$

dengan $a=3$.

```
>function f(x,a) := x^a-a^x;
```

Salah satu cara untuk meneruskan parameter tambahan ke $f()$ adalah dengan menggunakan daftar dengan nama fungsi dan parameter (sebaliknya adalah parameter titik koma).

```
>solve({{"f",3}},2,y=0.1)
```

2.54116291558

Ini juga bekerja dengan ekspresi. Tapi kemudian, elemen daftar bernama harus digunakan. (Lebih lanjut tentang daftar di tutorial tentang sintaks EMT).

```
>solve({{"x^a-a^x",a=3}},2,y=0.1)
```

2.54116291558

Menyelesaikan Pertidaksamaan

Untuk menyelesaikan pertidaksamaan, EMT tidak akan dapat melakukannya, melainkan dengan bantuan Maxima, artinya secara eksak (simbolik). Perintah Maxima yang digunakan adalah `fourier_elim()`, yang harus dipanggil dengan perintah "`load(fourier_elim)`" terlebih dahulu.

```
>load(fourier_elim)
```

```
C:/Program Files/Euler x64/maxima/share/maxima/5.35.1/share/f\
ourier_elim/fourier_elim.lisp
```

```
>$$fourier_elim([x^2 - 1>0],[x]) // x^2-1 > 0
>$$fourier_elim([x^2 - 1<0],[x]) // x^2-1 < 0
>$$fourier_elim([x^2 - 1 # 0],[x]) // x^2-1 <> 0
>$$fourier_elim([x # 6],[x])
>$$fourier_elim([x < 1, x > 1],[x]) // tidak memiliki penyelesaian
>$$fourier_elim([minf < x, x < inf],[x]) // solusinya R
>$$fourier_elim([x^3 - 1 > 0],[x])
>$$fourier_elim([cos(x) < 1/2],[x]) // ??? gagal
>$$fourier_elim([y-x < 5, x - y < 7, 10 < y],[x,y]) // sistem pertidaksamaan
>$$fourier_elim([y-x < 5, x - y < 7, 10 < y],[y,x])
>$$fourier_elim((x + y < 5) and (x - y > 8),[x,y])
>$$fourier_elim(((x + y < 5) and x < 1) or (x - y > 8),[x,y])
>&fourier_elim([max(x,y) > 6, x # 8, abs(y-1) > 12],[x,y])
```

$[6 < x, x < 8, y < -11]$ or $[8 < x, y < -11]$
or $[x < 8, 13 < y]$ or $[x = y, 13 < y]$ or $[8 < x, x < y, 13 < y]$
or $[y < x, 13 < y]$

```
>$fourier_elim([(x+6)/(x-9) <= 6],[x])
```

Bahasa Matriks

Dokumentasi inti EMT berisi diskusi terperinci tentang bahasa matriks Euler.

Vektor dan matriks dimasukkan dengan tanda kurung siku, elemen dipisahkan dengan koma, baris dipisahkan dengan titik koma.

```
>A=[1,2;3,4]
```

1	2
3	4

Produk matriks dilambangkan dengan titik.

```
>b=[3;4]
```

3
4

```
>b' // transpose b
```

[3, 4]

```
>inv(A) //inverse A
```

$$\begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$$

```
>A.b //perkalian matriks
```

$$\begin{pmatrix} 11 \\ 25 \end{pmatrix}$$

```
>A.inv(A)
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Poin utama dari bahasa matriks adalah bahwa semua fungsi dan operator bekerja elemen untuk elemen.

```
>A.A
```

$$\begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$$


```
>A^2 //perpangkatan elemen2 A
```

1	4
9	16

```
>A.A.A
```

37	54
81	118

```
>power(A,3) //perpangkatan matriks
```

37	54
81	118

```
>A/A //pembagian elemen-elemen matriks yang seletak
```

1	1
1	1

```
>A/b //pembagian elemen2 A oleh elemen2 b kolom demi kolom (karena b vektor kolom)
```

```
0.333333    0.666667
0.75        1
```

```
>A\b // hasilkali invers A dan b,  $A^{-1}b$ 
```

```
-2
2.5
```

```
>inv(A).b
```

```
-2
2.5
```

```
>A\A //  $A^{-1}A$ 
```

```
1    0
0    1
```

```
>inv(A).A
```

1	0
0	1

```
>A*A //perkalin elemen-elemen matriks seletak
```

1	4
9	16

Ini bukan produk matriks, tetapi perkalian elemen demi elemen. Hal yang sama berlaku untuk vektor.

```
>b^2 // perpangkatan elemen-elemen matriks/vektor
```

9
16

Jika salah satu operan adalah vektor atau skalar, itu diperluas secara alami.

```
>2*A
```

2	4
6	8

Misalnya, jika operan adalah vektor kolom, elemennya diterapkan ke semua baris A.

```
> [1,2]*A
```

1	4
3	8

Jika itu adalah vektor baris, itu diterapkan ke semua kolom A.

```
> A*[2,3]
```

2	6
6	12

Seseorang dapat membayangkan perkalian ini seolah-olah vektor baris v telah digandakan untuk membentuk matriks dengan ukuran yang sama dengan A.

```
> dup([1,2],2) // dup: menduplikasi/menggandakan vektor [1,2] sebanyak 2 kali (baris)
```

1	2
1	2

```
>A*dup([1,2],2)
```

1	4
3	8

Ini juga berlaku untuk dua vektor di mana satu adalah vektor baris dan yang lainnya adalah vektor kolom. Kami menghitung $i*j$ untuk i,j dari 1 hingga 5. Caranya adalah dengan mengalikan 1:5 dengan transposnya. Bahasa matriks Euler secara otomatis menghasilkan tabel nilai.

```
>(1:5)*(1:5)' // hasilkali elemen-elemen vektor baris dan vektor kolom
```

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

Sekali lagi, ingat bahwa ini bukan produk matriks!

```
>(1:5).(1:5)' // hasilkali vektor baris dan vektor kolom
```

```
>sum((1:5)*(1:5)) // sama hasilnya
```

55

Bahkan operator seperti < atau == bekerja dengan cara yang sama.

```
>(1:10)<6 // menguji elemen-elemen yang kurang dari 6
```

[1, 1, 1, 1, 1, 0, 0, 0, 0, 0]

Misalnya, kita dapat menghitung jumlah elemen yang memenuhi kondisi tertentu dengan fungsi sum().

```
>sum((1:10)<6) // banyak elemen yang kurang dari 6
```

5

Euler memiliki operator perbandingan, seperti "==", yang memeriksa kesetaraan. Kami mendapatkan vektor 0 dan 1, di mana 1 berarti benar.

```
>t=(1:10)^2; t==25 //menguji elemen2 t yang sama dengan 25 (hanya ada 1)
```

```
[0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
```

Dari vektor seperti itu, "bukan nol" memilih elemen bukan nol. Dalam hal ini, kami mendapatkan indeks semua elemen lebih besar dari 50.

```
>nonzeros(t>50) //indeks elemen2 t yang lebih besar daripada 50
```

```
[8, 9, 10]
```

Tentu saja, kita dapat menggunakan vektor indeks ini untuk mendapatkan nilai yang sesuai dalam t.

```
>t[nonzeros(t>50)] //elemen2 t yang lebih besar daripada 50
```

```
[64, 81, 100]
```

Sebagai contoh, mari kita cari semua kuadrat dari angka 1 hingga 1000, yaitu 5 modulo 11 dan 3 modulo 13.

```
>t=1:1000; nonzeros(mod(t^2,11)==5 && mod(t^2,13)==3)
```

```
[4, 48, 95, 139, 147, 191, 238, 282, 290, 334, 381, 425,  
433, 477, 524, 568, 576, 620, 667, 711, 719, 763, 810, 854,  
862, 906, 953, 997]
```

EMT tidak sepenuhnya efektif untuk perhitungan bilangan bulat. Ini menggunakan titik mengambang presisi ganda secara internal. Namun, seringkali sangat berguna.

Kita dapat memeriksa keutamaan. Mari kita cari tahu, berapa banyak kuadrat ditambah 1 adalah bilangan prima.

```
>t=1:1000; length(nonzeros(isprime(t^2+1)))
```

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Fungsi bukan nol() hanya berfungsi untuk vektor. Untuk matriks, ada mnonzeros().

```
>seed(2); A=random(3,4)
```

0.765761	0.401188	0.406347	0.267829
0.13673	0.390567	0.495975	0.952814
0.548138	0.006085	0.444255	0.539246

Ini mengembalikan indeks elemen, yang bukan nol.

```
>k=mnonzeros(A<0.4) //indeks elemen2 A yang kurang dari 0,4
```

1	4
2	1
2	2
3	2

Indeks ini dapat digunakan untuk mengatur elemen ke beberapa nilai.

```
>mset(A,k,0) //mengganti elemen2 suatu matriks pada indeks tertentu
```

0.765761	0.401188	0.406347	0
0	0	0.495975	0.952814
0.548138	0	0.444255	0.539246

Fungsi mset() juga dapat mengatur elemen pada indeks ke entri dari beberapa matriks lainnya.

```
>mset(A,k,-random(size(A)))
```

0.765761	0.401188	0.406347	-0.126917
-0.122404	-0.691673	0.495975	0.952814
0.548138	-0.483902	0.444255	0.539246

Dan dimungkinkan untuk mendapatkan elemen dalam vektor.

```
>mget(A,k)
```

```
[0.267829, 0.13673, 0.390567, 0.006085]
```

Fungsi lain yang berguna adalah `ekstrem`, yang mengembalikan nilai minimal dan maksimal di setiap baris matriks dan posisinya.

```
>ex=extrema(A)
```

0.267829	4	0.765761	1
0.13673	1	0.952814	4
0.006085	2	0.548138	1

Kita dapat menggunakan ini untuk mengekstrak nilai maksimal di setiap baris.

```
>ex[,3]
```

```
[0.765761, 0.952814, 0.548138]
```

Ini, tentu saja, sama dengan fungsi `max()`.

```
>max(A)'
```

```
[0.765761,  0.952814,  0.548138]
```

Tetapi dengan `mget()`, kita dapat mengekstrak indeks dan menggunakan informasi ini untuk mengekstrak elemen pada posisi yang sama dari matriks lain.

```
>j=(1:rows(A))'|ex[,4], mget(-A,j)
```

```
      1      1  
      2      4  
      3      1  
[-0.765761, -0.952814, -0.548138]
```

Fungsi Matriks Lainnya (Membangun Matriks)

Untuk membangun matriks, kita dapat menumpuk satu matriks di atas yang lain. Jika keduanya tidak memiliki jumlah kolom yang sama, kolom yang lebih pendek akan diisi dengan 0.

```
>v=1:3; v_v
```

1	2	3
1	2	3

Demikian juga, kita dapat melampirkan matriks ke yang lain secara berdampingan, jika keduanya memiliki jumlah baris yang sama.

```
>A=random(3,4); A|v'
```

0.032444	0.0534171	0.595713	0.564454	1
0.83916	0.175552	0.396988	0.83514	2
0.0257573	0.658585	0.629832	0.770895	3

Jika mereka tidak memiliki jumlah baris yang sama, matriks yang lebih pendek diisi dengan 0.

Ada pengecualian untuk aturan ini. Bilangan real yang dilampirkan pada matriks akan digunakan sebagai kolom yang diisi dengan bilangan real tersebut.

```
>A|1
```

0.032444	0.0534171	0.595713	0.564454	1
0.83916	0.175552	0.396988	0.83514	1
0.0257573	0.658585	0.629832	0.770895	1

Dimungkinkan untuk membuat matriks vektor baris dan kolom.

```
>[v;v]
```

1	2	3
1	2	3

```
>[v',v']
```

1	1
2	2
3	3

Tujuan utama dari ini adalah untuk menafsirkan vektor ekspresi untuk vektor kolom.

```
>"[x,x^2]"(v')
```

1	1
2	4
3	9

Untuk mendapatkan ukuran A, kita dapat menggunakan fungsi berikut.

```
>C=zeros(2,4); rows(C), cols(C), size(C), length(C)
```

```
2
4
[2, 4]
4
```

Untuk vektor, ada panjang().

```
>length(2:10)
```

Ada banyak fungsi lain, yang menghasilkan matriks.

```
>ones(2,2)
```

```
      1      1
      1      1
```

Ini juga dapat digunakan dengan satu parameter. Untuk mendapatkan vektor dengan angka selain 1, gunakan yang berikut ini.

```
>ones(5)*6
```

```
[6, 6, 6, 6, 6]
```

Juga matriks bilangan acak dapat dihasilkan dengan acak (distribusi seragam) atau normal (distribusi Gau).

```
>random(2,2)
```

```
0.66566      0.831835
0.977        0.544258
```

Berikut adalah fungsi lain yang berguna, yang merestrukturisasi elemen matriks menjadi matriks lain.

```
>redim(1:9,3,3) // menyusun elemen2 1, 2, 3, ..., 9 ke bentuk matriks 3x3
```

1	2	3
4	5	6
7	8	9

Dengan fungsi berikut, kita dapat menggunakan ini dan fungsi dup untuk menulis fungsi rep(), yang mengulang vektor n kali.

```
>function rep(v,n) := redim(dup(v,n),1,n*cols(v))
```

Mari kita uji.

```
>rep(1:3,5)
```

```
[1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3]
```

Fungsi multdup() menduplikasi elemen vektor.


```
>multdup(1:3,5), multdup(1:3,[2,3,2])
```

```
[1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3]  
[1, 1, 2, 2, 2, 3, 3]
```

Fungsi `flipx()` dan `flipy()` mengembalikan urutan baris atau kolom matriks. Yaitu, fungsi `flipx()` membalik secara horizontal.

```
>flipx(1:5) //membalik elemen2 vektor baris
```

```
[5, 4, 3, 2, 1]
```

Untuk rotasi, Euler memiliki `rotleft()` dan `rotright()`.

```
>rotleft(1:5) // memutar elemen2 vektor baris
```

```
[2, 3, 4, 5, 1]
```

Sebuah fungsi khusus adalah `drop(v,i)`, yang menghilangkan elemen dengan indeks di `i` dari vektor `v`.

```
>drop(10:20,3)
```

```
[10, 11, 13, 14, 15, 16, 17, 18, 19, 20]
```

Perhatikan bahwa vektor `i` di `drop(v,i)` mengacu pada indeks elemen di `v`, bukan nilai elemen. Jika Anda ingin menghapus elemen, Anda harus menemukan elemennya terlebih dahulu. Fungsi `indexof(v,x)` dapat digunakan untuk mencari elemen `x` dalam vektor terurut `v`.

```
>v=primes(50), i=indexof(v,10:20), drop(v,i)
```

```
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]  
[0, 5, 0, 6, 0, 0, 0, 7, 0, 8, 0]  
[2, 3, 5, 7, 23, 29, 31, 37, 41, 43, 47]
```

Seperti yang Anda lihat, tidak ada salahnya untuk memasukkan indeks di luar rentang (seperti 0), indeks ganda, atau indeks yang tidak diurutkan.

```
>drop(1:10,shuffle([0,0,5,5,7,12,12]))
```

```
[1, 2, 3, 4, 6, 8, 9, 10]
```

Ada beberapa fungsi khusus untuk mengatur diagonal atau untuk menghasilkan matriks diagonal. Kita mulai dengan matriks identitas.

```
>A=id(5) // matriks identitas 5x5
```

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Kemudian kita atur diagonal bawah (-1) menjadi 1:4.

```
>setdiag(A,-1,1:4) //mengganti diagonal di bawah diagonal utama
```

1	0	0	0	0
1	1	0	0	0
0	2	1	0	0
0	0	3	1	0
0	0	0	4	1

Perhatikan bahwa kami tidak mengubah matriks A. Kami mendapatkan matriks baru sebagai hasil dari `setdiag()`.

Berikut adalah fungsi, yang mengembalikan matriks tri-diagonal.

```
>function tridiag (n,a,b,c) := setdiag(setdiag(b*id(n),1,c),-1,a); ...  
>tridiag(5,1,2,3)
```

2	3	0	0	0
1	2	3	0	0
0	1	2	3	0
0	0	1	2	3
0	0	0	1	2

Diagonal suatu matriks juga dapat diekstraksi dari matriks tersebut. Untuk mendemonstrasikan ini, kami merestrukturisasi vektor 1:9 menjadi matriks 3x3.

```
>A=redim(1:9,3,3)
```

1	2	3
4	5	6
7	8	9

Sekarang kita dapat mengekstrak diagonal.

```
>d=getdiag(A,0)
```

```
[1, 5, 9]
```

Misalnya. Kita dapat membagi matriks dengan diagonalnya. Bahasa matriks memperhatikan bahwa vektor kolom d diterapkan ke matriks baris demi baris.

```
>fraction A/d'
```

1	2	3
4/5	1	6/5
7/9	8/9	1

Hampir semua fungsi di Euler juga berfungsi untuk input matriks dan vektor, kapan pun ini masuk akal. Misalnya, fungsi `sqrt()` menghitung akar kuadrat dari semua elemen vektor atau matriks.

```
>sqrt(1:3)
```

```
[1,  1.41421,  1.73205]
```

Jadi Anda dapat dengan mudah membuat tabel nilai. Ini adalah salah satu cara untuk memplot suatu fungsi (alternatifnya menggunakan ekspresi).

```
>x=1:0.01:5; y=log(x)/x^2; // terlalu panjang untuk ditampilkan
```

Dengan ini dan operator titik dua a:delta:b, vektor nilai fungsi dapat dihasilkan dengan mudah.

Pada contoh berikut, kita membangkitkan vektor nilai $t[i]$ dengan spasi 0,1 dari -1 hingga 1. Kemudian kita membangkitkan vektor nilai fungsi

lateks: $s = t^3 - t$

```
>t=-1:0.1:1; s=t^3-t
```

```
[0, 0.171, 0.288, 0.357, 0.384, 0.375, 0.336, 0.273, 0.192,  
0.099, 0, -0.099, -0.192, -0.273, -0.336, -0.375, -0.384,  
-0.357, -0.288, -0.171, 0]
```

EMT memperluas operator untuk skalar, vektor, dan matriks dengan cara yang jelas.

Misalnya, vektor kolom dikalikan vektor baris menjadi matriks, jika operator diterapkan. Berikut ini, v' adalah vektor yang ditransposisikan (vektor kolom).

```
>shortest (1:5)*(1:5)'
```

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

Perhatikan, bahwa ini sangat berbeda dari produk matriks. Produk matriks dilambangkan dengan titik "." di EMT.

```
>(1:5).(1:5)'
```

55

Secara default, vektor baris dicetak dalam format yang ringkas.

```
>[1,2,3,4]
```

[1, 2, 3, 4]

Untuk matriks operator khusus . menunjukkan perkalian matriks, dan A' menunjukkan transpos. Matriks 1x1 dapat digunakan seperti bilangan real.

```
>v:=[1,2]; v.v', %^2
```

5
25

Untuk mentranspos matriks kita menggunakan apostrof.

```
>v=1:4; v'
```

```
1  
2  
3  
4
```

Jadi kita dapat menghitung matriks A kali vektor b.

```
>A=[1,2,3,4;5,6,7,8]; A.v'
```

```
30  
70
```

Perhatikan bahwa v masih merupakan vektor baris. Jadi $v'.v$ berbeda dari $v.v'$.

```
>v'.v
```

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

$v \cdot v'$ menghitung norma v kuadrat untuk vektor baris v . Hasilnya adalah vektor 1×1 , yang bekerja seperti bilangan real.

```
>v.v'
```

30

Ada juga fungsi norma (bersama dengan banyak fungsi lain dari Aljabar Linier).

```
>norm(v)^2
```

30

Operator dan fungsi mematuhi bahasa matriks Euler.

Berikut ringkasan aturannya.

- Fungsi yang diterapkan ke vektor atau matriks diterapkan ke setiap elemen.
- Operator yang beroperasi pada dua matriks dengan ukuran yang sama diterapkan berpasangan ke elemen matriks.
- Jika kedua matriks memiliki dimensi yang berbeda, keduanya diperluas dengan cara yang masuk akal, sehingga memiliki ukuran yang sama.

Misalnya, nilai skalar kali vektor mengalikan nilai dengan setiap elemen vektor. Atau matriks kali vektor (dengan $*$, bukan $.$) memperluas vektor ke ukuran matriks dengan menduplikasinya.

Berikut ini adalah kasus sederhana dengan operator \wedge .

```
>[1,2,3]^2
```

```
[1, 4, 9]
```

Berikut adalah kasus yang lebih rumit. Vektor baris dikalikan dengan vektor kolom mengembang keduanya dengan menduplikasi.

```
>v:=[1,2,3]; v*v'
```

1	2	3
2	4	6
3	6	9

Perhatikan bahwa produk skalar menggunakan produk matriks, bukan *!

```
>v.v'
```

14

Ada banyak fungsi matriks. Kami memberikan daftar singkat. Anda harus berkonsultasi dengan dokumentasi untuk informasi lebih lanjut tentang perintah ini.

sum,prod menghitung jumlah dan produk dari baris
cumsum,cumprod melakukan hal yang sama secara kumulatif
menghitung nilai ekstrem dari setiap baris
extrema mengembalikan vektor dengan informasi ekstrim
diag(A,i) mengembalikan diagonal ke-i
setdiag(A,i,v) mengatur diagonal ke-i
id(n) matriks identitas
det(A) penentu
charpoly(A) polinomial karakteristik
nilai eigen(A) nilai eigen

```
>v*v, sum(v*v), cumsum(v*v)
```

```
[1, 4, 9]  
14  
[1, 5, 14]
```

Operator : menghasilkan vektor baris spasi yang sama, opsional dengan ukuran langkah.

```
>1:4, 1:2:10
```

```
[1, 2, 3, 4]  
[1, 3, 5, 7, 9]
```

Untuk menggabungkan matriks dan vektor ada operator "|" dan "_".

```
>[1,2,3] | [4,5], [1,2,3]_1
```

[1,	2,	3,	4,	5]			
		1			2		3
		1			1		1

Unsur-unsur matriks disebut dengan "A[i,j]".

```
>A:=[1,2,3;4,5,6;7,8,9]; A[2,3]
```

6

Untuk vektor baris atau kolom, v[i] adalah elemen ke-i dari vektor. Untuk matriks, ini mengembalikan baris ke-i lengkap dari matriks.

```
>v:=[2,4,6,8]; v[3], A[3]
```

6
[7, 8, 9]

Indeks juga bisa menjadi vektor baris dari indeks. : menunjukkan semua indeks.

```
>v[1:2], A[:,2]
```

[2, 4]

2
5
8

Bentuk singkat untuk : adalah menghilangkan indeks sepenuhnya.

```
>A[,2:3]
```

2	3
5	6
8	9

Untuk tujuan vektorisasi, elemen matriks dapat diakses seolah-olah mereka adalah vektor.

```
>A{4}
```

Matriks juga dapat diratakan, menggunakan fungsi `redim()`. Ini diimplementasikan dalam fungsi `flatten()`.

```
>redim(A,1,prod(size(A))), flatten(A)
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

Untuk menggunakan matriks untuk tabel, mari kita reset ke format default, dan menghitung tabel nilai sinus dan kosinus. Perhatikan bahwa sudut dalam radian secara default.

```
>defformat; w=0°:45°:360°; w=w'; deg(w)
```

```
0
45
90
135
180
225
270
315
360
```

Sekarang kita menambahkan kolom ke matriks.

```
>M = deg(w)|w|cos(w)|sin(w)
```

0	0	1	0
45	0.785398	0.707107	0.707107
90	1.5708	0	1
135	2.35619	-0.707107	0.707107
180	3.14159	-1	0
225	3.92699	-0.707107	-0.707107
270	4.71239	0	-1
315	5.49779	0.707107	-0.707107
360	6.28319	1	0

Dengan menggunakan bahasa matriks, kita dapat menghasilkan beberapa tabel dari beberapa fungsi sekaligus.

Dalam contoh berikut, kita menghitung $t[j]^i$ untuk i dari 1 hingga n . Kami mendapatkan matriks, di mana setiap baris adalah tabel t^i untuk satu i . Yaitu, matriks memiliki elemen latex: $a_{i,j} = t_j^i$, $\quad 1 \leq j \leq 101, \quad 1 \leq i \leq n$

Fungsi yang tidak berfungsi untuk input vektor harus "divektorkan". Ini dapat dicapai dengan kata kunci "peta" dalam definisi fungsi. Kemudian fungsi tersebut akan dievaluasi untuk setiap elemen dari parameter vektor.

Integrasi numerik terintegrasi() hanya berfungsi untuk batas interval skalar. Jadi kita perlu membuat vektor.

```
>function map f(x) := integrate("x^x",1,x)
```


Kata kunci "peta" membuat vektor fungsi. Fungsinya sekarang akan bekerja untuk vektor bilangan.

```
>f([1:5])
```

```
[0, 2.05045, 13.7251, 113.336, 1241.03]
```

Sub-Matriks dan Matriks-Elemen

Untuk mengakses elemen matriks, gunakan notasi braket.

```
>A=[1,2,3;4,5,6;7,8,9], A[2,2]
```

	1	2	3
	4	5	6
5	7	8	9

Kita dapat mengakses satu baris matriks yang lengkap.

```
>A[2]
```

```
[4, 5, 6]
```

Dalam kasus vektor baris atau kolom, ini mengembalikan elemen vektor.

```
>v=1:3; v[2]
```

Untuk memastikan, Anda mendapatkan baris pertama untuk matriks $1 \times n$ dan $m \times n$, tentukan semua kolom menggunakan indeks kedua kosong.

```
>A[2,]
```

```
[4, 5, 6]
```

Jika indeks adalah vektor indeks, Euler akan mengembalikan baris matriks yang sesuai. Di sini kita ingin baris pertama dan kedua dari A.

```
>A[[1,2]]
```

1	2	3
4	5	6

Kita bahkan dapat menyusun ulang A menggunakan vektor indeks. Tepatnya, kami tidak mengubah A di sini, tetapi menghitung versi A yang disusun ulang.

```
>A[[3,2,1]]
```

7	8	9
4	5	6
1	2	3

Trik indeks bekerja dengan kolom juga.

Contoh ini memilih semua baris A dan kolom kedua dan ketiga.

```
>A[1:3,2:3]
```

2	3
5	6
8	9

Untuk singkatan ":" menunjukkan semua indeks baris atau kolom.

```
>A[:,3]
```

3
6
9

Atau, biarkan indeks pertama kosong.

```
>A[,2:3]
```

2	3
5	6
8	9

Kita juga bisa mendapatkan baris terakhir dari A.

```
>A[-1]
```

```
[7, 8, 9]
```

Sekarang mari kita ubah elemen A dengan menetapkan submatriks A ke beberapa nilai. Ini sebenarnya mengubah matriks A yang disimpan.

```
>A[1,1]=4
```

4	2	3
4	5	6
7	8	9

Kami juga dapat menetapkan nilai ke baris A.

```
>A[1]=[-1,-1,-1]
```

-1	-1	-1
4	5	6
7	8	9

Kami bahkan dapat menetapkan sub-matriks jika memiliki ukuran yang tepat.

```
>A[1:2,1:2]=[5,6;7,8]
```

5	6	-1
7	8	6
7	8	9

Selain itu, beberapa jalan pintas diperbolehkan.

```
>A[1:2,1:2]=0
```

0	0	-1
0	0	6
7	8	9

Peringatan: Indeks di luar batas mengembalikan matriks kosong, atau pesan kesalahan, tergantung pada pengaturan sistem. Standarnya adalah pesan kesalahan. Ingat, bagaimanapun, bahwa indeks negatif dapat digunakan untuk mengakses elemen matriks yang dihitung dari akhir.

```
>A[4]
```

```
Row index 4 out of bounds!  
Error in:  
A[4] ...  
^
```

Menyortir dan Mengacak

Fungsi `sort()` mengurutkan vektor baris.

```
>sort([5,6,4,8,1,9])
```

```
[1, 4, 5, 6, 8, 9]
```

Seringkali perlu untuk mengetahui indeks dari vektor yang diurutkan dalam vektor aslinya. Ini dapat digunakan untuk menyusun ulang vektor lain dengan cara yang sama.

Mari kita mengocok vektor.

```
>v=shuffle(1:10)
```

```
[4, 5, 10, 6, 8, 9, 1, 7, 2, 3]
```

Indeks berisi urutan yang tepat dari `v`.

```
>{vs,ind}=sort(v); v[ind]
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Ini bekerja untuk vektor string juga.

```
>s=["a","d","e","a","aa","e"]
```

```
a  
d  
e  
a  
aa  
e
```

```
>{ss,ind}=sort(s); ss
```

```
a  
a  
aa  
d  
e  
e
```

Seperti yang Anda lihat, posisi entri ganda agak acak.

```
>ind
```

```
[4, 1, 5, 2, 6, 3]
```


Fungsi unik mengembalikan daftar elemen unik vektor yang diurutkan.

```
>intrandom(1,10,10), unique(%)
```

```
[4, 4, 9, 2, 6, 5, 10, 6, 5, 1]  
[1, 2, 4, 5, 6, 9, 10]
```

Ini bekerja untuk vektor string juga.

```
>unique(s)
```

```
a  
aa  
d  
e
```

EMT memiliki banyak fungsi untuk menyelesaikan sistem linier, sistem sparse, atau masalah regresi.

Untuk sistem linier $Ax=b$, Anda dapat menggunakan algoritma Gauss, matriks invers atau kecocokan linier. Operator $A \backslash b$ menggunakan versi algoritma Gauss.

```
>A=[1,2;3,4]; b=[5;6]; A\b
```

```
    -4  
    4.5
```

Untuk contoh lain, kami membuat matriks 200x200 dan jumlah barisnya. Kemudian kita selesaikan $Ax=b$ menggunakan matriks invers. Kami mengukur kesalahan sebagai deviasi maksimal semua elemen dari 1, yang tentu saja merupakan solusi yang benar.

```
>A=normal(200,200); b=sum(A); longest totalmax(abs(inv(A).b-1))
```

```
8.790745908981989e-13
```

Jika sistem tidak memiliki solusi, kecocokan linier meminimalkan norma kesalahan $Ax=b$.

```
>A=[1,2,3;4,5,6;7,8,9]
```

1	2	3
4	5	6
7	8	9

Determinan matriks ini adalah 0.

```
>det(A)
```

0

Matriks Simbolik

Maxima memiliki matriks simbolis. Tentu saja, Maxima dapat digunakan untuk masalah aljabar linier sederhana seperti itu. Kita dapat mendefinisikan matriks untuk Euler dan Maxima dengan `&:=`, dan kemudian menggunakannya dalam ekspresi simbolis. Bentuk [...] biasa untuk mendefinisikan matriks dapat digunakan di Euler untuk mendefinisikan matriks simbolik.

```
>A &= [a,1,1;1,a,1;1,1,a]; $A
>$&det(A), $&factor(%)
>$&invert(A) with a=0
>A &= [1,a;b,2]; $A
```

Seperti semua variabel simbolik, matriks ini dapat digunakan dalam ekspresi simbolik lainnya.

```
>$&det(A-x*ident(2)), $&solve(%,x)
```

Nilai eigen juga dapat dihitung secara otomatis. Hasilnya adalah vektor dengan dua vektor nilai eigen dan multiplisitas.

```
>$&eigenvalues([a,1;1,a])
```

Untuk mengekstrak vektor eigen tertentu perlu pengindeksan yang cermat.

```
>$eigenvectors([a,1;1,a]), &%[2] [1] [1]
```

$[1, -1]$

Matriks simbolik dapat dievaluasi dalam Euler secara numerik seperti ekspresi simbolik lainnya.

```
>A(a=4,b=5)
```

1	4
5	2

Dalam ekspresi simbolik, gunakan dengan.

```
>$&A with [a=4,b=5]
```

Akses ke baris matriks simbolik bekerja seperti halnya dengan matriks numerik.

```
>$&A[1]
```

Ekspresi simbolis dapat berisi tugas. Dan itu mengubah matriks A.

```
>&A[1,1]:=t+1; $&A
```

Ada fungsi simbolik di Maxima untuk membuat vektor dan matriks. Untuk ini, lihat dokumentasi Maxima atau tutorial tentang Maxima di EMT.

```
>v &= makelist(1/(i+j),i,1,3); $v
```

```
>B &:= [1,2;3,4]; $B, $&invert(B)
```

Hasilnya dapat dievaluasi secara numerik dalam Euler. Untuk informasi lebih lanjut tentang Maxima, lihat pengantar Maxima.

```
>$\text{invert}(B)()
```

$$\begin{array}{cc} -2 & 1 \\ 1.5 & -0.5 \end{array}$$

Euler juga memiliki fungsi `xinv()` yang kuat, yang membuat upaya lebih besar dan mendapatkan hasil yang lebih tepat.

Perhatikan, bahwa dengan `&:=` matriks B telah didefinisikan sebagai simbolik dalam ekspresi simbolik dan sebagai numerik dalam ekspresi numerik. Jadi kita bisa menggunakannya di sini.

```
>longest B.xinv(B)
```

$$\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

Misalnya. nilai eigen dari A dapat dihitung secara numerik.

```
>A=[1,2,3;4,5,6;7,8,9]; real(eigenvalues(A))
```

```
[16.1168, -1.11684, 0]
```

Atau secara simbolis. Lihat tutorial tentang Maxima untuk detailnya.

```
>$eigenvalues(@A)
```


Nilai Numerik dalam Ekspresi simbolis

Ekspresi simbolis hanyalah string yang berisi ekspresi. Jika kita ingin mendefinisikan nilai baik untuk ekspresi simbolik maupun ekspresi numerik, kita harus menggunakan "&:=".

```
>A &:= [1,pi;4,5]
```

$$\begin{array}{cc} 1 & 3.14159 \\ 4 & 5 \end{array}$$

Masih ada perbedaan antara bentuk numerik dan simbolik. Saat mentransfer matriks ke bentuk simbolis, pendekatan fraksional untuk real akan digunakan.

```
>$&A
```

Untuk menghindarinya, ada fungsi "mxmset(variable)".

```
>mxmset(A); $&A
```

Maxima juga dapat menghitung dengan angka floating point, dan bahkan dengan angka floating besar dengan 32 digit. Namun, evaluasinya jauh lebih lambat.

```
>$&bfloat(sqrt(2)), $&float(sqrt(2))
```

Ketepatan angka floating point besar dapat diubah.

```
>&fpprec:=100; &bfloat(pi)
```

```
3.14159265358979323846264338327950288419716939937510582097494\  
4592307816406286208998628034825342117068b0
```

Variabel numerik dapat digunakan dalam ekspresi simbolis apa pun menggunakan "@var".

Perhatikan bahwa ini hanya diperlukan, jika variabel telah didefinisikan dengan "==" atau "=" sebagai variabel numerik.

```
>B:=[1,pi;3,4]; $&det(@B)
```

Demo - Suku Bunga

Di bawah ini, kami menggunakan Euler Math Toolbox (EMT) untuk perhitungan suku bunga. Kami melakukannya secara numerik dan simbolis untuk menunjukkan kepada Anda bagaimana Euler dapat digunakan untuk memecahkan masalah kehidupan nyata.

Asumsikan Anda memiliki modal awal 5000 (katakanlah dalam dolar).

```
>K=5000
```

```
5000
```

Sekarang kita asumsikan tingkat bunga 3% per tahun. Mari kita tambahkan satu tarif sederhana dan hitung hasilnya.

```
>K*1.03
```

```
5150
```

Euler akan memahami sintaks berikut juga.

```
>K+K*3%
```

```
5150
```

Tetapi lebih mudah menggunakan faktornya

```
>q=1+3%, K*q
```

```
1.03  
5150
```

Selama 10 tahun, kita cukup mengalikan faktornya dan mendapatkan nilai akhir dengan suku bunga majemuk.

```
>K*q^10
```

```
6719.58189672
```

Untuk tujuan kita, kita dapat mengatur format menjadi 2 digit setelah titik desimal.

```
>format(12,2); K*q^10
```

```
6719.58
```

Mari kita cetak yang dibulatkan menjadi 2 digit dalam kalimat lengkap.

```
>"Starting from " + K + "$ you get " + round(K*q^10,2) + "$."
```

Starting from 5000\$ you get 6719.58\$.

Bagaimana jika kita ingin mengetahui hasil antara dari tahun 1 sampai tahun 9? Untuk ini, bahasa matriks Euler sangat membantu. Anda tidak harus menulis loop, tetapi cukup masukkan

```
>K*q^(0:10)
```

Real 1 x 11 matrix

5000.00	5150.00	5304.50	5463.64	...
---------	---------	---------	---------	-----

Bagaimana keajaiban ini bekerja? Pertama ekspresi 0:10 mengembalikan vektor bilangan bulat.

```
>short 0:10
```

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Kemudian semua operator dan fungsi dalam Euler dapat diterapkan pada elemen vektor untuk elemen. Jadi

```
>short q^(0:10)
```

```
[1, 1.03, 1.0609, 1.0927, 1.1255, 1.1593, 1.1941, 1.2299,  
1.2668, 1.3048, 1.3439]
```

adalah vektor faktor q^0 sampai q^{10} . Ini dikalikan dengan K , dan kami mendapatkan vektor nilai.

```
>VK=K*q^(0:10);
```

Tentu saja, cara realistis untuk menghitung suku bunga ini adalah dengan membulatkan ke sen terdekat setelah setiap tahun. Mari kita tambahkan fungsi untuk ini.

```
>function oneyear (K) := round(K*q,2)
```

Mari kita bandingkan dua hasil, dengan dan tanpa pembulatan.

```
>longest oneyear(1234.57), longest 1234.57*q
```

```
1271.61  
1271.6071
```

Sekarang tidak ada rumus sederhana untuk tahun ke-n, dan kita harus mengulang selama bertahun-tahun. Euler memberikan banyak solusi untuk ini.

Cara termudah adalah iterasi fungsi, yang mengulangi fungsi tertentu beberapa kali.

```
>VKr=iterate("oneyear",5000,10)
```

```
Real 1 x 11 matrix
```

```
5000.00    5150.00    5304.50    5463.64    ...
```

Kami dapat mencetaknya dengan cara yang ramah, menggunakan format kami dengan tempat desimal tetap.

```
>VKr'
```

```
5000.00  
5150.00  
5304.50  
5463.64  
5627.55  
5796.38  
5970.27  
6149.38  
6333.86  
6523.88  
6719.60
```

Untuk mendapatkan elemen tertentu dari vektor, kami menggunakan indeks dalam tanda kurung siku.

```
>VKr[2], VKr[1:3]
```

```
5150.00  
5000.00    5150.00    5304.50
```

Anehnya, kita juga bisa menggunakan vektor indeks. Ingat bahwa 1:3 menghasilkan vektor [1,2,3]. Mari kita bandingkan elemen terakhir dari nilai yang dibulatkan dengan nilai penuh.

```
>VKr[-1], VK[-1]
```

```
6719.60  
6719.58
```

Perbedaannya sangat kecil.

Memecahkan Persamaan

Sekarang kita mengambil fungsi yang lebih maju, yang menambahkan tingkat uang tertentu setiap tahun.

```
>function onepay (K) := K*q+R
```

Kita tidak perlu menentukan q atau R untuk definisi fungsi. Hanya jika kita menjalankan perintah, kita harus mendefinisikan nilai-nilai ini. Kami memilih $R=200$.

```
>R=200; iterate("onepay",5000,10)
```

Real 1 x 11 matrix

5000.00	5350.00	5710.50	6081.82	...
---------	---------	---------	---------	-----

Bagaimana jika kita menghapus jumlah yang sama setiap tahun?

```
>R=-200; iterate("onepay",5000,10)
```

Real 1 x 11 matrix

5000.00	4950.00	4898.50	4845.45	...
---------	---------	---------	---------	-----

Kami melihat bahwa uang berkurang. Jelas, jika kita hanya mendapatkan 150 bunga di tahun pertama, tetapi menghapus 200, kita kehilangan uang setiap tahun.

Bagaimana kita bisa menentukan berapa tahun uang itu akan bertahan? Kita harus menulis loop untuk ini. Cara termudah adalah dengan iterasi cukup lama.

```
>VKR=iterate("oneway",5000,50)
```

Real 1 x 51 matrix

5000.00	4950.00	4898.50	4845.45	...
---------	---------	---------	---------	-----

Dengan menggunakan bahasa matriks, kita dapat menentukan nilai negatif pertama dengan cara berikut.

```
>min(nonzeros(VKR<0))
```

48.00

Alasan untuk ini adalah bahwa bukan nol(VKR<0) mengembalikan vektor indeks i, di mana VKR[i]<0, dan min menghitung indeks minimal.

Karena vektor selalu dimulai dengan indeks 1, jawabannya adalah 47 tahun.

Fungsi iterate() memiliki satu trik lagi. Itu bisa mengambil kondisi akhir sebagai argumen. Kemudian akan mengembalikan nilai dan jumlah iterasi.

```
>{x,n}=iterate("onipay",5000,till="x<0"); x, n,
```

-19.83

47.00

Mari kita coba menjawab pertanyaan yang lebih ambigu. Asumsikan kita tahu bahwa nilainya adalah 0 setelah 50 tahun. Apa yang akan menjadi tingkat bunga?

Ini adalah pertanyaan yang hanya bisa dijawab dengan angka. Di bawah ini, kita akan mendapatkan formula yang diperlukan. Kemudian Anda akan melihat bahwa tidak ada formula yang mudah untuk tingkat bunga. Tapi untuk saat ini, kami bertujuan untuk solusi numerik.

Langkah pertama adalah mendefinisikan fungsi yang melakukan iterasi sebanyak n kali. Kami menambahkan semua parameter ke fungsi ini.

```
>function f(K,R,P,n) := iterate("x*(1+P/100)+R",K,n;P,R)[-1]
```

Iterasinya sama seperti di atas

$$x_{n+1} = x_n \cdot \left(1 + \frac{P}{100}\right) + R$$

Tapi kami tidak lagi menggunakan nilai global R dalam ekspresi kami. Fungsi seperti `iterate()` memiliki trik khusus di Euler. Anda dapat meneruskan nilai variabel dalam ekspresi sebagai parameter titik koma. Dalam hal ini P dan R.

Selain itu, kami hanya tertarik pada nilai terakhir. Jadi kita ambil indeks `[-1]`.

Mari kita coba tes.

```
>f(5000,-200,3,47)
```

-19.83

Sekarang kita bisa menyelesaikan masalah kita.

```
>solve("f(5000,-200,x,50)",3)
```

3.15

Rutin memecahkan memecahkan ekspresi=0 untuk variabel x. Jawabannya adalah 3,15% per tahun. Kami mengambil nilai awal 3% untuk algoritma. Fungsi `solve()` selalu membutuhkan nilai awal.

Kita dapat menggunakan fungsi yang sama untuk menyelesaikan pertanyaan berikut: Berapa banyak yang dapat kita keluarkan per tahun sehingga modal awal habis setelah 20 tahun dengan asumsi tingkat bunga 3% per tahun.

```
>solve("f(5000,x,3,20)",-200)
```

-336.08

Perhatikan bahwa Anda tidak dapat memecahkan jumlah tahun, karena fungsi kami mengasumsikan n sebagai nilai integer.

Solusi Simbolik untuk Masalah Suku Bunga

Kita dapat menggunakan bagian simbolik dari Euler untuk mempelajari masalah tersebut. Pertama kita mendefinisikan fungsi `onepay()` kita secara simbolis.

```
>function op(K) &= K*q+R; $&op(K)
```

Kita sekarang dapat mengulangi ini.

```
>$&op(op(op(op(K)))), $&expand(%)
```

Kami melihat sebuah pola. Setelah n periode yang kita miliki

$$K_n = q^n K + R(1 + q + \dots + q^{n-1}) = q^n K + \frac{q^n - 1}{q - 1} R$$

Rumusnya adalah rumus untuk jumlah geometri, yang diketahui Maxima.

```
>sum(q^k,k,0,n-1); %% = ev(%,simpsum)
```

Ini agak rumit. Jumlahnya dievaluasi dengan bendera "simpsum" untuk mengurangnya menjadi hasil bagi.

Mari kita membuat fungsi untuk ini.

```
>function fs(K,R,P,n) &= (1+P/100)^n*K + ((1+P/100)^n-1)/(P/100)*R; %%fs(K,R,P,n)
```

Fungsi tersebut melakukan hal yang sama seperti fungsi f kita sebelumnya. Tapi itu lebih efektif.

```
>longest f(5000,-200,3,47), longest fs(5000,-200,3,47)
```

```
-19.82504734650985
```

```
-19.82504734652684
```

Kita sekarang dapat menggunakannya untuk menanyakan waktu n . Kapan modal kita habis? Dugaan awal kami adalah 30 tahun.

```
>solve("fs(5000,-330,3,x)",30)
```

20.51

Jawaban ini mengatakan bahwa itu akan menjadi negatif setelah 21 tahun.

Kita juga dapat menggunakan sisi simbolis Euler untuk menghitung formula pembayaran.

Asumsikan kita mendapatkan pinjaman sebesar K , dan membayar n pembayaran sebesar R (dimulai setelah tahun pertama) meninggalkan sisa hutang sebesar K_n (pada saat pembayaran terakhir). Rumus untuk ini jelas

```
>equ &= fs(K,R,P,n)=Kn; $&equ
```

Biasanya rumus ini diberikan dalam bentuk

$$i = \frac{P}{100}$$

```
>equ &= (equ with P=100*i); $&equ
```

Kita dapat memecahkan tingkat R secara simbolis.

```
> solve(equ,R)
```

Seperti yang Anda lihat dari rumus, fungsi ini mengembalikan kesalahan titik mengambang untuk $i=0$. Euler tetap merencanakannya.

Tentu saja, kami memiliki batas berikut.

```
> limit(R(5000,0,x,10),x,0)
```

Jelas, tanpa bunga kita harus membayar kembali 10 tarif 500.

Persamaan juga dapat diselesaikan untuk n . Kelihatannya lebih bagus, jika kita menerapkan beberapa penyederhanaan untuk itu.

```
> fn <- solve(equ,n) | ratsimp; $fn
```


Menggambar Grafik 2D dengan EMT

Notebook ini menjelaskan tentang cara menggambar berbagai kurva dan grafik 2D dengan software EMT. EMT menyediakan fungsi `plot2d()` untuk menggambar berbagai kurva dan grafik dua dimensi (2D).

Plot Dasar

Ada fungsi yang sangat mendasar dari plot. Ada koordinat layar, yang selalu berkisar dari 0 hingga 1024 di setiap sumbu, tidak peduli apakah layarnya persegi atau tidak. Semut ada koordinat plot, yang dapat diatur dengan `setplot()`. Pemetaan antara koordinat tergantung pada jendela plot saat ini. Misalnya, `shrinkwindow()` default menyisakan ruang untuk label sumbu dan judul plot.

Dalam contoh, kita hanya menggambar beberapa garis acak dalam berbagai warna. Untuk detail tentang fungsi ini, pelajari fungsi inti EMT.

```
> clg; // clear screen
> window(0,0,1024,1024); // use all of the window
> setplot(0,1,0,1); // set plot coordinates
> hold on; // start overwrite mode
> n=100; X=random(n,2); Y=random(n,2); // get random points
> colors=rgb(random(n),random(n),random(n)); // get random colors
> loop 1 to n; color(colors[#]); plot(X[#],Y[#]); end; // plot
> hold off; // end overwrite mode
> insimg; // insert to notebook
> reset;
```

Grafik perlu ditahan, karena perintah `plot()` akan menghapus jendela plot.

Untuk menghapus semua yang kami lakukan, kami menggunakan `reset()`.

Untuk menampilkan gambar hasil plot di layar notebook, perintah `plot2d()` dapat diakhiri dengan titik dua (`:`). Cara lain adalah perintah `plot2d()` diakhiri dengan titik koma (`;`), kemudian menggunakan perintah `insimg()` untuk menampilkan gambar hasil plot.

Untuk contoh lain, kami menggambar plot sebagai sisipan di plot lain. Ini dilakukan dengan mendefinisikan jendela plot yang lebih kecil. Perhatikan bahwa jendela ini tidak menyediakan ruang untuk label sumbu di luar jendela plot. Kita harus menambahkan beberapa margin untuk ini sesuai kebutuhan. Perhatikan bahwa kami menyimpan dan memulihkan jendela penuh, dan menahan plot saat ini saat kami memplot inset.

```
>plot2d("x^3-x");  
>xw=200; yw=100; ww=300; hw=300;  
>ow=window();  
>window(xw,yw,xw+ww,yw+hw);  
>hold on;  
>barclear(xw-50,yw-10,ww+60,ww+60);  
>plot2d("x^4-x",grid=6):  
>hold off;  
>window(ow);
```

Plot dengan banyak angka dicapai dengan cara yang sama. Ada fungsi `figure()` utilitas untuk ini.

Plot default menggunakan jendela plot persegi. Anda dapat mengubah ini dengan fungsi `aspect()`. Jangan lupa untuk mengatur ulang aspek nanti. Anda juga dapat mengubah default ini di menu dengan "Set Aspect" ke rasio aspek tertentu atau ke ukuran jendela grafis saat ini.

Tetapi Anda juga dapat mengubahnya untuk satu plot. Untuk ini, ukuran area plot saat ini diubah, dan jendela diatur sehingga label memiliki cukup ruang.

```
>aspect(2); // rasio panjang dan lebar 2:1
>plot2d(["sin(x)","cos(x)"],0,2pi):
>aspect();
>reset;
```

Fungsi `reset()` mengembalikan default plot termasuk rasio aspek.

Plot 2D di Euler

EMT Math Toolbox memiliki plot dalam 2D, baik untuk data maupun fungsi. EMT menggunakan fungsi `plot2d`. Fungsi ini dapat memplot fungsi dan data.

Dimungkinkan untuk membuat plot di Maxima menggunakan Gnuplot atau dengan Python menggunakan Math Plot Lib.

Euler dapat memplot plot 2D dari

- ekspresi
- fungsi, variabel, atau kurva parameter,
- vektor nilai x-y,
- awan titik di pesawat,
- kurva implisit dengan level atau wilayah level.
- Fungsi kompleks

Gaya plot mencakup berbagai gaya untuk garis dan titik, plot batang dan plot berbayang.

Plot Ekspresi atau Variabel

Ekspresi tunggal dalam "x" (mis. $4x^2$) atau nama fungsi (mis. "f") menghasilkan grafik fungsi.

Berikut adalah contoh paling dasar, yang menggunakan rentang default dan menetapkan rentang y yang tepat agar sesuai dengan plot fungsi.

Catatan: Jika Anda mengakhiri baris perintah dengan titik dua ":", plot akan dimasukkan ke dalam jendela teks. Jika tidak, tekan TAB untuk melihat plot jika jendela plot tertutup.

```
>plot2d("x^2"):
>aspect(1.5); plot2d("x^3-x"):
>a:=5.6; plot2d("exp(-a*x^2)/a"); insimg(30); // menampilkan gambar hasil plot setinggi 25 baris
```

Dari beberapa contoh sebelumnya Anda dapat melihat bahwa Gambaran gambar plot menggunakan sumbu X dengan rentang nilai dari -2 sampai dengan 2. Untuk mengubah rentang nilai X dan Y, Anda dapat menambahkan nilai batas X (dan Y) di belakang ekspresi yang digambar.

Rentang plot diatur dengan parameter yang ditetapkan berikut:

- a,b: rentang-x (default -2,2)
- c,d: y-range (default: skala dengan nilai)
- r: sebagai alternatif radius di sekitar pusat plot
- cx,cy: koordinat pusat plot (default 0,0)

```
>plot2d("x^3-x",-1,2):
>plot2d("sin(x)",-2*pi,2*pi): // plot sin(x) pada interval [-2pi, 2pi]
>plot2d("cos(x)","sin(3*x)",xmin=0,xmax=2*pi):
```

Alternatif untuk titik dua adalah perintah `insimg(baris)`, yang menyisipkan plot yang menempati sejumlah baris teks tertentu.

Dalam opsi, plot dapat diatur untuk muncul

- di jendela terpisah yang dapat diubah ukurannya,
- di jendela buku catatan.

Lebih banyak gaya dapat dicapai dengan perintah plot tertentu.

Bagaimanapun, tekan tombol tabulator untuk melihat plot, jika disembunyikan.

Untuk membagi jendela menjadi beberapa plot, gunakan perintah `figure()`. Dalam contoh, kami memplot x^1 hingga x^4 menjadi 4 bagian jendela. `figure(0)` mengatur ulang jendela default.

```
>reset;  
>figure(2,2); ...  
>for n=1 to 4; figure(n); plot2d("x^"+n); end; ...  
>figure(0):
```

Di `plot2d()`, ada gaya alternatif yang tersedia dengan `grid=x`. Untuk gambaran umum, kami menunjukkan berbagai gaya kisi dalam satu gambar (lihat di bawah untuk perintah `figure()`). Gaya kisi=0 tidak disertakan. Ini menunjukkan tidak ada grid dan tidak ada bingkai.

```
>figure(3,3); ...  
>for k=1:9; figure(k); plot2d("x^3-x",-2,1,grid=k); end; ...  
>figure(0):
```

Jika argumen ke `plot2d()` adalah ekspresi yang diikuti oleh empat angka, angka-angka ini adalah rentang x dan y untuk plot.

Atau, a , b , c , d dapat ditentukan sebagai parameter yang ditetapkan sebagai $a=...$ dll.

Dalam contoh berikut, kita mengubah gaya kisi, menambahkan label, dan menggunakan label vertikal untuk sumbu y .

```
>aspect(1.5); plot2d("sin(x)",0,2pi,-1.2,1.2,grid=3,xl="x",yl="sin(x)":  
>plot2d("sin(x)+cos(2*x)",0,4pi):
```

Gambar yang dihasilkan dengan memasukkan plot ke dalam jendela teks disimpan di direktori yang sama dengan buku catatan, secara default di subdirektori bernama "gambar". Mereka juga digunakan oleh ekspor HTML.

Anda cukup menandai gambar apa saja dan menyalinnya ke clipboard dengan Ctrl-C. Tentu saja, Anda juga dapat mengekspor grafik saat ini dengan fungsi di menu File.

Fungsi atau ekspresi dalam `plot2d` dievaluasi secara adaptif. Untuk kecepatan lebih, matikan plot adaptif dengan `<adaptive` dan tentukan jumlah subinterval dengan `n=...`. Ini hanya diperlukan dalam kasus yang jarang terjadi.

```
>plot2d("sign(x)*exp(-x^2)",-1,1,<adaptive,n=10000):  
>plot2d("x^x",r=1.2,cx=1,cy=1):
```

Perhatikan bahwa x^x tidak didefinisikan untuk $x \leq 0$. Fungsi `plot2d` menangkap kesalahan ini, dan mulai merencanakan segera setelah fungsi didefinisikan. Ini berfungsi untuk semua fungsi yang mengembalikan NAN keluar dari jangkauan definisinya.

```
>plot2d("log(x)",-0.1,2):
```

Parameter `square=true` (atau `>square`) memilih y-range secara otomatis sehingga hasilnya adalah jendela plot persegi. Perhatikan bahwa secara default, Euler menggunakan ruang persegi di dalam jendela plot.

```
>plot2d("x^3-x",>square):  
>plot2d(''integrate("sin(x)*exp(-x^2)",0,x)'',0,2): // plot integral
```

Jika Anda membutuhkan lebih banyak ruang untuk label-y, panggil `shrinkwindow()` dengan parameter yang lebih kecil, atau tetapkan nilai positif untuk "lebih kecil" di `plot2d()`.

```
>plot2d("gamma(x)",1,10,yl="y-values",smaller=6,<vertical):
```

Ekspresi simbolik juga dapat digunakan, karena disimpan sebagai ekspresi string sederhana.

```
>x=linspace(0,2pi,1000); plot2d(sin(5x),cos(7x)):
>a:=5.6; expr &= exp(-a*x^2)/a; // define expression
>plot2d(expr,-2,2): // plot from -2 to 2
>plot2d(expr,r=1,thickness=2): // plot in a square around (0,0)
>plot2d(&diff(expr,x),>add,style="--",color=red): // add another plot
>plot2d(&diff(expr,x,2),a=-2,b=2,c=-2,d=1): // plot in rectangle
>plot2d(&diff(expr,x),a=-2,b=2,>square): // keep plot square
>plot2d("x^2",0,1,steps=1,color=red,n=10):
>plot2d("x^2",>add,steps=2,color=blue,n=10):
```


Fungsi dalam satu Parameter

Fungsi plot yang paling penting untuk plot planar adalah `plot2d()`. Fungsi ini diimplementasikan dalam bahasa Euler dalam file "plot.e", yang dimuat di awal program.

Berikut adalah beberapa contoh menggunakan fungsi. Seperti biasa di EMT, fungsi yang berfungsi untuk fungsi atau ekspresi lain, Anda dapat meneruskan parameter tambahan (selain x) yang bukan variabel global ke fungsi dengan parameter titik koma atau dengan koleksi panggilan.

```
>function f(x,a) := x^2/a+a*x^2-x; // define a function
>a=0.3; plot2d("f",0,1;a): // plot with a=0.3
>plot2d("f",0,1;0.4): // plot with a=0.4
>plot2d({"f",0.2}},0,1): // plot with a=0.2
>plot2d({"f(x,b)",b=0.1}},0,1): // plot with 0.1
>function f(x) := x^3-x; ...
>plot2d("f",r=1):
```

Berikut adalah ringkasan dari fungsi yang diterima

- ekspresi atau ekspresi simbolik dalam x
- fungsi atau fungsi simbolis dengan nama sebagai "f"
- fungsi simbolis hanya dengan nama f

Fungsi `plot2d()` juga menerima fungsi simbolis. Untuk fungsi simbolis, nama saja yang berfungsi.

```
>function f(x) &= diff(x^x,x)
```

$$x^{\log(x) + 1}$$

```
>plot2d(f,0,2):
```

Tentu saja, untuk ekspresi atau ekspresi simbolik, nama variabel sudah cukup untuk memplotnya.

```
>expr &= sin(x)*exp(-x)
```

$$E^{-x} \sin(x)$$

```
>plot2d(expr,0,3pi):
>function f(x) &= x^x;
>plot2d(f,r=1,cx=1,cy=1,color=blue,thickness=2);
>plot2d(&diff(f(x),x),>add,color=red,style="-.-"):
```

Untuk gaya garis ada berbagai pilihan.

- gaya="...". Pilih dari "-", "_", "-.", ".", "-.", "-.-".
- warna: Lihat di bawah untuk warna.
- ketebalan: Default adalah 1.

Warna dapat dipilih sebagai salah satu warna default, atau sebagai warna RGB.

- 0.15: indeks warna default.
- konstanta warna: putih, hitam, merah, hijau, biru, cyan, zaitun, abu-abu muda, abu-abu, abu-abu tua, oranye, hijau muda, pirus, biru muda, oranye terang, kuning
- rgb(merah, hijau, biru): parameter adalah real dalam [0,1].

```
>plot2d("exp(-x^2)",r=2,color=red,thickness=3,style="--"):
```

Berikut adalah tampilan warna EMT yang telah ditentukan sebelumnya.

```
>aspect(2); columnsplot(ones(1,16),lab=0:15,grid=0,color=0:15):
```

But you can use any color.

```
>columnsplot(ones(1,16),grid=0,color=rgb(0,0,linspace(0,1,15))):
```

Menggambar Beberapa Kurva pada bidang koordinat yang sama

Plot lebih dari satu fungsi (multiple function) ke dalam satu jendela dapat dilakukan dengan berbagai cara. Salah satu metode menggunakan `>add` untuk beberapa panggilan ke `plot2d` secara keseluruhan, tetapi panggilan pertama. Kami telah menggunakan fitur ini dalam contoh di atas.

```
>aspect(); plot2d("cos(x)",r=2,grid=6); plot2d("x",style=".",>add):  
>aspect(1.5); plot2d("sin(x)",0,2pi); plot2d("cos(x)",color=blue,style="--",>add):
```

Salah satu kegunaan `>add` adalah untuk menambahkan titik pada kurva.

```
>plot2d("sin(x)",0,pi); plot2d(2,sin(2),>points,>add):
```

Kami menambahkan titik persimpangan dengan label (pada posisi "cl" untuk kiri tengah), dan memasukkan hasilnya ke dalam notebook. Kami juga menambahkan judul ke plot.

```
>plot2d(["cos(x)","x"],r=1.1,cx=0.5,cy=0.5, ...  
> color=[black,blue],style=["-","."], ...  
> grid=1);  
>x0=solve("cos(x)-x",1); ...  
> plot2d(x0,x0,>points,>add,title="Intersection Demo"); ...  
> label("cos(x) = x",x0,x0,pos="cl",offset=20):
```

Dalam demo berikut, kami memplot fungsi $\text{sinc}(x)=\sin(x)/x$ dan ekspansi Taylor ke-8 dan ke-16. Kami menghitung ekspansi ini menggunakan Maxima melalui ekspresi simbolis.

Plot ini dilakukan dalam perintah multi-baris berikut dengan tiga panggilan ke `plot2d()`. Yang kedua dan yang ketiga memiliki set flag `>add`, yang membuat plot menggunakan rentang sebelumnya.

Kami menambahkan kotak label yang menjelaskan fungsi.

```
>$taylor(sin(x)/x,x,0,4)
>plot2d("sinc(x)",0,4pi,color=green,thickness=2); ...
> plot2d(&taylor(sin(x)/x,x,0,8),>add,color=blue,style="--"); ...
> plot2d(&taylor(sin(x)/x,x,0,16),>add,color=red,style="-.-"); ...
> labelbox(["sinc","T8","T16"],styles=["-","--","-.-"], ...
> colors=[black,blue,red]):
```

Dalam contoh berikut, kami menghasilkan Bernstein-Polinomial.

$$B_i(x) = \binom{n}{i} x^i (1-x)^{n-i}$$

```
>plot2d("(1-x)^10",0,1); // plot first function
>for i=1 to 10; plot2d("bin(10,i)*x^i*(1-x)^(10-i)",>add); end;
>insimg;
```

Metode kedua menggunakan pasangan matriks nilai-x dan matriks nilai-y yang berukuran sama.

Kami menghasilkan matriks nilai dengan satu Polinomial Bernstein di setiap baris. Untuk ini, kita cukup menggunakan vektor kolom i. Lihat pengantar tentang bahasa matriks untuk mempelajari lebih detail.

```
>x=linspace(0,1,500);  
>n=10; k=(0:n)'; // n is row vector, k is column vector  
>y=bin(n,k)*x^k*(1-x)^(n-k); // y is a matrix then  
>plot2d(x,y):
```

Perhatikan bahwa parameter warna dapat berupa vektor. Kemudian setiap warna digunakan untuk setiap baris matriks.

```
>x=linspace(0,1,200); y=x^(1:10)'; plot2d(x,y,color=1:10):
```

Metode lain adalah menggunakan vektor ekspresi (string). Anda kemudian dapat menggunakan larik warna, larik gaya, dan larik ketebalan dengan panjang yang sama.

```
>plot2d(["sin(x)","cos(x)"],0,2pi,color=4:5):  
>plot2d(["sin(x)","cos(x)"],0,2pi): // plot vector of expressions
```

Kita bisa mendapatkan vektor seperti itu dari Maxima menggunakan makelist() dan mxm2str().

```
>v &= makelist(binomial(10,i)*x^i*(1-x)^(10-i),i,0,10) // make list
```

$$\begin{bmatrix} (1-x)^{10}, 10(1-x)^9x, 45(1-x)^8x^2, 120(1-x)^7x^3, \\ 210(1-x)^6x^4, 252(1-x)^5x^5, 210(1-x)^4x^6, 120(1-x)^3x^7, \\ 45(1-x)^2x^8, 10(1-x)x^9, x^{10} \end{bmatrix}$$

```
>mxm2str(v) // get a vector of strings from the symbolic vector
```

```
(1-x)^10
10*(1-x)^9*x
45*(1-x)^8*x^2
120*(1-x)^7*x^3
210*(1-x)^6*x^4
252*(1-x)^5*x^5
210*(1-x)^4*x^6
120*(1-x)^3*x^7
45*(1-x)^2*x^8
10*(1-x)*x^9
x^10
```

```
>plot2d(mxm2str(v),0,1): // plot functions
```

Alternatif lain adalah dengan menggunakan bahasa matriks Euler.

Jika ekspresi menghasilkan matriks fungsi, dengan satu fungsi di setiap baris, semua fungsi ini akan diplot ke dalam satu plot.

Untuk ini, gunakan vektor parameter dalam bentuk vektor kolom. Jika array warna ditambahkan, itu akan digunakan untuk setiap baris plot.

```
>n=(1:10)'; plot2d("x^n",0,1,color=1:10):
```

Ekspresi dan fungsi satu baris dapat melihat variabel global.

Jika Anda tidak dapat menggunakan variabel global, Anda perlu menggunakan fungsi dengan parameter tambahan, dan meneruskan parameter ini sebagai parameter titik koma.

Berhati-hatilah, untuk meletakkan semua parameter yang ditetapkan di akhir perintah plot2d. Dalam contoh kita meneruskan $a=5$ ke fungsi f , yang kita plot dari -10 hingga 10.

```
>function f(x,a) := 1/a*exp(-x^2/a); ...  
>plot2d("f",-10,10;5,thickness=2,title="a=5"):
```


Atau, gunakan koleksi dengan nama fungsi dan semua parameter tambahan. Daftar khusus ini disebut koleksi panggilan, dan itu adalah cara yang lebih disukai untuk meneruskan argumen ke fungsi yang dengan sendirinya diteruskan sebagai argumen ke fungsi lain.

Dalam contoh berikut, kami menggunakan loop untuk memplot beberapa fungsi (lihat tutorial tentang pemrograman untuk loop).

```
>plot2d({{"f",1}},-10,10); ...  
>for a=2:10; plot2d({{"f",a}},>add); end:
```

Kami dapat mencapai hasil yang sama dengan cara berikut menggunakan bahasa matriks EMT. Setiap baris matriks $f(x,a)$ adalah satu fungsi. Selain itu, kita dapat mengatur warna untuk setiap baris matriks. Klik dua kali pada fungsi `getspectral()` untuk penjelasannya.

```
>x=-10:0.01:10; a=(1:10)'; plot2d(x,f(x,a),color=getspectral(a/10)):
```

Label Teks

Dekorasi sederhana bisa

- judul dengan judul="..."
- x- dan y-label dengan xl="...", yl="..."
- label teks lain dengan label("...",x,y)

Perintah label akan memplot ke dalam plot saat ini pada koordinat plot (x,y). Itu bisa mengambil argumen posisi.

```
>plot2d("x^3-x",-1,2,title="y=x^3-x",yl="y",xl="x"):  
>expr := "log(x)/x"; ...  
> plot2d(expr,0.5,5,title="y="+expr,xl="x",yl="y"); ...  
> label("(1,0)",1,0); label("Max",E,expr(E),pos="lc"):
```

Ada juga fungsi labelbox(), yang dapat menampilkan fungsi dan teks. Dibutuhkan vektor string dan warna, satu item untuk setiap fungsi.

```
>function f(x) &= x^2*exp(-x^2); ...  
>plot2d(&f(x),a=-3,b=3,c=-1,d=1); ...  
>plot2d(&diff(f(x),x),>add,color=blue,style="--"); ...  
>labelbox(["function","derivative"],styles=["-","--"], ...  
> colors=[black,blue],w=0.4):
```

Kotak ditambatkan di kanan atas secara default, tetapi `>` kiri menambatkannya di kiri atas. Anda dapat memindahkannya ke tempat yang Anda suka. Posisi jangkar adalah sudut kanan atas kotak, dan angkanya adalah pecahan dari ukuran jendela grafik. Lebar nya otomatis.

Untuk plot titik, kotak label juga berfungsi. Tambahkan parameter `>points`, atau vektor flag, satu untuk setiap label.

Dalam contoh berikut, hanya ada satu fungsi. Jadi kita bisa menggunakan string sebagai pengganti vektor string. Kami mengatur warna teks menjadi hitam untuk contoh ini.

```
>n=10; plot2d(0:n,bin(n,0:n),>addpoints); ...  
>labelbox("Binomials",styles="[]",>points,x=0.1,y=0.1, ...  
>tcolor=black,>left):
```

Gaya plot ini juga tersedia di `statplot()`. Seperti di `plot2d()` warna dapat diatur untuk setiap baris plot. Ada lebih banyak plot khusus untuk keperluan statistik (lihat tutorial tentang statistik).

```
>statplot(1:10,random(2,10),color=[red,blue]):
```

Fitur serupa adalah fungsi `textbox()`.

Lebar secara default adalah lebar maksimal dari baris teks. Tapi itu bisa diatur oleh pengguna juga.

```
>function f(x) &= exp(-x)*sin(2*pi*x); ...  
>plot2d("f(x)",0,2pi); ...  
>textbox(latex("\text{Example of a damped oscillation}\ f(x)=e^{-x}\sin(2\pi x)"),w=0.85):
```

Label teks, judul, kotak label, dan teks lainnya dapat berisi string Unicode (lihat sintaks EMT untuk mengetahui lebih lanjut tentang string Unicode).

```
>plot2d("x^3-x",title=u"x &rarr; x3 - x"):
```

Label pada sumbu x dan y bisa vertikal, begitu juga sumbunya.

```
>plot2d("sinc(x)",0,2pi,xl="x",yl=u"x &rarr; sinc(x)",>vertical):
```

LaTeX

Anda juga dapat memplot rumus LaTeX jika Anda telah menginstal sistem LaTeX. Saya merekomendasikan MiKTeX. Jalur ke biner "latex" dan "dvi2png" harus berada di jalur sistem, atau Anda harus mengatur LaTeX di menu opsi.

Perhatikan, bahwa penguraian LaTeX lambat. Jika Anda ingin menggunakan LaTeX dalam plot animasi, Anda harus memanggil `latex()` sebelum loop sekali dan menggunakan hasilnya (gambar dalam matriks RGB).

Dalam plot berikut, kami menggunakan LaTeX untuk label x dan y, label, kotak label, dan judul plot.

```

>plot2d("exp(-x)*sin(x)/x",a=0,b=2pi,c=0,d=1,grid=6,color=blue, ...
> title=latex("\text{Function $\Phi$}"), ...
> xl=latex("\phi"),yl=latex("\Phi(\phi)"); ...
>textbox( ...
> latex("\Phi(\phi) = e^{-\phi} \frac{\sin(\phi)}{\phi}"),x=0.8,y=0.5); ...
>label(latex("\Phi",color=blue),1,0.4):

```

Seringkali, kami menginginkan spasi dan label teks non-konformal pada sumbu x. Kita dapat menggunakan `xaxis()` dan `yaxis()` seperti yang akan kita tunjukkan nanti.

Cara termudah adalah dengan membuat plot kosong dengan bingkai menggunakan `grid=4`, lalu menambahkan grid dengan `ygrid()` dan `xgrid()`. Dalam contoh berikut, kami menggunakan tiga string LaTeX untuk label pada sumbu x dengan `xtick()`.

```

>plot2d("sinc(x)",0,2pi,grid=4,<ticks); ...
>ygrid(-2:0.5:2,grid=6); ...
>xgrid([0:2]*pi,<ticks,grid=6); ...
>xtick([0,pi,2pi],["0","\pi","2\pi"],>latex):

```

Tentu saja, fungsi juga dapat digunakan.

```

>function map f(x) ...

```

```

    if x>0 then return x^4
    else return x^2
  endif
endfunction

```

Parameter "peta" membantu menggunakan fungsi untuk vektor. Untuk plot, itu tidak perlu. Tetapi untuk mendemonstrasikan vektorisasi itu berguna, kami menambahkan beberapa poin kunci ke plot di $x=-1$, $x=0$ dan $x=1$.

Pada plot berikut, kami juga memasukkan beberapa kode LaTeX. Kami menggunakannya untuk dua label dan kotak teks. Tentu saja, Anda hanya akan dapat menggunakan LaTeX jika Anda telah menginstal LaTeX dengan benar.

```
>plot2d("f",-1,1,xl="x",yl="f(x)",grid=6); ...
>plot2d([-1,0,1],f([-1,0,1]),>points,>add); ...
>label(latex("x^3"),0.72,f(0.72)); ...
>label(latex("x^2"),-0.52,f(-0.52),pos="ll"); ...
>textbox( ...
> latex("f(x)=\begin{cases} x^3 & x>0 \\\ x^2 & x \leq 0\end{cases}"), ...
> x=0.7,y=0.2):
```

Interaksi pengguna

Saat memplot fungsi atau ekspresi, parameter `>user` memungkinkan pengguna untuk memperbesar dan menggeser plot dengan tombol kursor atau mouse. Pengguna dapat

- perbesar dengan `+` atau `-`
- pindahkan plot dengan tombol kursor
- pilih jendela plot dengan mouse
- atur ulang tampilan dengan spasi
- keluar dengan kembali

Tombol spasi akan mengatur ulang plot ke jendela plot asli.

Saat memplot data, flag `>user` hanya akan menunggu penekanan tombol.

```
>plot2d({{"x^3-a*x",a=1}},>user,title="Press any key!"):
>plot2d("exp(x)*sin(x)",user=true, ...
> title="+/- or cursor keys (return to exit)":
```

Berikut ini menunjukkan cara interaksi pengguna tingkat lanjut (lihat tutorial tentang pemrograman untuk detailnya).

Fungsi bawaan `mousedrag()` menunggu event mouse atau keyboard. Ini melaporkan mouse ke bawah, mouse dipindahkan atau mouse ke atas, dan penekanan tombol. Fungsi `dragpoints()` memanfaatkan ini, dan memungkinkan pengguna menyeret titik mana pun dalam plot.

Kita membutuhkan fungsi plot terlebih dahulu. Sebagai contoh, kita interpolasi dalam 5 titik dengan polinomial. Fungsi harus diplot ke area plot tetap.

```
>function plotf(xp,yp,select) ...

    d=interp(xp,yp);
    plot2d("interpval(xp,d,x)";d,xp,r=2);
    plot2d(xp,yp,>points,>add);
    if select>0 then
        plot2d(xp[select],yp[select],color=red,>points,>add);
    endif;
    title("Drag one point, or press space or return!");
endfunction
```

Perhatikan parameter titik koma di plot2d (d dan xp), yang diteruskan ke evaluasi fungsi interp(). Tanpa ini, kita harus menulis fungsi plotinterp() terlebih dahulu, mengakses nilai secara global.

Sekarang kita menghasilkan beberapa nilai acak, dan membiarkan pengguna menyeret poin.

```
>t=-1:0.5:1; dragpoints("plotf",t,random(size(t))-0.5):
```

Ada juga fungsi, yang memplot fungsi lain tergantung pada vektor parameter, dan memungkinkan pengguna menyesuaikan parameter ini.

Pertama kita membutuhkan fungsi plot.

```
>function plotf([a,b]) := plot2d("exp(a*x)*cos(2pi*b*x)",0,2pi;a,b);
```

Kemudian kita membutuhkan nama untuk parameter, nilai awal dan matriks rentang nx2, opsional baris judul.

Ada slider interaktif, yang dapat mengatur nilai oleh pengguna. Fungsi dragvalues() menyediakan ini.

```
>dragvalues("plotf",["a","b"],[-1,2],[[-2,2];[1,10]], ...  
> heading="Drag these values:",hcolor=black):
```


Dimungkinkan untuk membatasi nilai yang diseret ke bilangan bulat. Sebagai contoh, kita menulis fungsi plot, yang memplot polinomial Taylor derajat n ke fungsi kosinus.

```
>function plotf(n) ...
```

```
    plot2d("cos(x)",0,2pi,>square,grid=6);  
    plot2d(&"taylor(cos(x),x,0,@n)",color=blue,>add);  
    textbox("Taylor polynomial of degree "+n,0.1,0.02,style="t",>left);  
endfunction
```

Sekarang kami mengizinkan derajat n bervariasi dari 0 hingga 20 dalam 20 pemberhentian. Hasil drag-values() digunakan untuk memplot sketsa dengan n ini, dan untuk memasukkan plot ke dalam buku catatan.

```
>nd=dragvalues("plotf","degree",2,[0,20],20,y=0.8, ...  
>  heading="Drag the value:"); ...  
>plotf(nd):
```

Berikut ini adalah demonstrasi sederhana dari fungsi tersebut. Pengguna dapat menggambar di atas jendela plot, meninggalkan jejak poin.

```
>function dragtest ...
```

```

plot2d(none,r=1,title="Drag with the mouse, or press any key!");
start=0;
repeat
  {flag,m,time}=mousedrag();
  if flag==0 then return; endif;
  if flag==2 then
    hold on; mark(m[1],m[2]); hold off;
  endif;
end
endfunction

```

```
>dragtest // lihat hasilnya dan cobalah lakukan!
```

Gaya Plot 2D

Secara default, EMT menghitung tick sumbu otomatis dan menambahkan label ke setiap tick. Ini dapat diubah dengan parameter grid. Gaya default sumbu dan label dapat dimodifikasi. Selain itu, label dan judul dapat ditambahkan secara manual. Untuk mengatur ulang ke gaya default, gunakan reset().

```

>aspect();
>figure(3,4); ...
> figure(1); plot2d("x^3-x",grid=0); ... // no grid, frame or axis
> figure(2); plot2d("x^3-x",grid=1); ... // x-y-axis
> figure(3); plot2d("x^3-x",grid=2); ... // default ticks
> figure(4); plot2d("x^3-x",grid=3); ... // x-y- axis with labels inside
> figure(5); plot2d("x^3-x",grid=4); ... // no ticks, only labels

```

```

> figure(6); plot2d("x^3-x",grid=5); ... // default, but no margin
> figure(7); plot2d("x^3-x",grid=6); ... // axes only
> figure(8); plot2d("x^3-x",grid=7); ... // axes only, ticks at axis
> figure(9); plot2d("x^3-x",grid=8); ... // axes only, finer ticks at axis
> figure(10); plot2d("x^3-x",grid=9); ... // default, small ticks inside
> figure(11); plot2d("x^3-x",grid=10); ...// no ticks, axes only
> figure(0):

```

Parameter <frame mematikan frame, dan framecolor=blue mengatur frame ke warna biru.

Jika Anda ingin centang sendiri, Anda dapat menggunakan style=0, dan menambahkan semuanya nanti.

```

>aspect(1.5);
>plot2d("x^3-x",grid=0); // plot
>frame; xgrid([-1,0,1]); ygrid(0): // add frame and grid

```

Untuk judul plot dan label sumbu, lihat contoh berikut.

```

>plot2d("exp(x)",-1,1);
>textcolor(black); // set the text color to black
>title(latex("y=e^x")); // title above the plot
>xlabel(latex("x")); // "x" for x-axis
>ylabel(latex("y"),>vertical); // vertical "y" for y-axis
>label(latex("(0,1)"),0,1,color=blue): // label a point

```

Sumbu dapat digambar secara terpisah dengan `xaxis()` dan `yaxis()`.

```
>plot2d("x^3-x",<grid,<frame);  
>xaxis(0,xx=-2:1,style="->"); yaxis(0,yy=-5:5,style="->");
```

Teks pada plot dapat diatur dengan `label()`. Dalam contoh berikut, "lc" berarti tengah bawah. Ini mengatur posisi label relatif terhadap koordinat plot.

```
>function f(x) &= x^3-x
```

$$x^3 - x$$

```
>plot2d(f,-1,1,>square);  
>x0=fmin(f,0,1); // compute point of minimum  
>label("Rel. Min.",x0,f(x0),pos="lc"): // add a label there
```

Ada juga kotak teks.

```

>plot2d(&f(x),-1,1,-2,2); // function
>plot2d(&diff(f(x),x),>add,style="--",color=red); // derivative
>labelbox(["f","f'"],["-","--"],[black,red]): // label box
>plot2d(["exp(x)","1+x"],color=[black,blue],style=["-","-."]):
>gridstyle("->",color=gray,textcolor=gray,framecolor=gray); ...
> plot2d("x^3-x",grid=1); ...
> settitle("y=x^3-x",color=black); ...
> label("x",2,0,pos="bc",color=gray); ...
> label("y",0,6,pos="cl",color=gray); ...
> reset():

```

Untuk kontrol lebih, sumbu x dan sumbu y dapat dilakukan secara manual.

Perintah `fullwindow()` memperluas jendela plot karena kita tidak lagi membutuhkan tempat untuk label di luar jendela plot. Gunakan `shrinkwindow()` atau `reset()` untuk mengatur ulang ke default.

```

>fullwindow; ...
> gridstyle(color=darkgray,textcolor=darkgray); ...
> plot2d(["2^x","1","2^(-x)"],a=-2,b=2,c=0,d=4,<grid,color=4:6,<frame); ...
> xaxis(0,-2:1,style="->"); xaxis(0,2,"x",<axis); ...
> yaxis(0,4,"y",style="->"); ...
> yaxis(-2,1:4,>left); ...
> yaxis(2,2^(-2:2),style=".",<left); ...
> labelbox(["2^x","1","2^-x"],colors=4:6,x=0.8,y=0.2); ...
> reset:

```

Berikut adalah contoh lain, di mana string Unicode digunakan dan sumbu di luar area plot.

```
>aspect(1.5);  
>plot2d(["sin(x)", "cos(x)"], 0, 2pi, color=[red, green], <grid, <frame); ...  
> xaxis(-1.1, (0:2)*pi, xt=["0", u"&pi;", u"2&pi;"], style="-", >ticks, >zero); ...  
> xgrid((0:0.5:2)*pi, <ticks); ...  
> yaxis(-0.1*pi, -1:0.2:1, style="-", >zero, >grid); ...  
> labelbox(["sin", "cos"], colors=[red, green], x=0.5, y=0.2, >left); ...  
> xlabel(u"&phi;"); ylabel(u"f(&phi;)"):
```

Merencanakan Data 2D

Jika x dan y adalah vektor data, data ini akan digunakan sebagai koordinat x dan y dari suatu kurva. Dalam hal ini, a , b , c , dan d , atau radius r dapat ditentukan, atau jendela plot akan menyesuaikan secara otomatis dengan data. Atau, `>perseg` dapat diatur untuk menjaga rasio aspek persegi.

Memplot ekspresi hanyalah singkatan untuk plot data. Untuk plot data, Anda memerlukan satu atau beberapa baris nilai x , dan satu atau beberapa baris nilai y . Dari rentang dan nilai- x , fungsi `plot2d` akan menghitung data yang akan diplot, secara default dengan evaluasi fungsi yang adaptif. Untuk plot titik gunakan `">titik"`, untuk garis campuran dan titik gunakan `">tambahan"`.

Tapi Anda bisa memasukkan data secara langsung.

- Gunakan vektor baris untuk x dan y untuk satu fungsi.
- Matriks untuk x dan y diplot baris demi baris.

Berikut adalah contoh dengan satu baris untuk x dan y .

```
>x=-10:0.1:10; y=exp(-x^2)*x; plot2d(x,y):
```

Data juga dapat diplot sebagai titik. Gunakan `poin=true` untuk ini. Plotnya bekerja seperti poligon, tetapi hanya menggambar sudut-sudutnya.

- `style="..."`: Pilih dari `"[]"`, `"<>"`, `"o"`, `"."`, `".."`, `"+"`, `"*"`, `"[]"`, `"< >"`, `"o"`, `".."`, `"|"`, `"|"`.

Untuk memplot set poin gunakan `>points`. Jika warna adalah vektor warna, setiap titik mendapat warna yang berbeda. Untuk matriks koordinat dan vektor kolom, warna berlaku untuk baris matriks.

Parameter `>addpoints` menambahkan titik ke segmen garis untuk plot data.

```
>xdata=[1,1.5,2.5,3,4]; ydata=[3,3.1,2.8,2.9,2.7]; // data
>plot2d(xdata,ydata,a=0.5,b=4.5,c=2.5,d=3.5,style="."); // lines
>plot2d(xdata,ydata,>points,>add,style="o"): // add points
>p=polyfit(xdata,ydata,1); // get regression line
>plot2d("polyval(p,x)",>add,color=red): // add plot of line
```


Menggambar Daerah Yang Dibatasi Kurva

Plot data benar-benar poligon. Kita juga dapat memplot kurva atau kurva terisi.

- terisi=benar mengisi plot.
- style="...": Pilih dari " ", " / ", " \ ", " \ / ".
- fillcolor: Lihat di atas untuk warna yang tersedia.

Warna isian ditentukan oleh argumen "fillcolor", dan pada <outline opsional mencegah menggambar batas untuk semua gaya kecuali yang default.

```
>t=linspace(0,2pi,1000); // parameter for curve
>x=sin(t)*exp(t/pi); y=cos(t)*exp(t/pi); // x(t) and y(t)
>figure(1,2); aspect(16/9)
>figure(1); plot2d(x,y,r=10); // plot curve
>figure(2); plot2d(x,y,r=10,>filled,style="/",fillcolor=red); // fill curve
>figure(0):
```

Dalam contoh berikut kami memplot elips terisi dan dua segi enam terisi menggunakan kurva tertutup dengan 6 titik dengan gaya isian berbeda.

```
>x=linspace(0,2pi,1000); plot2d(sin(x),cos(x)*0.5,r=1,>filled,style="/"):
>t=linspace(0,2pi,6); ...
>plot2d(cos(t),sin(t),>filled,style="/",fillcolor=red,r=1.2):
>t=linspace(0,2pi,6); plot2d(cos(t),sin(t),>filled,style="#"):
```

Contoh lainnya adalah segi empat, yang kita buat dengan 7 titik pada lingkaran satuan.

```
>t=linspace(0,2pi,7); ...  
> plot2d(cos(t),sin(t),r=1,>filled,style="/",fillcolor=red):
```

Berikut ini adalah himpunan nilai maksimal dari empat kondisi linier yang kurang dari atau sama dengan 3. Ini adalah $A[k].v \leq 3$ untuk semua baris A. Untuk mendapatkan sudut yang bagus, kita menggunakan n yang relatif besar.

```
>A=[2,1;1,2;-1,0;0,-1];  
>function f(x,y) := max([x,y].A');  
>plot2d("f",r=4,level=[0;3],color=green,n=111):
```

Poin utama dari bahasa matriks adalah memungkinkan untuk menghasilkan tabel fungsi dengan mudah.

```
>t=linspace(0,2pi,1000); x=cos(3*t); y=sin(4*t);
```

Kami sekarang memiliki vektor x dan y nilai. `plot2d()` dapat memplot nilai-nilai ini sebagai kurva yang menghubungkan titik-titik. Plotnya bisa diisi. Pada kasus ini ini menghasilkan hasil yang bagus karena aturan lilitan, yang digunakan untuk isi.

```
>plot2d(x,y,<grid,<frame,>filled):
```

Sebuah vektor interval diplot terhadap nilai x sebagai daerah terisi antara nilai interval bawah dan atas.

Hal ini dapat berguna untuk memplot kesalahan perhitungan. Tapi itu bisa juga digunakan untuk memplot kesalahan statistik.

```
>t=0:0.1:1; ...  
> plot2d(t,interval(t-random(size(t)),t+random(size(t))),style="|"); ...  
> plot2d(t,t,add=true):
```

Jika x adalah vektor yang diurutkan, dan y adalah vektor interval, maka `plot2d` akan memplot rentang interval yang terisi dalam bidang. Gaya isian sama dengan gaya poligon.

```
>t=-1:0.01:1; x=~t-0.01,t+0.01~; y=x^3-x;  
>plot2d(t,y):
```

Jika x adalah vektor yang diurutkan, dan y adalah vektor interval, maka `plot2d` akan memplot rentang interval yang terisi dalam bidang. Gaya isian sama dengan gaya poligon.

```
>expr := "2*x^2+x*y+3*y^4+y"; // define an expression f(x,y)
>plot2d(expr,level=[0;1],style="-",color=blue): // 0 <= f(x,y) <= 1
```

Kami juga dapat mengisi rentang nilai seperti

$$-1 \leq (x^2 + y^2)^2 - x^2 + y^2 \leq 0.$$

```
>plot2d("(x^2+y^2)^2-x^2+y^2",r=1.2,level=[-1;0],style="/"):
>plot2d("cos(x)","sin(x)^3",xmin=0,xmax=2pi,>filled,style="/"):
```

Grafik Fungsi Parametrik

Nilai-x tidak perlu diurutkan. (x,y) hanya menggambarkan kurva. Jika x diurutkan, kurva tersebut merupakan grafik fungsi.

Dalam contoh berikut, kami memplot spiral

$$\gamma(t) = t \cdot (\cos(2\pi t), \sin(2\pi t))$$

Kita perlu menggunakan banyak titik untuk tampilan yang halus atau fungsi adaptif() untuk mengevaluasi ekspresi (lihat fungsi adaptif() untuk lebih jelasnya).

```
>t=linspace(0,1,1000); ...  
>plot2d(t*cos(2*pi*t),t*sin(2*pi*t),r=1):
```

Atau, dimungkinkan untuk menggunakan dua ekspresi untuk kurva. Berikut ini plot kurva yang sama seperti di atas.

```
>plot2d("x*cos(2*pi*x)","x*sin(2*pi*x)",xmin=0,xmax=1,r=1):  
>t=linspace(0,1,1000); r=exp(-t); x=r*cos(2*pi*t); y=r*sin(2*pi*t);  
>plot2d(x,y,r=1):
```

Dalam contoh berikutnya, kami memplot kurva

$$\gamma(t) = (r(t) \cos(t), r(t) \sin(t))$$

dengan

$$r(t) = 1 + \frac{\sin(3t)}{2}.$$

```
>t=linspace(0,2pi,1000); r=1+sin(3*t)/2; x=r*cos(t); y=r*sin(t); ...  
>plot2d(x,y,>filled,fillcolor=red,style="/",r=1.5):
```

Menggambar Grafik Bilangan Kompleks

Array bilangan kompleks juga dapat diplot. Kemudian titik-titik grid akan terhubung. Jika sejumlah garis kisi ditentukan (atau vektor garis kisi 1x2) dalam argumen `cgrid`, hanya garis kisi tersebut yang terlihat.

Matriks bilangan kompleks akan secara otomatis diplot sebagai kisi di bidang kompleks.

Dalam contoh berikut, kami memplot gambar lingkaran satuan di bawah fungsi eksponensial. Parameter `cgrid` menyembunyikan beberapa kurva grid.

```
>aspect(); r=linspace(0,1,50); a=linspace(0,2pi,80)'; z=r*exp(I*a);...  
>plot2d(z,a=-1.25,b=1.25,c=-1.25,d=1.25,cgrid=10):  
>aspect(1.25); r=linspace(0,1,50); a=linspace(0,2pi,200)'; z=r*exp(I*a);  
>plot2d(exp(z),cgrid=[40,10]):  
>r=linspace(0,1,10); a=linspace(0,2pi,40)'; z=r*exp(I*a);  
>plot2d(exp(z),>points,>add):
```

Sebuah vektor bilangan kompleks secara otomatis diplot sebagai kurva pada bidang kompleks dengan bagian real dan bagian imajiner.

Dalam contoh, kami memplot lingkaran satuan dengan

$$\gamma(t) = e^{it}$$

```
>t=linspace(0,2pi,1000); ...  
>plot2d(exp(I*t)+exp(4*I*t),r=2):
```

Plot Statistik

Ada banyak fungsi yang dikhususkan pada plot statistik. Salah satu plot yang sering digunakan adalah plot kolom.

Jumlah kumulatif dari nilai terdistribusi 0-1-normal menghasilkan jalan acak.

```
>plot2d(cumsum(randnormal(1,1000))):
```

Menggunakan dua baris menunjukkan jalan dalam dua dimensi.

```
>X=cumsum(randnormal(2,1000)); plot2d(X[1],X[2]):  
>columnplot(cumsum(random(10)),style="/",color=blue):
```

Itu juga dapat menampilkan string sebagai label.

```
>months=["Jan","Feb","Mar","Apr","May","Jun", ...  
>  "Jul","Aug","Sep","Oct","Nov","Dec"];  
>values=[10,12,12,18,22,28,30,26,22,18,12,8];  
>columnplot(values,lab=months,color=red,style="-");  
>title("Temperature");  
>k=0:10;  
>plot2d(k,bin(10,k),>bar):  
>plot2d(k,bin(10,k)); plot2d(k,bin(10,k),>points,>add):
```



```
>plot2d(normal(1000),normal(1000),>points,grid=6,style=".."):
>plot2d(normal(1,1000),>distribution,style="0"):
>plot2d("qnormal",0,5;2.5,0.5,>filled):
```

Untuk memplot distribusi statistik eksperimental, Anda dapat menggunakan `distribution=n` dengan `plot2d`.

```
>w=randexponential(1,1000); // exponential distribution
>plot2d(w,>distribution): // or distribution=n with n intervals
```

Atau Anda dapat menghitung distribusi dari data dan memplot hasilnya dengan `>bar` di `plot3d`, atau dengan plot kolom.

```
>w=normal(1000); // 0-1-normal distribution
>{x,y}=histo(w,10,v=[-6,-4,-2,-1,0,1,2,4,6]); // interval bounds v
>plot2d(x,y,>bar):
```

Fungsi `statplot()` menyetel gaya dengan string sederhana.

```
>statplot(1:10,cumsum(random(10)),"b"):
>n=10; i=0:n; ...
>plot2d(i,bin(n,i)/2^n,a=0,b=10,c=0,d=0.3); ...
>plot2d(i,bin(n,i)/2^n,points=true,style="ow",add=true,color=blue):
```

Selain itu, data dapat diplot sebagai batang. Dalam hal ini, x harus diurutkan dan satu elemen lebih panjang dari y. Bilah akan memanjang dari $x[i]$ ke $x[i+1]$ dengan nilai $y[i]$. Jika x memiliki ukuran yang sama dengan y, maka akan diperpanjang satu elemen dengan spasi terakhir.

Gaya isian dapat digunakan seperti di atas.

```
>n=10; k=bin(n,0:n); ...
>plot2d(-0.5:n+0.5,k,bar=true,fillcolor=lightgray):
```

Data untuk plot batang (`bar=1`) dan histogram (`histogram=1`) dapat dinyatakan secara eksplisit dalam `xv` dan `yv`, atau dapat dihitung dari distribusi empiris dalam `xv` dengan `>distribusi` (atau `distribusi=n`). Histogram nilai `xv` akan dihitung secara otomatis dengan `>histogram`. Jika `>genap` ditentukan, nilai `xv` akan dihitung dalam interval bilangan bulat.

```
>plot2d(normal(10000),distribution=50):
>k=0:10; m=bin(10,k); x=(0:11)-0.5; plot2d(x,m,>bar):
>columnplot(m,k):
>plot2d(random(600)*6,histogram=6):
```

Untuk distribusi, ada parameter `distribution=n`, yang menghitung nilai secara otomatis dan mencetak distribusi relatif dengan `n` sub-interval.

```
>plot2d(normal(1,1000),distribution=10,style="\/"): 
```

Dengan parameter `even=true`, ini akan menggunakan interval integer.

```
>plot2d(intrandom(1,1000,10),distribution=10,even=true): 
```

Perhatikan bahwa ada banyak plot statistik, yang mungkin berguna. Silahkan lihat tutorial tentang statistik.

```
>columnsplot(getmultiplicities(1:6,intrandom(1,6000,6))):  
>plot2d(normal(1,1000),>distribution); ...  
> plot2d("qnormal(x)",color=red,thickness=2,>add): 
```

Ada juga banyak plot khusus untuk statistik. Boxplot menunjukkan kuartil dari distribusi ini dan banyak outlier. Menurut definisi, outlier dalam boxplot adalah data yang melebihi 1,5 kali kisaran 50% tengah plot.

```
>M=normal(5,1000); boxplot(quantiles(M)): 
```

Fungsi Implisit

Plot implisit menunjukkan garis level yang menyelesaikan $f(x,y)=\text{level}$, di mana "level" dapat berupa nilai tunggal atau vektor nilai. Jika $\text{level}=\text{"auto"}$, akan ada garis level n_c , yang akan menyebar antara fungsi minimum dan maksimum secara merata. Warna yang lebih gelap atau lebih terang dapat ditambahkan dengan $>\text{hue}$ untuk menunjukkan nilai fungsi. Untuk fungsi implisit, xv harus berupa fungsi atau ekspresi dari parameter x dan y , atau, sebagai alternatif, xv dapat berupa matriks nilai.

Euler dapat menandai garis level

$$f(x,y) = c$$

dari fungsi apapun.

Untuk menggambar himpunan $f(x,y)=c$ untuk satu atau lebih konstanta c , Anda dapat menggunakan `plot2d()` dengan plot implisitnya di dalam bidang. Parameter untuk c adalah $\text{level}=c$, di mana c dapat berupa vektor garis level. Selain itu, skema warna dapat digambar di latar belakang untuk menunjukkan nilai fungsi untuk setiap titik dalam plot. Parameter " n " menentukan kehalusan plot.

```
>aspect(1.5);  
>plot2d("x^2+y^2-x*y-x",r=1.5,level=0,contourcolor=red):  
>expr := "2*x^2+x*y+3*y^4+y"; // define an expression f(x,y)  
>plot2d(expr,level=0): // Solutions of f(x,y)=0  
>plot2d(expr,level=0:0.5:20,>hue,contourcolor=white,n=200): // nice  
>plot2d(expr,level=0:0.5:20,>hue,>spectral,n=200,grid=4): // nicer
```

Ini berfungsi untuk plot data juga. Tetapi Anda harus menentukan rentangnya untuk label sumbu.

```
>x=-2:0.05:1; y=x'; z=expr(x,y);  
>plot2d(z,level=0,a=-1,b=2,c=-2,d=1,>hue):  
>plot2d("x^3-y^2",>contour,>hue,>spectral):  
>plot2d("x^3-y^2",level=0,contourwidth=3,>add,contourcolor=red):  
>z=z+normal(size(z))*0.2;  
>plot2d(z,level=0.5,a=-1,b=2,c=-2,d=1):  
>plot2d(expr,level=[0:0.2:5;0.05:0.2:5.05],color=lightgray):  
>plot2d("x^2+y^3+x*y",level=1,r=4,n=100):  
>plot2d("x^2+2*y^2-x*y",level=0:0.1:10,n=100,contourcolor=white,>hue):
```

Juga dimungkinkan untuk mengisi set

$$a \leq f(x,y) \leq b$$

dengan rentang tingkat.

Dimungkinkan untuk mengisi wilayah nilai untuk fungsi tertentu. Untuk ini, level harus berupa matriks 2xn. Baris pertama adalah batas bawah dan baris kedua berisi batas atas.

```
>plot2d(expr,level=[0;1],style="-",color=blue): // 0 <= f(x,y) <= 1
```

Plot implisit juga dapat menunjukkan rentang level. Kemudian level harus berupa matriks 2xn dari interval level, di mana baris pertama berisi awal dan baris kedua adalah akhir dari setiap interval. Atau, vektor baris sederhana dapat digunakan untuk level, dan parameter dl memperluas nilai level ke interval.

```
>plot2d("x^4+y^4",r=1.5,level=[0;1],color=blue,style="/"):
>plot2d("x^2+y^3+x*y",level=[0,2,4;1,3,5],style="/",r=2,n=100):
>plot2d("x^2+y^3+x*y",level=-10:20,r=2,style="-",dl=0.1,n=100):
>plot2d("sin(x)*cos(y)",r=pi,>hue,>levels,n=100):
```

Dimungkinkan juga untuk menandai suatu wilayah

$$a \leq f(x, y) \leq b.$$

Ini dilakukan dengan menambahkan level dengan dua baris.

```
>plot2d("(x^2+y^2-1)^3-x^2*y^3",r=1.3, ...
> style="#",color=red,<outline, ...
> level=[-2;0],n=100):
```

Dimungkinkan untuk menentukan level tertentu. Misalnya, kita dapat memplot solusi persamaan seperti

$$x^3 - xy + x^2y^2 = 6$$

```
>plot2d("x^3-x*y+x^2*y^2",r=6,level=1,n=100):  
>function starplot1 (v, style="/", color=green, lab=none) ...
```

```
    if !holding() then clg; endif;  
    w=window(); window(0,0,1024,1024);  
    h=holding(1);  
    r=max(abs(v))*1.2;  
    setplot(-r,r,-r,r);  
    n=cols(v); t=linspace(0,2pi,n);  
    v=v|v[1]; c=v*cos(t); s=v*sin(t);  
    cl=barcolor(color); st=barstyle(style);  
    loop 1 to n  
        polygon([0,c[#],c[#+1]], [0,s[#],s[#+1]],1);  
        if lab!=none then  
            rlab=v[#]+r*0.1;  
            {col,row}=toscreen(cos(t[#])*rlab,sin(t[#])*rlab);  
            ctext(""+lab[#],col,row-textheight()/2);  
        endif;  
    end;  
    barcolor(cl); barstyle(st);  
    holding(h);  
    window(w);  
endfunction
```

Tidak ada kotak atau sumbu kutu di sini. Selain itu, kami menggunakan jendela penuh untuk plot.

Kami memanggil reset sebelum kami menguji plot ini untuk mengembalikan default grafis. Ini tidak perlu, jika Anda yakin plot Anda berhasil.

```
>reset; starplot1(normal(1,10)+5,color=red,lab=1:10):
```

Terkadang, Anda mungkin ingin merencanakan sesuatu yang tidak dapat dilakukan plot2d, tetapi hampir. Dalam fungsi berikut, kami melakukan plot impuls logaritmik. plot2d dapat melakukan plot logaritmik, tetapi tidak untuk batang impuls.

```
>function logimpulseplot1 (x,y) ...  
  
    {x0,y0}=makeimpulse(x,log(y)/log(10));  
    plot2d(x0,y0,>bar,grid=0);  
    h=holding(1);  
    frame();  
    xgrid(ticks(x));  
    p=plot();  
    for i=-10 to 10;  
        if i<=p[4] and i>=p[3] then  
            ygrid(i,yt="10^"+i);  
        endif;  
    end;  
    holding(h);  
endfunction
```

Mari kita uji dengan nilai yang terdistribusi secara eksponensial.

```
>aspect(1.5); x=1:10; y=-log(random(size(x)))*200; ...  
>logimpulseplot1(x,y):
```


Mari kita menganimasikan kurva 2D menggunakan plot langsung. Perintah `plot(x,y)` hanya memplot kurva ke jendela plot. `setplot(a,b,c,d)` mengatur jendela ini.

Fungsi `wait(0)` memaksa plot untuk muncul di jendela grafik. Jika tidak, menggambar ulang terjadi dalam interval waktu yang jarang.

```
> function animliss (n,m) ...
```

```
    t=linspace(0,2pi,500);
    f=0;
    c=framecolor(0);
    l=linewidth(2);
    setplot(-1,1,-1,1);
    repeat
        clg;
        plot(sin(n*t),cos(m*t+f));
        wait(0);
        if testkey() then break; endif;
        f=f+0.02;
    end;
    framecolor(c);
    linewidth(l);
endfunction
```

Tekan sembarang tombol untuk menghentikan animasi ini.

```
>animliss(2,3); // lihat hasilnya, jika sudah puas, tekan ENTER
```

Plot Logaritmik

EMT menggunakan parameter "logplot" untuk skala logaritmik.

Plot logaritma dapat diplot baik menggunakan skala logaritma dalam y dengan logplot=1, atau menggunakan skala logaritma dalam x dan y dengan logplot=2, atau dalam x dengan logplot=3.

- logplot=1: y-logaritma
- logplot=2: x-y-logaritma
- logplot=3: x-logaritma

```
>plot2d("exp(x^3-x)*x^2",1,5,logplot=1):  
>plot2d("exp(x+sin(x))",0,100,logplot=1):  
>plot2d("exp(x+sin(x))",10,100,logplot=2):  
>plot2d("gamma(x)",1,10,logplot=1):  
>plot2d("log(x*(2+sin(x/100)))",10,1000,logplot=3):
```

Ini juga berfungsi dengan plot data.

```
>x=10^(1:20); y=x^2-x;  
>plot2d(x,y,logplot=2):
```

Buatlah fungsi grafik dari dua fungsi yaitu :

$$4\sin(x)$$

$$6\cos(x)$$

dengan domain mulai dari 0 hingga 8π .
lalu analisis kesalahan yang ada, dan perbaiki.

```
>aspect(4);  
>plot2d(["4sin(x)","6cos(x)"],0,8pi):  
>aspect();  
>reset;  
>aspect(4);  
>plot2d(["4sin(x)","4cos(x)"],0,8pi):  
>aspect();  
>reset;
```

Ini adalah pengenalan plot 3D di Euler. Kita membutuhkan plot 3D untuk memvisualisasikan fungsi dari dua variabel.

Euler menggambar fungsi tersebut menggunakan algoritma pengurutan untuk menyembunyikan bagian di latar belakang. Secara umum, Euler menggunakan proyeksi pusat. Standarnya adalah dari kuadran x-y positif menuju titik asal $x=y=z=0$, tetapi sudut= 0° terlihat dari arah sumbu y. Sudut pandang dan tinggi dapat diubah.

Euler dapat merencanakan

- permukaan dengan bayangan dan garis level atau rentang level,
- awan poin,
- kurva parametrik,
- permukaan implisit.

Plot 3D dari suatu fungsi menggunakan plot3d. Cara termudah adalah dengan memplot ekspresi dalam x dan y. Parameter r mengatur kisaran plot di sekitar (0,0).

```
>aspect(1.5); plot3d("x^2+sin(y)",-5,5,0,6*pi):  
>plot3d("x^2+x*sin(y)",-5,5,0,6*pi):
```

Fungsi dua Variabel

Untuk grafik fungsi, gunakan

- ekspresi sederhana dalam x dan y,
- nama fungsi dari dua variabel
- atau matriks data.

Standarnya adalah kotak kawat yang diisi dengan warna berbeda di kedua sisi. Perhatikan bahwa jumlah default interval grid adalah 10, tetapi plot menggunakan jumlah default 40x40 persegi panjang untuk membangun permukaan. Ini bisa diubah.

- `n=40`, `n=[40,40]`: jumlah garis grid di setiap arah
- `grid=10`, `grid=[10,10]`: jumlah garis grid di setiap arah.

Kami menggunakan default `n=40` dan `grid=10`.

```
>plot3d("x^2+y^2"):
```

Interaksi pengguna dimungkinkan dengan >parameter pengguna. Pengguna dapat menekan tombol berikut.

- kiri, kanan, atas, bawah: putar sudut pandang
- +,-: memperbesar atau memperkecil
- a: menghasilkan anaglyph (lihat di bawah)
- l: beralih memutar sumber cahaya (lihat di bawah)
- spasi: reset ke default
- kembali: akhiri interaksi

```
>plot3d("exp(-x^2+y^2)",>user, ...  
> title="Turn with the vector keys (press return to finish)":
```

Rentang plot untuk fungsi dapat ditentukan dengan

- a,b: rentang-x
- c,d: rentang-y
- r: persegi simetris di sekitar (0,0).
- n: jumlah subinterval untuk plot.

Ada beberapa parameter untuk menskalakan fungsi atau mengubah tampilan grafik.

fscale: skala ke nilai fungsi (defaultnya adalah <fscale).

skala: angka atau vektor 1x2 untuk skala ke arah x dan y.

bingkai: jenis bingkai (default 1).

```
>plot3d("exp(-(x^2+y^2)/5)",r=10,n=80,fscale=4,scale=1.2,frame=3):
```

Tampilan dapat diubah dengan berbagai cara.

- jarak: jarak pandang ke plot.
- zoom: nilai zoom.
- sudut: sudut terhadap sumbu y negatif dalam radian.
- tinggi: ketinggian tampilan dalam radian.

Nilai default dapat diperiksa atau diubah dengan fungsi view(). Ini mengembalikan parameter dalam urutan di atas.

```
>view
```

```
[5, 2.6, 2, 0.4]
```

Jarak yang lebih dekat membutuhkan lebih sedikit zoom. Efeknya lebih seperti lensa sudut lebar.

Dalam contoh berikut, sudut=0 dan tinggi=0 terlihat dari sumbu y negatif. Label sumbu untuk y disembunyikan dalam kasus ini.

```
>plot3d("x^2+y",distance=3,zoom=2,angle=pi/2,height=0):
```

Plot terlihat selalu ke pusat kubus plot. Anda dapat memindahkan pusat dengan parameter tengah.

```
>plot3d("x^4+y^2",a=0,b=1,c=-1,d=1,angle=-20°,height=20°, ...  
> center=[0.4,0,0],zoom=5):
```

Plot diskalakan agar sesuai dengan kubus satuan untuk dilihat. Jadi tidak perlu mengubah jarak atau zoom tergantung pada ukuran plot. Namun, label mengacu pada ukuran sebenarnya.

Jika Anda mematikannya dengan scale=false, Anda perlu berhati-hati, bahwa plot masih cocok dengan jendela plot, dengan mengubah jarak pandang atau zoom, dan memindahkan pusat.

```
>plot3d("5*exp(-x^2-y^2)",r=2,<fscale,<scale,distance=13,height=50°, ...  
> center=[0,0,-2],frame=3):
```

Sebuah plot kutub juga tersedia. Parameter `polar=true` menggambar plot polar. Fungsi tersebut harus tetap merupakan fungsi dari x dan y . Parameter "`fscale`" menskalakan fungsi dengan skala sendiri. Jika tidak, fungsi diskalakan agar sesuai dengan kubus.

```
>plot3d("1/(x^2+y^2+1)",r=5,>polar, ...  
>fscale=2,>hue,n=100,zoom=4,>contour,color=gray):  
>function f(r) := exp(-r/2)*cos(r); ...  
>plot3d("f(x^2+y^2)",>polar,scale=[1,1,0.4],r=pi,frame=3,zoom=4):
```

Rotasi parameter memutar fungsi dalam x di sekitar sumbu x .

- `rotate=1`: Menggunakan sumbu x
- `rotate=2`: Menggunakan sumbu z

```
>plot3d("x^2+1",a=-1,b=1,rotate=true,grid=5):  
>plot3d("x^2+1",a=-1,b=1,rotate=2,grid=5):  
>plot3d("sqrt(25-x^2)",a=0,b=5,rotate=1):  
>plot3d("x","x^2+y^2","y",r=2,zoom=3.5,frame=3):
```


Plot Kontur

Untuk plot, Euler menambahkan garis grid. Sebagai gantinya dimungkinkan untuk menggunakan garis level dan rona satu warna atau rona berwarna spektral. Euler dapat menggambar tinggi fungsi pada plot dengan bayangan. Di semua plot 3D, Euler dapat menghasilkan anaglyph merah/sian.

- > hue: Menyalakan bayangan cahaya alih-alih kabel.
- > kontur: Memplot garis kontur otomatis pada plot.
- level=... (atau level): Sebuah vektor nilai untuk garis kontur.

Standarnya adalah level="auto", yang menghitung beberapa garis level secara otomatis. Seperti yang Anda lihat di plot, level sebenarnya adalah rentang level.

Gaya default dapat diubah. Untuk plot kontur berikut, kami menggunakan grid yang lebih halus untuk 100x100 poin, skala fungsi dan plot, dan menggunakan sudut pandang yang berbeda.

```
>plot3d("exp(-x^2-y^2)",r=2,n=100,level="thin", ...  
> >contour,>spectral,fscale=1,scale=1.1,angle=45°,height=20°):  
>plot3d("exp(x*y)",angle=100°,>contour,color=green):
```

Bayangan default menggunakan warna abu-abu. Tetapi rentang warna spektral juga tersedia.

- > spektral: Menggunakan skema spektral default
- color=...: Menggunakan warna khusus atau skema spektral

Untuk plot berikut, kami menggunakan skema spektral default dan menambah jumlah titik untuk mendapatkan tampilan yang sangat halus.

```
>plot3d("x^2+y^2",>spectral,>contour,n=100):
```

Alih-alih garis level otomatis, kita juga dapat mengatur nilai garis level. Ini akan menghasilkan garis level tipis alih-alih rentang level.

```
>plot3d("x^2-y^2",0,5,0,5,level=-1:0.1:1,color=redgreen):
```

Dalam plot berikut, kami menggunakan dua pita level yang sangat luas dari -0,1 hingga 1, dan dari 0,9 hingga 1. Ini dimasukkan sebagai matriks dengan batas level sebagai kolom.

Selain itu, kami melapisi kisi dengan 10 interval di setiap arah.

```
>plot3d("x^2+y^3",level=[-0.1,0.9;0,1], ...  
> >spectral,angle=30°,grid=10,contourcolor=gray):
```

Dalam contoh berikut, kami memplot himpunan, di mana

$$f(x, y) = x^y - y^x = 0$$

Kami menggunakan satu garis tipis untuk garis level.

```
>plot3d("x^y-y^x",level=0,a=0,b=6,c=0,d=6,contourcolor=red,n=100):
```

Dimungkinkan untuk menunjukkan bidang kontur di bawah plot. Warna dan jarak ke plot dapat ditentukan.

```
>plot3d("x^2+y^4",>cp,cpcolor=green,cpdelta=0.2):
```

Here are a few more styles. We always turn off the frame, and use various color schemes for the plot and the grid.

```
>figure(2,2); ...
```

Dimungkinkan untuk menunjukkan bidang kontur di bawah plot. Warna dan jarak ke plot dapat ditentukan.

```
>expr="y^3-x^2"; ...  
>figure(1); ...  
> plot3d(expr,<frame,>cp,cpcolor=spectral); ...  
>figure(2); ...  
> plot3d(expr,<frame,>spectral,grid=10,cp=2); ...  
>figure(3); ...  
> plot3d(expr,<frame,>contour,color=gray,nc=5,cp=3,cpcolor=greenred); ...  
>figure(4); ...  
> plot3d(expr,<frame,>hue,grid=10,>transparent,>cp,cpcolor=gray); ...  
>figure(0):
```

Ada beberapa skema spektral lainnya, bernomor dari 1 hingga 9. Tetapi Anda juga dapat menggunakan warna=nilai, di mana nilai

- spektral: untuk rentang dari biru ke merah
- putih: untuk rentang yang lebih redup
- kuningbiru, ungu hijau, biru kuning, hijau merah
- biru kuning, hijau ungu, kuning biru, merah hijau

```
>figure(3,3); ...  
>for i=1:9; ...  
> figure(i); plot3d("x^2+y^2",spectral=i,>contour,>cp,<frame,zoom=4); ...  
>end; ...  
>figure(0):
```

Sumber cahaya dapat diubah dengan l dan tombol kursor selama interaksi pengguna. Itu juga dapat diatur dengan parameter.

- cahaya: arah untuk cahaya
- amb: cahaya sekitar antara 0 dan 1

Perhatikan bahwa program tidak membuat perbedaan antara sisi plot. Tidak ada bayangan. Untuk ini, Anda perlu Povray.

```
>plot3d("-x^2-y^2", ...  
> hue=true,light=[0,1,1],amb=0,user=true, ...  
> title="Press l and cursor keys (return to exit)");
```

Parameter warna mengubah warna permukaan. Warna garis level juga dapat diubah.

```
>plot3d("-x^2-y^2",color=rgb(0.2,0.2,0),hue=true,frame=false, ...  
>  zoom=3,contourcolor=red,level=-2:0.1:1,d1=0.01):
```

Warna 0 memberikan efek pelangi khusus.

```
>plot3d("x^2/(x^2+y^2+1)",color=0,hue=true,grid=10):
```

Permukaannya juga bisa transparan.

```
>plot3d("x^2+y^2",>transparent,grid=10,wirecolor=red):
```

Plot Implisit

Ada juga plot implisit dalam tiga dimensi. Euler menghasilkan pemotongan melalui objek. Fitur plot3d termasuk plot implisit. Plot-plot ini menunjukkan himpunan nol dari suatu fungsi dalam tiga variabel. Solusi dari

$$f(x, y, z) = 0$$

dapat divisualisasikan dalam potongan sejajar dengan bidang x-y-, x-z- dan y-z.

- implisit=1: potong sejajar dengan bidang y-z
- implisit=2: potong sejajar dengan bidang x-z
- implisit=4: potong sejajar dengan bidang x-y

Tambahkan nilai-nilai ini, jika Anda suka. Dalam contoh kita plot

$$M = \{(x, y, z) : x^2 + y^3 + zy = 1\}$$

```
>plot3d("x^2+y^3+z*y-1",r=5,implicit=3):  
>plot3d("x^2+y^2+4*x*z+z^3",>implicit,r=2,zoom=2.5):
```

Merencanakan Data 3D

Sama seperti plot2d, plot3d menerima data. Untuk objek 3D, Anda perlu menyediakan matriks nilai x-, y- dan z, atau tiga fungsi atau ekspresi $f_x(x,y)$, $f_y(x,y)$, $f_z(x,y)$.

$$\gamma(t, s) = (x(t, s), y(t, s), z(t, s))$$

Karena x,y,z adalah matriks, kita asumsikan bahwa (t,s) melalui sebuah kotak persegi. Hasilnya, Anda dapat memplot gambar persegi panjang di ruang angkasa.

Anda dapat menggunakan bahasa matriks Euler untuk menghasilkan koordinat secara efektif.

Dalam contoh berikut, kami menggunakan vektor nilai t dan vektor kolom nilai s untuk membuat parameter permukaan bola. Dalam gambar kita dapat menandai daerah, dalam kasus kita daerah kutub.

```
>t=linspace(0,2pi,180); s=linspace(-pi/2,pi/2,90)'; ...  
>x=cos(s)*cos(t); y=cos(s)*sin(t); z=sin(s); ...  
>plot3d(x,y,z,>hue, ...  
>color=blue,<frame,grid=[10,20], ...  
>values=s,contourcolor=red,level=[90°-24°;90°-22°], ...  
>scale=1.4,height=50°):
```

Berikut adalah contoh, yang merupakan grafik fungsi.

```
>t=-1:0.1:1; s=(-1:0.1:1)'; plot3d(t,s,t*s,grid=10):
```

Namun, kita bisa membuat segala macam permukaan. Berikut adalah permukaan yang sama dengan fungsi

$$x = y z$$

```
>plot3d(t*s,t,s,angle=180°,grid=10):
```

Dengan lebih banyak usaha, kami dapat menghasilkan banyak permukaan.

Dalam contoh berikut, kita membuat tampilan bayangan dari bola yang terdistorsi. Koordinat biasa untuk bola adalah

$$\gamma(t, s) = (\cos(t) \cos(s), \sin(t) \sin(s), \cos(s))$$

dengan

$$0 \leq t \leq 2\pi, \quad -\frac{\pi}{2} \leq s \leq \frac{\pi}{2}.$$

Kami mendistorsi ini dengan sebuah faktor

$$d(t, s) = \frac{\cos(4t) + \cos(8s)}{4}.$$

```
>t=linspace(0,2pi,320); s=linspace(-pi/2,pi/2,160)'; ...  
>d=1+0.2*(cos(4*t)+cos(8*s)); ...  
>plot3d(cos(t)*cos(s)*d,sin(t)*cos(s)*d,sin(s)*d,hue=1, ...  
> light=[1,0,1],frame=0,zoom=5):
```

Tentu saja, titik cloud juga dimungkinkan. Untuk memplot data titik dalam ruang, kita membutuhkan tiga vektor untuk koordinat titik-titik tersebut.

Gayanya sama seperti di plot2d dengan points=true;

```
>n=500; ...  
> plot3d(normal(1,n),normal(1,n),normal(1,n),points=true,style="."):
```

Dimungkinkan juga untuk memplot kurva dalam 3D. Dalam hal ini, lebih mudah untuk menghitung titik-titik kurva. Untuk kurva di pesawat kami menggunakan urutan koordinat dan parameter wire=true.

```
>t=linspace(0,8pi,500); ...  
>plot3d(sin(t),cos(t),t/10,>wire,zoom=3):  
>t=linspace(0,4pi,1000); plot3d(cos(t),sin(t),t/2pi,>wire, ...  
>linewidth=3,wirecolor=blue):  
>X=cumsum(normal(3,100)); ...  
> plot3d(X[1],X[2],X[3],>anaglyph,>wire):
```

EMT juga dapat memplot dalam mode anaglyph. Untuk melihat plot seperti itu, Anda memerlukan kacamata merah/sian.

```
> plot3d("x^2+y^3",>anaglyph,>contour,angle=30°):
```

Seringkali, skema warna spektral digunakan untuk plot. Ini menekankan ketinggian fungsi.

```
>plot3d("x^2*y^3-y",>spectral,>contour,zoom=3.2):
```

Euler juga dapat memplot permukaan berparameter, ketika parameternya adalah nilai x -, y -, dan z dari gambar kotak persegi panjang dalam ruang.

Untuk demo berikut, kami mengatur parameter u - dan v -, dan menghasilkan koordinat ruang dari ini.

```
>u=linspace(-1,1,10); v=linspace(0,2*pi,50)'; ...  
>X=(3+u*cos(v/2))*cos(v); Y=(3+u*cos(v/2))*sin(v); Z=u*sin(v/2); ...  
>plot3d(X,Y,Z,>anaglyph,<frame,>wire,scale=2.3):
```

Berikut adalah contoh yang lebih rumit, yang megah dengan kacamata merah/sian.

```
>u:=linspace(-pi,pi,160); v:=linspace(-pi,pi,400)'; ...  
>x:=(4*(1+.25*sin(3*v))+cos(u))*cos(2*v); ...  
>y:=(4*(1+.25*sin(3*v))+cos(u))*sin(2*v); ...  
> z=sin(u)+2*cos(3*v); ...  
>plot3d(x,y,z,frame=0,scale=1.5,hue=1,light=[1,0,-1],zoom=2.8,>anaglyph):
```

Plot bar juga dimungkinkan. Untuk ini, kita harus menyediakan

- x: vektor baris dengan $n+1$ elemen
- y: vektor kolom dengan $n+1$ elemen
- z: matriks nilai $n \times n$.

z bisa lebih besar, tetapi hanya nilai $n \times n$ yang akan digunakan.

Dalam contoh, pertama-tama kita menghitung nilainya. Kemudian kita sesuaikan x dan y, sehingga vektor berpusat pada nilai yang digunakan.

```
>x=-1:0.1:1; y=x'; z=x^2+y^2; ...  
>xa=(x|1.1)-0.05; ya=(y_1.1)-0.05; ...  
>plot3d(xa,ya,z,bar=true):
```

Dimungkinkan untuk membagi plot permukaan menjadi dua atau lebih bagian.

```
>x=-1:0.1:1; y=x'; z=x+y; d=zeros(size(x)); ...  
>plot3d(x,y,z,disconnect=2:2:20):
```

Jika memuat atau menghasilkan matriks data M dari file dan perlu memplotnya dalam 3D, Anda dapat menskalakan matriks ke $[-1,1]$ dengan `scale(M)`, atau menskalakan matriks dengan `>zscale`. Ini dapat dikombinasikan dengan faktor penskalaan individu yang diterapkan sebagai tambahan.

```
>i=1:20; j=i'; ...  
>plot3d(i*j^2+100*normal(20,20),>zscale,scale=[1,1,1.5],angle=-40°,zoom=1.8):  
>Z=intrandom(5,100,6); v=zeros(5,6); ...  
>loop 1 to 5; v[#]=getmultiplicities(1:6,Z[#]); end; ...  
>columnsp3d(v',scols=1:5,ccols=[1:5]):
```

Permukaan Benda Putar

```
>plot2d("(x^2+y^2-1)^3-x^2*y^3",r=1.3, ...  
>style="#",color=red,<outline, ...  
>level=[-2;0],n=100):  
>ekspresi &= (x^2+y^2-1)^3-x^2*y^3; $ekspresi
```

Kami ingin memutar kurva jantung di sekitar sumbu y. Berikut adalah ungkapan, yang mendefinisikan hati:

$$f(x,y) = (x^2 + y^2 - 1)^3 - x^2.y^3.$$

Selanjutnya kita atur

$$x = r.\cos(a), \quad y = r.\sin(a).$$

```
>function fr(r,a) &= ekspresi with [x=r*cos(a),y=r*sin(a)] | trigreduce; $fr(r,a)
```

Hal ini memungkinkan untuk mendefinisikan fungsi numerik, yang memecahkan r, jika a diberikan. Dengan fungsi itu kita dapat memplot jantung yang diputar sebagai permukaan parametrik.

```

>function map f(a) := bisect("fr",0,2;a); ...
>t=linspace(-pi/2,pi/2,100); r=f(t); ...
>s=linspace(pi,2pi,100)'; ...
>plot3d(r*cos(t)*sin(s),r*cos(t)*cos(s),r*sin(t), ...
>>hue,<frame,color=red,zoom=4,amb=0,max=0.7,grid=12,height=50°):

```

Berikut ini adalah plot 3D dari gambar di atas yang diputar di sekitar sumbu z. Kami mendefinisikan fungsi, yang menggambarkan objek.

```

>function f(x,y,z) ...

    r=x^2+y^2;
    return (r+z^2-1)^3-r*z^3;
endfunction

```

```

>plot3d("f(x,y,z)", ...
>xmin=0,xmax=1.2,ymin=-1.2,ymax=1.2,zmin=-1.2,zmax=1.4, ...
>implicit=1,angle=-30°,zoom=2.5,n=[10,60,60],>anaglyph):

```

Plot 3D Khusus

Fungsi `plot3d` bagus untuk dimiliki, tetapi tidak memenuhi semua kebutuhan. Selain rutinitas yang lebih mendasar, dimungkinkan untuk mendapatkan plot berbingkai dari objek apa pun yang Anda suka.

Meskipun Euler bukan program 3D, ia dapat menggabungkan beberapa objek dasar. Kami mencoba memvisualisasikan paraboloid dan garis singgungnya.

```
>function myplot ...
```

```
    y=0:0.01:1; x=(0.1:0.01:1)';  
    plot3d(x,y,0.2*(x-0.1)/2,<scale,<frame,>hue, ..  
        hues=0.5,>contour,color=orange);  
    h=holding(1);  
    plot3d(x,y,(x^2+y^2)/2,<scale,<frame,>contour,>hue);  
    holding(h);  
endfunction
```

Sekarang `framedplot()` menyediakan frame, dan mengatur tampilan.

```
>framedplot("myplot",[0.1,1,0,1,0,1],angle=-45°, ...  
> center=[0,0,-0.7],zoom=6):
```


Dengan cara yang sama, Anda dapat memplot bidang kontur secara manual. Perhatikan bahwa `plot3d()` menyetel jendela ke `fullwindow()` secara default, tetapi `plotcontourplane()` mengasumsikan itu.

```
>x=-1:0.02:1.1; y=x'; z=x^2-y^4;  
>function myplot (x,y,z) ...
```

```
    zoom(2);  
    wi=fullwindow();  
    plotcontourplane(x,y,z,level="auto",<scale);  
    plot3d(x,y,z,>hue,<scale,>add,color=white,level="thin");  
    window(wi);  
    reset();  
endfunction
```

```
>myplot(x,y,z):
```

Euler dapat menggunakan frame untuk menghitung animasi terlebih dahulu.

Salah satu fungsi yang memanfaatkan teknik ini adalah rotate. Itu dapat mengubah sudut pandang dan menggambar ulang plot 3D. Fungsi memanggil addpage() untuk setiap plot baru. Akhirnya itu menjiwai plot.

Silakan pelajari sumber rotasi untuk melihat lebih detail.

```
>function testplot () := plot3d("x^2+y^3"); ...  
>rotate("testplot"); testplot():
```

Dengan bantuan file Euler povray.e, Euler dapat menghasilkan file Povray. Hasilnya sangat bagus untuk dilihat.

Anda perlu menginstal Povray (32bit atau 64bit) dari <http://www.povray.org/>, dan meletakkan sub-direktori "bin" dari Povray ke jalur lingkungan, atau mengatur variabel "defaultpovray" dengan path lengkap yang menunjuk ke "pvengine.exe".

Antarmuka Povray dari Euler menghasilkan file Povray di direktori home pengguna, dan memanggil Povray untuk mengurai file-file ini. Nama file default adalah current.pov, dan direktori default adalah eulerhome(), biasanya c:\Users\Username\Euler. Povray menghasilkan file PNG, yang dapat dimuat oleh Euler ke dalam buku catatan. Untuk membersihkan file-file ini, gunakan povclear().

Fungsi pov3d memiliki semangat yang sama dengan plot3d. Ini dapat menghasilkan grafik fungsi $f(x,y)$, atau permukaan dengan koordinat X,Y,Z dalam matriks, termasuk garis level opsional. Fungsi ini memulai raytracer secara otomatis, dan memuat adegan ke dalam notebook Euler.

Selain pov3d(), ada banyak fungsi yang menghasilkan objek Povray. Fungsi-fungsi ini mengembalikan string, yang berisi kode Povray untuk objek. Untuk menggunakan fungsi ini, mulai file Povray dengan povstart(). Kemudian gunakan writeln(...) untuk menulis objek ke file adegan. Terakhir, akhiri file dengan povend(). Secara default, raytracer akan dimulai, dan PNG akan dimasukkan ke dalam notebook Euler.

Fungsi objek memiliki parameter yang disebut "look", yang membutuhkan string dengan kode Povray untuk tekstur dan hasil akhir objek. Fungsi povlook() dapat digunakan untuk menghasilkan string ini. Ini memiliki parameter untuk warna, transparansi, Phong Shading dll.

Perhatikan bahwa alam semesta Povray memiliki sistem koordinat lain. Antarmuka ini menerjemahkan semua koordinat ke sistem Povray. Jadi Anda dapat terus berpikir dalam sistem koordinat Euler dengan z menunjuk vertikal ke atas, and x,y,z sumbu dalam arti tangan kanan. Anda perlu memuat file povray.

```
>load povray;
```

Pastikan, direktori bin Povray ada di jalur. Jika tidak, edit variabel berikut sehingga berisi path ke povray yang dapat dieksekusi.

```
>defaultpovray="C:\Program Files\POV-Ray\v3.7\bin\pvengine.exe"
```

```
C:\Program Files\POV-Ray\v3.7\bin\pvengine.exe
```

Untuk kesan pertama, kami memplot fungsi sederhana. Perintah berikut menghasilkan file povray di direktori pengguna Anda, dan menjalankan Povray untuk ray tracing file ini.

Jika Anda memulai perintah berikut, GUI Povray akan terbuka, menjalankan file, dan menutup secara otomatis. Karena alasan keamanan, Anda akan ditanya, apakah Anda ingin mengizinkan file exe untuk dijalankan. Anda dapat menekan batal untuk menghentikan pertanyaan lebih lanjut. Anda mungkin harus menekan OK di jendela Povray untuk mengakui dialog awal Povray.

```
>pov3d("x^2+y^2",zoom=3);
```

Kita dapat membuat fungsi menjadi transparan dan menambahkan hasil akhir lainnya. Kami juga dapat menambahkan garis level ke plot fungsi.

```
>pov3d("x^2+y^3",axiscolor=red,angle=20°, ...  
> look=povlook(blue,0.2),level=-1:0.5:1,zoom=3.8);
```

Terkadang perlu untuk mencegah penskalaan fungsi, dan menskalakan fungsi dengan tangan.

Kami memplot himpunan titik di bidang kompleks, di mana produk dari jarak ke 1 dan -1 sama dengan 1.

```
>pov3d("((x-1)^2+y^2)*((x+1)^2+y^2)/40",r=1.5, ...  
>  angle=-120°,level=1/40,dlevel=0.005,light=[-1,1,1],height=45°,n=50, ...  
>  <fscale,zoom=3.8);
```

Merencanakan dengan Koordinat

Alih-alih fungsi, kita dapat memplot dengan koordinat. Seperti pada `plot3d`, kita membutuhkan tiga matriks untuk mendefinisikan objek.

Dalam contoh kita memutar fungsi di sekitar sumbu z .

```
>function f(x) := x^3-x+1; ...  
>x=-1:0.01:1; t=linspace(0,2pi,8)'; ...  
>Z=x; X=cos(t)*f(x); Y=sin(t)*f(x); ...  
>pov3d(X,Y,Z,angle=40°,height=20°,axis=0,zoom=4,light=[10,-5,5]);
```

Dalam contoh berikut, kami memplot gelombang teredam. Kami menghasilkan gelombang dengan bahasa matriks Euler.

Kami juga menunjukkan, bagaimana objek tambahan dapat ditambahkan ke adegan `pov3d`. Untuk pembuatan objek, lihat contoh berikut. Perhatikan bahwa `plot3d` menskalakan plot, sehingga cocok dengan kubus satuan.

```
>r=linspace(0,1,80); phi=linspace(0,2pi,80)'; ...  
>x=r*cos(phi); y=r*sin(phi); z=exp(-5*r)*cos(8*pi*r)/3; ...  
>pov3d(x,y,z,zoom=5,axis=0,add=povsphere([0,0,0.5],0.1,povlook(green)), ...  
> w=500,h=300);
```

Dengan metode bayangan canggih dari Povray, sangat sedikit titik yang dapat menghasilkan permukaan yang sangat halus. Hanya di perbatasan dan dalam bayang-bayang triknya mungkin menjadi jelas.

Untuk ini, kita perlu menambahkan vektor normal di setiap titik matriks.

```
>Z &= x^2*y^3
```

$$\begin{matrix} 2 & 3 \\ x & y \end{matrix}$$

Persamaan permukaannya adalah $[x,y,Z]$. Kami menghitung dua turunan ke x dan y ini dan mengambil produk silang sebagai normal.

```
>dx &= diff([x,y,Z],x); dy &= diff([x,y,Z],y);
```

Kami mendefinisikan normal sebagai produk silang dari turunan ini, dan mendefinisikan fungsi koordinat.

```
>N &= crossproduct(dx,dy); NX &= N[1]; NY &= N[2]; NZ &= N[3]; N,
```

$$[-2xy^3, -3x^2y^2, 1]$$

Kami hanya menggunakan 25 poin.

```
>x=-1:0.5:1; y=x';  
>pov3d(x,y,Z(x,y),angle=10°, ...  
>   xv=NX(x,y),yv=NY(x,y),zv=NZ(x,y),<shadow);
```

Berikut ini adalah simpul Trefoil yang dilakukan oleh A. Busser di Povray. Ada versi yang ditingkatkan dari ini dalam contoh.

Lihat: Contoh\Trefoil Simpul | Simpul trefoil

Untuk tampilan yang bagus dengan tidak terlalu banyak titik, kami menambahkan vektor normal di sini. Kami menggunakan Maxima untuk menghitung normal bagi kami. Pertama, ketiga fungsi koordinat sebagai ekspresi simbolik.

```
>X &= ((4+sin(3*y))+cos(x))*cos(2*y); ...  
>Y &= ((4+sin(3*y))+cos(x))*sin(2*y); ...  
>Z &= sin(x)+2*cos(3*y);
```

Kemudian kedua vektor turunan ke x dan y.

```
>dx &= diff([X,Y,Z],x); dy &= diff([X,Y,Z],y);
```


Sekarang normal, yang merupakan produk silang dari dua turunan.

```
>dn &= crossproduct(dx,dy);
```

Kami sekarang mengevaluasi semua ini secara numerik.

```
>x:=linspace(-%pi,%pi,40); y:=linspace(-%pi,%pi,100)';
```

Vektor normal adalah evaluasi dari ekspresi simbolik $dn[i]$ untuk $i=1,2,3$. Sintaks untuk ini adalah `&"expression"(parameters)`. Ini adalah alternatif dari metode pada contoh sebelumnya, di mana kita mendefinisikan ekspresi simbolik NX , NY , NZ terlebih dahulu.

```
>pov3d(X(x,y),Y(x,y),Z(x,y),axis=0,zoom=5,w=450,h=350, ...  
> <shadow,look=povlook(gray), ...  
> xv=&"dn[1]"(x,y), yv=&"dn[2]"(x,y), zv=&"dn[3]"(x,y));
```

Kami juga dapat menghasilkan grid dalam 3D.

```
>povstart(zoom=4); ...  
>x=-1:0.5:1; r=1-(x+1)^2/6; ...  
>t=(0°:30°:360°)'; y=r*cos(t); z=r*sin(t); ...  
>writeln(povgrid(x,y,z,d=0.02,dballs=0.05)); ...  
>povend();
```

Dengan povgrid(), kurva dimungkinkan.

```
>povstart(center=[0,0,1],zoom=3.6); ...  
>t=linspace(0,2,1000); r=exp(-t); ...  
>x=cos(2*pi*10*t)*r; y=sin(2*pi*10*t)*r; z=t; ...  
>writeln(povgrid(x,y,z,povlook(red))); ...  
>writeAxis(0,2,axis=3); ...  
>povend();
```

Objek Povray

Di atas, kami menggunakan pov3d untuk memplot permukaan. Antarmuka povray di Euler juga dapat menghasilkan objek Povray. Objek-objek ini disimpan sebagai string di Euler, dan perlu ditulis ke file Povray.

Kami memulai output dengan povstart().

```
>povstart(zoom=4);
```

Pertama kita mendefinisikan tiga silinder, dan menyimpannya dalam string di Euler.

Fungsi povx() dll. hanya mengembalikan vektor [1,0,0], yang dapat digunakan sebagai gantinya.

```
>c1=povcylinder(-povx,povx,1,povlook(red)); ...  
>c2=povcylinder(-povy,povy,1,povlook(green)); ...  
>c3=povcylinder(-povz,povz,1,povlook(blue)); ...
```

Pertama kita mendefinisikan tiga silinder, dan menyimpannya dalam string di Euler.

Fungsi `povx()` dll. hanya mengembalikan vektor $[1,0,0]$, yang dapat digunakan sebagai pengingat.

```
>c1
```

```
cylinder { <-1,0,0>, <1,0,0>, 1
  texture { pigment { color rgb <0.564706,0.0627451,0.0627451> } }
  finish { ambient 0.2 }
}
```

Seperti yang Anda lihat, kami menambahkan tekstur ke objek dalam tiga warna berbeda.

Itu dilakukan oleh `povlook()`, yang mengembalikan string dengan kode Povray yang relevan. Kita dapat menggunakan warna Euler default, atau menentukan warna kita sendiri. Kami juga dapat menambahkan transparansi, atau mengubah cahaya sekitar.

```
>povlook(rgb(0.1,0.2,0.3),0.1,0.5)
```

```
texture { pigment { color rgbf <0.101961,0.2,0.301961,0.1> } }
finish { ambient 0.5 }
```

Sekarang kita mendefinisikan objek persimpangan, dan menulis hasilnya ke file.

```
>writeln(povintersection([c1,c2,c3]));
```

Persimpangan tiga silinder sulit untuk divisualisasikan, jika Anda belum pernah melihatnya sebelumnya.

```
>povend;
```

Fungsi berikut menghasilkan fraktal secara rekursif.

Fungsi pertama menunjukkan, bagaimana Euler menangani objek Povray sederhana. Fungsi `povbox()` mengembalikan string, yang berisi koordinat kotak, tekstur, dan hasil akhir.

```
>function onebox(x,y,z,d) := povbox([x,y,z],[x+d,y+d,z+d],povlook());  
>function fractal (x,y,z,h,n) ...
```

```
    if n==1 then writeln(onebox(x,y,z,h));  
    else  
        h=h/3;  
        fractal(x,y,z,h,n-1);  
        fractal(x+2*h,y,z,h,n-1);  
        fractal(x,y+2*h,z,h,n-1);  
        fractal(x,y,z+2*h,h,n-1);  
        fractal(x+2*h,y+2*h,z,h,n-1);  
        fractal(x+2*h,y,z+2*h,h,n-1);  
        fractal(x,y+2*h,z+2*h,h,n-1);  
        fractal(x+2*h,y+2*h,z+2*h,h,n-1);  
        fractal(x+h,y+h,z+h,h,n-1);  
    endif;  
endfunction
```

```
>povstart(fade=10,<shadow);  
>fractal(-1,-1,-1,2,4);  
>povend();
```

Perbedaan memungkinkan memotong satu objek dari yang lain. Seperti persimpangan, ada bagian dari objek CSG Povray.

```
>povstart(light=[5,-5,5],fade=10);
```

Untuk demonstrasi ini, kami mendefinisikan objek di Povray, alih-alih menggunakan string di Euler. Definisi ditulis ke file segera.

Koordinat kotak -1 berarti [-1,-1,-1].

```
>povdefine("mycube",povbox(-1,1));
```

Kita dapat menggunakan objek ini di povobject(), yang mengembalikan string seperti biasa.

```
>c1=povobject("mycube",povlook(red));
```

Kami menghasilkan kubus kedua, dan memutar dan menskalakannya sedikit.

```
>c2=povobject("mycube",povlook(yellow),translate=[1,1,1], ...  
> rotate=xrotate(10°)+yrotate(10°), scale=1.2);
```

Kemudian kita ambil selisih kedua benda tersebut.

```
>writeln(povdifference(c1,c2));
```

Sekarang tambahkan tiga sumbu.

```
>writeAxis(-1.2,1.2,axis=1); ...  
>writeAxis(-1.2,1.2,axis=2); ...  
>writeAxis(-1.2,1.2,axis=4); ...  
>povend();
```

Fungsi Implisit

Povray dapat memplot himpunan di mana $f(x,y,z)=0$, seperti parameter implisit di plot3d. Namun, hasilnya terlihat jauh lebih baik.

Sintaks untuk fungsinya sedikit berbeda. Anda tidak dapat menggunakan output dari ekspresi Maxima atau Euler.

```
>povstart(angle=70°,height=50°,zoom=4);
```

Buat permukaan implisit. Perhatikan sintaks yang berbeda dalam ekspresi.

```
>writeln(povsurface("pow(x,2)*y-pow(y,3)-pow(z,2)",povlook(green))); ...  
>writeAxes(); ...  
>povend();
```


Objek Jala

Dalam contoh ini, kami menunjukkan cara membuat objek mesh, dan menggambarinya dengan informasi tambahan.

Kami ingin memaksimalkan xy di bawah kondisi $x+y=1$ dan menunjukkan sentuhan tangensial dari garis level.

```
>povstart(angle=-10°,center=[0.5,0.5,0.5],zoom=7);
```

Kami tidak dapat menyimpan objek dalam string seperti sebelumnya, karena terlalu besar. Jadi kita mendefinisikan objek dalam file Povray menggunakan declare. Fungsi povtriangle() melakukan ini secara otomatis. Itu dapat menerima vektor normal seperti pov3d().

Berikut ini mendefinisikan objek mesh, dan langsung menulisnya ke dalam file.

```
>x=0:0.02:1; y=x'; z=x*y; vx=-y; vy=-x; vz=1;  
>mesh=povtriangles(x,y,z,"",vx,vy,vz);
```

Sekarang kita mendefinisikan dua buah cakram, yang akan berpotongan dengan permukaan.

```
>c1=povdisc([0.5,0.5,0],[1,1,0],2); ...  
>l1=povdisc([0,0,1/4],[0,0,1],2);
```

Tuliskan permukaan dikurangi dengan dua cakram.

```
>writeln(povdifference(mesh,povunion([c1,l1]),povlook(green)));
```

Tuliskan dua persimpangan

```
>writeln(povintersection([mesh,c1],povlook(red))); ...  
>writeln(povintersection([mesh,l1],povlook(gray)));
```

Menuliskan sebuah titik secara maksimal.

```
>writeln(povpoint([1/2,1/2,1/4],povlook(gray),size=2*defaultpointsize));
```

Tambahkan sumbu dan selesaikan.

```
>writeAxes(0,1,0,1,0,1,d=0.015); ...  
>povend();
```

Anaglyphs di Povray

Untuk menghasilkan anaglyph untuk kacamata merah/cyan, Povray harus dijalankan dua kali dari posisi kamera yang berbeda. Ini menghasilkan dua file Povray dan dua file PNG, yang dimuat dengan fungsi `loadanaglyph()`.

Tentu saja, Anda membutuhkan kacamata merah/cyan untuk melihat contoh berikut dengan benar.

Fungsi `pov3d()` memiliki sebuah saklar sederhana untuk menghasilkan anaglyph.

```
>pov3d("-exp(-x^2-y^2)/2",r=2,height=45°,>anaglyph, ...  
> center=[0,0,0.5],zoom=3.5);
```

Jika Anda membuat adegan dengan objek, Anda perlu memasukkan pembuatan adegan ke dalam sebuah fungsi, dan jalankan dua kali dengan nilai yang berbeda untuk parameter `anaglyph`.

```
>function myscene ...
```

```
    s=povsphere(povc,1);  
    cl=povcylinder(-povz,povz,0.5);  
    clx=povobject(cl,rotate=xrotate(90°));  
    cly=povobject(cl,rotate=yrotate(90°));  
    c=povbox([-1,-1,0],1);  
    un=povunion([cl,clx,cly,c]);  
    obj=povdifference(s,un,povlook(red));  
    writeln(obj);  
    writeAxes();  
endfunction
```

Fungsi `povanaglyph()` melakukan semua ini. Parameter-parameternya adalah seperti di dalam `povstart()` dan `povend()` digabungkan.

```
>povanaglyph("myscene",zoom=4.5);
```

Mendefinisikan Objek Sendiri

Antarmuka povray Euler berisi banyak objek. Tapi Anda tidak terbatas pada objek-objek tersebut. Anda dapat membuat objek sendiri, yang menggabungkan objek lain, atau objek yang benar-benar baru.

Kami mendemonstrasikan sebuah torus. Perintah Povray untuk ini adalah “torus”. Jadi kami mengembalikan sebuah string dengan perintah ini dan parameternya. Perhatikan bahwa torus selalu berpusat pada titik asal.

```
>function povdonat (r1,r2,look="") ...  
  
    return "torus {" +r1+", "+r2+look+"}";  
endfunction
```

Ini adalah torus pertama kita.

```
>t1=povdonat(0.8,0.2)
```

```
torus {0.8,0.2}
```

Mari kita gunakan objek ini untuk membuat torus kedua, ditranslasikan dan diputar.

```
>t2=povobject(t1,rotate=xrotate(90°),translate=[0.8,0,0])
```

```
object { torus {0.8,0.2}  
  rotate 90 *x  
  translate <0.8,0,0>  
}
```

Sekarang kita tempatkan objek-objek ini ke dalam sebuah scene. Untuk tampilan, kita menggunakan Phong

```
>povstart(center=[0.4,0,0],angle=0°,zoom=3.8,aspect=1.5); ...  
>writeln(povobject(t1,povlook(green,phong=1))); ...  
>writeln(povobject(t2,povlook(green,phong=1))); ...
```

```
>povend();
```

memanggil program Povray. Namun, jika terjadi kesalahan program ini tidak menampilkan kesalahan. Oleh karena itu, Anda harus menggunakan

```
>povend(<exit>);
```

jika ada yang tidak berhasil. Ini akan membiarkan jendela Povray terbuka.

```
>povend(h=320,w=480);
```

Berikut adalah contoh yang lebih rumit. Kami menyelesaikan

$$Ax \leq b, \quad x \geq 0, \quad c.x \rightarrow \text{Max.}$$

dan menunjukkan titik-titik yang layak dan optimal dalam plot 3D.

```
>A=[10,8,4;5,6,8;6,3,2;9,5,6];  
>b=[10,10,10,10]';  
>c=[1,1,1];
```

Pertama, mari kita periksa, apakah contoh ini memiliki solusi atau tidak.

```
>x=simplex(A,b,c,>max,>check)'
```

```
[0, 1, 0.5]
```

Ya, ada.

Selanjutnya kita mendefinisikan dua objek. Yang pertama adalah pesawat

$$a \cdot x \leq b$$

```
>function oneplane (a,b,look="") ...
```

```
    return povplane(a,b,look)
endfunction
```

Kemudian kita mendefinisikan perpotongan dari semua setengah ruang dan sebuah kubus.

```
>function adm (A, b, r, look="") ...
```

```
    ol=[];
    loop 1 to rows(A); ol=ol|oneplane(A[#],b[#]); end;
    ol=ol|povbox([0,0,0],[r,r,r]);
    return povintersection(ol,look);
endfunction
```

Kita sekarang dapat memplot adegan.

```
>povstart(angle=120°,center=[0.5,0.5,0.5],zoom=3.5); ...
>writeln(adm(A,b,2,povlook(green,0.4))); ...
>writeAxes(0,1.3,0,1.6,0,1.5); ...
```


Berikut ini adalah sebuah lingkaran di sekitar titik optimal.

```
>writeln(povintersection([povsphere(x,0.5),povplane(c,c.x')], ...  
>  povlook(red,0.9)));
```

Dan kesalahan ke arah optimum

```
>writeln(povarrow(x,c*0.5,povlook(red)));
```

Kami menambahkan teks ke layar. Teks hanyalah sebuah objek 3D. Kita perlu menempatkan dan mengubahnya sesuai dengan pandangan kita.

```
>writeln(povtext("Linear Problem",[0,0.2,1.3],size=0.05,rotate=125°)); ...  
>povend();
```

Contoh Lainnya

Anda dapat menemukan beberapa contoh lain untuk Povray di Euler dalam file-file berikut.

Lihat: Examples/Dandelin Spheres

Lihat: Contoh/Contoh/Donat Matematika

Lihat: Contoh/Simpul Trefoil

Lihat: Contoh/Optimalisasi dengan Penskalaan Affine

Contoh Soal

Grafik dari fungsi f dengan dua variabel yang dimaksud adalah grafik dari persamaan $z = f(x,y)$. Biasanya grafik ini berupa permukaan dan karena setiap (x,y) di daerah asal hanya berpadanan dengan satu nilai z , maka setiap garis tegaklurus bidang- xy memotong permukaan pada paling banyak satu titik.

Soal :

1. Grafik merupakan sebuah paraboloida $f(x,y) = y^2 - x^2$
2. $z = -4x^3y^2$
3. $z = xy \exp(-x^2 - y^2)$
4. $z = x - 1/8x^3 - 1/3y^2$

```
> aspect(1.5); plot3d("y^2-x^2"):
> aspect(1.5); plot3d("-4x^3*y^2"):
> plot3d("x*y*exp(-x^2-y^2)",r=2,<fscale,<scale,distance=13,height=20°, ...
> center=[0,0,-0.2],frame=3):
> aspect(1.5); plot3d("x-1/8*x^3-1/3*y^2"):
```

Kalkulus dengan EMT

Materi Kalkulus mencakup di antaranya:

- Fungsi (fungsi aljabar, trigonometri, eksponensial, logaritma, komposisi fungsi)
- Limit Fungsi,
- Turunan Fungsi,
- Integral Tak Tentu,
- Integral Tentu dan Aplikasinya,
- Barisan dan Deret (kekonvergenan barisan dan deret).

EMT (bersama Maxima) dapat digunakan untuk melakukan semua perhitungan di dalam kalkulus, baik secara numerik maupun analitik (eksak).

Mendefinisikan Fungsi

Terdapat beberapa cara mendefinisikan fungsi pada EMT, yakni:

- Menggunakan format `nama_fungsi := rumus fungsi` (untuk fungsi numerik),
- Menggunakan format `nama_fungsi &= rumus fungsi` (untuk fungsi simbolik, namun dapat dihitung secara numerik),
- Menggunakan format `nama_fungsi &&= rumus fungsi` (untuk fungsi simbolik murni, tidak dapat dihitung langsung),
- Fungsi sebagai program EMT.

Setiap format harus diawali dengan perintah `function` (bukan sebagai ekspresi).

Berikut adalah beberapa contoh cara mendefinisikan fungsi.

```
>function f(x) := 2*x^2+exp(sin(x)) // fungsi numerik  
>f(0), f(1), f(pi)
```

```
1  
4.31977682472  
20.7392088022
```

```
>f(a) // tidak dapat dihitung nilainya
```

Real 41 x 1 matrix

```
1  
1.21868  
1.55948  
2.01872  
2.58957  
3.26182  
4.02223  
4.85563  
5.74672  
6.68221  
7.65308  
8.65613  
9.69456  
10.7774  
11.9179  
13.1314  
14.4331  
15.8362  
17.3508
```

18.984

...

```
>function g(x) := sqrt(x^2-3*x)/(x+1)
>g(3)
```

0

```
>g(0)
```

0

```
>g(1) // kompleks, tidak dapat dihitung oleh fungsi numerik
```

Floating point error!

Error in sqrt

Try "trace errors" to inspect local variables after errors.

g:

```
  useglobal; return sqrt(x^2-3*x)/(x+1)
```

Error in:

```
g(1) // kompleks, tidak dapat dihitung oleh fungsi numerik ...
```

^

Silakan Anda plot kurva fungsi di atas

```
>f(g(5)) // komposisi fungsi
```

2.20920171961

```
>g(f(5))
```

0.950898070639

```
>function h(x) := f(g(x)) // definisi komposisi fungsi  
>h(5) // sama dengan f(g(5))
```

2.20920171961

Silakan Anda plot kurva fungsi komposisi fungsi f dan g:

$$h(x) = f(g(x))$$

dan

$$u(x) = g(f(x))$$

bersama-sama kurva fungsi f dan g dalam satu bidang koordinat.

```
>f(0:10) // nilai-nilai f(1), f(2), ..., f(10)
```

```
[1, 4.31978, 10.4826, 19.1516, 32.4692, 50.3833, 72.7562,  
99.929, 130.69, 163.51, 200.58]
```

```
>fmap(0:10) // sama dengan f(0:10), berlaku untuk semua fungsi
```

```
[1, 4.31978, 10.4826, 19.1516, 32.4692, 50.3833, 72.7562,  
99.929, 130.69, 163.51, 200.58]
```

Misalkan kita akan mendefinisikan fungsi

$$f(x) = \begin{cases} x^3 & x > 0 \\ x^2 & x \leq 0. \end{cases}$$

Fungsi tersebut tidak dapat didefinisikan sebagai fungsi numerik secara "inline" menggunakan format `:=`, melainkan didefinisikan sebagai program. Perhatikan, kata "map" digunakan agar fungsi dapat menerima vektor sebagai input, dan hasilnya berupa vektor. Jika tanpa kata "map" fungsinya hanya dapat menerima input satu nilai.

```
>function map f(x) ...
```

```
    if x>0 then return x^3
    else return x^2
    endif;
endfunction
```

```
>f(1)
```

1

```
>f(-2)
```

4

```
>f(-5:5)
```

[25, 16, 9, 4, 1, 0, 1, 8, 27, 64, 125]

```
>aspect(1.5); plot2d("f(x)",-5,5):
>function f(x) &= 2*E^x // fungsi simbolik
```



```
>$f(a) // nilai fungsi secara simbolik  
>f(E)
```

30.308524483

```
>$f(E)  
>function g(x) &= 3*x+1
```

$$3x + 1$$

```
>function h(x) &= f(g(x)) // komposisi fungsi
```

$$3x + 1$$
$$2E$$

```
>plot2d("h(x)",-1,1):
```

Bukalah buku Kalkulus. Cari dan pilih beberapa (paling sedikit 5 fungsi berbeda tipe/bentuk/jenis) fungsi dari buku tersebut, kemudian definisikan fungsi-sungsi tersebut dan komposisinya di EMT pada baris-baris perintah berikut (jika perlu tambahkan lagi). Untuk setiap fungsi, hitung beberapa nilainya, baik untuk satu nilai maupun vektor. Gambar grafik fungsi-fungsi tersebut dan komposisi - komposisi 2 fungsi.

Juga, carilah fungsi beberapa (dua) variabel. Lakukan hal sama seperti di atas.

Nomor 1

$$a(x) = x^3 - 4x^2 + 5x - 6$$

```
>function a(x) &= (x^3+4*x^2+5*x-6) // fungsi simbolik
```

$$x^3 + 4x^2 + 5x - 6$$

```
>function a(x) := (x^3+4*x^2+5*x-6) // fungsi numerik  
>a(5)
```

```
>a(-5:5)
```

```
[-56, -26, -12, -8, -8, -6, 4, 28, 72, 142, 244]
```

```
>aspect(2); plot2d("a(x)",-5,5):
```

Nomor 2

$$g(x) = \sqrt{x^2 + 4}$$

```
>function g(x) := (sqrt(x^2+4)) // fungsi numerik  
>g(2)
```

```
2.82842712475
```

```
>g=(-2:2)
```

```
[-2, -1, 0, 1, 2]
```

```
>aspect(2); plot2d("g(x)",-2,2):
```

Nomor 3

$$d(x) = \frac{x^3 + 2}{3x}$$

```
>function d(x) := ((x^3+2)/(3*x)) // fungsi numerik  
>d(4)
```

5.5

```
>d=(-4:4)
```

```
[-4, -3, -2, -1, 0, 1, 2, 3, 4]
```

```
>aspect(2); plot2d("d(x)",-4,4):
```

Nomor 4

$$f(x) = \cos x$$

$$g(x) = \sin x$$

```
>function f(x) &= (cos(x)) // fungsi numerik
```

`cos(x)`

```
>f(pi)
```

-1

```
>f(2*pi)
```

1

```
>function g(x) &= (sin(x)) // fungsi numerik
```

$\sin(x)$

```
>g(pi)
```

0

```
>g(2*pi)
```

0

```
>f(g(pi)) // komposisi fungsi
```

1

```
>g(f(pi))
```

-0.841470984808

```
>function h(x) &= f(g(pi))
```

1

```
>plot2d("h(x)",-2,2):
```

Nomor 5

$$e(x) = \begin{cases} -x^2 + 4 & x \leq 1 \\ 3x & x > 1. \end{cases}$$

```
>function map e(x) ...
```

```
  if x<=1 then return -x^2+4
  else return 3*x
endif;
endfunction
```

```
>e(5)
```

15

```
>e=(-5:5)
```

```
[-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5]
```

```
>aspect(2); plot2d("e(x)",-5,5):
```


Menghitung Limit

Perhitungan limit pada EMT dapat dilakukan dengan menggunakan fungsi Maxima, yakni "limit". Fungsi "limit" dapat digunakan untuk menghitung limit fungsi dalam bentuk ekspresi maupun fungsi yang sudah didefinisikan sebelumnya. Nilai limit dapat dihitung pada sebarang nilai atau pada tak hingga (-inf, minf, dan inf). Limit kiri dan limit kanan juga dapat dihitung, dengan cara memberi opsi "plus" atau "minus". Hasil limit dapat berupa nilai, "und" (tak definisi), "ind" (tak tentu namun terbatas), "infinity" (kompleks tak hingga).

Perhatikan beberapa contoh berikut. Perhatikan cara menampilkan perhitungan secara lengkap, tidak hanya menampilkan hasilnya saja.

```
>$limit((x^3-13*x^2+51*x-63)/(x^3-4*x^2-3*x+18),x,3)
```

```
maxima: 'limit((x^3-13*x^2+51*x-63)/(x^3-4*x^2-3*x+18),x,3)=limit((x^3-13*x^2+51*x-63)/(x^3-4*x^2-3*x+18),x,3)
```

Fungsi tersebut diskontinu di titik $x=3$. Berikut adalah grafik fungsinya.

```
>aspect(1.5); plot2d("(x^3-13*x^2+51*x-63)/(x^3-4*x^2-3*x+18)",0,4); plot2d(3,-4/5,>points,style="ow  
>$limit(2*x*sin(x)/(1-cos(x)),x,0)
```

maxima: `'limit(2*x*sin(x)/(1-cos(x)),x,0)=limit(2*x*sin(x)/(1-cos(x)),x,0)`

Fungsi tersebut diskontinu di titik $x=3$. Berikut adalah grafiknya.

```
>plot2d("2*x*sin(x)/(1-cos(x))",-pi,pi); plot2d(0,4,>points,style="ow",>add):  
>$limit(cot(7*h)/cot(5*h),h,0)
```

maxima: `showev('limit(cot(7*h)/cot(5*h),h,0))`

Fungsi tersebut juga diskontinu (karena tidak terdefinisi) di $x=0$. Berikut adalah grafiknya.

```
>plot2d("cot(7*x)/cot(5*x)",-0.001,0.001); plot2d(0,5/7,>points,style="ow",>add):  
>$showev('limit(((x/8)^(1/3)-1)/(x-8),x,8))  
>aspect(1.5); plot2d("((x/8)^(1/3)-1)/(x-8)",-1,1); plot2d(8,1/24,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit(1/(2*x-1),x,0))  
>plot2d("1/(2*x-1)",-2,2); plot2d(0,-1,>points,style="ow",>add):  
>$showev('limit((x^2-3*x-10)/(x-5),x,5))  
>plot2d("(x^2-3*x-10)/(x-5)",-8,8); plot2d(5,7,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit(sqrt(x^2+x)-x,x,inf))  
>plot2d("(sqrt(x^2+x)-x)",-1,1); plot2d(0,1/2,>points,style="ow",>add):  
>$showev('limit(abs(x-1)/(x-1),x,1,minus))
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit(sin(x)/x,x,0))  
>plot2d("sin(x)/x",-pi,pi); plot2d(0,1,>points,style="ow",>add):  
>$showev('limit(sin(x^3)/x,x,0))  
>plot2d("sin(x^3)/x",-pi,pi); plot2d(0,0,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit(log(x), x, minf))  
>$showev('limit((-2)^x,x, inf))  
>$showev('limit(t-sqrt(2-t),t,2,minus))  
>$showev('limit(t-sqrt(2-t),t,5,plus)) // Perhatikan hasilnya  
>plot2d("x-sqrt(2-x)",0,2):  
>$showev('limit((x^2-9)/(2*x^2-5*x-3),x,3))  
>plot2d("(x^2-9)/(2*x^2-5*x-3)",-1,1); plot2d(3,6/7,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit((1-cos(x))/x,x,0))  
>plot2d("(1-cos(x))/x",-pi,pi); plot2d(0,0,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit((x^2+abs(x))/(x^2-abs(x)),x,0))  
>plot2d("(x^2+abs(x))",-pi,pi); plot2d(0,-1,>points,style="ow",>add):
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit((1+1/x)^x,x,inf))  
>plot2d("(1+1/x)^x",0,1000):  
>$showev('limit((1+k/x)^x,x,inf))  
>$showev('limit((1+x)^(1/x),x,0))
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit((x/(x+k))^x,x,inf))  
>$showev('limit((E^x-E^2)/(x-2),x,2))
```

Tunjukkan limit tersebut dengan grafik, seperti contoh-contoh sebelumnya.

```
>$showev('limit(sin(1/x),x,0))  
>$showev('limit(sin(1/x),x,inf))  
>plot2d("sin(1/x)",-5,5):
```

Bukalah buku Kalkulus. Cari dan pilih beberapa (paling sedikit 5 fungsi berbeda tipe/bentuk/jenis) fungsi dari buku tersebut, kemudian definisikan di EMT pada baris-baris perintah berikut (jika perlu tambahkan lagi). Untuk setiap fungsi, hitung nilai limit fungsi tersebut di beberapa nilai dan di tak hingga. Gambar grafik fungsi tersebut untuk mengkonfirmasi nilai-nilai limit tersebut.

Nomor 1

```
>$showev('limit((x^2+2*x-1),x,2))  
>$showev('limit((x^2+2*x-1),x,4))  
>$showev('limit((x^2+2*x-1),x,inf))  
>plot2d("(x^2+2*x-1)",-8,8); plot2d(2,7,>points,style="ow",>add):
```

Nomor 2

```
>$showev('limit((x^4 + 2*x^3 - x^2)/(x^2),x,0))  
>$showev('limit((x^4 + 2*x^3 - x^2)/(x^2),x,1))  
>$showev('limit((x^4 + 2*x^3 - x^2)/(x^2),x,inf))  
>plot2d("(x^4 + 2*x^3 - x^2)",-1,1); plot2d(0,-1,>points,style="ow",>add):
```

Nomor 3

```
>$showev('limit(sqrt(3*x-5),x,inf))
>$showev('limit(sqrt(3*x-5),x,0))
>$showev('limit(sqrt(3*x-5),x,2))
>plot2d("sqrt(3*x-5)",-5,5); plot2d(2,1,>points,style="ow",>add):
```

Nomor 4

```
>$showev('limit(((x^2)+(2*x)),x,0,plus))
>$showev('limit(((x^2)+(2*x)),x,2,minus))
>$showev('limit(((x^2)+(2*x)),x,inf))
>plot2d("((x^2)+(2*x))",0,8):
```

Nomor 5

```
>$showev('limit((1-cos(x))/x^2,x,0))
>$showev('limit((1-cos(x))/x^2,x,inf))
>$showev('limit((1-cos(x))/x^2,x,2))
>plot2d("(1-cos(x))/x^2",-5,5):
```

Turunan Fungsi

Definisi turunan:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Berikut adalah contoh-contoh menentukan turunan fungsi dengan menggunakan definisi turunan (limit).

```
>$showev('limit(((x+h)^n-x^n)/h,h,0)) // turunan x^n
```

Mengapa hasilnya seperti itu? Tuliskan atau tunjukkan bahwa hasil limit tersebut benar, sehingga benar turunan fungsinya benar. Tulis penjelasan Anda di komentar ini.

Sebagai petunjuk, ekspansikan $(x+h)^n$ dengan menggunakan teorema binomial.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Untuk

$$f(x) = x^n$$

maka

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\f'(x) &= \lim_{h \rightarrow 0} \frac{(x^n + \frac{n}{1!}x^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^3 + \dots) - x^n}{h} \\f'(x) &= \lim_{h \rightarrow 0} \frac{n.x^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^3 + \dots}{h} \\f'(x) &= \lim_{h \rightarrow 0} n.x^{n-1} + \frac{n(n-1)}{2!}.x^{n-2}h + \frac{n(n-1)(n-2)}{3!}.x^{n-3}h^2 + \dots \\f'(x) &= n.x^{n-1} + 0 + 0 + \dots + 0 \\f'(x) &= n.x^{n-1}\end{aligned}$$

Jadi, terbukti bahwa

$$\lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} = n.x^{n-1}$$

```
>$showev('limit((sin(x+h)-sin(x))/h,h,0)) // turunan sin(x)
```

Mengapa hasilnya seperti itu? Tuliskan atau tunjukkan bahwa hasil limit tersebut benar, sehingga benar turunan fungsinya benar. Tulis penjelasan Anda di komentar ini. Sebagai petunjuk, ekspansikan $\sin(x+h)$ dengan menggunakan rumus jumlah dua sudut.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Untuk

$$f(x) = \sin(x)$$

maka

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{\sin(x)(\cos(h) - 1) + \cos(x)\sin(h)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \sin(x) \cdot \frac{\cos(h) - 1}{h} + \lim_{h \rightarrow 0} \cos(x) \cdot \frac{\sin(h)}{h} \\ f'(x) &= \sin(x) \cdot 0 + \cos(x) \cdot 1 \\ f'(x) &= \cos(x) \end{aligned}$$

Jadi, terbukti bahwa

$$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} = \cos(x)$$

```
>$showev('limit((log(x+h)-log(x))/h,h,0)) // turunan log(x)
```

Mengapa hasilnya seperti itu? Tuliskan atau tunjukkan bahwa hasil limit tersebut benar, sehingga benar turunan fungsinya benar. Tulis penjelasan Anda di komentar ini.

Sebagai petunjuk, gunakan sifat-sifat logaritma dan hasil limit pada bagian sebelumnya di atas.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

untuk

$$f(x) = \log(x)$$

maka

$$f'(x) = \lim_{h \rightarrow 0} \frac{\log(x+h) - \log(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\log \frac{x+h}{x}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \log \left(\frac{x+h}{x} \right)^{\frac{1}{h}}$$

$$f'(x) = \lim_{h \rightarrow 0} \log \left(1 + \frac{h}{x} \right)^{\frac{1}{h}}$$

$$f'(x) = \log \lim_{h \rightarrow 0} \left(1 + \frac{h}{x} \right)^{\frac{1}{h}}$$

$$f'(x) = \log e^{\frac{1}{x}}$$

$$f'(x) = \frac{1}{x} \log e$$

$$f'(x) = \frac{1}{x}$$

Jadi, terbukti bahwa

$$\lim_{h \rightarrow 0} \frac{\log(x+h) - \log(x)}{h} = \frac{1}{x}$$

```
>$showev('limit((1/(x+h)-1/x)/h,h,0)) // turunan 1/x
>$showev('limit((E^(x+h)-E^x)/h,h,0))// turunan f(x)=e^x
```

```
Answering "Is x an integer?" with "integer"
Answering "Is x an integer?" with "integer"
Answering "Is x an integer?" with "integer"
Answering "Is x an integer?" with "integer"
Answering "Is x an integer?" with "integer"
Maxima is asking
Acceptable answers are: yes, y, Y, no, n, N, unknown, uk
Is x an integer?
```

Use assume!

Error in:

```
$showev('limit((E^(x+h)-E^x)/h,h,0))// turunan f(x)=e^x ...
^
```

Maxima bermasalah dengan limit:

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}.$$

Oleh karena itu diperlukan trik khusus agar hasilnya benar.

```
>.,$showev('limit((E^h-1)/h,h,0))
```

Syntax error in expression, or unfinished expression!

Error in:

```
.,$showev('limit((E^h-1)/h,h,0)) ...
```

```
>$showev('factor(E^(x+h)-E^x))
>$showev('limit(factor((E^(x+h)-E^x)/h),h,0)) // turunan f(x)=e^x
>function f(x) &= x^x
```

x
x

```
>$showev('limit((f(x+h)-f(x))/h,h,0)) // turunan f(x)=x^x
```

Di sini Maxima juga bermasalah terkait limit:

$$\lim_{h \rightarrow 0} \frac{(x+h)^{x+h} - x^x}{h}.$$

Dalam hal ini diperlukan asumsi nilai x .

```
>&assume(x>0); $showev('limit((f(x+h)-f(x))/h,h,0)) // turunan f(x)=x^x  
>&forget(x>0) // jangan lupa, lupakan asumsi untuk kembali ke semula
```

$[x > 0]$

```
>&forget(x<0)
```

$[x < 0]$

```
>&facts()
```

```
[kind(sinh, one_to_one), kind(log, one_to_one),  
 kind(tanh, one_to_one), kind(log, increasing)]
```

```
>$showev('limit((asin(x+h)-asin(x))/h,h,0)) // turunan arcsin(x)  
>$showev('limit((tan(x+h)-tan(x))/h,h,0)) // turunan tan(x)  
>function f(x) &= sinh(x) // definisikan f(x)=sinh(x)
```

$\sinh(x)$

```
>function df(x) &= limit((f(x+h)-f(x))/h,h,0); $df(x) // df(x) = f'(x)
```

Hasilnya adalah $\cosh(x)$, karena

$$\frac{e^x + e^{-x}}{2} = \cosh(x).$$

```
>plot2d(["f(x)","df(x)"],-pi,pi,color=[blue,red]):  
>function f(x) &= sin(3*x^5+7)^2
```

$$\sin^2(3x^5 + 7)$$

```
>diff(f,3), diffc(f,3)
```

```
1198.32948904  
1198.72863721
```


Apakah perbedaan diff dan diffc?

```
>$showev('diff(f(x),x))  
>$% with x=3  
>$float(%)  
>plot2d(f,0,3.1):  
>function f(x) &=5*cos(2*x)-2*x*sin(2*x) // mendefinisikan fungsi f
```

$$5 \cos(2 x) - 2 x \sin(2 x)$$

```
>function df(x) &=diff(f(x),x) // fd(x) = f'(x)
```

$$- 12 \sin(2 x) - 4 x \cos(2 x)$$

```
>$'f(1)=f(1), $float(f(1)), $'f(2)=f(2), $float(f(2)) // nilai f(1) dan f(2)  
>xp=solve("df(x)",1,2,0) // solusi f'(x)=0 pada interval [1, 2]
```

1.35822987384

```
>df(xp), f(xp) // cek bahwa  $f'(x_p)=0$  dan nilai ekstrim di titik tersebut
```

```
0  
-5.67530133759
```

```
>plot2d(["f(x)","df(x)"],0,2*pi,color=[blue,red]): //grafik fungsi dan turunannya
```

Perhatikan titik-titik "puncak" grafik $y=f(x)$ dan nilai turunan pada saat grafik fungsinya mencapai titik "puncak" tersebut.

Latihan

Bukalah buku Kalkulus. Cari dan pilih beberapa (paling sedikit 5 fungsi berbeda tipe/bentuk/jenis) fungsi dari buku tersebut, kemudian definisikan di EMT pada baris-baris perintah berikut (jika perlu tambahkan lagi). Untuk setiap fungsi, tentukan turunannya dengan menggunakan definisi turunan (limit), seperti contoh-contoh tersebut. Gambar grafik fungsi asli dan fungsi turunannya pada sumbu koordinat yang sama.

Nomor 1

```
>function f(x) := cos(x^2)
>$showev('limit((cos((x+h)^2)- cos(x^2))/h,h,0))
>plot2d(["f(x)","df(x)"], -pi, pi, color=[green,red]):
```

Nomor 2

```
>function f(x) := sqrt(x^2+6)
>$showev('limit((sqrt((x+h)^2+6)-sqrt(x^2+6))/h,h,0))
>plot2d(["f(x)","df(x)"], -pi, pi, color=[green,red]):
```

Nomor 3

```
>function f(x) :=(2-x)^4  
>$showev('limit(((2-(x+h))^4-(2-x)^4)/h,h,0))  
>plot2d(["f(x)","df(x)"],-pi,pi,color=[green,red]):
```

Nomor 4

```
>function f(x) :=2*sin(x)+3*cos(x)  
>$showev('limit((2*sin(x+h)+3*cos(x+h)-(2*sin(x)+3*cos(x)))/h,h,0))  
>plot2d(["f(x)","df(x)"],-pi,pi,color=[green,red]):
```

Nomor 5

```
>function f(x) :=5*x-4  
>$showev('limit(((5*(x+h)-4)-(5*x-4))/h,h,0))  
>plot2d(["f(x)","df(x)"],-pi,pi,color=[green,red]):
```

EMT dapat digunakan untuk menghitung integral, baik integral tak tentu maupun integral tentu. Untuk integral tak tentu (simbolik) sudah tentu EMT menggunakan Maxima, sedangkan untuk perhitungan integral tentu EMT sudah menyediakan beberapa fungsi yang mengimplementasikan algoritma kuadratur (perhitungan integral tentu menggunakan metode numerik).

Pada notebook ini akan ditunjukkan perhitungan integral tentu dengan menggunakan Teorema Dasar Kalkulus:

$$\int_a^b f(x) \, dx = F(b) - F(a), \quad \text{dengan } F'(x) = f(x).$$

Fungsi untuk menentukan integral adalah `integrate`. Fungsi ini dapat digunakan untuk menentukan, baik integral tentu maupun tak tentu (jika fungsinya memiliki antiderivatif). Untuk perhitungan integral tentu fungsi `integrate` menggunakan metode numerik (kecuali fungsinya tidak integrabel, kita tidak akan menggunakan metode ini).

```
>$showev('integrate(x^n,x))
```

```
Answering "Is n equal to -1?" with "no"
```

```
>$showev('integrate(1/(1+x),x))
>$showev('integrate(1/(1+x^2),x))
>$showev('integrate(1/sqrt(1-x^2),x))
>$showev('integrate(sin(x),x,0,pi))
>$showev('integrate(sin(x),x,a,b))
>$showev('integrate(x^n,x,a,b))
```

Answering "Is n positive, negative or zero?" with "positive"

```
>$showev('integrate(x^2*sqrt(2*x+1),x))
>$showev('integrate(x^2*sqrt(2*x+1),x,0,2))
>$ratsimp(%)
>$showev('integrate((sin(sqrt(x)+a)*E^sqrt(x))/sqrt(x),x,0,pi^2))
>$factor(%)
>function map f(x) &= E^(-x^2)
```

$$E^{-x^2}$$

```
>$showev('integrate(f(x),x))
```

Fungsi f tidak memiliki antiturunan, integralnya masih memuat integral lain.

$$erf(x) = \int \frac{e^{-x^2}}{\sqrt{\pi}} dx.$$

Kita tidak dapat menggunakan teorema Dasar kalkulus untuk menghitung integral tentu fungsi tersebut jika semua batasnya berhingga. Dalam hal ini dapat digunakan metode numerik (rumus kuadratur).

Misalkan kita akan menghitung:

maxima: `'integrate(f(x),x,0,pi)`

```
>x=0:0.1:pi-0.1; plot2d(x,f(x+0.1),>bar); plot2d("f(x)",0,pi,>add):
```

Integral tentu

maxima: `'integrate(f(x),x,0,pi)`

dapat dihampiri dengan jumlah luas persegi-persegi panjang di bawah kurva $y=f(x)$ tersebut. Langkah-langkahnya adalah sebagai berikut.

```
>t &= makelist(a,a,0,pi-0.1,0.1); // t sebagai list untuk menyimpan nilai-nilai x
>fx &= makelist(f(t[i]+0.1),i,1,length(t)); // simpan nilai-nilai f(x)
>// jangan menggunakan x sebagai list, kecuali Anda pakar Maxima!
```

Hasilnya adalah:

maxima: `'integrate(f(x),x,0,pi) = 0.1*sum(fx[i],i,1,length(fx))`

Jumlah tersebut diperoleh dari hasil kali lebar sub-subinterval ($=0.1$) dan jumlah nilai-nilai $f(x)$ untuk $x = 0.1, 0.2, 0.3, \dots, 3.2$.

```
>0.1*sum(f(x+0.1)) // cek langsung dengan perhitungan numerik EMT
```

0.836219610253

Untuk mendapatkan nilai integral tentu yang mendekati nilai sebenarnya, lebar sub-intervalnya dapat diperkecil lagi, sehingga daerah di bawah kurva tertutup semuanya, misalnya dapat digunakan lebar subinterval 0.001. (Silakan dicoba!)

Meskipun Maxima tidak dapat menghitung integral tentu fungsi tersebut untuk batas-batas yang berhingga, namun integral tersebut dapat dihitung secara eksak jika batas-batasnya tak hingga. Ini adalah salah satu keajaiban di dalam matematika, yang terbatas tidak dapat dihitung secara eksak, namun yang tak hingga malah dapat dihitung secara eksak.

```
>$showev('integrate(f(x),x,0,inf))
```


Berikut adalah contoh lain fungsi yang tidak memiliki antiderivatif, sehingga integral tentunya hanya dapat dihitung dengan metode numerik.

```
>function f(x) &= x^x
```

x
x

```
>$showev('integrate(f(x),x,0,1))  
>x=0:0.1:1-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Maxima gagal menghitung integral tentu tersebut secara langsung menggunakan perintah integrate. Berikut kita lakukan seperti contoh sebelumnya untuk mendapat hasil atau pendekatan nilai integral tentu tersebut.

```
>t &= makelist(a,a,0,1-0.01,0.01);  
>fx &= makelist(f(t[i]+0.01),i,1,length(t));
```

maxima: `'integrate(f(x),x,0,1) = 0.01*sum(fx[i],i,1,length(fx))`

Apakah hasil tersebut cukup baik? perhatikan gambarnya.

```
>function f(x) &= sin(3*x^5+7)^2
```

$$\sin^2(3x^5 + 7)$$

```
>integrate(f,0,1)
```

0.542581176074

```
>&showev('integrate(f(x),x,0,1))
```

$$\int_0^1 \sin^2(3x^5 + 7) dx = \left(\int_0^1 \gamma_{\text{incomplete}}\left(-, 6\right) dx \right)$$

$$\begin{aligned}
& - \frac{1}{5} \Gamma_{\text{incomplete}}(-, -6I) \sin(7) \\
& + (-2 \frac{1}{5} \Gamma_{\text{incomplete}}(-, 6I) - 2 \frac{1}{5} \Gamma_{\text{incomplete}}(-, -6I) \\
& + 4 \frac{1}{5} \Gamma(-) \cos(7) \sin(7) + (I \frac{1}{5} \Gamma_{\text{incomplete}}(-, -6I) \\
& - I \frac{1}{5} \Gamma_{\text{incomplete}}(-, 6I) \cos(7)) \sin\left(\frac{\pi}{10}\right) + 10 \frac{1}{6} \sin^2(7) \\
& + 10 \frac{1}{6} \cos^2(7)) / (20 \frac{1}{6})
\end{aligned}$$

```
>&float(%)
```

$$\begin{aligned}
& \frac{1.0}{[I \sin(3.0 x^2 + 7.0) dx = 0.0} \\
& 0.03494135593857896 (0.3090169943749474 \\
& (0.4316313908960832 (I \Gamma_{\text{incomplete}}(0.2, 6.0 I) \\
& - 1.0 I \Gamma_{\text{incomplete}}(0.2, -6.0 I) \\
& + 0.5683686091039167 (I \Gamma_{\text{incomplete}}(0.2, -6.0 I) \\
& - 1.0 I \Gamma_{\text{incomplete}}(0.2, 6.0 I) \\
& + 0.4953036778474351 (-2.0 \Gamma_{\text{incomplete}}(0.2, 6.0 I) \\
& - 2.0 \Gamma_{\text{incomplete}}(0.2, -6.0 I) + 18.36337484799522)) \\
& + 14.30969081105255)
\end{aligned}$$

```
>$showev('integrate(x*exp(-x),x,0,1)) // Integral tentu (eksak)
```

Aplikasi Integral Tentu

```
>plot2d("x^3-x",-0.1,1.1); plot2d("-x^2",>add); ...  
>b=solve("x^3-x+x^2",0.5); x=linspace(0,b,200); xi=flipx(x); ...  
>plot2d(x|xi,x^3-x|-xi^2,>filled,style="|",fillcolor=1,>add): // Plot daerah antara 2 kurva  
>a=solve("x^3-x+x^2",0), b=solve("x^3-x+x^2",1) // absis titik-titik potong kedua kurva
```

```
0  
0.61803398875
```

```
>integrate("(-x^2)-(x^3-x)",a,b) // luas daerah yang diarsir
```

```
0.0758191713542
```

Hasil tersebut akan kita bandingkan dengan perhitungan secara analitik.

```
>a &= solve((-x^2)-(x^3-x),x); $a // menentukan absis titik potong kedua kurva secara eksak  
>$showev('integrate(-x^2-x^3+x,x,0,(sqrt(5)-1)/2)) // Nilai integral secara eksak  
>$float(%)
```

Hitunglah panjang kurva berikut ini dan luas daerah di dalam kurva tersebut.

$$\gamma(t) = (r(t) \cos(t), r(t) \sin(t))$$

dengan

$$r(t) = 1 + \frac{\sin(3t)}{2}, \quad 0 \leq t \leq 2\pi.$$

```
>t=linspace(0,2pi,1000); r=1+sin(3*t)/2; x=r*cos(t); y=r*sin(t); ...  
>plot2d(x,y,>filled,fillcolor=red,style="/",r=1.5): // Kita gambar kurvanya terlebih dahulu  
>function r(t) &= 1+sin(3*t)/2; $'r(t)=r(t)  
>function fx(t) &= r(t)*cos(t); $'fx(t)=fx(t)  
>\function fy(t) &= r(t)*sin(t); $'fy(t)=fy(t)
```

Syntax error in expression, or unfinished expression!

Error in:

```
\function fy(t) &= r(t)*sin(t); $'fy(t)=fy(t) ...  
^
```

```
>function ds(t) &= trigreduce(radcan(sqrt(diff(fx(t),t)^2+diff(fy(t),t)^2))); $'ds(t)=ds(t)
```

```
diff: second argument must be a variable; found errexp1
-- an error. To debug this try: debugmode(true);
```

```
... e(radcan(sqrt(diff(fx(t),t)^2+diff(fy(t),t)^2))); $'ds(t)=ds( ...
```

[illegible]

Berikut kita hitung integralnya secara umerik dengan perintah EMT.

```
Function ds not found.  
Try list ... to find functions!  
Error in expression: ds(x)  
  %mapexpression1:  
    return expr(x,args());  
Error in map.  
  %evalexpression:  
    if maps then return %mapexpression1(x,f$;args());  
gauss:
```

```

        if maps then y=%evalexpression(f$,a+h-(h*xn)',maps;args());
adaptivegauss:
    t1=gauss(f$,c,c+h;args(),=maps);
Try "trace errors" to inspect local variables after errors.
integrate:
    return adaptivegauss(f$,a,b,eps*1000;args(),=maps);

```

Spiral Logaritmik

$$x = e^{ax} \cos x, \quad y = e^{ax} \sin x.$$

```

>a=0.1; plot2d("exp(a*x)*cos(x)","exp(a*x)*sin(x)",r=2,xmin=0,xmax=2*pi):
>&kill(a) // hapus ekspresi a

```

done

```

>function fx(t) &= exp(a*t)*cos(t); $'fx(t)=fx(t)
>function fy(t) &= exp(a*t)*sin(t); $'fy(t)=fy(t)
>function df(t) &= trigreduce(radcan(sqrt(diff(fx(t),t)^2+diff(fy(t),t)^2))); $'df(t)=df(t)

```

Maxima said:

```

diff: second argument must be a variable; found errexp1
-- an error. To debug this try: debugmode(true);

```

Error in:

```

... e(radcan(sqrt(diff(fx(t),t)^2+diff(fy(t),t)^2))); $'df(t)=df(t) ...

```



```
>S &=integrate(df(t),t,0,2*%pi); $S // panjang kurva (spiral)
```

Maxima said:

```
defint: variable of integration cannot be a constant; found errexp1
-- an error. To debug this try: debugmode(true);
```

Error in:

```
S &=integrate(df(t),t,0,2*%pi); $S // panjang kurva (spiral) ...
^
```

```
>S(a=0.1) // Panjang kurva untuk a=0.1
```

Function S not found.

Try list ... to find functions!

Error in:

```
S(a=0.1) // Panjang kurva untuk a=0.1 ...
^
```

Berikut adalah contoh menghitung panjang parabola.

```
>plot2d("x^2",xmin=-1,xmax=1):
>$showev('integrate(sqrt(1+diff(x^2,x)^2),x,-1,1))
>$float(%)
>x=-1:0.2:1; y=x^2; plot2d(x,y); ...
> plot2d(x,y,points=1,style="o#",add=1):
```

Panjang tersebut dapat dihamperi dengan menggunakan jumlah panjang ruas-ruas garis yang menghubungkan titik-titik pada parabola tersebut.

```
>i=1:cols(x)-1; sum(sqrt((x[i+1]-x[i])^2+(y[i+1]-y[i])^2))
```

2.95191957027

Hasilnya mendekati panjang yang dihitung secara eksak. Untuk mendapatkan hampiran yang cukup akurat, jarak antar titik dapat diperkecil, misalnya 0.1, 0.05, 0.01, dan seterusnya. Cobalah Anda ulangi perhitungannya dengan nilai-nilai tersebut.

Koordinat Kartesius

Berikut diberikan contoh perhitungan panjang kurva menggunakan koordinat Kartesius. Kita akan hitung panjang kurva dengan persamaan implisit:

$$x^3 + y^3 - 3xy = 0.$$

```
>z &= x^3+y^3-3*x*y; $z  
>plot2d(z,r=2,level=0,n=100):
```

Kita tertarik pada kurva di kuadran pertama.

```
>plot2d(z,a=0,b=2,c=0,d=2,level=[-10;0],n=100,contourwidth=3,style="/"):
```

Kita selesaikan persamaannya untuk x .

```
>$z with y=l*x, sol &= solve(%,x); $sol
```

Kita gunakan solusi tersebut untuk mendefinisikan fungsi dengan Maxima.

```
>function f(l) &= rhs(sol[1]); $'f(l)=f(l)
```

Fungsi tersebut juga dapat digunaka untuk menggambar kurvanya. Ingat, bahwa fungsi tersebut adalah nilai x dan nilai $y=l*x$, yakni $x=f(l)$ dan $y=l*f(l)$.

```
>plot2d(&f(x),&x*f(x),xmin=-0.5,xmax=2,a=0,b=2,c=0,d=2,r=1.5):
```

Elemen panjang kurva adalah:

$$ds = \sqrt{f'(l)^2 + (lf'(l) + f(l))^2}.$$

```
>function ds(l) &= ratsimp(sqrt(diff(f(l),l)^2+diff(l*f(l),l)^2)); $'ds(l)=ds(l)
>$integrate(ds(l),l,0,1)
```

Integral tersebut tidak dapat dihitung secara eksak menggunakan Maxima. Kita hitung integral tersebut secara numerik dengan Euler. Karena kurva simetris, kita hitung untuk nilai variabel integrasi dari 0 sampai 1, kemudian hasilnya dikalikan 2.

```
>2*integrate("ds(x)",0,1)
```

4.91748872168

```
>2*romberg(&ds(x),0,1)// perintah Euler lain untuk menghitung nilai hampiran integral yang sama
```

4.91748872168

Perhitungan di atas dapat dilakukan untuk sebarang fungsi x dan y dengan mendefinisikan fungsi EMT, misalnya kita beri nama panjangkurva. Fungsi ini selalu memanggil Maxima untuk menurunkan fungsi yang diberikan.

```
>function panjangkurva(fx,fy,a,b) ...
```

```
    ds=mxm("sqrt(diff(@fx,x)^2+diff(@fy,x)^2)");  
    return romberg(ds,a,b);  
endfunction
```

```
>panjangkurva("x","x^2",-1,1) // cek untuk menghitung panjang kurva parabola sebelumnya
```

```
2.95788571509
```

Bandingkan dengan nilai eksak di atas.

```
>2*panjangkurva(mxm("f(x)"),mxm("x*f(x)"),0,1) // cek contoh terakhir, bandingkan hasilnya!
```

```
4.91748872168
```

Kita hitung panjang spiral Archimides berikut ini dengan fungsi tersebut.

```
>plot2d("x*cos(x)","x*sin(x)",xmin=0,xmax=2*pi,square=1):
>panjangkurva("x*cos(x)","x*sin(x)",0,2*pi)
```

21.2562941482

Berikut kita definisikan fungsi yang sama namun dengan Maxima, untuk perhitungan eksak.

```
>&kill(ds,x,fx,fy)
```

done

```
>function ds(fx,fy) &&= sqrt(diff(fx,x)^2+diff(fy,x)^2)
```

$$\sqrt{(\text{diff}(f_y, x))^2 + (\text{diff}(f_x, x))^2}$$

```
>sol &= ds(x*cos(x),x*sin(x)); $sol // Kita gunakan untuk menghitung panjang kurva terakhir di atas
>$sol | trigreduce | expand, $integrate(%,x,0,2*pi), %()
```

21.2562941482

Hasilnya sama dengan perhitungan menggunakan fungsi EMT.

Berikut adalah contoh lain penggunaan fungsi Maxima tersebut.

```
>plot2d("3*x^2-1","3*x^3-1",xmin=-1/sqrt(3),xmax=1/sqrt(3),square=1):  
>sol &= radcan(ds(3*x^2-1,3*x^3-1)); $sol  
>$showev('integrate(sol,x,0,1/sqrt(3))), $2*float(%) // panjang kurva di atas
```

Sikloid

Berikut kita akan menghitung panjang kurva lintasan (sikloid) suatu titik pada lingkaran yang berputar ke kanan pada permukaan datar. Misalkan jari-jari lingkaran tersebut adalah r . Posisi titik pusat lingkaran pada saat t adalah:

$$(rt, r).$$

Misalkan posisi titik pada lingkaran tersebut mula-mula $(0,0)$ dan posisinya pada saat t adalah:

$$(r(t - \sin(t)), r(1 - \cos(t))).$$

Berikut kita plot lintasan tersebut dan beberapa posisi lingkaran ketika $t=0$, $t=\pi/2$, $t=\pi$.

```
>x &= r*(t-sin(t))
```

```
[0, 1.66665833335744e-7 r, 1.33330666692022e-6 r,  
4.499797504338432e-6 r, 1.066581336583994e-5 r,  
2.083072932167196e-5 r, 3.599352055540239e-5 r,  
5.71526624672386e-5 r, 8.530603082730626e-5 r,  
1.214508019889565e-4 r, 1.665833531718508e-4 r,  
2.216991628251896e-4 r, 2.877927110806339e-4 r,  
3.658573803051457e-4 r, 4.568853557635201e-4 r,  
5.618675264007778e-4 r, 6.817933857540259e-4 r,  
8.176509330039827e-4 r, 9.704265741758145e-4 r,  
0.001141105023499428 r, 0.001330669204938795 r,  
0.001540100153900437 r, 0.001770376919130678 r,  
0.002022476464811601 r, 0.002297373572865413 r,  
0.002596040745477063 r, 0.002919448107844891 r,  
0.003268563311168871 r, 0.003644351435886262 r,  
0.004047774895164447 r, 0.004479793338660443 r, 0.0049413635565565 r,  
0.005433439383882244 r, 0.005956971605131645 r,  
0.006512907859185624 r, 0.007102192544548636 r,  
0.007725766724910044 r, 0.00838456803503801 r,  
0.009079530587017326 r, 0.009811584876838586 r, 0.0105816576913495 r,  
0.01139067201557714 r, 0.01223954694042984 r, 0.01312919757078923 r,  
0.01406053493400045 r, 0.01503446588876983 r, 0.01605189303448024 r,  
0.01711371462093175 r, 0.01822082445851714 r, 0.01937411182884202 r,  
0.02057446139579705 r, 0.02182275311709253 r, 0.02311986215626333 r,  
0.02446665879515308 r, 0.02586400834688696 r, 0.02731277106934082 r,  
0.02881380207911666 r, 0.03036795126603076 r, 0.03197606320812652 r,  
0.0336389770872163 r, 0.03535752660496472 r, 0.03713253989951881 r,  
0.03896483946269502 r, 0.0408552420577305 r, 0.04280455863760801 r,  
0.04481359426396048 r, 0.04688314802656623 r, 0.04901401296344043 r,
```



```
0.05120697598153157 r, 0.05346281777803219 r, 0.05578231276230905 r,  
0.05816622897846346 r, 0.06061532802852698 r, 0.0631303649963022 r,  
0.06571208837185505 r, 0.06836123997666599 r, 0.07107855488944881 r,  
0.07386476137264342 r, 0.07672058079958999 r, 0.07964672758239233 r,  
0.08264390910047736 r, 0.0857128256298576 r, 0.08885417027310427 r,  
0.09206862889003742 r, 0.09535688002914089 r, 0.0987195948597075 r,  
0.1021574371047232 r, 0.1056710629744951 r, 0.1092611211010309 r,  
0.1129282524731764 r, 0.1166730903725168 r, 0.1204962603100498 r,  
0.1243983799636342 r, 0.1283800591162231 r, 0.1324418995948859 r,  
0.1365844952106265 r, 0.140808431699002 r, 0.1451142866615502 r,  
0.1495026295080298 r, 0.1539740213994798 r]
```

```
>y &= r*(1-cos(t))
```

```
[0, 4.999958333473664e-5 r, 1.999933334222437e-4 r,  
4.499662510124569e-4 r, 7.998933390220841e-4 r,  
0.001249739605033717 r, 0.00179946006479581 r,  
0.002448999746720415 r, 0.003198293697380561 r,  
0.004047266988005727 r, 0.004995834721974179 r,  
0.006043902043303184 r, 0.00719136414613375 r, 0.00843810628521191 r,  
0.009784003787362772 r, 0.01122892206395776 r, 0.01277271662437307 r,  
0.01441523309043924 r, 0.01615630721187855 r, 0.01799576488272969 r,  
0.01993342215875837 r, 0.02196908527585173 r, 0.02410255066939448 r,  
0.02633360499462523 r, 0.02866202514797045 r, 0.03108757828935527 r,  
0.03361002186548678 r, 0.03622910363410947 r, 0.03894456168922911 r,  
0.04175612448730281 r, 0.04466351087439402 r, 0.04766643011428662 r,  
0.05076458191755917 r, 0.0539576564716131 r, 0.05724533447165381 r,  
0.06062728715262111 r, 0.06410317632206519 r, 0.06767265439396564 r,  
0.07133536442348987 r, 0.07509094014268702 r, 0.07893900599711501 r,  
0.08287917718339499 r, 0.08691105968769186 r, 0.09103425032511492 r,
```

```

0.09524833678003664 r, 0.09955289764732322 r, 0.1039475024744748 r,
0.1084317118046711 r, 0.113005077220716 r, 0.1176671413898787 r,
0.1224174381096274 r, 0.1272554923542488 r, 0.1321808203223502 r,
0.1371929294852391 r, 0.1422913186361759 r, 0.1474754779404944 r,
0.152744888986584 r, 0.1580990248377314 r, 0.1635373500848132 r,
0.1690593208998367 r, 0.1746643850903219 r, 0.1803519821545206 r,
0.1861215433374662 r, 0.1919724916878484 r, 0.1979042421157076 r,
0.2039162014509444 r, 0.2100077685026351 r, 0.216178334119151 r,
0.2224272812490723 r, 0.2287539850028937 r, 0.2351578127155118 r,
0.2416381240094921 r, 0.2481942708591053 r, 0.2548255976551299 r,
0.2615314412704124 r, 0.2683111311261794 r, 0.2751639892590951 r,
0.2820893303890569 r, 0.2890864619877229 r, 0.2961546843477643 r,
0.3032932906528349 r, 0.3105015670482534 r, 0.3177787927123868 r,
0.3251242399287333 r, 0.3325371741586922 r, 0.3400168541150183 r,
0.3475625318359485 r, 0.3551734527599992 r, 0.3628488558014202 r,
0.3705879734263036 r, 0.3783900317293359 r, 0.3862542505111889 r,
0.3941798433565377 r, 0.4021660177127022 r, 0.4102119749689023 r,
0.418316910536117 r, 0.4264800139275439 r, 0.4347004688396462 r,
0.4429774532337832 r, 0.451310139418413 r]

```

Berikut kita gambar sikloid untuk $r=1$.

```

>ex &= x-sin(x); ey &= 1-cos(x); aspect(1);
>plot2d(ex,ey,xmin=0,xmax=4pi,square=1); ...
> plot2d("2+cos(x)","1+sin(x)",xmin=0,xmax=2pi,>add,color=blue); ...
> plot2d([2,ex(2)], [1,ey(2)],color=red,>add); ...
> plot2d(ex(2),ey(2),>points,>add,color=red); ...
> plot2d("2pi+cos(x)","1+sin(x)",xmin=0,xmax=2pi,>add,color=blue); ...
> plot2d([2pi,ex(2pi)], [1,ey(2pi)],color=red,>add); ...
> plot2d(ex(2pi),ey(2pi),>points,>add,color=red):

```

```
Error : [0,1.66665833335744e-7*r-sin(1.66665833335744e-7*r),1.33330666692022e-6*r-sin(1.3333066669
```

Error generated by error() command

adaptiveeval:

```
error(f$|" does not produce a real or column vector");
```

Try "trace errors" to inspect local variables after errors.

plot2d:

```
dw/n,dw/n^2,dw/n;args());
```

Berikut dihitung panjang lintasan untuk 1 putaran penuh. (Jangan salah menduga bahwa panjang lintasan 1 putaran penuh sama dengan keliling lingkaran!)

```
>ds &= radcan(sqrt(diff(ex,x)^2+diff(ey,x)^2)); $ds=trigsimp(ds) // elemen panjang kurva sikloid
```

Maxima said:

```
diff: second argument must be a variable; found errexp1
```

```
-- an error. To debug this try: debugmode(true);
```

Error in:

```
ds &= radcan(sqrt(diff(ex,x)^2+diff(ey,x)^2)); $ds=trigsimp(ds ...
```

```
>ds &= trigsimp(ds); $ds
```

```
>$showev('integrate(ds,x,0,2*pi)) // hitung panjang sikloid satu putaran penuh
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate(ds,[0,1.66665833335744e-7*r,1.33330666692022e-6*r,4.499797504338432e-6*r,1
-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(ds,x,0,2*pi)) // hitung panjang sikloid sat ...
^

```
>integrate(mxm("ds"),0,2*pi) // hitung secara numerik
```

Illegal function result in map.

%evalexpression:

if maps then return %mapexpression1(x,f\$;args());

gauss:

if maps then y=%evalexpression(f\$,a+h-(h*xn)',maps;args());

adaptivegauss:

t1=gauss(f\$,c,c+h;args(),=maps);

Try "trace errors" to inspect local variables after errors.

integrate:

return adaptivegauss(f\$,a,b,eps*1000;args(),=maps);

```
>romberg(mxm("ds"),0,2*pi) // cara lain hitung secara numerik
```

Wrong argument!

Cannot combine a symbolic expression here.

Did you want to create a symbolic expression?

Then start with &.

Try "trace errors" to inspect local variables after errors.

romberg:

```
if cols(y)==1 then return y*(b-a); endif;
```

Error in:

```
romberg(mxm("ds"),0,2*pi) // cara lain hitung secara numerik ...
```

Perhatikan, seperti terlihat pada gambar, panjang sikloid lebih besar daripada keliling lingkarannya, yakni:

$$2\pi.$$

Kurvatur (Kelengkungan) Kurva

image: Osculating.png

Aslinya, kelengkungan kurva diferensiabel (yakni, kurva mulus yang tidak lancip) di titik P didefinisikan melalui lingkaran oskulasi (yaitu, lingkaran yang melalui titik P dan terbaik memperkirakan, paling banyak menyinggung kurva di sekitar P). Pusat dan radius kelengkungan kurva di P adalah pusat dan radius lingkaran oskulasi. Kelengkungan adalah kebalikan dari radius kelengkungan:

$$\kappa = \frac{1}{R}$$

dengan R adalah radius kelengkungan. (Setiap lingkaran memiliki kelengkungan ini pada setiap titiknya, dapat diartikan, setiap lingkaran berputar 2π sejauh $2\pi R$.)
Definisi ini sulit dimanipulasi dan dinyatakan ke dalam rumus untuk kurva umum. Oleh karena itu digunakan definisi lain yang ekuivalen.

Definisi Kurvatur dengan Fungsi Parametrik Panjang Kurva

Setiap kurva diferensiabel dapat dinyatakan dengan persamaan parametrik terhadap panjang kurva s :

$$\gamma(s) = (x(s), y(s)),$$

dengan x dan y adalah fungsi riil yang diferensiabel, yang memenuhi:

$$\|\gamma'(s)\| = \sqrt{x'(s)^2 + y'(s)^2} = 1.$$

Ini berarti bahwa vektor singgung

$$\mathbf{T}(s) = (x'(s), y'(s))$$

memiliki norm 1 dan merupakan vektor singgung satuan.

Apabila kurvanya memiliki turunan kedua, artinya turunan kedua x dan y ada, maka $\mathbf{T}'(s)$ ada. Vektor ini merupakan normal kurva yang arahnya menuju pusat kurvatur, norm-nya merupakan nilai kurvatur (kelengkungan):

$$\mathbf{T}(s) = \gamma'(s),$$

$$\|\mathbf{T}(s)\| = 1 \text{ (konstanta)} \Rightarrow \mathbf{T}'(s) \cdot \mathbf{T}(s) = 0$$

$$\kappa(s) = \|\mathbf{T}'(s)\| = \|\gamma''(s)\| = \sqrt{x''(s)^2 + y''(s)^2}.$$

Nilai

$$R(s) = \frac{1}{\kappa(s)}$$

disebut jari-jari (radius) kelengkungan kurva.

Bilangan riil

$$k(s) = \pm \kappa(s)$$

disebut nilai kelengkungan bertanda.

Contoh:

Akan ditentukan kurvatur lingkaran

$$x = r \cos t, \quad y = r \sin t.$$

```
>fx &= r*cos(t); fy &=r*sin(t);  
>&assume(t>0,r>0); s &=integrate(sqrt(diff(fx,t)^2+diff(fy,t)^2),t,0,t); s // elemen panjang kurva,
```

Maxima said:

```
diff: second argument must be a variable; found errexp1  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
... =integrate(sqrt(diff(fx,t)^2+diff(fy,t)^2),t,0,t); s // elemen ...  
^
```

```
>&kill(s); fx &= r*cos(s/r); fy &=r*sin(s/r); // definisi ulang persamaan parametrik terhadap s deng  
>k &= trigsimp(sqrt(diff(fx,s,2)^2+diff(fy,s,2)^2)); $k // nilai kurvatur lingkaran dengan menggunak
```

Untuk representasi parametrik umum, misalkan

$$x = x(t), \quad y = y(t)$$

merupakan persamaan parametrik untuk kurva bidang yang terdiferensialkan dua kali. Kurvatur untuk kurva tersebut didefinisikan sebagai

$$\begin{aligned}\kappa &= \frac{d\phi}{ds} = \frac{\frac{d\phi}{dt}}{\frac{ds}{dt}} \quad (\phi \text{ adalah sudut kemiringan garis singgung dan } s \text{ adalah panjang kurva}) \\ &= \frac{\frac{d\phi}{dt}}{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}} = \frac{\frac{d\phi}{dt}}{\sqrt{x'(t)^2 + y'(t)^2}}.\end{aligned}$$

Selanjutnya, pembilang pada persamaan di atas dapat dicari sebagai berikut.

$$\sec^2 \phi \frac{d\phi}{dt} = \frac{d}{dt} (\tan \phi) = \frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy/dt}{dx/dt} \right) = \frac{d}{dt} \left(\frac{y'(t)}{x'(t)} \right) = \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2}.$$

$$\begin{aligned}\frac{d\phi}{dt} &= \frac{1}{\sec^2 \phi} \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2} \\ &= \frac{1}{1 + \tan^2 \phi} \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2} \\ &= \frac{1}{1 + \left(\frac{y'(t)}{x'(t)}\right)^2} \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2} \\ &= \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2 + y'(t)^2}.\end{aligned}$$

Jadi, rumus kurvatur untuk kurva parametrik

$$x = x(t), \quad y = y(t)$$

adalah

$$\kappa(t) = \frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}.$$

Jika kurvanya dinyatakan dengan persamaan parametrik pada koordinat kutub

$$x = r(\theta) \cos \theta, \quad y = r(\theta) \sin \theta,$$

maka rumus kurvturnya adalah

$$\kappa(\theta) = \frac{r(\theta)^2 + 2r'(\theta)^2 - r(\theta)r''(\theta)}{(r'(\theta)^2 + r''(\theta)^2)^{3/2}}.$$

(Silakan Anda turunkan rumus tersebut!)

Contoh:

Lingkar dengan pusat (0,0) dan jari-jari r dapat dinyatakan dengan persamaan parametrik

$$x = r \cos t, \quad y = r \sin t.$$

Nilai kelengkungan lingkaran tersebut adalah

$$\kappa(t) = \frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}} = \frac{r^2}{r^3} = \frac{1}{r}.$$

Hasil cocok dengan definisi kurvatur suatu kelengkungan.

Kurva

$$y = f(x)$$

dapat dinyatakan ke dalam persamaan parametrik

$$x = t, \quad y = f(t), \quad \text{dengan } x'(t) = 1, \quad x''(t) = 0,$$

sehingga kurvturnya adalah

$$\kappa(t) = \frac{y''(t)}{(1 + y'(t)^2)^{3/2}}.$$

Contoh:

Akan ditentukan kurvatur parabola

$$y = ax^2 + bx + c.$$

```
>function f(x) &= a*x^2+b*x+c; $y=f(x)
>function k(x) &= (diff(f(x),x,2))/(1+diff(f(x),x)^2)^(3/2); $'k(x)=k(x) // kelengkungan parabola
```

Maxima said:

```
diff: second argument must be a variable; found errexp1
-- an error. To debug this try: debugmode(true);
```

Error in:

```
... (x) &= (diff(f(x),x,2))/(1+diff(f(x),x)^2)^(3/2); $'k(x)=k(x) ...
```

```
>function f(x) &= x^2+x+1; $y=f(x) // akan kita plot kelengkungan parabola untuk a=b=c=1
>function k(x) &= (diff(f(x),x,2))/(1+diff(f(x),x)^2)^(3/2); $'k(x)=k(x) // kelengkungan parabola
```

Maxima said:

```
diff: second argument must be a variable; found errexp1
-- an error. To debug this try: debugmode(true);
```

Error in:

```
... (x) &= (diff(f(x),x,2))/(1+diff(f(x),x)^2)^(3/2); $'k(x)=k(x) ...
```

Berikut kita gambar parabola tersebut beserta kurva kelengkungan, kurva jari-jari kelengkungan dan salah satu lingkaran oskulasi di titik puncak parabola. Perhatikan, puncak parabola dan jari-jari lingkaran oskulasi di puncak parabola adalah

$$(-1/2, 3/4), 1/k(2) = 1/2,$$

sehingga pusat lingkaran oskulasi adalah $(-1/2, 5/4)$.

```
>plot2d(["f(x)", "k(x)"],-2,1, color=[blue,red]); plot2d("1/k(x)",-1.5,1,color=green,>add); ...
>plot2d("-1/2+1/k(-1/2)*cos(x)", "5/4+1/k(-1/2)*sin(x)",xmin=0,xmax=2pi,>add,color=blue):
```

Error : f(x) does not produce a real or column vector

Error generated by error() command

%ploteval:

```
error(f$|" does not produce a real or column vector");
```

```

adaptiveevalone:
    s=%ploteval(g$,t,args());
Try "trace errors" to inspect local variables after errors.
plot2d:
    dw/n,dw/n^2,dw/n,auto,args());

```

Untuk kurva yang dinyatakan dengan fungsi implisit

$$F(x, y) = 0$$

dengan turunan-turunan parsial

$$F_x = \frac{\partial F}{\partial x}, \quad F_y = \frac{\partial F}{\partial y}, \quad F_{xy} = \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial x} \right), \quad F_{xx} = \frac{\partial}{\partial x} \left(\frac{\partial F}{\partial x} \right), \quad F_{yy} = \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial y} \right),$$

berlaku

$$F_x dx + F_y dy = 0 \text{ atau } \frac{dy}{dx} = -\frac{F_x}{F_y},$$

sehingga kurvturnya adalah

$$\kappa = \frac{F_y^2 F_{xx} - 2F_x F_y F_{xy} + F_x^2 F_{yy}}{(F_x^2 + F_y^2)^{3/2}}.$$

(Silakan Anda turunkan sendiri!)

Contoh 1:

Parabola

$$y = ax^2 + bx + c$$

dapat dinyatakan ke dalam persamaan implisit

$$ax^2 + bx + c - y = 0.$$

```
>function F(x,y) &=a*x^2+b*x+c-y; $F(x,y)
>Fx &= diff(F(x,y),x), Fxx &=diff(F(x,y),x,2), Fy &=diff(F(x,y),y), Fxy &=diff(diff(F(x,y),x),y), Fy
```

Maxima said:

```
diff: second argument must be a variable; found errexpl
-- an error. To debug this try: debugmode(true);
```

Error in:

```
Fx &= diff(F(x,y),x), Fxx &=diff(F(x,y),x,2), Fy &=diff(F(x,y) ...
^
```

```
>function k(x) &= (Fy^2*Fxx-2*Fx*Fy*Fxy+Fx^2*Fyy)/(Fx^2+Fy^2)^(3/2); $'k(x)=k(x) // kurvatur parabol
```

Hasilnya sama dengan sebelumnya yang menggunakan persamaan parabola biasa.

Latihan

-
- Bukalah buku Kalkulus.
 - Cari dan pilih beberapa (paling sedikit 5 fungsi berbeda tipe/bentuk/jenis) fungsi dari buku tersebut, kemudian definisikan di EMT pada baris-baris perintah berikut (jika perlu tambahkan lagi).
 - Untuk setiap fungsi, tentukan anti turunannya (jika ada), hitunglah integral tentu dengan batas-batas yang menarik (Anda tentukan sendiri), seperti contoh-contoh tersebut.
 - Lakukan hal yang sama untuk fungsi-fungsi yang tidak dapat diintegrasikan (cari sedikitnya 3 fungsi).
 - Gambar grafik fungsi dan daerah integrasinya pada sumbu koordinat yang sama.
 - Gunakan integral tentu untuk mencari luas daerah yang dibatasi oleh dua kurva yang berpotongan di dua titik. (Cari dan gambar kedua kurva dan arsir (warnai) daerah yang dibatasi oleh keduanya.)
 - Gunakan integral tentu untuk menghitung volume benda putar kurva $y=f(x)$ yang diputar mengelilingi sumbu x dari $x=a$ sampai $x=b$, yakni

$$V = \int_a^b \pi(f(x))^2 dx.$$

- (Pilih fungsinya dan gambar kurva dan benda putar yang dihasilkan. Anda dapat mencari contoh-contoh bagaimana cara menggambar benda hasil perputaran suatu kurva.)
- Gunakan integral tentu untuk menghitung panjang kurva $y=f(x)$ dari $x=a$ sampai $x=b$ dengan menggunakan rumus:

$$S = \int_a^b \sqrt{1 + (f'(x))^2} dx.$$

(Pilih fungsi dan gambar kurvanya.)

- Apabila fungsi dinyatakan dalam koordinat kutub $x=f(r,t)$, $y=g(r,t)$, $r=h(t)$, $x=a$ bersesuaian dengan $t=t_0$ dan $x=b$ bersesuaian dengan $t=t_1$, maka rumus di atas akan menjadi:

$$S = \int_{t_0}^{t_1} \sqrt{x'(t)^2 + y'(t)^2} dt.$$

- Pilih beberapa kurva menarik (selain lingkaran dan parabola) dari buku kalkulus. Nyatakan setiap kurva tersebut dalam bentuk:

- a. koordinat Kartesius (persamaan $y=f(x)$)
- b. koordinat kutub ($r=r(\theta)$)
- c. persamaan parametrik $x=x(t)$, $y=y(t)$
- d. persamaan implit $F(x,y)=0$

- Tentukan kurvatur masing-masing kurva dengan menggunakan keempat representasi tersebut (hasilnya harus sama).
- Gambarlah kurva asli, kurva kurvatur, kurva jari-jari lingkaran oskulasi, dan salah satu lingkaran oskulasinya.

Nomor 1

```
>function f(x):=sin(3x)
>$showev('integrate(sin(2*x),x))
```

```
Maxima output too long!
Error in:
  $showev('integrate(sin(2*x),x)) ...
      ^
```

```
>$showev('integrate(sin(2*x),x,0,pi/2))
```


Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([0,sin(3.333316666714881e-7*r),sin(2.66661333384044e-6*r),sin(8.9995950086

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(sin(2*x),x,0,pi/2)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Nomor 2

```
>function f(x):=sqrt(2*x^3+3*x)
>$showev('integrate(f(x),x))
```

Maxima output too long!

Error in:

\$showev('integrate(f(x),x)) ...
^

```
>$showev('integrate(sqrt(2*x^3+3*x),x,0,2))
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([0,sqrt(9.259120371466593e-21*r^3+4.999975000072321e-7*r),sqrt(4.740456304

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(sqrt(2*x^3+3*x),x,0,2)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Nomor 3

```
>function f(x):=(x^5+x)  
>$showev('integrate((x^5+x),x))
```

Maxima output too long!

Error in:

\$showev('integrate((x^5+x),x)) ...
^

```
>$showev('integrate(x^5+5,x,0,3))
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([5,1.285976080661417e-34*r^5+5,4.213570391232446e-30*r^5+5,1.8448661079775

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(x^5+5,x,0,3)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Nomor 4

```
>function f(x):=cos(2x)
>$showev('integrate(cos(2*x),x))
```

Maxima output too long!

Error in:

\$showev('integrate(cos(2*x),x)) ...
^

```
>$showev('integrate(cos(2*x),x,0,pi/2))
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([1,cos(3.333316666714881e-7*r),cos(2.66661333384044e-6*r),cos(8.9995950086

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(cos(2*x),x,0,pi/2)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Nomor 5

```
>function f(x):=5*x^3+4*x+2  
>$showev('integrate(5*x^3+4*x+2,x))
```

Maxima output too long!

Error in:

\$showev('integrate(5*x^3+4*x+2,x)) ...
^

```
>$showev('integrate(5*x^3+4*x+2,x,0,pi/2))
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([2,2.314780092866648e-20*r^3+6.666633333429761e-7*r+2,1.185114076172401e-1

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(5*x^3+4*x+2,x,0,pi/2)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Nomor 6

```
>function f(x):=cos(x)
>$showev('integrate(cos(x),x))
```

Maxima output too long!

Error in:

\$showev('integrate(cos(x),x)) ...
^

```
>$showev('integrate(cos(x),x,0,pi/2))
```

Maxima said:

defint: variable of integration must be a simple or subscripted variable.

defint: found errexp1

#0: showev(f='integrate([1,cos(1.66665833335744e-7*r),cos(1.333306666692022e-6*r),cos(4.49979750433

-- an error. To debug this try: debugmode(true);

Error in:

\$showev('integrate(cos(x),x,0,pi/2)) ...
^

```
>x=0:0.1:pi-0.01; plot2d(x,f(x+0.01),>bar); plot2d("f(x)",0,1,>add):
```

Barisan dan Deret

(Catatan: bagian ini belum lengkap. Anda dapat membaca contoh-contoh penggunaan EMT dan Maxima untuk menghitung limit barisan, rumus jumlah parsial suatu deret, jumlah tak hingga suatu deret konvergen, dan sebagainya. Anda dapat mengeksplor contoh-contoh di EMT atau berbagai panduan penggunaan Maxima di software Maxima atau dari Internet.)

Barisan dapat didefinisikan dengan beberapa cara di dalam EMT, di antaranya:

- dengan cara yang sama seperti mendefinisikan vektor dengan elemen-elemen beraturan (menggunakan titik dua ":");
- menggunakan perintah "sequence" dan rumus barisan (suku ke -n);
- menggunakan perintah "iterate" atau "niterate";
- menggunakan fungsi Maxima "create_list" atau "makelist" untuk menghasilkan barisan simbolik;
- menggunakan fungsi biasa yang inputnya vektor atau barisan;
- menggunakan fungsi rekursif.

EMT menyediakan beberapa perintah (fungsi) terkait barisan, yakni:

- sum: menghitung jumlah semua elemen suatu barisan
- cumsum: jumlah kumulatif suatu barisan
- differences: selisih antar elemen-elemen berturutan

EMT juga dapat digunakan untuk menghitung jumlah deret berhingga maupun deret tak hingga, dengan menggunakan perintah (fungsi) "sum". Perhitungan dapat dilakukan secara numerik maupun simbolik dan eksak.

Berikut adalah beberapa contoh perhitungan barisan dan deret menggunakan EMT.

```
>1:10 // barisan sederhana
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

```
>1:2:30
```

```
[1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29]
```


Iterasi dan Barisan

EMT menyediakan fungsi `iterate("g(x)", x0, n)` untuk melakukan iterasi

$$x_{k+1} = g(x_k), \quad x_0 = x_0, k = 1, 2, 3, \dots, n.$$

Berikut ini disajikan contoh-contoh penggunaan iterasi dan rekursi dengan EMT. Contoh pertama menunjukkan pertumbuhan dari nilai awal 1000 dengan laju pertambahan 5%, selama 10 periode.

```
>q=1.05; iterate("x*q",1000,n=10)'
```

```
1000
1050
1102.5
1157.63
1215.51
1276.28
1340.1
1407.1
1477.46
1551.33
1628.89
```

Contoh berikutnya memperlihatkan bahaya menabung di bank pada masa sekarang! Dengan bunga tabungan sebesar 6% per tahun atau 0.5% per bulan dipotong pajak 20%, dan biaya administrasi 10000 per bulan, tabungan sebesar 1 juta tanpa diambil selama sekitar 10 tahunan akan habis diambil oleh bank!

```
>r=0.005; plot2d(iterate("(1+0.8*r)*x-10000",1000000,n=130)):
```

Silakan Anda coba-coba, dengan tabungan minimal berapa agar tidak akan habis diambil oleh bank dengan ketentuan bunga dan biaya administrasi seperti di atas.

Berikut adalah perhitungan minimal tabungan agar aman di bank dengan bunga sebesar r dan biaya administrasi a , pajak bunga 20%.

```
>$solve(0.8*r*A-a,A), $% with [r=0.005, a=10]
```

Berikut didefinisikan fungsi untuk menghitung saldo tabungan, kemudian dilakukan iterasi.

```
>function saldo(x,r,a) := round((1+0.8*r)*x-a,2);  
>iterate({{"saldo",0.005,10}},1000,n=6)
```

```
[1000, 994, 987.98, 981.93, 975.86, 969.76, 963.64]
```

```
>iterate({{"saldo",0.005,10}},2000,n=6)
```

```
[2000, 1998, 1995.99, 1993.97, 1991.95, 1989.92, 1987.88]
```

```
>iterate({{"saldo",0.005,10}},2500,n=6)
```

```
[2500, 2500, 2500, 2500, 2500, 2500, 2500]
```

Tabungan senilai 2,5 juta akan aman dan tidak akan berubah nilai (jika tidak ada penarikan), sedangkan jika tabungan awal kurang dari 2,5 juta, lama kelamaan akan berkurang meskipun tidak pernah dilakukan penarikan uang tabungan.

```
>iterate({{"saldo",0.005,10}},3000,n=6)
```

```
[3000, 3002, 3004.01, 3006.03, 3008.05, 3010.08, 3012.12]
```

Tabungan yang lebih dari 2,5 juta baru akan bertambah jika tidak ada penarikan.

Untuk barisan yang lebih kompleks dapat digunakan fungsi "sequence()". Fungsi ini menghitung nilai-nilai $x[n]$ dari semua nilai sebelumnya, $x[1], \dots, x[n-1]$ yang diketahui.

Berikut adalah contoh barisan Fibonacci.

$$x_n = x_{n-1} + x_{n-2}, \quad x_1 = 1, \quad x_2 = 1$$

```
>sequence("x[n-1]+x[n-2]", [1,1], 15)
```

```
[1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610]
```

Barisan Fibonacci memiliki banyak sifat menarik, salah satunya adalah akar pangkat ke-n suku ke-n akan konvergen ke pecahan emas:

```
>$'(1+sqrt(5))/2=float((1+sqrt(5))/2)
>plot2d(sequence("x[n-1]+x[n-2]", [1,1], 250)^(1/(1:250))):
```

Barisan yang sama juga dapat dihasilkan dengan menggunakan loop.

```
>x=ones(500); for k=3 to 500; x[k]=x[k-1]+x[k-2]; end;
```

Rekursi dapat dilakukan dengan menggunakan rumus yang tergantung pada semua elemen sebelumnya. Pada contoh berikut, elemen ke-n merupakan jumlah (n-1) elemen sebelumnya, dimulai dengan 1 (elemen ke-1). Jelas, nilai elemen ke-n adalah $2^{(n-2)}$, untuk n=2, 4, 5,

```
>sequence("sum(x)", 1, 10)
```

[1, 1, 2, 4, 8, 16, 32, 64, 128, 256]

Selain menggunakan ekspresi dalam x dan n, kita juga dapat menggunakan fungsi.

Pada contoh berikut, digunakan iterasi

$$x_n = A \cdot x_{n-1},$$

dengan A suatu matriks 2x2, dan setiap $x[n]$ merupakan matriks/vektor 2x1.

```
>A=[1,1;1,2]; function suku(x,n) := A.x[,n-1]
>sequence("suku",[1;1],6)
```

Real 2 x 6 matrix

1	2	5	13	...
1	3	8	21	...

Hasil yang sama juga dapat diperoleh dengan menggunakan fungsi perpangkatan matriks "matrix-power()". Cara ini lebih cepat, karena hanya menggunakan perkalian matriks sebanyak $\log_2(n)$.

$$x_n = A.x_{n-1} = A^2.x_{n-2} = A^3.x_{n-3} = \dots = A^{n-1}.x_1.$$

```
>sequence("matrixpower(A,n).[1;1]",1,6)
```

Real 2 x 6 matrix

1	5	13	34	...
1	8	21	55	...

Spiral Theodorus

image: Spiral_of_Theodorus.png

Spiral Theodorus (spiral segitiga siku-siku) dapat digambar secara rekursif. Rumus rekursifnya adalah:

$$x_n = \left(1 + \frac{i}{\sqrt{n-1}}\right) x_{n-1}, \quad x_1 = 1,$$

yang menghasilkan barisan bilangan kompleks.

```
>function g(n) := 1+I/sqrt(n)
```

Rekursinya dapat dijalankan sebanyak 17 untuk menghasilkan barisan 17 bilangan kompleks, kemudian digambar bilangan-bilangan kompleksnya.

```
>x=sequence("g(n-1)*x[n-1]",1,17); plot2d(x,r=3.5); textbox(latex("Spiral\ Theodorus"),0.4):
```

Selanjutnya dihubungkan titik 0 dengan titik-titik kompleks tersebut menggunakan loop.

```
>for i=1:cols(x); plot2d([0,x[i]],>add); end:  
>
```

Spiral tersebut juga dapat didefinisikan menggunakan fungsi rekursif, yang tidak memerlukan indeks dan bilangan kompleks. Dalam hal ini digunakan vektor kolom pada bidang.

```
>function gstep (v) ...
```

```
    w=[-v[2];v[1]];
    return v+w/norm(w);
endfunction
```

Jika dilakukan iterasi 16 kali dimulai dari $[1;0]$ akan didapatkan matriks yang memuat vektor-vektor dari setiap iterasi.

```
>x=iterate("gstep",[1;0],16); plot2d(x[1],x[2],r=3.5,>points):
```

Kekonvergenan

Terkadang kita ingin melakukan iterasi sampai konvergen. Apabila iterasinya tidak konvergen setelah ditunggu lama, Anda dapat menghentikannya dengan menekan tombol [ESC].

```
>iterate("cos(x)",1) // iterasi  $x(n+1)=\cos(x(n))$ , dengan  $x(0)=1$ .
```

```
0.739085133216
```

Iterasi tersebut konvergen ke penyelesaian persamaan

$$x = \cos(x).$$

Iterasi ini juga dapat dilakukan pada interval, hasilnya adalah barisan interval yang memuat akar tersebut.

```
>hasil := iterate("cos(x)",~1,2~) //iterasi  $x(n+1)=\cos(x(n))$ , dengan interval awal (1, 2)
```

```
~0.739085133211,0.7390851332133~
```


Jika interval hasil tersebut sedikit diperlebar, akan terlihat bahwa interval tersebut memuat akar persamaan $x=\cos(x)$.

```
>h=expand(hasil,100), cos(h) << h
```

```
~0.73908513309,0.7390851333~  
1
```

Iterasi juga dapat digunakan pada fungsi yang didefinisikan.

```
>function f(x) := (x+2/x)/2
```

Iterasi $x_{(n+1)}=f(x_{(n)})$ akan konvergen ke akar kuadrat 2.

```
>iterate("f",2), sqrt(2)
```

```
1.41421356237  
1.41421356237
```

Jika pada perintah iterate diberikan tambahan parameter n, maka hasil iterasinya akan ditampilkan mulai dari iterasi pertama sampai ke-n.

```
>iterate("f",2,5)
```

```
[2, 1.5, 1.41667, 1.41422, 1.41421, 1.41421]
```

Untuk iterasi ini tidak dapat dilakukan terhadap interval.

```
>niterate("f",~1,2~,5)
```

```
[ ~1,2~, ~1,2~, ~1,2~, ~1,2~, ~1,2~, ~1,2~ ]
```

Perhatikan, hasil iterasinya sama dengan interval awal. Alasannya adalah perhitungan dengan interval bersifat terlalu longgar. Untuk meningkatkan perhitungan pada ekspresi dapat digunakan pembagian intervalnya, menggunakan fungsi `ieval()`.

```
>function s(x) := ieval("(x+2/x)/2",x,10)
```

Selanjutnya dapat dilakukan iterasi hingga diperoleh hasil optimal, dan intervalnya tidak semakin mengecil. Hasilnya berupa interval yang memuat akar persamaan:

$$x = \frac{1}{2} \left(x + \frac{2}{x} \right).$$

Satu-satunya solusi adalah

$$x = \sqrt{2}.$$

```
>iterate("s",~1,2~)
```

```
~1.41421356236,1.41421356239~
```

Fungsi "iterate()" juga dapat bekerja pada vektor. Berikut adalah contoh fungsi vektor, yang menghasilkan rata-rata aritmetika dan rata-rata geometri.

$$(a_{n+1}, b_{n+1}) = \left(\frac{a_n + b_n}{2}, \sqrt{a_n b_n} \right)$$

Iterasi ke-n disimpan pada vektor kolom x[n].

```
>function g(x) := [(x[1]+x[2])/2;sqrt(x[1]*x[2])]
```

Iterasi dengan menggunakan fungsi tersebut akan konvergen ke rata-rata aritmetika dan geometri dari nilai-nilai awal.

```
>iterate("g",[1;5])
```

```
2.60401
2.60401
```

Hasil tersebut konvergen agak cepat, seperti kita cek sebagai berikut.

```
>iterate("g",[1;5],4)
```

1	3	2.61803	2.60403	2.60401
5	2.23607	2.59002	2.60399	2.60401

Iterasi pada interval dapat dilakukan dan stabil, namun tidak menunjukkan bahwa limitnya pada batas-batas yang dihitung.

```
>iterate("g",[~1~;~5~],4)
```

Interval 2 x 5 matrix

```
~0.999999999999999778,1.000000000000000022~ ...
~4.99999999999999911,5.000000000000000089~ ...
```

Iterasi berikut konvergen sangat lambat.

$$x_{n+1} = \sqrt{x_n}.$$

```
>iterate("sqrt(x)",2,10)
```

```
[2, 1.41421, 1.18921, 1.09051, 1.04427, 1.0219, 1.01089,  
1.00543, 1.00271, 1.00135, 1.00068]
```

Kekonvergenan iterasi tersebut dapat dipercepat dengan percepatan Steffenson:

```
>steffenson("sqrt(x)",2,10)
```

```
[1.04888, 1.00028, 1, 1]
```

Iterasi menggunakan Loop yang ditulis Langsung

Berikut adalah beberapa contoh penggunaan loop untuk melakukan iterasi yang ditulis langsung pada baris perintah.

```
>x=2; repeat x=(x+2/x)/2; until x^2~=2; end; x,
```

```
1.41421356237
```

Penggabungan matriks menggunakan tanda "|" dapat digunakan untuk menyimpan semua hasil iterasi.

```
>v=[1]; for i=2 to 8; v=v|(v[i-1]*i); end; v,
```

```
[1, 2, 6, 24, 120, 720, 5040, 40320]
```

hasil iterasi juga dapat disimpan pada vektor yang sudah ada.

```
>v=ones(1,100); for i=2 to cols(v); v[i]=v[i-1]*i; end; ...  
>plot2d(v,logplot=1); textbox(latex(&log(n)),x=0.5):  
>A =[0.5,0.2;0.7,0.1]; b=[2;2]; ...  
>x=[1;1]; repeat xnew=A.x-b; until all(xnew~=x); x=xnew; end; ...  
>x,
```

-7.09677

-7.74194

Iterasi di dalam Fungsi

Fungsi atau program juga dapat menggunakan iterasi dan dapat digunakan untuk melakukan iterasi. Berikut adalah beberapa contoh iterasi di dalam fungsi.

Contoh berikut adalah suatu fungsi untuk menghitung berapa lama suatu iterasi konvergen. Nilai fungsi tersebut adalah hasil akhir iterasi dan banyak iterasi sampai konvergen.

```
>function map hiter(f$,x0) ...
```

```
  x=x0;
  maxiter=0;
  repeat
    xnew=f$(x);
    maxiter=maxiter+1;
    until xnew~=x;
    x=xnew;
  end;
  return maxiter;
endfunction
```

Misalnya, berikut adalah iterasi untuk mendapatkan hampiran akar kuadrat 2, cukup cepat, konvergen pada iterasi ke-5, jika dimulai dari hampiran awal 2.

```
>hiter("(x+2/x)/2",2)
```


Karena fungsinya didefinisikan menggunakan "map". maka nilai awalnya dapat berupa vektor.

```
>x=1.5:0.1:10; hasil=hiter("(x+2/x)/2",x); ...  
> plot2d(x,hasil):
```

Dari gambar di atas terlihat bahwa kekonvergenan iterasinya semakin lambat, untuk nilai awal semakin besar, namun penambahannya tidak kontinu. Kita dapat menemukan kapan maksimum iterasinya bertambah.

```
>hasil[1:10]
```

```
[4, 5, 5, 5, 5, 5, 6, 6, 6, 6]
```

```
>x[nonzeros(differences(hasil))]
```

```
[1.5, 2, 3.4, 6.6]
```

maksimum iterasi sampai konvergen meningkat pada saat nilai awalnya 1.5, 2, 3.4, dan 6.6.
Contoh berikutnya adalah metode Newton pada polinomial kompleks berderajat 3.

```
>p &= x^3-1; newton &= x-p/diff(p,x); $newton
```

Maxima said:

```
diff: second argument must be a variable; found errexpl  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
p &= x^3-1; newton &= x-p/diff(p,x); $newton ...  
^
```

Selanjutnya didefinisikan fungsi untuk melakukan iterasi (aslinya 10 kali).

```
>function iterasi(f$,x,n=10) ...
```

```
loop 1 to n; x=f$(x); end;  
return x;  
endfunction
```

Kita mulai dengan menentukan titik-titik grid pada bidang kompleksnya.

```
>r=1.5; x=linspace(-r,r,501); Z=x+I*x'; W=iterasi(newton,Z);
```

```
Function newton needs at least 3 arguments!  
Use: newton (f$: call, df$: call, x: scalar complex {, y: number, eps: none})  
Error in:  
... x=linspace(-r,r,501); Z=x+I*x'; W=iterasi(newton,Z); ...  
^
```

Berikut adalah akar-akar polinomial di atas.

```
>z=&solve(p)()
```

```
Maxima said:  
solve: more equations than unknowns.  
Unknowns given :  
[r]  
Equations given:  
errexp1  
-- an error. To debug this try: debugmode(true);
```

```
Error in:  
z=&solve(p)() ...  
^
```

Untuk menggambar hasil iterasinya, dihitung jarak dari hasil iterasi ke-10 ke masing-masing akar, kemudian digunakan untuk menghitung warna yang akan digambar, yang menunjukkan limit untuk masing-masing nilai awal.

Fungsi `plotrgb()` menggunakan jendela gambar terkini untuk menggambar warna RGB sebagai matriks.

```
>C=rgb(max(abs(W-z[1]),1),max(abs(W-z[2]),1),max(abs(W-z[3]),1)); ...  
> plot2d(none,-r,r,-r,r); plotrgb(C):
```

Variable W not found!

Error in:

```
C=rgb(max(abs(W-z[1]),1),max(abs(W-z[2]),1),max(abs(W-z[3]),1) ...  
      ^
```

Iterasi Simbolik

Seperti sudah dibahas sebelumnya, untuk menghasilkan barisan ekspresi simbolik dengan Maxima dapat digunakan fungsi `makelist()`.

```
>deret &= makelist(taylor(exp(x),x,0,k),k,1,3); $deret // barisan deret Taylor untuk e^x
```

Maxima said:

```
taylor: 0.1539740213994798*r cannot be a variable.  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
deret &= makelist(taylor(exp(x),x,0,k),k,1,3); $deret // baris ...  
^
```

Untuk mengubah barisan deret tersebut menjadi vektor string di EMT digunakan fungsi `mxm2str()`. Selanjutnya, vektor string/ekspresi hasilnya dapat digambar seperti menggambar vektor ekspresi pada EMT.

```
>plot2d("exp(x)",0,3); // plot fungsi aslinya, e^x  
>plot2d(mxm2str("deret"),>add,color=4:6): // plot ketiga deret taylor hampiran fungsi tersebut
```

Maxima said:

```
length: argument cannot be a symbol; found deret
-- an error. To debug this try: debugmode(true);
```

mxmeval:

```
return evaluate(mxm(s));
Try "trace errors" to inspect local variables after errors.
mxm2str:
n=mxmeval("length(VVV)");
```

Selain cara di atas dapat juga dengan cara menggunakan indeks pada vektor/list yang dihasilkan.

```
>$deret[3]
>plot2d(["exp(x)",&deret[1],&deret[2],&deret[3]],0,3,color=1:4):
```

deret is not a variable!

Error in expression: deret[1]

%ploteval:

```
y0=f$(x[1],args());
Try "trace errors" to inspect local variables after errors.
plot2d:
u=u_(%ploteval(xx[#],t,args()));
```

```
>$sum(sin(k*x)/k,k,1,5)
```

Berikut adalah cara menggambar kurva

$$y = \sin(x) + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots$$

```
>plot2d(&sum(sin((2*k+1)*x)/(2*k+1),k,0,20),0,2pi):
```

Maxima output too long!

Error in:

```
plot2d(&sum(sin((2*k+1)*x)/(2*k+1),k,0,20),0,2pi): ...
```

Hal serupa juga dapat dilakukan dengan menggunakan matriks, misalkan kita akan menggambar kurva

$$y = \sum_{k=1}^{100} \frac{\sin(kx)}{k}, \quad 0 \leq x \leq 2\pi.$$

```
>x=linspace(0,2pi,1000); k=1:100; y=sum(sin(k*x')/k)'; plot2d(x,y):
```

Tabel Fungsi

Terdapat cara menarik untuk menghasilkan barisan dengan ekspresi Maxima. Perintah `mxmtable()` berguna untuk menampilkan dan menggambar barisan dan menghasilkan barisan sebagai vektor kolom.

Sebagai contoh berikut adalah barisan turunan ke- n x^x di $x=1$.

```
>mxmtable("diffat(x^x,x=1,n)","n",1,8,frac=1);
```

```
Maxima said:
```

```
diff: second argument must be a variable; found errexpl
```

```
#0: diffat(expr=[0,1.66665833335744e-7*r,1.33330666692022e-6*r,4.499797504338432e-6*r,1.0665813365  
-- an error. To debug this try: debugmode(true);
```

```
%mxmevtable:
```

```
return mxm("@expr,@var=@value")();
```

```
Try "trace errors" to inspect local variables after errors.
```

```
mxmtable:
```

```
y[#,1]=%mxmevtable(expr,var,x[#]);
```

```
>$'sum(k, k, 1, n) = factor(ev(sum(k, k, 1, n),simpsum=true)) // simpsum:menghitung deret secara sim  
>$'sum(1/(3^k+k), k, 0, inf) = factor(ev(sum(1/(3^k+k), k, 0, inf),simpsum=true))
```


Di sini masih gagal, hasilnya tidak dihitung.

```
>$'sum(1/x^2, x, 1, inf)= ev(sum(1/x^2, x, 1, inf),simpsum=true) // ev: menghitung nilai ekspresi
>$'sum((-1)^(k-1)/k, k, 1, inf) = factor(ev(sum((-1)^(x-1)/x, x, 1, inf),simpsum=true))
```

Di sini masih gagal, hasilnya tidak dihitung.

```
>$'sum((-1)^k/(2*k-1), k, 1, inf) = factor(ev(sum((-1)^k/(2*k-1), k, 1, inf),simpsum=true))
>$ev(sum(1/n!, n, 0, inf),simpsum=true)
```

Di sini masih gagal, hasilnya tidak dihitung, harusnya hasilnya e.

```
>&assume(abs(x)<1); $'sum(a*x^k, k, 0, inf)=ev(sum(a*x^k, k, 0, inf),simpsum=true), &forget(abs(x)<1)
```

Answering "Is 15819*r-94914474571 positive, negative or zero?" with "positive"

Maxima said:

sum: sum is divergent.

-- an error. To debug this try: debugmode(true);

Error in:

```
... k, 0, inf)=ev(sum(a*x^k, k, 0, inf),simpsum=true), &forget(abs ...
```

Deret geometri tak hingga, dengan asumsi rasional antara -1 dan 1.

```
>$'sum(x^k/k!,k,0,inf)=ev(sum(x^k/k!,k,0,inf),simpsum=true)
>$limit(sum(x^k/k!,k,0,n),n,inf)
>function d(n) &= sum(1/(k^2-k),k,2,n); $'d(n)=d(n)
>$d(10)=ev(d(10),simpsum=true)
>$d(100)=ev(d(100),simpsum=true)
```

Deret Taylor

Deret Taylor suatu fungsi f yang diferensiabel sampai tak hingga di sekitar $x=a$ adalah:

$$f(x) = \sum_{k=0}^{\infty} \frac{(x-a)^k f^{(k)}(a)}{k!}.$$

```
>$'e^x =taylor(exp(x),x,0,10) // deret Taylor e^x di sekitar x=0, sampai suku ke-11
```

Maxima said:

```
taylor: 0.1539740213994798*r cannot be a variable.  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
$'e^x =taylor(exp(x),x,0,10) // deret Taylor e^x di sekitar x= ...  
^
```

```
>$'log(x)=taylor(log(x),x,1,10)// deret log(x) di sekitar x=1
```

Maxima said:

```
log: encountered log(0).  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
$'log(x)=taylor(log(x),x,1,10)// deret log(x) di sekitar x=1 ...  
^
```

Visualisasi dan Perhitungan Geometri dengan EMT

Euler menyediakan beberapa fungsi untuk melakukan visualisasi dan perhitungan geometri, baik secara numerik maupun analitik (seperti biasanya tentunya, menggunakan Maxima). Fungsi-fungsi untuk visualisasi dan perhitungan geometri tersebut disimpan di dalam file program "geometry.e", sehingga file tersebut harus dipanggil sebelum menggunakan fungsi-fungsi atau perintah-perintah untuk geometri.

```
>load geometry
```

Numerical and symbolic geometry.

Fungsi-fungsi Geometri

Fungsi-fungsi untuk Menggambar Objek Geometri:

```
defaultd:=textheight()*1.5: nilai asli untuk parameter d  
setPlotrange(x1,x2,y1,y2): menentukan rentang x dan y pada bidang
```

koordinat

`setPlotRange(r)`: pusat bidang koordinat (0,0) dan batas-batas

sumbu-x dan y adalah -r sd r

`plotPoint (P, "P")`: menggambar titik P dan diberi label "P"

`plotSegment (A,B, "AB", d)`: menggambar ruas garis AB, diberi label

"AB" sejauh d

`plotLine (g, "g", d)`: menggambar garis g diberi label "g" sejauh d

`plotCircle (c,"c",v,d)`: Menggambar lingkaran c dan diberi label "c"

`plotLabel (label, P, V, d)`: menuliskan label pada posisi P

Fungsi-fungsi Geometri Analitik (numerik maupun simbolik):

`turn(v, phi)`: memutar vektor v sejauh phi

`turnLeft(v)`: memutar vektor v ke kiri

`turnRight(v)`: memutar vektor v ke kanan

`normalize(v)`: normal vektor v

`crossProduct(v, w)`: hasil kali silang vektor v dan w.

`lineThrough(A, B)`: garis melalui A dan B, hasilnya [a,b,c] sdh.

$ax+by=c$.

`lineWithDirection(A,v)`: garis melalui A searah vektor v
`getLineDirection(g)`: vektor arah (gradien) garis g
`getNormal(g)`: vektor normal (tegak lurus) garis g
`getPointOnLine(g)`: titik pada garis g
`perpendicular(A, g)`: garis melalui A tegak lurus garis g
`parallel (A, g)`: garis melalui A sejajar garis g
`lineIntersection(g, h)`: titik potong garis g dan h
`projectToLine(A, g)`: proyeksi titik A pada garis g
`distance(A, B)`: jarak titik A dan B
`distanceSquared(A, B)`: kuadrat jarak A dan B
`quadrance(A, B)`: kuadrat jarak A dan B
`areaTriangle(A, B, C)`: luas segitiga ABC
`computeAngle(A, B, C)`: besar sudut $\angle ABC$
`angleBisector(A, B, C)`: garis bagi sudut $\angle ABC$
`circleWithCenter (A, r)`: lingkaran dengan pusat A dan jari-jari r
`getCircleCenter(c)`: pusat lingkaran c
`getCircleRadius(c)`: jari-jari lingkaran c
`circleThrough(A,B,C)`: lingkaran melalui A, B, C
`middlePerpendicular(A, B)`: titik tengah AB
`lineCircleIntersections(g, c)`: titik potong garis g dan lingkaran c
`circleCircleIntersections (c1, c2)`: titik potong lingkaran $c1$ dan

c2

`planeThrough(A, B, C)`: bidang melalui titik A, B, C

Fungsi-fungsi Khusus Untuk Geometri Simbolik:

`getLineEquation (g,x,y)`: persamaan garis g dinyatakan dalam x dan y
`getHesseForm (g,x,y,A)`: bentuk Hesse garis g dinyatakan dalam x dan

y dengan titik A pada

sisi positif (kanan/atas) garis

quad(A,B): kuadrat jarak AB

spread(a,b,c): Spread segitiga dengan panjang sisi-sisi a,b,c, yakni

$\sin(\alpha)^2$ dengan

α sudut yang menghadap sisi a.

crosslaw(a,b,c,sa): persamaan 3 quads dan 1 spread pada segitiga

dengan panjang sisi a, b, c.

triplespread(sa,sb,sc): persamaan 3 spread sa,sb,sc yang membentuk

suatu segitiga

doublespread(sa): Spread sudut rangkap Spread 2ϕ , dengan

$sa = \sin(\phi)^2$ spread a.

Contoh 1: Luas, Lingkaran Luar, Lingkaran Dalam Segitiga

Untuk menggambar objek-objek geometri, langkah pertama adalah menentukan rentang sumbu-sumbu koordinat. Semua objek geometri akan digambar pada satu bidang koordinat, sampai didefinisikan bidang koordinat yang baru.

```
>setPlotRange(-0.5,2.5,-0.5,2.5); // mendefinisikan bidang koordinat baru
```

Sekarang tetapkan tiga poin dan plot mereka.

```
>A=[1,0]; plotPoint(A,"A"); // definisi dan gambar tiga titik  
>B=[0,1]; plotPoint(B,"B");  
>C=[2,2]; plotPoint(C,"C");
```

Kemudian tiga segmen.

```
>plotSegment(A,B,"c"); // c=AB  
>plotSegment(B,C,"a"); // a=BC  
>plotSegment(A,C,"b"); // b=AC
```


Fungsi geometri meliputi fungsi untuk membuat garis dan lingkaran. Format garis adalah $[a,b,c]$, yang mewakili garis dengan persamaan $ax+by=c$.

```
>lineThrough(B,C) // garis yang melalui B dan C
```

```
[-1, 2, 2]
```

Hitunglah garis tegak lurus yang melalui A pada BC.

```
>h=perpendicular(A,lineThrough(B,C)); // garis h tegak lurus BC melalui A
```

Dan persimpangannya dengan BC.

```
>D=lineIntersection(h,lineThrough(B,C)); // D adalah titik potong h dan BC
```

Plot itu.

```
>plotPoint(D,value=1); // koordinat D ditampilkan  
>aspect(1); plotSegment(A,D): // tampilkan semua gambar hasil plot...()
```

Hitung luas ABC:

$$L_{\triangle ABC} = \frac{1}{2} AD \cdot BC.$$

```
>norm(A-D)*norm(B-C)/2 // AD=norm(A-D) , BC=norm(B-C)
```

1.5

Bandingkan dengan rumus determinan.

```
>areaTriangle(A,B,C) // hitung luas segitiga langsung dengan fungsi
```

1.5

Cara lain menghitung luas segitiga ABC:

```
>distance(A,D)*distance(B,C)/2
```

1.5

Sudut di C

```
>degprint(computeAngle(B,C,A))
```

36°52'11.63''

Sekarang lingkaran luar segitiga.

```
>c=circleThrough(A,B,C); // lingkaran luar segitiga ABC  
>R=getCircleRadius(c); // jari2 lingkaran luar  
>O=getCircleCenter(c); // titik pusat lingkaran c  
>plotPoint(O,"O"); // gambar titik "O"  
>plotCircle(c,"Lingkaran luar segitiga ABC"):
```

Tampilkan koordinat titik pusat dan jari-jari lingkaran luar.

```
>O, R
```

```
[1.16667, 1.16667]  
1.17851130198
```

Sekarang akan digambar lingkaran dalam segitiga ABC. Titik pusat lingkaran dalam adalah titik potong garis-garis bagi sudut.

```
>l=angleBisector(A,C,B); // garis bagi <ACB  
>g=angleBisector(C,A,B); // garis bagi <CAB  
>P=lineIntersection(l,g) // titik potong kedua garis bagi sudut
```

```
[0.86038, 0.86038]
```

Tambahkan semuanya ke plot.

```
>color(5); plotLine(l); plotLine(g); color(1); // gambar kedua garis bagi sudut  
>plotPoint(P,"P"); // gambar titik potongnya  
>r=norm(P-projectToLine(P,lineThrough(A,B))) // jari-jari lingkaran dalam
```

```
0.509653732104
```

```
>plotCircle(circleWithCenter(P,r),"Lingkaran dalam segitiga ABC"): // gambar lingkaran dalam
```

1. Tentukan ketiga titik singgung lingkaran dalam dengan sisi-sisi segitiga ABC.

```
>setPlotRange(-2.5,4.5,-2.5,4.5);  
>A=[-2,1]; plotPoint(A,"A");  
>B=[1,-2]; plotPoint(B,"B");  
>C=[4,4]; plotPoint(C,"C"):
```

2. Gambar segitiga dengan titik-titik sudut ketiga titik singgung tersebut.

```
>plotSegment(A,B,"c")  
>plotSegment(B,C,"a")  
>plotSegment(A,C,"b")  
>aspect(1):
```

3. Tunjukkan bahwa garis bagi sudut yang ke tiga juga melalui titik pusat lingkaran dalam.

```
>l=angleBisector(A,C,B);  
>g=angleBisector(C,A,B);  
>P=lineIntersection(l,g)
```

[0.581139, 0.581139]

```
>color(5); plotLine(l); plotLine(g); color(1);  
>plotPoint(P,"P");  
>r=norm(P-projectToLine(P,lineThrough(A,B)))
```

1.52896119631

```
>plotCircle(circleWithCenter(P,r),"Lingkaran dalam segitiga ABC"):
```

Jadi, terbukti bahwa garis bagi sudut yang ketiga juga melalui titik pusat lingkaran dalam.

4. Gambar jari-jari lingkaran dalam.

```
>r=norm(P-projectToLine(P,lineThrough(A,B)))
```

1.52896119631

```
>plotCircle(circleWithCenter(P,r),"Lingkaran dalam segitiga ABC"):
```

Contoh 2: Geometri Simbolik

Kita dapat menghitung geometri eksak dan simbolik menggunakan Maxima.

File `geometri.e` menyediakan fungsi yang sama (dan lebih banyak lagi) di Maxima. Namun, kita dapat menggunakan perhitungan simbolis sekarang.

```
>A &= [1,0]; B &= [0,1]; C &= [2,2]; // menentukan tiga titik A, B, C
```

Fungsi untuk garis dan lingkaran bekerja seperti fungsi Euler, tetapi memberikan perhitungan simbolis.

```
>c &= lineThrough(B,C) // c=BC
```

```
[- 1, 2, 2]
```

Kita bisa mendapatkan persamaan garis dengan mudah.

```
>$getLineEquation(c,x,y), $solve(%,y) | expand // persamaan garis c
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

\$getlineEquation(c,x,y), \$solve(%,y) | expand // persamaan gar ...
^

```
>$getlineEquation(lineThrough(A,[x1,y1]),x,y) // persamaan garis melalui A dan (x1, y1)
>h &= perpendicular(A,lineThrough(B,C)) // h melalui A tegak lurus BC
```

[2, 1, 2]

```
>Q &= lineIntersection(c,h) // Q titik potong garis c=BC dan h
```

Maxima said:

rat: replaced 9.983250083613754e-5 by 612914/6139423483 = 9.983250083613756e-5

rat: replaced 3.986533601775671e-4 by 220554/553247563 = 3.986533601775666e-4

rat: replaced 8.954327045205754e-4 by 584699/652979277 = 8.954327045205756e-4

rat: replaced 0.001589120864678328 by 740868/466212493 = 0.00158912086467833

rat: replaced 0.002478648480745763 by 878917/354595259 = 0.002478648480745762

rat: replaced 0.003562926609036218 by 2735717/767828614 = 0.003562926609036219

rat: replaced 0.004840846830973591 by $1164348/240525685 = 0.004840846830973582$

rat: replaced 0.006311281363933816 by $16515210/2616776063 = 0.006311281363933816$

rat: replaced 0.007973083174022497 by $2414321/302808957 = 0.007973083174022491$

rat: replaced 0.009825086090776508 by $1144049/116441626 = 0.009825086090776506$

rat: replaced 0.01186610492378118 by $1659683/139867548 = 0.01186610492378118$

rat: replaced 0.01409493558118687 by $986877/70016425 = 0.01409493558118684$

rat: replaced 0.01651035519011868 by $1738361/105289134 = 0.01651035519011867$

rat: replaced 0.01911112221896202 by $1475047/77182647 = 0.01911112221896199$

rat: replaced 0.02189597660151474 by $7711274/352177669 = 0.02189597660151473$

rat: replaced 0.02486363986299212 by $3887839/156366446 = 0.02486363986299209$

rat: replaced 0.0280128152478745 by $2263313/80795628 = 0.02801281524787455$

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rat: replaced 0.7419827122498429 by $23520179/31699093 = 0.7419827122498426$

rat: replaced 0.7592581822173726 by $16709871/22008154 = 0.7592581822173727$

part: invalid index of list or matrix.

```
#0: lineIntersection(g=[-1,2,2],h=[2,1,2])
-- an error. To debug this try: debugmode(true);

Error in:
... ersection(c,h) // Q titik potong garis c=BC dan h ...
```

```
>$projectToLine(A,lineThrough(B,C)) // proyeksi A pada BC
```

Maxima said:

```
rat: replaced 5.033291500140813e-5 by 263336/5231884543 = 5.033291500140813e-5

rat: replaced 2.026599467560841e-4 by 407727/2011877564 = 2.02659946756084e-4

rat: replaced 4.589658460211338e-4 by 352373/767754296 = 4.589658460211339e-4

rat: replaced 8.21224965753764e-4 by 219501/267284860 = 8.212249657537654e-4

rat: replaced 0.001291401063677061 by 174589/135193477 = 0.001291401063677059

rat: replaced 0.001871447105906615 by 1078337/576204904 = 0.001871447105906617

rat: replaced 0.002563305071654892 by 1323915/516487489 = 0.002563305071654891

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rat: replaced 0.005329001428317881 by 3020890/566877311 = 0.005329001428317882

rat: replaced 0.006487300368953564 by 2580732/397812935 = 0.006487300368953564
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rat: replaced 0.007766949568295017 by $1049181/135082762 = 0.007766949568295028$

rat: replaced 0.009169821045822202 by $2408608/262666849 = 0.009169821045822193$

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rat: replaced 0.1564153650475627 by $30759845/196654881 = 0.1564153650475627$

rat: replaced 0.1635663609012215 by $11970848/73186491 = 0.1635663609012215$

rat: replaced 0.1709009985884339 by $3726835/21806982 = 0.1709009985884339$

rat: replaced 0.1784205446348769 by $7050541/39516419 = 0.178420544634877$

rat: replaced 0.1861262470755453 by $7913431/42516470 = 0.1861262470755451$

rat: replaced 0.1940193353299499 by $15356416/79148895 = 0.19401933532995$

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rat: replaced 0.2188349273697929 by $14393696/65774217 = 0.2188349273697929$

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rat: replaced 0.3037740645557676 by $12785981/42090430 = 0.3037740645557672$

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rat: replaced 0.3248412332121354 by $13048490/40168823 = 0.3248412332121357$

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rat: replaced 0.4819272183079686 by $11623658/24119115 = 0.4819272183079686$

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rat: replaced 0.3332782472122374 by $5743591/17233621 = 0.333278247212237$

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rat: replaced 0.3630188086379282 by $51253958/141188161 = 0.3630188086379282$

rat: replaced 0.373132388978704 by $9370061/25111894 = 0.3731323889787047$

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rat: replaced 0.393647586516613 by $9153768/23253713 = 0.3936475865166135$

rat: replaced 0.4040451522277552 by $16634707/41170416 = 0.404045152227755$

rat: replaced 0.4145333126687146 by $2088920/5039209 = 0.4145333126687145$

rat: replaced 0.4251100190405208 by $24667763/58026774 = 0.4251100190405209$

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rat: replaced 0.4573507941689697 by $20158688/44077081 = 0.4573507941689696$

rat: replaced 0.4682610222756929 by $12818601/27374905 = 0.4682610222756937$

rat: replaced 0.4792494236287415 by $13652513/28487281 = 0.4792494236287416$

rat: replaced 0.4903138994054704 by $35114711/71616797 = 0.4903138994054705$

rat: replaced 0.5014523431758559 by $15102855/30118226 = 0.5014523431758564$

rat: replaced 0.5126626411131362 by $31697340/61828847 = 0.5126626411131361$

rat: replaced 0.5239426722051925 by $27432767/52358337 = 0.5239426722051924$

rat: replaced 0.5352903084666492 by $6124470/11441399 = 0.5352903084666482$

rat: replaced 0.5467034151516694 by $41717397/76307182 = 0.5467034151516694$

rat: replaced 0.5581798509674292 by $7494380/13426461 = 0.5581798509674292$

rat: replaced 0.5697174682882435 by $14609183/25642856 = 0.5697174682882438$

rat: replaced 0.581314113370329 by $14367580/24715691 = 0.5813141133703282$

rat: replaced 0.5929676265671738 by $9820294/16561265 = 0.5929676265671735$

rat: replaced 0.6046758425455033 by $23593213/39017952 = 0.6046758425455031$

```

rat: replaced 0.6164365905018095 by 15720181/25501700 = 0.6164365905018097
rat: replaced 0.6282476943794307 by 53974636/85912987 = 0.6282476943794306
rat: replaced 0.640106973086155 by 20459615/31962806 = 0.6401069730861552
rat: replaced 0.652012240712328 by 51645100/79208789 = 0.652012240712328
rat: replaced 0.6639613067494411 by 12215999/18398661 = 0.6639613067494422
rat: replaced 0.6759519763091814 by 18558734/27455699 = 0.6759519763091808
rat: replaced 0.6879820503429186 by 23500536/34158647 = 0.687982050342919
rat: replaced 0.7000493258616074 by 29992669/42843651 = 0.7000493258616078
rat: replaced 0.7121515961560857 by 10685401/15004391 = 0.7121515961560853
rat: replaced 0.7242866510177421 by 11795807/16286103 = 0.7242866510177419
rat: replaced 0.7364522769595366 by 14940657/20287339 = 0.7364522769595362
rat: replaced 0.7486462574373463 by 42508133/56779998 = 0.7486462574373461
part: invalid index of list or matrix.
#0: lineIntersection(g=[2,1,2],h=[-1,2,2])
#1: projectToLine(a=[1,0],g=[-1,2,2])
-- an error. To debug this try: debugmode(true);

```

Error in:

```

$projectToLine(A,lineThrough(B,C)) // proyeksi A pada BC ...
^

```

```
>$distance(A,Q) // jarak AQ  
>cc &= circleThrough(A,B,C); $cc // (titik pusat dan jari-jari) lingkaran melalui A, B, C
```

Maxima said:

```
rat: replaced -4.98329175014009e-5 by -86001/1725786976 = -4.983291750140082e-5  
  
rat: replaced -1.986600267553235e-4 by -1133306/5704751069 = -1.986600267553234e-4  
  
rat: replaced -4.454664535081185e-4 by -474290/1064704191 = -4.454664535081181e-4  
  
rat: replaced -7.892275256562442e-4 by -1190199/1508055613 = -7.892275256562439e-4  
  
rat: replaced -0.001228908875712045 by -259907/211494119 = -0.001228908875712047  
  
rat: replaced -0.001763466544240408 by -5854594/3319934829 = -0.001763466544240408  
  
rat: replaced -0.002391847084253176 by -866601/362314550 = -0.002391847084253172  
  
rat: replaced -0.003112987666553255 by -5049204/1621980085 = -0.003112987666553255  
  
rat: replaced -0.00392581618601677 by -1241039/316122544 = -0.003925816186016774  
  
rat: replaced -0.004829251368802329 by -3015690/624463249 = -0.00482925136880233  
  
rat: replaced -0.005822202880477995 by -2532373/434951006 = -0.005822202880477991  
  
rat: replaced -0.006903571435053116 by -1331361/192851050 = -0.006903571435053115  
  
rat: replaced -0.008072248904906765 by -7953293/985263598 = -0.008072248904906766  
  
rat: replaced -0.009327118431599252 by -432515/46371771 = -0.009327118431599259  
  
rat: replaced -0.01066705453755698 by -2950074/276559381 = -0.01066705453755698
```


rat: replaced -0.01209092323861904 by $-1254816/103781653 = -0.01209092323861907$

rat: replaced -0.01359758215743526 by $-1827823/134422648 = -0.01359758215743526$

rat: replaced -0.01518588063770274 by $-9199276/605778237 = -0.01518588063770274$

rat: replaced -0.01685465985923026 by $-2516580/149310637 = -0.01685465985923026$

rat: replaced -0.01860275295381958 by $-2032371/109251088 = -0.01860275295381955$

rat: replaced -0.02042898512195129 by $-1413911/69211025 = -0.02042898512195131$

rat: replaced -0.02233217375026381 by $-3647892/163346929 = -0.02233217375026377$

rat: replaced -0.02431112852981362 by $-1377268/56651751 = -0.02431112852981367$

rat: replaced -0.02636465157510504 by $-2533336/96088355 = -0.02636465157510502$

rat: replaced -0.0284915375438782 by $-9699307/340427644 = -0.02849153754387819$

rat: replaced -0.03069057375764189 by $-7938451/258660886 = -0.03069057375764189$

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rat: replaced -0.05637740959715515 by $-11093364/196769665 = -0.05637740959715513$

rat: replaced -0.05928808635892763 by $-3489209/58851773 = -0.05928808635892754$

rat: replaced -0.06225583383647254 by $-3380435/54299088 = -0.06225583383647254$

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rat: replaced -0.07148850516781785 by $-5513427/77123266 = -0.07148850516781798$

rat: replaced -0.07467151274726203 by $-2259975/30265558 = -0.07467151274726208$

rat: replaced -0.07790505275432569 by $-657797/8443573 = -0.07790505275432569$

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rat: replaced -0.1018429767138303 by $-3912367/38415678 = -0.1018429767138302$

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rat: replaced -0.1201627068711536 by $-9494831/79016454 = -0.1201627068711537$

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rat: replaced -0.1916952326588277 by $-16014703/83542521 = -0.1916952326588277$

rat: replaced -0.1958193528985573 by $-21279927/108671215 = -0.1958193528985574$

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rat: replaced -0.2040854343696463 by $-17847769/87452439 = -0.2040854343696464$

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rat: replaced -0.2247887414183958 by $-11437558/50881365 = -0.2247887414183955$

rat: replaced -0.2289246224392826 by $-17547464/76651711 = -0.2289246224392825$

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rat: replaced -0.2412972592553108 by $-36037383/149348497 = -0.2412972592553108$

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rat: replaced -0.7067159597263644 by $-7239052/10243227 = -0.7067159597263636$

rat: replaced -0.724411755697878 by $-65966965/91062803 = -0.7244117556978781$

rat: replaced -0.742317108747504 by $-29643877/39934250 = -0.7423171087475037$

rat: replaced -0.7604312283465253 by $-19521554/25671689 = -0.760431228346526$

rat: replaced -0.778753303089744 by $-17717453/22751047 = -0.7787533030897436$

rat: replaced -0.7972825007766203 by $-6544613/8208650 = -0.7972825007766198$

rat: replaced -0.8160179684944936 by $-15744063/19293770 = -0.8160179684944933$

rat: replaced -0.8349588327038714 by $-31965589/38284030 = -0.8349588327038716$

rat: replaced -0.8541041993257835 by $-22076179/25847173 = -0.8541041993257832$

rat: replaced -0.8734531538311887 by $-59286729/67876255 = -0.8734531538311888$

rat: replaced -0.8930047613324276 by $-6137127/6872446 = -0.8930047613324281$

```

rat: replaced -0.9127580666767096 by -13137137/14392792 = -0.9127580666767088
rat: replaced -0.9327120945416275 by -15972523/17124816 = -0.932712094541629
rat: replaced -0.9528658495326905 by -44894507/47115244 = -0.9528658495326905
rat: replaced -0.9732183162828605 by -25482581/26183828 = -0.9732183162828598
rat: replaced -0.9937684595540898 by -23211595/23357146 = -0.9937684595540911
rat: replaced -1.014515224340843 by -33401253/32923363 = -1.014515224340843
rat: replaced -1.035457535975596 by -9211102/8895683 = -1.035457535975596

rat: replaced -1.056594300236306 by -24469996/23159311 = -1.056594300236307
part: invalid index of list or matrix.
#0: lineIntersection(g=[1,-1,0],h=[-1,-2,-7/2])
#1: circleThrough(a=[1,0],b=[0,1],c=[2,2])
-- an error. To debug this try: debugmode(true);

Error in:
cc &= circleThrough(A,B,C); $cc // (titik pusat dan jari-jari) ...
      ^

```

```

>r:=getCircleRadius(cc); $r , $float(r) // tampilkan nilai jari-jari
>$computeAngle(A,C,B) // nilai <ACB
>$solve(getLineEquation(angleBisector(A,C,B),x,y),y)[1] // persamaan garis bagi <ACB

```

```

Maxima said:
solve: all variables must not be numbers.
-- an error. To debug this try: debugmode(true);

```

Error in:

```
... (getLineEquation(angleBisector(A,C,B),x,y),y)[1] // persamaan ...  
^
```

```
>P &= lineIntersection(angleBisector(A,C,B),angleBisector(C,B,A)); $P // titik potong 2 garis bagi s
```

Maxima said:

```
rat: replaced -4.98329175014009e-5 by -86001/1725786976 = -4.983291750140082e-5
```

```
rat: replaced -1.986600267553235e-4 by -1133306/5704751069 = -1.986600267553234e-4
```

```
rat: replaced -4.454664535081185e-4 by -474290/1064704191 = -4.454664535081181e-4
```

```
rat: replaced -7.892275256562442e-4 by -1190199/1508055613 = -7.892275256562439e-4
```

```
rat: replaced -0.001228908875712045 by -259907/211494119 = -0.001228908875712047
```

```
rat: replaced -0.001763466544240408 by -5854594/3319934829 = -0.001763466544240408
```

```
rat: replaced -0.002391847084253176 by -866601/362314550 = -0.002391847084253172
```

```
rat: replaced -0.003112987666553255 by -5049204/1621980085 = -0.003112987666553255
```

```
rat: replaced -0.00392581618601677 by -1241039/316122544 = -0.003925816186016774
```

```
rat: replaced -0.004829251368802329 by -3015690/624463249 = -0.00482925136880233
```

```
rat: replaced -0.005822202880477995 by -2532373/434951006 = -0.005822202880477991
```

```
rat: replaced -0.006903571435053116 by -1331361/192851050 = -0.006903571435053115
```

rat: replaced -0.008072248904906765 by $-7953293/985263598 = -0.008072248904906766$

rat: replaced -0.009327118431599252 by $-432515/46371771 = -0.009327118431599259$

rat: replaced -0.01066705453755698 by $-2950074/276559381 = -0.01066705453755698$

rat: replaced -0.01209092323861904 by $-1254816/103781653 = -0.01209092323861907$

rat: replaced -0.01359758215743526 by $-1827823/134422648 = -0.01359758215743526$

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rat: replaced -0.01860275295381958 by $-2032371/109251088 = -0.01860275295381955$

rat: replaced -0.02042898512195129 by $-1413911/69211025 = -0.02042898512195131$

rat: replaced -0.02233217375026381 by $-3647892/163346929 = -0.02233217375026377$

rat: replaced -0.02431112852981362 by $-1377268/56651751 = -0.02431112852981367$

rat: replaced -0.02636465157510504 by $-2533336/96088355 = -0.02636465157510502$

rat: replaced -0.0284915375438782 by $-9699307/340427644 = -0.02849153754387819$

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rat: replaced -0.05352509460807248 by $-3894269/72755948 = -0.05352509460807246$

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rat: replaced -0.05928808635892763 by $-3489209/58851773 = -0.05928808635892754$

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rat: replaced -0.08118780184603619 by $-4832180/59518547 = -0.08118780184603633$

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rat: replaced -0.116427310289289 by $-22239618/191017193 = -0.116427310289289$

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rat: replaced -0.2371802941295513 by $-11052217/46598378 = -0.237180294129551$

rat: replaced -0.2412972592553108 by $-36037383/149348497 = -0.2412972592553108$

rat: replaced -0.2454050947312253 by $-4652365/18957899 = -0.2454050947312252$

rat: replaced -0.2495023897855041 by $-6175634/24751803 = -0.2495023897855037$

rat: replaced -0.2535877347003893 by $-11299519/44558618 = -0.2535877347003895$

rat: replaced -0.2576597209531272 by $-6871877/26670358 = -0.2576597209531271$

rat: replaced -0.2617169413568191 by $-2245730/8580759 = -0.2617169413568194$

rat: replaced -0.2657579902011391 by $-10500993/39513367 = -0.2657579902011388$

rat: replaced -0.2697814633929034 by $-21050552/78028163 = -0.2697814633929034$

rat: replaced -0.2737859585964791 by $-1510231/5516101 = -0.2737859585964796$

rat: replaced -0.2777700753740163 by $-9819093/35349715 = -0.2777700753740164$

rat: replaced -0.2817324153254904 by $-10837378/38466919 = -0.2817324153254905$

rat: replaced -0.2856715822285418 by $-17041418/59653879 = -0.2856715822285421$

rat: replaced -0.289586182178096 by $-721506/2491507 = -0.2895861821780955$

rat: replaced -0.2934748237257534 by $-11793110/40184401 = -0.2934748237257537$

rat: replaced -0.2973361180189332 by $-15390047/51759763 = -0.2973361180189329$

rat: replaced $1.66665833335744e-7$ by $15819/94914474571 = 1.66665833335744e-7$

rat: replaced $4.999958333473664e-5$ by $201389/4027813565 = 4.99995833347366e-5$

rat: replaced $1.33330666692022e-6$ by $31771/23828726570 = 1.333306666920221e-6$

rat: replaced $1.999933334222437e-4$ by $200030/1000183339 = 1.999933334222437e-4$

rat: replaced $4.499797504338432e-6$ by $24036/5341573699 = 4.499797504338431e-6$

rat: replaced $4.499662510124569e-4$ by $1162901/2584418270 = 4.499662510124571e-4$

rat: replaced $1.066581336583994e-5$ by $58861/5518660226 = 1.066581336583993e-5$

rat: replaced $7.998933390220841e-4$ by $1137431/1421978337 = 7.998933390220838e-4$

rat: replaced $2.083072932167196e-5$ by $35635/1710693824 = 2.0830729321672e-5$

rat: replaced 0.001249739605033717 by $567943/454449069 = 0.001249739605033716$

rat: replaced $3.599352055540239e-5$ by $98277/2730408098 = 3.599352055540234e-5$

rat: replaced 0.00179946006479581 by $479561/266502719 = 0.001799460064795812$

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rat: replaced 0.002448999746720415 by $1946227/794702818 = 0.002448999746720415$

rat: replaced $8.530603082730626e-5$ by $121691/1426522824 = 8.530603082730627e-5$

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rat: replaced $1.665833531718508e-4$ by $142521/855553675 = 1.66583353171851e-4$

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rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced $2.877927110806339e-4$ by $1167733/4057548906 = 2.877927110806339e-4$

rat: replaced 0.00719136414613375 by $2476362/344352191 = 0.007191364146133747$

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rat: replaced 0.009784003787362772 by $1752551/179124113 = 0.009784003787362787$

rat: replaced $5.618675264007778e-4$ by $150595/268025812 = 5.618675264007782e-4$

rat: replaced 0.01122892206395776 by $5450241/485375263 = 0.01122892206395776$

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rat: replaced 0.01277271662437307 by $3258991/255152533 = 0.01277271662437308$

rat: replaced $8.176509330039827e-4$ by $105841/129445214 = 8.176509330039812e-4$

rat: replaced 0.01441523309043924 by $2330472/161667313 = 0.01441523309043925$

rat: replaced $9.704265741758145e-4$ by $651321/671169790 = 9.704265741758132e-4$

rat: replaced 0.01615630721187855 by $19391318/1200232067 = 0.01615630721187855$

rat: replaced 0.001141105023499428 by $1259907/1104111343 = 0.001141105023499428$

rat: replaced 0.01799576488272969 by $4765614/264818641 = 0.01799576488272969$

rat: replaced 0.001330669204938795 by $1231154/925214167 = 0.001330669204938796$

rat: replaced 0.01993342215875837 by $2504519/125644206 = 0.01993342215875836$

rat: replaced 0.001540100153900437 by $276884/179783113 = 0.001540100153900439$

rat: replaced 0.02196908527585173 by $1298306/59096953 = 0.0219690852758517$

rat: replaced 0.001770376919130678 by $644389/363984072 = 0.001770376919130681$

rat: replaced 0.02410255066939448 by $2001286/83032125 = 0.02410255066939453$

rat: replaced 0.002022476464811601 by $1271955/628909667 = 0.002022476464811599$

rat: replaced 0.02633360499462523 by $2978115/113091808 = 0.02633360499462525$

rat: replaced 0.002297373572865413 by $1020913/444382669 = 0.002297373572865417$

rat: replaced 0.02866202514797045 by $1770713/61779061 = 0.02866202514797044$

rat: replaced 0.002596040745477063 by $1097643/422814242 = 0.002596040745477065$

rat: replaced 0.03108757828935527 by $5034207/161936287 = 0.03108757828935525$

rat: replaced 0.002919448107844891 by $906221/310408326 = 0.002919448107844891$

rat: replaced 0.03361002186548678 by $4553215/135471944 = 0.03361002186548678$

rat: replaced 0.003268563311168871 by $1379071/421919623 = 0.003268563311168867$

rat: replaced 0.03622910363410947 by $3082649/85087642 = 0.0362291036341094$

rat: replaced 0.003644351435886262 by $5966577/1637212301 = 0.003644351435886261$

rat: replaced 0.03894456168922911 by $4913415/126164342 = 0.03894456168922911$

rat: replaced 0.004047774895164447 by $572425/141417202 = 0.004047774895164451$

rat: replaced 0.04175612448730281 by $1734727/41544253 = 0.04175612448730273$

rat: replaced 0.004479793338660443 by $2952779/659132861 = 0.004479793338660444$

rat: replaced 0.04466351087439402 by $4691119/105032473 = 0.04466351087439405$

rat: replaced 0.0049413635565565 by $2524919/510976165 = 0.004941363556556498$

rat: replaced 0.04766643011428662 by $3536207/74186529 = 0.04766643011428665$

rat: replaced 0.005433439383882244 by $1361584/250593391 = 0.005433439383882235$

rat: replaced 0.05076458191755917 by $7710025/151878036 = 0.05076458191755916$

rat: replaced 0.005956971605131645 by $1447422/242979503 = 0.005956971605131648$

rat: replaced 0.0539576564716131 by $3377975/62604183 = 0.05395765647161309$

rat: replaced 0.006512907859185624 by $3695063/567344584 = 0.006512907859185626$

rat: replaced 0.05724533447165381 by $2560865/44734912 = 0.05724533447165382$

rat: replaced 0.007102192544548636 by $1363981/192050693 = 0.007102192544548642$

rat: replaced 0.06062728715262111 by $8274761/136485754 = 0.06062728715262107$

rat: replaced 0.007725766724910044 by $1464384/189545459 = 0.007725766724910038$

rat: replaced 0.06410317632206519 by $5287663/82486755 = 0.06410317632206528$

rat: replaced 0.00838456803503801 by $1113589/132814117 = 0.008384568035038023$

rat: replaced 0.06767265439396564 by $2921400/43169579 = 0.06767265439396572$

rat: replaced 0.009079530587017326 by $433906/47789475 = 0.00907953058701733$

rat: replaced 0.07133536442348987 by $7236103/101437808 = 0.07133536442348991$

rat: replaced 0.009811584876838586 by $1363090/138926587 = 0.009811584876838586$

rat: replaced 0.07509094014268702 by $9209133/122639735 = 0.07509094014268704$

rat: replaced 0.0105816576913495 by $1163729/109976058 = 0.01058165769134951$

rat: replaced 0.07893900599711501 by $5197067/65836489 = 0.07893900599711506$

rat: replaced 0.01139067201557714 by $13426050/1178688139 = 0.01139067201557714$

rat: replaced 0.08287917718339499 by $11217158/135343501 = 0.082879177183395$

rat: replaced 0.01223954694042984 by $2283101/186534764 = 0.01223954694042983$

rat: replaced 0.08691105968769186 by $5213115/59982182 = 0.08691105968769192$

rat: replaced 0.01312919757078923 by $3499615/266552086 = 0.01312919757078922$

rat: replaced 0.09103425032511492 by $5893225/64736349 = 0.09103425032511488$

rat: replaced 0.01406053493400045 by $2280713/162206702 = 0.01406053493400045$

rat: replaced 0.09524833678003664 by $9601787/100807923 = 0.09524833678003662$

rat: replaced 0.01503446588876983 by $200490/13335359 = 0.01503446588876985$

rat: replaced 0.09955289764732322 by $5687088/57126293 = 0.09955289764732328$

rat: replaced 0.01605189303448024 by $951971/59305840 = 0.01605189303448025$

rat: replaced 0.1039475024744748 by $10260011/98703776 = 0.1039475024744747$

rat: replaced 0.01711371462093175 by $9432386/551159477 = 0.01711371462093176$

rat: replaced 0.1084317118046711 by $14939691/137779721 = 0.1084317118046712$

rat: replaced 0.01822082445851714 by $2559788/140486947 = 0.01822082445851713$

rat: replaced 0.113005077220716 by $8478529/75027859 = 0.1130050772207161$

rat: replaced 0.01937411182884202 by $2983799/154009589 = 0.01937411182884203$

rat: replaced 0.1176671413898787 by $7123715/60541243 = 0.1176671413898786$

rat: replaced 0.02057446139579705 by $7167743/348380590 = 0.02057446139579705$

rat: replaced 0.1224174381096274 by $12172179/99431741 = 0.1224174381096274$

rat: replaced 0.02182275311709253 by $7415562/339808729 = 0.02182275311709253$

rat: replaced 0.1272554923542488 by $7277933/57191504 = 0.127255492354249$

rat: replaced 0.02311986215626333 by $2988661/129268115 = 0.02311986215626336$

rat: replaced 0.1321808203223502 by $3633064/27485561 = 0.1321808203223503$

rat: replaced 0.02446665879515308 by $1991976/81415939 = 0.02446665879515312$

rat: replaced 0.1371929294852391 by $56235017/409897341 = 0.1371929294852391$

rat: replaced 0.02586400834688696 by $5000736/193347293 = 0.02586400834688697$

rat: replaced 0.1422913186361759 by $9349741/65708443 = 0.1422913186361759$

rat: replaced 0.02731277106934082 by $858413/31428997 = 0.02731277106934084$

rat: replaced 0.1474754779404944 by $1549881/10509415 = 0.1474754779404943$

rat: replaced 0.02881380207911666 by $3754753/130310918 = 0.02881380207911666$

rat: replaced 0.152744888986584 by $5264425/34465474 = 0.1527448889865841$

rat: replaced 0.03036795126603076 by $4118329/135614318 = 0.03036795126603077$

rat: replaced 0.1580990248377314 by $5442776/34426373 = 0.1580990248377312$

rat: replaced 0.03197606320812652 by $3497683/109384416 = 0.03197606320812647$

rat: replaced 0.1635373500848132 by $12328488/75386375 = 0.1635373500848131$

rat: replaced 0.0336389770872163 by $3971799/118071337 = 0.03363897708721635$

rat: replaced 0.1690593208998367 by $20896917/123607009 = 0.1690593208998367$

rat: replaced 0.03535752660496472 by $1815732/51353479 = 0.03535752660496478$

rat: replaced 0.1746643850903219 by $2841592/16268869 = 0.1746643850903219$

rat: replaced 0.03713253989951881 by $3333721/89778965 = 0.03713253989951878$

rat: replaced 0.1803519821545206 by $4461007/24735004 = 0.1803519821545208$

rat: replaced 0.03896483946269502 by $8785771/225479461 = 0.03896483946269501$

rat: replaced 0.1861215433374662 by $4381209/23539505 = 0.1861215433374661$

rat: replaced 0.0408552420577305 by $3189084/78058135 = 0.04085524205773043$

rat: replaced 0.1919724916878484 by $72809759/379271834 = 0.1919724916878484$

rat: replaced 0.04280455863760801 by $7646593/178639688 = 0.04280455863760801$

rat: replaced 0.1979042421157076 by $26318167/132984350 = 0.1979042421157076$

rat: replaced 0.04481359426396048 by $20610430/459914683 = 0.04481359426396048$

rat: replaced 0.2039162014509444 by $8519416/41779005 = 0.2039162014509441$

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rat: replaced 0.05120697598153157 by $4148974/81023609 = 0.0512069759815315$

rat: replaced 0.2224272812490723 by $23234851/104460437 = 0.2224272812490723$

rat: replaced 0.05346281777803219 by $11998448/224426031 = 0.05346281777803218$

rat: replaced 0.2287539850028937 by $8185268/35781969 = 0.2287539850028935$

rat: replaced 0.05578231276230905 by $1398019/25062048 = 0.05578231276230897$

rat: replaced 0.2351578127155118 by $12642104/53760085 = 0.2351578127155119$

rat: replaced 0.05816622897846346 by $4451048/76522891 = 0.05816622897846345$

rat: replaced 0.2416381240094921 by $8002142/33116223 = 0.2416381240094923$

rat: replaced 0.06061532802852698 by $2146337/35409146 = 0.06061532802852686$

rat: replaced 0.2481942708591053 by $8882901/35790113 = 0.2481942708591057$

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rat: replaced 0.2615314412704124 by $8212450/31401387 = 0.2615314412704127$

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rat: replaced 0.07107855488944881 by $3146673/44270357 = 0.07107855488944893$

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rat: replaced 0.07386476137264342 by $12898997/174629915 = 0.0738647613726434$

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rat: replaced 0.2961546843477643 by $11052271/37319251 = 0.2961546843477647$

rat: replaced 0.08264390910047736 by $4686067/56701904 = 0.08264390910047748$

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rat: replaced 0.3400168541150183 by $13391981/39386227 = 0.3400168541150184$

rat: replaced 0.1021574371047232 by $8336413/81603584 = 0.1021574371047232$

rat: replaced 0.3475625318359485 by $10097818/29053241 = 0.347562531835949$

rat: replaced 0.1056710629744951 by $5741011/54329074 = 0.105671062974495$

rat: replaced 0.3551734527599992 by $15867851/44676343 = 0.3551734527599987$

rat: replaced 0.1092611211010309 by $5551873/50812887 = 0.1092611211010309$

rat: replaced 0.3628488558014202 by $6897641/19009681 = 0.3628488558014203$

rat: replaced 0.1129282524731764 by $11548693/102265755 = 0.1129282524731764$

rat: replaced 0.3705879734263036 by $23358661/63031352 = 0.3705879734263038$

rat: replaced 0.1166730903725168 by $5656228/48479285 = 0.1166730903725168$

rat: replaced 0.3783900317293359 by $14241382/37636779 = 0.3783900317293358$

rat: replaced 0.1204962603100498 by $4057613/33674182 = 0.12049626031005$

rat: replaced 0.3862542505111889 by $3461217/8960981 = 0.3862542505111884$

rat: replaced 0.1243983799636342 by $7966447/64039797 = 0.1243983799636342$

rat: replaced 0.3941798433565377 by $5314214/13481699 = 0.3941798433565384$

rat: replaced 0.1283800591162231 by $796346/6203035 = 0.1283800591162229$

rat: replaced 0.4021660177127022 by $11567173/28762184 = 0.4021660177127022$

rat: replaced 0.1324418995948859 by $4716124/35609003 = 0.1324418995948862$

rat: replaced 0.4102119749689023 by $11320633/27597032 = 0.4102119749689024$

rat: replaced 0.1365844952106265 by $612971/4487852 = 0.1365844952106264$

rat: replaced 0.418316910536117 by $12225195/29224721 = 0.4183169105361177$

rat: replaced 0.140808431699002 by $10431632/74083859 = 0.1408084316990021$

rat: replaced 0.4264800139275439 by $7978696/18708253 = 0.4264800139275431$

rat: replaced 0.1451142866615502 by $3554077/24491572 = 0.1451142866615504$

```
rat: replaced 0.4347004688396462 by 20489554/47134879 = 0.4347004688396463
```

```
rat: replaced 0.1495026295080298 by 26759297/178988805 = 0.1495026295080298
```

```
rat: replaced 0.4429774532337832 by 23449796/52936771 = 0.4429774532337834
```

```
rat: replaced 0.1539740213994798 by 16145763/104860306 = 0.1539740213994798
```

```
rat: replaced 0.451310139418413 by 8841241/19590167 = 0.4513101394184133
```

```
part: invalid index of list or matrix.
```

```
#0: lineIntersection(g=[1,-1,0],h=[2-sqrt(5)/sqrt(2),sqrt(5)/sqrt(2)+1,(2-sqrt(5)/sqrt(2))*(sqrt(5)
-- an error. To debug this try: debugmode(true);
```

```
Error in:
```

```
... ection(angleBisector(A,C,B),angleBisector(C,B,A)); $P // titik ...
      ^
```

```
>P() // hasilnya sama dengan perhitungan sebelumnya
```

```
Function P needs at least 2 arguments!
```

```
Use: P (x, n)
```

```
Error in:
```

```
P() // hasilnya sama dengan perhitungan sebelumnya ...
      ^
```

Garis dan Lingkaran yang Berpotongan

Tentu saja, kita juga dapat memotong garis dengan lingkaran, dan lingkaran dengan lingkaran.

```
>A &:= [1,0]; c=circleWithCenter(A,4);  
>B &:= [1,2]; C &:= [2,1]; l=lineThrough(B,C);  
>setPlotRange(5); plotCircle(c); plotLine(l);
```

Perpotongan garis dengan lingkaran menghasilkan dua titik dan jumlah titik potong.

```
>{P1,P2,f}=lineCircleIntersections(l,c);  
>P1, P2,
```

```
[4.64575, -1.64575]  
[-0.645751, 3.64575]
```

```
>plotPoint(P1); plotPoint(P2):
```

Begitu pula di Maxima.

```
>c &= circleWithCenter(A,4) // lingkaran dengan pusat A jari-jari 4
```

[1, 0, 4]

```
>l &= lineThrough(B,C) // garis l melalui B dan C
```

[1, 1, 3]

```
>$lineCircleIntersections(l,c) | radcan, // titik potong lingkaran c dan garis l
```

Maxima said:

rat: replaced -4.98329175014009e-5 by -86001/1725786976 = -4.983291750140082e-5

rat: replaced -1.986600267553235e-4 by -1133306/5704751069 = -1.986600267553234e-4

rat: replaced -4.454664535081185e-4 by -474290/1064704191 = -4.454664535081181e-4

rat: replaced -7.892275256562442e-4 by -1190199/1508055613 = -7.892275256562439e-4

rat: replaced -0.001228908875712045 by -259907/211494119 = -0.001228908875712047

rat: replaced -0.001763466544240408 by $-5854594/3319934829 = -0.001763466544240408$

rat: replaced -0.002391847084253176 by $-866601/362314550 = -0.002391847084253172$

rat: replaced -0.003112987666553255 by $-5049204/1621980085 = -0.003112987666553255$

rat: replaced -0.00392581618601677 by $-1241039/316122544 = -0.003925816186016774$

rat: replaced -0.004829251368802329 by $-3015690/624463249 = -0.00482925136880233$

rat: replaced -0.005822202880477995 by $-2532373/434951006 = -0.005822202880477991$

rat: replaced -0.006903571435053116 by $-1331361/192851050 = -0.006903571435053115$

rat: replaced -0.008072248904906765 by $-7953293/985263598 = -0.008072248904906766$

rat: replaced -0.009327118431599252 by $-432515/46371771 = -0.009327118431599259$

rat: replaced -0.01066705453755698 by $-2950074/276559381 = -0.01066705453755698$

rat: replaced -0.01209092323861904 by $-1254816/103781653 = -0.01209092323861907$

rat: replaced -0.01359758215743526 by $-1827823/134422648 = -0.01359758215743526$

rat: replaced -0.01518588063770274 by $-9199276/605778237 = -0.01518588063770274$

rat: replaced -0.01685465985923026 by $-2516580/149310637 = -0.01685465985923026$

rat: replaced -0.01860275295381958 by $-2032371/109251088 = -0.01860275295381955$

rat: replaced -0.02042898512195129 by $-1413911/69211025 = -0.02042898512195131$

rat: replaced -0.02233217375026381 by $-3647892/163346929 = -0.02233217375026377$

rat: replaced -0.02431112852981362 by $-1377268/56651751 = -0.02431112852981367$

rat: replaced -0.02636465157510504 by $-2533336/96088355 = -0.02636465157510502$

rat: replaced -0.0284915375438782 by $-9699307/340427644 = -0.02849153754387819$

rat: replaced -0.03069057375764189 by $-7938451/258660886 = -0.03069057375764189$

rat: replaced -0.0329605403229406 by $-2936449/89089832 = -0.03296054032294056$

rat: replaced -0.03530021025334285 by $-5224432/148000025 = -0.03530021025334287$

rat: replaced -0.03770834959213837 by $-2448749/64939172 = -0.03770834959213832$

rat: replaced -0.04018371753573358 by $-2461511/61256428 = -0.04018371753573356$

rat: replaced -0.04272506655773012 by $-13954421/326609696 = -0.04272506655773012$

rat: replaced -0.04533114253367693 by $-2051558/45257143 = -0.04533114253367695$

rat: replaced -0.04800068486648146 by $-16995415/354066094 = -0.04800068486648145$

rat: replaced -0.05073242661246818 by $-3970295/78259513 = -0.05073242661246818$

rat: replaced -0.05352509460807248 by $-3894269/72755948 = -0.05352509460807246$

rat: replaced -0.05637740959715515 by $-11093364/196769665 = -0.05637740959715513$

rat: replaced -0.05928808635892763 by $-3489209/58851773 = -0.05928808635892754$

rat: replaced -0.06225583383647254 by $-3380435/54299088 = -0.06225583383647254$

rat: replaced -0.06527935526584844 by $-7571267/115982564 = -0.06527935526584841$

rat: replaced -0.06835734830576551 by $-8050241/117767017 = -0.06835734830576544$

rat: replaced -0.07148850516781785 by $-5513427/77123266 = -0.07148850516781798$

rat: replaced -0.07467151274726203 by $-2259975/30265558 = -0.07467151274726208$

rat: replaced -0.07790505275432569 by $-657797/8443573 = -0.07790505275432569$

rat: replaced -0.08118780184603619 by $-4832180/59518547 = -0.08118780184603633$

rat: replaced -0.08451843175855339 by $-3076049/36395008 = -0.08451843175855327$

rat: replaced -0.08789560943999458 by $-7150621/81353563 = -0.08789560943999465$

rat: replaced -0.0913179971837394 by $-20067867/219758072 = -0.0913179971837394$

rat: replaced -0.09478425276219882 by $-5487749/57897265 = -0.09478425276219869$

rat: replaced -0.09829302956103664 by $-4406725/44832528 = -0.09829302956103658$

rat: replaced -0.1018429767138303 by $-3912367/38415678 = -0.1018429767138302$

rat: replaced -0.1054327392371563 by $-8451941/80164293 = -0.1054327392371564$

rat: replaced -0.1090609581660869 by $-13126833/120362348 = -0.1090609581660869$

rat: replaced -0.112726270690086 by $-2754747/24437489 = -0.112726270690086$

rat: replaced -0.116427310289289 by $-22239618/191017193 = -0.116427310289289$

rat: replaced -0.1201627068711536 by $-9494831/79016454 = -0.1201627068711537$

rat: replaced -0.1239310869074673 by $-3190398/25743323 = -0.1239310869074672$

rat: replaced -0.1277310735717007 by $-15999330/125257931 = -0.1277310735717006$

rat: replaced -0.1315612868766867 by $-13929723/105880106 = -0.1315612868766867$

rat: replaced -0.1354203438126204 by $-28035370/207024803 = -0.1354203438126204$

rat: replaced -0.1393068584853572 by $-11590983/83204683 = -0.1393068584853571$

rat: replaced -0.1432194422550018 by $-12738764/88945773 = -0.1432194422550018$

rat: replaced -0.1471567038747712 by $-5246589/35653075 = -0.147156703874771$

rat: replaced -0.1511172496301179 by $-4676629/30947023 = -0.1511172496301179$

rat: replaced -0.1550996834780995 by $-15854305/102220099 = -0.1550996834780995$

rat: replaced -0.1591026071869839 by $-9026555/56734174 = -0.159102607186984$

rat: replaced -0.1631246204760689 by $-10435073/63969945 = -0.1631246204760689$

rat: replaced -0.1671643211557106 by $-164873401/986295400 = -0.1671643211557106$

rat: replaced -0.1712203052675407 by $-7017638/40986015 = -0.1712203052675406$

rat: replaced -0.1752911672248615 by $-3184915/18169284 = -0.1752911672248615$

rat: replaced -0.1793754999532028 by $-2646709/14755131 = -0.1793754999532027$

rat: replaced -0.1834718950310287 by $-8392143/45740755 = -0.1834718950310287$

rat: replaced -0.1875789428305783 by $-12888313/68708741 = -0.1875789428305781$

rat: replaced -0.1916952326588277 by $-16014703/83542521 = -0.1916952326588277$

rat: replaced -0.1958193528985573 by $-21279927/108671215 = -0.1958193528985574$

rat: replaced -0.1999498911495134 by $-5994245/29978736 = -0.1999498911495134$

rat: replaced -0.2040854343696463 by $-17847769/87452439 = -0.2040854343696464$

rat: replaced -0.2082245690164135 by $-5203892/24991729 = -0.2082245690164134$

rat: replaced -0.2123658811881329 by $-20393053/96027916 = -0.2123658811881328$

rat: replaced -0.2165079567653719 by $-8489188/39209589 = -0.2165079567653719$

rat: replaced -0.2206493815523576 by $-14881929/67446049 = -0.2206493815523575$

rat: replaced -0.2247887414183958 by $-11437558/50881365 = -0.2247887414183955$

rat: replaced -0.2289246224392826 by $-17547464/76651711 = -0.2289246224392825$

rat: replaced -0.2330556110386959 by $-11148764/47837355 = -0.2330556110386956$

rat: replaced -0.2371802941295513 by $-11052217/46598378 = -0.237180294129551$

rat: replaced -0.2412972592553108 by $-36037383/149348497 = -0.2412972592553108$

rat: replaced -0.2454050947312253 by $-4652365/18957899 = -0.2454050947312252$

rat: replaced -0.2495023897855041 by $-6175634/24751803 = -0.2495023897855037$

rat: replaced -0.2535877347003893 by $-11299519/44558618 = -0.2535877347003895$

rat: replaced -0.2576597209531272 by $-6871877/26670358 = -0.2576597209531271$

rat: replaced -0.2617169413568191 by $-2245730/8580759 = -0.2617169413568194$

rat: replaced -0.2657579902011391 by $-10500993/39513367 = -0.2657579902011388$

rat: replaced -0.2697814633929034 by $-21050552/78028163 = -0.2697814633929034$

rat: replaced -0.2737859585964791 by $-1510231/5516101 = -0.2737859585964796$

rat: replaced -0.2777700753740163 by $-9819093/35349715 = -0.2777700753740164$

rat: replaced -0.2817324153254904 by $-10837378/38466919 = -0.2817324153254905$

rat: replaced -0.2856715822285418 by $-17041418/59653879 = -0.2856715822285421$

rat: replaced -0.289586182178096 by $-721506/2491507 = -0.2895861821780955$

rat: replaced -0.2934748237257534 by $-11793110/40184401 = -0.2934748237257537$

rat: replaced -0.2973361180189332 by $-15390047/51759763 = -0.2973361180189329$

rat: replaced $5.016624916807239e-5$ by $153117/3052191514 = 5.016624916807235e-5$

rat: replaced $2.013266400891639e-4$ by $232411/1154397649 = 2.013266400891639e-4$

rat: replaced $4.544660485167953e-4$ by $444871/978887205 = 4.544660485167952e-4$

rat: replaced $8.105591523879241e-4$ by $1425236/1758336817 = 8.105591523879239e-4$

rat: replaced 0.001270570334355389 by $696221/547959433 = 0.00127057033435539$

rat: replaced 0.001835453585351213 by $1018402/554850315 = 0.001835453585351213$

rat: replaced 0.002506152409187654 by $484773/193433168 = 0.002506152409187653$

rat: replaced 0.003283599728207867 by $1007483/306822720 = 0.003283599728207872$

rat: replaced 0.004168717789994683 by $897113/215201183 = 0.004168717789994677$

rat: replaced 0.00516241807514603 by $757433/146720585 = 0.005162418075146034$

rat: replaced 0.006265601206128374 by $1194190/190594639 = 0.006265601206128363$

rat: replaced 0.007479156857214384 by $1971251/263565939 = 0.007479156857214391$

rat: replaced 0.008803963665517056 by $365844/41554465 = 0.008803963665517051$

rat: replaced 0.01024088914312629 by $1345773/131411734 = 0.01024088914312629$

rat: replaced 0.01179078959035854 by $1519715/128890011 = 0.01179078959035856$

rat: replaced 0.0134545100101271 by $2242921/166704027 = 0.01345451001012711$

rat: replaced 0.01523288402344322 by $1950407/128039247 = 0.01523288402344322$

rat: replaced 0.01712673378605437 by $1362867/79575418 = 0.01712673378605438$

rat: replaced 0.01913686990622912 by $1694449/88543686 = 0.01913686990622911$

rat: replaced 0.02126409136369717 by $9814128/461535263 = 0.02126409136369716$

rat: replaced 0.02350918542975217 by $2315819/98506986 = 0.02350918542975216$

rat: replaced 0.02587292758852516 by $3386321/130882792 = 0.02587292758852516$

rat: replaced 0.02835608145943683 by $10230271/360778728 = 0.02835608145943682$

rat: replaced 0.03095939872083586 by $14307719/462144602 = 0.03095939872083587$

rat: replaced 0.03368361903483233 by $4712088/139892569 = 0.03368361903483236$

rat: replaced 0.03652946997333167 by $4111522/112553563 = 0.03652946997333172$

rat: replaced 0.03949766694527834 by $8626745/218411508 = 0.03949766694527836$

rat: replaced 0.04258891312511537 by $3115258/73147159 = 0.04258891312511536$

rat: replaced 0.04580389938246726 by $2358579/51492974 = 0.04580389938246721$

rat: replaced 0.04914330421305446 by $2180747/44375262 = 0.04914330421305456$

rat: replaced 0.05260779367084312 by $4975224/94571995 = 0.05260779367084304$

rat: replaced 0.05619802130144141 by $1396735/24853811 = 0.05619802130144146$

rat: replaced 0.05991462807674475 by $6603037/110207427 = 0.05991462807674477$

rat: replaced 0.06375824233083943 by $6198842/97224167 = 0.0637582423308394$

rat: replaced 0.06772947969716975 by $4012504/59243095 = 0.06772947969716978$

rat: replaced 0.07182894304697524 by $5813372/80933559 = 0.07182894304697511$

rat: replaced 0.07605722242900365 by $14672328/192911699 = 0.07605722242900365$

rat: replaced 0.08041489501050719 by $3507279/43614793 = 0.0804148950105071$

rat: replaced 0.08490252501952561 by $2460362/28978667 = 0.08490252501952557$

rat: replaced 0.08952066368846451 by $4304415/48082921 = 0.08952066368846436$

rat: replaced 0.09426984919897213 by $3898288/41352437 = 0.09426984919897224$

rat: replaced 0.0991506066281217 by $11428253/115261554 = 0.09915060662812164$

rat: replaced 0.1041634478959041 by $7209817/69216382 = 0.1041634478959042$

rat: replaced 0.1093088717140371 by $3826731/35008421 = 0.109308871714037$

rat: replaced 0.1145873635360931 by $5173172/45146095 = 0.1145873635360932$

rat: replaced 0.1199993955089551 by $23218093/193485083 = 0.1199993955089551$

rat: replaced 0.1255454264256029 by $2445819/19481546 = 0.125545426425603$

rat: replaced 0.1312259016792331 by $9111136/69430927 = 0.131225901679233$

rat: replaced 0.1370412532187207 by $16597683/121114501 = 0.1370412532187207$

rat: replaced 0.1429918995054244 by $34253454/239548213 = 0.1429918995054244$

rat: replaced 0.1490782454713414 by $11997679/80479073 = 0.1490782454713414$

rat: replaced 0.1553006824786136 by $13065213/84128497 = 0.1553006824786136$

rat: replaced 0.1616595882803922 by $12686167/78474572 = 0.1616595882803923$

rat: replaced 0.1681553269830629 by $4527449/26924208 = 0.168155326983063$

rat: replaced 0.1747882490098353 by $23565700/134824281 = 0.1747882490098353$

rat: replaced 0.1815586910657007 by $4563713/25136296 = 0.1815586910657004$

rat: replaced 0.1884669761037622 by $8213146/43578701 = 0.1884669761037623$

rat: replaced 0.1955134132929397 by $7172626/36686107 = 0.1955134132929395$

rat: replaced 0.202698297987053 by $17668607/87167022 = 0.2026982979870529$

rat: replaced 0.2100219116952866 by $8269584/39374863 = 0.2100219116952864$

rat: replaced 0.2174845220540395 by $56596301/260231397 = 0.2174845220540395$

rat: replaced 0.2250863828001612 by $8187128/36373271 = 0.2250863828001611$

rat: replaced 0.2328277337455789 by $10320856/44328293 = 0.2328277337455787$

rat: replaced 0.2407088007533156 by $16964872/70478819 = 0.2407088007533157$

rat: replaced 0.2487297957149048 by $11063220/44478869 = 0.2487297957149045$

rat: replaced 0.2568909165292014 by $17200949/66958183 = 0.2568909165292015$

rat: replaced 0.2651923470825914 by $8866093/33432688 = 0.2651923470825918$

rat: replaced 0.2736342572306039 by $12664159/46281336 = 0.2736342572306037$

rat: replaced 0.2822168027809259 by $8116045/28758192 = 0.2822168027809259$

rat: replaced 0.2909401254778209 by $24764749/85119744 = 0.290940125477821$

rat: replaced 0.2998043529879556 by $28498628/95057419 = 0.2998043529879556$

rat: replaced 0.3088095988876323 by $13390352/43361191 = 0.308809598887632$

rat: replaced 0.3179559626514321 by $26241235/82531036 = 0.3179559626514321$

rat: replaced 0.3272435296422674 by $8247573/25203166 = 0.3272435296422679$

rat: replaced 0.3366723711028454 by $10805861/32096073 = 0.3366723711028449$

rat: replaced 0.3462425441485439 by $20967050/60555961 = 0.3462425441485438$

rat: replaced 0.3559540917617003 by $19053013/53526602 = 0.3559540917617001$

rat: replaced 0.3658070427873129 by $10401097/28433288 = 0.3658070427873132$

rat: replaced 0.3758014119301566 by $5923743/15762961 = 0.375801411930157$

rat: replaced 0.3859371997533123 by $2934328/7603123 = 0.3859371997533119$

rat: replaced 0.396214392678111 by $30414315/76762267 = 0.396214392678111$

rat: replaced 0.4066329629854911 by $13711485/33719561 = 0.4066329629854908$

rat: replaced 0.4171928688187707 by $20838614/49949593 = 0.4171928688187709$

rat: replaced 0.4278940541878331 by $16106690/37641771 = 0.427894054187833$

rat: replaced 0.4387364489747257 by $4869080/11097961 = 0.4387364489747261$

rat: replaced 0.4497199689406718 by $4550581/10118699 = 0.4497199689406711$

rat: replaced 0.4608445157344944 by $7970699/17295853 = 0.4608445157344943$

rat: replaced 0.4721099769024512 by $25424083/53852035 = 0.4721099769024513$

rat: replaced 0.48351622589948 by $17675673/36556525 = 0.4835162258994803$

rat: replaced 0.4950631221018528 by $7053395/14247466 = 0.495063122101853$

rat: replaced 0.5067505108212387 by $13754758/27143057 = 0.5067505108212388$

rat: replaced 0.5185782233201719 by $21662467/41772805 = 0.518578223320172$

rat: replaced 0.5305460768289253 by $10488897/19770002 = 0.530546076828925$

rat: replaced 0.5426538745637882 by $22388393/41257225 = 0.5426538745637886$

rat: replaced 0.5549014057467435 by $9960301/17949677 = 0.5549014057467441$

rat: replaced 0.5672884456265459 by $28078535/49496046 = 0.5672884456265456$

rat: replaced 0.5798147555011964 by $18086313/31193261 = 0.5798147555011962$

rat: replaced 0.5924800827418131 by $20592707/34756792 = 0.5924800827418134$

```

rat: replaced 0.6052841608178928 by 26813845/44299598 = 0.6052841608178927
part: invalid index of list or matrix.
#0: lineIntersection(g=[1,-1,1],h=[1,1,3])
#1: projectToLine(a=[1,0],g=[1,1,3])
#2: lineCircleIntersections(g=[1,1,3],c=[1,0,4])
-- an error. To debug this try: debugmode(true);

Error in:
  $lineCircleIntersections(l,c) | radcan, // titik potong lingk ...
                                ^

```

Akan ditunjukkan bahwa sudut-sudut yang menghadap busur yang sama adalah sama besar.

```

>C=A+normalize([-2,-3])*4; plotPoint(C); plotSegment(P1,C); plotSegment(P2,C);
>degprint(computeAngle(P1,C,P2))

```

69°17'42.68''

```

>C=A+normalize([-4,-3])*4; plotPoint(C); plotSegment(P1,C); plotSegment(P2,C);
>degprint(computeAngle(P1,C,P2))

```

69°17'42.68''

```

>insimg;

```

Berikut adalah langkah-langkah menggambar garis sumbu ruas garis AB:

1. Gambar lingkaran dengan pusat A melalui B.
2. Gambar lingkaran dengan pusat B melalui A.
3. Tarik garis melalui kedua titik potong kedua lingkaran tersebut. Garis ini merupakan garis sumbu (melalui titik tengah dan tegak lurus) AB.

```
>A=[2,2]; B=[-1,-2];  
>c1=circleWithCenter(A,distance(A,B));  
>c2=circleWithCenter(B,distance(A,B));  
>{P1,P2,f}=circleCircleIntersections(c1,c2);  
>l=lineThrough(P1,P2);  
>setPlotRange(5); plotCircle(c1); plotCircle(c2);  
>plotPoint(A); plotPoint(B); plotSegment(A,B); plotLine(l):
```

Selanjutnya, kami melakukan hal yang sama di Maxima dengan koordinat umum.

```
>A &= [a1,a2]; B &= [b1,b2];  
>c1 &= circleWithCenter(A,distance(A,B));  
>c2 &= circleWithCenter(B,distance(A,B));  
>P &= circleCircleIntersections(c1,c2); P1 &= P[1]; P2 &= P[2];
```

Persamaan untuk persimpangan cukup terlibat. Tetapi kita dapat menyederhanakannya, jika kita memecahkan y .

```
>g &= getLineEquation(lineThrough(P1,P2),x,y);  
>$solve(g,y)
```

```
Maxima said:  
solve: all variables must not be numbers.  
-- an error. To debug this try: debugmode(true);  
  
Error in:  
$solve(g,y) ...  
      ^
```

Ini memang sama dengan tegak lurus tengah, yang dihitung dengan cara yang sama sekali berbeda.

```
>$solve(getLineEquation(middlePerpendicular(A,B),x,y),y)
```

```
Maxima said:  
solve: all variables must not be numbers.  
-- an error. To debug this try: debugmode(true);  
  
Error in:  
... (getLineEquation(middlePerpendicular(A,B),x,y),y) ...  
      ^
```

```
>h &=getLineEquation(lineThrough(A,B),x,y);
>$solve(h,y)
```

```
Maxima said:
solve: all variables must not be numbers.
-- an error. To debug this try: debugmode(true);
```

```
Error in:
$solve(h,y) ...
^
```

```
>setPlotRange(-1,10,-1,8); plotPoint([0,0], "C(0,0)"); plotPoint([5.5,0], "B(a,0)"); ...
>plotPoint([7.5,6], "A(x,y)");
>plotSegment([0,0],[5.5,0], "a",25); plotSegment([5.5,0],[7.5,6],"c",15); ...
>plotSegment([0,0],[7.5,6],"b",25);
>plotSegment([7.5,6],[7.5,0],"t=y",25):
>&assume(a>0); sol &= solve([x^2+y^2=b^2,(x-a)^2+y^2=c^2],[x,y])
```

```
Maxima said:
fullmap: arguments must have same formal structure.
-- an error. To debug this try: debugmode(true);
```

```
Error in:
... sol &= solve([x^2+y^2=b^2,(x-a)^2+y^2=c^2],[x,y]) ...
^
```

Ekstrak solusi y.

```
>ysol &= y with sol[2][2]; $'y=sqrt(factor(ysol^2))
```

Maxima said:

at: improper argument: (3*x*sqrt(9*x^2+4))[2][2]

#0: with(expr=[0,4.999958333473664e-5*r,1.999933334222437e-4*r,4.499662510124569e-4*r,7.9989333902
-- an error. To debug this try: debugmode(true);

Error in:

```
ysol &= y with sol[2][2]; $'y=sqrt(factor(ysol^2)) ...  
^
```

Kami mendapatkan rumus Heron.

```
>function H(a,b,c) &= sqrt(factor((ysol*a/2)^2)); $'H(a,b,c)=H(a,b,c)  
>$'Luas=H(2,5,6) // luas segitiga dengan panjang sisi-sisi 2, 5, 6
```

Tentu saja, setiap segitiga persegi panjang adalah kasus yang terkenal.

```
>H(3,4,5) //luas segitiga siku-siku dengan panjang sisi 3, 4, 5
```

```

Variable or function ysol not found.
Try "trace errors" to inspect local variables after errors.
H:
    useglobal; return a*abs(ysol)/2
Error in:
H(3,4,5) //luas segitiga siku-siku dengan panjang sisi 3, 4, 5 ...
    ^

```

Dan juga jelas, bahwa ini adalah segitiga dengan luas maksimal dan dua sisi 3 dan 4.

```
>aspect (1.5); plot2d(&H(3,4,x),1,7): // Kurva luas segitiga sengan panjang sisi 3, 4, x (1<= x <=7)
```

```

Variable or function ysol not found.
Error in expression: 3*abs(ysol)/2
%ploteval:
    y0=f$(x[1],args());
adaptiveevalone:
    s=%ploteval(g$,t,args());
Try "trace errors" to inspect local variables after errors.
plot2d:
    dw/n,dw/n^2,dw/n,auto,args());

```


Kasus umum juga berfungsi.

```
>$solve(diff(H(a,b,c)^2,c)=0,c)
```

Maxima said:

```
diff: second argument must be a variable; found [1,0,4]
-- an error. To debug this try: debugmode(true);
```

Error in:

```
$solve(diff(H(a,b,c)^2,c)=0,c) ...
^
```

Sekarang mari kita cari himpunan semua titik di mana $b+c=d$ untuk beberapa konstanta d . Diketahui bahwa ini adalah elips.

```
>s1 &= subst(d-c,b,sol[2]); $s1
```

Dan buat fungsi ini.

```
>function fx(a,c,d) &= rhs(s1[1]); $fx(a,c,d), function fy(a,c,d) &= rhs(s1[2]); $fy(a,c,d)
```

Sekarang kita bisa menggambar setnya. Sisi b bervariasi dari 1 hingga 4. Diketahui bahwa kita mendapatkan elips.

```
>aspect(1); plot2d(&fx(3,x,5),&fy(3,x,5),xmin=1,xmax=4,square=1):
```

Kita dapat memeriksa persamaan umum untuk elips ini, yaitu.

$$\frac{(x - x_m)^2}{u^2} + \frac{(y - y_m)^2}{v^2} = 1,$$

di mana (xm,ym) adalah pusat, dan u dan v adalah setengah sumbu.

```
>$ratsimp((fx(a,c,d)-a/2)^2/u^2+fy(a,c,d)^2/v^2 with [u=d/2,v=sqrt(d^2-a^2)/2])
```

Kita lihat bahwa tinggi dan luas segitiga adalah maksimal untuk x=0. Jadi luas segitiga dengan a+b+c=d maksimal jika segitiga sama sisi. Kami ingin menurunkan ini secara analitis.

```
>eqns &= [diff(H(a,b,d-(a+b))^2,a)=0,diff(H(a,b,d-(a+b))^2,b)=0]; $eqns
```

Kami mendapatkan beberapa minima, yang termasuk dalam segitiga dengan satu sisi 0, dan solusinya $a=b=c=d/3$.

```
>$solve(eqns,[a,b])
```

Ada juga metode Lagrange, memaksimalkan $H(a,b,c)^2$ terhadap $a+b+c=d$.

```
>&solve([diff(H(a,b,c)^2,a)=1a,diff(H(a,b,c)^2,b)=1a, ...  
> diff(H(a,b,c)^2,c)=1a,a+b+c=d],[a,b,c,1a])
```

Maxima said:

```
diff: second argument must be a variable; found [1,0,4]  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
... 1a, diff(H(a,b,c)^2,c)=1a,a+b+c=d],[a,b,c,1a]) ...  
^
```

Pertama-tama atur poin di Maxima.

```
>A &= at([x,y],sol[2]); $A
```

```
Maxima said:
at: improper argument: (3*x*sqrt(9*x^2+4))[2]
-- an error. To debug this try: debugmode(true);
```

```
Error in:
A &= at([x,y],sol[2]); $A ...
      ^
```

```
>B &= [0,0]; $B, C &= [a,0]; $C
```

Kemudian atur rentang plot, dan plot titik-titiknya.

```
>setPlotRange(0,5,-2,3); ...
>a=4; b=3; c=2; ...
>plotPoint(mxmeval("B"),"B"); plotPoint(mxmeval("C"),"C"); ...
>plotPoint(mxmeval("A"),"A");
```

```
Variable a1 not found!
Use global variables or parameters for string evaluation.
Error in Evaluate, superfluous characters found.
Try "trace errors" to inspect local variables after errors.
mxmeval:
      return evaluate(mxm(s));
Error in:
... otPoint(mxmeval("C"),"C"); plotPoint(mxmeval("A"),"A"): ...
      ^
```

Plot segmen.

```
>plotSegment(mxmeval("A"),mxmeval("C")); ...  
>plotSegment(mxmeval("B"),mxmeval("C")); ...  
>plotSegment(mxmeval("B"),mxmeval("A")):
```

Variable a1 not found!

Use global variables or parameters for string evaluation.

Error in Evaluate, superfluous characters found.

Try "trace errors" to inspect local variables after errors.

mxmeval:

```
    return evaluate(mxm(s));
```

Error in:

```
plotSegment(mxmeval("A"),mxmeval("C")); plotSegment(mxmeval("B ...  
^
```

Hitung tegak lurus tengah di Maxima.

```
>h &= middlePerpendicular(A,B); g &= middlePerpendicular(B,C);
```

Dan pusat lingkaran.

```
>U &= lineIntersection(h,g);
```

Maxima said:

rat: replaced $1.66665833335744e-7$ by $15819/94914474571$ = $1.66665833335744e-7$

rat: replaced $4.999958333473664e-5$ by $201389/4027813565$ = $4.99995833347366e-5$

rat: replaced $1.33330666692022e-6$ by $31771/23828726570$ = $1.333306666920221e-6$

rat: replaced $1.999933334222437e-4$ by $200030/1000183339$ = $1.999933334222437e-4$

rat: replaced $4.499797504338432e-6$ by $24036/5341573699$ = $4.499797504338431e-6$

rat: replaced $4.499662510124569e-4$ by $1162901/2584418270$ = $4.499662510124571e-4$

rat: replaced $1.066581336583994e-5$ by $58861/5518660226$ = $1.066581336583993e-5$

rat: replaced $7.998933390220841e-4$ by $1137431/1421978337$ = $7.998933390220838e-4$

rat: replaced $2.083072932167196e-5$ by $35635/1710693824$ = $2.0830729321672e-5$

rat: replaced 0.001249739605033717 by $567943/454449069$ = 0.001249739605033716

rat: replaced $3.599352055540239e-5$ by $98277/2730408098$ = $3.599352055540234e-5$

rat: replaced 0.00179946006479581 by $479561/266502719$ = 0.001799460064795812

rat: replaced $5.71526624672386e-5$ by $51154/895041417$ = $5.715266246723866e-5$

rat: replaced 0.002448999746720415 by $1946227/794702818$ = 0.002448999746720415

rat: replaced $8.530603082730626e-5$ by $121691/1426522824 = 8.530603082730627e-5$

rat: replaced 0.003198293697380561 by $2986741/933854512 = 0.003198293697380562$

rat: replaced $1.214508019889565e-4$ by $158455/1304684674 = 1.214508019889563e-4$

rat: replaced 0.004047266988005727 by $2125334/525128193 = 0.004047266988005727$

rat: replaced $1.665833531718508e-4$ by $142521/855553675 = 1.66583353171851e-4$

rat: replaced 0.004995834721974179 by $1957223/391770967 = 0.004995834721974179$

rat: replaced $2.216991628251896e-4$ by $179571/809975995 = 2.216991628251896e-4$

rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced $2.877927110806339e-4$ by $1167733/4057548906 = 2.877927110806339e-4$

rat: replaced 0.00719136414613375 by $2476362/344352191 = 0.007191364146133747$

rat: replaced $3.658573803051457e-4$ by $386279/1055818526 = 3.658573803051454e-4$

rat: replaced 0.00843810628521191 by $2079855/246483622 = 0.008438106285211924$

rat: replaced $4.5688535576352e-4$ by $262978/575588595 = 4.568853557635206e-4$

rat: replaced 0.009784003787362772 by $1752551/179124113 = 0.009784003787362787$

rat: replaced $5.618675264007778e-4$ by $150595/268025812 = 5.618675264007782e-4$

rat: replaced 0.01122892206395776 by $5450241/485375263 = 0.01122892206395776$

rat: replaced $6.817933857540259e-4$ by $192316/282073725 = 6.817933857540258e-4$

rat: replaced 0.01277271662437307 by $3258991/255152533 = 0.01277271662437308$

rat: replaced $8.176509330039827e-4$ by $105841/129445214 = 8.176509330039812e-4$

rat: replaced 0.01441523309043924 by $2330472/161667313 = 0.01441523309043925$

rat: replaced $9.704265741758145e-4$ by $651321/671169790 = 9.704265741758132e-4$

rat: replaced 0.01615630721187855 by $19391318/1200232067 = 0.01615630721187855$

rat: replaced 0.001141105023499428 by $1259907/1104111343 = 0.001141105023499428$

rat: replaced 0.01799576488272969 by $4765614/264818641 = 0.01799576488272969$

rat: replaced 0.001330669204938795 by $1231154/925214167 = 0.001330669204938796$

rat: replaced 0.01993342215875837 by $2504519/125644206 = 0.01993342215875836$

rat: replaced 0.001540100153900437 by $276884/179783113 = 0.001540100153900439$

rat: replaced 0.02196908527585173 by $1298306/59096953 = 0.0219690852758517$

rat: replaced 0.001770376919130678 by $644389/363984072 = 0.001770376919130681$

rat: replaced 0.02410255066939448 by $2001286/83032125 = 0.02410255066939453$

rat: replaced 0.002022476464811601 by $1271955/628909667 = 0.002022476464811599$

rat: replaced 0.02633360499462523 by $2978115/113091808 = 0.02633360499462525$

rat: replaced 0.002297373572865413 by $1020913/444382669 = 0.002297373572865417$

rat: replaced 0.02866202514797045 by $1770713/61779061 = 0.02866202514797044$

rat: replaced 0.002596040745477063 by $1097643/422814242 = 0.002596040745477065$

rat: replaced 0.03108757828935527 by $5034207/161936287 = 0.03108757828935525$

rat: replaced 0.002919448107844891 by $906221/310408326 = 0.002919448107844891$

rat: replaced 0.03361002186548678 by $4553215/135471944 = 0.03361002186548678$

rat: replaced 0.003268563311168871 by $1379071/421919623 = 0.003268563311168867$

rat: replaced 0.03622910363410947 by $3082649/85087642 = 0.0362291036341094$

rat: replaced 0.003644351435886262 by $5966577/1637212301 = 0.003644351435886261$

rat: replaced 0.03894456168922911 by $4913415/126164342 = 0.03894456168922911$

rat: replaced 0.004047774895164447 by $572425/141417202 = 0.004047774895164451$

rat: replaced 0.04175612448730281 by $1734727/41544253 = 0.04175612448730273$

rat: replaced 0.004479793338660443 by $2952779/659132861 = 0.004479793338660444$

rat: replaced 0.04466351087439402 by $4691119/105032473 = 0.04466351087439405$

rat: replaced 0.0049413635565565 by $2524919/510976165 = 0.004941363556556498$

rat: replaced 0.04766643011428662 by $3536207/74186529 = 0.04766643011428665$

rat: replaced 0.005433439383882244 by $1361584/250593391 = 0.005433439383882235$

rat: replaced 0.05076458191755917 by $7710025/151878036 = 0.05076458191755916$

rat: replaced 0.005956971605131645 by $1447422/242979503 = 0.005956971605131648$

rat: replaced 0.0539576564716131 by $3377975/62604183 = 0.05395765647161309$

rat: replaced 0.006512907859185624 by $3695063/567344584 = 0.006512907859185626$

rat: replaced 0.05724533447165381 by $2560865/44734912 = 0.05724533447165382$

rat: replaced 0.007102192544548636 by $1363981/192050693 = 0.007102192544548642$

rat: replaced 0.06062728715262111 by $8274761/136485754 = 0.06062728715262107$

rat: replaced 0.007725766724910044 by $1464384/189545459 = 0.007725766724910038$

rat: replaced 0.06410317632206519 by $5287663/82486755 = 0.06410317632206528$

rat: replaced 0.00838456803503801 by $1113589/132814117 = 0.008384568035038023$

rat: replaced 0.06767265439396564 by $2921400/43169579 = 0.06767265439396572$

rat: replaced 0.009079530587017326 by $433906/47789475 = 0.00907953058701733$

rat: replaced 0.07133536442348987 by $7236103/101437808 = 0.07133536442348991$

rat: replaced 0.009811584876838586 by $1363090/138926587 = 0.009811584876838586$

rat: replaced 0.07509094014268702 by $9209133/122639735 = 0.07509094014268704$

rat: replaced 0.0105816576913495 by $1163729/109976058 = 0.01058165769134951$

rat: replaced 0.07893900599711501 by $5197067/65836489 = 0.07893900599711506$

rat: replaced 0.01139067201557714 by $13426050/1178688139 = 0.01139067201557714$

rat: replaced 0.08287917718339499 by $11217158/135343501 = 0.082879177183395$

rat: replaced 0.01223954694042984 by $2283101/186534764 = 0.01223954694042983$

rat: replaced 0.08691105968769186 by $5213115/59982182 = 0.08691105968769192$

rat: replaced 0.01312919757078923 by $3499615/266552086 = 0.01312919757078922$

rat: replaced 0.09103425032511492 by $5893225/64736349 = 0.09103425032511488$

rat: replaced 0.01406053493400045 by $2280713/162206702 = 0.01406053493400045$

rat: replaced 0.09524833678003664 by $9601787/100807923 = 0.09524833678003662$

rat: replaced 0.01503446588876983 by $200490/13335359 = 0.01503446588876985$

rat: replaced 0.09955289764732322 by $5687088/57126293 = 0.09955289764732328$

rat: replaced 0.01605189303448024 by $951971/59305840 = 0.01605189303448025$

rat: replaced 0.1039475024744748 by $10260011/98703776 = 0.1039475024744747$

rat: replaced 0.01711371462093175 by $9432386/551159477 = 0.01711371462093176$

rat: replaced 0.1084317118046711 by $14939691/137779721 = 0.1084317118046712$

rat: replaced 0.01822082445851714 by $2559788/140486947 = 0.01822082445851713$

rat: replaced 0.113005077220716 by $8478529/75027859 = 0.1130050772207161$

rat: replaced 0.01937411182884202 by $2983799/154009589 = 0.01937411182884203$

rat: replaced 0.1176671413898787 by $7123715/60541243 = 0.1176671413898786$

rat: replaced 0.02057446139579705 by $7167743/348380590 = 0.02057446139579705$

rat: replaced 0.1224174381096274 by $12172179/99431741 = 0.1224174381096274$

rat: replaced 0.02182275311709253 by $7415562/339808729 = 0.02182275311709253$

rat: replaced 0.1272554923542488 by $7277933/57191504 = 0.127255492354249$

rat: replaced 0.02311986215626333 by $2988661/129268115 = 0.02311986215626336$

rat: replaced 0.1321808203223502 by $3633064/27485561 = 0.1321808203223503$

rat: replaced 0.02446665879515308 by $1991976/81415939 = 0.02446665879515312$

rat: replaced 0.1371929294852391 by $56235017/409897341 = 0.1371929294852391$

rat: replaced 0.02586400834688696 by $5000736/193347293 = 0.02586400834688697$

rat: replaced 0.1422913186361759 by $9349741/65708443 = 0.1422913186361759$

rat: replaced 0.02731277106934082 by $858413/31428997 = 0.02731277106934084$

rat: replaced 0.1474754779404944 by $1549881/10509415 = 0.1474754779404943$

rat: replaced 0.02881380207911666 by $3754753/130310918 = 0.02881380207911666$

rat: replaced 0.152744888986584 by $5264425/34465474 = 0.1527448889865841$

rat: replaced 0.03036795126603076 by $4118329/135614318 = 0.03036795126603077$

rat: replaced 0.1580990248377314 by $5442776/34426373 = 0.1580990248377312$

rat: replaced 0.03197606320812652 by $3497683/109384416 = 0.03197606320812647$

rat: replaced 0.1635373500848132 by $12328488/75386375 = 0.1635373500848131$

rat: replaced 0.0336389770872163 by $3971799/118071337 = 0.03363897708721635$

rat: replaced 0.1690593208998367 by $20896917/123607009 = 0.1690593208998367$

rat: replaced 0.03535752660496472 by $1815732/51353479 = 0.03535752660496478$

rat: replaced 0.1746643850903219 by $2841592/16268869 = 0.1746643850903219$

rat: replaced 0.03713253989951881 by $3333721/89778965 = 0.03713253989951878$

rat: replaced 0.1803519821545206 by $4461007/24735004 = 0.1803519821545208$

rat: replaced 0.03896483946269502 by $8785771/225479461 = 0.03896483946269501$

rat: replaced 0.1861215433374662 by $4381209/23539505 = 0.1861215433374661$

rat: replaced 0.0408552420577305 by $3189084/78058135 = 0.04085524205773043$

rat: replaced 0.1919724916878484 by $72809759/379271834 = 0.1919724916878484$

rat: replaced 0.04280455863760801 by $7646593/178639688 = 0.04280455863760801$

rat: replaced 0.1979042421157076 by $26318167/132984350 = 0.1979042421157076$

rat: replaced 0.04481359426396048 by $20610430/459914683 = 0.04481359426396048$

rat: replaced 0.2039162014509444 by $8519416/41779005 = 0.2039162014509441$

rat: replaced 0.04688314802656623 by $3439140/73355569 = 0.04688314802656633$

rat: replaced 0.2100077685026351 by $50962787/242670961 = 0.2100077685026351$

rat: replaced 0.04901401296344043 by $4006732/81746663 = 0.04901401296344048$

rat: replaced 0.216178334119151 by $1347531/6233423 = 0.2161783341191509$

rat: replaced 0.05120697598153157 by $4148974/81023609 = 0.0512069759815315$

rat: replaced 0.2224272812490723 by $23234851/104460437 = 0.2224272812490723$

rat: replaced 0.05346281777803219 by $11998448/224426031 = 0.05346281777803218$

rat: replaced 0.2287539850028937 by $8185268/35781969 = 0.2287539850028935$

rat: replaced 0.05578231276230905 by $1398019/25062048 = 0.05578231276230897$

rat: replaced 0.2351578127155118 by $12642104/53760085 = 0.2351578127155119$

rat: replaced 0.05816622897846346 by $4451048/76522891 = 0.05816622897846345$

rat: replaced 0.2416381240094921 by $8002142/33116223 = 0.2416381240094923$

rat: replaced 0.06061532802852698 by $2146337/35409146 = 0.06061532802852686$

rat: replaced 0.2481942708591053 by $8882901/35790113 = 0.2481942708591057$

rat: replaced 0.0631303649963022 by $14651447/232082406 = 0.06313036499630222$

rat: replaced 0.2548255976551299 by $868346/3407609 = 0.25482559765513$

rat: replaced 0.06571208837185505 by $4240309/64528599 = 0.06571208837185509$

rat: replaced 0.2615314412704124 by $8212450/31401387 = 0.2615314412704127$

rat: replaced 0.06836123997666599 by $2716643/39739522 = 0.06836123997666604$

rat: replaced 0.2683111311261794 by $34459769/128432126 = 0.2683111311261794$

rat: replaced 0.07107855488944881 by $3146673/44270357 = 0.07107855488944893$

rat: replaced 0.2751639892590951 by $12552159/45617012 = 0.2751639892590949$

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rat: replaced -0.1539740213994798 by -16145763/104860306 = -0.1539740213994798
part: invalid index of list or matrix.
#0: lineIntersection(g=[a1,a2,a2^2/2+a1^2/2],h=[-a,0,-a^2/2])
-- an error. To debug this try: debugmode(true);

Error in:
U &= lineIntersection(h,g); ...
      ^

```

Kami mendapatkan rumus untuk jari-jari lingkaran.

```
>&assume(a>0,b>0,c>0); $distance(U,B) | radcan
```

Mari kita tambahkan ini ke plot.

```

>plotPoint(U()); ...
>plotCircle(circleWithCenter(mxmeval("U"),mxmeval("distance(U,C)"))):

```

```

Function U not found.
Try list ... to find functions!
Error in:
plotPoint(U()); plotCircle(circleWithCenter(mxmeval("U"),mxmev ...
      ^

```

Menggunakan geometri, kami memperoleh rumus sederhana

$$\frac{a}{\sin(\alpha)} = 2r$$

untuk radiusnya. Kami dapat memeriksa, apakah ini benar dengan Maxima. Maxima akan memfaktorkan ini hanya jika kita kuadratkan.

```
>$c^2/sin(computeAngle(A,B,C))^2 | factor
```

Contoh 4: Garis Euler dan Parabola

Garis Euler adalah garis yang ditentukan dari sembarang segitiga yang tidak sama sisi. Ini adalah garis tengah segitiga, dan melewati beberapa titik penting yang ditentukan dari segitiga, termasuk orthocenter, circumcenter, centroid, titik Exeter dan pusat lingkaran sembilan titik segitiga.

Untuk demonstrasi, kami menghitung dan memplot garis Euler dalam sebuah segitiga.

Pertama, kita mendefinisikan sudut-sudut segitiga di Euler. Kami menggunakan definisi, yang terlihat dalam ekspresi simbolis.

```
>A:=[-1,-1]; B:=[2,0]; C:=[1,2];
```


Untuk memplot objek geometris, kami menyiapkan area plot, dan menambahkan titik ke sana. Semua plot objek geometris ditambahkan ke plot saat ini.

```
>setPlotRange(3); plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C");
```

Kita juga bisa menambahkan sisi segitiga.

```
>plotSegment(A,B,""); plotSegment(B,C,""); plotSegment(C,A,""):
```

Berikut adalah luas segitiga, menggunakan rumus determinan. Tentu saja, kita harus mengambil nilai absolut dari hasil ini.

```
>$areaTriangle(A,B,C)
```

Kita dapat menghitung koefisien sisi c.

```
>c &= lineThrough(A,B)
```

$[-1, 3, -2]$

Dan juga dapatkan rumus untuk baris ini.

```
>$getLineEquation(c,x,y)
```

Untuk bentuk Hesse, kita perlu menentukan sebuah titik, sehingga titik tersebut berada di sisi positif dari bentuk Hesse. Memasukkan titik menghasilkan jarak positif ke garis.

```
>$getHesseForm(c,x,y,C), $at(%, [x=C[1],y=C[2]])
```

Sekarang kita hitung lingkaran luar ABC.

```
>LL &= circleThrough(A,B,C); $getCircleEquation(LL,x,y)
```

Maxima said:

rat: replaced -5.049958083474387e-5 by -102157/2022927682 = -5.049958083474385e-5

rat: replaced -2.039932534230044e-4 by -284619/1395237319 = -2.039932534230043e-4

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rat: replaced -0.1757894768764047 by $-15940893/90681725 = -0.1757894768764048$

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rat: replaced -0.1927237517055264 by $-1392861/7227241 = -0.1927237517055264$

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rat: replaced -0.2105929058706983 by $-10627754/50465869 = -0.2105929058706985$

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rat: replaced -0.239186295223934 by $-27692337/115777273 = -0.239186295223934$

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rat: replaced -0.3030160617255513 by $-18597622/61375037 = -0.3030160617255514$

rat: replaced -0.3145382178610399 by $-11102944/35299189 = -0.3145382178610392$

rat: replaced -0.3263179180285316 by $-13053510/40002431 = -0.3263179180285318$

rat: replaced -0.3383569842428258 by $-13796661/40775458 = -0.3383569842428257$

rat: replaced -0.3506572125823338 by $-33496033/95523582 = -0.3506572125823338$

rat: replaced -0.3632203730094723 by $-23086207/63559780 = -0.3632203730094724$

rat: replaced -0.376048209193667 by $-22674222/60296051 = -0.3760482091936668$

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rat: replaced -0.4161368109448825 by $-10481453/25187517 = -0.4161368109448819$

rat: replaced -0.4300402549446862 by $-19565443/45496771 = -0.4300402549446861$

rat: replaced -0.4442166926440365 by $-16102633/36249500 = -0.4442166926440365$

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rat: replaced -0.4733948510561774 by $-10262860/21679281 = -0.4733948510561766$

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rat: replaced -0.551225017954267 by $-14288533/25921416 = -0.551225017954266$

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rat: replaced -0.5843413035316997 by $-18888222/32323955 = -0.5843413035316997$

rat: replaced -0.6013301265988455 by $-5789399/9627655 = -0.6013301265988447$

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rat: replaced -0.6361756386941407 by $-13914515/21872128 = -0.6361756386941407$

rat: replaced -0.6540348431501183 by $-48160581/73636109 = -0.6540348431501181$

rat: replaced -0.6721866416834846 by $-6617334/9844489 = -0.6721866416834841$

rat: replaced -0.690632219104513 by $-16840135/24383651 = -0.6906322191045139$

rat: replaced -0.7093727308458327 by $-29189494/41148317 = -0.7093727308458326$

rat: replaced -0.7284093028468864 by $-13153959/18058472 = -0.7284093028468854$

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rat: replaced -0.7673749832474404 by $-39576757/51574208 = -0.7673749832474402$

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rat: replaced -0.84890530902455 by $-25231431/29722315 = -0.8489053090245494$

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rat: replaced -0.8914853417578728 by $-33469619/37543656 = -0.8914853417578725$

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rat: replaced -0.9160185005549473 by $-21232969/23179629 = -0.9160185005549485$

rat: replaced -0.9416558733318703 by $-11919739/12658275 = -0.9416558733318718$

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rat: replaced -1.047757508208077 by $-16616467/15859077 = -1.047757508208075$

rat: replaced -1.07516769015946 by $-21797467/20273551 = -1.07516769015946$

rat: replaced -1.102930352404475 by $-30072842/27266311 = -1.102930352404474$

rat: replaced -1.131044718683369 by $-7906291/6990255 = -1.131044718683367$

rat: replaced -1.159509977566275 by $-26421893/22787120 = -1.159509977566274$

rat: replaced -1.188325282534358 by $-19245269/16195287 = -1.188325282534357$

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rat: replaced -1.247002469717292 by $-21540268/17273637 = -1.247002469717292$

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rat: replaced -1.337620425225263 by $-48765573/36456959 = -1.337620425225263$

rat: replaced -1.368516275933041 by $-117856634/86120009 = -1.368516275933041$

rat: replaced -1.399755272153666 by $-29694085/21213769 = -1.399755272153666$

rat: replaced -1.431336289996881 by $-23110861/16146353 = -1.43133628999688$

rat: replaced -1.463258171370553 by $-19245288/13152353 = -1.463258171370553$

rat: replaced -1.495519724096479 by $-43164951/28862843 = -1.495519724096479$

rat: replaced -1.528119722029604 by $-31224680/20433399 = -1.528119722029605$

rat: replaced -1.561056905180636 by $-23576644/15103001 = -1.561056905180633$

rat: replaced -1.594329979842039 by $-26705354/16750205 = -1.594329979842038$

rat: replaced -1.627937618717409 by $-52734804/32393627 = -1.62793761871741$

rat: replaced -1.661878461054199 by $-42611978/25640851 = -1.661878461054198$

```

part: invalid index of list or matrix.
#0: lineIntersection(g=[-3,-1,-1],h=[-2,-3,-3/2])
#1: circleThrough(a=[-1,-1],b=[2,0],c=[1,2])
-- an error. To debug this try: debugmode(true);

Error in:
LL &= circleThrough(A,B,C); $getCircleEquation(LL,x,y) ...
      ^

```

```
>0 &= getCircleCenter(LL); $0
```

Gambarkan lingkaran dan pusatnya. Cu dan U adalah simbolis. Kami mengevaluasi ekspresi ini untuk Euler.

```
>plotCircle(LL()); plotPoint(0(),"0"):
```

```

Function LL not found.
Try list ... to find functions!
Error in:
plotCircle(LL()); plotPoint(0(),"0"): ...
      ^

```

Kita dapat menghitung perpotongan ketinggian di ABC (orthocenter) secara numerik dengan perintah berikut.

```
>H &= lineIntersection(perpendicular(A,lineThrough(C,B)),...  
> perpendicular(B,lineThrough(A,C))); $H
```

Maxima said:

```
rat: replaced -9.983250083613754e-5 by -612914/6139423483 = -9.983250083613756e-5  
  
rat: replaced -3.986533601775671e-4 by -220554/553247563 = -3.986533601775666e-4  
  
rat: replaced -8.954327045205754e-4 by -584699/652979277 = -8.954327045205756e-4  
  
rat: replaced -0.001589120864678328 by -740868/466212493 = -0.00158912086467833  
  
rat: replaced -0.002478648480745763 by -878917/354595259 = -0.002478648480745762  
  
rat: replaced -0.003562926609036218 by -2735717/767828614 = -0.003562926609036219  
  
rat: replaced -0.004840846830973591 by -1164348/240525685 = -0.004840846830973582  
  
rat: replaced -0.006311281363933816 by -16515210/2616776063 = -0.006311281363933816  
  
rat: replaced -0.007973083174022497 by -2414321/302808957 = -0.007973083174022491  
  
rat: replaced -0.009825086090776508 by -1144049/116441626 = -0.009825086090776506  
  
rat: replaced -0.01186610492378118 by -1659683/139867548 = -0.01186610492378118  
  
rat: replaced -0.01409493558118687 by -986877/70016425 = -0.01409493558118684  
  
rat: replaced -0.01651035519011868 by -1738361/105289134 = -0.01651035519011867  
  
rat: replaced -0.01911112221896202 by -1475047/77182647 = -0.01911112221896199  
  
rat: replaced -0.02189597660151474 by -7711274/352177669 = -0.02189597660151473
```

rat: replaced -0.02486363986299212 by $-3887839/156366446 = -0.02486363986299209$

rat: replaced -0.0280128152478745 by $-2263313/80795628 = -0.02801281524787455$

rat: replaced -0.03134218784958129 by $-1116362/35618509 = -0.03134218784958124$

rat: replaced -0.03485042474195996 by $-3920507/112495243 = -0.03485042474195998$

rat: replaced -0.03853617511257795 by $-5379408/139593719 = -0.03853617511257795$

rat: replaced -0.04239807039780302 by $-3385918/79860191 = -0.04239807039780308$

rat: replaced -0.04643472441965829 by $-10918553/235137672 = -0.04643472441965828$

rat: replaced -0.05064473352443885 by $-5036501/99447675 = -0.05064473352443886$

rat: replaced -0.05502667672307548 by $-2932521/53292715 = -0.05502667672307557$

rat: replaced -0.05957911583323347 by $-6320819/106091185 = -0.05957911583323346$

rat: replaced -0.06430059562312868 by $-9893260/153859539 = -0.0643005956231287$

rat: replaced -0.06918964395705007 by $-6012189/86894348 = -0.06918964395705$

rat: replaced -0.07424477194257195 by $-6096479/82113243 = -0.07424477194257204$

rat: replaced -0.07946447407944118 by $-5389689/67825139 = -0.07946447407944125$

rat: replaced -0.0848472284101276 by $-9595393/113090235 = -0.08484722841012754$

rat: replaced -0.09039149667201674 by $-3773144/41742245 = -0.09039149667201657$

rat: replaced -0.0960957244512361 by $-5162056/53717853 = -0.09609572445123597$

rat: replaced -0.1019583413380946 by $-1082663/10618680 = -0.1019583413380948$

rat: replaced -0.107977761084122 by $-1922059/17800508 = -0.1079777610841219$

rat: replaced -0.1141523817606936 by $-5923297/51889386 = -0.1141523817606938$

rat: replaced -0.1204805859192203 by $-17634703/146369665 = -0.1204805859192204$

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rat: replaced -0.2159601709509153 by $-11348921/52550991 = -0.2159601709509151$

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rat: replaced -0.2499192001753251 by $-11309023/45250717 = -0.2499192001753254$

rat: replaced -0.2587186289254649 by $-7582961/29309683 = -0.2587186289254647$

rat: replaced -0.267638184811648 by $-17912865/66929407 = -0.2676381848116479$

rat: replaced -0.2766759758940514 by $-27538925/99534934 = -0.2766759758940514$

rat: replaced -0.2858300984094321 by $-29258587/102363562 = -0.2858300984094321$

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rat: replaced -0.3235714244095225 by $-178371467/551258404 = -0.3235714244095225$

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rat: replaced 1.399755272153666 by $29694085/21213769 = 1.399755272153666$

```

rat: replaced 1.431336289996881 by 23110861/16146353 = 1.43133628999688
rat: replaced 1.463258171370553 by 19245288/13152353 = 1.463258171370553
rat: replaced 1.495519724096479 by 43164951/28862843 = 1.495519724096479
rat: replaced 1.528119722029604 by 31224680/20433399 = 1.528119722029605
rat: replaced 1.561056905180636 by 23576644/15103001 = 1.561056905180633
rat: replaced 1.594329979842039 by 26705354/16750205 = 1.594329979842038
rat: replaced 1.627937618717409 by 52734804/32393627 = 1.62793761871741

rat: replaced 1.661878461054199 by 42611978/25640851 = 1.661878461054198
part: invalid index of list or matrix.
#0: lineIntersection(g=[1,-2,1],h=[2,3,4])
-- an error. To debug this try: debugmode(true);

Error in:
  perpendicular(B,lineThrough(A,C)); $H ...
               ^

```

Sekarang kita dapat menghitung garis Euler dari segitiga.

```
>el &= lineThrough(H,O); $getLineEquation(el,x,y)
```

Tambahkan ke plot kami.

```
>plotPoint(H(),"H"); plotLine(e1(),"Garis Euler"):
```

```
Function H needs at least 3 arguments!  
Use: H (a, b, c)  
Error in:  
plotPoint(H(),"H"); plotLine(e1(),"Garis Euler"): ...  
~
```

Pusat gravitasi harus berada di garis ini.

```
>M &= (A+B+C)/3; $getLineEquation(e1,x,y) with [x=M[1],y=M[2]]  
>plotPoint(M(),"M"): // titik berat
```

Teorinya memberitahu kita $MH=2*MO$. Kita perlu menyederhanakan dengan radcan untuk mencapai ini.

```
>$distance(M,H)/distance(M,O)|radcan
```

Fungsi termasuk fungsi untuk sudut juga.

```
>$computeAngle(A,C,B), degprint(%())
```

60°15'18.43''

Persamaan untuk pusat incircle tidak terlalu bagus.

```
>Q &= lineIntersection(angleBisector(A,C,B),angleBisector(C,B,A))|radcan; $Q
```

Maxima said:

rat: replaced 1.66665833335744e-7 by 15819/94914474571 = 1.66665833335744e-7

rat: replaced 4.999958333473664e-5 by 201389/4027813565 = 4.99995833347366e-5

rat: replaced 1.33330666692022e-6 by 31771/23828726570 = 1.333306666920221e-6

rat: replaced 1.999933334222437e-4 by 200030/1000183339 = 1.999933334222437e-4

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rat: replaced 2.083072932167196e-5 by 35635/1710693824 = 2.0830729321672e-5

rat: replaced 0.001249739605033717 by 567943/454449069 = 0.001249739605033716

rat: replaced $3.599352055540239\text{e-}5$ by $98277/2730408098 = 3.599352055540234\text{e-}5$

rat: replaced 0.00179946006479581 by $479561/266502719 = 0.001799460064795812$

rat: replaced $5.71526624672386\text{e-}5$ by $51154/895041417 = 5.715266246723866\text{e-}5$

rat: replaced 0.002448999746720415 by $1946227/794702818 = 0.002448999746720415$

rat: replaced $8.530603082730626\text{e-}5$ by $121691/1426522824 = 8.530603082730627\text{e-}5$

rat: replaced 0.003198293697380561 by $2986741/933854512 = 0.003198293697380562$

rat: replaced $1.214508019889565\text{e-}4$ by $158455/1304684674 = 1.214508019889563\text{e-}4$

rat: replaced 0.004047266988005727 by $2125334/525128193 = 0.004047266988005727$

rat: replaced $1.665833531718508\text{e-}4$ by $142521/855553675 = 1.66583353171851\text{e-}4$

rat: replaced 0.004995834721974179 by $1957223/391770967 = 0.004995834721974179$

rat: replaced $2.216991628251896\text{e-}4$ by $179571/809975995 = 2.216991628251896\text{e-}4$

rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced $2.877927110806339\text{e-}4$ by $1167733/4057548906 = 2.877927110806339\text{e-}4$

rat: replaced 0.00719136414613375 by $2476362/344352191 = 0.007191364146133747$

rat: replaced $3.658573803051457\text{e-}4$ by $386279/1055818526 = 3.658573803051454\text{e-}4$

rat: replaced 0.00843810628521191 by $2079855/246483622 = 0.008438106285211924$

rat: replaced $4.5688535576352\text{e-}4$ by $262978/575588595 = 4.568853557635206\text{e-}4$

rat: replaced 0.009784003787362772 by $1752551/179124113 = 0.009784003787362787$

rat: replaced $5.618675264007778e-4$ by $150595/268025812 = 5.618675264007782e-4$

rat: replaced 0.01122892206395776 by $5450241/485375263 = 0.01122892206395776$

rat: replaced $6.817933857540259e-4$ by $192316/282073725 = 6.817933857540258e-4$

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rat: replaced 0.001141105023499428 by $1259907/1104111343 = 0.001141105023499428$

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rat: replaced 0.01993342215875837 by $2504519/125644206 = 0.01993342215875836$

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rat: replaced 0.1474754779404944 by $1549881/10509415 = 0.1474754779404943$

rat: replaced 0.02731277106934082 by $858413/31428997 = 0.02731277106934084$

rat: replaced 0.152744888986584 by $5264425/34465474 = 0.1527448889865841$

rat: replaced 0.02881380207911666 by $3754753/130310918 = 0.02881380207911666$

rat: replaced 0.1580990248377314 by $5442776/34426373 = 0.1580990248377312$

rat: replaced 0.03036795126603076 by $4118329/135614318 = 0.03036795126603077$

rat: replaced 0.1635373500848132 by $12328488/75386375 = 0.1635373500848131$

rat: replaced 0.03197606320812652 by $3497683/109384416 = 0.03197606320812647$

rat: replaced 0.1690593208998367 by $20896917/123607009 = 0.1690593208998367$

rat: replaced 0.0336389770872163 by $3971799/118071337 = 0.03363897708721635$

rat: replaced 0.1746643850903219 by $2841592/16268869 = 0.1746643850903219$

rat: replaced 0.03535752660496472 by $1815732/51353479 = 0.03535752660496478$

rat: replaced 0.1803519821545206 by $4461007/24735004 = 0.1803519821545208$

rat: replaced 0.03713253989951881 by $3333721/89778965 = 0.03713253989951878$

rat: replaced 0.1861215433374662 by $4381209/23539505 = 0.1861215433374661$

rat: replaced 0.03896483946269502 by $8785771/225479461 = 0.03896483946269501$

rat: replaced 0.1919724916878484 by $72809759/379271834 = 0.1919724916878484$

rat: replaced 0.0408552420577305 by $3189084/78058135 = 0.04085524205773043$

rat: replaced 0.1979042421157076 by $26318167/132984350 = 0.1979042421157076$

rat: replaced 0.04280455863760801 by $7646593/178639688 = 0.04280455863760801$

rat: replaced 0.2039162014509444 by $8519416/41779005 = 0.2039162014509441$

rat: replaced 0.04481359426396048 by $20610430/459914683 = 0.04481359426396048$

rat: replaced 0.2100077685026351 by $50962787/242670961 = 0.2100077685026351$

rat: replaced 0.04688314802656623 by $3439140/73355569 = 0.04688314802656633$

rat: replaced 0.216178334119151 by $1347531/6233423 = 0.2161783341191509$

rat: replaced 0.04901401296344043 by $4006732/81746663 = 0.04901401296344048$

rat: replaced 0.2224272812490723 by $23234851/104460437 = 0.2224272812490723$

rat: replaced 0.05120697598153157 by $4148974/81023609 = 0.0512069759815315$

rat: replaced 0.2287539850028937 by $8185268/35781969 = 0.2287539850028935$

rat: replaced 0.05346281777803219 by $11998448/224426031 = 0.05346281777803218$

rat: replaced 0.2351578127155118 by $12642104/53760085 = 0.2351578127155119$

rat: replaced 0.05578231276230905 by $1398019/25062048 = 0.05578231276230897$

rat: replaced 0.2416381240094921 by $8002142/33116223 = 0.2416381240094923$

rat: replaced 0.05816622897846346 by $4451048/76522891 = 0.05816622897846345$

rat: replaced 0.2481942708591053 by $8882901/35790113 = 0.2481942708591057$

rat: replaced 0.06061532802852698 by $2146337/35409146 = 0.06061532802852686$

rat: replaced 0.2548255976551299 by $868346/3407609 = 0.25482559765513$

rat: replaced 0.0631303649963022 by $14651447/232082406 = 0.06313036499630222$

rat: replaced 0.2615314412704124 by $8212450/31401387 = 0.2615314412704127$

rat: replaced 0.06571208837185505 by $4240309/64528599 = 0.06571208837185509$

rat: replaced 0.2683111311261794 by $34459769/128432126 = 0.2683111311261794$

rat: replaced 0.06836123997666599 by $2716643/39739522 = 0.06836123997666604$

rat: replaced 0.2751639892590951 by $12552159/45617012 = 0.2751639892590949$

rat: replaced 0.07107855488944881 by $3146673/44270357 = 0.07107855488944893$

rat: replaced 0.2820893303890569 by $11134456/39471383 = 0.2820893303890568$

rat: replaced 0.07386476137264342 by $12898997/174629915 = 0.0738647613726434$

rat: replaced 0.2890864619877229 by $9583357/33150487 = 0.2890864619877228$

rat: replaced 0.07672058079958999 by $5073506/66129661 = 0.07672058079959007$

rat: replaced 0.2961546843477643 by $11052271/37319251 = 0.2961546843477647$

rat: replaced 0.07964672758239233 by $5672399/71219486 = 0.07964672758239227$

rat: replaced 0.3032932906528349 by $9918077/32701274 = 0.3032932906528351$

rat: replaced 0.08264390910047736 by $4686067/56701904 = 0.08264390910047748$

rat: replaced 0.3105015670482534 by $9320011/30015987 = 0.3105015670482533$

rat: replaced 0.0857128256298576 by $3585977/41837111 = 0.08571282562985766$

rat: replaced 0.3177787927123868 by $248395525/781661743 = 0.3177787927123868$

rat: replaced 0.08885417027310427 by $5751353/64728003 = 0.0888541702731042$

rat: replaced 0.3251242399287333 by $13842845/42577093 = 0.3251242399287335$

rat: replaced 0.09206862889003742 by $7305460/79347983 = 0.09206862889003745$

rat: replaced 0.3325371741586922 by $9318229/28021616 = 0.3325371741586923$

rat: replaced 0.09535688002914089 by $5971998/62627867 = 0.09535688002914103$

rat: replaced 0.3400168541150183 by $13391981/39386227 = 0.3400168541150184$

rat: replaced 0.0987195948597075 by $9821211/99485933 = 0.09871959485970745$

rat: replaced 0.3475625318359485 by $10097818/29053241 = 0.347562531835949$

rat: replaced 0.1021574371047232 by $8336413/81603584 = 0.1021574371047232$

rat: replaced 0.3551734527599992 by $15867851/44676343 = 0.3551734527599987$

rat: replaced 0.1056710629744951 by $5741011/54329074 = 0.105671062974495$

rat: replaced 0.3628488558014202 by $6897641/19009681 = 0.3628488558014203$

rat: replaced 0.1092611211010309 by $5551873/50812887 = 0.1092611211010309$

rat: replaced 0.3705879734263036 by $23358661/63031352 = 0.3705879734263038$

rat: replaced 0.1129282524731764 by $11548693/102265755 = 0.1129282524731764$

rat: replaced 0.3783900317293359 by $14241382/37636779 = 0.3783900317293358$

rat: replaced 0.1166730903725168 by $5656228/48479285 = 0.1166730903725168$

rat: replaced 0.3862542505111889 by $3461217/8960981 = 0.3862542505111884$

rat: replaced 0.1204962603100498 by $4057613/33674182 = 0.12049626031005$

rat: replaced 0.3941798433565377 by $5314214/13481699 = 0.3941798433565384$

rat: replaced 0.1243983799636342 by $7966447/64039797 = 0.1243983799636342$

rat: replaced 0.4021660177127022 by $11567173/28762184 = 0.4021660177127022$

rat: replaced 0.1283800591162231 by $796346/6203035 = 0.1283800591162229$

rat: replaced 0.4102119749689023 by $11320633/27597032 = 0.4102119749689024$

rat: replaced 0.1324418995948859 by $4716124/35609003 = 0.1324418995948862$

rat: replaced 0.418316910536117 by $12225195/29224721 = 0.4183169105361177$

rat: replaced 0.1365844952106265 by $612971/4487852 = 0.1365844952106264$

rat: replaced 0.4264800139275439 by $7978696/18708253 = 0.4264800139275431$

rat: replaced 0.140808431699002 by $10431632/74083859 = 0.1408084316990021$

rat: replaced 0.4347004688396462 by $20489554/47134879 = 0.4347004688396463$

rat: replaced 0.1451142866615502 by $3554077/24491572 = 0.1451142866615504$

rat: replaced 0.4429774532337832 by $23449796/52936771 = 0.4429774532337834$

rat: replaced 0.1495026295080298 by $26759297/178988805 = 0.1495026295080298$

rat: replaced 0.451310139418413 by $8841241/19590167 = 0.4513101394184133$

```

rat: replaced 0.1539740213994798 by 16145763/104860306 = 0.1539740213994798
part: invalid index of list or matrix.
#0: lineIntersection(g=[-sqrt(13)/sqrt(5)-2,sqrt(5)*sqrt(13)/2-sqrt(13)/(2*sqrt(5))-3,(1-2*sqrt(13)
-- an error. To debug this try: debugmode(true);

Error in:
... angleBisector(A,C,B),angleBisector(C,B,A))|radcan; $Q ...

```

Mari kita hitung juga ekspresi untuk jari-jari lingkaran yang tertulis.

```
>r &= distance(Q,projectToLine(Q,lineThrough(A,B)))|ratsimp; $r
```

```

Maxima said:
rat: replaced 1.498320841708742e-4 by 1329822/8875415485 = 1.498320841708742e-4

rat: replaced 5.986466935998108e-4 by 398723/666040595 = 5.986466935998098e-4

rat: replaced 0.001345398955533032 by 4525441/3363642421 = 0.001345398955533032

rat: replaced 0.002389014203700413 by 1071627/448564516 = 0.00238901420370041

rat: replaced 0.00372838808577948 by 661903/177530607 = 0.003728388085779485

rat: replaced 0.005362386673832029 by 5230891/975478144 = 0.005362386673832028

rat: replaced 0.007289846577694006 by 32241346/4422774287 = 0.007289846577694006

rat: replaced 0.009509575061314376 by 2146493/225719129 = 0.009509575061314364

rat: replaced 0.01202035016202822 by 1789188/148846579 = 0.01202035016202825

```

rat: replaced 0.01482092081275069 by $2581665/174190594 = 0.01482092081275066$

rat: replaced 0.01791000696708436 by $5107285/285163764 = 0.01791000696708436$

rat: replaced 0.02128629972732062 by $3323295/156123659 = 0.02128629972732064$

rat: replaced 0.02494846147533059 by $4548287/182307314 = 0.02494846147533061$

rat: replaced 0.02889512600632479 by $3147802/108938857 = 0.02889512600632481$

rat: replaced 0.0331248986654725 by $5858625/176864692 = 0.03312489866547248$

rat: replaced 0.03763635648736519 by $10043830/266865099 = 0.03763635648736518$

rat: replaced 0.04242804833831373 by $4635713/109260576 = 0.04242804833831372$

rat: replaced 0.04749849506145984 by $5610259/118114458 = 0.04749849506145979$

rat: replaced 0.05284618962468965 by $4237503/80185592 = 0.05284618962468968$

rat: replaced 0.05846959727133633 by $3317197/56733707 = 0.05846959727133642$

rat: replaced 0.06436715567365475 by $13427433/208606903 = 0.06436715567365477$

rat: replaced 0.07053727508905278 by $8025659/113778977 = 0.07053727508905269$

rat: replaced 0.07697833851906408 by $6306881/81930594 = 0.07697833851906408$

rat: replaced 0.08368870187104593 by $4282086/51166835 = 0.08368870187104596$

rat: replaced 0.09066669412258874 by $2175091/23989967 = 0.09066669412258883$

rat: replaced 0.09791061748861546 by $8290049/84669561 = 0.09791061748861554$

rat: replaced 0.1054187475911595 by $8501563/80645646 = 0.1054187475911595$

rat: replaced 0.1131893336318011 by $6539019/57770629 = 0.113189333631801$

rat: replaced 0.121220598566744 by $5779101/47674249 = 0.1212205985667441$

rat: replaced 0.1295107392845216 by $7134865/55090914 = 0.1295107392845216$

rat: replaced 0.1380579267863034 by $6113057/44278928 = 0.1380579267863034$

rat: replaced 0.1468603063687953 by $6311140/42973763 = 0.1468603063687953$

rat: replaced 0.1559159978097077 by $4027079/25828517 = 0.1559159978097078$

rat: replaced 0.1652230955557758 by $10597125/64138279 = 0.1652230955557757$

rat: replaced 0.1747796689133147 by $9649007/55206690 = 0.1747796689133147$

rat: replaced 0.1845837622412855 by $6871913/37229239 = 0.1845837622412857$

rat: replaced 0.1946333951468589 by $39341769/202132676 = 0.1946333951468589$

rat: replaced 0.2049265626834523 by $10758647/52500012 = 0.2049265626834523$

rat: replaced 0.2154612355512225 by $33702610/156420759 = 0.2154612355512225$

rat: replaced 0.2262353602999955 by $2338161/10335082 = 0.2262353602999957$

rat: replaced 0.2372468595346078 by $7573078/31920667 = 0.2372468595346081$

rat: replaced 0.2484936321226457 by $3764353/15148690 = 0.2484936321226456$

rat: replaced 0.2599735534045555 by $26335713/101301508 = 0.2599735534045554$

rat: replaced 0.2716844754061095 by $29831699/109802737 = 0.2716844754061094$

rat: replaced 0.2836242270531998 by $15100773/53242183 = 0.2836242270531995$

rat: replaced 0.2957906143889442 by $2942977/9949528 = 0.2957906143889439$

rat: replaced 0.3081814207930817 by $12077608/39189929 = 0.3081814207930818$

rat: replaced 0.3207944072036307 by $9185023/28632117 = 0.3207944072036308$

rat: replaced 0.333627312340794 by $5228336/15671187 = 0.3336273123407946$

rat: replaced 0.346677852933085 by $15615111/45042136 = 0.3466778529330847$

rat: replaced 0.3599437239456539 by $7564465/21015688 = 0.3599437239456543$

rat: replaced 0.3734225988107874 by $7702871/20627758 = 0.3734225988107869$

rat: replaced 0.3871121296605642 by $97723109/252441351 = 0.3871121296605642$

rat: replaced 0.4010099475616409 by $3146543/7846546 = 0.4010099475616405$

rat: replaced 0.4151136627521425 by $6219049/14981557 = 0.4151136627521425$

rat: replaced 0.4294208648806354 by $26148647/60892819 = 0.4294208648806356$

rat: replaced 0.4439291232471635 by $19525684/43983787 = 0.4439291232471638$

rat: replaced 0.458635987046313 by $38604672/84172793 = 0.4586359870463132$

rat: replaced 0.4735389856122937 by $11146199/23538081 = 0.4735389856122935$

rat: replaced 0.488635628666001 by $13946471/28541658 = 0.4886356286660011$

rat: replaced 0.5039234065640431 by $5948069/11803518 = 0.503923406564043$

rat: replaced 0.5193997905497036 by $24027011/46259185 = 0.5193997905497038$

rat: replaced 0.5350622330058146 by $7363779/13762472 = 0.5350622330058147$

rat: replaced 0.5509081677095147 by $8130825/14758948 = 0.5509081677095142$

rat: replaced 0.5669350100888726 by $10250363/18080314 = 0.5669350100888735$

rat: replaced 0.5831401574813392 by $37655026/64572857 = 0.5831401574813393$

rat: replaced 0.5995209893940125 by $30778651/51338738 = 0.5995209893940128$

rat: replaced 0.6160748677656853 by $23698401/38466755 = 0.616074867765685$

rat: replaced 0.6327991372306488 by $5052598/7984521 = 0.6327991372306492$

rat: replaced 0.6496911253842265 by $60646047/93345968 = 0.6496911253842266$

rat: replaced 0.666748143050013 by $30125566/45182827 = 0.6667481430500132$

rat: replaced 0.6839674845487889 by $8953739/13090884 = 0.6839674845487899$

rat: replaced 0.7013464279690875 by $7888577/11247761 = 0.7013464279690864$

rat: replaced 0.7188822354393821 by $16662338/23178119 = 0.7188822354393815$

rat: replaced 0.7365721534018723 by $13899283/18870226 = 0.7365721534018723$

rat: replaced 0.7544134128878366 by $16270763/21567436 = 0.754413412887837$

rat: replaced 0.7724032297945274 by $8203205/10620366 = 0.7724032297945287$

rat: replaced 0.7905388051635788 by $10794522/13654639 = 0.7905388051635784$

rat: replaced 0.8088173254609005 by $16745047/20703126 = 0.808817325460899$

rat: replaced 0.8272359628580275 by $20291194/24528907 = 0.827235962858027$

rat: replaced 0.8457918755149025 by $10996366/13001267 = 0.8457918755149018$

rat: replaced 0.8644822078640563 by $9158500/10594203 = 0.8644822078640555$

rat: replaced 0.8833040908961625 by $13759446/15577247 = 0.8833040908961641$

rat: replaced 0.9022546424469358 by $19827819/21975857 = 0.9022546424469362$

rat: replaced 0.9213309674853474 by $60458149/65620446 = 0.9213309674853475$

rat: replaced 0.9405301584031224 by $11658841/12396031 = 0.9405301584031212$

rat: replaced 0.9598492953055026 by $26214088/27310629 = 0.9598492953055018$

rat: replaced 0.9792854463032298 by $35089005/35831233 = 0.9792854463032293$

rat: replaced 0.9988356678057343 by $15735752/15754095 = 0.9988356678057356$

rat: replaced 1.018497004815491 by $16202286/15908035 = 1.018497004815491$

rat: replaced 1.038266491223517 by $17763365/17108676 = 1.038266491223517$

rat: replaced 1.058141150105979 by $33730321/31876958 = 1.058141150105979$

rat: replaced 1.078117994021884 by $51996446/48228901 = 1.078117994021883$

rat: replaced 1.098194025311821 by $124719922/113568203 = 1.098194025311821$

rat: replaced 1.118366236397724 by $92837336/83011569 = 1.118366236397724$

rat: replaced 1.13863161008363 by $20601995/18093644 = 1.138631610083629$


```

rat: replaced 1.158987119857388 by 20626233/17796775 = 1.15898711985739

rat: replaced 1.17942973019332 by 4098089/3474636 = 1.179429730193321

rat: replaced 1.199956396855759 by 17442145/14535649 = 1.199956396855758
part: invalid index of list or matrix.
#0: lineIntersection(g=[3,1,[3,1] . Q],h=[-1,3,-2])
#1: projectToLine(a=Q,g=[-1,3,-2])
    -- an error. To debug this try: debugmode(true);

Error in:
... ance(Q,projectToLine(Q,lineThrough(A,B)))|ratsimp; $r ...

```

```
>LD &= circleWithCenter(Q,r); // Lingkaran dalam
```

Mari kita tambahkan ini ke plot.

```
>color(5); plotCircle(LD()):
```

```

Q is not a variable!
Error in expression: [Q[1],Q[2],cc[3]]
Error in:
color(5); plotCircle(LD()): ...

```

Selanjutnya akan dicari persamaan tempat kedudukan titik-titik yang berjarak sama ke titik C dan ke garis AB.

```
>p &= getHesseForm(lineThrough(A,B),x,y,C)-distance([x,y],C); $p='0
```

Persamaan tersebut dapat digambar menjadi satu dengan gambar sebelumnya.

```
>plot2d(p,level=0,add=1,contourcolor=6):
```

Wrong argument!

Cannot combine a symbolic expression here.

Did you want to create a symbolic expression?

Then start with &.

Error in expression: (if [2/sqrt(10),(1.498320841708742e-4*r+2)/sqrt(10),(5.986466935998108e-4*r+2

%ploteval2:

if maps then return %mapexpression2(x,y,f\$;args());

fcontour:

Z=%ploteval2(f\$,X,Y,maps;args());

Try "trace errors" to inspect local variables after errors.

plot2d:

=style,=outline,=frame);

Ini seharusnya menjadi beberapa fungsi, tetapi pemecah default Maxima hanya dapat menemukan solusinya, jika kita kuadratkan persamaannya. Akibatnya, kami mendapatkan solusi palsu.

```
>akar &= solve(getHesseForm(lineThrough(A,B),x,y,C)^2-distance([x,y],C)^2,y)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
... (lineThrough(A,B),x,y,C)^2-distance([x,y],C)^2,y) ...
```

Solusi pertama adalah

maxima: akar[1]

Menambahkan solusi pertama ke plot menunjukkan, bahwa itu memang jalan yang kita cari. Teorinya memberi tahu kita bahwa itu adalah parabola yang diputar.

```
>plot2d(&rhs(akar[1]),add=1):  
>function g(x) &= rhs(akar[1]); $'g(x)= g(x)// fungsi yang mendefinisikan kurva di atas  
>T &=[-1, g(-1)]; // ambil sebarang titik pada kurva tersebut  
>dTC &= distance(T,C); $fullratsimp(dTC), $float(%) // jarak T ke C  
>U &= projectToLine(T,lineThrough(A,B)); $U // proyeksi T pada garis AB
```

Maxima said:

rat: replaced 5.049958083474387e-5 by 102157/2022927682 = 5.049958083474385e-5

rat: replaced $2.039932534230044e-4$ by $284619/1395237319 = 2.039932534230043e-4$

rat: replaced $4.634656435254722e-4$ by $573493/1237401322 = 4.634656435254721e-4$

rat: replaced $8.31890779119604e-4$ by $332331/399488741 = 8.318907791196046e-4$

rat: replaced 0.001312231792998733 by $448125/341498356 = 0.001312231792998734$

rat: replaced 0.001907440626462018 by $276030/144712237 = 0.001907440626462018$

rat: replaced 0.002620457734122131 by $2586613/987084419 = 0.002620457734122131$

rat: replaced 0.00345421178986248 by $3402379/984994322 = 0.00345421178986248$

rat: replaced 0.004411619393972596 by $966955/219183686 = 0.004411619393972597$

rat: replaced 0.005495584781489732 by $2798484/509224061 = 0.005495584781489734$

rat: replaced 0.006708999531778753 by $6054060/902378957 = 0.006708999531778753$

rat: replaced 0.008054742279375651 by $806546/100133061 = 0.00805474227937564$

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part: invalid index of list or matrix.
#0: lineIntersection(g=[3,1,-3],h=[-1,3,-2])
#1: projectToLine(a=[-1,0],g=[-1,3,-2])
-- an error. To debug this try: debugmode(true);

Error in:
U &= projectToLine(T,lineThrough(A,B)); $U // proyeksi T pada ...

```

```

>dU2AB &= distance(T,U); $fullratsimp(dU2AB), $float(%) // jarak T ke AB

```

Ternyata jarak T ke C sama dengan jarak T ke AB. Coba Anda pilih titik T yang lain dan ulangi perhitungan-perhitungan di atas untuk menunjukkan bahwa hasilnya juga sama.

Contoh 5: Trigonometri Rasional

Ini terinspirasi dari ceramah N.J.Wildberger. Dalam bukunya "Divine Proportions", Wildberger mengusulkan untuk mengganti pengertian klasik tentang jarak dan sudut dengan kuadrat dan penyebaran. Dengan menggunakan ini, memang mungkin untuk menghindari fungsi trigonometri dalam banyak contoh, dan tetap "rasional".

Berikut ini, saya memperkenalkan konsep, dan memecahkan beberapa masalah. Saya menggunakan perhitungan simbolik Maxima di sini, yang menyembunyikan keuntungan utama dari trigonometri rasional bahwa perhitungan hanya dapat dilakukan dengan kertas dan pensil. Anda diundang untuk memeriksa hasil tanpa komputer.

Intinya adalah bahwa perhitungan rasional simbolis sering kali menghasilkan hasil yang sederhana. Sebaliknya, trigonometri klasik menghasilkan hasil trigonometri yang rumit, yang hanya mengevaluasi perkiraan numerik.

```
>load geometry;
```

Untuk pengenalan pertama, kami menggunakan segitiga persegi panjang dengan proporsi Mesir terkenal 3, 4 dan 5. Perintah berikut adalah perintah Euler untuk merencanakan geometri bidang yang terdapat dalam file Euler "geometry.e".

```
>C&:=[0,0]; A&:=[4,0]; B&:=[0,3]; ...  
>setPlotRange(-1,5,-1,5); ...  
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...  
>plotSegment(B,A,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...  
>insimg(30);
```


Tentu saja,

$$\sin(w_a) = \frac{a}{c},$$

di mana w_a adalah sudut di A. Cara yang biasa untuk menghitung sudut ini, adalah dengan mengambil invers dari fungsi sinus. Hasilnya adalah sudut yang tidak dapat dicerna, yang hanya dapat dicetak kira-kira.

```
>wa := arcsin(3/5); degprint(wa)
```

36°52'11.63''

Trigonometri rasional mencoba menghindari hal ini.

Gagasan pertama trigonometri rasional adalah kuadran, yang menggantikan jarak. Sebenarnya, itu hanya jarak kuadrat. Berikut ini, a , b , dan c menunjukkan kuadrat dari sisi-sisinya.

Teorema Pythagoras menjadi $a+b=c$.

```
>a &= 3^2; b &= 4^2; c &= 5^2; &a+b=c
```

Pengertian kedua dari trigonometri rasional adalah penyebaran. Spread mengukur pembukaan antar baris. Ini adalah 0, jika garis-garisnya sejajar, dan 1, jika garis-garisnya persegi panjang. Ini adalah kuadrat sinus sudut antara dua garis.

Penyebaran garis AB dan AC pada gambar di atas didefinisikan sebagai:

$$s_a = \sin(\alpha)^2 = \frac{a}{c},$$

di mana a dan c adalah kuadrat dari sembarang segitiga siku-siku dengan salah satu sudut di A.

```
>sa &= a/c; $sa
```

Ini lebih mudah dihitung daripada sudut, tentu saja. Tetapi Anda kehilangan properti bahwa sudut dapat ditambahkan dengan mudah.

Tentu saja, kita dapat mengonversi nilai perkiraan untuk sudut wa menjadi sprad, dan mencetaknya sebagai pecahan.

```
>fracprint(sin(wa)^2)
```

9/25

Hukum kosinus trigonometri klasik diterjemahkan menjadi "hukum silang" berikut.

$$(c + b - a)^2 = 4bc(1 - s_a)$$

Di sini a , b , dan c adalah kuadrat dari sisi-sisi segitiga, dan sa adalah penyebaran sudut A . Sisi a , seperti biasa, berhadapan dengan sudut A .

Hukum ini diimplementasikan dalam file `geometri.e` yang kami muat ke Euler.

```
>$crosslaw(aa,bb,cc,saa)
```

Dalam kasus kami, kami mendapatkan

```
>$crosslaw(a,b,c,sa)
```

Mari kita gunakan `crosslaw` ini untuk mencari spread di A . Untuk melakukan ini, kita buat `crosslaw` untuk kuadran a , b , dan c , dan selesaikan untuk spread yang tidak diketahui sa .

Anda dapat melakukannya dengan tangan dengan mudah, tetapi saya menggunakan Maxima. Tentu saja, kami mendapatkan hasilnya, kami sudah memilikinya.

```
>$crosslaw(a,b,c,x), $solve(%,x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: `debugmode(true);`

Error in:

```
$crosslaw(a,b,c,x), $solve(%,x) ...
```

Kita sudah tahu ini. Definisi spread adalah kasus khusus dari crosslaw.

Kita juga dapat menyelesaikan ini untuk umum a, b, c . Hasilnya adalah rumus yang menghitung penyebaran sudut segitiga yang diberikan kuadrat dari ketiga sisinya.

```
>$solve(crosslaw(aa,bb,cc,x),x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$solve(crosslaw(aa,bb,cc,x),x) ...  
^
```

Kita bisa membuat fungsi dari hasilnya. Fungsi seperti itu sudah didefinisikan dalam file `geometri.e` dari Euler.

```
>$spread(a,b,c)
```

Sebagai contoh, kita dapat menggunakannya untuk menghitung sudut segitiga dengan sisi

$$a, \quad a, \quad \frac{4a}{7}$$

Hasilnya rasional, yang tidak begitu mudah didapat jika kita menggunakan trigonometri klasik.

```
>$spread(a,a,4*a/7)
```

Ini adalah sudut dalam derajat.

```
>degprint(arcsin(sqrt(6/7)))
```

67°47'32.44''

Contoh lain

Sekarang, mari kita coba contoh yang lebih maju.

Kami mengatur tiga sudut segitiga sebagai berikut.

```
>A&:=[1,2]; B&:=[4,3]; C&:=[0,4]; ...  
>setPlotRange(-1,5,1,7); ...  
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...  
>plotSegment(B,A,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...  
>insimg;
```

Menggunakan Pythagoras, mudah untuk menghitung jarak antara dua titik. Saya pertama kali menggunakan jarak fungsi file Euler untuk geometri. Jarak fungsi menggunakan geometri klasik.

```
>$distance(A,B)
```

Euler juga mengandung fungsi untuk kuadran antara dua titik.

Dalam contoh berikut, karena c+b bukan a, maka segitiga itu bukan persegi panjang.

```
>c &= quad(A,B); $c, b &= quad(A,C); $b, a &= quad(B,C); $a,
```

Pertama, mari kita hitung sudut tradisional. Fungsi computeAngle menggunakan metode biasa berdasarkan hasil kali titik dua vektor. Hasilnya adalah beberapa pendekatan floating point.

$$A = \langle 1, 2 \rangle \quad B = \langle 4, 3 \rangle, \quad C = \langle 0, 4 \rangle$$

$$\mathbf{a} = C - B = \langle -4, 1 \rangle, \quad \mathbf{c} = A - B = \langle -3, -1 \rangle, \quad \beta = \angle ABC$$

$$\mathbf{a} \cdot \mathbf{c} = |\mathbf{a}| \cdot |\mathbf{c}| \cos \beta$$

$$\cos \angle ABC = \cos \beta = \frac{\mathbf{a} \cdot \mathbf{c}}{|\mathbf{a}| \cdot |\mathbf{c}|} = \frac{12 - 1}{\sqrt{17}\sqrt{10}} = \frac{11}{\sqrt{17}\sqrt{10}}$$

```
>wb &= computeAngle(A,B,C); $wb, $(wb/pi*180)()
```

Dengan menggunakan pensil dan kertas, kita dapat melakukan hal yang sama dengan hukum silang. Kami memasukkan kuadran a, b, dan c ke dalam hukum silang dan menyelesaikan x.

```
>$crosslaw(a,b,c,x), $solve(%,x), //(b+c-a)^=4b.c(1-x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$crosslaw(a,b,c,x), $solve(%,x), //(b+c-a)^=4b.c(1-x) ...  
^
```

Yaitu, apa yang dilakukan oleh penyebaran fungsi yang didefinisikan dalam "geometry.e".

```
>sb &= spread(b,a,c); $sb
```

Maxima mendapatkan hasil yang sama menggunakan trigonometri biasa, jika kita memaksanya. Itu menyelesaikan istilah $\sin(\arccos(\dots))$ menjadi hasil pecahan. Sebagian besar siswa tidak dapat melakukan ini.

```
>$sin(computeAngle(A,B,C))^2
```

Setelah kita memiliki spread di B, kita dapat menghitung tinggi h_a di sisi a. Ingat bahwa

$$s_b = \frac{h_a}{c}$$

Menurut definisi.

```
>ha <= c*sb; $ha
```

Gambar berikut telah dihasilkan dengan program geometri C.a.R., yang dapat menggambar kuadrat dan menyebar.

image: (20) Rational_Geometry_CaR.png

Menurut definisi, panjang h_a adalah akar kuadrat dari kuadratnya.

```
>$sqrt(ha)
```

Sekarang kita dapat menghitung luas segitiga. Jangan lupa, bahwa kita berhadapan dengan kuadrat!

```
>$sqrt(ha)*sqrt(a)/2
```


Rumus determinan biasa menghasilkan hasil yang sama.

```
>$areaTriangle(B,A,C)
```

Rumus Heron

Sekarang, mari kita selesaikan masalah ini secara umum!

```
>&remvalue(a,b,c,sb,ha);
```

Pertama kita hitung spread di B untuk segitiga dengan sisi a, b, dan c. Kemudian kita menghitung luas kuadrat ("quadrea"?), faktorkan dengan Maxima, dan kita mendapatkan rumus Heron yang terkenal.

Memang, ini sulit dilakukan dengan pensil dan kertas.

```
>$spread(b^2,c^2,a^2), $factor(%*c^2*a^2/4)
```

Aturan Triple Spread

Kerugian dari spread adalah mereka tidak lagi hanya menambahkan sudut yang sama. Namun, tiga spread dari sebuah segitiga memenuhi aturan "triple spread" berikut.

```
>remvalue(sa,sb,sc); $triplespread(sa,sb,sc)
```

Aturan ini berlaku untuk setiap tiga sudut yang menambah 180 °.

$$\alpha + \beta + \gamma = \pi$$

Sejak menyebar

$$\alpha, \pi - \alpha$$

sama, aturan triple spread juga benar, jika

$$\alpha + \beta = \gamma$$

Karena penyebaran sudut negatif adalah sama, aturan penyebaran rangkap tiga juga berlaku, jika

$$\alpha + \beta + \gamma = 0$$

Misalnya, kita dapat menghitung penyebaran sudut 60° . Ini $3/4$. Persamaan memiliki solusi kedua, bagaimanapun, di mana semua spread adalah 0.

```
>$solve(triplespread(x,x,x),x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$solve(triplespread(x,x,x),x) ...  
^
```

Sebaran 90° jelas 1. Jika dua sudut dijumlahkan menjadi 90° , sebarannya menyelesaikan persamaan sebaran rangkap tiga dengan a,b,1. Dengan perhitungan berikut kita mendapatkan $a+b=1$.

```
>$triplespread(x,y,1), $solve(%,x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$triplespread(x,y,1), $solve(%,x) ...  
^
```

Karena sebaran 180° -t sama dengan sebaran t, rumus sebaran rangkap tiga juga berlaku, jika satu sudut adalah jumlah atau selisih dua sudut lainnya.

Jadi kita dapat menemukan penyebaran sudut berlipat ganda. Perhatikan bahwa ada dua solusi lagi. Kami membuat ini fungsi.

```
>$solve(triplespread(a,a,x),x), function doublespread(a) &= factor(rhs(%[1]))
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$solve(triplespread(a,a,x),x), function doublespread(a) &= fac ...  
^
```

Pembagi Sudut

Ini situasinya, kita sudah tahu.

```
>C&:=[0,0]; A&:=[4,0]; B&:=[0,3]; ...  
>setPlotRange(-1,5,-1,5); ...  
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...  
>plotSegment(B,A,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...  
>insimg;
```

Mari kita hitung panjang garis bagi sudut di A. Tetapi kita ingin menyelesaikannya untuk umum a, b, c .

```
>&remvalue(a,b,c);
```

Jadi pertama-tama kita hitung penyebaran sudut yang dibagi dua di A, dengan menggunakan rumus sebaran rangkap tiga.

Masalah dengan rumus ini muncul lagi. Ini memiliki dua solusi. Kita harus memilih yang benar. Solusi lainnya mengacu pada sudut terbelah 180° -wa.

```
>$triplespread(x,x,a/(a+b)), $solve(%,x), sa2 &= rhs(#[1]); $sa2
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$triplespread(x,x,a/(a+b)), $solve(%,x), sa2 &= rhs(#[1]); $sa ...  
^
```

Mari kita periksa persegi panjang Mesir.

```
>$sa2 with [a=3^2,b=4^2]
```

Kami dapat mencetak sudut dalam Euler, setelah mentransfer penyebaran ke radian.

```
>wa2 := arcsin(sqrt(1/10)); degprint(wa2)
```

18°26'5.82''

Titik P adalah perpotongan garis bagi sudut dengan sumbu y.

```
>P := [0,tan(wa2)*4]
```

[0, 1.33333]

```
>plotPoint(P,"P"); plotSegment(A,P):
```

Mari kita periksa sudut dalam contoh spesifik kita.

```
>computeAngle(C,A,P), computeAngle(P,A,B)
```

0.321750554397

0.321750554397

Sekarang kita hitung panjang garis bagi AP.

Kami menggunakan teorema sinus dalam segitiga APC. Teorema ini menyatakan bahwa

$$\frac{BC}{\sin(w_a)} = \frac{AC}{\sin(w_b)} = \frac{AB}{\sin(w_c)}$$

berlaku dalam segitiga apa pun. Kuadratkan, itu diterjemahkan ke dalam apa yang disebut "hukum penyebaran"

$$\frac{a}{s_a} = \frac{b}{s_b} = \frac{c}{s_b}$$

di mana a,b,c menunjukkan qudrances.

Karena spread CPA adalah $1-sa^2$, kita dapatkan darinya $bis_a/1=b/(1-sa^2)$ dan dapat menghitung bis_a (kuadran dari garis-bagi sudut).

```
>&factor(ratsimp(b/(1-sa2))); bis_a &= %; $bis_a
```

Mari kita periksa rumus ini untuk nilai-nilai Mesir kita.

```
>sqrt(mxmeval("at(bisa,[a=3^2,b=4^2])")), distance(A,P)
```

```
Variable sa2 not found!  
Use global variables or parameters for string evaluation.  
Error in Evaluate, superfluous characters found.  
Try "trace errors" to inspect local variables after errors.  
mxmeval:  
  return evaluate(mxm(s));  
Error in:  
sqrt(mxmeval("at(bisa,[a=3^2,b=4^2])")), distance(A,P) ...  
      ^
```

Kita juga dapat menghitung P menggunakan rumus spread.

```
>py<=factor(ratsimp(sa2*bisa)); $py
```

Nilainya sama dengan yang kita dapatkan dengan rumus trigonometri.

```
>sqrt(mxmeval("at(py,[a=3^2,b=4^2])"))
```



```

Variable sa2 not found!
Use global variables or parameters for string evaluation.
Error in Evaluate, superfluous characters found.
Try "trace errors" to inspect local variables after errors.
mxmeval:
    return evaluate(mxm(s));
Error in:
sqrt(mxmeval("at(py,[a=3^2,b=4^2])")) ...

```

Sudut Akord

Perhatikan situasi berikut.

```

>setPlotRange(1.2); ...
>color(1); plotCircle(circleWithCenter([0,0],1)); ...
>A:=[cos(1),sin(1)]; B:=[cos(2),sin(2)]; C:=[cos(6),sin(6)]; ...
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...
>color(3); plotSegment(A,B,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...
>color(1); O:=[0,0]; plotPoint(O,"O"); ...
>plotSegment(A,O); plotSegment(B,O); plotSegment(C,O,"r"); ...
>insimg;

```

Kita dapat menggunakan Maxima untuk menyelesaikan rumus penyebaran rangkap tiga untuk sudut-sudut di pusat O untuk r. Jadi kita mendapatkan rumus untuk jari-jari kuadrat dari pericircle dalam hal kuadrat dari sisi.

Kali ini, Maxima menghasilkan beberapa nol kompleks, yang kita abaikan.

```
>&remvalue(a,b,c,r); // hapus nilai-nilai sebelumnya untuk perhitungan baru  
>rabc &= rhs(solve(triplespread(spread(b,r,r),spread(a,r,r),spread(c,r,r)),r)[4]); $rabc
```

Kita dapat menjadikannya sebagai fungsi Euler.

```
>function periradius(a,b,c) &= rabc;
```

Mari kita periksa hasilnya untuk poin A,B,C.

```
>a:=quadrance(B,C); b:=quadrance(A,C); c:=quadrance(A,B);
```

Jari-jarinya memang 1.

```
>periradius(a,b,c)
```

Faktanya, spread CBA hanya bergantung pada b dan c . Ini adalah teorema sudut chord.

```
>$spread(b,a,c)*rabc | ratsimp
```

Sebenarnya spreadnya adalah $b/(4r)$, dan kita melihat bahwa sudut chord dari chord b adalah setengah dari sudut pusat.

```
>$doublespread(b/(4*r))-spread(b,r,r) | ratsimp
```

Contoh 6: Jarak Minimal pada Bidang

Catatan awal

Fungsi yang, ke titik M di bidang, menetapkan jarak AM antara titik tetap A dan M , memiliki garis level yang agak sederhana: lingkaran berpusat di A .

```
>&remvalue();
>A=[-1,-1];
>function d1(x,y):=sqrt((x-A[1])^2+(y-A[2])^2)
>fcontour("d1",xmin=-2,xmax=0,ymin=-2,ymax=0,hue=1, ...
>title="If you see ellipses, please set your window square");
```

dan grafiknya juga agak sederhana: bagian atas kerucut:

```
>plot3d("d1",xmin=-2,xmax=0,ymin=-2,ymax=0):
```

Tentu saja minimal 0 dicapai di A.

Dua poin

Sekarang kita lihat fungsi $MA+MB$ dimana A dan B adalah dua titik (tetap). Ini adalah "fakta yang diketahui" bahwa kurva level adalah elips, titik fokusnya adalah A dan B; kecuali untuk AB minimum yang konstan pada segmen $[AB]$:

```
>B=[1,-1];
>function d2(x,y):=d1(x,y)+sqrt((x-B[1])^2+(y-B[2])^2)
>fcontour("d2",xmin=-2,xmax=2,ymin=-3,ymax=1,hue=1):
```

Grafiknya lebih menarik:

```
>plot3d("d2",xmin=-2,xmax=2,ymin=-3,ymax=1):
```

Pembatasan garis (AB) lebih terkenal:

```
>plot2d("abs(x+1)+abs(x-1)",xmin=-3,xmax=3):
```

Tiga poin

Sekarang hal-hal yang kurang sederhana: Ini sedikit kurang terkenal bahwa $MA+MB+MC$ mencapai minimum pada satu titik pesawat tetapi untuk menentukan itu kurang sederhana:

1) Jika salah satu sudut segitiga ABC lebih dari 120° (katakanlah di A), maka minimum dicapai pada titik ini (misalnya $AB+AC$).

Contoh:

```

>C=[-4,1];
>function d3(x,y):=d2(x,y)+sqrt((x-C[1])^2+(y-C[2])^2)
>plot3d("d3",xmin=-5,xmax=3,ymin=-4,ymax=4);
>insimg;
>fcontour("d3",xmin=-4,xmax=1,ymin=-2,ymax=2,hue=1,title="The minimum is on A");
>P=(A_B_C_A)'; plot2d(P[1],P[2],add=1,color=12);
>insimg;

```

2) Tetapi jika semua sudut segitiga ABC kurang dari 120° , minimumnya adalah pada titik F di bagian dalam segitiga, yang merupakan satu-satunya titik yang melihat sisi-sisi ABC dengan sudut yang sama (maka masing-masing 120°):

```

>C=[-0.5,1];
>plot3d("d3",xmin=-2,xmax=2,ymin=-2,ymax=2):
>fcontour("d3",xmin=-2,xmax=2,ymin=-2,ymax=2,hue=1,title="The Fermat point");
>P=(A_B_C_A)'; plot2d(P[1],P[2],add=1,color=12);
>insimg;

```

Merupakan kegiatan yang menarik untuk mewujudkan gambar di atas dengan perangkat lunak geometri; misalnya, saya tahu soft yang ditulis di Jawa yang memiliki instruksi "garis kontur" ...

Semua ini di atas telah ditemukan oleh seorang hakim Perancis bernama Pierre de Fermat; dia menulis surat kepada dilettants lain seperti pendeta Marin Mersenne dan Blaise Pascal yang bekerja di pajak penghasilan. Jadi titik unik F sedemikian rupa sehingga $FA+FB+FC$ minimal, disebut titik Fermat segitiga. Tetapi tampaknya beberapa tahun sebelumnya, Torricelli Italia telah menemukan titik ini sebelum Fermat melakukannya! Bagaimanapun tradisinya adalah mencatat poin ini F...

Langkah selanjutnya adalah menambahkan 4 titik D dan mencoba meminimalkan $MA+MB+MC+MD$; katakan bahwa Anda adalah operator TV kabel dan ingin mencari di bidang mana Anda harus meletakkan antena sehingga Anda dapat memberi makan empat desa dan menggunakan panjang kabel sesedikit mungkin!

```
>D=[1,1];  
>function d4(x,y):=d3(x,y)+sqrt((x-D[1])^2+(y-D[2])^2)  
>plot3d("d4",xmin=-1.5,xmax=1.5,ymin=-1.5,ymax=1.5):  
>fcontour("d4",xmin=-1.5,xmax=1.5,ymin=-1.5,ymax=1.5,hue=1);  
>P=(A_B_C_D)'; plot2d(P[1],P[2],points=1,add=1,color=12);  
>insimg;
```

Masih ada minimum dan tidak tercapai di salah satu simpul A, B, C atau D:

```
>function f(x):=d4(x[1],x[2])  
>neldermin("f",[0.2,0.2])
```

[0.142858, 0.142857]

Tampaknya dalam kasus ini, koordinat titik optimal adalah rasional atau mendekati rasional...
Sekarang ABCD adalah persegi, kami berharap bahwa titik optimal akan menjadi pusat ABCD:

```
>C=[-1,1];  
>plot3d("d4",xmin=-1,xmax=1,ymin=-1,ymax=1):  
>fcontour("d4",xmin=-1.5,xmax=1.5,ymin=-1.5,ymax=1.5,hue=1);  
>P=(A_B_C_D)'; plot2d(P[1],P[2],add=1,color=12,points=1);  
>insimg;
```

Contoh 7: Bola Dandelin dengan Povray

Anda dapat menjalankan demonstrasi ini, jika Anda telah menginstal Povray, dan pvengine.exe di jalur program.

Pertama kita hitung jari-jari bola.

Jika Anda melihat gambar di bawah, Anda melihat bahwa kita membutuhkan dua lingkaran yang menentukan dua garis yang membentuk kerucut, dan satu garis yang membentuk bidang yang memotong kerucut.

Kami menggunakan file geometri.e dari Euler untuk ini.

```
>load geometry;
```


Pertama dua garis yang membentuk kerucut.

```
>g1 &= lineThrough([0,0],[1,a])
```

$$[-a, 1, 0]$$

```
>g2 &= lineThrough([0,0],[-1,a])
```

$$[-a, -1, 0]$$

Kemudian saya baris ketiga.

```
>g &= lineThrough([-1,0],[1,1])
```

$$[-1, 2, 1]$$

Kami merencanakan semuanya sejauh ini.

```
>setPlotRange(-1,1,0,2);  
>color(black); plotLine(g(),"")  
>a:=2; color(blue); plotLine(g1(),""), plotLine(g2(),""):
```

Sekarang kita ambil titik umum pada sumbu y.

```
>P &= [0,u]
```

[0, u]

Hitung jarak ke g1.

```
>d1 &= distance(P,projectToLine(P,g1)); $d1
```

Maxima said:

rat: replaced 1.66665833335744e-7 by 15819/94914474571 = 1.66665833335744e-7

rat: replaced 4.999958333473664e-5 by 201389/4027813565 = 4.99995833347366e-5

rat: replaced 1.33330666692022e-6 by 31771/23828726570 = 1.333306666920221e-6

rat: replaced $1.999933334222437e-4$ by $200030/1000183339 = 1.999933334222437e-4$

rat: replaced $4.499797504338432e-6$ by $24036/5341573699 = 4.499797504338431e-6$

rat: replaced $4.499662510124569e-4$ by $1162901/2584418270 = 4.499662510124571e-4$

rat: replaced $1.066581336583994e-5$ by $58861/5518660226 = 1.066581336583993e-5$

rat: replaced $7.998933390220841e-4$ by $1137431/1421978337 = 7.998933390220838e-4$

rat: replaced $2.083072932167196e-5$ by $35635/1710693824 = 2.0830729321672e-5$

rat: replaced 0.001249739605033717 by $567943/454449069 = 0.001249739605033716$

rat: replaced $3.599352055540239e-5$ by $98277/2730408098 = 3.599352055540234e-5$

rat: replaced 0.00179946006479581 by $479561/266502719 = 0.001799460064795812$

rat: replaced $5.71526624672386e-5$ by $51154/895041417 = 5.715266246723866e-5$

rat: replaced 0.002448999746720415 by $1946227/794702818 = 0.002448999746720415$

rat: replaced $8.530603082730626e-5$ by $121691/1426522824 = 8.530603082730627e-5$

rat: replaced 0.003198293697380561 by $2986741/933854512 = 0.003198293697380562$

rat: replaced $1.214508019889565e-4$ by $158455/1304684674 = 1.214508019889563e-4$

rat: replaced 0.004047266988005727 by $2125334/525128193 = 0.004047266988005727$

rat: replaced $1.665833531718508e-4$ by $142521/855553675 = 1.66583353171851e-4$

rat: replaced 0.004995834721974179 by $1957223/391770967 = 0.004995834721974179$

rat: replaced $2.216991628251896e-4$ by $179571/809975995 = 2.216991628251896e-4$

rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced $2.877927110806339e-4$ by $1167733/4057548906 = 2.877927110806339e-4$

rat: replaced 0.00719136414613375 by $2476362/344352191 = 0.007191364146133747$

rat: replaced $3.658573803051457e-4$ by $386279/1055818526 = 3.658573803051454e-4$

rat: replaced 0.00843810628521191 by $2079855/246483622 = 0.008438106285211924$

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rat: replaced 0.4264800139275439 by 7978696/18708253 = 0.4264800139275431
rat: replaced -0.140808431699002 by -10431632/74083859 = -0.1408084316990021
rat: replaced 0.4347004688396462 by 20489554/47134879 = 0.4347004688396463
rat: replaced -0.1451142866615502 by -3554077/24491572 = -0.1451142866615504
rat: replaced 0.4429774532337832 by 23449796/52936771 = 0.4429774532337834
rat: replaced -0.1495026295080298 by -26759297/178988805 = -0.1495026295080298
rat: replaced 0.451310139418413 by 8841241/19590167 = 0.4513101394184133

rat: replaced -0.1539740213994798 by -16145763/104860306 = -0.1539740213994798
part: invalid index of list or matrix.
#0: lineIntersection(g=[1,a,a*u],h=[-a,1,0])
#1: projectToLine(a=[0,u],g=[-a,1,0])
-- an error. To debug this try: debugmode(true);

Error in:
d1 &= distance(P,projectToLine(P,g1)); $d1 ...
      ^

```

Hitung jarak ke g.

```
>d &= distance(P,projectToLine(P,g)); $d
```

Maxima said:

```
rat: replaced 5.033291500140813e-5 by 263336/5231884543 = 5.033291500140813e-5
```

rat: replaced $2.026599467560841e-4$ by $407727/2011877564 = 2.02659946756084e-4$

rat: replaced $4.589658460211338e-4$ by $352373/767754296 = 4.589658460211339e-4$

rat: replaced $8.21224965753764e-4$ by $219501/267284860 = 8.212249657537654e-4$

rat: replaced 0.001291401063677061 by $174589/135193477 = 0.001291401063677059$

rat: replaced 0.001871447105906615 by $1078337/576204904 = 0.001871447105906617$

rat: replaced 0.002563305071654892 by $1323915/516487489 = 0.002563305071654891$

rat: replaced 0.003368905759035173 by $820537/243561874 = 0.003368905759035176$

rat: replaced 0.00429016859198364 by $7572857/1765165363 = 0.00429016859198364$

rat: replaced 0.005329001428317881 by $3020890/566877311 = 0.005329001428317882$

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rat: replaced 0.009169821045822202 by $2408608/262666849 = 0.009169821045822193$

rat: replaced 0.01069777449888981 by $2325322/217365023 = 0.01069777449888982$

rat: replaced 0.01235265711675931 by $7449711/603085711 = 0.01235265711675931$

rat: replaced 0.01413630339588112 by $3774568/267012379 = 0.01413630339588113$

rat: replaced 0.0160505349564472 by $2619104/163178611 = 0.0160505349564472$

rat: replaced 0.01809716036023018 by $3107690/171722521 = 0.01809716036023021$

rat: replaced 0.02027797492972855 by $6791343/334912289 = 0.02027797492972854$

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rat: replaced 0.04623326456100163 by $7175941/155211644 = 0.04623326456100162$

rat: replaced 0.04985167427763171 by $2856261/57295187 = 0.04985167427763173$

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rat: replaced 0.5092614977088081 by $27060617/53136978 = 0.5092614977088084$

rat: replaced 0.523250934216974 by $57357723/109618004 = 0.5232509342169741$

rat: replaced 0.5374560438344332 by $19984722/37183919 = 0.5374560438344328$

rat: replaced 0.551877406045395 by $10637804/19275665 = 0.5518774060453946$

rat: replaced 0.5665155787089895 by $22241852/39260795 = 0.5665155787089895$

rat: replaced 0.5813710980034821 by $10844268/18652919 = 0.5813710980034814$

rat: replaced 0.5964444783726564 by $13079224/21928653 = 0.596444478372657$

rat: replaced 0.6117362124743696 by $11199699/18308053 = 0.6117362124743685$

rat: replaced 0.6272467711312885 by $11338738/18076997 = 0.6272467711312891$

rat: replaced 0.6429766032838061 by $10161473/15803799 = 0.6429766032838053$

rat: replaced 0.6589261359451484 by $11120191/16876233 = 0.6589261359451484$

rat: replaced 0.6750957741586742 by $11234073/16640710 = 0.6750957741586747$

rat: replaced 0.6914859009573701 by $9571673/13842181 = 0.6914859009573708$

rat: replaced 0.7080968773255479 by $20218829/28553761 = 0.7080968773255474$

rat: replaced 0.7249290421627467 by $10945526/15098755 = 0.7249290421627479$

rat: replaced 0.7419827122498429 by $23520179/31699093 = 0.7419827122498426$

rat: replaced 0.7592581822173726 by $16709871/22008154 = 0.7592581822173727$

rat: replaced $9.983250083613754e-5$ by $612914/6139423483 = 9.983250083613756e-5$

rat: replaced $3.986533601775671e-4$ by $220554/553247563 = 3.986533601775666e-4$

rat: replaced $8.954327045205754e-4$ by $584699/652979277 = 8.954327045205756e-4$

rat: replaced 0.001589120864678328 by $740868/466212493 = 0.00158912086467833$

rat: replaced 0.002478648480745763 by $878917/354595259 = 0.002478648480745762$

rat: replaced 0.003562926609036218 by $2735717/767828614 = 0.003562926609036219$

rat: replaced 0.004840846830973591 by $1164348/240525685 = 0.004840846830973582$

rat: replaced 0.006311281363933816 by $16515210/2616776063 = 0.006311281363933816$

rat: replaced 0.007973083174022497 by $2414321/302808957 = 0.007973083174022491$

rat: replaced 0.009825086090776508 by $1144049/116441626 = 0.009825086090776506$

rat: replaced 0.01186610492378118 by $1659683/139867548 = 0.01186610492378118$

rat: replaced 0.01409493558118687 by $986877/70016425 = 0.01409493558118684$

rat: replaced 0.01651035519011868 by $1738361/105289134 = 0.01651035519011867$

rat: replaced 0.01911112221896202 by $1475047/77182647 = 0.01911112221896199$

rat: replaced 0.02189597660151474 by $7711274/352177669 = 0.02189597660151473$

rat: replaced 0.02486363986299212 by $3887839/156366446 = 0.02486363986299209$

rat: replaced 0.0280128152478745 by $2263313/80795628 = 0.02801281524787455$

rat: replaced 0.03134218784958129 by $1116362/35618509 = 0.03134218784958124$

rat: replaced 0.03485042474195996 by $3920507/112495243 = 0.03485042474195998$

rat: replaced 0.03853617511257795 by $5379408/139593719 = 0.03853617511257795$

rat: replaced 0.04239807039780302 by $3385918/79860191 = 0.04239807039780308$

rat: replaced 0.04643472441965829 by $10918553/235137672 = 0.04643472441965828$

rat: replaced 0.05064473352443885 by $5036501/99447675 = 0.05064473352443886$

rat: replaced 0.05502667672307548 by $2932521/53292715 = 0.05502667672307557$

rat: replaced 0.05957911583323347 by $6320819/106091185 = 0.05957911583323346$

rat: replaced 0.06430059562312868 by $9893260/153859539 = 0.0643005956231287$

rat: replaced 0.06918964395705007 by $6012189/86894348 = 0.06918964395705$

rat: replaced 0.07424477194257195 by $6096479/82113243 = 0.07424477194257204$

rat: replaced 0.07946447407944118 by $5389689/67825139 = 0.07946447407944125$

rat: replaced 0.0848472284101276 by $9595393/113090235 = 0.08484722841012754$

rat: replaced 0.09039149667201674 by $3773144/41742245 = 0.09039149667201657$

rat: replaced 0.0960957244512361 by $5162056/53717853 = 0.09609572445123597$

rat: replaced 0.1019583413380946 by $1082663/10618680 = 0.1019583413380948$

rat: replaced 0.107977761084122 by $1922059/17800508 = 0.1079777610841219$

rat: replaced 0.1141523817606936 by $5923297/51889386 = 0.1141523817606938$

rat: replaced 0.1204805859192203 by $17634703/146369665 = 0.1204805859192204$

rat: replaced 0.1269607407528933 by $11368220/89541223 = 0.1269607407528932$

rat: replaced 0.1335911982599624 by $4657902/34866833 = 0.1335911982599624$

rat: replaced 0.1403702954085355 by $8528456/60756843 = 0.1403702954085353$

rat: replaced 0.1472963543028805 by $11128453/75551449 = 0.1472963543028804$

rat: replaced 0.1543676823512128 by $8170760/52930509 = 0.1543676823512126$

rat: replaced 0.1615825724349539 by $188109817/1164171446 = 0.1615825724349539$

rat: replaced 0.1689393030794406 by $5046974/29874481 = 0.1689393030794409$

rat: replaced 0.1764361386260728 by $6530305/37012287 = 0.176436138626073$

rat: replaced 0.1840713294058766 by $25189859/136848357 = 0.1840713294058766$

rat: replaced 0.1918431119144694 by $24326967/126806570 = 0.1918431119144694$

rat: replaced 0.1997497089884105 by $14902039/74603558 = 0.1997497089884104$

rat: replaced 0.2077893299829148 by $7281351/35041987 = 0.2077893299829145$

rat: replaced 0.2159601709509153 by $11348921/52550991 = 0.2159601709509151$

rat: replaced 0.2242604148234577 by $22385730/99820247 = 0.2242604148234576$

rat: replaced 0.2326882315914051 by $25615030/110083049 = 0.2326882315914051$

rat: replaced 0.2412417784884371 by $14523232/60201977 = 0.2412417784884373$

rat: replaced 0.2499192001753251 by $11309023/45250717 = 0.2499192001753254$

rat: replaced 0.2587186289254649 by $7582961/29309683 = 0.2587186289254647$

rat: replaced 0.267638184811648 by $17912865/66929407 = 0.2676381848116479$

rat: replaced 0.2766759758940514 by $27538925/99534934 = 0.2766759758940514$

rat: replaced 0.2858300984094321 by $29258587/102363562 = 0.2858300984094321$

rat: replaced 0.2950986369614998 by $7877677/26695064 = 0.2950986369614997$

rat: replaced 0.304479664712457 by $14469542/47522195 = 0.304479664712457$

rat: replaced 0.3139712435756791 by $8375733/26676752 = 0.3139712435756797$

rat: replaced 0.3235714244095225 by $178371467/551258404 = 0.3235714244095225$

rat: replaced 0.3332782472122374 by $5743591/17233621 = 0.333278247212237$

rat: replaced 0.3430897413179662 by $15588245/45434891 = 0.3430897413179664$

rat: replaced 0.3530039255938071 by $6523425/18479752 = 0.3530039255938067$

rat: replaced 0.3630188086379282 by $51253958/141188161 = 0.3630188086379282$

rat: replaced 0.373132388978704 by $9370061/25111894 = 0.3731323889787047$

rat: replaced 0.3833426552748616 by $11820697/30835851 = 0.3833426552748617$

rat: replaced 0.393647586516613 by $9153768/23253713 = 0.3936475865166135$

rat: replaced 0.4040451522277552 by $16634707/41170416 = 0.404045152227755$

rat: replaced 0.4145333126687146 by $2088920/5039209 = 0.4145333126687145$

rat: replaced 0.4251100190405208 by $24667763/58026774 = 0.4251100190405209$

rat: replaced 0.4357732136896836 by $10448574/23977091 = 0.435773213689684$

rat: replaced 0.4465208303139576 by $8346266/18691773 = 0.4465208303139568$

rat: replaced 0.4573507941689697 by $20158688/44077081 = 0.4573507941689696$

rat: replaced 0.4682610222756929 by $12818601/27374905 = 0.4682610222756937$

rat: replaced 0.4792494236287415 by $13652513/28487281 = 0.4792494236287416$

rat: replaced 0.4903138994054704 by $35114711/71616797 = 0.4903138994054705$

rat: replaced 0.5014523431758559 by $15102855/30118226 = 0.5014523431758564$

rat: replaced 0.5126626411131362 by $31697340/61828847 = 0.5126626411131361$

rat: replaced 0.5239426722051925 by $27432767/52358337 = 0.5239426722051924$

rat: replaced 0.5352903084666492 by $6124470/11441399 = 0.5352903084666482$

rat: replaced 0.5467034151516694 by $41717397/76307182 = 0.5467034151516694$

rat: replaced 0.5581798509674292 by $7494380/13426461 = 0.5581798509674292$

rat: replaced 0.5697174682882435 by $14609183/25642856 = 0.5697174682882438$

rat: replaced 0.581314113370329 by $14367580/24715691 = 0.5813141133703282$

rat: replaced 0.5929676265671738 by $9820294/16561265 = 0.5929676265671735$

rat: replaced 0.6046758425455033 by $23593213/39017952 = 0.6046758425455031$

rat: replaced 0.6164365905018095 by $15720181/25501700 = 0.6164365905018097$

rat: replaced 0.6282476943794307 by $53974636/85912987 = 0.6282476943794306$

rat: replaced 0.640106973086155 by $20459615/31962806 = 0.6401069730861552$

rat: replaced 0.652012240712328 by $51645100/79208789 = 0.652012240712328$

rat: replaced 0.6639613067494411 by $12215999/18398661 = 0.6639613067494422$

rat: replaced 0.6759519763091814 by $18558734/27455699 = 0.6759519763091808$

rat: replaced 0.6879820503429186 by $23500536/34158647 = 0.687982050342919$

```

rat: replaced 0.7000493258616074 by 29992669/42843651 = 0.7000493258616078
rat: replaced 0.7121515961560857 by 10685401/15004391 = 0.7121515961560853
rat: replaced 0.7242866510177421 by 11795807/16286103 = 0.7242866510177419
rat: replaced 0.7364522769595366 by 14940657/20287339 = 0.7364522769595362
rat: replaced 0.7486462574373463 by 42508133/56779998 = 0.7486462574373461
part: invalid index of list or matrix.
#0: lineIntersection(g=[2,1,u],h=[-1,2,1])
#1: projectToLine(a=[0,u],g=[-1,2,1])
-- an error. To debug this try: debugmode(true);

Error in:
d &= distance(P,projectToLine(P,g)); $d ...
      ^

```

Dan temukan pusat kedua lingkaran yang jaraknya sama.

```
>sol &= solve(d1^2=d^2,u); $sol
```

Ada dua solusi.

Kami mengevaluasi solusi simbolis, dan menemukan kedua pusat, dan kedua jarak.

```
>u := sol()
```

```
[]
```

```
>dd := d()
```

```
Function d needs at least one argument!  
Use: d (n)  
Error in:  
dd := d() ...  
      ^
```

Plot lingkaran ke dalam gambar.

```
>color(red);  
>plotCircle(circleWithCenter([0,u[1]],dd[1]),"");
```

```
Index 1 out of bounds!  
Error in:  
plotCircle(circleWithCenter([0,u[1]],dd[1]),""); ...  
      ^
```

```
>plotCircle(circleWithCenter([0,u[2]],dd[2]),"");
```

```
Index 2 out of bounds!  
Error in:  
plotCircle(circleWithCenter([0,u[2]],dd[2]),""); ...  
      ^
```

```
>insimg;
```

Plot dengan Povray

Selanjutnya kami merencanakan semuanya dengan Povray. Perhatikan bahwa Anda mengubah perintah apa pun dalam urutan perintah Povray berikut, dan menjalankan kembali semua perintah dengan Shift-Return.

Pertama kita memuat fungsi povray.

```
>load povray;  
>defaultpovray="C:\Program Files\POV-Ray\v3.7\bin\pvengine.exe"
```

```
C:\Program Files\POV-Ray\v3.7\bin\pvengine.exe
```

Kami mengatur adegan dengan tepat.

```
>povstart(zoom=11,center=[0,0,0.5],height=10°,angle=140°);
```

Selanjutnya kita menulis dua bidang ke file Povray.

```
>writeln(povsphere([0,0,u[1]],dd[1],povlook(red)));
```

```
Index 1 out of bounds!  
Error in:  
writeln(povsphere([0,0,u[1]],dd[1],povlook(red))); ...  
      ^
```

```
>writeln(povsphere([0,0,u[2]],dd[2],povlook(red)));
```

```
Index 2 out of bounds!  
Error in:  
writeln(povsphere([0,0,u[2]],dd[2],povlook(red))); ...  
      ^
```

Dan kerucutnya, transparan.

```
>writeln(povcone([0,0,0],0,[0,0,a],1,povlook(lightgray,1)));
```

Kami menghasilkan bidang terbatas pada kerucut.


```

>gp=g();
>pc=povcone([0,0,0],0,[0,0,a],1,"");
>vp=[gp[1],0,gp[2]]; dp=gp[3];
>writeln(povplane(vp,dp,povlook(blue,0.5),pc));

```

Sekarang kita menghasilkan dua titik pada lingkaran, di mana bola menyentuh kerucut.

```

>function turnz(v) := return [-v[2],v[1],v[3]]
>P1=projectToLine([0,u[1]],g1()); P1=turnz([P1[1],0,P1[2]]);

```

```

Index 1 out of bounds!
Error in:
P1=projectToLine([0,u[1]],g1()); P1=turnz([P1[1],0,P1[2]]); ...
      ^

```

```

>writeln(povpoint(P1,povlook(yellow)));

```

```

Function povpoint needs a vector for P
Error in:
writeln(povpoint(P1,povlook(yellow))); ...
      ^

```

```

>P2=projectToLine([0,u[2]],g1()); P2=turnz([P2[1],0,P2[2]]);

```

```
Index 2 out of bounds!  
Error in:  
P2=projectToLine([0,u[2]],g1()); P2=turnz([P2[1],0,P2[2]]); ...  
      ^
```

```
>writeln(povpoint(P2,povlook(yellow)));
```

```
Function povpoint needs a vector for P  
Error in:  
writeln(povpoint(P2,povlook(yellow))); ...  
      ^
```

Kemudian kami menghasilkan dua titik di mana bola menyentuh bidang. Ini adalah fokus dari elips.

```
>P3=projectToLine([0,u[1]],g()); P3=[P3[1],0,P3[2]];
```

```
Index 1 out of bounds!  
Error in:  
P3=projectToLine([0,u[1]],g()); P3=[P3[1],0,P3[2]]; ...  
      ^
```

```
>writeln(povpoint(P3,povlook(yellow)));
```

```
Variable or function P3 not found.  
Error in:  
writeln(povpoint(P3,povlook(yellow))); ...  
^
```

```
>P4=projectToLine([0,u[2]],g()); P4=[P4[1],0,P4[2]];
```

```
Index 2 out of bounds!  
Error in:  
P4=projectToLine([0,u[2]],g()); P4=[P4[1],0,P4[2]]; ...  
^
```

```
>writeln(povpoint(P4,povlook(yellow)));
```

```
Variable or function P4 not found.  
Error in:  
writeln(povpoint(P4,povlook(yellow))); ...  
^
```

Selanjutnya kita hitung perpotongan P1P2 dengan bidang.

```
>t1=scalp(vp,P1)-dp; t2=scalp(vp,P2)-dp; P5=P1+t1/(t1-t2)*(P2-P1);
```

Matrix expected in scalp!

Error in:

```
t1=scalp(vp,P1)-dp; t2=scalp(vp,P2)-dp; P5=P1+t1/(t1-t2)*(P2-P ...  
^
```

```
>writeln(povpoint(P5,povlook(yellow)));
```

Variable or function P5 not found.

Error in:

```
writeln(povpoint(P5,povlook(yellow))); ...  
^
```

Kami menghubungkan titik-titik dengan segmen garis.

```
>writeln(povsegment(P1,P2,povlook(yellow)));
```

Function povsegment needs a vector for P1

Error in:

```
writeln(povsegment(P1,P2,povlook(yellow))); ...  
^
```

```
>writeln(povsegment(P5,P3,povlook(yellow)));
```

```
Variable or function P5 not found.  
Error in:  
writeln(povsegment(P5,P3,povlook(yellow))); ...  
^
```

```
>writeln(povsegment(P5,P4,povlook(yellow)));
```

```
Variable or function P5 not found.  
Error in:  
writeln(povsegment(P5,P4,povlook(yellow))); ...  
^
```

Sekarang kita menghasilkan pita abu-abu, di mana bola menyentuh kerucut.

```
>pcw=povcone([0,0,0],0,[0,0,a],1.01);  
>pc1=povcylinder([0,0,P1[3]-defaultpointsize/2],[0,0,P1[3]+defaultpointsize/2],1);
```

```
Index 3 out of range for string (need string vector).  
Error in:  
pc1=povcylinder([0,0,P1[3]-defaultpointsize/2],[0,0,P1[3]+defa ...  
^
```

```
>writeln(povintersection([pcw,pc1],povlook(gray)));
```

```
Variable pc1 not found!  
Error in:  
writeln(povintersection([pcw,pc1],povlook(gray))); ...  
      ^
```

```
>pc2=povcylinder([0,0,P2[3]-defaultpointsiz/2],[0,0,P2[3]+defaultpointsiz/2],1);
```

```
Index 3 out of range for string (need string vector).  
Error in:  
pc2=povcylinder([0,0,P2[3]-defaultpointsiz/2],[0,0,P2[3]+defa ...  
      ^
```

```
>writeln(povintersection([pcw,pc2],povlook(gray)));
```

```
Variable pc2 not found!  
Error in:  
writeln(povintersection([pcw,pc2],povlook(gray))); ...  
      ^
```

Mulai program Povray.

```
>povend();
```

Untuk mendapatkan Anaglyph ini kita perlu memasukkan semuanya ke dalam fungsi scene. Fungsi ini akan digunakan dua kali kemudian.

```
>function scene () ...
```

```
global a,u,dd,g,g1,defaultpointsize;
writeln(povsphere([0,0,u[1]],dd[1],povlook(red)));
writeln(povsphere([0,0,u[2]],dd[2],povlook(red)));
writeln(povcone([0,0,0],0,[0,0,a],1,povlook(lightgray,1)));
gp=g();
pc=povcone([0,0,0],0,[0,0,a],1,"");
vp=[gp[1],0,gp[2]]; dp=gp[3];
writeln(povplane(vp,dp,povlook(blue,0.5),pc));
P1=projectToLine([0,u[1]],g1()); P1=turnz([P1[1],0,P1[2]]);
writeln(povpoint(P1,povlook(yellow)));
P2=projectToLine([0,u[2]],g1()); P2=turnz([P2[1],0,P2[2]]);
writeln(povpoint(P2,povlook(yellow)));
P3=projectToLine([0,u[1]],g()); P3=[P3[1],0,P3[2]];
writeln(povpoint(P3,povlook(yellow)));
P4=projectToLine([0,u[2]],g()); P4=[P4[1],0,P4[2]];
writeln(povpoint(P4,povlook(yellow)));
t1=scalp(vp,P1)-dp; t2=scalp(vp,P2)-dp; P5=P1+t1/(t1-t2)*(P2-P1);
writeln(povpoint(P5,povlook(yellow)));
writeln(povsegment(P1,P2,povlook(yellow)));
writeln(povsegment(P5,P3,povlook(yellow)));
writeln(povsegment(P5,P4,povlook(yellow)));
pcw=povcone([0,0,0],0,[0,0,a],1.01);
pc1=povcylinder([0,0,P1[3]-defaultpointsize/2],[0,0,P1[3]+defaultpointsize/2],1);
writeln(povintersection([pcw,pc1],povlook(gray)));
pc2=povcylinder([0,0,P2[3]-defaultpointsize/2],[0,0,P2[3]+defaultpointsize/2],1);
writeln(povintersection([pcw,pc2],povlook(gray)));
endfunction
```

Anda membutuhkan kacamata merah/sian untuk menghargai efek berikut.

```
>povanaglyph("scene",zoom=11,center=[0,0,0.5],height=10°,angle=140°);
```

```
Global variable dd not found in "global" command.
```

```
scene:
```

```
    global a,u,dd,g,g1,defaultpointsizes;
```

```
Try "trace errors" to inspect local variables after errors.
```

```
povanaglyph:
```

```
    scene$(args());
```

Contoh 8: Geometri Bumi

Dalam buku catatan ini, kami ingin melakukan beberapa perhitungan sferis. Fungsi-fungsi tersebut terdapat dalam file "spherical.e" di folder contoh. Kita perlu memuat file itu terlebih dahulu.

```
>load "spherical.e";
```


Untuk memasukkan posisi geografis, kami menggunakan vektor dengan dua koordinat dalam radian (utara dan timur, nilai negatif untuk selatan dan barat). Berikut koordinat Kampus FMIPA UNY.

```
>FMIPA=[rad(-7,-46.467),rad(110,23.05)]
```

```
[-0.13569, 1.92657]
```

Anda dapat mencetak posisi ini dengan `sposprint` (cetak posisi spherical).

```
>sposprint(FMIPA) // posisi garis lintang dan garis bujur FMIPA UNY
```

```
S 7°46.467' E 110°23.050'
```

Mari kita tambahkan dua kota lagi, Solo dan Semarang.

```
>Solo=[rad(-7,-34.333),rad(110,49.683)]; Semarang=[rad(-6,-59.05),rad(110,24.533)];  
>sposprint(Solo), sposprint(Semarang),
```

```
S 7°34.333' E 110°49.683'
```

```
S 6°59.050' E 110°24.533'
```

Pertama kita menghitung vektor dari satu ke yang lain pada bola ideal. Vektor ini [pos,jarak] dalam radian. Untuk menghitung jarak di bumi, kita kalikan dengan jari-jari bumi pada garis lintang 7°.

```
>br=svector(FMIPA,Solo); degprint(br[1]), br[2]*rearth(7°)->km // perkiraan jarak FMIPA-Solo
```

```
65°20'26.60''  
53.8945384608
```

Ini adalah perkiraan yang baik. Rutinitas berikut menggunakan perkiraan yang lebih baik. Pada jarak yang begitu pendek hasilnya hampir sama.

```
>esdist(FMIPA,Semarang)->" km", // perkiraan jarak FMIPA-Semarang
```

```
88.0114026318 km
```

Ada fungsi untuk heading, dengan mempertimbangkan bentuk elips bumi. Sekali lagi, kami mencetak dengan cara yang canggih.

```
>sdegprint(esdir(FMIPA,Solo))
```

```
65.34°
```

Sudut segitiga melebihi 180° pada bola.

```
>asum=sangle(Solo,FMIPA,Semarang)+sangle(FMIPA,Solo,Semarang)+sangle(FMIPA,Semarang,Solo); degprint(
```

```
180°0'10.77''
```

Ini dapat digunakan untuk menghitung luas segitiga. Catatan: Untuk segitiga kecil, ini tidak akurat karena kesalahan pengurangan dalam $\text{asum}-\pi$.

```
>(asum-pi)*rearth(48°)^2->" km^2", // perkiraan luas segitiga FMIPA-Solo-Semarang
```

```
2116.02948749 km^2
```

Ada fungsi untuk ini, yang menggunakan garis lintang rata-rata segitiga untuk menghitung jari-jari bumi, dan menangani kesalahan pembulatan untuk segitiga yang sangat kecil.

```
>esarea(Solo,FMIPA,Semarang)->" km^2", //perkiraan yang sama dengan fungsi esarea()
```

```
2123.64310526 km^2
```

Kita juga dapat menambahkan vektor ke posisi. Sebuah vektor berisi heading dan jarak, keduanya dalam radian. Untuk mendapatkan vektor, kami menggunakan `vector`. Untuk menambahkan vektor ke posisi, kami menggunakan `sadd`.

```
>v=svector(FMIPA,Solo); sposprint(saddvector(FMIPA,v)), sposprint(Solo),
```

```
S 7°34.333' E 110°49.683'  
S 7°34.333' E 110°49.683'
```

Fungsi-fungsi ini mengasumsikan bola yang ideal. Hal yang sama di bumi.

```
>sposprint(esadd(FMIPA,esdir(FMIPA,Solo),esdist(FMIPA,Solo))), sposprint(Solo),
```

```
S 7°34.333' E 110°49.683'  
S 7°34.333' E 110°49.683'
```

Mari kita beralih ke contoh yang lebih besar, Tugu Jogja dan Monas Jakarta (menggunakan Google Earth untuk mencari koordinatnya).

```
>Tugu=[-7.7833°,110.3661°]; Monas=[-6.175°,106.811944°];  
>sposprint(Tugu), sposprint(Monas)
```

```
S 7°46.998' E 110°21.966'  
S 6°10.500' E 106°48.717'
```

Menurut Google Earth, jaraknya adalah 429,66 km. Kami mendapatkan pendekatan yang baik.

```
>esdist(Tugu,Monas)->" km", // perkiraan jarak Tugu Jogja - Monas Jakarta
```

431.565659488 km

Judulnya sama dengan judul yang dihitung di Google Earth.

```
>degprint(esdir(Tugu,Monas))
```

294°17'2.85''

Namun, kita tidak lagi mendapatkan posisi target yang tepat, jika kita menambahkan heading dan jarak ke posisi semula. Hal ini terjadi, karena kita tidak menghitung fungsi invers secara tepat, tetapi mengambil perkiraan jari-jari bumi di sepanjang jalan.

```
>sposprint(esadd(Tugu,esdir(Tugu,Monas),esdist(Tugu,Monas)))
```

S 6°10.500' E 106°48.717'

Namun, kesalahannya tidak besar.

```
>sposprint(Monas),
```

S 6°10.500' E 106°48.717'

Tentu kita tidak bisa berlayar dengan tujuan yang sama dari satu tujuan ke tujuan lainnya, jika kita ingin menempuh jalur terpendek. Bayangkan, Anda terbang NE mulai dari titik mana pun di bumi. Kemudian Anda akan berputar ke kutub utara. Lingkaran besar tidak mengikuti heading yang konstan! Perhitungan berikut menunjukkan bahwa kami jauh dari tujuan yang benar, jika kami menggunakan pos yang sama selama perjalanan kami.

```
>dist=esdist(Tugu,Monas); hd=esdir(Tugu,Monas);
```

Sekarang kita tambahkan 10 kali sepersepuluh dari jarak, menggunakan pos ke Monas, kita sampai di Tugu.

```
>p=Tugu; loop 1 to 10; p=esadd(p,hd,dist/10); end;
```

Hasilnya jauh.

```
>sposprint(p), skmprint(esdist(p,Monas))
```

S 6°11.250' E 106°48.372'
1.529km

Sebagai contoh lain, mari kita ambil dua titik di bumi pada garis lintang yang sama.

```
>P1=[30°,10°]; P2=[30°,50°];
```

Jalur terpendek dari P1 ke P2 bukanlah lingkaran garis lintang 30°, melainkan jalur terpendek yang dimulai 10° lebih jauh ke utara di P1.

```
>sdegprint(esdir(P1,P2))
```

79.69°

Tapi, jika kita mengikuti pembacaan kompas ini, kita akan berputar ke kutub utara! Jadi kita harus menyesuaikan arah kita di sepanjang jalan. Untuk tujuan kasar, kami menyesuaikannya pada 1/10 dari total jarak.

```
>p=P1; dist=esdist(P1,P2); ...  
> loop 1 to 10; dir=esdir(p,P2); sdegprint(dir), p=esadd(p,dir,dist/10); end;
```

79.69°
81.67°
83.71°
85.78°
87.89°
90.00°
92.12°
94.22°
96.29°
98.33°

Jaraknya tidak tepat, karena kita akan menambahkan sedikit kesalahan, jika kita mengikuti heading yang sama terlalu lama.

```
>skmprint(esdist(p,P2))
```

0.203km

Kami mendapatkan perkiraan yang baik, jika kami menyesuaikan pos setelah setiap 1/100 dari total jarak dari Tugu ke Monas.


```
>p=Tugu; dist=esdist(Tugu,Monas); ...  
> loop 1 to 100; p=esadd(p,esdir(p,Monas),dist/100); end;  
>skmprint(esdist(p,Monas))
```

0.000km

Untuk keperluan navigasi, kita bisa mendapatkan urutan posisi GPS di sepanjang lingkaran besar menuju Monas dengan fungsi navigasi.

```
>load spherical; v=navigate(Tugu,Monas,10); ...  
> loop 1 to rows(v); sposprint(v[#]), end;
```

```
S 7°46.998' E 110°21.966'  
S 7°37.422' E 110°0.573'  
S 7°27.829' E 109°39.196'  
S 7°18.219' E 109°17.834'  
S 7°8.592' E 108°56.488'  
S 6°58.948' E 108°35.157'  
S 6°49.289' E 108°13.841'  
S 6°39.614' E 107°52.539'  
S 6°29.924' E 107°31.251'  
S 6°20.219' E 107°9.977'  
S 6°10.500' E 106°48.717'
```

Kami menulis sebuah fungsi, yang memplot bumi, dua posisi, dan posisi di antaranya.

```
>function testplot ...  
  
    useglobal;  
    plotearth;  
    plotpos(Tugu,"Tugu Jogja"); plotpos(Monas,"Tugu Monas");  
    plotposline(v);  
endfunction
```

Sekarang rencanakan semuanya.

```
>plot3d("testplot",angle=25, height=6,>own,>user,zoom=4):
```

Atau gunakan plot3d untuk mendapatkan tampilan anaglyph. Ini terlihat sangat bagus dengan kacamata merah/sian.

```
>plot3d("testplot",angle=25,height=6,distance=5,own=1,anaglyph=1,zoom=4):
```

MENCOBA RUMUS-RUMUS PADA MATERI DI ATAS

```
>A &= [2,0]; B &= [0,2]; C &= [3,3]; // menentukan tiga titik A, B, C  
>c &= lineThrough(B,C) // c=BC
```

$[-1, 3, 6]$

```
>$getLineEquation(c,x,y), $solve(%,y) | expand // persamaan garis c
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$getLineEquation(c,x,y), $solve(%,y) | expand // persamaan gar ...
```

```
>h &= perpendicular(A,lineThrough(B,C)) // h melalui A tegak lurus BC
```

$[3, 1, 6]$

```
>Q &= lineIntersection(c,h) // Q titik potong garis c=BC dan h
```

Maxima said:

```
rat: replaced 1.498320841708742e-4 by 1329822/8875415485 = 1.498320841708742e-4  
  
rat: replaced 5.986466935998108e-4 by 398723/666040595 = 5.986466935998098e-4  
  
rat: replaced 0.001345398955533032 by 4525441/3363642421 = 0.001345398955533032  
  
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rat: replaced 0.0331248986654725 by 5858625/176864692 = 0.03312489866547248
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rat: replaced 0.7673749832474404 by 39576757/51574208 = 0.7673749832474402
rat: replaced 0.7873061950613714 by 6818881/8661028 = 0.7873061950613714
rat: replaced 0.8075376737535601 by 20498953/25384516 = 0.807537673753559
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rat: replaced 0.84890530902455 by 25231431/29722315 = 0.8489053090245494
rat: replaced 0.8700433288242969 by 9721738/11173855 = 0.8700433288242957
rat: replaced 0.8914853417578728 by 33469619/37543656 = 0.8914853417578725
rat: replaced 0.9132322036168524 by 21961040/24047597 = 0.913232203616852
part: invalid index of list or matrix.
#0: lineIntersection(g=[-1,3,6],h=[3,1,6])
-- an error. To debug this try: debugmode(true);

Error in:
... ersection(c,h) // Q titik potong garis c=BC dan h ...

```

```

>$projectToLine(A,lineThrough(B,C)) // proyeksi A pada BC

```

```

Maxima said:
rat: replaced 5.049958083474387e-5 by 102157/2022927682 = 5.049958083474385e-5

rat: replaced 2.039932534230044e-4 by 284619/1395237319 = 2.039932534230043e-4

rat: replaced 4.634656435254722e-4 by 573493/1237401322 = 4.634656435254721e-4

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rat: replaced $8.31890779119604e-4$ by $332331/399488741 = 8.318907791196046e-4$

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rat: replaced 0.7905388051635788 by $10794522/13654639 = 0.7905388051635784$

rat: replaced 0.8088173254609005 by $16745047/20703126 = 0.808817325460899$

rat: replaced 0.8272359628580275 by $20291194/24528907 = 0.827235962858027$

rat: replaced 0.8457918755149025 by $10996366/13001267 = 0.8457918755149018$

rat: replaced 0.8644822078640563 by $9158500/10594203 = 0.8644822078640555$

rat: replaced 0.8833040908961625 by $13759446/15577247 = 0.8833040908961641$

rat: replaced 0.9022546424469358 by $19827819/21975857 = 0.9022546424469362$

rat: replaced 0.9213309674853474 by $60458149/65620446 = 0.9213309674853475$

rat: replaced 0.9405301584031224 by $11658841/12396031 = 0.9405301584031212$

rat: replaced 0.9598492953055026 by $26214088/27310629 = 0.9598492953055018$

rat: replaced 0.9792854463032298 by $35089005/35831233 = 0.9792854463032293$

rat: replaced 0.9988356678057343 by $15735752/15754095 = 0.9988356678057356$

rat: replaced 1.018497004815491 by $16202286/15908035 = 1.018497004815491$

rat: replaced 1.038266491223517 by $17763365/17108676 = 1.038266491223517$

rat: replaced 1.058141150105979 by $33730321/31876958 = 1.058141150105979$

rat: replaced 1.078117994021884 by $51996446/48228901 = 1.078117994021883$

rat: replaced 1.098194025311821 by $124719922/113568203 = 1.098194025311821$

rat: replaced 1.118366236397724 by $92837336/83011569 = 1.118366236397724$

rat: replaced 1.13863161008363 by $20601995/18093644 = 1.138631610083629$

rat: replaced 1.158987119857388 by $20626233/17796775 = 1.15898711985739$

```
rat: replaced 1.17942973019332 by 4098089/3474636 = 1.179429730193321
```

```
rat: replaced 1.199956396855759 by 17442145/14535649 = 1.199956396855758
```

```
part: invalid index of list or matrix.
```

```
#0: lineIntersection(g=[3,1,6],h=[-1,3,6])
```

```
#1: projectToLine(a=[2,0],g=[-1,3,6])
```

```
-- an error. To debug this try: debugmode(true);
```

```
Error in:
```

```
$projectToLine(A,lineThrough(B,C)) // proyeksi A pada BC ...
```

```
>$distance(A,Q) // jarak AQ
```

```
>cc &= circleThrough(A,B,C); $cc // (titik pusat dan jari-jari) lingkaran melalui A, B, C
```

```
Maxima said:
```

```
rat: replaced -9.96658350028018e-5 by -86001/862893488 = -9.966583500280164e-5
```

```
rat: replaced -3.973200535106469e-4 by -1080775/2720162223 = -3.973200535106468e-4
```

```
rat: replaced -8.90932907016237e-4 by -1194571/1340809157 = -8.90932907016237e-4
```

```
rat: replaced -0.001578455051312488 by -2522953/1598368606 = -0.001578455051312488
```

```
rat: replaced -0.002457817751424091 by -519814/211494119 = -0.002457817751424095
```

```
rat: replaced -0.003526933088480816 by -11813191/3349423055 = -0.003526933088480816
```

```
rat: replaced -0.004783694168506353 by -866601/181157275 = -0.004783694168506343
```

```
rat: replaced -0.006225975333106509 by -4878061/783501498 = -0.00622597533310651
```

rat: replaced -0.00785163237203354 by $-1241039/158061272 = -0.007851632372033549$

rat: replaced -0.009658502737604657 by $-6031380/624463249 = -0.009658502737604659$

rat: replaced -0.01164440576095599 by $-2532373/217475503 = -0.01164440576095598$

rat: replaced -0.01380714287010623 by $-1331361/96425525 = -0.01380714287010623$

rat: replaced -0.01614449780981353 by $-7953293/492631799 = -0.01614449780981353$

rat: replaced -0.0186542368631985 by $-865030/46371771 = -0.01865423686319852$

rat: replaced -0.02133410907511396 by $-2814913/131944249 = -0.02133410907511399$

rat: replaced -0.02418184647723809 by $-2509632/103781653 = -0.02418184647723813$

rat: replaced -0.02719516431487051 by $-1827823/67211324 = -0.02719516431487051$

rat: replaced -0.03037176127540547 by $-9190485/302599672 = -0.03037176127540548$

rat: replaced -0.03370931971846053 by $-3905653/115862706 = -0.03370931971846057$

rat: replaced -0.03720550590763916 by $-2032371/54625544 = -0.03720550590763911$

rat: replaced -0.04085797024390259 by $-2827822/69211025 = -0.04085797024390261$

rat: replaced -0.04466434750052761 by $-3719233/83270734 = -0.04466434750052762$

rat: replaced -0.04862225705962725 by $-2754536/56651751 = -0.04862225705962733$

rat: replaced -0.05272930315021007 by $-5066672/96088355 = -0.05272930315021003$

rat: replaced -0.05698307508775641 by $-9699307/170213822 = -0.05698307508775639$

rat: replaced -0.06138114751528378 by $-7938451/129330443 = -0.06138114751528378$

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rat: replaced -0.07060042050668569 by $-4716201/66801316 = -0.07060042050668583$

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rat: replaced -0.1127548191943103 by $-9512927/84368252 = -0.1127548191943102$

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rat: replaced -0.5781884863630807 by $-55545197/96067629 = -0.5781884863630807$

rat: replaced -0.5973294694750937 by $-15924011/26658673 = -0.5973294694750936$

rat: replaced -0.6167727171212756 by $-14518184/23538953 = -0.6167727171212756$

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rat: replaced -1.144845032612569 by  $-13646839/11920250 = -1.144845032612571$ 
rat: replaced -1.171191421254493 by  $-12158144/10381005 = -1.171191421254493$ 
rat: replaced -1.197807688505292 by  $-18610041/15536752 = -1.197807688505294$ 
rat: replaced -1.224692172752087 by  $-1607117/1312262 = -1.224692172752088$ 
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rat: replaced -1.33487811225433 by  $-18675990/13990783 = -1.334878112254332$ 
rat: replaced -1.363077824501593 by  $-60815834/44616553 = -1.363077824501592$ 
rat: replaced -1.391535226818977 by  $-16386165/11775602 = -1.391535226818977$ 
rat: replaced -1.420248473481634 by  $-17316077/12192287 = -1.420248473481636$ 
rat: replaced -1.449215693180489 by  $-19585953/13514864 = -1.449215693180486$ 
rat: replaced -1.478434989209379 by  $-157494279/106527700 = -1.478434989209379$ 

rat: replaced -1.507904439654719 by  $-39585536/26252019 = -1.507904439654718$ 
part: invalid index of list or matrix.
#0: lineIntersection(g=[2,-2,0],h=[-1,-3,-7])
#1: circleThrough(a=[2,0],b=[0,2],c=[3,3])
-- an error. To debug this try: debugmode(true);

```

Error in:

```
cc &= circleThrough(A,B,C); $cc // (titik pusat dan jari-jari) ...  
^
```

```
>r:=getCircleRadius(cc); $r , $float(r) // tampilkan nilai jari-jari  
>$computeAngle(A,C,B) // nilai <ACB  
>$solve(getLineEquation(angleBisector(A,C,B),x,y),y)[1] // persamaan garis bagi <ACB
```

Maxima said:

```
solve: all variables must not be numbers.  
-- an error. To debug this try: debugmode(true);
```

Error in:

```
... (getLineEquation(angleBisector(A,C,B),x,y),y)[1] // persamaan ...  
^
```

```
>P &= lineIntersection(angleBisector(A,C,B),angleBisector(C,B,A)); $P // titik potong 2
```

Maxima said:

```
rat: replaced -9.96658350028018e-5 by -86001/862893488 = -9.966583500280164e-5
```

```
rat: replaced -3.973200535106469e-4 by -1080775/2720162223 = -3.973200535106468e-4
```

```
rat: replaced -8.90932907016237e-4 by -1194571/1340809157 = -8.90932907016237e-4
```

```
rat: replaced -0.001578455051312488 by -2522953/1598368606 = -0.001578455051312488
```

```
rat: replaced -0.002457817751424091 by -519814/211494119 = -0.002457817751424095
```

rat: replaced -0.003526933088480816 by $-11813191/3349423055 = -0.003526933088480816$

rat: replaced -0.004783694168506353 by $-866601/181157275 = -0.004783694168506343$

rat: replaced -0.006225975333106509 by $-4878061/783501498 = -0.00622597533310651$

rat: replaced -0.00785163237203354 by $-1241039/158061272 = -0.007851632372033549$

rat: replaced -0.009658502737604657 by $-6031380/624463249 = -0.009658502737604659$

rat: replaced -0.01164440576095599 by $-2532373/217475503 = -0.01164440576095598$

rat: replaced -0.01380714287010623 by $-1331361/96425525 = -0.01380714287010623$

rat: replaced -0.01614449780981353 by $-7953293/492631799 = -0.01614449780981353$

rat: replaced -0.0186542368631985 by $-865030/46371771 = -0.01865423686319852$

rat: replaced -0.02133410907511396 by $-2814913/131944249 = -0.02133410907511399$

rat: replaced -0.02418184647723809 by $-2509632/103781653 = -0.02418184647723813$

rat: replaced -0.02719516431487051 by $-1827823/67211324 = -0.02719516431487051$

rat: replaced -0.03037176127540547 by $-9190485/302599672 = -0.03037176127540548$

rat: replaced -0.03370931971846053 by $-3905653/115862706 = -0.03370931971846057$

rat: replaced -0.03720550590763916 by $-2032371/54625544 = -0.03720550590763911$

rat: replaced -0.04085797024390259 by $-2827822/69211025 = -0.04085797024390261$

rat: replaced -0.04466434750052761 by $-3719233/83270734 = -0.04466434750052762$

rat: replaced -0.04862225705962725 by $-2754536/56651751 = -0.04862225705962733$

rat: replaced -0.05272930315021007 by $-5066672/96088355 = -0.05272930315021003$

rat: replaced -0.05698307508775641 by $-9699307/170213822 = -0.05698307508775639$

rat: replaced -0.06138114751528378 by $-7938451/129330443 = -0.06138114751528378$

rat: replaced -0.0659210806458812 by $-2936449/44544916 = -0.06592108064588112$

rat: replaced -0.07060042050668569 by $-4716201/66801316 = -0.07060042050668583$

rat: replaced -0.07541669918427674 by $-2448749/32469586 = -0.07541669918427664$

rat: replaced -0.08036743507146715 by $-2461511/30628214 = -0.08036743507146711$

rat: replaced -0.08545013311546024 by $-13954421/163304848 = -0.08545013311546024$

rat: replaced -0.09066228506735385 by $-4103116/45257143 = -0.0906622850673539$

rat: replaced -0.09600136973296292 by $-16995415/177033047 = -0.0960013697329629$

rat: replaced -0.1014648532249364 by $-7634177/75239620 = -0.1014648532249365$

rat: replaced -0.107050189216145 by $-3894269/36377974 = -0.1070501892161449$

rat: replaced -0.1127548191943103 by $-9512927/84368252 = -0.1127548191943102$

rat: replaced -0.1185761727178553 by $-6978418/58851773 = -0.1185761727178551$

rat: replaced -0.1245116676729451 by $-3380435/27149544 = -0.1245116676729451$

rat: replaced -0.1305587105316969 by $-7571267/57991282 = -0.1305587105316968$

rat: replaced -0.136714696611531 by $-8109727/59318619 = -0.136714696611531$

rat: replaced -0.1429770103356357 by $-5513427/38561633 = -0.142977010335636$

rat: replaced -0.1493430254945241 by $-2259975/15132779 = -0.1493430254945242$

rat: replaced -0.1558101055086514 by $-1315594/8443573 = -0.1558101055086514$

rat: replaced -0.1623756036920724 by $-5159837/31777169 = -0.1623756036920721$

rat: replaced -0.1690368635171068 by $-3076049/18197504 = -0.1690368635171065$

rat: replaced -0.1757912188799892 by $-8356449/47536214 = -0.1757912188799891$

rat: replaced -0.1826359943674788 by $-20067867/109879036 = -0.1826359943674788$

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rat: replaced -0.1965860591220733 by $-4406725/22416264 = -0.1965860591220732$

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rat: replaced 0.01277271662437307 by $3258991/255152533 = 0.01277271662437308$

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rat: replaced 0.001540100153900437 by $276884/179783113 = 0.001540100153900439$

rat: replaced 0.02196908527585173 by $1298306/59096953 = 0.0219690852758517$

rat: replaced 0.001770376919130678 by $644389/363984072 = 0.001770376919130681$

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rat: replaced 0.002297373572865413 by 1020913/444382669 = 0.002297373572865417

rat: replaced 0.02866202514797045 by 1770713/61779061 = 0.02866202514797044

rat: replaced 0.002596040745477063 by 1097643/422814242 = 0.002596040745477065

rat: replaced 0.03108757828935527 by 5034207/161936287 = 0.03108757828935525

rat: replaced 0.002919448107844891 by 906221/310408326 = 0.002919448107844891

rat: replaced 0.03361002186548678 by 4553215/135471944 = 0.03361002186548678

rat: replaced 0.003268563311168871 by 1379071/421919623 = 0.003268563311168867

rat: replaced 0.03622910363410947 by 3082649/85087642 = 0.0362291036341094

rat: replaced 0.003644351435886262 by 5966577/1637212301 = 0.003644351435886261

rat: replaced 0.03894456168922911 by 4913415/126164342 = 0.03894456168922911

rat: replaced 0.004047774895164447 by 572425/141417202 = 0.004047774895164451

rat: replaced 0.04175612448730281 by 1734727/41544253 = 0.04175612448730273

rat: replaced 0.004479793338660443 by 2952779/659132861 = 0.004479793338660444

rat: replaced 0.04466351087439402 by 4691119/105032473 = 0.04466351087439405

rat: replaced 0.0049413635565565 by 2524919/510976165 = 0.004941363556556498

rat: replaced 0.04766643011428662 by 3536207/74186529 = 0.04766643011428665

rat: replaced 0.005433439383882244 by 1361584/250593391 = 0.005433439383882235

rat: replaced 0.05076458191755917 by 7710025/151878036 = 0.05076458191755916

rat: replaced 0.005956971605131645 by $1447422/242979503 = 0.005956971605131648$

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rat: replaced 0.006512907859185624 by $3695063/567344584 = 0.006512907859185626$

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rat: replaced 0.01312919757078923 by $3499615/266552086 = 0.01312919757078922$

rat: replaced 0.09103425032511492 by $5893225/64736349 = 0.09103425032511488$

rat: replaced 0.01406053493400045 by $2280713/162206702 = 0.01406053493400045$

rat: replaced 0.09524833678003664 by $9601787/100807923 = 0.09524833678003662$

rat: replaced 0.01503446588876983 by $200490/13335359 = 0.01503446588876985$

rat: replaced 0.09955289764732322 by $5687088/57126293 = 0.09955289764732328$

rat: replaced 0.01605189303448024 by $951971/59305840 = 0.01605189303448025$

rat: replaced 0.1039475024744748 by $10260011/98703776 = 0.1039475024744747$

rat: replaced 0.01711371462093175 by $9432386/551159477 = 0.01711371462093176$

rat: replaced 0.1084317118046711 by $14939691/137779721 = 0.1084317118046712$

rat: replaced 0.01822082445851714 by $2559788/140486947 = 0.01822082445851713$

rat: replaced 0.113005077220716 by $8478529/75027859 = 0.1130050772207161$

rat: replaced 0.01937411182884202 by $2983799/154009589 = 0.01937411182884203$

rat: replaced 0.1176671413898787 by $7123715/60541243 = 0.1176671413898786$

rat: replaced 0.02057446139579705 by $7167743/348380590 = 0.02057446139579705$

rat: replaced 0.1224174381096274 by $12172179/99431741 = 0.1224174381096274$

rat: replaced 0.02182275311709253 by $7415562/339808729 = 0.02182275311709253$

rat: replaced 0.1272554923542488 by $7277933/57191504 = 0.127255492354249$

rat: replaced 0.02311986215626333 by $2988661/129268115 = 0.02311986215626336$

rat: replaced 0.1321808203223502 by $3633064/27485561 = 0.1321808203223503$

rat: replaced 0.02446665879515308 by $1991976/81415939 = 0.02446665879515312$

rat: replaced 0.1371929294852391 by $56235017/409897341 = 0.1371929294852391$

rat: replaced 0.02586400834688696 by $5000736/193347293 = 0.02586400834688697$

rat: replaced 0.1422913186361759 by $9349741/65708443 = 0.1422913186361759$

rat: replaced 0.02731277106934082 by $858413/31428997 = 0.02731277106934084$

rat: replaced 0.1474754779404944 by $1549881/10509415 = 0.1474754779404943$

rat: replaced 0.02881380207911666 by $3754753/130310918 = 0.02881380207911666$

rat: replaced 0.152744888986584 by $5264425/34465474 = 0.1527448889865841$

rat: replaced 0.03036795126603076 by $4118329/135614318 = 0.03036795126603077$

rat: replaced 0.1580990248377314 by $5442776/34426373 = 0.1580990248377312$

rat: replaced 0.03197606320812652 by $3497683/109384416 = 0.03197606320812647$

rat: replaced 0.1635373500848132 by $12328488/75386375 = 0.1635373500848131$

rat: replaced 0.0336389770872163 by $3971799/118071337 = 0.03363897708721635$

rat: replaced 0.1690593208998367 by $20896917/123607009 = 0.1690593208998367$

rat: replaced 0.03535752660496472 by $1815732/51353479 = 0.03535752660496478$

rat: replaced 0.1746643850903219 by $2841592/16268869 = 0.1746643850903219$

rat: replaced 0.03713253989951881 by $3333721/89778965 = 0.03713253989951878$

rat: replaced 0.1803519821545206 by $4461007/24735004 = 0.1803519821545208$

rat: replaced 0.03896483946269502 by $8785771/225479461 = 0.03896483946269501$

rat: replaced 0.1861215433374662 by $4381209/23539505 = 0.1861215433374661$

rat: replaced 0.0408552420577305 by $3189084/78058135 = 0.04085524205773043$

rat: replaced 0.1919724916878484 by $72809759/379271834 = 0.1919724916878484$

rat: replaced 0.04280455863760801 by $7646593/178639688 = 0.04280455863760801$

rat: replaced 0.1979042421157076 by $26318167/132984350 = 0.1979042421157076$

rat: replaced 0.04481359426396048 by $20610430/459914683 = 0.04481359426396048$

rat: replaced 0.2039162014509444 by $8519416/41779005 = 0.2039162014509441$

rat: replaced 0.04688314802656623 by $3439140/73355569 = 0.04688314802656633$

rat: replaced 0.2100077685026351 by $50962787/242670961 = 0.2100077685026351$

rat: replaced 0.04901401296344043 by $4006732/81746663 = 0.04901401296344048$

rat: replaced 0.216178334119151 by $1347531/6233423 = 0.2161783341191509$

rat: replaced 0.05120697598153157 by $4148974/81023609 = 0.0512069759815315$

rat: replaced 0.2224272812490723 by $23234851/104460437 = 0.2224272812490723$

rat: replaced 0.05346281777803219 by $11998448/224426031 = 0.05346281777803218$

rat: replaced 0.2287539850028937 by $8185268/35781969 = 0.2287539850028935$

rat: replaced 0.05578231276230905 by $1398019/25062048 = 0.05578231276230897$

rat: replaced 0.2351578127155118 by $12642104/53760085 = 0.2351578127155119$

rat: replaced 0.05816622897846346 by $4451048/76522891 = 0.05816622897846345$

rat: replaced 0.2416381240094921 by $8002142/33116223 = 0.2416381240094923$

rat: replaced 0.06061532802852698 by $2146337/35409146 = 0.06061532802852686$

rat: replaced 0.2481942708591053 by $8882901/35790113 = 0.2481942708591057$

rat: replaced 0.0631303649963022 by $14651447/232082406 = 0.06313036499630222$

rat: replaced 0.2548255976551299 by $868346/3407609 = 0.25482559765513$

rat: replaced 0.06571208837185505 by $4240309/64528599 = 0.06571208837185509$

rat: replaced 0.2615314412704124 by $8212450/31401387 = 0.2615314412704127$

rat: replaced 0.06836123997666599 by $2716643/39739522 = 0.06836123997666604$

rat: replaced 0.2683111311261794 by $34459769/128432126 = 0.2683111311261794$

rat: replaced 0.07107855488944881 by $3146673/44270357 = 0.07107855488944893$

rat: replaced 0.2751639892590951 by $12552159/45617012 = 0.2751639892590949$

rat: replaced 0.07386476137264342 by $12898997/174629915 = 0.0738647613726434$

rat: replaced 0.2820893303890569 by $11134456/39471383 = 0.2820893303890568$

rat: replaced 0.07672058079958999 by $5073506/66129661 = 0.07672058079959007$

rat: replaced 0.2890864619877229 by $9583357/33150487 = 0.2890864619877228$

rat: replaced 0.07964672758239233 by $5672399/71219486 = 0.07964672758239227$

rat: replaced 0.2961546843477643 by $11052271/37319251 = 0.2961546843477647$

rat: replaced 0.08264390910047736 by $4686067/56701904 = 0.08264390910047748$

rat: replaced 0.3032932906528349 by $9918077/32701274 = 0.3032932906528351$

rat: replaced 0.0857128256298576 by $3585977/41837111 = 0.08571282562985766$

rat: replaced 0.3105015670482534 by $9320011/30015987 = 0.3105015670482533$

rat: replaced 0.08885417027310427 by $5751353/64728003 = 0.0888541702731042$

rat: replaced 0.3177787927123868 by $248395525/781661743 = 0.3177787927123868$

rat: replaced 0.09206862889003742 by $7305460/79347983 = 0.09206862889003745$

rat: replaced 0.3251242399287333 by $13842845/42577093 = 0.3251242399287335$

rat: replaced 0.09535688002914089 by $5971998/62627867 = 0.09535688002914103$

rat: replaced 0.3325371741586922 by $9318229/28021616 = 0.3325371741586923$

rat: replaced 0.0987195948597075 by $9821211/99485933 = 0.09871959485970745$

rat: replaced 0.3400168541150183 by $13391981/39386227 = 0.3400168541150184$

rat: replaced 0.1021574371047232 by $8336413/81603584 = 0.1021574371047232$

rat: replaced 0.3475625318359485 by $10097818/29053241 = 0.347562531835949$

rat: replaced 0.1056710629744951 by $5741011/54329074 = 0.105671062974495$

rat: replaced 0.3551734527599992 by $15867851/44676343 = 0.3551734527599987$

rat: replaced 0.1092611211010309 by $5551873/50812887 = 0.1092611211010309$

rat: replaced 0.3628488558014202 by $6897641/19009681 = 0.3628488558014203$

rat: replaced 0.1129282524731764 by $11548693/102265755 = 0.1129282524731764$

rat: replaced 0.3705879734263036 by $23358661/63031352 = 0.3705879734263038$

rat: replaced 0.1166730903725168 by $5656228/48479285 = 0.1166730903725168$

rat: replaced 0.3783900317293359 by $14241382/37636779 = 0.3783900317293358$

rat: replaced 0.1204962603100498 by $4057613/33674182 = 0.12049626031005$

rat: replaced 0.3862542505111889 by $3461217/8960981 = 0.3862542505111884$

rat: replaced 0.1243983799636342 by $7966447/64039797 = 0.1243983799636342$

rat: replaced 0.3941798433565377 by $5314214/13481699 = 0.3941798433565384$

rat: replaced 0.1283800591162231 by $796346/6203035 = 0.1283800591162229$

rat: replaced 0.4021660177127022 by $11567173/28762184 = 0.4021660177127022$

```

rat: replaced 0.1324418995948859 by 4716124/35609003 = 0.1324418995948862
rat: replaced 0.4102119749689023 by 11320633/27597032 = 0.4102119749689024
rat: replaced 0.1365844952106265 by 612971/4487852 = 0.1365844952106264
rat: replaced 0.418316910536117 by 12225195/29224721 = 0.4183169105361177
rat: replaced 0.140808431699002 by 10431632/74083859 = 0.1408084316990021
rat: replaced 0.4264800139275439 by 7978696/18708253 = 0.4264800139275431
rat: replaced 0.1451142866615502 by 3554077/24491572 = 0.1451142866615504
rat: replaced 0.4347004688396462 by 20489554/47134879 = 0.4347004688396463
rat: replaced 0.1495026295080298 by 26759297/178988805 = 0.1495026295080298
rat: replaced 0.4429774532337832 by 23449796/52936771 = 0.4429774532337834
rat: replaced 0.1539740213994798 by 16145763/104860306 = 0.1539740213994798

```

```

rat: replaced 0.451310139418413 by 8841241/19590167 = 0.4513101394184133
part: invalid index of list or matrix.

```

```

#0: lineIntersection(g=[2,-2,0],h=[3-sqrt(10)/sqrt(2),sqrt(10)/sqrt(2)+1,(3-sqrt(10)/sqrt(2))*(sqrt(2)-1)])
-- an error. To debug this try: debugmode(true);

```

```

Error in:

```

```

... ection(angleBisector(A,C,B),angleBisector(C,B,A)); $P // titik ...

```

```

>P() //

```


0

Garis dan Lingkaran yang berpotongan

```
>A &:= [2,0]; c=circleWithCenter(A,4);  
>B &:= [2,3]; C &:= [3,2]; l=lineThrough(B,C);  
>setPlotRange(5); plotCircle(c); plotLine(l);  
>{P1,P2,f}=lineCircleIntersections(l,c);  
>P1, P2,
```

```
[5.89792, -0.897916]  
[1.10208, 3.89792]
```

```
>plotPoint(P1); plotPoint(P2):
```

```
>c &= circleWithCenter(A,4) // lingkaran dengan pusat A jari-jari 4
```

[2, 0, 4]

```
>l &= lineThrough(B,C) // garis l melalui B dan C
```

```
[1, 1, 5]
```

```
>$lineCircleIntersections(l,c) | radcan, // titik potong lingkaran c dan garis l
```

Maxima said:

```
rat: replaced -4.98329175014009e-5 by -86001/1725786976 = -4.983291750140082e-5
```

```
rat: replaced -1.986600267553235e-4 by -1133306/5704751069 = -1.986600267553234e-4
```

```
rat: replaced -4.454664535081185e-4 by -474290/1064704191 = -4.454664535081181e-4
```

```
rat: replaced -7.892275256562442e-4 by -1190199/1508055613 = -7.892275256562439e-4
```

```
rat: replaced -0.001228908875712045 by -259907/211494119 = -0.001228908875712047
```

```
rat: replaced -0.001763466544240408 by -5854594/3319934829 = -0.001763466544240408
```

```
rat: replaced -0.002391847084253176 by -866601/362314550 = -0.002391847084253172
```

```
rat: replaced -0.003112987666553255 by -5049204/1621980085 = -0.003112987666553255
```

```
rat: replaced -0.00392581618601677 by -1241039/316122544 = -0.003925816186016774
```

```
rat: replaced -0.004829251368802329 by -3015690/624463249 = -0.00482925136880233
```

```
rat: replaced -0.005822202880477995 by -2532373/434951006 = -0.005822202880477991
```

rat: replaced -0.006903571435053116 by $-1331361/192851050 = -0.006903571435053115$

rat: replaced -0.008072248904906765 by $-7953293/985263598 = -0.008072248904906766$

rat: replaced -0.009327118431599252 by $-432515/46371771 = -0.009327118431599259$

rat: replaced -0.01066705453755698 by $-2950074/276559381 = -0.01066705453755698$

rat: replaced -0.01209092323861904 by $-1254816/103781653 = -0.01209092323861907$

rat: replaced -0.01359758215743526 by $-1827823/134422648 = -0.01359758215743526$

rat: replaced -0.01518588063770274 by $-9199276/605778237 = -0.01518588063770274$

rat: replaced -0.01685465985923026 by $-2516580/149310637 = -0.01685465985923026$

rat: replaced -0.01860275295381958 by $-2032371/109251088 = -0.01860275295381955$

rat: replaced -0.02042898512195129 by $-1413911/69211025 = -0.02042898512195131$

rat: replaced -0.02233217375026381 by $-3647892/163346929 = -0.02233217375026377$

rat: replaced -0.02431112852981362 by $-1377268/56651751 = -0.02431112852981367$

rat: replaced -0.02636465157510504 by $-2533336/96088355 = -0.02636465157510502$

rat: replaced -0.0284915375438782 by $-9699307/340427644 = -0.02849153754387819$

rat: replaced -0.03069057375764189 by $-7938451/258660886 = -0.03069057375764189$

rat: replaced -0.0329605403229406 by $-2936449/89089832 = -0.03296054032294056$

rat: replaced -0.03530021025334285 by $-5224432/148000025 = -0.03530021025334287$

rat: replaced -0.03770834959213837 by $-2448749/64939172 = -0.03770834959213832$

rat: replaced -0.04018371753573358 by $-2461511/61256428 = -0.04018371753573356$

rat: replaced -0.04272506655773012 by $-13954421/326609696 = -0.04272506655773012$

rat: replaced -0.04533114253367693 by $-2051558/45257143 = -0.04533114253367695$

rat: replaced -0.04800068486648146 by $-16995415/354066094 = -0.04800068486648145$

rat: replaced -0.05073242661246818 by $-3970295/78259513 = -0.05073242661246818$

rat: replaced -0.05352509460807248 by $-3894269/72755948 = -0.05352509460807246$

rat: replaced -0.05637740959715515 by $-11093364/196769665 = -0.05637740959715513$

rat: replaced -0.05928808635892763 by $-3489209/58851773 = -0.05928808635892754$

rat: replaced -0.06225583383647254 by $-3380435/54299088 = -0.06225583383647254$

rat: replaced -0.06527935526584844 by $-7571267/115982564 = -0.06527935526584841$

rat: replaced -0.06835734830576551 by $-8050241/117767017 = -0.06835734830576544$

rat: replaced -0.07148850516781785 by $-5513427/77123266 = -0.07148850516781798$

rat: replaced -0.07467151274726203 by $-2259975/30265558 = -0.07467151274726208$

rat: replaced -0.07790505275432569 by $-657797/8443573 = -0.07790505275432569$

rat: replaced -0.08118780184603619 by $-4832180/59518547 = -0.08118780184603633$

rat: replaced -0.08451843175855339 by $-3076049/36395008 = -0.08451843175855327$

rat: replaced -0.08789560943999458 by $-7150621/81353563 = -0.08789560943999465$

rat: replaced -0.0913179971837394 by $-20067867/219758072 = -0.0913179971837394$

rat: replaced -0.09478425276219882 by $-5487749/57897265 = -0.09478425276219869$

rat: replaced -0.09829302956103664 by $-4406725/44832528 = -0.09829302956103658$

rat: replaced -0.1018429767138303 by $-3912367/38415678 = -0.1018429767138302$

rat: replaced -0.1054327392371563 by $-8451941/80164293 = -0.1054327392371564$

rat: replaced -0.1090609581660869 by $-13126833/120362348 = -0.1090609581660869$

rat: replaced -0.112726270690086 by $-2754747/24437489 = -0.112726270690086$

rat: replaced -0.116427310289289 by $-22239618/191017193 = -0.116427310289289$

rat: replaced -0.1201627068711536 by $-9494831/79016454 = -0.1201627068711537$

rat: replaced -0.1239310869074673 by $-3190398/25743323 = -0.1239310869074672$

rat: replaced -0.1277310735717007 by $-15999330/125257931 = -0.1277310735717006$

rat: replaced -0.1315612868766867 by $-13929723/105880106 = -0.1315612868766867$

rat: replaced -0.1354203438126204 by $-28035370/207024803 = -0.1354203438126204$

rat: replaced -0.1393068584853572 by $-11590983/83204683 = -0.1393068584853571$

rat: replaced -0.1432194422550018 by $-12738764/88945773 = -0.1432194422550018$

rat: replaced -0.1471567038747712 by $-5246589/35653075 = -0.147156703874771$

rat: replaced -0.1511172496301179 by $-4676629/30947023 = -0.1511172496301179$

rat: replaced -0.1550996834780995 by $-15854305/102220099 = -0.1550996834780995$

rat: replaced -0.1591026071869839 by $-9026555/56734174 = -0.159102607186984$

rat: replaced -0.1631246204760689 by $-10435073/63969945 = -0.1631246204760689$

rat: replaced -0.1671643211557106 by $-164873401/986295400 = -0.1671643211557106$

rat: replaced -0.1712203052675407 by $-7017638/40986015 = -0.1712203052675406$

rat: replaced -0.1752911672248615 by $-3184915/18169284 = -0.1752911672248615$

rat: replaced -0.1793754999532028 by $-2646709/14755131 = -0.1793754999532027$

rat: replaced -0.1834718950310287 by $-8392143/45740755 = -0.1834718950310287$

rat: replaced -0.1875789428305783 by $-12888313/68708741 = -0.1875789428305781$

rat: replaced -0.1916952326588277 by $-16014703/83542521 = -0.1916952326588277$

rat: replaced -0.1958193528985573 by $-21279927/108671215 = -0.1958193528985574$

rat: replaced -0.1999498911495134 by $-5994245/29978736 = -0.1999498911495134$

rat: replaced -0.2040854343696463 by $-17847769/87452439 = -0.2040854343696464$

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rat: replaced -0.2123658811881329 by $-20393053/96027916 = -0.2123658811881328$

rat: replaced -0.2165079567653719 by $-8489188/39209589 = -0.2165079567653719$

rat: replaced -0.2206493815523576 by $-14881929/67446049 = -0.2206493815523575$

rat: replaced -0.2247887414183958 by $-11437558/50881365 = -0.2247887414183955$

rat: replaced -0.2289246224392826 by $-17547464/76651711 = -0.2289246224392825$

rat: replaced -0.2330556110386959 by $-11148764/47837355 = -0.2330556110386956$

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rat: replaced -0.2412972592553108 by $-36037383/149348497 = -0.2412972592553108$

rat: replaced -0.2454050947312253 by $-4652365/18957899 = -0.2454050947312252$

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rat: replaced -0.2856715822285418 by $-17041418/59653879 = -0.2856715822285421$

rat: replaced -0.289586182178096 by $-721506/2491507 = -0.2895861821780955$

rat: replaced -0.2934748237257534 by $-11793110/40184401 = -0.2934748237257537$

rat: replaced -0.2973361180189332 by $-15390047/51759763 = -0.2973361180189329$

rat: replaced $5.016624916807239e-5$ by $153117/3052191514 = 5.016624916807235e-5$

rat: replaced $2.013266400891639e-4$ by $232411/1154397649 = 2.013266400891639e-4$

rat: replaced $4.544660485167953e-4$ by $444871/978887205 = 4.544660485167952e-4$

rat: replaced $8.105591523879241e-4$ by $1425236/1758336817 = 8.105591523879239e-4$

rat: replaced 0.001270570334355389 by $696221/547959433 = 0.00127057033435539$

rat: replaced 0.001835453585351213 by $1018402/554850315 = 0.001835453585351213$

rat: replaced 0.002506152409187654 by $484773/193433168 = 0.002506152409187653$

rat: replaced 0.003283599728207867 by $1007483/306822720 = 0.003283599728207872$

rat: replaced 0.004168717789994683 by $897113/215201183 = 0.004168717789994677$

rat: replaced 0.00516241807514603 by $757433/146720585 = 0.005162418075146034$

rat: replaced 0.006265601206128374 by $1194190/190594639 = 0.006265601206128363$

rat: replaced 0.007479156857214384 by $1971251/263565939 = 0.007479156857214391$

rat: replaced 0.008803963665517056 by $365844/41554465 = 0.008803963665517051$

rat: replaced 0.01024088914312629 by $1345773/131411734 = 0.01024088914312629$

rat: replaced 0.01179078959035854 by $1519715/128890011 = 0.01179078959035856$

rat: replaced 0.0134545100101271 by $2242921/166704027 = 0.01345451001012711$

rat: replaced 0.01523288402344322 by $1950407/128039247 = 0.01523288402344322$

rat: replaced 0.01712673378605437 by $1362867/79575418 = 0.01712673378605438$

rat: replaced 0.01913686990622912 by $1694449/88543686 = 0.01913686990622911$

rat: replaced 0.02126409136369717 by $9814128/461535263 = 0.02126409136369716$

rat: replaced 0.02350918542975217 by $2315819/98506986 = 0.02350918542975216$

rat: replaced 0.02587292758852516 by $3386321/130882792 = 0.02587292758852516$

rat: replaced 0.02835608145943683 by $10230271/360778728 = 0.02835608145943682$

rat: replaced 0.03095939872083586 by $14307719/462144602 = 0.03095939872083587$

rat: replaced 0.03368361903483233 by $4712088/139892569 = 0.03368361903483236$

rat: replaced 0.03652946997333167 by $4111522/112553563 = 0.03652946997333172$

rat: replaced 0.03949766694527834 by $8626745/218411508 = 0.03949766694527836$

rat: replaced 0.04258891312511537 by $3115258/73147159 = 0.04258891312511536$

rat: replaced 0.04580389938246726 by $2358579/51492974 = 0.04580389938246721$

rat: replaced 0.04914330421305446 by $2180747/44375262 = 0.04914330421305456$

rat: replaced 0.05260779367084312 by $4975224/94571995 = 0.05260779367084304$

rat: replaced 0.05619802130144141 by $1396735/24853811 = 0.05619802130144146$

rat: replaced 0.05991462807674475 by $6603037/110207427 = 0.05991462807674477$

rat: replaced 0.06375824233083943 by $6198842/97224167 = 0.0637582423308394$

rat: replaced 0.06772947969716975 by $4012504/59243095 = 0.06772947969716978$

rat: replaced 0.07182894304697524 by $5813372/80933559 = 0.07182894304697511$

rat: replaced 0.07605722242900365 by $14672328/192911699 = 0.07605722242900365$

rat: replaced 0.08041489501050719 by $3507279/43614793 = 0.0804148950105071$

rat: replaced 0.08490252501952561 by $2460362/28978667 = 0.08490252501952557$

rat: replaced 0.08952066368846451 by $4304415/48082921 = 0.08952066368846436$

rat: replaced 0.09426984919897213 by $3898288/41352437 = 0.09426984919897224$

rat: replaced 0.0991506066281217 by $11428253/115261554 = 0.09915060662812164$

rat: replaced 0.1041634478959041 by $7209817/69216382 = 0.1041634478959042$

rat: replaced 0.1093088717140371 by $3826731/35008421 = 0.109308871714037$

rat: replaced 0.1145873635360931 by $5173172/45146095 = 0.1145873635360932$

rat: replaced 0.1199993955089551 by $23218093/193485083 = 0.1199993955089551$

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rat: replaced 0.1370412532187207 by $16597683/121114501 = 0.1370412532187207$

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rat: replaced 0.1616595882803922 by $12686167/78474572 = 0.1616595882803923$

rat: replaced 0.1681553269830629 by $4527449/26924208 = 0.168155326983063$

rat: replaced 0.1747882490098353 by $23565700/134824281 = 0.1747882490098353$

rat: replaced 0.1815586910657007 by $4563713/25136296 = 0.1815586910657004$

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rat: replaced 0.1955134132929397 by $7172626/36686107 = 0.1955134132929395$

rat: replaced 0.202698297987053 by $17668607/87167022 = 0.2026982979870529$

rat: replaced 0.2100219116952866 by $8269584/39374863 = 0.2100219116952864$

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rat: replaced 0.2328277337455789 by $10320856/44328293 = 0.2328277337455787$

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rat: replaced 0.2909401254778209 by $24764749/85119744 = 0.290940125477821$

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rat: replaced 0.3179559626514321 by $26241235/82531036 = 0.3179559626514321$

rat: replaced 0.3272435296422674 by $8247573/25203166 = 0.3272435296422679$

rat: replaced 0.3366723711028454 by $10805861/32096073 = 0.3366723711028449$

rat: replaced 0.3462425441485439 by $20967050/60555961 = 0.3462425441485438$

rat: replaced 0.3559540917617003 by $19053013/53526602 = 0.3559540917617001$

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rat: replaced 0.4497199689406718 by $4550581/10118699 = 0.4497199689406711$

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rat: replaced 0.4721099769024512 by 25424083/53852035 = 0.4721099769024513
rat: replaced 0.48351622589948 by 17675673/36556525 = 0.4835162258994803
rat: replaced 0.4950631221018528 by 7053395/14247466 = 0.495063122101853
rat: replaced 0.5067505108212387 by 13754758/27143057 = 0.5067505108212388
rat: replaced 0.5185782233201719 by 21662467/41772805 = 0.518578223320172
rat: replaced 0.5305460768289253 by 10488897/19770002 = 0.530546076828925
rat: replaced 0.5426538745637882 by 22388393/41257225 = 0.5426538745637886
rat: replaced 0.5549014057467435 by 9960301/17949677 = 0.5549014057467441
rat: replaced 0.5672884456265459 by 28078535/49496046 = 0.5672884456265456
rat: replaced 0.5798147555011964 by 18086313/31193261 = 0.5798147555011962
rat: replaced 0.5924800827418131 by 20592707/34756792 = 0.5924800827418134

rat: replaced 0.6052841608178928 by 26813845/44299598 = 0.6052841608178927
part: invalid index of list or matrix.
#0: lineIntersection(g=[1,-1,2],h=[1,1,5])
#1: projectToLine(a=[2,0],g=[1,1,5])
#2: lineCircleIntersections(g=[1,1,5],c=[2,0,4])
-- an error. To debug this try: debugmode(true);

Error in:
  $lineCircleIntersections(l,c) | radcan, // titik potong lingkla ...
                                ^

```

```
>C=A+normalize([-3,-4])*4; plotPoint(C); plotSegment(P1,C); plotSegment(P2,C);  
>degprint(computeAngle(P1,C,P2))
```

57°58'20.06''

```
>C=A+normalize([-4,-5])*4; plotPoint(C); plotSegment(P1,C); plotSegment(P2,C);  
>degprint(computeAngle(P1,C,P2))
```

57°58'20.06''

```
>insimg;
```

Garis Sumbu

```
>A=[3,3]; B=[-2,-3];  
>c1=circleWithCenter(A,distance(A,B));  
>c2=circleWithCenter(B,distance(A,B));  
>{P1,P2,f}=circleCircleIntersections(c1,c2);  
>l=lineThrough(P1,P2);  
>setPlotRange(5); plotCircle(c1); plotCircle(c2);  
>plotPoint(A); plotPoint(B); plotSegment(A,B); plotLine(l):
```

```
>A &= [a1,a2]; B &= [b1,b2];
>c1 &= circleWithCenter(A,distance(A,B));
>c2 &= circleWithCenter(B,distance(A,B));
>P &= circleCircleIntersections(c1,c2); P1 &= P[1]; P2 &= P[2];
>g &= getLineEquation(lineThrough(P1,P2),x,y);
>$solve(g,y)
```

```
Maxima said:
solve: all variables must not be numbers.
  -- an error. To debug this try: debugmode(true);

Error in:
  $solve(g,y) ...
    ^
```

```
>$solve(getLineEquation(middlePerpendicular(A,B),x,y),y)
```

```
Maxima said:
solve: all variables must not be numbers.
  -- an error. To debug this try: debugmode(true);

Error in:
... (getLineEquation(middlePerpendicular(A,B),x,y),y) ...
```

```
>h &=getLineEquation(lineThrough(A,B),x,y);
>$solve(h,y)
```

```
Maxima said:  
solve: all variables must not be numbers.  
-- an error. To debug this try: debugmode(true);
```

```
Error in:  
$solve(h,y) ...  
^
```

Garis Euler dan Parabola

```
>A:=[-1.5,-1.5]; B:=[3,0]; C:=[1.5,3];  
>setPlotRange(3); plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C");
```

```
>plotSegment(A,B,""); plotSegment(B,C,""); plotSegment(C,A,"");  
>$areaTriangle(A,B,C)
```



```
>c &= lineThrough(A,B)
```

$$\left[-\frac{3}{2}, -\frac{9}{2}, -\frac{9}{2}\right]$$

```
>$getLineEquation(c,x,y)  
>$getHesseForm(c,x,y,C), $at(%,[x=C[1],y=C[2]])
```

```
>LL &= circleThrough(A,B,C); $getCircleEquation(LL,x,y)
```

Maxima said:

rat: replaced -7.57493712521158e-5 by -291512/3848375177 = -7.574937125211583e-5

rat: replaced -3.059898801345065e-4 by -367004/1199399143 = -3.059898801345067e-4

rat: replaced -6.951984652882083e-4 by -649868/934794929 = -6.951984652882086e-4

rat: replaced -0.001247836168679406 by -996993/798977482 = -0.001247836168679407

rat: replaced -0.0019683476894981 by -1171852/595348071 = -0.001968347689498099

rat: replaced -0.002861160939693026 by $-414045/144712237 = -0.002861160939693027$

rat: replaced -0.003930686601183196 by $-1414939/359972479 = -0.003930686601183198$

rat: replaced -0.00518131768479372 by $-1585327/305969851 = -0.005181317684793722$

rat: replaced -0.006617429090958894 by $-2655242/401249785 = -0.00661742909095889$

rat: replaced -0.008243377172234598 by $-1494085/181246711 = -0.00824337717223459$

rat: replaced -0.01006349929766813 by $-2785964/276838495 = -0.01006349929766812$

rat: replaced -0.01208211341906348 by $-403273/33377687 = -0.01208211341906346$

rat: replaced -0.01430351763919102 by $-2688199/187939713 = -0.01430351763919103$

rat: replaced -0.01673198978198 by $-3399597/203179481 = -0.01673198978198$

rat: replaced -0.01937178696474014 by $-4095384/211409717 = -0.01937178696474013$

rat: replaced -0.02222714517245272 by $-1488848/66983321 = -0.02222714517245271$

rat: replaced -0.02530227883417678 by $-11141142/440321683 = -0.02530227883417678$

rat: replaced -0.02860138040160899 by $-5896067/206146239 = -0.02860138040160898$

rat: replaced -0.03212861992984196 by $-3474579/108145915 = -0.03212861992984201$

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rat: replaced -0.03988407860632956 by $-5523906/138499025 = -0.03988407860632954$

rat: replaced -0.04412052214017978 by $-4053557/91874638 = -0.04412052214017975$

rat: replaced -0.04860155158359004 by $-3943740/81144323 = -0.04860155158359014$

rat: replaced -0.05333121879985003 by $-1834427/34396870 = -0.0533312187998501$

rat: replaced -0.05831355078867968 by $-2465946/42287701 = -0.05831355078867967$

rat: replaced -0.06355254928353218 by $-4583196/72116635 = -0.06355254928353216$

rat: replaced -0.06905219035142413 by $-9155887/132593723 = -0.0690521903514241$

rat: replaced -0.07481642399533184 by $-2967077/39658097 = -0.0748164239953319$

rat: replaced -0.08084917375919423 by $-6800433/84112585 = -0.0808491737591943$

rat: replaced -0.08715433633556302 by $-5843645/67049389 = -0.08715433633556303$

rat: replaced -0.09373578117593417 by $-7402616/78973215 = -0.09373578117593415$

rat: replaced -0.1005973501038089 by $-1640864/16311205 = -0.100597350103809$

rat: replaced -0.1077428569305121 by $-20150833/187027090 = -0.107742856930512$

rat: replaced -0.115176087073816 by $-3594765/31211036 = -0.1151760870738158$

rat: replaced -0.1229007971794005 by $-4862770/39566627 = -0.1229007971794007$

rat: replaced -0.130920714745193 by $-4199712/32078285 = -0.1309207147451929$

rat: replaced -0.1392395377486195 by $-36213847/260083074 = -0.1392395377486195$

rat: replaced -0.1478609342768128 by $-4198057/28391928 = -0.1478609342768128$

rat: replaced -0.1567885421598042 by $-14899832/95031383 = -0.1567885421598042$

rat: replaced -0.1660259686067453 by $-12607897/75939307 = -0.1660259686067454$

rat: replaced -0.1755767898451896 by $-9911603/56451670 = -0.1755767898451897$

rat: replaced -0.185444550763472 by $-7194550/38796233 = -0.1854445507634723$

rat: replaced -0.1956327645562239 by $-14925693/76294444 = -0.1956327645562238$

rat: replaced -0.206144912373057 by $-10508817/50977814 = -0.206144912373057$

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rat: replaced -0.2396592835011996 by $-18130307/75650343 = -0.2396592835011997$

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rat: replaced -0.2636842153146071 by $-16839380/63861919 = -0.2636842153146071$

rat: replaced -0.2762112334455278 by $-9903377/35854360 = -0.2762112334455279$

rat: replaced -0.2890856275582896 by $-4178583/14454482 = -0.2890856275582895$

rat: replaced -0.3023106101867103 by $-5206455/17222204 = -0.3023106101867101$

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rat: replaced -0.3298250155152552 by $-20176073/61172052 = -0.3298250155152552$

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rat: replaced -0.3738043179537355 by $-4462655/11938479 = -0.373804317953736$

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rat: replaced -0.5448305595142084 by $-33841376/62113579 = -0.5448305595142087$

rat: replaced -0.5640723137905005 by $-15307610/27137673 = -0.5640723137905007$

rat: replaced -0.5837136575054853 by $-47878079/82023229 = -0.5837136575054854$

rat: replaced -0.6037571265036585 by $-12602624/20873665 = -0.6037571265036591$

rat: replaced -0.6242052164173237 by $-10481453/16791678 = -0.624205216417323$

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rat: replaced -0.07561466217598092 by $-14346317/189729301 = -0.07561466217598092$

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rat: replaced -0.09369240732922907 by $-5174175/55225126 = -0.0936924073292291$

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rat: replaced -2.492817691581298 by $-26575204/10660709 = -2.492817691581301$

```

part: invalid index of list or matrix.
#0: lineIntersection(g=[-9/2,-3/2,-9/4],h=[-3,-9/2,-27/8])
#1: circleThrough(a=[-3/2,-3/2],b=[3,0],c=[3/2,3])
-- an error. To debug this try: debugmode(true);

Error in:
LL &= circleThrough(A,B,C); $getCircleEquation(LL,x,y) ...
      ^

```

```

>0 &= getCircleCenter(LL); $0
>plotCircle(LL()); plotPoint(0(),"0"):

```

```

Function LL not found.
Try list ... to find functions!
Error in:
plotCircle(LL()); plotPoint(0(),"0"): ...
      ^

```

```

>H &= lineIntersection(perpendicular(A,lineThrough(C,B)),...
> perpendicular(B,lineThrough(A,C))); $H

```

```

Maxima said:
rat: replaced -1.497487512542063e-4 by -299560/2000417349 = -1.497487512542064e-4

rat: replaced -5.979800402663507e-4 by -330831/553247563 = -5.979800402663499e-4

rat: replaced -0.001343149056780863 by -584699/435319518 = -0.001343149056780863

rat: replaced -0.002383681297017493 by -756665/317435473 = -0.002383681297017489

```

rat: replaced -0.003717972721118644 by $-2149606/578166157 = -0.003717972721118646$

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rat: replaced 0.6907681590690352 by $10003471/14481662 = 0.690768159069035$

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rat: replaced 0.8318462650060389 by $30346636/36481063 = 0.8318462650060389$

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rat: replaced 1.085684402341557 by $38378733/35349806 = 1.085684402341557$

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rat: replaced 1.298720202951555 by $33824443/26044442 = 1.298720202951555$

rat: replaced 1.336106284436991 by $5396397/4038898 = 1.336106284436992$

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rat: replaced 1.451473616334275 by $8780993/6049709 = 1.451473616334273$

rat: replaced 1.490996270868687 by $44673937/29962474 = 1.490996270868687$

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rat: replaced 1.612751535239189 by $65392401/40547102 = 1.612751535239189$

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rat: replaced 1.739264966349412 by $21773512/12518801 = 1.739264966349413$

rat: replaced 1.782487923801538 by $19245269/10796858 = 1.782487923801536$

rat: replaced 1.826234628096705 by $18221771/9977782 = 1.826234628096705$

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rat: replaced 1.915293726343482 by $16780009/8761063 = 1.915293726343481$

rat: replaced 1.960603214409484 by $45722957/23320862 = 1.960603214409484$

rat: replaced 2.006430637837895 by $146296719/72913918 = 2.006430637837895$

rat: replaced 2.052774413899562 by $58550872/28522799 = 2.052774413899562$

rat: replaced 2.099632908230499 by $93949097/44745487 = 2.099632908230499$

rat: replaced 2.147004434995322 by $25274650/11772053 = 2.147004434995323$

rat: replaced 2.194887257055829 by $28867932/13152353 = 2.194887257055829$

rat: replaced 2.243279586144718 by $38403199/17119221 = 2.24327958614472$

rat: replaced 2.292179583044406 by $15612340/6811133 = 2.292179583044407$

rat: replaced 2.341585357770954 by $20809175/8886789 = 2.341585357770956$

rat: replaced 2.391494969763059 by $22142156/9258709 = 2.391494969763063$


```
rat: replaced 2.441906428076114 by 36070003/14771247 = 2.441906428076113
```

```
rat: replaced 2.492817691581298 by 26575204/10660709 = 2.492817691581301
```

```
part: invalid index of list or matrix.
```

```
#0: lineIntersection(g=[3/2,-3,9/4],h=[3,9/2,9])
```

```
-- an error. To debug this try: debugmode(true);
```

```
Error in:
```

```
perpendicular(B,lineThrough(A,C)); $H ...  
^
```

```
>el &= lineThrough(H,0); $getLineEquation(el,x,y)
```

```
>plotPoint(H(),"H"); plotLine(el(),"Garis Euler"):
```

```
Function H needs at least 3 arguments!
```

```
Use: H (a, b, c)
```

```
Error in:
```

```
plotPoint(H(),"H"); plotLine(el(),"Garis Euler"): ...  
^
```

```
>M &= (A+B+C)/3; $getLineEquation(e1,x,y) with [x=M[1],y=M[2]]
>plotPoint(M(),"M"): // titik berat
>$distance(M,H)/distance(M,O)|radcan
```

```
>$computeAngle(A,C,B), degprint(%())
```

60°15'18.43''

```
>Q &= lineIntersection(angleBisector(A,C,B),angleBisector(C,B,A))|radcan; $Q
```

Maxima said:

rat: replaced 1.66665833335744e-7 by 15819/94914474571 = 1.66665833335744e-7

rat: replaced 4.999958333473664e-5 by 201389/4027813565 = 4.99995833347366e-5

rat: replaced 1.33330666692022e-6 by 31771/23828726570 = 1.333306666920221e-6

rat: replaced 1.999933334222437e-4 by 200030/1000183339 = 1.999933334222437e-4

rat: replaced 4.499797504338432e-6 by 24036/5341573699 = 4.499797504338431e-6

rat: replaced 4.499662510124569e-4 by 1162901/2584418270 = 4.499662510124571e-4

rat: replaced 1.066581336583994e-5 by 58861/5518660226 = 1.066581336583993e-5

rat: replaced $7.998933390220841e-4$ by $1137431/1421978337 = 7.998933390220838e-4$

rat: replaced $2.083072932167196e-5$ by $35635/1710693824 = 2.0830729321672e-5$

rat: replaced 0.001249739605033717 by $567943/454449069 = 0.001249739605033716$

rat: replaced $3.599352055540239e-5$ by $98277/2730408098 = 3.599352055540234e-5$

rat: replaced 0.00179946006479581 by $479561/266502719 = 0.001799460064795812$

rat: replaced $5.71526624672386e-5$ by $51154/895041417 = 5.715266246723866e-5$

rat: replaced 0.002448999746720415 by $1946227/794702818 = 0.002448999746720415$

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rat: replaced 0.003198293697380561 by $2986741/933854512 = 0.003198293697380562$

rat: replaced $1.214508019889565e-4$ by $158455/1304684674 = 1.214508019889563e-4$

rat: replaced 0.004047266988005727 by $2125334/525128193 = 0.004047266988005727$

rat: replaced $1.665833531718508e-4$ by $142521/855553675 = 1.66583353171851e-4$

rat: replaced 0.004995834721974179 by $1957223/391770967 = 0.004995834721974179$

rat: replaced $2.216991628251896e-4$ by $179571/809975995 = 2.216991628251896e-4$

rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced $2.877927110806339e-4$ by $1167733/4057548906 = 2.877927110806339e-4$

rat: replaced 0.00719136414613375 by $2476362/344352191 = 0.007191364146133747$

rat: replaced $3.658573803051457e-4$ by $386279/1055818526 = 3.658573803051454e-4$

rat: replaced 0.00843810628521191 by $2079855/246483622 = 0.008438106285211924$

rat: replaced $4.5688535576352e-4$ by $262978/575588595 = 4.568853557635206e-4$

rat: replaced 0.009784003787362772 by $1752551/179124113 = 0.009784003787362787$

rat: replaced $5.618675264007778e-4$ by $150595/268025812 = 5.618675264007782e-4$

rat: replaced 0.01122892206395776 by $5450241/485375263 = 0.01122892206395776$

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rat: replaced 0.01277271662437307 by $3258991/255152533 = 0.01277271662437308$

rat: replaced $8.176509330039827e-4$ by $105841/129445214 = 8.176509330039812e-4$

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rat: replaced 0.1451142866615502 by $3554077/24491572 = 0.1451142866615504$

rat: replaced 0.4429774532337832 by $23449796/52936771 = 0.4429774532337834$

rat: replaced 0.1495026295080298 by $26759297/178988805 = 0.1495026295080298$

rat: replaced 0.451310139418413 by $8841241/19590167 = 0.4513101394184133$

rat: replaced 0.1539740213994798 by $16145763/104860306 = 0.1539740213994798$

part: invalid index of list or matrix.

#0: lineIntersection($g = [-3\sqrt{13}/(2\sqrt{5}) - 3, 3\sqrt{5}\sqrt{13}/4 - 3\sqrt{13}/(4\sqrt{5}) - 9/2$),
-- an error. To debug this try: debugmode(true);

Error in:

... angleBisector(A,C,B),angleBisector(C,B,A))|radcan; \$Q ...
^

```
>r &= distance(Q,projectToLine(Q,lineThrough(A,B)))|ratsimp; $r
```

Maxima said:

rat: replaced $2.247481262563113e-4$ by $615301/2737735839 = 2.247481262563112e-4$

rat: replaced $8.979700403997162e-4$ by $1151181/1281981523 = 8.979700403997164e-4$

rat: replaced 0.002018098433299548 by $4408921/2184690760 = 0.002018098433299548$

rat: replaced 0.003583521305550619 by $3332605/929980518 = 0.003583521305550618$

rat: replaced 0.00559258212866922 by $661903/118353738 = 0.005592582128669227$

rat: replaced 0.008043580010748043 by $5032876/625700993 = 0.008043580010748041$

rat: replaced 0.01093476986654101 by $48362019/4422774287 = 0.01093476986654101$

rat: replaced 0.01426436259197156 by $4450653/312012049 = 0.01426436259197157$

rat: replaced 0.01803052524304233 by $2302457/127697722 = 0.01803052524304231$

rat: replaced 0.02223138121912603 by $2788127/125414025 = 0.02223138121912601$

rat: replaced 0.02686501045062654 by $3971066/147815539 = 0.02686501045062658$

rat: replaced 0.03192944959098092 by $3972053/124400923 = 0.03192944959098093$

rat: replaced 0.03742269221299588 by $17164291/458659973 = 0.03742269221299588$

rat: replaced 0.04334268900948719 by $4721703/108938857 = 0.04334268900948722$

rat: replaced 0.04968734799820874 by $11739417/236265719 = 0.04968734799820874$

rat: replaced 0.05645453473104778 by $5021915/88955033 = 0.05645453473104776$

rat: replaced 0.0636420725074706 by $4635713/72840384 = 0.06364207250747059$

rat: replaced 0.07124774259218976 by $5610259/78742972 = 0.07124774259218969$

rat: replaced 0.07926928443703447 by $10758163/135716666 = 0.07926928443703443$

rat: replaced 0.08770439590700449 by $11027461/125734416 = 0.08770439590700449$

rat: replaced 0.09655073351048213 by $7645228/79183531 = 0.09655073351048213$

rat: replaced 0.1058059126335792 by $4020133/37995353 = 0.1058059126335792$

rat: replaced 0.1154675077785961 by $6306881/54620396 = 0.1154675077785961$

rat: replaced 0.1255330528065689 by $6423129/51166835 = 0.1255330528065689$

rat: replaced 0.1360000411838831 by $6525273/47979934 = 0.1360000411838833$

rat: replaced 0.1468659262329232 by $8290049/56446374 = 0.1468659262329233$

rat: replaced 0.1581281213867393 by $8280149/52363545 = 0.1581281213867396$

rat: replaced 0.1697840004477016 by $16992749/100084513 = 0.1697840004477016$

rat: replaced 0.181830897850116 by $12921406/71062763 = 0.181830897850116$

rat: replaced 0.1942661089267824 by $7134865/36727276 = 0.1942661089267824$

rat: replaced 0.207086890179455 by $7822594/37774453 = 0.2070868901794554$

rat: replaced 0.2202904595531929 by $9466710/42973763 = 0.220290459553193$

rat: replaced 0.2338739967145615 by $9446678/40392169 = 0.2338739967145612$

rat: replaced 0.2478346433336637 by $16355222/65992477 = 0.2478346433336636$

rat: replaced 0.262169503369972 by $9649007/36804460 = 0.262169503369972$

rat: replaced 0.2768756433619283 by $5642794/20380247 = 0.2768756433619278$

rat: replaced 0.2919500927202884 by $121438203/415955350 = 0.2919500927202884$

rat: replaced 0.3073898440251784 by $10758647/35000008 = 0.3073898440251785$

rat: replaced 0.3231918533268338 by $16851305/52140253 = 0.3231918533268337$

rat: replaced 0.3393530404499933 by $7014483/20670164 = 0.3393530404499935$

rat: replaced 0.3558702893019118 by $11359617/31920667 = 0.3558702893019122$

rat: replaced 0.3727404481839686 by $11293059/30297380 = 0.3727404481839684$

rat: replaced 0.3899603301068333 by $25988611/66644243 = 0.3899603301068331$

rat: replaced 0.4075267131091642 by $99927009/245203580 = 0.4075267131091642$

rat: replaced 0.4254363405797997 by $23297981/54762555 = 0.4254363405797995$

rat: replaced 0.4436859215834164 by $8828931/19899056 = 0.4436859215834158$

rat: replaced 0.4622721311896225 by $18116412/39189929 = 0.4622721311896227$

rat: replaced 0.481191610805446 by $9185023/19088078 = 0.4811916108054462$

rat: replaced 0.5004409685111909 by $2614168/5223729 = 0.5004409685111919$

rat: replaced 0.5200167793996274 by $27157128/52223561 = 0.5200167793996276$

rat: replaced 0.5399155859184809 by $17073709/31622923 = 0.5399155859184808$

rat: replaced 0.5601338982161811 by $23162159/41351111 = 0.560133898216181$

rat: replaced 0.5806681944908463 by $93819617/161571820 = 0.5806681944908463$

rat: replaced 0.6015149213424613 by $9439629/15693092 = 0.6015149213424608$

rat: replaced 0.6226704941282136 by $18657147/29963114 = 0.6226704941282137$

rat: replaced 0.6441312973209531 by $13192294/20480753 = 0.644131297320953$

rat: replaced 0.6658936848707453 by $22412057/33657110 = 0.665893684870745$

rat: replaced 0.6879539805694695 by $62531575/90894997 = 0.6879539805694697$

rat: replaced 0.7103084784184406 by $11146199/15692054 = 0.7103084784184404$

rat: replaced 0.7329534429990014 by $13946471/19027772 = 0.7329534429990017$

rat: replaced 0.7558851098460647 by $5948069/7869012 = 0.7558851098460645$

rat: replaced 0.7790996858245554 by $4386064/5629657 = 0.7790996858245538$

rat: replaced 0.8025933495087219 by $22091337/27524944 = 0.802593349508722$

rat: replaced 0.826362251564272 by $24392475/29517896 = 0.8263622515642713$

rat: replaced 0.8504025151333089 by $26363177/31000822 = 0.8504025151333084$

rat: replaced 0.8747102362220087 by $20629426/23584297 = 0.8747102362220083$

rat: replaced 0.8992814840910188 by $92877137/103279272 = 0.8992814840910187$

rat: replaced 0.924112301648528 by $10835177/11724957 = 0.9241123016485263$

rat: replaced 0.9491987058459732 by $2526299/2661507 = 0.9491987058459738$

rat: replaced 0.9745366880763398 by $57495659/58997942 = 0.97453668807634$

rat: replaced 1.000122214575019 by $49451864/49445821 = 1.000122214575019$

rat: replaced 1.025951226823183 by $8953739/8727256 = 1.025951226823185$

rat: replaced 1.052019641953631 by $13350751/12690591 = 1.052019641953633$

rat: replaced 1.078323353159073 by $15380966/14263779 = 1.078323353159075$

rat: replaced 1.104858230102808 by $39992086/36196577 = 1.104858230102808$

rat: replaced 1.131620119331755 by $12144847/10732265 = 1.131620119331753$

rat: replaced 1.158604844691791 by $8203205/7080244 = 1.158604844691793$

rat: replaced 1.185808207745368 by $16191783/13654639 = 1.185808207745368$

rat: replaced 1.213225988191351 by $16745047/13802084 = 1.213225988191349$

rat: replaced 1.240853944287041 by $22797805/18372674 = 1.240853944287043$

rat: replaced 1.268687813272354 by $16494549/13001267 = 1.268687813272353$

rat: replaced 1.296723311796085 by $4579250/3531401 = 1.296723311796083$

rat: replaced 1.324956136344244 by $20639169/15577247 = 1.324956136344246$

rat: replaced 1.353381963670404 by $12810146/9465285 = 1.353381963670402$

rat: replaced 1.381996451228021 by $69697753/50432657 = 1.381996451228021$

rat: replaced 1.410795237604684 by $16557131/11736027 = 1.410795237604685$

rat: replaced 1.439773942958254 by $13107044/9103543 = 1.439773942958253$

rat: replaced 1.468928169454845 by $13138474/8944259 = 1.468928169454842$

rat: replaced 1.498253501708601 by $7867876/5251365 = 1.498253501708603$

rat: replaced 1.527745507223237 by $24303429/15908035 = 1.527745507223237$

rat: replaced 1.557399736835275 by $17763365/11405784 = 1.557399736835276$

rat: replaced 1.587211725158968 by $18758111/11818279 = 1.587211725158968$

rat: replaced 1.617176991032825 by $77994669/48228901 = 1.617176991032825$

rat: replaced 1.647291037967731 by $186985994/113511207 = 1.647291037967731$

rat: replaced 1.677549354596587 by $41868463/24958111 = 1.677549354596588$

```

rat: replaced 1.707947415125444 by 18975622/11110191 = 1.707947415125447
rat: replaced 1.738480679786083 by 70736079/40688447 = 1.738480679786083
rat: replaced 1.76914459528998 by 4098089/2316424 = 1.769144595289981
rat: replaced 1.799934595283639 by 19693277/10941107 = 1.79993459528364
part: invalid index of list or matrix.
#0: lineIntersection(g=[9/2,3/2,[9/2,3/2] . Q],h=[-3/2,9/2,-9/2])
#1: projectToLine(a=Q,g=[-3/2,9/2,-9/2])
-- an error. To debug this try: debugmode(true);

Error in:
... ance(Q,projectToLine(Q,lineThrough(A,B)))|ratsimp; $r ...

```

```

>LD &= circleWithCenter(Q,r); // Lingkaran dalam

```

```

>color(5); plotCircle(LD()):

```

```

Q is not a variable!
Error in expression: [Q[1],Q[2],cc[3]]
Error in:
color(5); plotCircle(LD()): ...

```

contoh lain dari materi trigonometri rasional

```
>A&:=[2,3]; B&:=[5,4]; C&:=[0,5]; ...
>setPlotRange(-1,5,1,7); ...
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...
>plotSegment(B,A,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...
>insimg;
>$distance(A,B)
>c &= quad(A,B); $c, b &= quad(A,C); $b, a &= quad(B,C); $a,
```

```
>wb &= computeAngle(A,B,C); $wb, $(wb/pi*180)()
```

29.7448812969

```
>$crosslaw(a,b,c,x), $solve(%,x), //(b+c-a)^=4b.c(1-x)
```

Maxima said:

solve: all variables must not be numbers.

-- an error. To debug this try: debugmode(true);

Error in:

```
$crosslaw(a,b,c,x), $solve(%,x), //(b+c-a)^=4b.c(1-x) ...
```



```

>sb &= spread(b,a,c); $sb
>$sin(computeAngle(A,B,C))^2
>ha &= c*sb; $ha
>$sqrt(ha)
>$sqrt(ha)*sqrt(a)/2

```

```

>$areaTriangle(B,A,C)

```

Aturan penyebaran 3 kali lipat

```

>setPlotRange(1); ...
>color(1); plotCircle(circleWithCenter([0,0],1)); ...
>A:=[cos(1),sin(1)]; B:=[cos(2),sin(2)]; C:=[cos(6),sin(6)]; ...
>plotPoint(A,"A"); plotPoint(B,"B"); plotPoint(C,"C"); ...
>color(3); plotSegment(A,B,"c"); plotSegment(A,C,"b"); plotSegment(C,B,"a"); ...
>color(1); O:=[0,0]; plotPoint(O,"O"); ...
>plotSegment(A,O); plotSegment(B,O); plotSegment(C,O,"r"); ...
>insimg;
>&remvalue(a,b,c,r); // hapus nilai-nilai sebelumnya untuk perhitungan baru
>rabc &= rhs(solve(triplespread(spread(b,r,r),spread(a,r,r),spread(c,r,r)),r)[4]); $rabc

```

```
>function periradius(a,b,c) &= rabc;
```

```
>a:=quadrance(B,C); b:=quadrance(A,C); c:=quadrance(A,B);
```

```
>periradius(a,b,c)
```

1

```
>$spread(b,a,c)*rabc | ratsimp  
>$doublespread(b/(4*r))-spread(b,r,r) | ratsimp
```

Contoh 6: Jarak Minimal pada Bidang

Fungsi yang, ke titik M di bidang, menetapkan jarak AM antara titik tetap A dan M, memiliki garis level yang agak sederhana: lingkaran berpusat di A.

```
>&remvalue();  
>A=[-2,-2];  
>function d1(x,y):=sqrt((x-A[1])^2+(y-A[2])^2)  
>fcontour("d1",xmin=-2,xmax=0,ymin=-2,ymax=0,hue=1, ...  
>title="If you see ellipses, please set your window square"):
```

dan grafiknya juga agak sederhana: bagian atas kerucut:

```
>plot3d("d1",xmin=-2,xmax=0,ymin=-2,ymax=0):
```

Ternyata setelah mencoba yang bisa hanya dengan memasukkan angka 1, karena ketika memakai angka 2, plot tidak membentuk kerucut diatas.

```
>B=[2,-2];  
>function d2(x,y):=d1(x,y)+sqrt((x-B[1])^2+(y-B[2])^2)  
>fcontour("d2",xmin=-2,xmax=2,ymin=-3,ymax=1,hue=1):
```

Grafiknya lebih menarik:

```
>plot3d("d2",xmin=-2,xmax=2,ymin=-3,ymax=1):
```

Pembatasan garis (AB) lebih terkenal:

```
>plot2d("abs(x+1)+abs(x-1)",xmin=-3,xmax=3):
```

Tiga poin

Contoh:

```

>C=[-3,2];
>function d3(x,y):=d2(x,y)+sqrt((x-C[1])^2+(y-C[2])^2)
>plot3d("d3",xmin=-5,xmax=3,ymin=-4,ymax=4);
>insimg;
>fcontour("d3",xmin=-4,xmax=1,ymin=-2,ymax=2,hue=1,title="The minimum is on A");
>P=(A_B_C_A)'; plot2d(P[1],P[2],add=1,color=12);
>insimg;

```

Tetapi jika semua sudut segitiga ABC kurang dari 120° , minimumnya adalah pada titik F di bagian dalam segitiga, yang merupakan satu-satunya titik yang melihat sisi-sisi ABC dengan sudut yang sama (maka masing-masing 120°):

```

>C=[-1,2];
>plot3d("d3",xmin=-2,xmax=2,ymin=-2,ymax=2):
>fcontour("d3",xmin=-2,xmax=2,ymin=-2,ymax=2,hue=1,title="The Fermat point");
>P=(A_B_C_A)'; plot2d(P[1],P[2],add=1,color=12);
>insimg;

```

Empat poin

Langkah selanjutnya adalah menambahkan 4 titik D dan mencoba meminimalkan $MA+MB+MC+MD$; katakan bahwa Anda adalah operator TV kabel dan ingin mencari di bidang mana Anda harus meletakkan antenna sehingga Anda dapat memberi makan empat desa dan menggunakan panjang kabel sesedikit mungkin!

```
>D=[2,21];  
>function d4(x,y):=d3(x,y)+sqrt((x-D[1])^2+(y-D[2])^2)  
>plot3d("d4",xmin=-1.5,xmax=1.5,ymin=-1.5,ymax=1.5):  
>fcontour("d4",xmin=-1.5,xmax=1.5,ymin=-1.5,ymax=1.5,hue=1);  
>P=(A_B_C_D)'; plot2d(P[1],P[2],points=1,add=1,color=12);  
>insimg;
```

Contoh 7: Bola Dandelin dengan Povray

```
>load geometry;
```

Pertama dua garis yang membentuk kerucut.

```
>g1 &= lineThrough([0,0],[2,a])
```

$[-a, 2, 0]$

```
>g2 &= lineThrough([0,0],[-2,a])
```

$[-a, -2, 0]$

```
>g &= lineThrough([-2,0],[2,2])
```

$[-2, 4, 4]$

```
>setPlotRange(-2,2,0,3);  
>color(black); plotLine(g(),"")  
>a:=2; color(blue); plotLine(g1(),""), plotLine(g2(),""):
```

Sekarang kita ambil titik umum pada sumbu y.

```
>P &= [0,u]
```

[0, u]

Hitung jarak ke g1.

```
>d1 &= distance(P,projectToLine(P,g1)); $d1
```

Maxima said:

rat: replaced 3.333316666714881e-7 by 31638/94914474571 = 3.333316666714881e-7

rat: replaced 4.999958333473664e-5 by 201389/4027813565 = 4.99995833347366e-5

rat: replaced 2.66661333384044e-6 by 31771/11914363285 = 2.666613333840441e-6

rat: replaced 1.999933334222437e-4 by 200030/1000183339 = 1.999933334222437e-4

rat: replaced $8.999595008676864e-6$ by $48072/5341573699 = 8.999595008676863e-6$

rat: replaced $4.499662510124569e-4$ by $1162901/2584418270 = 4.499662510124571e-4$

rat: replaced $2.133162673167988e-5$ by $58861/2759330113 = 2.133162673167985e-5$

rat: replaced $7.998933390220841e-4$ by $1137431/1421978337 = 7.998933390220838e-4$

rat: replaced $4.166145864334392e-5$ by $35635/855346912 = 4.1661458643344e-5$

rat: replaced 0.001249739605033717 by $567943/454449069 = 0.001249739605033716$

rat: replaced $7.198704111080478e-5$ by $98277/1365204049 = 7.198704111080467e-5$

rat: replaced 0.00179946006479581 by $479561/266502719 = 0.001799460064795812$

rat: replaced $1.143053249344772e-4$ by $102308/895041417 = 1.143053249344773e-4$

rat: replaced 0.002448999746720415 by $1946227/794702818 = 0.002448999746720415$

rat: replaced $1.706120616546125e-4$ by $121691/713261412 = 1.706120616546125e-4$

rat: replaced 0.003198293697380561 by $2986741/933854512 = 0.003198293697380562$

rat: replaced $2.42901603977913e-4$ by $158455/652342337 = 2.429016039779126e-4$

rat: replaced 0.004047266988005727 by $2125334/525128193 = 0.004047266988005727$

rat: replaced $3.331667063437016e-4$ by $285042/855553675 = 3.331667063437019e-4$

rat: replaced 0.004995834721974179 by $1957223/391770967 = 0.004995834721974179$

rat: replaced $4.433983256503793e-4$ by $359142/809975995 = 4.433983256503793e-4$

rat: replaced 0.006043902043303184 by $1800665/297930871 = 0.006043902043303193$

rat: replaced 5.755854221612677e-4 by 1167733/2028774453 = 5.755854221612677e-4

rat: replaced 0.00719136414613375 by 2476362/344352191 = 0.007191364146133747

rat: replaced 7.317147606102914e-4 by 386279/527909263 = 7.317147606102907e-4

rat: replaced 0.00843810628521191 by 2079855/246483622 = 0.008438106285211924

rat: replaced 9.137707115270399e-4 by 525956/575588595 = 9.137707115270413e-4

rat: replaced 0.009784003787362772 by 1752551/179124113 = 0.009784003787362787

rat: replaced 0.001123735052801556 by 150595/134012906 = 0.001123735052801556

rat: replaced 0.01122892206395776 by 5450241/485375263 = 0.01122892206395776

rat: replaced 0.001363586771508052 by 384632/282073725 = 0.001363586771508052

rat: replaced 0.01277271662437307 by 3258991/255152533 = 0.01277271662437308

rat: replaced 0.001635301866007965 by 105841/64722607 = 0.001635301866007962

rat: replaced 0.01441523309043924 by 2330472/161667313 = 0.01441523309043925

rat: replaced 0.001940853148351629 by 651321/335584895 = 0.001940853148351627

rat: replaced 0.01615630721187855 by 19391318/1200232067 = 0.01615630721187855

rat: replaced 0.002282210046998856 by 1081775/474003259 = 0.002282210046998854

rat: replaced 0.01799576488272969 by 4765614/264818641 = 0.01799576488272969

rat: replaced 0.002661338409877589 by 1101107/413741821 = 0.002661338409877594

rat: replaced 0.01993342215875837 by $2504519/125644206 = 0.01993342215875836$

rat: replaced 0.003080200307800873 by $553768/179783113 = 0.003080200307800878$

rat: replaced 0.02196908527585173 by $1298306/59096953 = 0.0219690852758517$

rat: replaced 0.003540753838261357 by $644389/181992036 = 0.003540753838261362$

rat: replaced 0.02410255066939448 by $2001286/83032125 = 0.02410255066939453$

rat: replaced 0.004044952929623202 by $2584223/638875914 = 0.004044952929623201$

rat: replaced 0.02633360499462523 by $2978115/113091808 = 0.02633360499462525$

rat: replaced 0.004594747145730826 by $1274677/277420489 = 0.004594747145730826$

rat: replaced 0.02866202514797045 by $1770713/61779061 = 0.02866202514797044$

rat: replaced 0.005192081490954126 by $1097643/211407121 = 0.005192081490954129$

rat: replaced 0.03108757828935527 by $5034207/161936287 = 0.03108757828935525$

rat: replaced 0.005838896215689782 by $906221/155204163 = 0.005838896215689782$

rat: replaced 0.03361002186548678 by $4553215/135471944 = 0.03361002186548678$

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rat: replaced 0.902620278836826 by  $17682482/19590167 = 0.9026202788368266$ 

rat: replaced -0.1539740213994798 by  $-16145763/104860306 = -0.1539740213994798$ 
part: invalid index of list or matrix.
#0: lineIntersection(g=[2,a,a*u],h=[-a,2,0])
#1: projectToLine(a=[0,u],g=[-a,2,0])
-- an error. To debug this try: debugmode(true);

Error in:
d1 &= distance(P,projectToLine(P,g1)); $d1 ...
      ^

```

Hitung jarak ke g.

```
>d &= distance(P,projectToLine(P,g)); $d
```

```

Maxima said:
rat: replaced 1.006658300028163e-4 by  $526672/5231884543 = 1.006658300028163e-4$ 

rat: replaced 4.053198935121682e-4 by  $407727/1005938782 = 4.053198935121681e-4$ 

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rat: replaced 0.02373220984756236 by $1659683/69933774 = 0.02373220984756235$

rat: replaced 0.02818987116237373 by $1973754/70016425 = 0.02818987116237369$

rat: replaced 0.03302071038023735 by $1738361/52644567 = 0.03302071038023734$

rat: replaced 0.03822224443792405 by $2950094/77182647 = 0.03822224443792398$

rat: replaced 0.04379195320302948 by $7345187/167729148 = 0.04379195320302945$

rat: replaced 0.04972727972598423 by $3887839/78183223 = 0.04972727972598418$

rat: replaced 0.05602563049574899 by $2263313/40397814 = 0.0560256304957491$

rat: replaced 0.06268437569916258 by $2232724/35618509 = 0.06268437569916248$

rat: replaced 0.06970084948391991 by $4074381/58455256 = 0.0697008494839198$

rat: replaced 0.0770723502251559 by $10758816/139593719 = 0.0770723502251559$

rat: replaced 0.08479614079560605 by $6073253/71621809 = 0.08479614079560599$

rat: replaced 0.09286944883931658 by $10918553/117568836 = 0.09286944883931657$

rat: replaced 0.1012894670488777 by $8842549/87299788 = 0.1012894670488776$

rat: replaced 0.110053353446151 by $5865042/53292715 = 0.1100533534461511$

rat: replaced 0.1191582316664669 by $12641638/106091185 = 0.1191582316664669$

rat: replaced 0.1286011912462574 by $9582619/74514232 = 0.1286011912462575$

rat: replaced 0.1383792879141001 by $6012189/43447174 = 0.1383792879141$

rat: replaced 0.1484895438851439 by $14736907/99245419 = 0.148489543885144$

rat: replaced 0.1589289481588824 by $9158215/57624587 = 0.1589289481588823$

rat: replaced 0.1696944568202552 by $9021593/53163746 = 0.1696944568202549$

rat: replaced 0.1807829933440335 by $7546288/41742245 = 0.1807829933440331$

rat: replaced 0.1921914489024722 by $10324112/53717853 = 0.1921914489024719$

rat: replaced 0.2039166826761891 by $1082663/5309340 = 0.2039166826761895$

rat: replaced 0.215955522168244 by $1922059/8900254 = 0.2159555221682437$

rat: replaced 0.2283047635213872 by $5923297/25944693 = 0.2283047635213876$

rat: replaced 0.2409611718384407 by $15101087/62670209 = 0.2409611718384408$

rat: replaced 0.2539214815057865 by $11143913/43887240 = 0.2539214815057862$

rat: replaced 0.2671823965199248 by $9315804/34866833 = 0.2671823965199248$

rat: replaced 0.2807405908170709 by $9623555/34279172 = 0.2807405908170711$

rat: replaced 0.294592708605761 by $14699749/49898550 = 0.2945927086057611$

rat: replaced 0.3087353647024257 by $17677509/57257804 = 0.3087353647024255$

rat: replaced 0.3231651448699078 by $188109817/582085723 = 0.3231651448699078$

rat: replaced 0.3378786061588812 by $18176431/53795744 = 0.3378786061588813$

rat: replaced 0.3528722772521456 by 13060610/37012287 = 0.352872277252146

rat: replaced 0.3681426588117532 by 50379718/136848357 = 0.3681426588117532

rat: replaced 0.3836862238289388 by 24326967/63403285 = 0.3836862238289389

rat: replaced 0.3994994179768211 by 14902039/37301779 = 0.3994994179768209

rat: replaced 0.4155786599658295 by 12170593/29285895 = 0.41557865996583

rat: replaced 0.4319203419018306 by 28311791/65548640 = 0.4319203419018305

rat: replaced 0.4485208296469153 by 18739477/41780617 = 0.448520829646915

rat: replaced 0.4653764631828102 by 51230060/110083049 = 0.4653764631828103

rat: replaced 0.4824835569768743 by 17595895/36469419 = 0.4824835569768742

rat: replaced 0.4998384003506502 by 16209213/32428907 = 0.4998384003506501

rat: replaced 0.5174372578509299 by 15165922/29309683 = 0.5174372578509293

rat: replaced 0.5352763696232961 by 14519447/27125141 = 0.5352763696232953

rat: replaced 0.5533519517881027 by 27538925/49767467 = 0.5533519517881028

rat: replaced 0.5716601968188642 by 29258587/51181781 = 0.5716601968188642

rat: replaced 0.5901972739229997 by 7877677/13347532 = 0.5901972739229994

rat: replaced 0.608959329424914 by 28939084/47522195 = 0.6089593294249139

rat: replaced 0.6279424871513581 by 8375733/13338376 = 0.6279424871513594

rat: replaced 0.6471428488190449 by 178371467/275629202 = 0.6471428488190449

rat: replaced 0.6665564944244748 by $11487182/17233621 = 0.6665564944244741$

rat: replaced 0.6861794826359324 by $31176490/45434891 = 0.6861794826359328$

rat: replaced 0.7060078511876142 by $6523425/9239876 = 0.7060078511876133$

rat: replaced 0.7260376172758565 by $49516379/68200845 = 0.7260376172758564$

rat: replaced 0.7462647779574081 by $9370061/12555947 = 0.7462647779574093$

rat: replaced 0.7666853105497231 by $13840503/18052391 = 0.7666853105497217$

rat: replaced 0.787295173033226 by $18307536/23253713 = 0.787295173033227$

rat: replaced 0.8080903044555103 by $16634707/20585208 = 0.80809030445551$

rat: replaced 0.8290666253374293 by $4177840/5039209 = 0.8290666253374289$

rat: replaced 0.8502200380810416 by $24667763/29013387 = 0.8502200380810417$

rat: replaced 0.8715464273793672 by $16836757/19318256 = 0.8715464273793659$

rat: replaced 0.8930416606279152 by $15817553/17711999 = 0.8930416606279167$

rat: replaced 0.9147015883379395 by $29667113/32433652 = 0.9147015883379399$

rat: replaced 0.9365220445513858 by $15635627/16695418 = 0.9365220445513853$

rat: replaced 0.9584988472574829 by $25326901/26423507 = 0.9584988472574818$

rat: replaced 0.9806277988109409 by $70229422/71616797 = 0.980627798810941$

rat: replaced 1.002904686351712 by $15102855/15059113 = 1.002904686351713$

rat: replaced 1.025325282226272 by $63394680/61828847 = 1.025325282226272$

rat: replaced 1.047885344410385 by $54865534/52358337 = 1.047885344410385$

rat: replaced 1.070580616933298 by $12248940/11441399 = 1.070580616933296$

rat: replaced 1.093406830303339 by $41717397/38153591 = 1.093406830303339$

rat: replaced 1.116359701934858 by $14988760/13426461 = 1.116359701934858$

rat: replaced 1.139434936576487 by $14609183/12821428 = 1.139434936576488$

rat: replaced 1.162628226740658 by $15190011/13065235 = 1.16262822674066$

rat: replaced 1.185935253134348 by $19640588/16561265 = 1.185935253134347$

rat: replaced 1.209351685091007 by $23593213/19508976 = 1.209351685091006$

rat: replaced 1.232873181003619 by $15720181/12750850 = 1.232873181003619$

rat: replaced 1.256495388758861 by $51017285/40602843 = 1.256495388758861$

rat: replaced 1.28021394617231 by $20459615/15981403 = 1.28021394617231$

rat: replaced 1.304024481424656 by $102407691/78532031 = 1.304024481424656$

rat: replaced 1.327922613498882 by $33383689/25139785 = 1.327922613498882$

rat: replaced 1.351903952618363 by $21561239/15948795 = 1.351903952618364$

rat: replaced 1.375964100685837 by $49074793/35665751 = 1.375964100685837$

rat: replaced 1.400098651723215 by $29355433/20966689 = 1.400098651723217$

rat: replaced 1.424303192312171 by $21370802/15004391 = 1.424303192312171$

```

rat: replaced 1.448573302035484 by 23591614/16286103 = 1.448573302035484
rat: replaced 1.472904553919073 by 29881314/20287339 = 1.472904553919072
rat: replaced 1.497292514874693 by 42508133/28389999 = 1.497292514874692
part: invalid index of list or matrix.
#0: lineIntersection(g=[4,2,2*u],h=[-2,4,4])
#1: projectToLine(a=[0,u],g=[-2,4,4])
    -- an error. To debug this try: debugmode(true);

Error in:
d &= distance(P,projectToLine(P,g)); $d ...
      ^

```

Dan temukan pusat kedua lingkaran yang jaraknya sama.

```
>sol &= solve(d1^2=d^2,u); $sol
```

Ada dua solusi.

```
>u := sol()
```

[]

```
>dd := d()
```

Function d needs at least one argument!

Use: d (n)

Error in:

```
dd := d() ...  
      ^
```

Plot lingkaran ke dalam gambar.

```
>color(red);  
>plotCircle(circleWithCenter([0,u[1]],dd[1]),"");
```

Index 1 out of bounds!

Error in:

```
plotCircle(circleWithCenter([0,u[1]],dd[1]),""); ...  
                        ^
```

```
>plotCircle(circleWithCenter([0,u[2]],dd[2]),"");
```

Index 2 out of bounds!

Error in:

```
plotCircle(circleWithCenter([0,u[2]],dd[2]),""); ...  
                        ^
```

```
>insimg;
```

Latihan

1. Gambarkanlah segi-n beraturan jika diketahui titik pusat O , n , dan jarak titik pusat ke titik-titik sudut segi-n tersebut (jari-jari lingkaran luar segi-n), r .

Petunjuk:

- Besar sudut pusat yang menghadap masing-masing sisi segi-n adalah $(360/n)$.
- Titik-titik sudut segi-n merupakan perpotongan lingkaran luar segi-n dan garis-garis yang melalui pusat dan saling membentuk sudut sebesar kelipatan $(360/n)$.
- Untuk n ganjil, pilih salah satu titik sudut adalah di atas.
- Untuk n genap, pilih 2 titik di kanan dan kiri lurus dengan titik pusat.
- Anda dapat menggambar segi-3, 4, 5, 6, 7, dst beraturan.

Penyelesaian :

```
>load geometry
```

Numerical and symbolic geometry.

```

>setPlotRange(-3.5,3.5,-3.5,3.5);
>A=[-2,-2]; plotPoint(A,"A");
>B=[2,-2]; plotPoint(B,"B");
>C=[0,3]; plotPoint(C,"C");
>plotSegment(A,B,"c");
>plotSegment(B,C,"a");
>plotSegment(A,C,"b");
>aspect(1):
>c=circleThrough(A,B,C);
>R=getCircleRadius(c);
>O=getCircleCenter(c);
>plotPoint(O,"O");
>l=angleBisector(A,C,B);
>color(2); plotLine(l); color(1);
>plotCircle(c,"Lingkaran luar segitiga ABC"):

```

2. Gambarkanlah suatu parabola yang melalui 3 titik yang diketahui.

Petunjuk:

- Misalkan persamaan parabolanya $y = ax^2 + bx + c$.
- Substitusikan koordinat titik-titik yang diketahui ke persamaan tersebut.
- Selesaikan SPL yang terbentuk untuk mendapatkan nilai-nilai a , b , c .

Penyelesaian :

```

>load geometry;
>setPlotRange(5); P=[2,0]; Q=[4,0]; R=[0,-4];
>plotPoint(P,"P"); plotPoint(Q,"Q"); plotPoint(R,"R"):
>sol &= solve([a+b=-c,16*a+4*b=-c,c=-4],[a,b,c])

```

```
[[a = - 1, b = 5, c = - 4]]
```

Sehingga didapatkan nilai $a = -1$, $b = 5$ dan $c = -4$

```
>function y&=-x^2+5*x-4
```

```
[- 4, - 2.7777500001498e-14 r2 + 8.333291666787201e-7 r - 4,  
- 1.777706668053906e-12 r2 + 6.666533334601099e-6 r - 4,  
- 2.024817758005038e-11 r2 + 2.249898752169216e-5 r - 4,  
- 1.137595747549299e-10 r2 + 5.332906682919969e-5 r - 4,  
- 4.339192840727639e-10 r2 + 1.041536466083598e-4 r - 4,  
- 1.295533521972174e-9 r2 + 1.79967602777012e-4 r - 4,  
- 3.266426827094104e-9 r2 + 2.85763312336193e-4 r - 4,  
- 7.277118895509326e-9 r2 + 4.265301541365313e-4 r - 4,  
- 1.475029730376073e-8 r2 + 6.072540099447826e-4 r - 4,  
- 2.775001355397757e-8 r2 + 8.32916765859254e-4 r - 4,  
- 4.915051879738995e-8 r2 + 0.001108495814125948 r - 4,
```

$$\begin{aligned}
& - 8.28246445511412e-8 r^2 + 0.001438963555403169 r - 4, \\
& - 1.33851622723744e-7 r^2 + 0.001829286901525728 r - 4, \\
& - 2.087442283111582e-7 r^2 + 0.0022844267788176 r - 4, \\
& - 3.156951172237287e-7 r^2 + 0.002809337632003889 r - 4, \\
& - 4.64842220857938e-7 r^2 + 0.00340896692877013 r - 4, \\
& - 6.685530482422835e-7 r^2 + 0.004088254665019914 r - 4, \\
& - 9.417277358666075e-7 r^2 + 0.004852132870879072 r - 4, \\
& - 1.30212067465563e-6 r^2 + 0.00570552511749714 r - 4, \\
& - 1.770680532972444e-6 r^2 + 0.006653346024693974 r - 4, \\
& - 2.371908484044149e-6 r^2 + 0.007700500769502183 r - 4, \\
& - 3.134234435790633e-6 r^2 + 0.008851884595653392 r - 4, \\
& - 4.090411050716832e-6 r^2 + 0.01011238232405801 r - 4, \\
& - 5.277925333300395e-6 r^2 + 0.01148686786432707 r - 4, \\
& - 6.739427552177103e-6 r^2 + 0.01298020372738531 r - 4, \\
& - 8.523177254399114e-6 r^2 + 0.01459724053922445 r - 4, \\
& - 1.068350611911921e-5 r^2 + 0.01634281655584435 r - 4, \\
& - 1.328129738824626e-5 r^2 + 0.01822175717943131 r - 4,
\end{aligned}$$

$$\begin{aligned}
& - 1.638448160192355e-5 \, r^2 + 0.02023887447582223 \, r - 4, \\
& - 2.006854835710647e-5 \, r^2 + 0.02239896669330221 \, r - 4, \\
& - 2.44170737980647e-5 \, r^2 + 0.0247068177827825 \, r - 4, \\
& - 2.952226353832265e-5 \, r^2 + 0.02716719691941122 \, r - 4, \\
& - 3.548551070434468e-5 \, r^2 + 0.02978485802565822 \, r - 4, \\
& - 4.241796878224187e-5 \, r^2 + 0.03256453929592812 \, r - 4, \\
& - 5.044113893984222e-5 \, r^2 + 0.03551096272274318 \, r - 4, \\
& - 5.968747148772726e-5 \, r^2 + 0.03862883362455022 \, r - 4, \\
& - 7.030098113418114e-5 \, r^2 + 0.04192284017519005 \, r - 4, \\
& - 8.243787568058321e-5 \, r^2 + 0.04539765293508663 \, r - 4, \\
& - 9.626719779540763e-5 \, r^2 + 0.04905792438419293 \, r - 4, \\
& - 1.11971479496896e-4 \, r^2 + 0.0529082884567475 \, r - 4, \\
& - 1.297474089664522e-4 \, r^2 + 0.05695336007788571 \, r - 4, \\
& - 1.498065093069853e-4 \, r^2 + 0.06119773470214918 \, r - 4, \\
& - 1.723758288528179e-4 \, r^2 + 0.06564598785394615 \, r - 4, \\
& - 1.976986426302469e-4 \, r^2 + 0.07030267467000223 \, r - 4, \\
& - 2.260351645605837e-4 \, r^2 + 0.07517232944384916 \, r - 4, \\
& - 2.576632699903951e-4 \, r^2 + 0.08025946517240118 \, r - 4,
\end{aligned}$$

$$\begin{aligned}
& - 2.928792281266932e-4 r^2 + 0.08556857310465876 r - 4, \\
& - 3.319984439480964e-4 r^2 + 0.09110412229258569 r - 4, \\
& - 3.753562091564763e-4 r^2 + 0.09687055914421011 r - 4, \\
& - 4.233084617271431e-4 r^2 + 0.1028723069789853 r - 4, \\
& - 4.762325536095718e-4 r^2 + 0.1091137655854627 r - 4, \\
& - 5.34528026124617e-4 r^2 + 0.1155993107813166 r - 4, \\
& - 5.986173925984417e-4 r^2 + 0.1223332939757654 r - 4, \\
& - 6.689469277678383e-4 r^2 + 0.1293200417344348 r - 4, \\
& - 7.459874634862211e-4 r^2 + 0.1365638553467041 r - 4, \\
& - 8.302351902545073e-4 r^2 + 0.1440690103955833 r - 4, \\
& - 9.222124640960191e-4 r^2 + 0.1518397563301538 r - 4, \\
& - 0.001022468618290102 r^2 + 0.1598803160406326 r - 4, \\
& - 0.001131580779474263 r^2 + 0.1681948854360815 r - 4, \\
& - 0.001250154687620788 r^2 + 0.1767876330248236 r - 4, \\
& - 0.001378825519389357 r^2 + 0.1856626994975941 r - 4, \\
& - 0.001518258714353595 r^2 + 0.1948241973134751 r - 4, \\
& - 0.001669150803595751 r^2 + 0.2042762102886525 r - 4,
\end{aligned}$$

$$\begin{aligned}
& - 0.001832230240160423 \, r^2 + 0.2140227931880401 \, r - 4, \\
& - 0.002008258230854871 \, r^2 + 0.2240679713198024 \, r - 4, \\
& - 0.002198029568880921 \, r^2 + 0.2344157401328312 \, r - 4, \\
& - 0.002402373466780307 \, r^2 + 0.2450700648172022 \, r - 4, \\
& - 0.002622154389173151 \, r^2 + 0.2560348799076578 \, r - 4, \\
& - 0.002858272884767075 \, r^2 + 0.267314088890161 \, r - 4, \\
& - 0.003111666417112067 \, r^2 + 0.2789115638115452 \, r - 4, \\
& - 0.003383310193575043 \, r^2 + 0.2908311448923173 \, r - 4, \\
& - 0.003674217992005929 \, r^2 + 0.3030766401426349 \, r - 4, \\
& - 0.003985442984566339 \, r^2 + 0.315651824981511 \, r - 4, \\
& - 0.004318078558190487 \, r^2 + 0.3285604418592752 \, r - 4, \\
& - 0.004673259131147316 \, r^2 + 0.3418061998833299 \, r - 4, \\
& - 0.005052160965172387 \, r^2 + 0.355392774447244 \, r - 4, \\
& - 0.005456002972637555 \, r^2 + 0.3693238068632171 \, r - 4, \\
& - 0.005886047518226416 \, r^2 + 0.3836029039979499 \, r - 4, \\
& - 0.006343601214583815 \, r^2 + 0.3982336379119616 \, r - 4, \\
& - 0.006830015711407966 \, r^2 + 0.4132195455023868 \, r - 4, \\
& - 0.007346688477454374 \, r^2 + 0.428564128149288 \, r - 4,
\end{aligned}$$

$$\begin{aligned}
& - 0.007895063574921807 r^2 + 0.4442708513655214 r - 4, \\
& - 0.008476632425691433 r^2 + 0.4603431444501871 r - 4, \\
& - 0.009092934568891969 r^2 + 0.4767844001457044 r - 4, \\
& - 0.009745558409264787 r^2 + 0.4935979742985375 r - 4, \\
& - 0.01043614195580549 r^2 + 0.5107871855236162 r - 4, \\
& - 0.01116637355015972 r^2 + 0.5283553148724757 r - 4, \\
& - 0.01193799258425414 r^2 + 0.5463056055051546 r - 4, \\
& - 0.01275279020664547 r^2 + 0.564641262365882 r - 4, \\
& - 0.01361261001707348 r^2 + 0.5833654518625842 r - 4, \\
& - 0.01451934874870728 r^2 + 0.6024813015502489 r - 4, \\
& - 0.01547495693757671 r^2 + 0.6219918998181712 r - 4, \\
& - 0.01648143957868493 r^2 + 0.6419002955811154 r - 4, \\
& - 0.01754085676830185 r^2 + 0.6622094979744297 r - 4, \\
& - 0.01865532433194167 r^2 + 0.6829224760531327 r - 4, \\
& - 0.01982701443753252 r^2 + 0.7040421584950102 r - 4, \\
& - 0.02105815619329058 r^2 + 0.7255714333077512 r - 4, \\
& - 0.02235103622981523 r^2 + 0.7475131475401492 r - 4,
\end{aligned}$$

$$- 0.02370799926592746 r + 0.769870106997399 r - 4]$$

```
>plot2d("-x^2+5*x-4",-5,5,-5,5):
```

3. Gambarkanlah suatu segi-4 yang diketahui keempat titik sudutnya, misalnya A, B, C, D.

- Tentukan apakah segi-4 tersebut merupakan segi-4 garis singgung

(sisinya-sisintya merupakan garis singgung lingkaran yang sama yakni lingkaran dalam segi-4 tersebut).

- Suatu segi-4 merupakan segi-4 garis singgung apabila keempat

garis bagi sudutnya bertemu di satu titik.

- Jika segi-4 tersebut merupakan segi-4 garis singgung, gambar

lingkaran dalamnya.

- Tunjukkan bahwa syarat suatu segi-4 merupakan segi-4 garis

singgung apabila hasil kali panjang sisi-sisi yang berhadapan sama.

Penyelesaian :

```
>load geometry
```

Numerical and symbolic geometry.

```
>setPlotRange(-4.5,4.5,-4.5,4.5);  
>A=[-3,-3]; plotPoint(A,"A");  
>B=[3,-3]; plotPoint(B,"B");  
>C=[3,3]; plotPoint(C,"C");  
>D=[-3,3]; plotPoint(D,"D");  
>plotSegment(A,B,"");  
>plotSegment(B,C,"");  
>plotSegment(C,D,"");  
>plotSegment(A,D,"");  
>aspect(1):  
>l=angleBisector(A,B,C);  
>m=angleBisector(B,C,D);  
>P=lineIntersection(l,m);  
>color(5); plotLine(l); plotLine(m); color(1);  
>plotPoint(P,"P");
```

Dari gambar diatas terlihat bahwa keempat garis bagi sudutnya bertemu di satu titik yaitu titik P.

```
>r=norm(P-projectToLine(P,lineThrough(A,B)));  
>plotCircle(circleWithCenter(P,r),"Lingkaran dalam segiempat ABCD");
```

Dari gambar diatas, terlihat bahwa sisi-sisinya merupakan garis singgung lingkaran yang sama yaitu lingkaran dalam segiempat.

Akan ditunjukkan bahwa hasil kali panjang sisi-sisi yang berhadapan sama.

```
>AB=norm(A-B) //panjang sisi AB
```

6

```
>CD=norm(C-D) //panjang sisi CD
```

6

```
>AD=norm(A-D) //panjang sisi AD
```

6

```
>BC=norm(B-C) //panjang sisi BC
```

6

```
>AB.CD
```

36

```
>AD.BC
```

36

Terbukti bahwa hasil kali panjang sisi-sisi yang berhadapan sama yaitu 36. Jadi dapat dipastikan bahwa segiempat tersebut merupakan segiempat garis singgung.

4. Gambarkanlah suatu ellips jika diketahui kedua titik fokusnya, misalnya P dan Q. Ingat ellips dengan fokus P dan Q adalah tempat kedudukan titik-titik yang jumlah jarak ke P dan ke Q selalu sama (konstan).

Penyelesaian :

Diketahui kedua titik fokus $P = [-1,-1]$ dan $Q = [1,-1]$

```
>P=[-1,-1]; Q=[1,-1];  
>function d1(x,y):=sqrt((x-P[1])^2+(y-P[2])^2)  
>Q=[1,-1]; function d2(x,y):=sqrt((x-P[1])^2+(y-P[2])^2)+sqrt((x-Q[1])^2+(y-Q[2])^2)  
>fcontour("d2",xmin=-2,xmax=2,ymin=-3,ymax=1,hue=1):
```


Grafik yang lebih menarik

```
>plot3d("d2",xmin=-2,xmax=2,ymin=-3,ymax=1):
```

Batasan ke garis PQ

```
>plot2d("abs(x+1)+abs(x-1)",xmin=-3,xmax=3):
```

5. Gambarkanlah suatu hiperbola jika diketahui kedua titik fokusnya, misalnya P dan Q. Ingat ellips dengan fokus P dan Q adalah tempat kedudukan titik-titik yang selisih jarak ke P dan ke Q selalu sama (konstan).

Penyelesaian :

```
>P=[-1,-1]; Q=[1,-1];  
>function d1(x,y):=sqrt((x-p[1])^2+(y-p[2])^2)  
>Q=[1,-1]; function d2(x,y):=sqrt((x-P[1])^2+(y-P[2])^2)+sqrt((x+Q[1])^2+(y+Q[2])^2)  
>fcontour("d2",xmin=-2,xmax=2,ymin=-3,ymax=1,hue=1):
```

Grafik yang lebih menarik

```
>plot3d("d2",xmin=-2,xmax=2,ymin=-3,ymax=1):  
>plot2d("abs(x+1)+abs(x-1)",xmin=-3,xmax=3):
```

Di buku catatan ini, kami mendemonstrasikan plot statistik utama, pengujian, dan distribusi di Euler.

Mari kita mulai dengan beberapa statistik deskriptif. Ini bukan pengantar statistik. Jadi, Anda mungkin memerlukan latar belakang untuk memahami detailnya.

Asumsikan pengukuran berikut. Kami ingin menghitung nilai rata-rata dan deviasi standar yang diukur.

```
>M=[1000,1004,998,997,1002,1001,998,1004,998,997]; ...  
>median(M), mean(M), dev(M),
```

```
999  
999.9  
2.72641400622
```

Kita dapat memplot plot kotak-dan-kumis untuk datanya. Dalam kasus kami, tidak ada outlier.

```
>aspect(1.75); boxplot(M):
```

Kami menghitung probabilitas suatu nilai lebih besar dari 1005, dengan asumsi nilai terukur berdistribusi normal.

Semua fungsi untuk distribusi di Euler diakhiri dengan ...dis dan menghitung distribusi probabilitas kumulatif (CPF).

$$\text{normaldis}(x,m,d) = \int_{-\infty}^x \frac{1}{d\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-m}{d}\right)^2} dt.$$

Kami mencetak hasilnya dalam % dengan akurasi 2 digit menggunakan fungsi print.

```
>print((1-normaldis(1005,mean(M),dev(M)))*100,2,unit=" %")
```

3.07 %

Untuk contoh berikutnya, kita asumsikan jumlah pria berikut dalam rentang ukuran tertentu.

```
>r=155.5:4:187.5; v=[22,71,136,169,139,71,32,8];
```

Berikut adalah alur pendistribusiannya.

```
>plot2d(r,v,a=150,b=200,c=0,d=190,bar=1,style="\ /"):
```

Kita bisa memasukkan data mentah tersebut ke dalam tabel.

Tabel adalah metode untuk menyimpan data statistik. Tabel kita harus berisi tiga kolom: Awal jangkauan, akhir jangkauan, jumlah pria dalam jangkauan.

Tabel dapat dicetak dengan header. Kami menggunakan vektor string untuk mengatur header.

```
>T:=r[1:8]' | r[2:9]' | v'; writetable(T,labc=["BB","BA","Frek"])
```

BB	BA	Frek
155.5	159.5	22
159.5	163.5	71
163.5	167.5	136
167.5	171.5	169
171.5	175.5	139
175.5	179.5	71
179.5	183.5	32
183.5	187.5	8

Jika kita memerlukan nilai rata-rata dan statistik ukuran lainnya, kita perlu menghitung titik tengah rentang tersebut. Kita bisa menggunakan dua kolom pertama tabel kita untuk ini.

Symbol "|" digunakan untuk memisahkan kolom, fungsi "writetable" digunakan untuk menulis tabel, dengan opsi "labc" untuk menentukan header kolom.

```
>(T[,1]+T[,2])/2 // the midpoint of each interval
```

```
157.5  
161.5  
165.5  
169.5  
173.5  
177.5  
181.5  
185.5
```

Namun akan lebih mudah jika menjumlahkan rentang dengan vektor $[1/2, 1/2]$.

```
>M=fold(r,[0.5,0.5])
```

```
[157.5, 161.5, 165.5, 169.5, 173.5, 177.5, 181.5, 185.5]
```

Sekarang kita dapat menghitung mean dan deviasi sampel dengan frekuensi tertentu.

```
>{m,d}=meandev(M,v); m, d,
```

```
169.901234568  
5.98912964449
```

Mari kita tambahkan distribusi nilai normal ke diagram batang di atas. Rumus distribusi normal dengan mean m dan simpangan baku d adalah:

$$y = \frac{1}{d\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2d^2}}.$$

Karena nilainya antara 0 dan 1, maka untuk memplotnya pada bar plot harus dikalikan dengan 4 kali jumlah data.

```
>plot2d("qnormal(x,m,d)*sum(v)*4", ...  
>  xmin=min(r),xmax=max(r),thickness=3,add=1):
```

Di direktori buku catatan ini Anda menemukan file dengan tabel. Data tersebut merupakan hasil survei. Berikut adalah empat baris pertama file tersebut. Datanya berasal dari buku online Jerman "Einführung in die Statistik mit R" oleh A. Handl.

```
>printfile("table.dat",4);
```

```
Could not open the file
table.dat
for reading!
Try "trace errors" to inspect local variables after errors.
printfile:
  open(filename,"r");
```

Tabel berisi 7 kolom angka atau token (string). Kami ingin membaca tabel dari file. Pertama, kami menggunakan terjemahan kami sendiri untuk tokennya.

Untuk ini, kami mendefinisikan kumpulan token. Fungsi `strtokens()` mendapatkan vektor string token dari string tertentu.

```
>mf=["m","f"]; yn=["y","n"]; ev:=strtokens("g vg m b vb");
```


Sekarang kita membaca tabel dengan terjemahan ini.

Argumen tok2, tok4 dll. adalah terjemahan dari kolom tabel. Argumen ini tidak ada dalam daftar parameter readtable(), jadi Anda perlu menyediakannya dengan ":=".

```
>{MT,hd}=readtable("table.dat",tok2:=mf,tok4:=yn,tok5:=ev,tok7:=yn);
```

```
Could not open the file
table.dat
for reading!
Try "trace errors" to inspect local variables after errors.
readtable:
  if filename!=none then open(filename,"r"); endif;
```

```
>load over statistics;
```

Untuk mencetak, kita perlu menentukan kumpulan token yang sama. Kami mencetak empat baris pertama saja.

```
>writetable(MT[1:10],labc=hd,wc=5,tok2:=mf,tok4:=yn,tok5:=ev,tok7:=yn);
```

```
MT is not a variable!
Error in:
writetable(MT[1:10],labc=hd,wc=5,tok2:=mf,tok4:=yn,tok5:=ev,to ...
^
```

Titik "." mewakili nilai-nilai, yang tidak tersedia.

Jika kita tidak ingin menentukan token yang akan diterjemahkan terlebih dahulu, kita hanya perlu menentukan, kolom mana yang berisi token dan bukan angka.

```
>ctok=[2,4,5,7]; {MT,hd,tok}=readtable("table.dat",ctok=ctok);
```

```
Could not open the file
table.dat
for reading!
Try "trace errors" to inspect local variables after errors.
readtable:
  if filename!=none then open(filename,"r"); endif;
```

Fungsi readtable() kini mengembalikan sekumpulan token.

```
>tok
```

```
Variable tok not found!
Error in:
tok ...
  ^
```

Tabel berisi entri dari file dengan token yang diterjemahkan ke dalam angka.

String khusus NA = "." diartikan sebagai "Tidak Tersedia", dan mendapatkan NAN (bukan angka) di tabel. Terjemahan ini dapat diubah dengan parameter NA, dan NAval.

```
>MT[1]
```

```
MT is not a variable!  
Error in:  
MT[1] ...  
      ^
```

Berikut isi tabel dengan nomor yang belum diterjemahkan.

```
>writetable(MT,wc=5)
```

```
Variable or function MT not found.  
Error in:  
writetable(MT,wc=5) ...  
      ^
```

Untuk kenyamanan, Anda dapat memasukkan keluaran readtable() ke dalam daftar.

```
>Table={{readtable("table.dat",ctok=ctok)}};
```

```
Could not open the file
table.dat
for reading!
Try "trace errors" to inspect local variables after errors.
readtable:
  if filename!=none then open(filename,"r"); endif;
```

Dengan menggunakan kolom token yang sama dan token yang dibaca dari file, kita dapat mencetak tabel. Kita dapat menentukan ctok, tok, dll. atau menggunakan tabel daftar.

```
>writetable(Table,ctok=ctok,wc=5);
```

```
Variable or function Table not found.
Error in:
writetable(Table,ctok=ctok,wc=5); ...
      ^
```

Fungsi `tablecol()` mengembalikan nilai kolom tabel, melewati baris apa pun dengan nilai NAN (”.” dalam file), dan indeks kolom, yang berisi nilai-nilai ini.

```
>{c,i}=tablecol(MT,[5,6]);
```

```
Variable or function MT not found.  
Error in:  
{c,i}=tablecol(MT,[5,6]); ...  
                ^
```

Kita bisa menggunakan ini untuk mengekstrak kolom dari tabel untuk tabel baru.

```
>j=[1,5,6]; writetable(MT[i,j],lab=hd[j],ctok=[2],tok=tok)
```

```
Variable or function i not found.  
Error in:  
j=[1,5,6]; writetable(MT[i,j],lab=hd[j],ctok=[2],tok=tok) ...  
                        ^
```

Tentu saja, kita perlu mengekstrak tabel itu sendiri dari daftar Tabel dalam kasus ini.

```
>MT=Table[1];
```

```
Table is not a variable!  
Error in:  
MT=Table[1]; ...  
      ^
```

Tentu saja, kita juga dapat menggunakannya untuk menentukan nilai rata-rata suatu kolom atau nilai statistik lainnya.

```
>mean(tablecol(MT,6))
```

```
Variable or function MT not found.  
Error in:  
mean(tablecol(MT,6)) ...  
      ^
```

Fungsi `getstatistics()` mengembalikan elemen dalam vektor, dan jumlahnya. Kami menerapkannya pada nilai "m" dan "f" di kolom kedua tabel kami.

```
>{xu,count}=getstatistics(tablecol(MT,2)); xu, count,
```

```
Variable or function MT not found.  
Error in:  
{xu,count}=getstatistics(tablecol(MT,2)); xu, count, ...  
^
```

Kita bisa mencetak hasilnya di tabel baru.

```
>writetable(count',labr=tok[xu])
```

```
Variable count not found!  
Error in:  
writetable(count',labr=tok[xu]) ...  
^
```

Fungsi `selecttable()` mengembalikan tabel baru dengan nilai dalam satu kolom yang dipilih dari vektor indeks. Pertama kita mencari indeks dari dua nilai kita di tabel token.

```
>v:=indexof(tok,["g","vg"])
```

```
Variable or function tok not found.  
Error in:  
v:=indexof(tok,["g","vg"]) ...  
      ^
```

Sekarang kita dapat memilih baris tabel, yang memiliki salah satu nilai `v` pada baris ke-5.

```
>MT1:=MT[selectrows(MT,5,v)]; i:=sortedrows(MT1,5);
```

```
Variable or function MT not found.  
Error in:  
MT1:=MT[selectrows(MT,5,v)]; i:=sortedrows(MT1,5); ...  
      ^
```


Sekarang kita dapat mencetak tabel, dengan nilai yang diekstraksi dan diurutkan di kolom ke-5.

```
>writetable(MT1[i],labc=hd,ctok=ctok,tok=tok,wc=7);
```

```
Variable or function i not found.  
Error in:  
writetable(MT1[i],labc=hd,ctok=ctok,tok=tok,wc=7); ...  
      ^
```

Untuk statistik selanjutnya, kami ingin menghubungkan dua kolom tabel. Jadi kita ekstrak kolom 2 dan 4 dan urutkan tabelnya.

```
>i=sortedrows(MT,[2,4]); ...  
> writetable(tablecol(MT[i],[2,4]))',ctok=[1,2],tok=tok)
```

```
Variable or function MT not found.  
Error in:  
i=sortedrows(MT,[2,4]); writetable(tablecol(MT[i],[2,4]))',c ...  
      ^
```

Dengan `getstatistics()`, kita juga bisa menghubungkan jumlah dalam dua kolom tabel satu sama lain.

```
>MT24=tablecol(MT,[2,4]); ...  
>{xu1,xu2,count}=getstatistics(MT24[1],MT24[2]); ...  
>writetable(count,labr=tok[xu1],labc=tok[xu2])
```

```
Variable or function MT not found.  
Error in:  
MT24=tablecol(MT,[2,4]); {xu1,xu2,count}=getstatistics(MT24[1] ...  
~
```

Sebuah tabel dapat ditulis ke file.

```
>filename="test.dat"; ...  
>writetable(count,labr=tok[xu1],labc=tok[xu2],file=filename);
```

```
Variable or function count not found.  
Error in:  
filename="test.dat"; writetable(count,labr=tok[xu1],labc=tok[x ...  
~
```

Kemudian kita bisa membaca tabel dari file tersebut.

```
>{MT2,hd,tok2,hdr}=readtable(filename,>clabs,>rlabs); ...  
>writetable(MT2,labr=hdr,labc=hd)
```

```
Could not open the file  
test.dat  
for reading!  
Try "trace errors" to inspect local variables after errors.  
readtable:  
    if filename!=none then open(filename,"r"); endif;
```

Dan hapus file tersebut.

```
>fileremove(filename);
```

Dengan `plot2d`, ada metode yang sangat mudah untuk memplot sebaran data eksperimen.

```
>p=normal(1,1000); //1000 random normal-distributed sample p
>plot2d(p,distribution=20,style="\"); // plot the random sample p
>plot2d("qnormal(x,0,1)",add=1): // add the standard normal distribution plot
```

Perlu diperhatikan perbedaan antara bar plot (sampel) dan kurva normal (distribusi sebenarnya). Masukkan kembali ketiga perintah untuk melihat hasil pengambilan sampel lainnya.

Berikut adalah perbandingan 10 simulasi dari 1000 nilai terdistribusi normal menggunakan apa yang disebut plot kotak. Plot ini menunjukkan median, kuartil 25% dan 75%, nilai minimal dan maksimal, serta outlier.

```
>p=normal(10,1000); boxplot(p):
```

Untuk menghasilkan bilangan bulat acak, Euler memiliki `intrandom`. Mari kita simulasikan lemparan dadu dan plot distribusinya.

Kita menggunakan fungsi `getmultiplicities(v,x)`, yang menghitung seberapa sering elemen v muncul di x . Kemudian kita plot hasilnya menggunakan `kolomplot()`.

```
>k=intrandom(1,6000,6); ...  
>columnplot(getmultiplicities(1:6,k)); ...  
>ygrid(1000,color=red):
```

Meskipun `intrandom(n,m,k)` mengembalikan bilangan bulat yang terdistribusi secara seragam dari 1 hingga k, distribusi bilangan bulat lainnya dapat digunakan dengan `randpint()`.

Dalam contoh berikut, probabilitas untuk 1,2,3 masing-masing adalah 0,4,0.1,0.5.

```
>randpint(1,1000,[0.4,0.1,0.5]); getmultiplicities(1:3,%)
```

```
[378, 102, 520]
```

Euler dapat menghasilkan nilai acak dari lebih banyak distribusi. Lihat referensinya.

Misalnya, kita mencoba distribusi eksponensial. Variabel acak kontinu X dikatakan berdistribusi eksponensial, jika PDF-nya diberikan oleh

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0,$$

dengan parameter

$$\lambda = \frac{1}{\mu}, \quad \mu \text{ adalah mean, dan dilambangkan dengan } X \sim \text{Exponential}(\lambda).$$

```
>plot2d(randexponential(1,1000,2),>distribution):
```

Untuk banyak distribusi, Euler dapat menghitung fungsi distribusi dan inversnya.

```
>plot2d("normaldis",-4,4):
```

Berikut ini adalah salah satu cara untuk memplot kuantil.

```
>plot2d("qnormal(x,1,1.5)",-4,6); ...  
>plot2d("qnormal(x,1,1.5)",a=2,b=5,>add,>filled):
```

$$\text{normaldis}(x,m,d) = \int_{-\infty}^x \frac{1}{d\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-m}{d}\right)^2} dt.$$

Peluang berada di kawasan hijau adalah sebagai berikut.

```
>normaldis(5,1,1.5)-normaldis(2,1,1.5)
```

0.248662156979

Ini dapat dihitung secara numerik dengan integral berikut.

$$\int_2^5 \frac{1}{1.5\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-1}{1.5}\right)^2} dx.$$

```
>gauss("qnormal(x,1,1.5)",2,5)
```

```
0.248662156979
```

Mari kita bandingkan distribusi binomial dengan distribusi normal yang mean dan deviasinya sama. Fungsi `invbindis()` menyelesaikan interpolasi linier antara nilai integer.

```
>invbindis(0.95,1000,0.5), invnormaldis(0.95,500,0.5*sqrt(1000))
```

```
525.516721219  
526.007419394
```

Fungsi `qdis()` adalah kepadatan distribusi chi-kuadrat. Seperti biasa, Euler memetakan vektor ke fungsi ini. Dengan demikian kita mendapatkan plot semua distribusi chi-kuadrat dengan derajat 5 sampai 30 dengan mudah dengan cara berikut.

```
>plot2d("qchidis(x,(5:5:50)')",0,50):
```

Euler memiliki fungsi akurat untuk mengevaluasi distribusi. Mari kita periksa `chidis()` dengan `integral`.

Penamaannya mencoba untuk konsisten. Misalnya.,

- distribusi chi-kuadratnya adalah `chidis()`,
- fungsi kebalikannya adalah `invchidis()`,
- kepadatannya adalah `qchidis()`.

Pelengkap distribusi (ekor atas) adalah `chicdis()`.

```
>chidis(1.5,2), integrate("qchidis(x,2)",0,1.5)
```

```
0.527633447259
```

```
0.527633447259
```


Distribusi Diskrit

Untuk menentukan distribusi diskrit Anda sendiri, Anda dapat menggunakan metode berikut. Pertama kita atur fungsi distribusinya.

```
>wd = 0|((1:6)+[-0.01,0.01,0,0,0,0])/6
```

```
[0, 0.165, 0.335, 0.5, 0.666667, 0.833333, 1]
```

Artinya dengan probabilitas $wd[i+1]-wd[i]$ kita menghasilkan nilai acak i .

Ini hampir merupakan distribusi yang seragam. Mari kita tentukan generator nomor acak untuk ini. Fungsi `find(v,x)` mencari nilai x pada vektor v . Fungsi ini juga berfungsi untuk vektor x .

```
>function wrongdice (n,m) := find(wd,random(n,m))
```

Kesalahannya sangat halus sehingga kita hanya melihatnya dengan banyak iterasi.

```
>columnplot(getmultiplicities(1:6,wrongdice(1,1000000))):
```

Berikut adalah fungsi sederhana untuk memeriksa keseragaman distribusi nilai 1...K dalam v. Kita menerima hasilnya, jika untuk semua frekuensi

$$\left| f_i - \frac{1}{K} \right| < \frac{\delta}{\sqrt{n}}$$

```
>function checkrandom (v, delta=1) ...
```

```
    K=max(v); n=cols(v);  
    fr=getfrequencies(v,1:K);  
    return max(fr/n-1/K)<delta/sqrt(n);  
endfunction
```

Memang fungsinya menolak distribusi seragam.

```
>checkrandom(wrongdice(1,1000000))
```

0

Dan ia menerima generator acak bawaan.

```
>checkrandom(intrandom(1,1000000,6))
```

1

Kita dapat menghitung distribusi binomial. Pertama ada `binomsum()`, yang mengembalikan probabilitas i atau kurang hit dari n percobaan.

```
>bindis(410,1000,0.4)
```

```
0.751401349654
```

Fungsi Beta terbalik digunakan untuk menghitung interval kepercayaan Clopper-Pearson untuk parameter p . Tingkat defaultnya adalah alfa.

Arti dari interval ini adalah jika p berada di luar interval, hasil pengamatan 410 dalam 1000 jarang terjadi.

```
>clopperpearson(410,1000)
```

```
[0.37932, 0.441212]
```

Perintah berikut adalah cara langsung untuk mendapatkan hasil di atas. Namun untuk n yang besar, penjumlahan langsungnya tidak akurat dan lambat.

```
>p=0.4; i=0:410; n=1000; sum(bin(n,i)*p^i*(1-p)^(n-i))
```

```
0.751401349655
```

`invbinsum()` menghitung kebalikan dari `binomsum()`.

```
>invbindis(0.75,1000,0.4)
```

409.932733047

Di Bridge, kami mengasumsikan 5 kartu beredar (dari 52) di dua tangan (26 kartu). Mari kita hitung probabilitas distribusi yang lebih buruk dari 3:2 (misalnya 0:5, 1:4, 4:1, atau 5:0).

```
>2*hypergeomsum(1,5,13,26)
```

0.321739130435

Ada juga simulasi distribusi multinomial.

```
>randmultinomial(10,1000,[0.4,0.1,0.5])
```

381	100	519
376	91	533
417	80	503
440	94	466
406	112	482
408	94	498
395	107	498

399	96	505
428	87	485
400	99	501

Merencanakan Data

Untuk memetakan data, kami mencoba hasil pemilu Jerman sejak tahun 1990, diukur dalam jumlah kursi.

```
>BW := [ ...  
>1990,662,319,239,79,8,17; ...  
>1994,672,294,252,47,49,30; ...  
>1998,669,245,298,43,47,36; ...  
>2002,603,248,251,47,55,2; ...  
>2005,614,226,222,61,51,54; ...  
>2009,622,239,146,93,68,76; ...  
>2013,631,311,193,0,63,64];
```

Untuk pesta, kami menggunakan rangkaian nama.

```
>P:=["CDU/CSU", "SPD", "FDP", "Gr", "Li"];
```

Mari kita cetak persentasenya dengan baik.

Pertama kita mengekstrak kolom yang diperlukan. Kolom 3 sampai 7 adalah kursi masing-masing partai, dan kolom 2 adalah jumlah kursi seluruhnya. kolom adalah tahun pemilihan.

```
>BT:=BW[,3:7]; BT:=BT/sum(BT); YT:=BW[,1]';
```

Kemudian statistiknya kita cetak dalam bentuk tabel. Kami menggunakan nama sebagai header kolom, dan tahun sebagai header untuk baris. Lebar default untuk kolom adalah wc=10, tetapi kami lebih memilih keluaran yang lebih padat. Kolom akan diperluas untuk label kolom, jika perlu.

```
>writetable(BT*100,wc=6,dc=0,>fixed,labc=P,labr=YT)
```

	CDU/CSU	SPD	FDP	Gr	Li
1990	48	36	12	1	3
1994	44	38	7	7	4
1998	37	45	6	7	5
2002	41	42	8	9	0
2005	37	36	10	8	9
2009	38	23	15	11	12
2013	49	31	0	10	10

Perkalian matriks berikut ini menjumlahkan persentase dua partai besar yang menunjukkan bahwa partai-partai kecil berhasil memperoleh suara di parlemen hingga tahun 2009.

```
>BT1:=(BT.[1;1;0;0;0])'*100
```

```
[84.29, 81.25, 81.1659, 82.7529, 72.9642, 61.8971, 79.8732]
```

Ada juga plot statistik sederhana. Kami menggunakannya untuk menampilkan garis dan titik secara bersamaan. Alternatifnya adalah memanggil plot2d dua kali dengan >add.

```
>statplot(YT,BT1,"b"):
```

Tentukan beberapa warna untuk setiap pesta.

```
>CP:=[rgb(0.5,0.5,0.5),red,yellow,green,rgb(0.8,0,0)];
```


Sekarang kita bisa memplot hasil pemilu 2009 dan perubahannya menjadi satu plot dengan menggunakan gambar. Kita dapat menambahkan vektor kolom ke setiap plot.

```
>figure(2,1); ...  
>figure(1); columnspot(BW[6,3:7],P,color=CP); ...  
>figure(2); columnspot(BW[6,3:7]-BW[5,3:7],P,color=CP); ...  
>figure(0):
```

Plot data menggabungkan deretan data statistik dalam satu plot.

```
>J:=BW[,1]'; DP:=BW[,3:7]'; ...  
>dataplot(YT,BT',color=CP); ...  
>labelbox(P,colors=CP,styles="[]",>points,w=0.2,x=0.3,y=0.4):
```

Plot kolom 3D memperlihatkan baris data statistik dalam bentuk kolom. Kami memberikan label untuk baris dan kolom. sudut adalah sudut pandang.

```
>columnspot3d(BT,scols=P,srows=YT, ...  
> angle=30°,ccols=CP):
```

Representasi lainnya adalah plot mosaik. Perhatikan bahwa kolom plot mewakili kolom matriks di sini. Karena panjang label CDU/CSU, kami mengambil jendela yang lebih kecil dari biasanya.

```
>shrinkwindow(>smaller); ...  
>mosaicplot(BT',srows=YT,scols=P,color=CP,style="#"); ...  
>shrinkwindow():
```

Kita juga bisa membuat diagram lingkaran. Karena hitam dan kuning membentuk koalisi, kami menyusun ulang elemen-elemennya.

```
>i=[1,3,5,4,2]; piechart(BW[6,3:7][i],color=CP[i],lab=P[i]):
```

Ini adalah jenis plot lainnya.

```
>starplot(normal(1,10)+4,lab=1:10,>rays):
```

Beberapa plot di plot2d bagus untuk statika. Berikut adalah plot impuls dari data acak, terdistribusi secara seragam di $[0,1]$.

```
>plot2d(makeimpulse(1:10,random(1,10)),>bar):
```

Namun untuk data yang terdistribusi secara eksponensial, kita mungkin memerlukan plot logaritmik.

```
>logimpulseplot(1:10,-log(random(1,10))*10):
```

Fungsi Columnplot() lebih mudah digunakan, karena hanya memerlukan vektor nilai. Selain itu, ia dapat mengatur labelnya ke apa pun yang kita inginkan, kami telah mendemonstrasikannya di tutorial ini.

Ini adalah aplikasi lain, di mana kita menghitung karakter dalam sebuah kalimat dan membuat statistik.

```
>v=strtochar("the quick brown fox jumps over the lazy dog"); ...  
>w=ascii("a"):ascii("z"); x=getmultiplicities(w,v); ...  
>cw=[]; for k=w; cw=cw|char(k); end; ...  
>columnplot(x,lab=cw,width=0.05):
```

Dimungkinkan juga untuk mengatur sumbu secara manual.

```
>n=10; p=0.4; i=0:n; x=bin(n,i)*p^i*(1-p)^(n-i); ...  
>columnplot(x,lab=i,width=0.05,<frame,<grid); ...  
>yaxis(0,0:0.1:1,style="->",>left); xaxis(0,style="."); ...  
>label("p",0,0.25), label("i",11,0); ...  
>textbox(["Binomial distribution","with p=0.4"]):
```

Berikut ini cara memplot frekuensi bilangan dalam suatu vektor.

Kami membuat vektor bilangan acak bilangan bulat 1 hingga 6.

```
>v:=inrandom(1,10,10)
```

```
[8, 5, 8, 8, 6, 8, 8, 3, 5, 5]
```

Kemudian ekstrak nomor unik di v.

```
>vu:=unique(v)
```

```
[3, 5, 6, 8]
```

Dan plot frekuensi dalam plot kolom.

```
>columnplot(getmultiplicities(vu,v),lab=vu,style="/"):
```

Kami ingin mendemonstrasikan fungsi distribusi nilai empiris.

```
>x=normal(1,20);
```

Fungsi `empdist(x,vs)` memerlukan array nilai yang diurutkan. Jadi kita harus mengurutkan `x` sebelum kita dapat menggunakannya.

```
>xs=sort(x);
```

Kemudian kita plot distribusi empiris dan beberapa batang kepadatan ke dalam satu plot. Alih-alih plot batang untuk distribusi kali ini kami menggunakan plot gigi gergaji.

```
>figure(2,1); ...  
>figure(1); plot2d("empdist",-4,4;xs); ...  
>figure(2); plot2d(histo(x,v=-4:0.2:4,<bar)); ...  
>figure(0):
```

Plot sebar mudah dilakukan di Euler dengan plot titik biasa. Grafik berikut menunjukkan bahwa X dan $X+Y$ jelas berkorelasi positif.

```
>x=normal(1,100); plot2d(x,x+rotright(x),>points,style=".."):
```

Seringkali kita ingin membandingkan dua sampel dengan distribusi yang berbeda. Hal ini dapat dilakukan dengan plot kuantil-kuantil.

Untuk pengujiannya, kami mencoba distribusi student-t dan distribusi eksponensial.

```
>x=randt(1,1000,5); y=randnormal(1,1000,mean(x),dev(x)); ...  
>plot2d("x",r=6,style="--",yl="normal",xl="student-t",>vertical); ...  
>plot2d(sort(x),sort(y),>points,color=red,style="x",>add):
```

Plot tersebut dengan jelas menunjukkan bahwa nilai terdistribusi normal cenderung lebih kecil di ujung ekstrim.

Jika kita mempunyai dua distribusi yang ukurannya berbeda, kita dapat memperluas distribusi yang lebih kecil atau mengecilkan distribusi yang lebih besar. Fungsi berikut ini baik untuk keduanya. Dibutuhkan nilai median dengan persentase antara 0 dan 1.

```
>function medianexpand (x,n) := median(x,p=linspace(0,1,n-1));
```

Mari kita bandingkan dua distribusi yang sama.

```
>x=random(1000); y=random(400); ...  
>plot2d("x",0,1,style="--"); ...  
>plot2d(sort(medianexpand(x,400)),sort(y),>points,color=red,style="x",>add):
```

Regresi dan Korelasi

Regresi linier dapat dilakukan dengan fungsi `polyfit()` atau berbagai fungsi fit.

Sebagai permulaan kita menemukan garis regresi untuk data univariat dengan `polyfit(x,y,1)`.

```
>x=1:10; y=[2,3,1,5,6,3,7,8,9,8]; writetable(x'|y',labc=["x","y"])
```

x	y
1	2
2	3
3	1
4	5
5	6
6	3
7	7
8	8
9	9
10	8

Kami ingin membandingkan kecocokan yang tidak berbobot dan berbobot. Pertama koefisien kecocokan linier.

```
>p=polyfit(x,y,1)
```

```
[0.733333, 0.812121]
```

Sekarang koefisien dengan bobot yang menekankan nilai terakhir.

```
>w &= "exp(-(x-10)^2/10)"; pw=polyfit(x,y,1,w=w(x))
```

```
[4.71566, 0.38319]
```

Kami memasukkan semuanya ke dalam satu plot untuk titik dan garis regresi, dan untuk bobot yang digunakan.

```
>figure(2,1); ...  
>figure(1); statplot(x,y,"b",xl="Regression"); ...  
> plot2d("evalpoly(x,p)",>add,color=blue,style="--"); ...  
> plot2d("evalpoly(x,pw)",5,10,>add,color=red,style="--"); ...  
>figure(2); plot2d(w,1,10,>filled,style="/",fillcolor=red,xl=w); ...  
>figure(0):
```

Contoh lain kita membaca survei siswa, usia mereka, usia orang tua mereka dan jumlah saudara kandung dari sebuah file.

Tabel ini berisi "m" dan "f" di kolom kedua. Kami menggunakan variabel tok2 untuk mengatur terjemahan yang tepat alih-alih membiarkan readtable() mengumpulkan terjemahannya.

```
>{MS,hd}:=readtable("table1.dat",tok2:=["m","f"]); ...  
>writetable(MS,labc=hd,tok2:=["m","f"]);
```



```
Could not open the file
table1.dat
for reading!
Try "trace errors" to inspect local variables after errors.
readtable:
    if filename!=none then open(filename,"r"); endif;
```

Bagaimana usia bergantung satu sama lain? Kesan pertama muncul dari plot sebar berpasangan.

```
>scatterplots(tablecol(MS,3:5),hd[3:5]):
```

```
Variable or function MS not found.
Error in:
scatterplots(tablecol(MS,3:5),hd[3:5]): ...
                        ^
```

Jelas terlihat bahwa usia ayah dan ibu saling bergantung satu sama lain. Mari kita tentukan dan plot garis regresinya.

```
>cs:=MS[,4:5]'; ps:=polyfit(cs[1],cs[2],1)
```

```
MS is not a variable!
Error in:
cs:=MS[,4:5]'; ps:=polyfit(cs[1],cs[2],1) ...
                        ^
```

Ini jelas merupakan model yang salah. Garis regresinya adalah $s=17+0,74t$, dengan t adalah umur ibu dan s adalah umur ayah. Perbedaan usia mungkin sedikit bergantung pada usia, tapi tidak terlalu banyak. Sebaliknya, kami mencurigai fungsi seperti $s=a+t$. Maka a adalah mean dari $s-t$. Ini adalah perbedaan usia rata-rata antara ayah dan ibu.

```
>da:=mean(cs[2]-cs[1])
```

```
cs is not a variable!  
Error in:  
da:=mean(cs[2]-cs[1]) ...  
          ^
```

Mari kita plot ini menjadi satu plot sebar.

```
>plot2d(cs[1],cs[2],>points); ...  
>plot2d("evalpoly(x,ps)",color=red,style=".",>add); ...  
>plot2d("x+da",color=blue,>add):
```

```
cs is not a variable!  
Error in:  
plot2d(cs[1],cs[2],>points); plot2d("evalpoly(x,ps)",color=re ...  
          ^
```

Berikut adalah plot kotak dari dua zaman tersebut. Ini hanya menunjukkan, bahwa usianya berbeda-beda.

```
>boxplot(cs,["mothers","fathers"]):
```

```
Variable or function cs not found.  
Error in:  
boxplot(cs,["mothers","fathers"]): ...  
^
```

Menariknya, perbedaan median tidak sebesar perbedaan mean.

```
>median(cs[2])-median(cs[1])
```

```
cs is not a variable!  
Error in:  
median(cs[2])-median(cs[1]) ...  
^
```

Koefisien korelasi menunjukkan korelasi positif.

```
>correl(cs[1],cs[2])
```

```
cs is not a variable!  
Error in:  
correl(cs[1],cs[2]) ...  
      ^
```

Korelasi pangkat merupakan ukuran keteraturan yang sama pada kedua vektor. Hal ini juga cukup positif.

```
>rankcorrel(cs[1],cs[2])
```

```
cs is not a variable!  
Error in:  
rankcorrel(cs[1],cs[2]) ...  
      ^
```

Membuat Fungsi baru

Tentu saja, bahasa EMT dapat digunakan untuk memprogram fungsi-fungsi baru. Misalnya, kita mendefinisikan fungsi skewness.

$$\text{sk}(x) = \frac{\sqrt{n} \sum_i (x_i - m)^3}{(\sum_i (x_i - m)^2)^{3/2}}$$

dimana m adalah mean dari x .

```
>function skew (x:vector) ...  
  
    m=mean(x);  
    return sqrt(cols(x))*sum((x-m)^3)/(sum((x-m)^2))^(3/2);  
endfunction
```

Seperti yang Anda lihat, kita dapat dengan mudah menggunakan bahasa matriks untuk mendapatkan implementasi yang sangat singkat dan efisien. Mari kita coba fungsi ini.

```
>data=normal(20); skew(normal(10))
```

```
-0.198710316203
```

Berikut adalah fungsi lainnya, yang disebut koefisien skewness Pearson.

```
>function skew1 (x) := 3*(mean(x)-median(x))/dev(x)  
>skew1(data)
```

-0.0801873249135

Simulasi Monte Carlo

Euler dapat digunakan untuk mensimulasikan kejadian acak. Kita telah melihat contoh sederhana di atas. Ini satu lagi, yang mensimulasikan 1000 kali lemparan 3 dadu, dan menanyakan pembagian jumlahnya.

```
>ds:=sum(intrandom(1000,3,6))'; fs=getmultiplicities(3:18,ds)
```

```
[5, 17, 35, 44, 75, 97, 114, 116, 143, 116, 104, 53, 40,  
22, 13, 6]
```

Kita bisa merencanakannya sekarang.

```
>columnspot(fs,lab=3:18):
```

Untuk menentukan distribusi yang diharapkan tidaklah mudah. Kami menggunakan rekursi tingkat lanjut untuk ini.

Fungsi berikut menghitung banyaknya cara bilangan k dapat direpresentasikan sebagai jumlah dari n bilangan dalam rentang 1 sampai m. Ia bekerja secara rekursif dengan cara yang jelas.

```
>function map countways (k; n, m) ...  
  
    if n==1 then return k>=1 && k<=m  
    else  
        sum=0;  
        loop 1 to m; sum=sum+countways(k-#,n-1,m); end;  
        return sum;  
    end;  
endfunction
```

Berikut ini hasil dari tiga kali lemparan dadu.

```
>countways(5:25,5,5)
```

```
[1, 5, 15, 35, 70, 121, 185, 255, 320, 365, 381, 365, 320,  
255, 185, 121, 70, 35, 15, 5, 1]
```

```
>cw=countways(3:18,3,6)
```



```
[1, 3, 6, 10, 15, 21, 25, 27, 27, 25, 21, 15, 10, 6, 3, 1]
```

Kami menambahkan nilai yang diharapkan ke plot.

```
>plot2d(cw/6^3*1000,>add); plot2d(cw/6^3*1000,>points,>add):
```

Untuk simulasi lain, deviasi nilai rata-rata dari n variabel acak berdistribusi normal 0-1 adalah $1/\sqrt{n}$.

```
>longformat; 1/sqrt(10)
```

```
0.316227766017
```

Mari kita periksa ini dengan simulasi. Kita hasilkan 10000 kali 10 vektor acak.

```
>M=normal(10000,10); dev(mean(M)')
```

```
0.319493614817
```

```
>plot2d(mean(M)',>distribution):
```

Median dari 10 bilangan acak berdistribusi normal 0-1 memiliki deviasi yang lebih besar.

```
>dev(median(M)')
```

```
0.374460271535
```

Karena kita dapat dengan mudah menghasilkan lintasan acak, kita dapat mensimulasikan proses Wiener. Kita mengambil 1000 langkah dari 1000 proses. Kemudian kita memetakan deviasi standar dan rata-rata langkah ke-n dari proses ini bersama dengan nilai yang diharapkan dalam warna merah.

```
>n=1000; m=1000; M=cumsum(normal(n,m)/sqrt(m)); ...  
>t=(1:n)/n; figure(2,1); ...  
>figure(1); plot2d(t,mean(M')'); plot2d(t,0,color=red,>add); ...  
>figure(2); plot2d(t,dev(M')'); plot2d(t,sqrt(t),color=red,>add); ...  
>figure(0):
```

Pengujian

Pengujian merupakan alat penting dalam statistik. Dalam Euler, banyak pengujian yang diterapkan. Semua pengujian ini menghasilkan galat yang kita terima jika kita menolak hipotesis nol.

Sebagai contoh, kita menguji lemparan dadu untuk distribusi seragam. Pada 600 lemparan, kita memperoleh nilai berikut, yang kita masukkan ke dalam uji chi-kuadrat.

```
>chitest([90,103,114,101,103,89],dup(100,6)')
```

```
0.498830517952
```

Uji chi-square juga memiliki modus, yang menggunakan simulasi Monte Carlo untuk menguji statistik. Hasilnya harus hampir sama. Parameter `>p` menginterpretasikan vektor `y` sebagai vektor probabilitas.

```
>chitest([90,103,114,101,103,89],dup(1/6,6)',>p,>montecarlo)
```

```
0.526
```

Kesalahan ini terlalu besar. Jadi kita tidak dapat menolak distribusi seragam. Ini tidak membuktikan bahwa dadu kita adil. Namun, kita tidak dapat menolak hipotesis kita.

Selanjutnya, kita menghasilkan 1000 lemparan dadu menggunakan generator angka acak, dan melakukan pengujian yang sama.

```
>n=1000; t=random([1,n*6]); chitest(count(t*6,6),dup(n,6)')
```

0.528028118442

Mari kita uji nilai rata-rata 100 dengan uji-t.

```
>s=200+normal([1,100])*10; ...  
>ttest(mean(s),dev(s),100,200)
```

0.0218365848476

Fungsi ttest() memerlukan nilai rata-rata, deviasi, jumlah data, dan nilai rata-rata yang akan diuji.

Sekarang mari kita periksa dua pengukuran untuk nilai rata-rata yang sama. Kita tolak hipotesis bahwa keduanya memiliki nilai rata-rata yang sama, jika hasilnya $< 0,05$.

```
>tcomparedata(normal(1,10),normal(1,10))
```

0.38722000942

Jika kita menambahkan bias pada satu distribusi, kita akan mendapatkan lebih banyak penolakan. Ulangi simulasi ini beberapa kali untuk melihat efeknya.

```
>tcomparedata(normal(1,10),normal(1,10)+2)
```

5.60009101758e-07

Pada contoh berikutnya, kita buat 20 lemparan dadu acak sebanyak 100 kali dan hitung angka-angka yang ada di dalamnya. Rata-rata harus ada $20/6=3,3$ angka.

```
>R=random(100,20); R=sum(R*6<=1)'; mean(R)
```

3.28

Sekarang kita bandingkan jumlah angka satu dengan distribusi binomial. Pertama kita gambarkan distribusi angka satu.

```
>plot2d(R,distribution=max(R)+1,even=1,style="\/"):
>t=count(R,21);
```

Lalu kami hitung nilai yang diharapkan.

```
>n=0:20; b=bin(20,n)*(1/6)^n*(5/6)^(20-n)*100;
```

Kita harus mengumpulkan beberapa angka untuk mendapatkan kategori yang cukup besar.

```
>t1=sum(t[1:2])|t[3:7]|sum(t[8:21]); ...  
>b1=sum(b[1:2])|b[3:7]|sum(b[8:21]);
```

Uji chi-kuadrat menolak hipotesis bahwa distribusi kami adalah distribusi binomial, jika hasilnya $< 0,05$.

```
>chitest(t1,b1)
```

0.53921579764

Contoh berikut berisi hasil dari dua kelompok orang (misalnya pria dan wanita) yang memilih satu dari enam partai.

```
>A=[23,37,43,52,64,74;27,39,41,49,63,76]; ...  
> writetable(A,wc=6,labr=["m","f"],labc=1:6)
```

	1	2	3	4	5	6
m	23	37	43	52	64	74
f	27	39	41	49	63	76

Kami ingin menguji independensi suara dari jenis kelamin. Uji tabel χ^2 melakukan hal ini. Hasilnya terlalu besar untuk menolak independensi. Jadi, kami tidak dapat mengatakan, apakah pemungutan suara bergantung pada jenis kelamin dari data ini.

```
>tabletest(A)
```

0.990701632326

Berikut ini adalah tabel yang diharapkan, jika kita mengasumsikan frekuensi pemungutan suara yang diamati.

```
>writetable(expectedtable(A),wc=6,dc=1,labr=["m","f"],labc=1:6)
```

	1	2	3	4	5	6
m	24.9	37.9	41.9	50.3	63.3	74.7
f	25.1	38.1	42.1	50.7	63.7	75.3

Kita dapat menghitung koefisien kontingensi yang dikoreksi. Karena sangat mendekati 0, kita simpulkan bahwa pemungutan suara tidak bergantung pada jenis kelamin.

```
>contingency(A)
```

0.0427225484717

Beberapa Pengujian Lainnya

Selanjutnya, kami menggunakan analisis varians (uji F) untuk menguji tiga sampel data berdistribusi normal untuk nilai rata-rata yang sama. Metode ini disebut ANOVA (analisis varians). Dalam Euler, fungsi `varanalysis()` digunakan.

```
>x1=[109,111,98,119,91,118,109,99,115,109,94]; mean(x1),
```

```
106.545454545
```

```
>x2=[120,124,115,139,114,110,113,120,117]; mean(x2),
```

```
119.111111111
```

```
>x3=[120,112,115,110,105,134,105,130,121,111]; mean(x3)
```

```
116.3
```

```
>varanalysis(x1,x2,x3)
```

```
0.0138048221371
```

Artinya, kita menolak hipotesis nilai rata-rata yang sama. Kita melakukan ini dengan probabilitas kesalahan sebesar 1,3%.

Ada juga uji median, yang menolak sampel data dengan distribusi rata-rata yang berbeda dengan menguji median sampel gabungan.

```
>a=[56,66,68,49,61,53,45,58,54];  
>b=[72,81,51,73,69,78,59,67,65,71,68,71];  
>mediantest(a,b)
```

```
0.0241724220052
```

Uji kesetaraan lainnya adalah uji peringkat. Uji peringkat jauh lebih tajam daripada uji median.

```
>ranktest(a,b)
```

```
0.00199969612469
```

Dalam contoh berikut, kedua distribusi memiliki rata-rata yang sama.

```
>ranktest(random(1,100),random(1,50)*3-1)
```

```
0.129608141484
```

Sekarang, mari kita coba simulasikan dua perawatan a dan b yang diterapkan pada orang yang berbeda.

```
>a=[8.0,7.4,5.9,9.4,8.6,8.2,7.6,8.1,6.2,8.9];  
>b=[6.8,7.1,6.8,8.3,7.9,7.2,7.4,6.8,6.8,8.1];
```

Uji signum memutuskan, apakah a lebih baik dari b.

```
>signtest(a,b)
```

```
0.0546875
```

Ini adalah kesalahan yang sangat besar. Kita tidak dapat menolak bahwa a sama baiknya dengan b.
Uji Wilcoxon lebih tajam daripada uji ini, tetapi bergantung pada nilai kuantitatif perbedaannya.

```
>wilcoxon(a,b)
```

```
0.0296680599405
```

Mari kita coba dua pengujian lagi menggunakan seri yang dihasilkan.

```
>wilcoxon(normal(1,20),normal(1,20)-1)
```

0.0068706451766

```
>wilcoxon(normal(1,20),normal(1,20))
```

0.275145971064

Angka Acak

Berikut ini adalah pengujian untuk generator angka acak. Euler menggunakan generator yang sangat bagus, jadi kita tidak perlu mengharapkan masalah apa pun.

Pertama-tama kita menghasilkan sepuluh juta angka acak dalam $[0,1]$.

```
>n:=10000000; r:=random(1,n);
```

Berikutnya kita hitung jarak antara dua angka kurang dari 0,05.

```
>a:=0.05; d:=differences(nonzeros(r<a));
```

Terakhir, kami memplot berapa kali setiap jarak terjadi, dan membandingkannya dengan nilai yang diharapkan.

```
>m=getmultiplicities(1:100,d); plot2d(m); ...  
> plot2d("n*(1-a)^(x-1)*a^2",color=red,>add):
```

Hapus data.

```
>remvalue n;
```

Pendahuluan bagi Pengguna Proyek R

Jelas, EMT tidak bersaing dengan R sebagai paket statistik. Akan tetapi, ada banyak prosedur dan fungsi statistik yang tersedia di EMT juga. Jadi, EMT dapat memenuhi kebutuhan dasar. Lagi pula, EMT dilengkapi dengan paket numerik dan sistem aljabar komputer.

Buku catatan ini ditujukan bagi Anda yang sudah familier dengan R, tetapi perlu mengetahui perbedaan sintaksis EMT dan R. Kami mencoba memberikan gambaran umum tentang hal-hal yang jelas dan kurang jelas yang perlu Anda ketahui.

Selain itu, kami melihat cara untuk bertukar data antara kedua sistem.

Harap dicatat bahwa ini adalah pekerjaan yang masih dalam tahap pengerjaan.

Sintaksis

Dasar

Hal pertama yang Anda pelajari di R adalah membuat vektor. Dalam EMT, perbedaan utamanya adalah operator : dapat mengambil ukuran langkah. Selain itu, operator ini memiliki daya pengikatan yang rendah.

```
>n=10; 0:n/20:n-1
```

```
[0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5,  
7, 7.5, 8, 8.5, 9]
```

Fungsi `c()` tidak ada. Dimungkinkan untuk menggunakan vektor guna menggabungkan berbagai hal.

Contoh berikut ini, seperti banyak contoh lainnya, berasal dari "Interoduction to R" yang disertakan dalam proyek R. Jika Anda membaca PDF ini, Anda akan menemukan bahwa saya mengikuti alurnya dalam tutorial ini.

```
>x=[10.4, 5.6, 3.1, 6.4, 21.7]; [x,0,x]
```

```
[10.4, 5.6, 3.1, 6.4, 21.7, 0, 10.4, 5.6, 3.1, 6.4, 21.7]
```


Operator titik dua dengan ukuran langkah EMT digantikan oleh fungsi `seq()` di R. Kita dapat menulis fungsi ini dalam EMT.

```
>function seq(a,b,c) := a:b:c; ...  
>seq(0,-0.1,-1)
```

```
[0, -0.1, -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9, -1]
```

Fungsi `rep()` dari R tidak ada dalam EMT. Untuk input vektor, dapat ditulis sebagai berikut.

```
>function rep(x:vector,n:index) := flatten(dup(x,n)); ...  
>rep(x,2)
```

```
[10.4, 5.6, 3.1, 6.4, 21.7, 10.4, 5.6, 3.1, 6.4, 21.7]
```

Perhatikan bahwa "=" atau "==" digunakan untuk penugasan. Operator "->" digunakan untuk unit dalam EMT.

```
>125km -> " miles"
```

```
77.6713990297 miles
```

Operator "<-" untuk penugasan menyesatkan dan bukan ide yang baik untuk R. Berikut ini akan membandingkan a dan -4 dalam EMT.

```
>a=2; a<-4
```

```
0
```

Dalam R, "a<-4<3" berfungsi, tetapi "a<-4<-3" tidak. Saya juga mengalami ambiguitas serupa dalam EMT, tetapi mencoba menghilangkannya sedikit demi sedikit.

EMT dan R memiliki vektor bertipe boolean. Namun dalam EMT, angka 0 dan 1 digunakan untuk mewakili false dan true. Dalam R, nilai true dan false tetap dapat digunakan dalam aritmatika biasa seperti dalam EMT.

```
>x<5, %*x
```

```
[0, 0, 1, 0, 0]  
[0, 0, 3.1, 0, 0]
```

EMT memunculkan kesalahan atau menghasilkan NAN, tergantung pada tanda "kesalahan".

```
>errors off; 0/0, isNAN(sqrt(-1)), errors on;
```

```
NAN  
1
```

String sama di R dan EMT. Keduanya berada di lokal saat ini, bukan di Unicode.

Di R ada paket untuk Unicode. Di EMT, string dapat berupa string Unicode. String unicode dapat diterjemahkan ke pengodean lokal dan sebaliknya. Selain itu, u"..." dapat berisi entitas HTML.

```
>u"&#169; Ren&eacute; Grothmann"
```

© René Grothmann

Berikut ini mungkin atau mungkin tidak ditampilkan dengan benar pada sistem Anda sebagai A dengan titik dan garis di atasnya. Hal ini bergantung pada font yang Anda gunakan.

```
>chartoutf([480])
```

Penggabungan string dilakukan dengan "+" atau "|". String dapat menyertakan angka, yang akan dicetak dalam format saat ini.

```
>"pi = "+pi
```

```
pi = 3.14159265359
```

Pengindeksan

Sering kali, ini akan berfungsi seperti di R.

Namun EMT akan menginterpretasikan indeks negatif dari belakang vektor, sementara R menginterpretasikan $x[n]$ sebagai x tanpa elemen ke- n .

```
>x, x[1:3], x[-2]
```

```
[10.4,  5.6,  3.1,  6.4, 21.7]  
[10.4,  5.6,  3.1]  
6.4
```

Perilaku R dapat dicapai dalam EMT dengan `drop()`.

```
>drop(x,2)
```

```
[10.4,  3.1,  6.4, 21.7]
```

Vektor logika tidak diperlakukan secara berbeda sebagai indeks dalam EMT, berbeda dengan R. Anda perlu mengekstrak elemen bukan nol terlebih dahulu dalam EMT.

```
>x, x>5, x[nonzeros(x>5)]
```

```
[10.4,  5.6,  3.1,  6.4,  21.7]  
[1,  1,  0,  1,  1]  
[10.4,  5.6,  6.4,  21.7]
```

Sama seperti di R, vektor indeks dapat berisi pengulangan.

```
>x[[1,2,2,1]]
```

```
[10.4,  5.6,  5.6,  10.4]
```

Namun, nama untuk indeks tidak dimungkinkan dalam EMT. Untuk paket statistik, hal ini mungkin sering diperlukan untuk memudahkan akses ke elemen vektor.

Untuk meniru perilaku ini, kita dapat mendefinisikan fungsi sebagai berikut.

```
>function sel (v,i,s) := v[indexof(s,i)]; ...  
>s=["first","second","third","fourth"]; sel(x,["first","third"],s)
```

```
Trying to overwrite protected function sel!  
Error in:  
function sel (v,i,s) := v[indexof(s,i)]; ... ...  
^
```

```
Trying to overwrite protected function sel!  
Error in:  
function sel (v,i,s) := v[indexof(s,i)]; ... ...  
^
```

```
Trying to overwrite protected function sel!  
Error in:  
function sel (v,i,s) := v[indexof(s,i)]; ... ...  
^
```

```
[10.4, 3.1]
```

Tipe Data

EMT memiliki lebih banyak tipe data tetap daripada R. Jelas, di R terdapat vektor yang terus bertambah. Anda dapat menetapkan vektor numerik kosong `v` dan menetapkan nilai ke elemen `v[17]`. Hal ini tidak mungkin dilakukan di EMT.

Berikut ini agak tidak efisien.

```
>v=[]; for i=1 to 10000; v=v|i; end;
```

EMT sekarang akan membuat vektor dengan `v` dan `i` yang ditambahkan pada tumpukan dan menyalin vektor itu kembali ke variabel global `v`.

Yang lebih efisien mendefinisikan vektor terlebih dahulu.

```
>v=zeros(10000); for i=1 to 10000; v[i]=i; end;
```

Yang lebih efisien mendefinisikan vektor terlebih dahulu.

```
>complex(1:4)
```

```
[ 1+0i , 2+0i , 3+0i , 4+0i ]
```

Konversi ke string hanya dimungkinkan untuk tipe data dasar. Format saat ini digunakan untuk penggabungan string sederhana. Namun, ada fungsi seperti `print()` atau `frac()`.

Untuk vektor, Anda dapat dengan mudah menulis fungsi Anda sendiri.

```
>function tostr (v) ...  
  
    s="[";  
    loop 1 to length(v);  
        s=s+print(v[#],2,0);  
        if #<length(v) then s=s+","; endif;  
    end;  
    return s+"]";  
endfunction
```

```
>tostr(linspace(0,1,10))
```

```
[0.00,0.10,0.20,0.30,0.40,0.50,0.60,0.70,0.80,0.90,1.00]
```

Untuk komunikasi dengan Maxima, terdapat fungsi `convertmxm()`, yang juga dapat digunakan untuk memformat vektor untuk keluaran.

```
>convertmxm(1:10)
```

```
[1,2,3,4,5,6,7,8,9,10]
```


Untuk Latex perintah tex dapat digunakan untuk mendapatkan perintah Latex.

```
>tex(&[1,2,3])
```

```
\left[ 1 , 2 , 3 \right]
```

Faktor dan Tabel

Dalam pengantar R terdapat contoh dengan apa yang disebut faktor.

Berikut ini adalah daftar wilayah dari 30 negara bagian.

```
>austates = ["tas", "sa", "qld", "nsw", "nsw", "nt", "wa", "wa", ...  
>"qld", "vic", "nsw", "vic", "qld", "qld", "sa", "tas", ...  
>"sa", "nt", "wa", "vic", "qld", "nsw", "nsw", "wa", ...  
>"sa", "act", "nsw", "vic", "vic", "act"];
```

Asumsikan, kita memiliki pendapatan yang sesuai di setiap negara bagian.

```
>incomes = [60, 49, 40, 61, 64, 60, 59, 54, 62, 69, 70, 42, 56, ...  
>61, 61, 61, 58, 51, 48, 65, 49, 49, 41, 48, 52, 46, ...  
>59, 46, 58, 43];
```

Sekarang, kita ingin menghitung rata-rata pendapatan di wilayah tersebut. Sebagai program statistik, R memiliki `factor()` dan `tapply()` untuk ini.

EMT dapat melakukan ini dengan menemukan indeks wilayah dalam daftar wilayah yang unik.

```
>auterr=sort(unique(austates)); f=indexofsorted(auterr,austates)
```

```
[6, 5, 4, 2, 2, 3, 8, 8, 4, 7, 2, 7, 4, 4, 5, 6, 5, 3,  
8, 7, 4, 2, 2, 8, 5, 1, 2, 7, 7, 1]
```

Pada titik tersebut, kita dapat menulis fungsi loop kita sendiri untuk melakukan sesuatu hanya untuk satu faktor.

Atau kita dapat meniru fungsi `tapply()` dengan cara berikut.

```
>function map tappl (i; f$:call, cat, x) ...
```

```
u=sort(unique(cat));  
f=indexof(u,cat);  
return f$(x[nonzeros(f==indexof(u,i))]);  
endfunction
```

Agak tidak efisien, karena menghitung wilayah unik untuk setiap i, tetapi berhasil.

```
>tappl(auterr,"mean",austates,incomes)
```

```
[44.5, 57.3333333333, 55.5, 53.6, 55, 60.5, 56, 52.25]
```

Perhatikan bahwa ini berfungsi untuk setiap vektor wilayah.

```
>tappl(["act","nsw"],"mean",austates,incomes)
```

```
[44.5, 57.3333333333]
```

Sekarang, paket statistik EMT mendefinisikan tabel seperti di R. Fungsi readtable() dan writetable() dapat digunakan untuk input dan output.

Jadi kita dapat mencetak pendapatan negara rata-rata di wilayah dengan cara yang mudah.

```
>writetable(tappl(auterr,"mean",austates,incomes),labc=auterr,wc=7)
```

act	nsw	nt	qld	sa	tas	vic	wa
44.5	57.33	55.5	53.6	55	60.5	56	52.25

Kita juga dapat mencoba meniru perilaku R sepenuhnya.

Faktor-faktor tersebut harus disimpan dalam suatu koleksi dengan jenis dan kategori (negara bagian dan teritori dalam contoh kita). Untuk EMT, kita tambahkan indeks yang telah dihitung sebelumnya.

```
>function makef (t) ...  
  
## Factor data  
## Returns a collection with data t, unique data, indices.  
## See: tapply  
u=sort(unique(t));  
return {{t,u,indexofsorted(u,t)}};  
endfunction
```

```
>statef=makef(austates);
```

Sekarang elemen ketiga dari koleksi akan berisi indeks.

```
>statef[3]
```

```
[6, 5, 4, 2, 2, 3, 8, 8, 4, 7, 2, 7, 4, 4, 5, 6, 5, 3,  
8, 7, 4, 2, 2, 8, 5, 1, 2, 7, 7, 1]
```

Sekarang kita dapat meniru `tapply()` dengan cara berikut. Fungsi ini akan mengembalikan tabel sebagai kumpulan data tabel dan judul kolom.

```
>function tapply (t:vector,tf,f$:call) ...
```

```
## Makes a table of data and factors
## tf : output of makef()
## See: makef
uf=tf[2]; f=tf[3]; x=zeros(length(uf));
for i=1 to length(uf);
    ind=nonzeros(f==i);
    if length(ind)==0 then x[i]=NAN;
    else x[i]=f$(t[ind]);
    endif;
end;
return {{x,uf}};
endfunction
```

Kami tidak menambahkan banyak pemeriksaan tipe di sini. Satu-satunya tindakan pencegahan menyangkut kategori (faktor) tanpa data. Namun, seseorang harus memeriksa panjang `t` yang benar dan kebenaran koleksi `tf`.

Tabel ini dapat dicetak sebagai tabel dengan `writetable()`.

```
>writetable(tapply(incomes,statef,"mean"),wc=7)
```

act	nsw	nt	qld	sa	tas	vic	wa
44.5	57.33	55.5	53.6	55	60.5	56	52.25

Array

EMT hanya memiliki dua dimensi untuk array. Tipe data ini disebut matriks. Akan mudah untuk menulis fungsi untuk dimensi yang lebih tinggi atau pustaka C untuk ini.

R memiliki lebih dari dua dimensi. Dalam R, array adalah vektor dengan bidang dimensi.

Dalam EMT, vektor adalah matriks dengan satu baris. Vektor dapat dibuat menjadi matriks dengan `redim()`.

```
>shortformat; X=redim(1:20,4,5)
```

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Ekstraksi baris dan kolom, atau sub-matriks, sangat mirip di R.

```
>X[,2:3]
```

2	3
7	8
12	13
17	18

Namun, dalam R dimungkinkan untuk menetapkan daftar indeks vektor tertentu ke suatu nilai. Hal yang sama dimungkinkan dalam EMT hanya dengan loop.

```
>function setmatrixvalue (M, i, j, v) ...
```

```
  loop 1 to max(length(i),length(j),length(v))
    M[i{#},j{#}] = v{#};
  end;
endfunction
```

Kami mendemonstrasikan ini untuk menunjukkan bahwa matriks dilewatkan dengan referensi dalam EMT. Jika Anda tidak ingin mengubah matriks asli M, Anda perlu menyalinnya dalam fungsi tersebut.

```
>setmatrixvalue(X,1:3,3:-1:1,0); X,
```

1	2	0	4	5
6	0	8	9	10
0	12	13	14	15
16	17	18	19	20

Produk luar dalam EMT hanya dapat dilakukan antara vektor. Hal ini dilakukan secara otomatis karena bahasa matriks. Satu vektor harus berupa vektor kolom dan yang lainnya berupa vektor baris.

```
>(1:5)*(1:5)'
```

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

Dalam pengantar PDF untuk R terdapat sebuah contoh, yang menghitung distribusi ab-cd untuk a,b,c,d yang dipilih secara acak dari 0 hingga n. Solusi dalam R adalah membentuk matriks 4 dimensi dan menjalankan table() di atasnya.

Tentu saja, ini dapat dicapai dengan loop. Namun, loop tidak efektif dalam EMT atau R. Dalam EMT, kita dapat menulis loop dalam C dan itu akan menjadi solusi tercepat.

Namun, kita ingin meniru perilaku R. Untuk ini, kita perlu meratakan perkalian ab dan membuat matriks ab-cd.

```
>a=0:6; b=a'; p=flatten(a*b); q=flatten(p-p'); ...  
>u=sort(unique(q)); f=getmultiplicities(u,q); ...  
>statplot(u,f,"h"):
```

Selain multiplisitas yang tepat, EMT dapat menghitung frekuensi dalam vektor.

```
>getfrequencies(q,-50:10:50)
```

```
[0, 23, 132, 316, 602, 801, 333, 141, 53, 0]
```

Cara termudah untuk memplot ini sebagai distribusi adalah sebagai berikut.

```
>plot2d(q,distribution=11):
```

Namun, Anda juga dapat menghitung terlebih dahulu jumlah dalam interval yang dipilih. Tentu saja, berikut ini menggunakan `getfrequencies()` secara internal.

Karena fungsi `histo()` mengembalikan frekuensi, kita perlu menskalakannya sehingga integral di bawah grafik batang adalah 1.

```
>{x,y}=histo(q,v=-55:10:55); y=y/sum(y)/differences(x); ...  
>plot2d(x,y,>bar,style="/"):
```

Daftar

EMT memiliki dua jenis daftar. Satu adalah daftar global yang dapat diubah, dan yang lainnya adalah jenis daftar yang tidak dapat diubah. Kami tidak peduli dengan daftar global di sini.

Jenis daftar yang tidak dapat diubah disebut koleksi dalam EMT. Ia berperilaku seperti struktur dalam C, tetapi elemennya hanya diberi nomor dan tidak diberi nama.

```
>L={{ "Fred", "Flintstone", 40, [1990, 1992] }}
```

```
Fred  
Flintstone  
40  
[1990, 1992]
```

Saat ini unsur-unsur tersebut tidak memiliki nama, meskipun nama dapat ditetapkan untuk tujuan khusus. Unsur-unsur tersebut diakses dengan angka.

```
>(L[4])[2]
```

```
1992
```

Input dan Output File (Membaca dan Menulis Data)

Anda sering kali ingin mengimpor matriks data dari sumber lain ke EMT. Tutorial ini memberi tahu Anda tentang berbagai cara untuk mencapainya. Fungsi sederhana adalah `writematrix()` dan `readmatrix()`.

Mari kita tunjukkan cara membaca dan menulis vektor bilangan real ke dalam file.

```
>a=random(1,100); mean(a), dev(a),
```

```
0.49815
```

```
0.28037
```

Untuk menulis data ke dalam berkas, kami menggunakan fungsi `writematrix()`.

Karena pengantar ini kemungkinan besar berada di dalam direktori, tempat pengguna tidak memiliki akses tulis, kami menulis data ke direktori beranda pengguna. Untuk buku catatan sendiri, ini tidak diperlukan, karena berkas data akan ditulis ke dalam direktori yang sama.

```
>filename="test.dat";
```

Sekarang kita tulis vektor kolom `a'` ke dalam berkas. Ini menghasilkan satu angka di setiap baris berkas.

```
>writematrix(a',filename);
```

Untuk membaca data, kita menggunakan `readmatrix()`.

```
>a=readmatrix(filename)';
```

Dan hapus berkasnya.

```
>fileremove(filename);  
>mean(a), dev(a),
```

```
0.49815
```

```
0.28037
```

Fungsi `writematrix()` atau `writetable()` dapat dikonfigurasi untuk bahasa lain.

Misalnya, jika Anda memiliki sistem bahasa Indonesia (titik desimal dengan koma), Excel Anda memerlukan nilai dengan koma desimal yang dipisahkan oleh titik koma dalam file csv (nilai default dipisahkan dengan koma). File berikut "test.csv" akan muncul di folder Anda saat ini.

```
>filename="test.csv"; ...  
>writematrix(random(5,3),file=filename,separator=",");
```

Anda sekarang dapat membuka berkas ini langsung dengan Excel Indonesia.

```
>fileremove(filename);
```

Terkadang kita memiliki string dengan token seperti berikut.

```
>s1:="f m m f m m m f f f m m f"; ...  
>s2:="f f f m m f f";
```

Untuk menokenisasi ini, kami mendefinisikan vektor token.

```
>tok:=["f","m"]
```

```
f  
m
```

Lalu kita dapat menghitung berapa kali setiap token muncul dalam string, dan memasukkan hasilnya ke dalam tabel.

```
>M:=getmultiplicities(tok,strtokens(s1))_ ...  
> getmultiplicities(tok,strtokens(s2));
```

Tulis tabel dengan tajuk token.

```
>writetable(M,labc=tok,labr=1:2,wc=8)
```

	f	m
1	6	7
2	5	2

Untuk statika, EMT dapat membaca dan menulis tabel.

```
>file="test.dat"; open(file,"w"); ...  
>writeIn("A,B,C"); writematrix(random(3,3)); ...  
>close();
```

Berkasnya tampak seperti ini.

```
>printfile(file)
```

```
A,B,C  
0.7003664386138074,0.1875530821001213,0.3262339279660414  
0.5926249243193858,0.1522927283984059,0.368140583062521  
0.8065535209872989,0.7265910840408142,0.7332619844597152
```

Fungsi `readtable()` dalam bentuk yang paling sederhana dapat membacanya dan mengembalikan kumpulan nilai dan baris judul.

```
>L=readtable(file,>list);
```

Koleksi ini dapat dicetak dengan `writetable()` ke buku catatan, atau ke berkas.

```
>writetable(L,wc=10,dc=5)
```

A	B	C
0.70037	0.18755	0.32623
0.59262	0.15229	0.36814
0.80655	0.72659	0.73326

Matriks nilai adalah elemen pertama L. Perhatikan bahwa `mean()` dalam EMT menghitung nilai rata-rata baris matriks.

```
>mean(L[1])
```

```
0.40472  
0.37102  
0.75547
```


Berkas CSV

Pertama, mari kita tulis matriks ke dalam berkas. Untuk output, kita buat berkas di direktori kerja saat ini.

```
>file="test.csv"; ...  
>M=random(3,3); writematrix(M,file);
```

Berikut ini isi berkas tersebut.

```
>printfile(file)
```

```
0.8221197733097619,0.821531098722547,0.7771240608094004  
0.8482947121863489,0.3237767724883862,0.6501422353377985  
0.1482301827518109,0.3297459716109594,0.6261901074210923
```

CVS ini dapat dibuka pada sistem bahasa Inggris ke Excel dengan mengklik dua kali. Jika Anda mendapatkan berkas tersebut pada sistem bahasa Jerman, Anda perlu mengimpor data ke Excel dengan memperhatikan titik desimal.

Namun, titik desimal juga merupakan format default untuk EMT. Anda dapat membaca matriks dari berkas dengan `readmatrix()`.

```
>readmatrix(file)
```

0.82212	0.82153	0.77712
0.84829	0.32378	0.65014
0.14823	0.32975	0.62619

Dimungkinkan untuk menulis beberapa matriks ke dalam satu berkas. Perintah `open()` dapat membuka berkas untuk ditulis dengan parameter "w". Nilai default untuk membaca adalah "r".

```
>open(file,"w"); writematrix(M); writematrix(M'); close();
```

Matriks dipisahkan oleh baris kosong. Untuk membaca matriks, buka berkas dan panggil `readmatrix()` beberapa kali.

```
>open(file); A=readmatrix(); B=readmatrix(); A==B, close();
```

1	0	0
0	1	0
0	0	1

Di Excel atau lembar kerja serupa, Anda dapat mengekspor matriks sebagai CSV (nilai yang dipisahkan koma). Di Excel 2007, gunakan "simpan sebagai" dan "format lain", lalu pilih "CSV". Pastikan, tabel saat ini hanya berisi data yang ingin Anda ekspor.

Berikut ini contohnya.

```
>printfile("excel-data.csv")
```

```
Could not open the file
excel-data.csv
for reading!
Try "trace errors" to inspect local variables after errors.
printfile:
    open(filename,"r");
```

Seperti yang Anda lihat, sistem Jerman saya menggunakan titik koma sebagai pemisah dan koma desimal. Anda dapat mengubahnya di pengaturan sistem atau di Excel, tetapi tidak diperlukan untuk membaca matriks ke EMT.

Cara termudah untuk membaca ini ke Euler adalah readmatrix(). Semua koma diganti dengan titik dengan parameter >comma. Untuk CSV bahasa Inggris, cukup abaikan parameter ini.

```
>M=readmatrix("excel-data.csv",>comma)
```

```
Could not open the file
excel-data.csv
for reading!
Try "trace errors" to inspect local variables after errors.
readmatrix:
    if filename<>"" then open(filename,"r"); endif;
```

Mari kita plot ini.

```
>plot2d(M'[1],M'[2:3],>points,color=[red,green]'):
```

Ada cara yang lebih mendasar untuk membaca data dari sebuah berkas. Anda dapat membuka berkas dan membaca angka baris demi baris. Fungsi `getvectorline()` akan membaca angka dari sebaris data. Secara default, fungsi ini mengharapkan titik desimal. Namun, fungsi ini juga dapat menggunakan koma desimal, jika Anda memanggil `setdecimaldot(",")` sebelum menggunakan fungsi ini.

Fungsi berikut adalah contohnya. Fungsi ini akan berhenti di akhir berkas atau baris kosong.

```
>function myload (file) ...
```

```
    open(file);
    M=[];
    repeat
        until eof();
        v=getvectorline(3);
        if length(v)>0 then M=M_v; else break; endif;
    end;
    return M;
    close(file);
endfunction
```

```
>myload(file)
```

0.82212	0	0.82153	0	0.77712
0.84829	0	0.32378	0	0.65014
0.14823	0	0.32975	0	0.62619

Semua angka dalam berkas itu juga dapat dibaca dengan `getvector()`.

```
>open(file); v=getvector(10000); close(); redim(v[1:9],3,3)
```

0.82212	0	0.82153
0	0.77712	0.84829
0	0.32378	0

Jadi sangat mudah untuk menyimpan vektor nilai, satu nilai di setiap baris dan membaca kembali vektor ini.

```
>v=random(1000); mean(v)
```

0.50303

```
>writematrix(v',file); mean(readmatrix(file)')
```

0.50303

Menggunakan Tabel

Tabel dapat digunakan untuk membaca atau menulis data numerik. Misalnya, kita menulis tabel dengan tajuk baris dan kolom ke dalam sebuah berkas.

```
>file="test.tab"; M=random(3,3); ...  
>open(file,"w"); ...  
>writetable(M,separator=",",labc=["one","two","three"]); ...  
>close(); ...  
>printfile(file)
```

one	two	three
0.09,	0.39,	0.86
0.39,	0.86,	0.71
0.2,	0.02,	0.83

Ini dapat diimpor ke Excel.

Untuk membaca berkas di EMT, kami menggunakan `readtable()`.

```
>{M,headings}=readtable(file,>clabs); ...  
>writetable(M,labc=headings)
```

one	two	three
0.09	0.39	0.86
0.39	0.86	0.71
0.2	0.02	0.83

Menganalisis Garis

Anda bahkan dapat mengevaluasi setiap garis secara manual. Misalkan, kita memiliki garis dengan format berikut.

```
>line="2020-11-03,Tue,1'114.05"
```

```
2020-11-03,Tue,1'114.05
```

Pertama, kita dapat membuat token pada baris tersebut.

```
>vt=strtokens(line)
```

```
2020-11-03
```

```
Tue
```

```
1'114.05
```

Kemudian kita dapat mengevaluasi setiap elemen garis menggunakan evaluasi yang tepat.

```
>day(vt[1]), ...  
>indexof(["mon","tue","wed","thu","fri","sat","sun"],tolower(vt[2])), ...  
>strrepl(vt[3],"',"")()
```

```
7.3816e+05  
2  
1114
```

Dengan menggunakan ekspresi reguler, dimungkinkan untuk mengekstrak hampir semua informasi dari sebaris data.

Asumsikan kita memiliki baris berikut sebagai dokumen HTML.

```
>line("<tr><td>1145.45</td><td>5.6</td><td>-4.5</td><tr>"
```

```
<tr><td>1145.45</td><td>5.6</td><td>-4.5</td><tr>
```


Untuk mengekstraknya, kami menggunakan ekspresi reguler, yang mencari

- tanda kurung tutup > ,
- string apa pun yang tidak mengandung tanda kurung dengan sub-kecocokan "(...)",
- tanda kurung buka dan tutup menggunakan solusi terpendek,
- lagi-lagi string apa pun yang tidak mengandung tanda kurung,
- dan tanda kurung buka < .

Ekspresi reguler agak sulit dipelajari tetapi sangat ampuh.

```
>{pos,s,vt}=strxfind(line,">([<>]+)<.+?>([<>]+)<");
```

Hasilnya adalah posisi kecocokan, string yang cocok, dan vektor string untuk sub-kecocokan.

```
>for k=1:length(vt); vt[k](), end;
```

```
1145.5  
5.6
```

Berikut adalah fungsi yang membaca semua item numerik antara <td> dan </td>.

```
>function readtd (line) ...
```

```
v=[]; cp=0;
repeat
    {pos,s,vt}=strxfind(line,"<td.*?>(.*?)</td>",cp);
    until pos==0;
    if length(vt)>0 then v=v|vt[1]; endif;
    cp=pos+strlen(s);
end;
return v;
endfunction
```

```
>readtd(line+"<td>non-numerical</td>")
```

```
1145.45
5.6
-4.5
non-numerical
```

Membaca dari Web

Situs web atau berkas dengan URL dapat dibuka di EMT dan dapat dibaca baris demi baris.

Dalam contoh ini, kami membaca versi terkini dari situs EMT. Kami menggunakan ekspresi reguler untuk memindai "Versi ..." dalam judul.

```
>function readversion () ...  
  
    urlopen("http://www.euler-math-toolbox.de/Programs/Changes.html");  
    repeat  
        until urleof();  
        s=urlgetline();  
        k=strfind(s,"Version ",1);  
        if k>0 then substring(s,k,strfind(s,"<",k)-1), break; endif;  
    end;  
    urlclose();  
endfunction
```

```
>readversion
```

Version 2024-01-12

Input dan Output Variabel

Anda dapat menulis variabel dalam bentuk definisi Euler ke dalam file atau ke baris perintah.

```
>writevar(pi,"mypi");
```

```
mypi = 3.141592653589793;
```

Untuk pengujian, kami membuat file Euler di direktori kerja EMT.

```
>file="test.e"; ...  
>writevar(random(2,2),"M",file); ...  
>printfile(file,3)
```

```
M = [ ..  
0.5991820585590205, 0.7960280262224293;  
0.5167243983231363, 0.2996684599070898];
```

Sekarang kita dapat memuat berkas tersebut. Berkas tersebut akan mendefinisikan matriks M.

```
>load(file); show M,
```

```
M =  
  0.59918  0.79603  
  0.51672  0.29967
```

Ngomong-ngomong, jika `writevar()` digunakan pada suatu variabel, ia akan mencetak definisi variabel dengan nama variabel ini.

```
>writevar(M); writevar(inch$)
```

```
M = [ ..  
  0.5991820585590205, 0.7960280262224293;  
  0.5167243983231363, 0.2996684599070898];  
inch$ = 0.0254;
```

Kita juga dapat membuka berkas baru atau menambahkannya ke berkas yang sudah ada. Dalam contoh ini, kita menambahkannya ke berkas yang dibuat sebelumnya.

```
>open(file,"a"); ...  
>writevar(random(2,2),"M1"); ...  
>writevar(random(3,1),"M2"); ...  
>close();  
>load(file); show M1; show M2;
```

```
M1 =  
  0.30287  0.15372  
  0.7504   0.75401  
M2 =  
  0.27213  
  0.053211  
  0.70249
```

Untuk menghapus file apa pun gunakan `fileremove()`.

```
>fileremove(file);
```

Vektor baris dalam sebuah berkas tidak memerlukan koma, jika setiap angka berada di baris baru. Mari kita buat berkas seperti itu, tulis setiap baris satu per satu dengan `writeln()`.

```
>open(file,"w"); writeln("M = ["); ...  
>for i=1 to 5; writeln(""+random()); end; ...  
>writeln("]"); close(); ...  
>printfile(file)
```

```
M = [  
0.344851384551  
0.0807510017715  
0.876519562911  
0.754157709472  
0.688392638934  
];
```

```
>load(file); M
```

```
[0.34485, 0.080751, 0.87652, 0.75416, 0.68839]
```

catatan : ketika mengenter perintah-perintah diatas ternyata hasil yang didapatkan berbeda-beda

Latihan soal

Nomor 1 Carilah rata-rata dan standar deviasi beserta plot dari data berikut
 $X = 2000, 2500, 2700, 3500, 4500, 5000$

```
>X=[2000,2500,2700,3500,4500,5000]; ...  
>mean(X), dev(X),
```

```
3366.7  
1186
```

```
>aspect(1.5); boxplot(X):
```

Nomor 2
Misalkan diberikan data skor hasil statistika dari 20 orang mahasiswa sebagai berikut:
70,65,79,90,60,79,86,95,100,70,60,91,68,84,59,90,88,84,86,90
Tentukan rata-rata dari data tersebut!

```
>X=[70,65,79,90,60,79,86,95,100,70,60,91,68,84,59,90,88,84,86,90]
```

```
[70, 65, 79, 90, 60, 79, 86, 95, 100, 70, 60, 91, 68, 84,  
59, 90, 88, 84, 86, 90]
```



```
>mean(X)
```

```
79.7
```