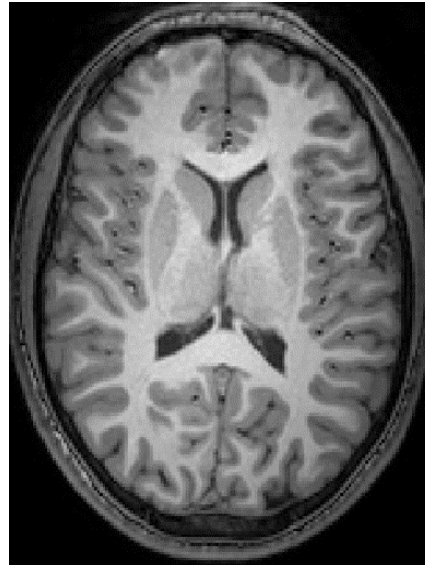
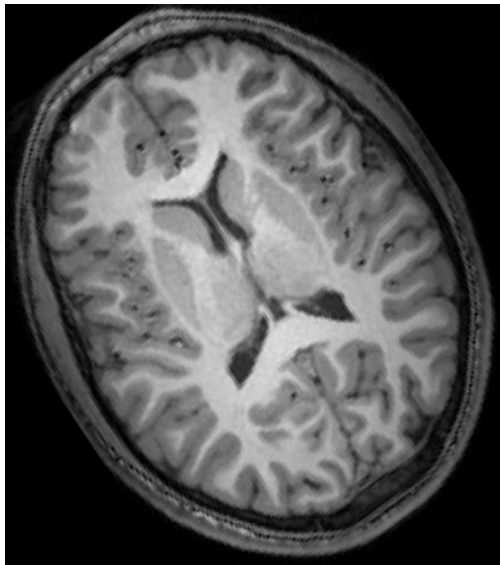

Image Registration



Image Registration

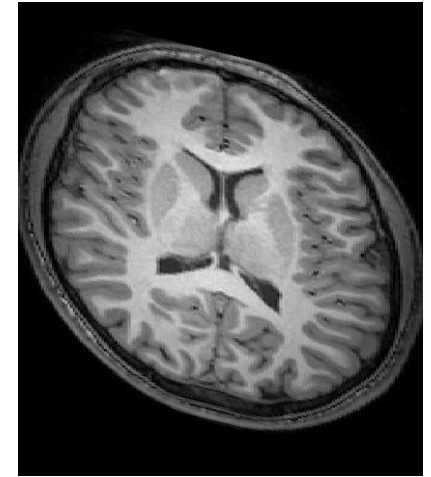
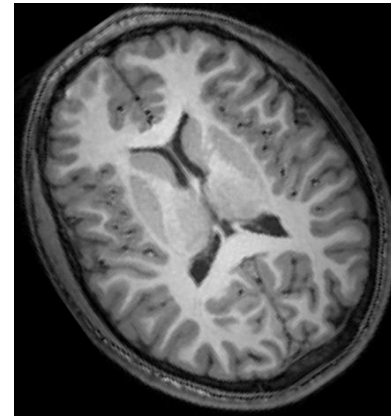
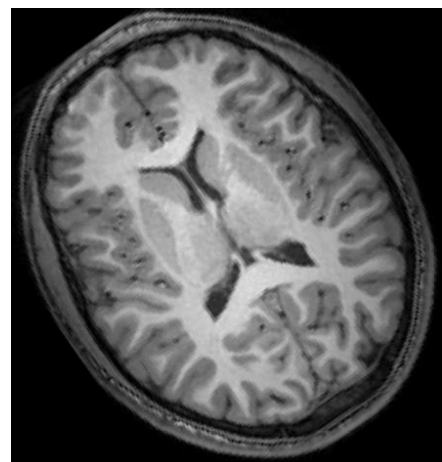
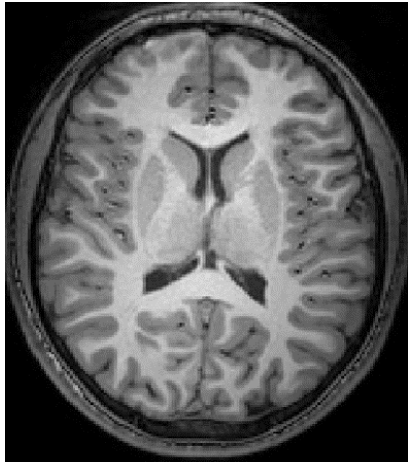
	Conventional methods	Deep Learning methods
Registration	Transformation matrix Iterative closest point (ICP) Non rigid ICP Deformable models	FlowNet CNN for Registration

Overview



- Find correspondences
- Estimate transformation matrix
- Transform the moving image

Transformation



- Rigid transformation
- Similarity transformation
- Affine transformation
- Projective transformation (Homography)

Transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Pseudo Inverse

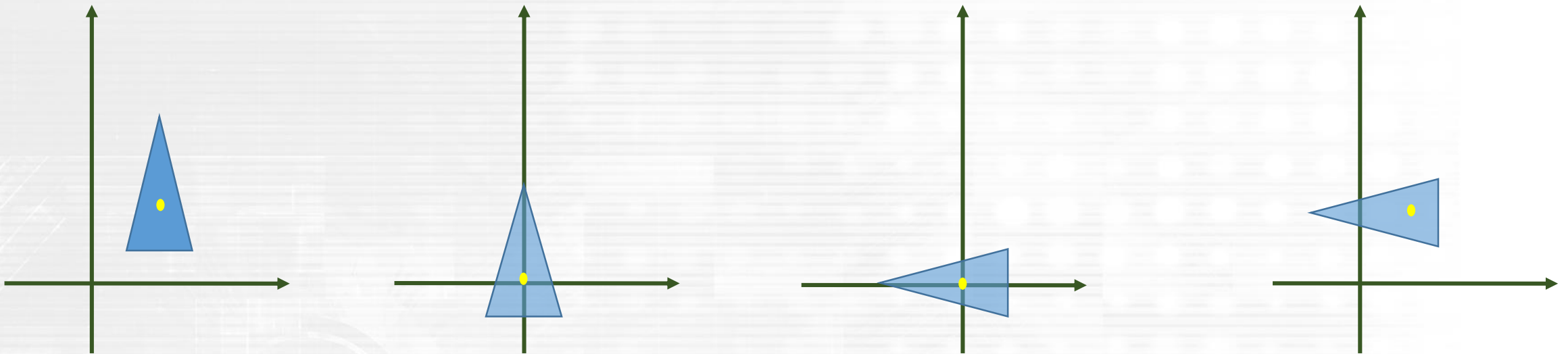
Transformation Matrix

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

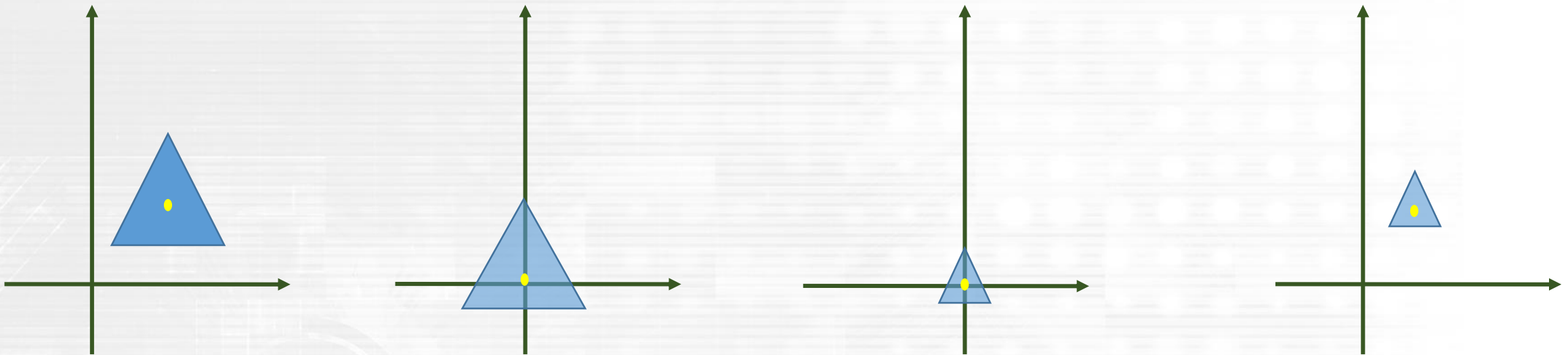
$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example



Example



Transformation in 3D

Transformation in 3D

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

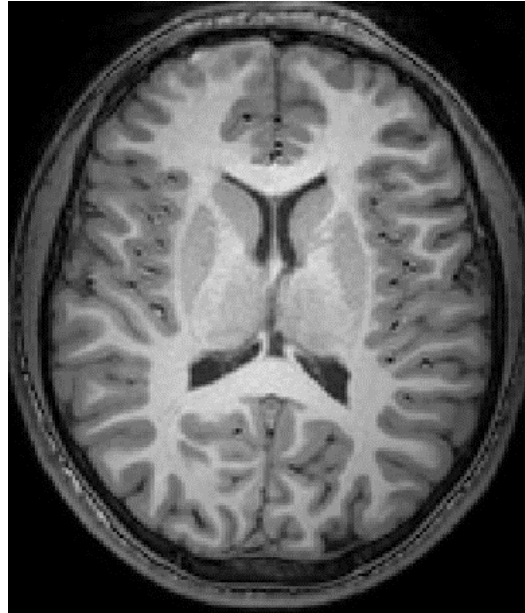
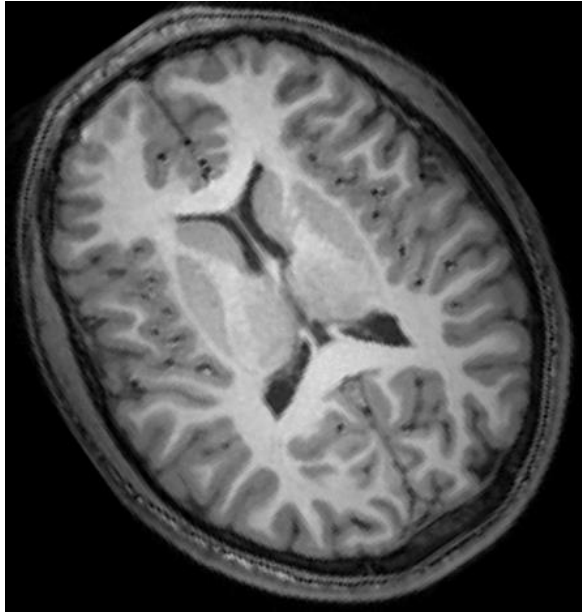
$$R = R_z(\theta_3)R_y(\theta_2)R_x(\theta_1)$$

Transformation in 3D

$$\begin{bmatrix} ll(1 - \cos \theta) + \cos \theta & ml(1 - \cos \theta) - n \sin \theta & nl(1 - \cos \theta) + m \sin \theta \\ lm(1 - \cos \theta) + n \sin \theta & mm(1 - \cos \theta) + \cos \theta & nm(1 - \cos \theta) - l \sin \theta \\ ln(1 - \cos \theta) - m \sin \theta & mn(1 - \cos \theta) + l \sin \theta & nn(1 - \cos \theta) + \cos \theta \end{bmatrix}$$

Transformation Matrix

Backward Warping



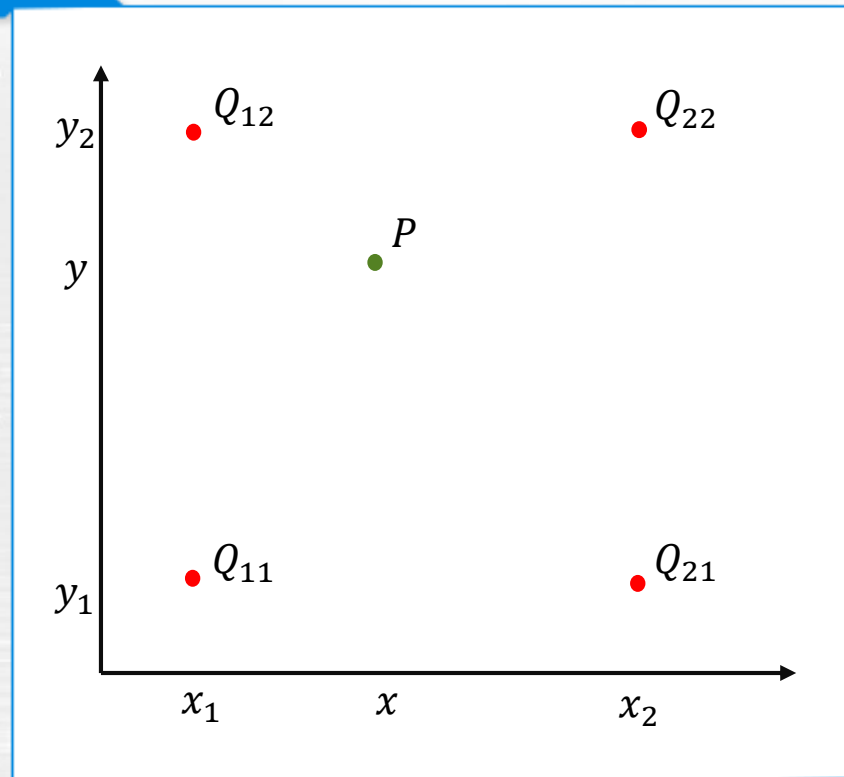
Interpolation

Nearest-neighborhood

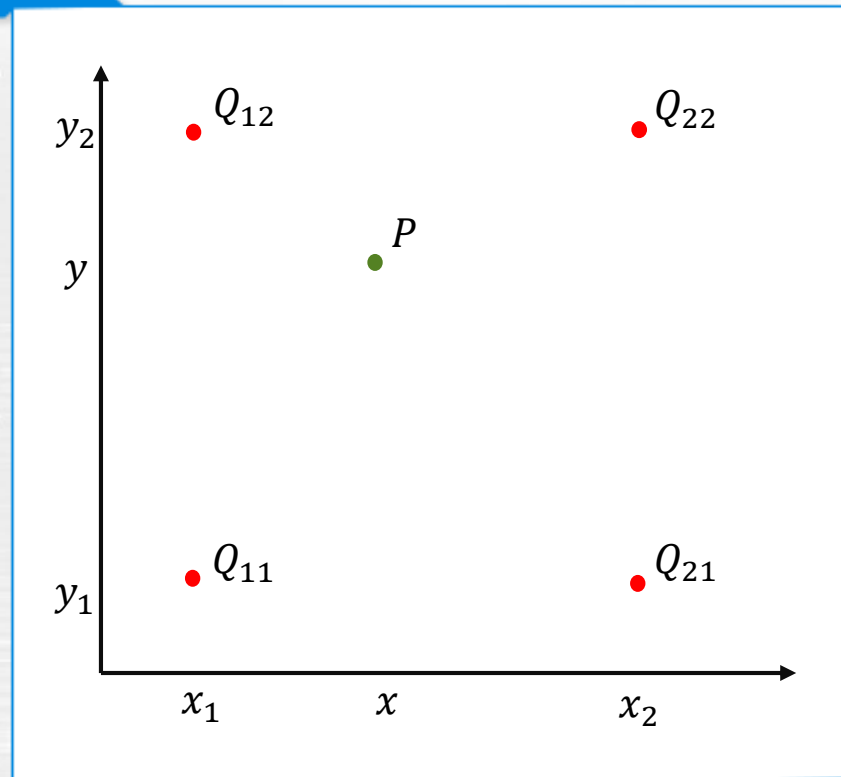
Linear

Cubic

Bilinear Interpolation



Bilinear Interpolation



$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

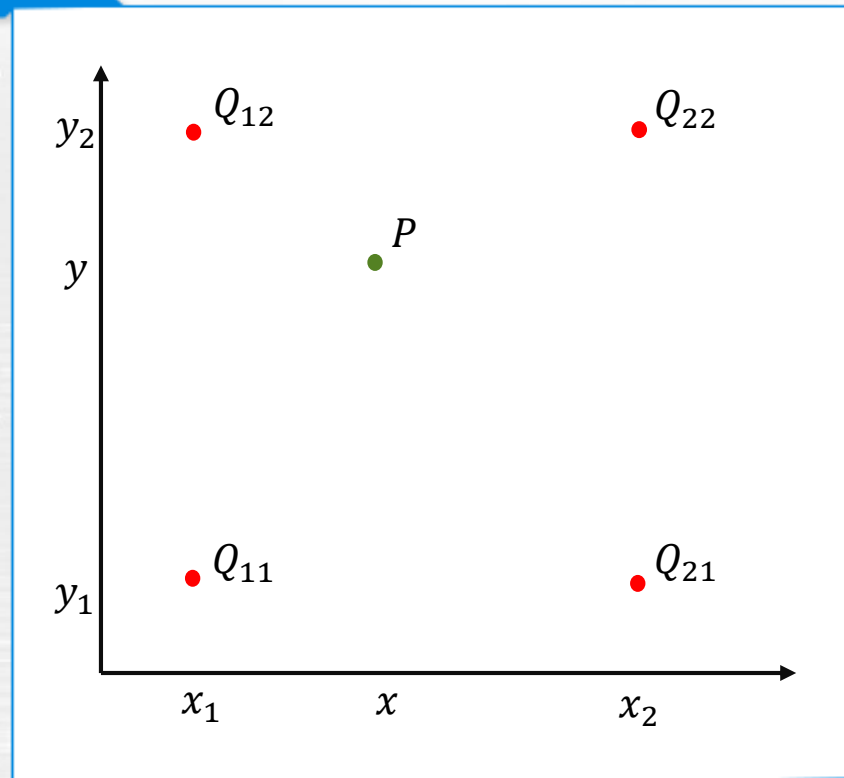
$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2)$$

$$= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 - x & x - x_1 \end{bmatrix} \begin{bmatrix} f(Q_{11}) & f(Q_{12}) \\ f(Q_{21}) & f(Q_{22}) \end{bmatrix} \begin{bmatrix} y_2 - y \\ y - y_1 \end{bmatrix}$$

Bicubic Interpolation

$$p(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$



Bicubic Interpolation

$$p(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$

1. $f(0,0) = p(0,0) = a_{00}$,
2. $f(1,0) = p(1,0) = a_{00} + a_{10} + a_{20} + a_{30}$,
3. $f(0,1) = p(0,1) = a_{00} + a_{01} + a_{02} + a_{03}$,
4. $f(1,1) = p(1,1) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij}$.

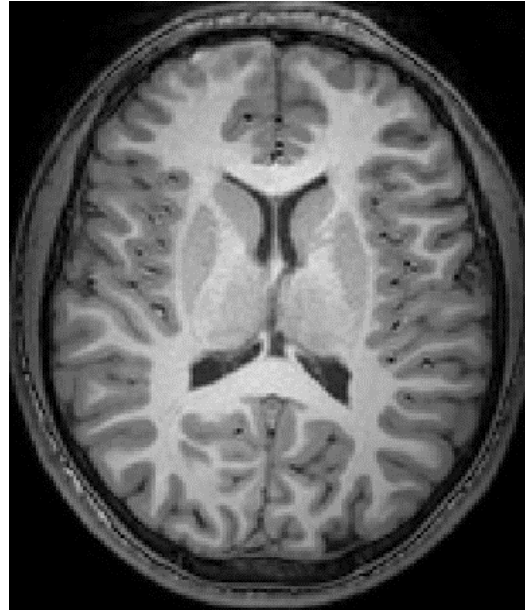
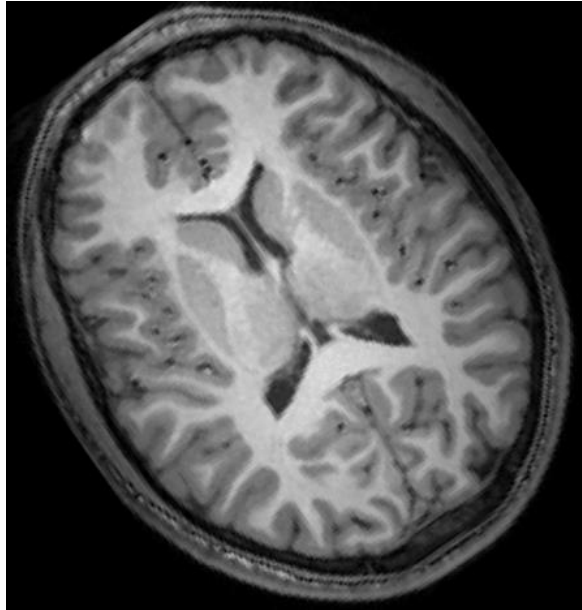
1. $f_{xy}(0,0) = p_{xy}(0,0) = a_{11}$,
2. $f_{xy}(1,0) = p_{xy}(1,0) = a_{11} + 2a_{21} + 3a_{31}$,
3. $f_{xy}(0,1) = p_{xy}(0,1) = a_{11} + 2a_{12} + 3a_{13}$,
4. $f_{xy}(1,1) = p_{xy}(1,1) = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} ij$.

1. $f_x(0,0) = p_x(0,0) = a_{10}$,
2. $f_x(1,0) = p_x(1,0) = a_{10} + 2a_{20} + 3a_{30}$,
3. $f_x(0,1) = p_x(0,1) = a_{10} + a_{11} + a_{12} + a_{13}$,
4. $f_x(1,1) = p_x(1,1) = \sum_{i=1}^3 \sum_{j=0}^3 a_{ij} i$,
5. $f_y(0,0) = p_y(0,0) = a_{01}$,
6. $f_y(1,0) = p_y(1,0) = a_{01} + a_{11} + a_{21} + a_{31}$,
7. $f_y(0,1) = p_y(0,1) = a_{01} + 2a_{02} + 3a_{03}$,
8. $f_y(1,1) = p_y(1,1) = \sum_{i=0}^3 \sum_{j=1}^3 a_{ij} j$.

Bicubic Interpolation

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} f(0,0) & f(0,1) & f_y(0,0) & f_y(0,1) \\ f(1,0) & f(1,1) & f_y(1,0) & f_y(1,1) \\ f_x(0,0) & f_x(0,1) & f_{xy}(0,0) & f_{xy}(0,1) \\ f_x(1,0) & f_x(1,1) & f_{xy}(1,0) & f_{xy}(1,1) \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Similarity Measures

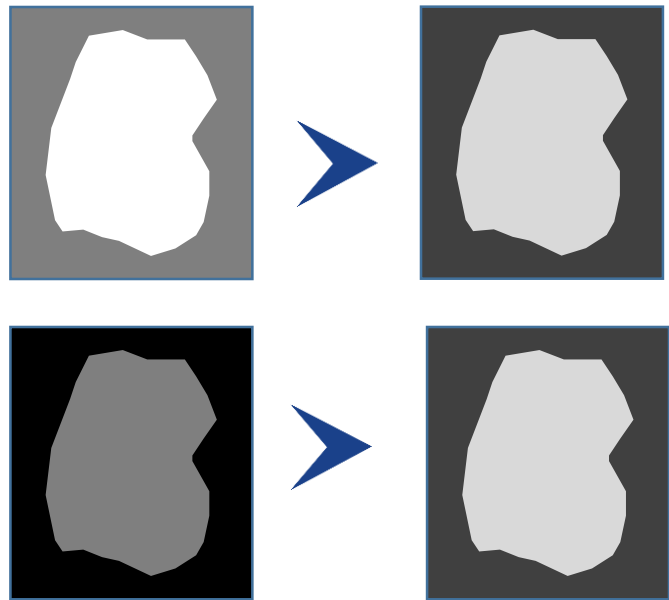


Sum of Squared Distance

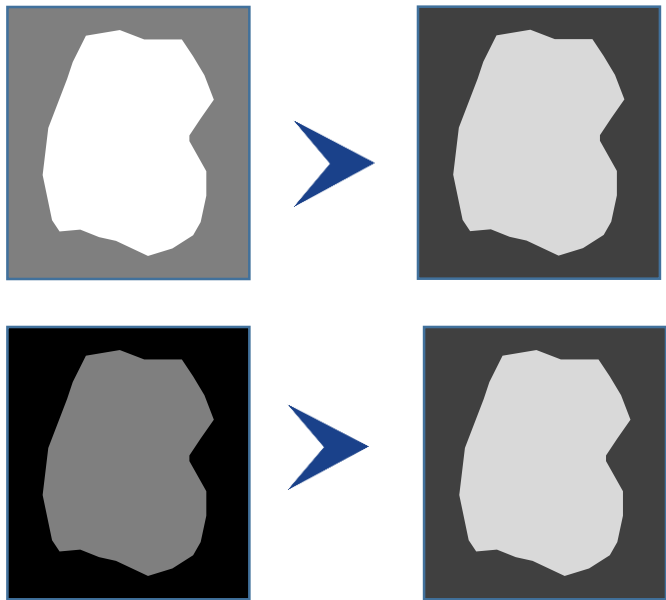
$$\text{SSD} = \sum_{xy} (I_1(x, y) - I_2(x, y))^2$$

$$\text{SAD} = \sum_{xy} |(I_1(x, y) - I_2(x, y))|$$

Normalized Cross Correlation

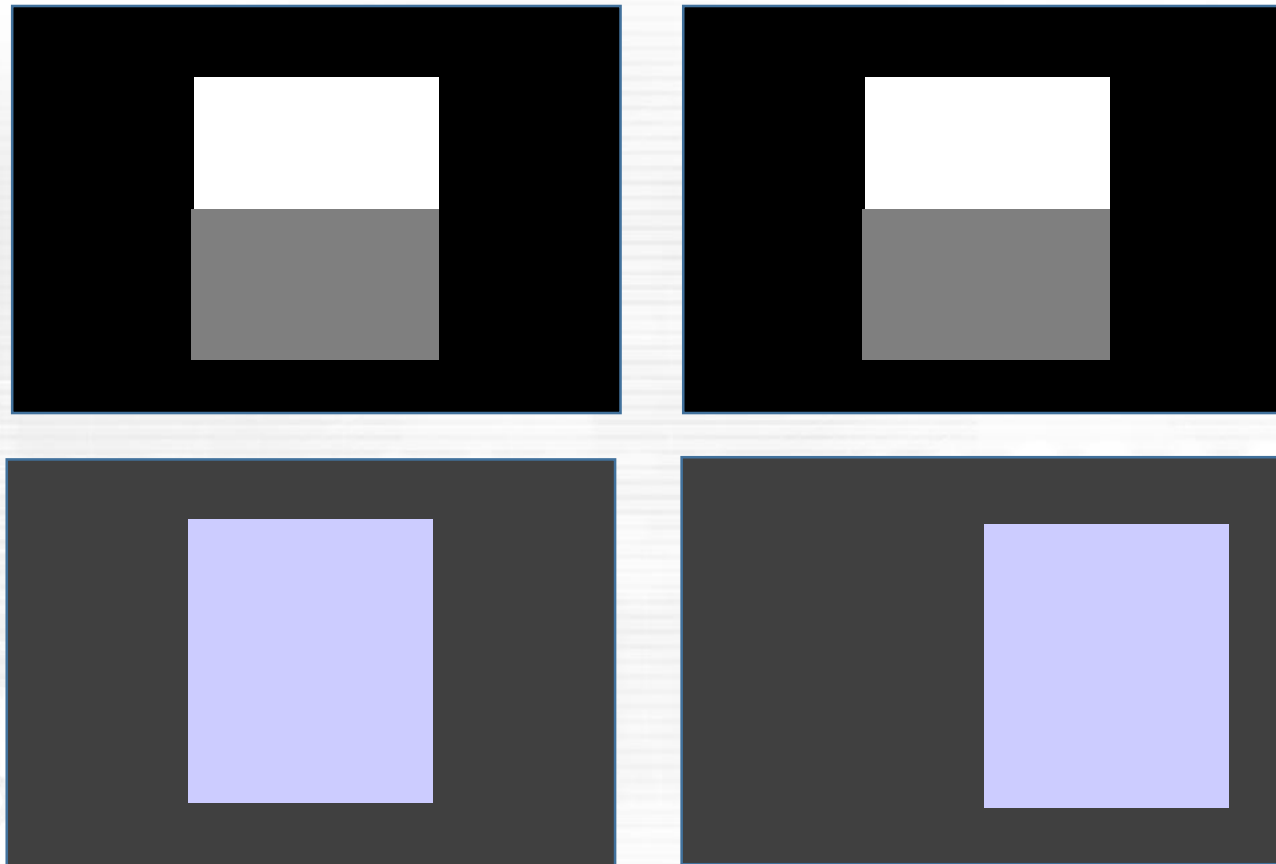


Normalized Cross Correlation



$$NCC = \frac{1}{N} \sum_{xy} \frac{(I_1(x, y) - m_1)(I_2(x, y) - m_2)}{\sigma_1 \sigma_2}$$

Similarity in Different Modality



Mutual Information

Mutual Information

Maximize

$$I(F, W) = H(F) + H(W) - H(F, W)$$

where

$$H(F) = \int_f -p(f) \log(p(f))$$

$$H(W) = \int_w -p(w) \log(p(w))$$

$$H(F, W) = \int_f \int_w -p(f, w) \log(p(f, w))$$