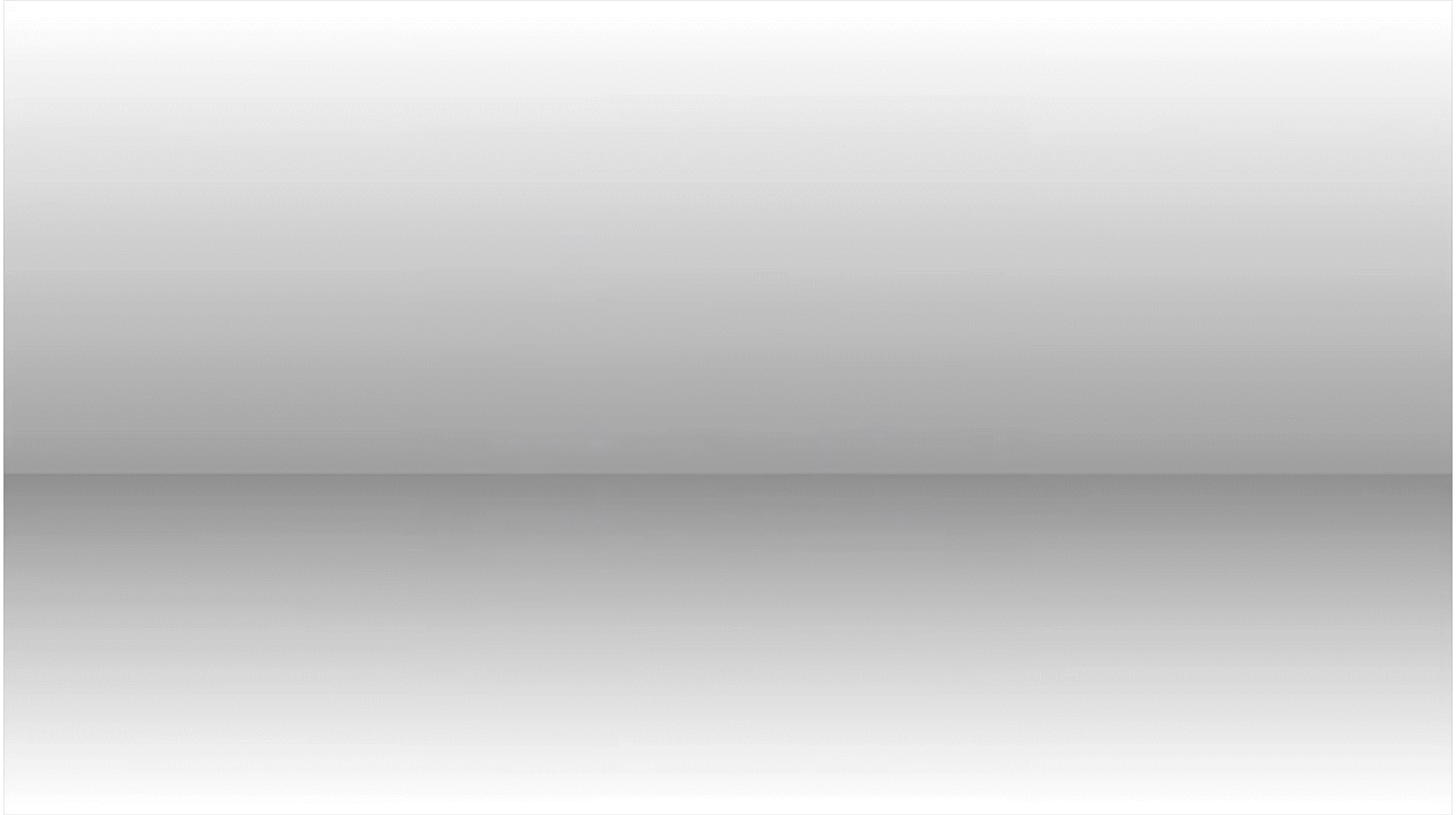
Medical Imaging
(RT516 – Spring 2021)
Lecture 9: Ultrasound Imaging

Jaesok Yu, Ph.D. (jaesok.yu@dgist.ac.kr)

Assistant Professor
Department of Robotics Engineering, DGIST

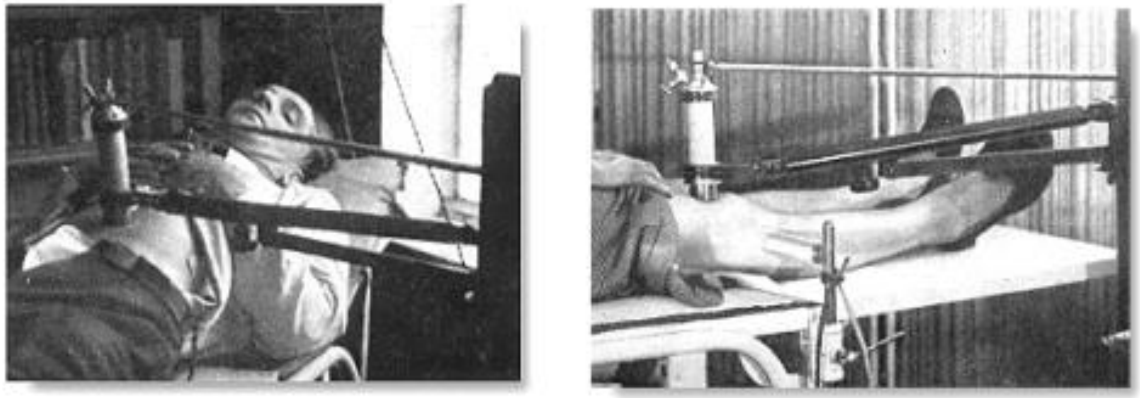
Ultrasound Imaging



History of Development of Ultrasound

Sonar
Radar
Metal Flaw Detector

The underwater **SONAR**, the **RADAR** and the ultrasonic **Metal Flaw Detector** were each, in their unique ways, a precursor of medical ultrasonic equipments. The modern ultrasound scanner embraces the concepts and science of all these modalities.



Therapy

Uses of ultrasonic energy in the 1940s. Left, in gastric ulcers. Right, in arthritis

A short History of the development of Ultrasound in Obstetrics and Gynecology : Dr. Joseph Woo
http://www.ob-ultrasound.net/history1.html

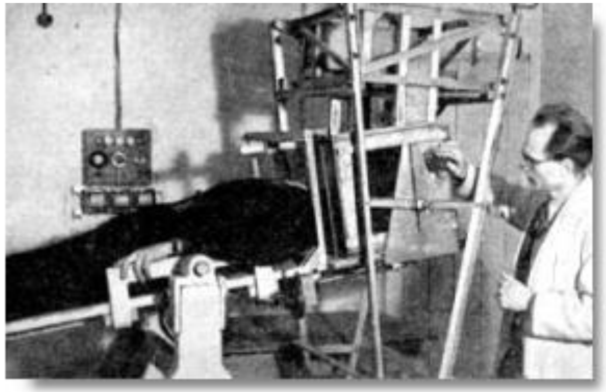
History of Medical Ultrasound

1942:The Austrian neurologist Dr. Karl Theodore Dussik published on transmission ultrasound investigation of the brain –he succeeded in displaying the cerebral ventricle by means of ultrasound waves with a method called Hyperphonographie.

1949:The potential of ultrasound imaging for the detection of gallstones has been described by Ludwig and Struthers.

1947-1950:Howryand Bliss succeeded in analyzing and displaying **the anatomic details of an organ** in water-bath (**B-Mode-Technology**)

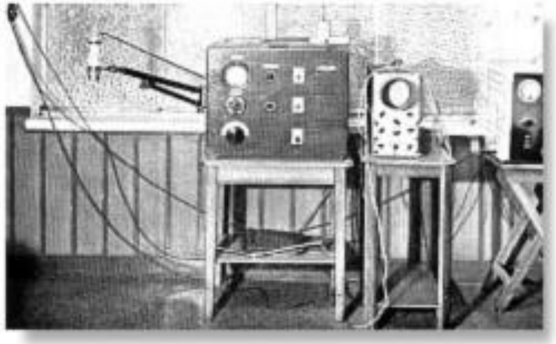
1954:Beating heart -the echocardiography was described by Edlerand Hertz.



Dussik and his ultrasonic apparatus in 1946

History of Development of Ultrasound

Dussik and his apparatus in 1946

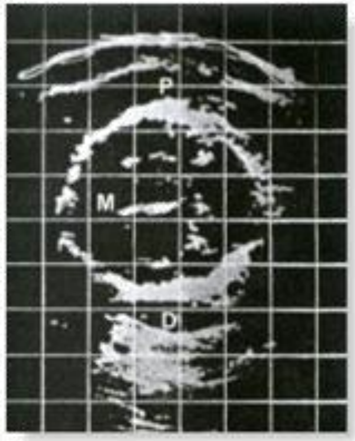


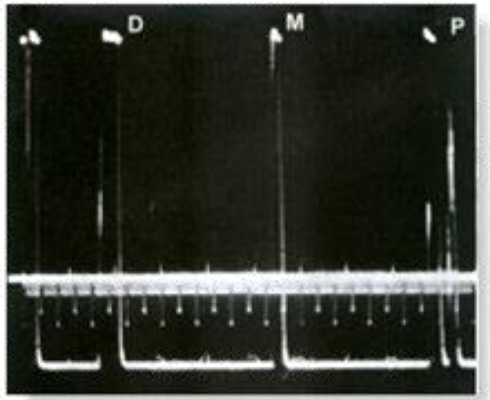
Denier's ultrasonic apparatus in 1946

The pan-scanner in 1957

A & B scan

The articulated arm scanner that Wright and Meyerdirk built in 1962, the earliest of such design in the U.S.

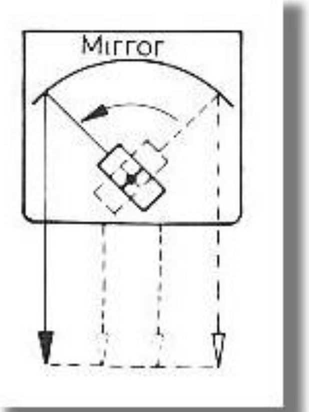


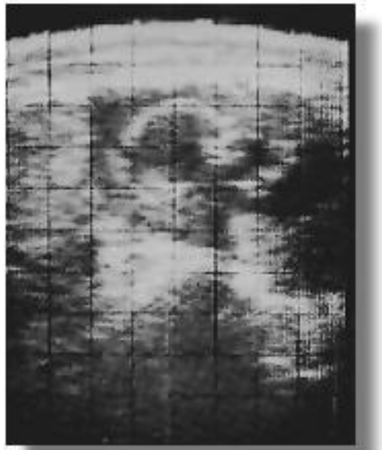


The early bistable oscilloscopic B-scan image at the level of the BPD and the A-scan tracing showing cephalic (P and D) and midline echoes (M). The distance between the 2 cephalic echoes is the BPD. Without scan converters on-screen (oscilloscope) measurements on the B-mode image are not possible. Very accurate measurements can however be made using the A-scan calipers.

History of Development of Ultrasound

Vidoson® by Siemens Medical Systems
of Germany in 1965





Real-time imaging

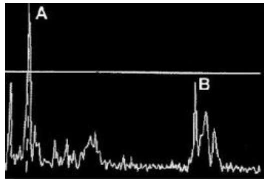
The Vidoson*, its working mechanism and the resultant image of a fetal face and hand.
The transducer housing is mounted on a mobile gantry and rigidly connected to the main console.
The scanning frequency was 2.25 MHz. Scaling and caliper functions were not present.

Combison 100 from Kretztechnik®
of Austria (1977),

The large hand-held circular rotating transducer (Combison 100) from KretzTechnik® and the resultant sector image.
The transducer is connected to the main console by a flexible cable.

History of Medical Ultrasound

A-scan (1-D)
1st generation

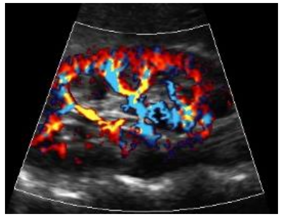


Bistable
Static 2-D
2nd generation

(Array transducers technology)

Real-time
B-scan (2-D)
3rd generation

Digital system /
Doppler (2-D)
4th generation



Static 3-D
5th generation

Functional /
Real-time 3-D
6th generation

Feature implementation

What's the next?

?

Finding 'Value'

1950 **1960** **1970** **1980** **1990** **2000** **2010**

History of Medical Ultrasound

Current 7th generation

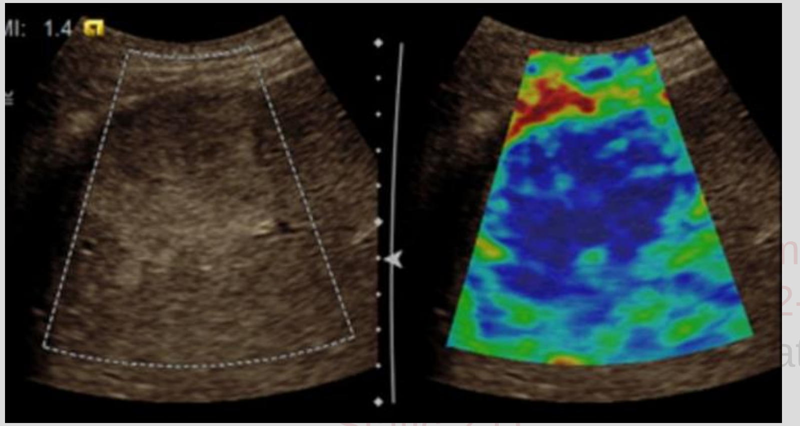
"Beyond Ultrasound"

Multi-modal imaging,

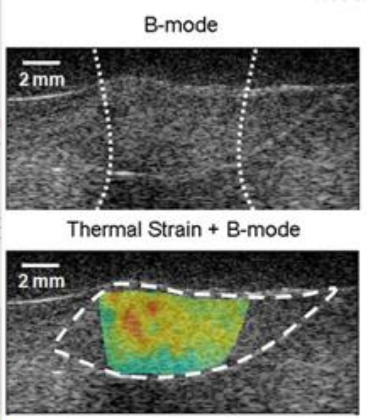
(Functional + Anatomical)

Theranostic (Nano)

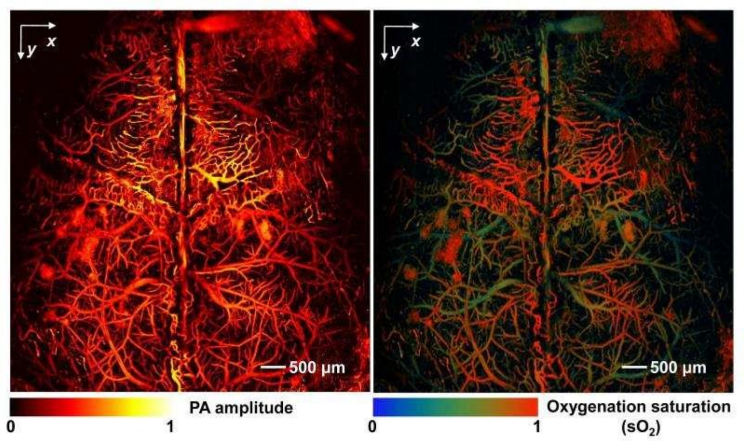
Elastography



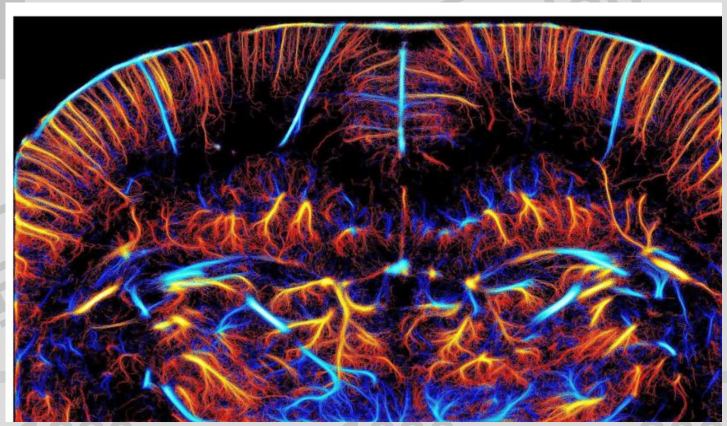
Thermal strain imaging



Photoacoustic imaging

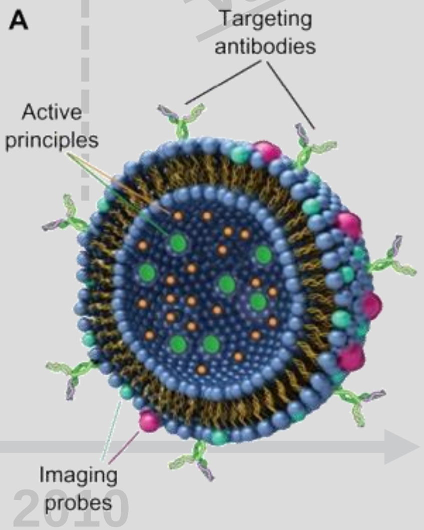


Super-resolution imaging



Theranostic

(Therapy + Diagnostic)

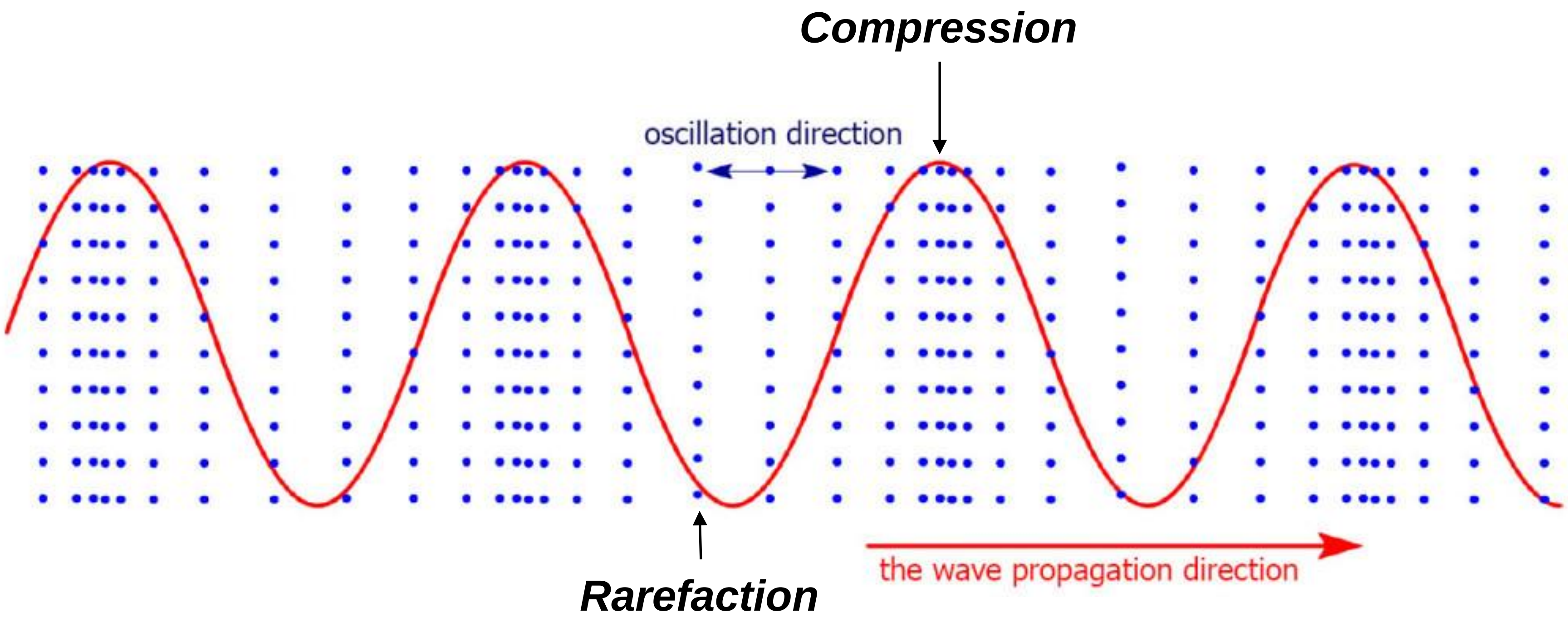


The type of waves

- **Longitudinal waves**

– particles move along the longitudinal (axial) direction

– wave propagates in the same direction (axial, or longitudinal)



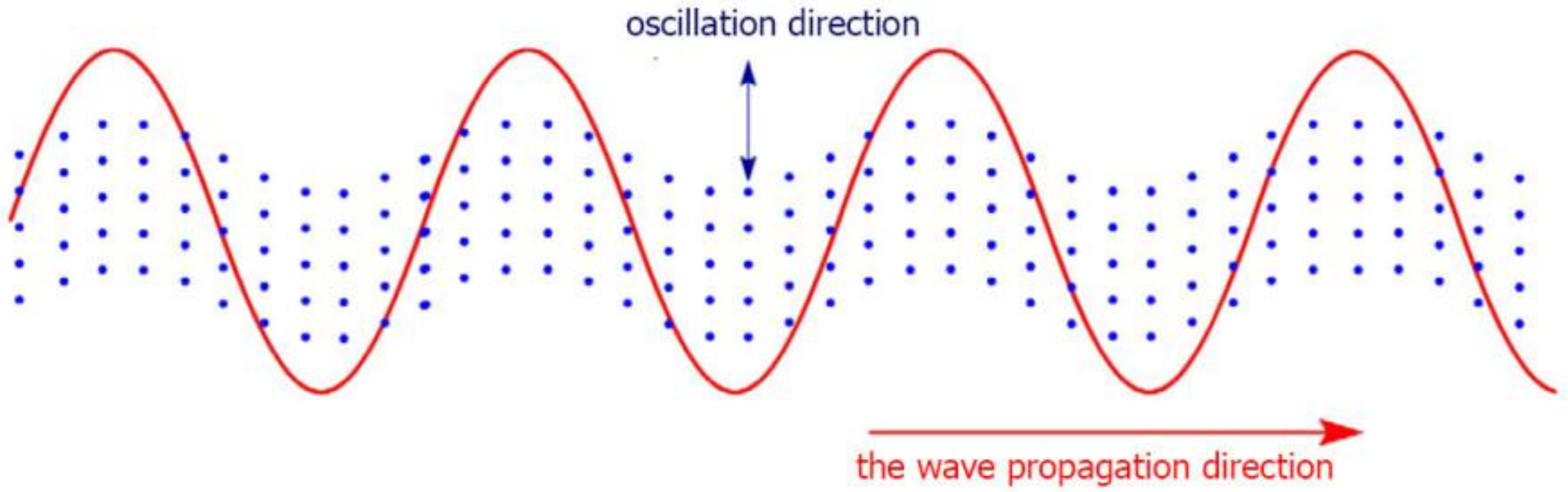
The type of waves

- **Transverse (Shear) waves**

– Characterized by particle motion orthogonal to the direction of the wave propagation

– Shear waves were not often used in ultrasound (**low speed, high attenuation**)

but it's widely used to measure elasticity of tissue, recently.

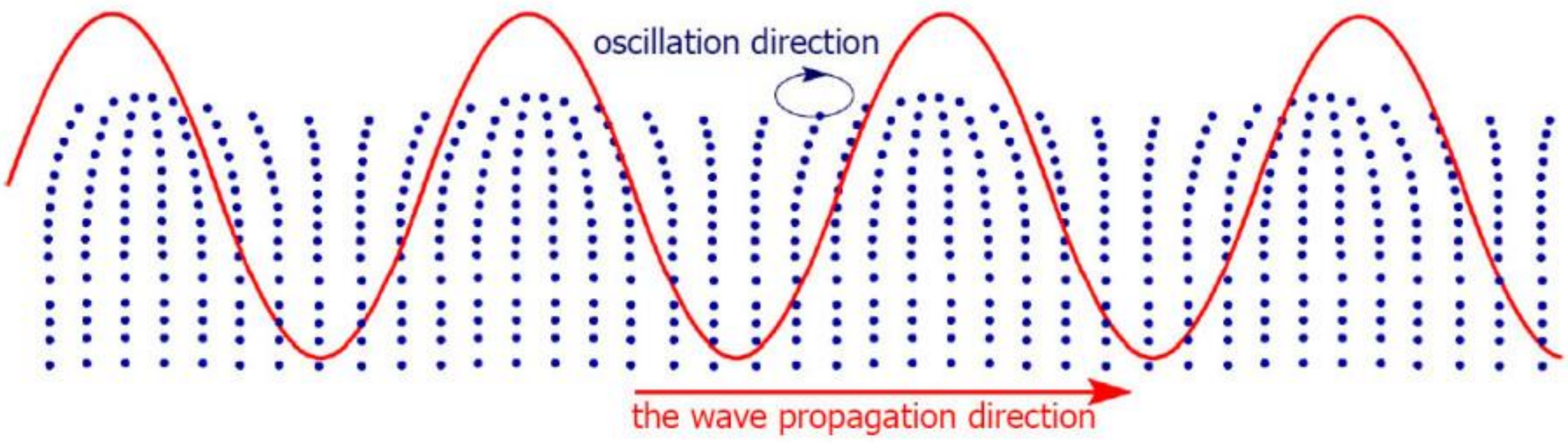


The type of waves

- **Surface waves**

– Wave propagate along the boundary between two media

– Longitudinal + Transverse vibration (**low speed, high attenuation**)

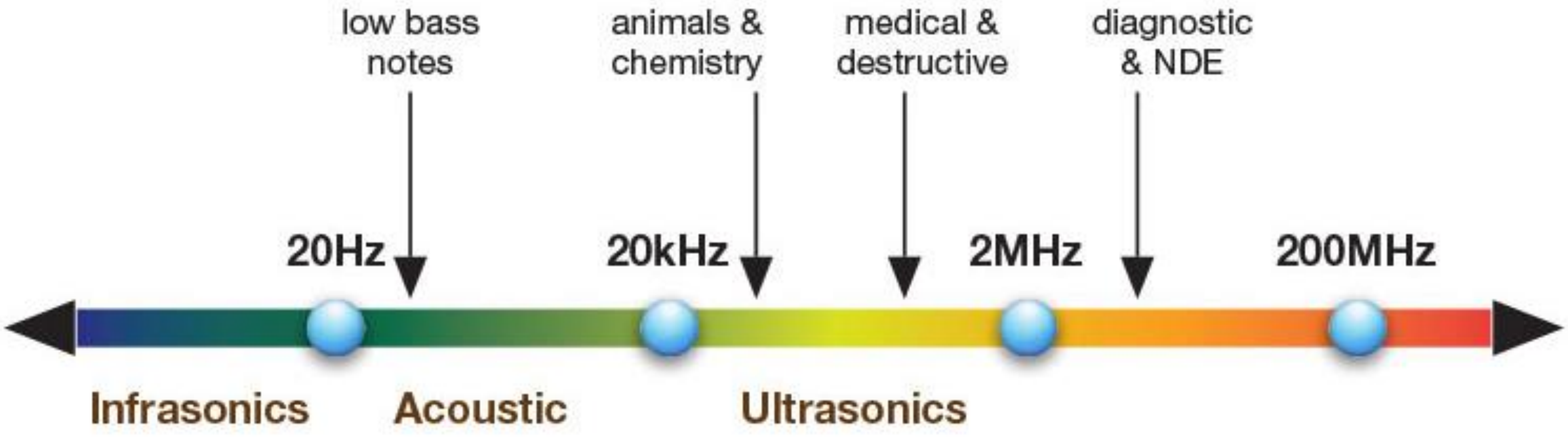


Sound waves (~ any longitudinal wave)

- A type of energy propagation through a medium by means of adiabatic compression and decompression. *(e.g., Audible sound, Earthquake, and Medical ultrasound)*
- A sound wave is similar in nature to a slinky wave

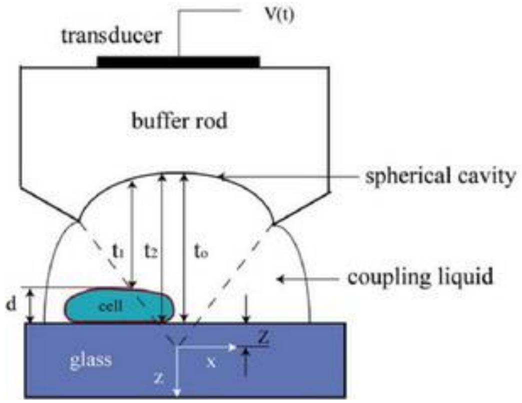
What is Ultrasound?

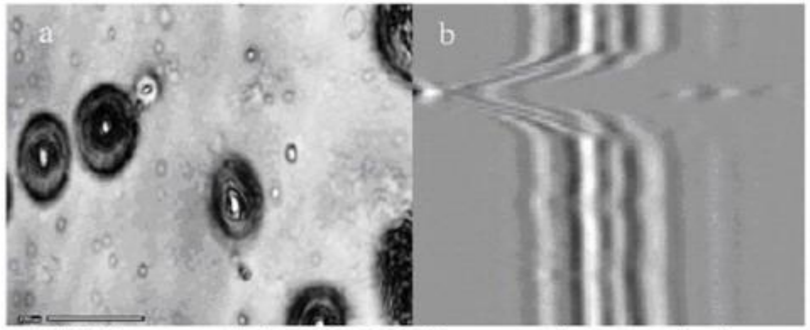
- ***"Sound"*** is the rhythmic compression and decompression of the air around us caused by a vibrating object.
- Audible Frequency range ~ 20 KHz for human
- Ultrasound: > 20 KHz (1-15 MHz are widely used in medical ultrasound)



What is Ultrasound?

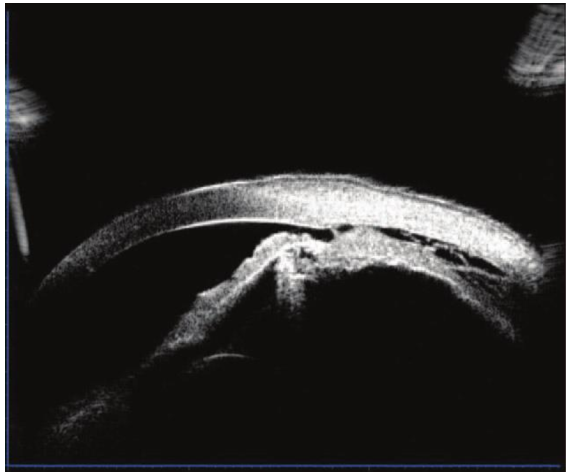
Wavelength (λ)	Frequency (f)	
1.5 μm	1 GHz	Acoustic Microscopy
15 μm	100 MHz	Ultrasound Biomicroscopy
150 μm	10 MHz	Medical Ultrasound Imaging
1.5 mm	1.0 MHz	Ultrasound Therapy
15 mm	0.1 MHZ	
17 mm	0.02 MHZ	Audible sound (340 m/s)





SAM image made by OXSAM of an epoxy layer on aluminum at 300 MHz showing subsurface defects: (a) C-scan; (b) B-scan.

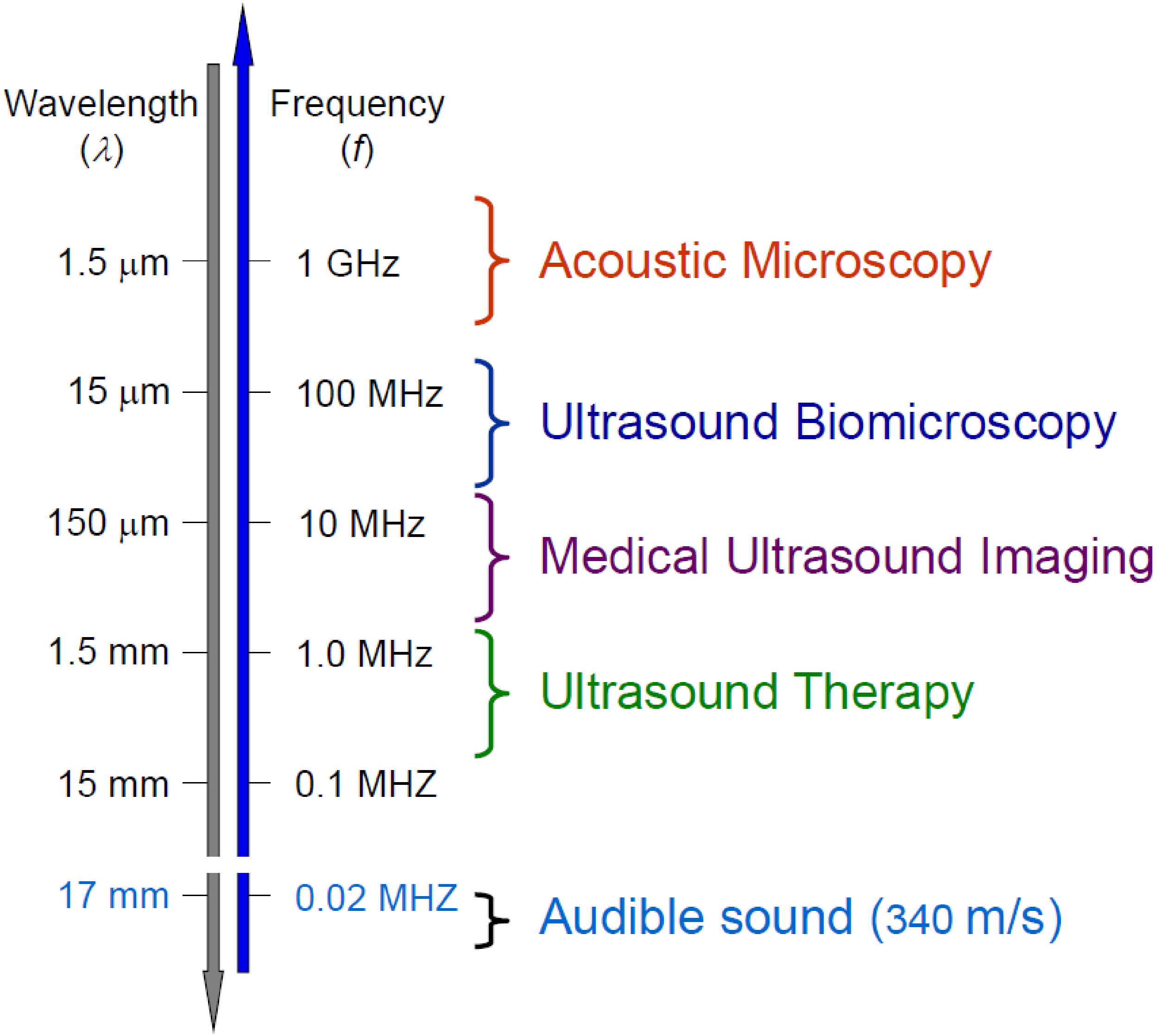
Acoustic microscopy
(1 μm resolution, 10 μm depth)

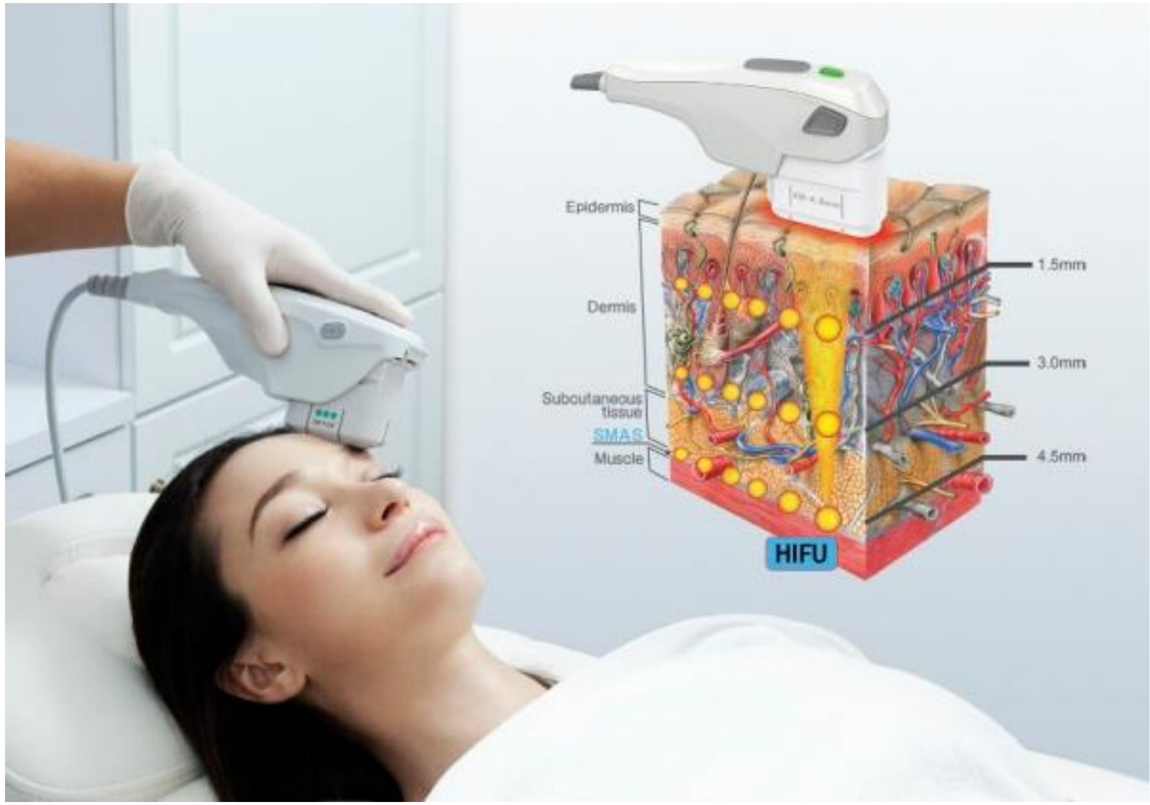


Ultrasound Biomicroscopy
(2-30 μm resolution, <5 mm depth)

Medical Ultrasound
(100~300 μm resolution, ~ 20 cm depth)

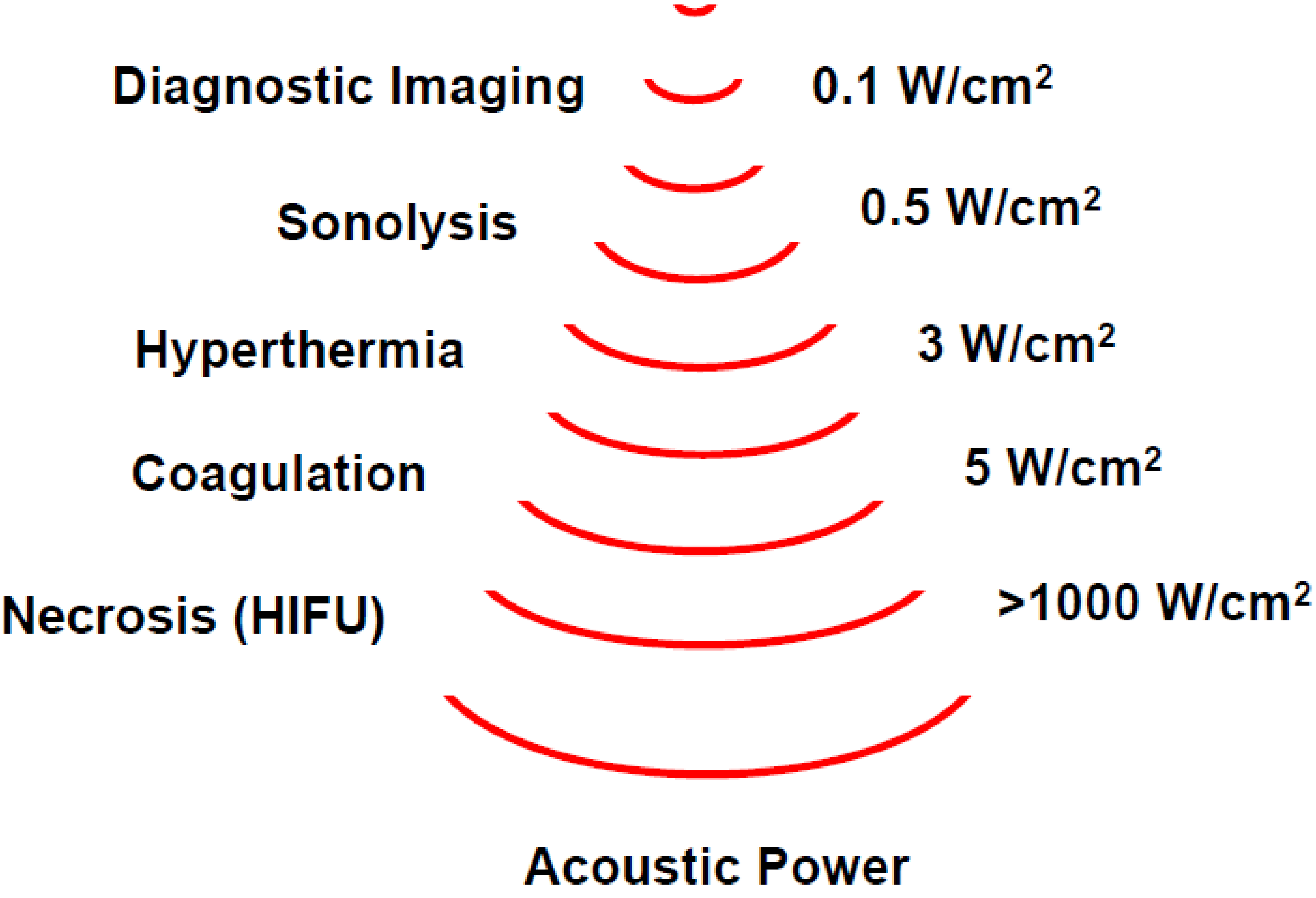
What is Ultrasound?



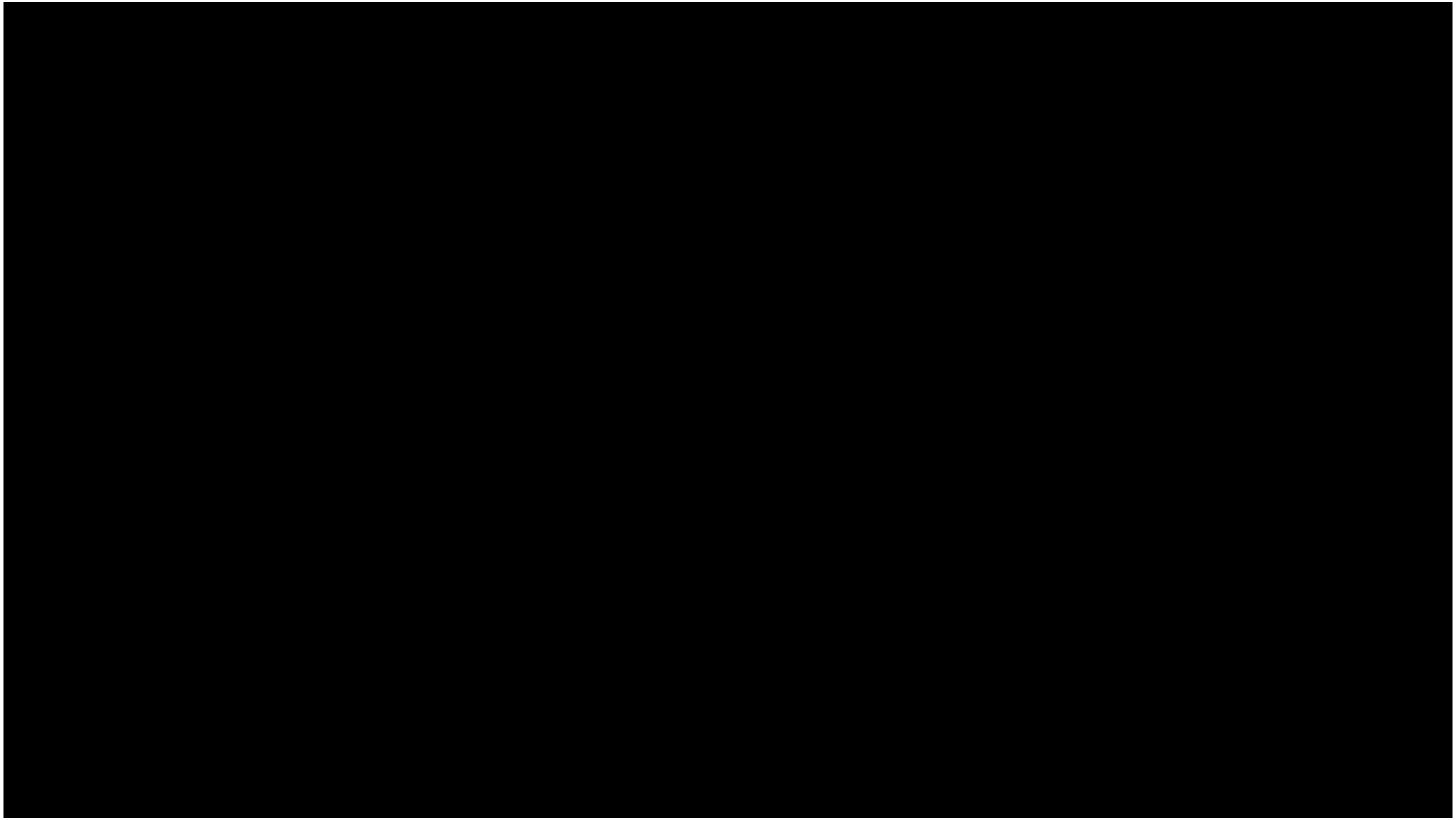


Ultrasound Therapy
Medical – Targeted cell therapy
Cosmetic – Anti-aging, Fat-removal

Acoustic power for different applications



State-of-the-art: Blood-brain barrier (BBB) openings



Ultrasound Imaging – Overview

- What are we imaging?

  Variation in ‘Acoustic impedance’ of tissue

- Difference of acoustic impedances between tissues determines the reflection ratio of transmitted ultrasound wave.

- Transmitted wave interacts with variation in medium, then we can form image of backscattered acoustic wave or “echo”

Ultrasound Imaging – Overview

- Advantages
  - **No ionizing** radiation
  - **Real-time** Imaging (Available to ultrafast imaging up to <100,000 Hz)
  - Relatively inexpensive, portable
  - High spatial resolution (shallow depths, high frequency) or high penetration depth (low frequency)
  - Some functional imaging capabilities

- Disadvantages
  - Not whole body imaging

Current research trend

  - **Low contrast**
  - Speckle –Constructive/destructive interference between US beam and sub-resolution scattering particles
  - Resolution limited by beam width (diffraction)

Ultrasound Imaging – Overview

- Advantages
  - **No ionizing** radiation
  - **Real-time** Imaging (Available to ultrafast imaging up to <100,000 Hz)
  - Relatively inexpensive, portable
  - High spatial resolution (shallow depths, high frequency) or high penetration depth (low frequency)
  - Some functional imaging capabilities

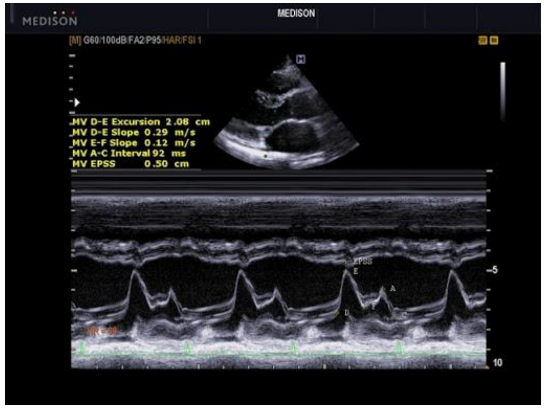
- Disadvantages
  - Not whole body imaging
  - **Low contrast**
  - Speckle
  - Limited Resolution

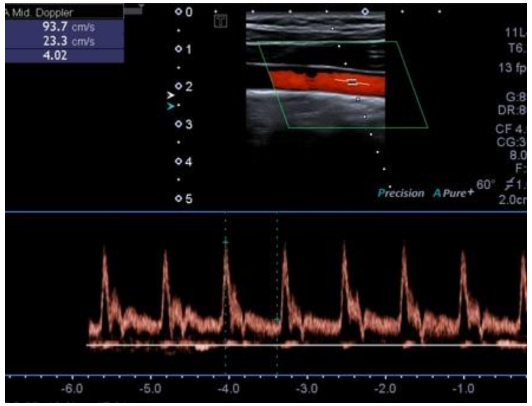
Current research trend

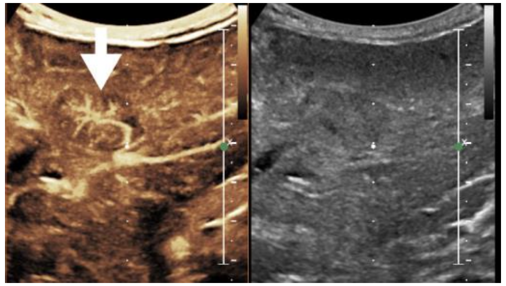
> Functional Imaging technique
> Elastography
> Super-resolution imaging

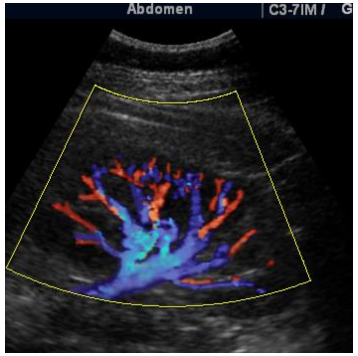
Imaging modes

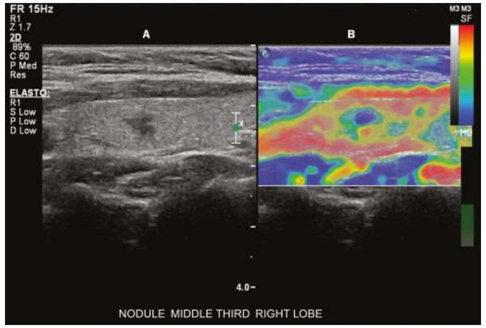
- B-mode / M-mode
- Doppler
  - 1-D Spectral Doppler
  - 2-D Color / Power Doppler
- 3-D & 4-D Imaging
- Contrast enhanced (Harmonic) imaging
- Elastography
- Thermal Strain Imaging
- Photoacoustic Imaging
- Super-resolution Imaging
- Molecular Imaging

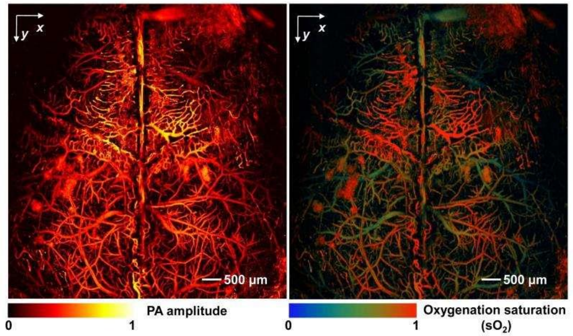


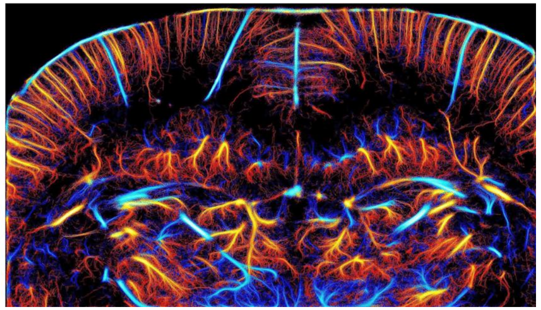












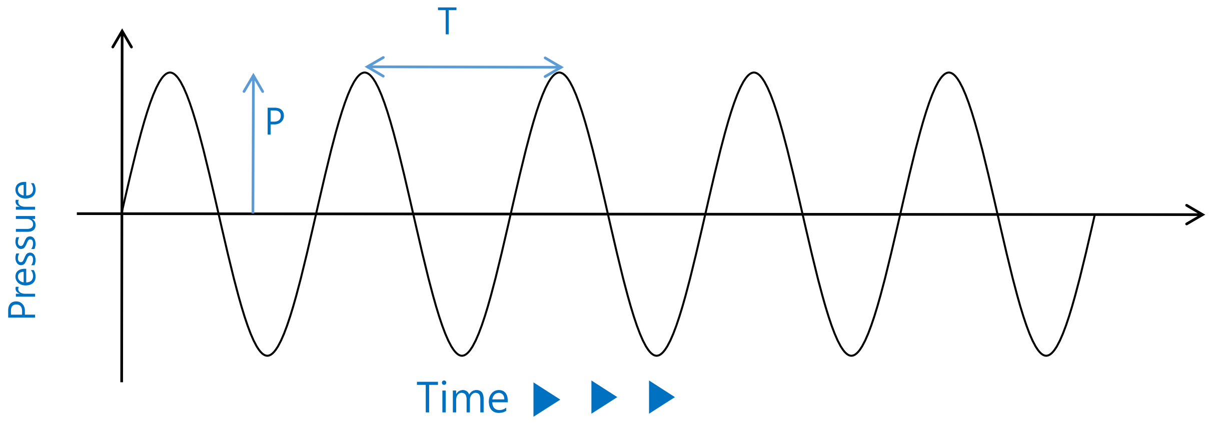
Basic Principles

Basic principle

- Acoustic pressure
  - Strength of a wave
  - Atmospheric pressure : 14.7psi (Pounds per square inch) = 100,000 Pa
  - Some diagnostic ultrasound beams : 10 Mpa

- Compressions and rarefactions
  - regions of compression ≥ 14.7psi
  - regions of rarefactions ≤ 14.7psi

- Period and frequency
  - $T = 1/f$



Basic principle

- Speed (of sound)
  - Bulk modulus : (stiffness)
  - $C \propto \sqrt{\frac{B}{\rho}}$ (B : bulk modulus, ρ : density)

- Wavelength
  - $\lambda = c/f$
  - related to imaging factor (spatial resolution)

Table 1-1. Speed of sound in some nonbiological materials*

Material	Sound speed (m/s)
Air	330
Water	1480
Lead	2400
Aluminum	6400

*From Wells PNT: *Biomedical ultrasonics*, New York, 1977, Academic Press.

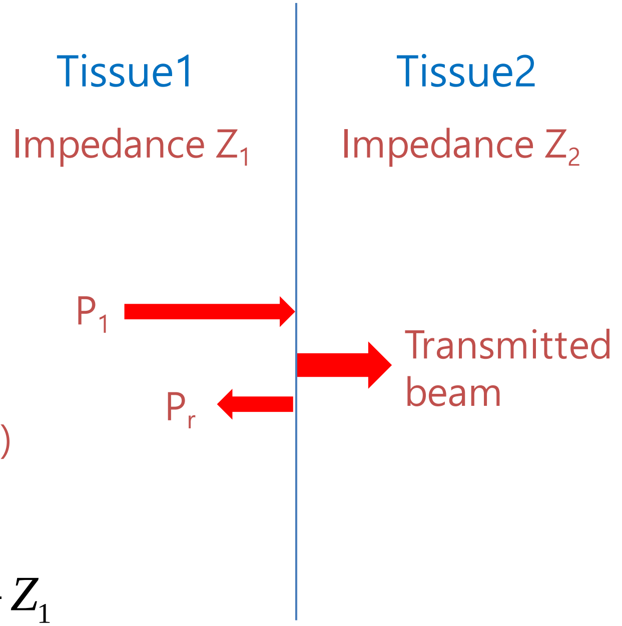
Table 1-2. Speed of sound in selected tissues[2-5]

Material	Sound speed (m/s)
Lung	600
Fat	1460
Aqueous humor	1510
Liver	1555
Blood	1560
Kidney	1565
Muscle	1600
Lens of eye	1620
Skull bone	4080

Basic principle

- Amplitude and intensity
  - $I = \dfrac{P^2}{2c\rho}$ (W/㎡) : ultrasonic power per unit area

- Reflection and transmission
  - Acoustic impedance $Z = \dfrac{\text{Acoustic pressure}}{\text{Flow}\left(\text{velocity}\right)}$
  - Z = ρc (ρ : kg/m, c : m/s) = $\sqrt{\rho B}$ (plane wave)
  - Reflection coefficient
    - Amplitude reflection coefficient : $R = \dfrac{P_r}{P_i} = \dfrac{Z_2 - Z_1}{Z_2 + Z_1}$
    - intensity reflection coefficient : $\dfrac{I_r}{I_i} = \left(\dfrac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$



Basic principle

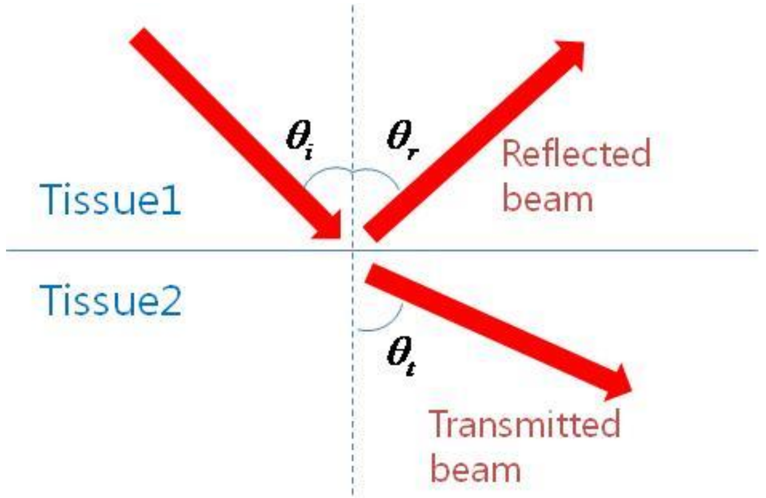
- Reflection and transmission
  - Reflection coefficient
    - Example of Reflection coefficients

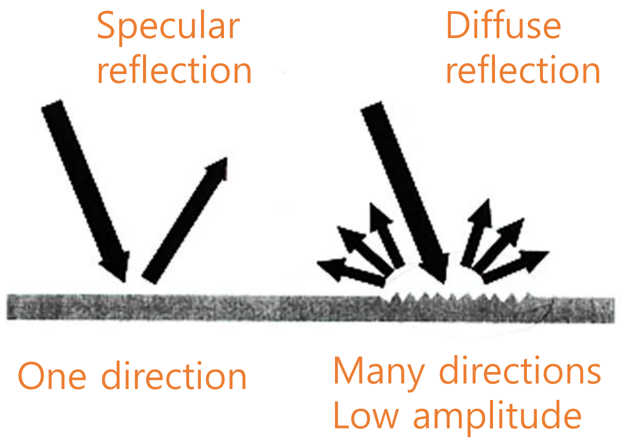
interface	Pr/Pi
fat – muscle	0.10
fat – kidney	0.08
blood – muscle	0.03
kidney – liver	0.01
liver – muscle	0.02
soft tissue – water	0.05
fat – bone	0.69
muscle – bone	0.64
soft tissue – air	0.99

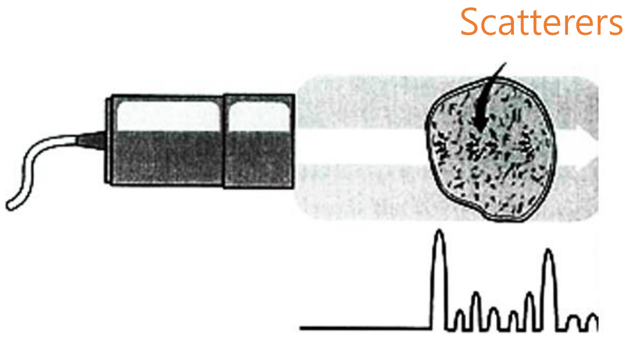
Coupling medium: Ultrasound gel

Basic principle

- Reflection and transmission
  - Nonperpendicular sound beam incidence (refraction)
    - Snell's law : $\frac{\sin\theta_t}{\sin\theta_i} = \frac{C_2}{C_1}$
  - Specular and diffuse scattering



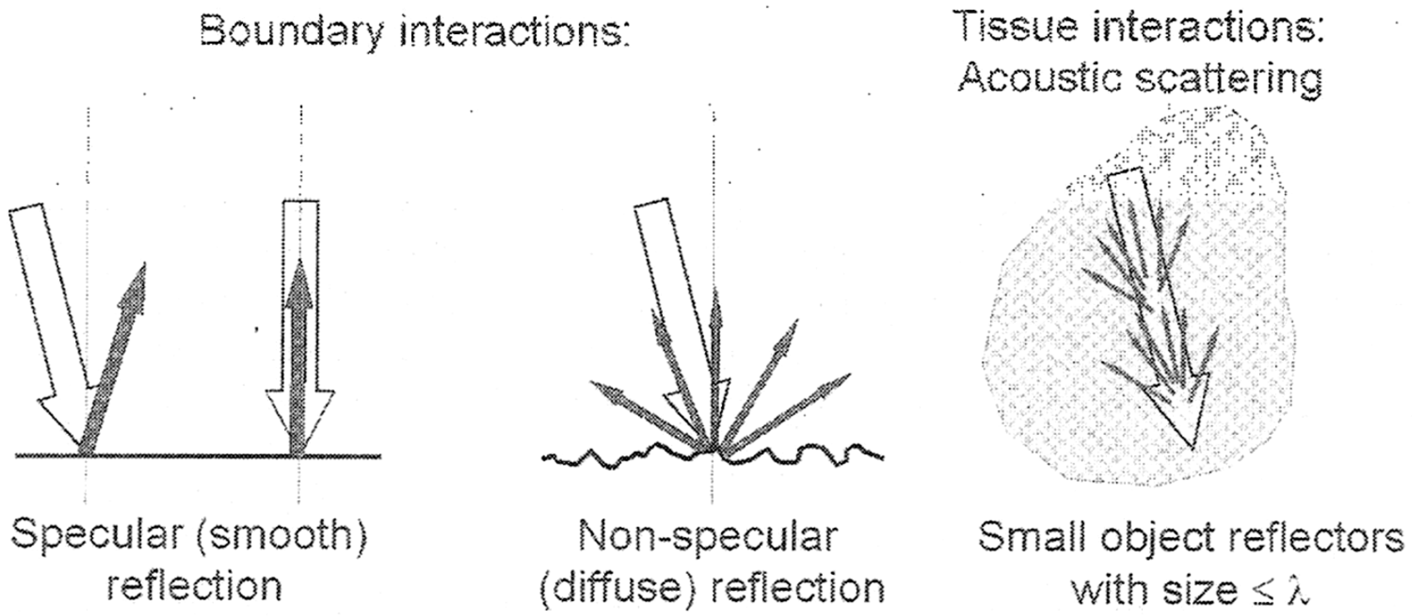




Basic principle

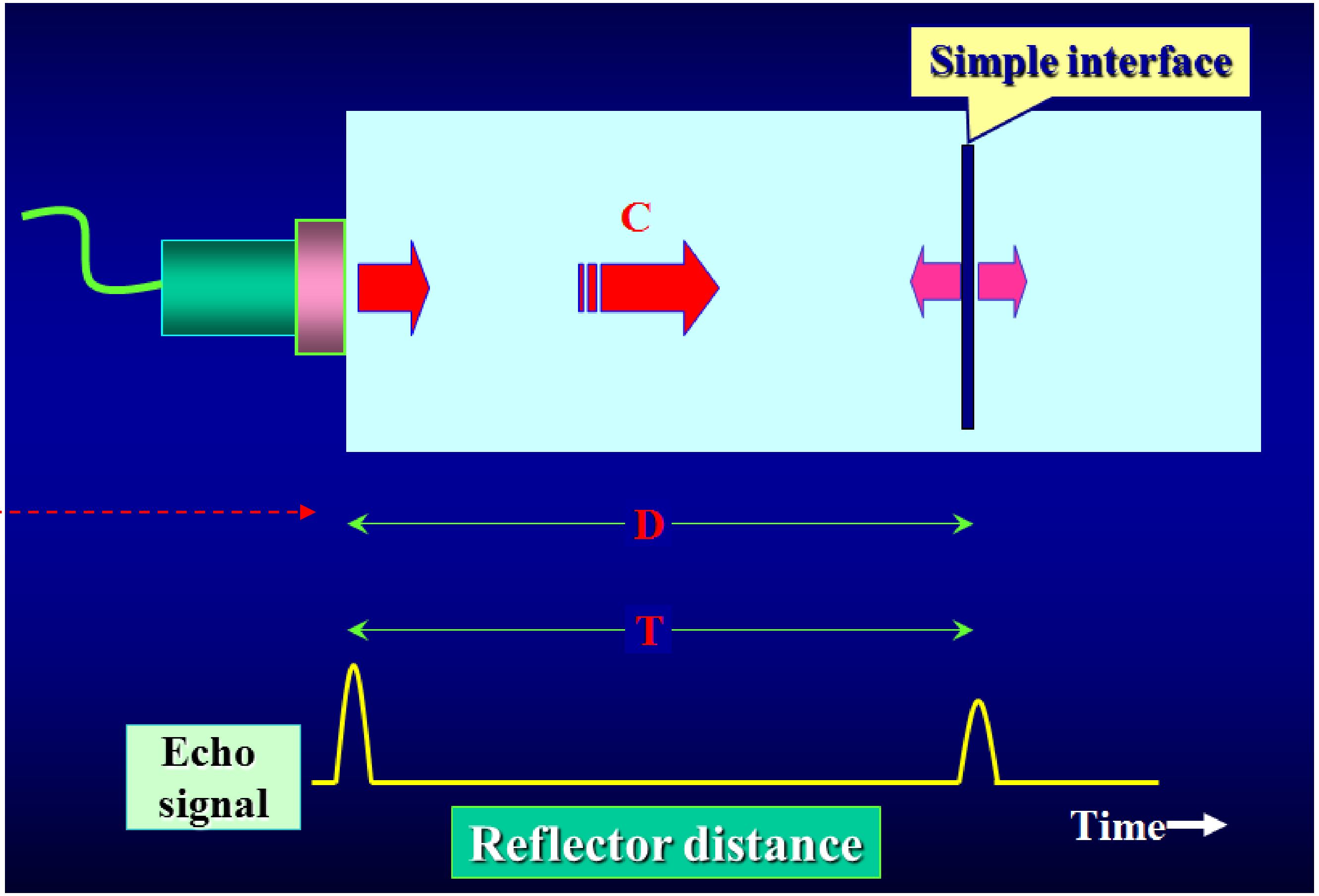
- Images reconstruct Reflectivity in the body, R(x, y, z)

- Surface reflection @ boundary, (Strong signal)
- Volume scatter (backscatter, Weak signal)



Basic principle

Pulsed Ultrasound – range information



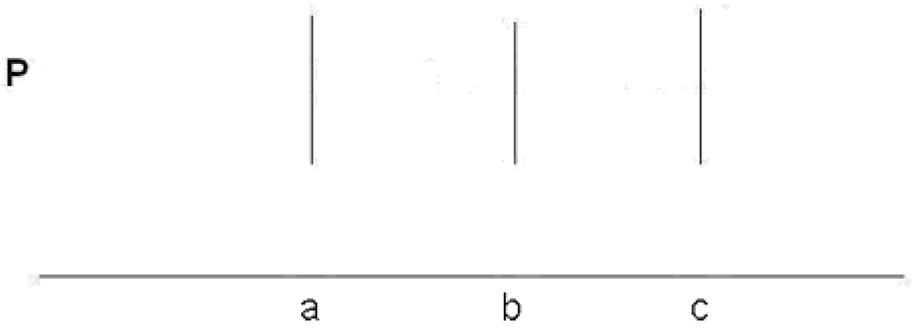
Roundtrip

$$T = \frac{2D}{C}$$

Basic principle

- Step 1: The ultrasonic (pressure) pulse is transmitted into the body
- Step 2: A backscattered ultrasonic wave is received
- Step 3: The received signal is used to form an image

RV/LV (Right/Left Ventricle)
(filled with fluid > No echo)



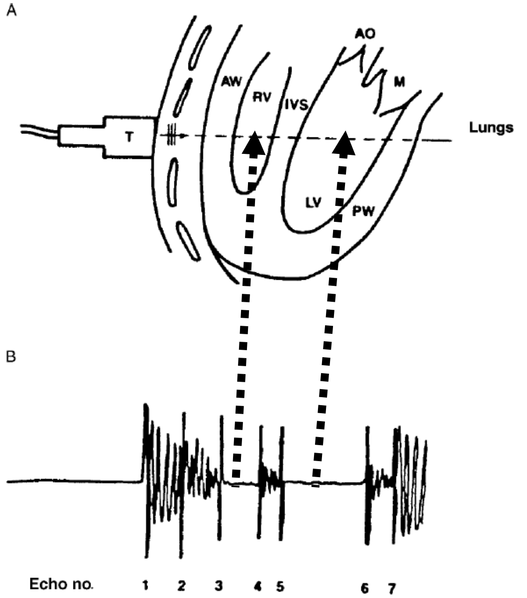


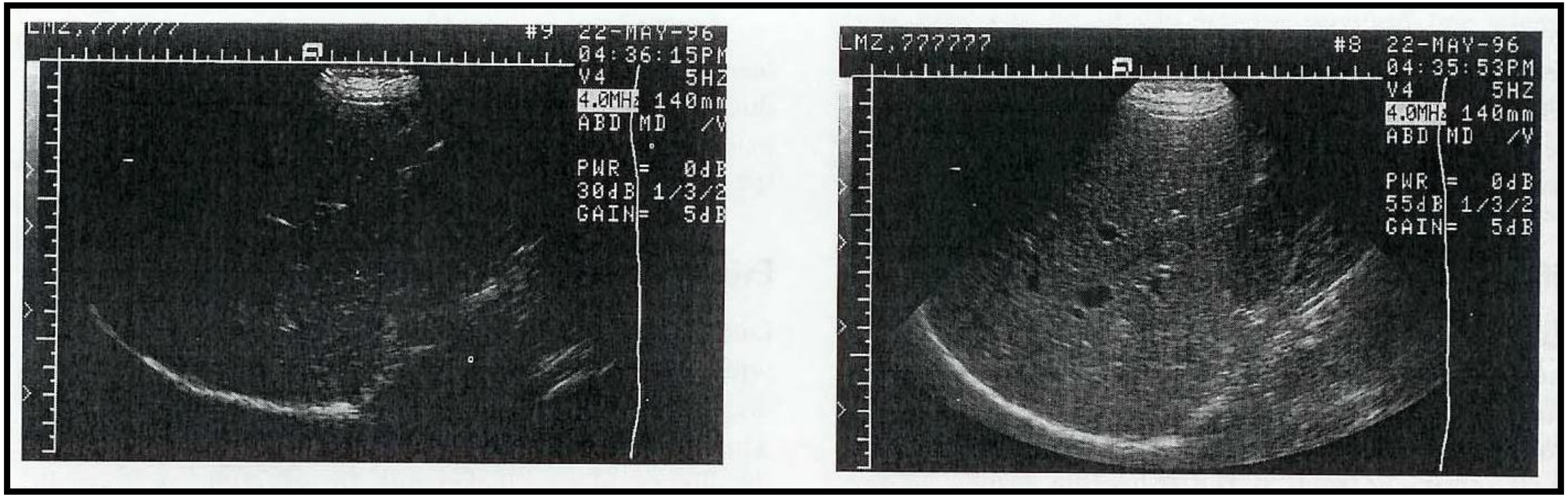
Figure 10.12 (A) Echo path through the heart. AW = anterior wall, RV = right ventricle, IVS = intraventricular septum, LV = left ventricle, AO = aortic valve, M = mitral valve, PW = posterior wall. (B) Amplified echoes corresponding to path in (A) (from Shoup and Hart, 1988, *IEEE*).

Time of flight → depth information

Pulse echo amplitude = Reflectivity

Basic principle

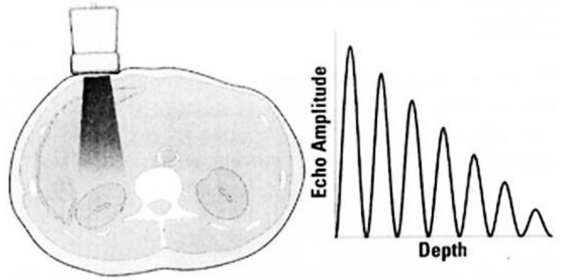
- Echoes from scattering
  - For very small scatterers, echo signals depend on the following
    - *The number of scatterers per unit volume*
    - *Acoustic impedance changes at the interface (reflection)*
    - *The size of scatterer*
    - *The ultrasonic frequency. (spatial resolution)*

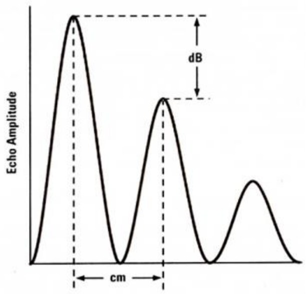


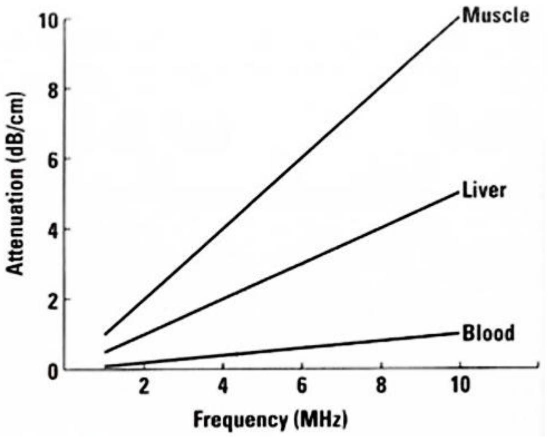
Basic principle

- Attenuation
  - Sources of attenuation
    - Reflection and scatter at interfaces
    - Absorption (=> heat energy)
  - Attenuation coefficient (dB/(MHz·cm))

Low frequency for deeper imaging depth







- Attenuation in soft tissue is highly dependent on the ultrasonic frequency

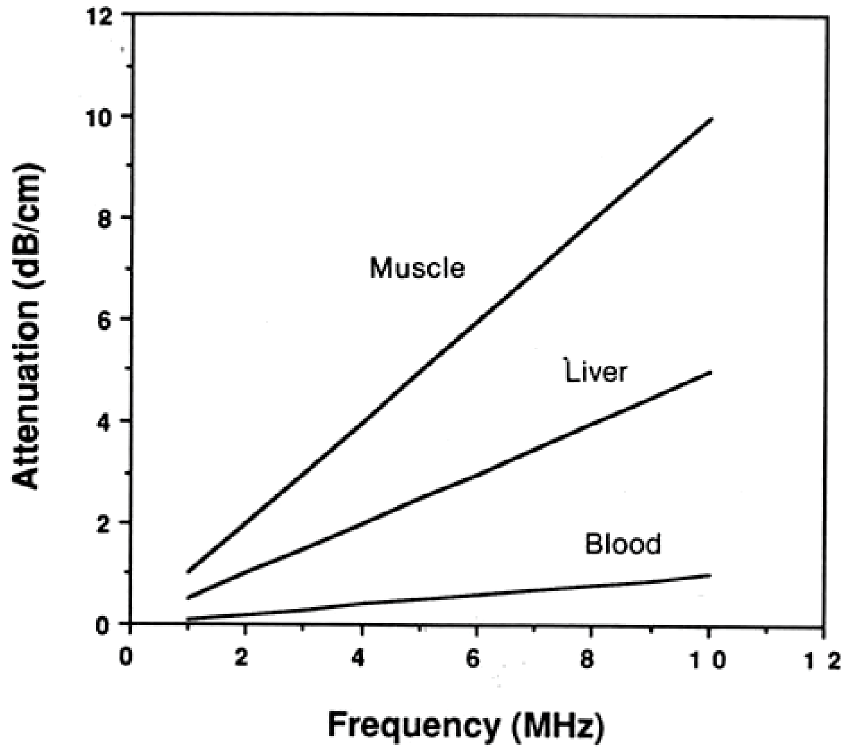
Basic principle

- Attenuation: example

$$\beta = 0.6dB/MHz/cm, \qquad \mathrm{f = 3.5MHz,\ d = 20cm}$$

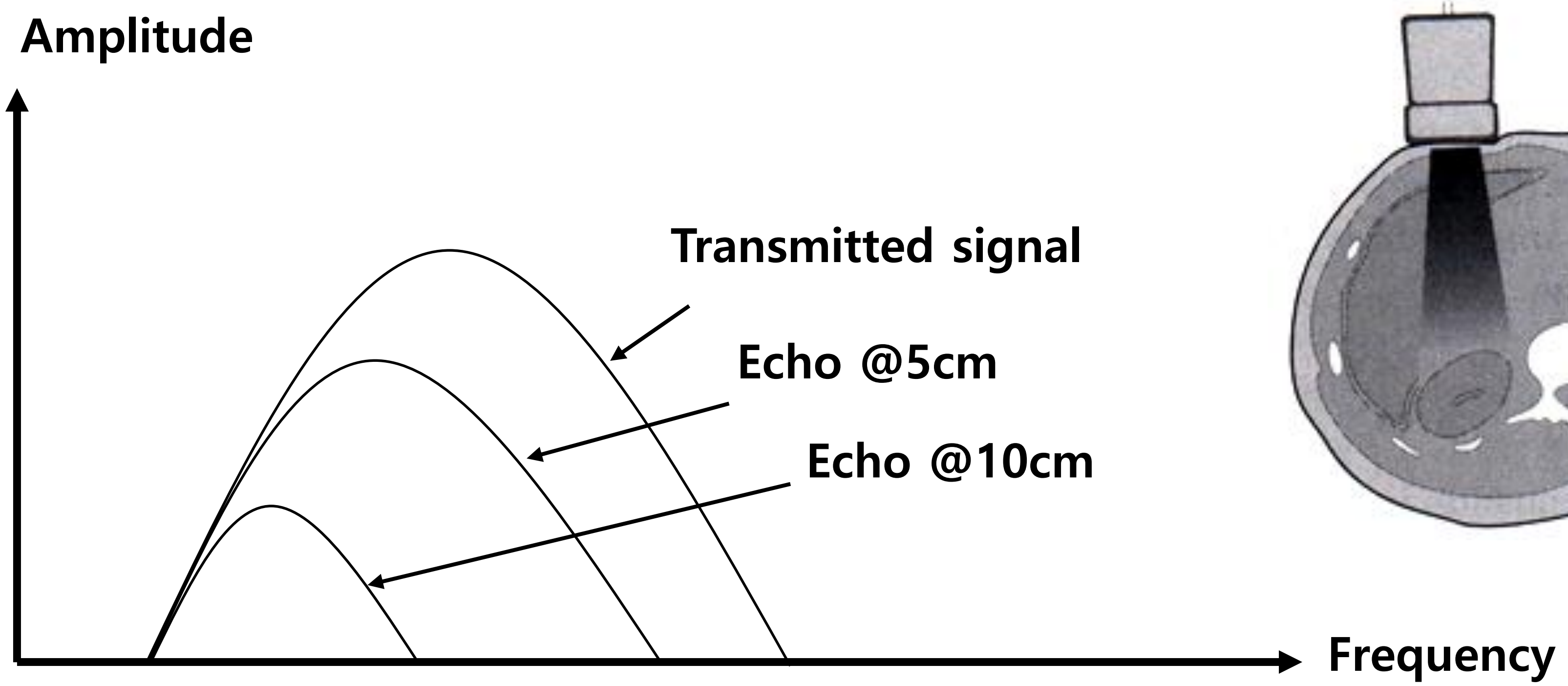
$$0.6[dB/MHz/cm] \times 3.5[\mathrm{MHz}] \times 20[\mathrm{cm}] = 42dB$$

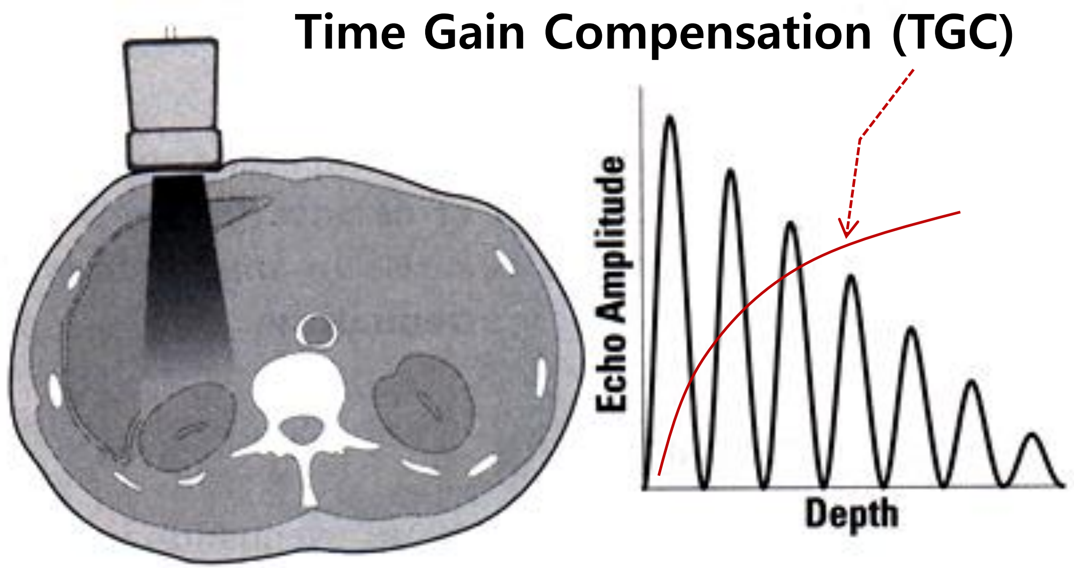
$\frac{A}{A_0} =$	1/2	1/10	1/100	1/1000
	⇕	⇕	⇕	⇕
$20log\left(\frac{A}{A_0}\right) =$	-6 dB	-20 dB	-40 dB	-60 dB
	6 dB	12 dB	20 dB	40 dB
	⇕	⇕	⇕	⇕
$\frac{A}{A_0} =$	2	4	10	100



Basic principle

- Attenuation: example





Signal amplitude is decreased at deeper depth
Echo frequency is decreased at deeper depth (why?)

Basic principle

- Attenuation coefficient

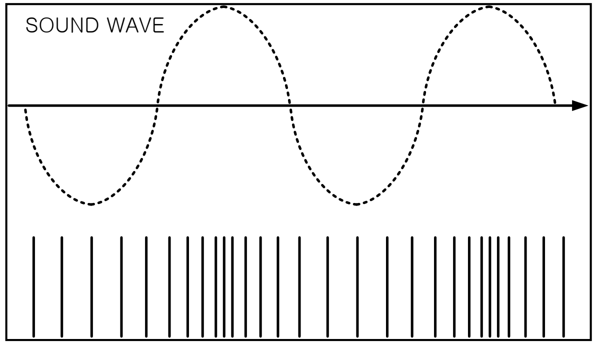
	Velocity $m/\sec$	Attenuation $dB/(MHz \cdot cm)$	Characteristic Impedance $10^6\, kg/(m^2 \cdot \sec)$
Water	1480	0.0025	1.48
Air	330	12.000	0.0004
Amniotic	1510	0.007	1.5
Fat	1410 – 1470	0.35 – 0.78	1.34 – 1.39
Soft tissue	1540	0.81	1.62
Liver	1550	0.95	1.66
Kidney	1560	1.1	1.63
Muscle	1590	1.5 – 3.3	1.71
Spleen	1550	0.52	1.65
Bone	4080	12.0	7.8
Vitreous of eye	1520	0.1	1.52

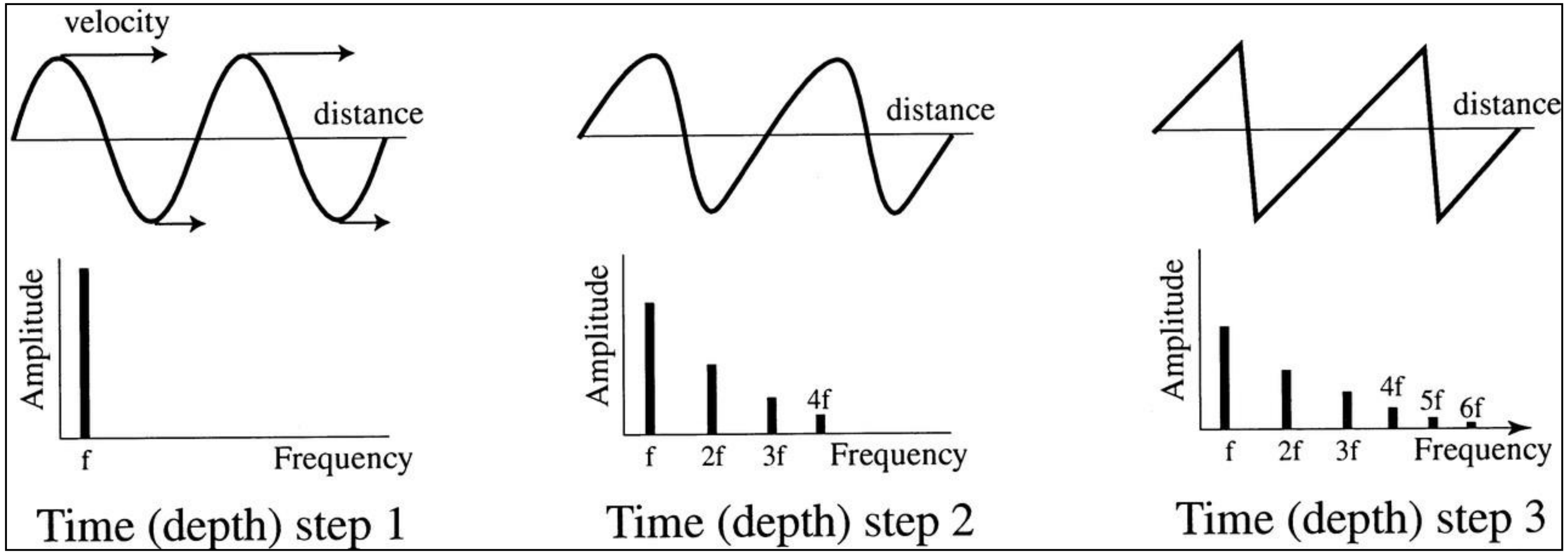
Basic principle

- Nonlinear propagation of Ultrasound

$$c = c_o + \frac{1}{2\rho_o c_o} \cdot (B/A) \cdot p(t)$$

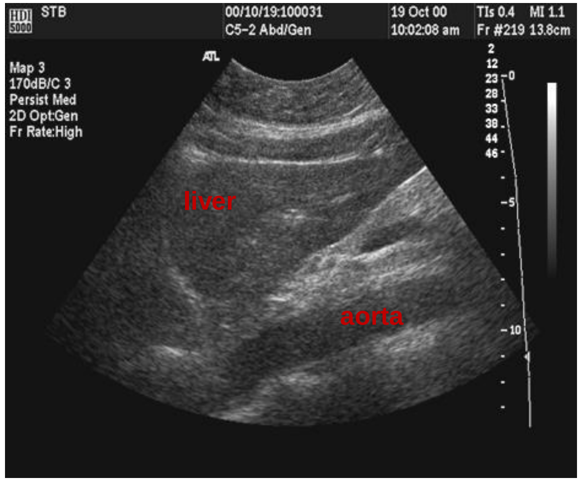
$$B/A = 2\rho_o c_o \frac{\partial c}{\partial p}$$

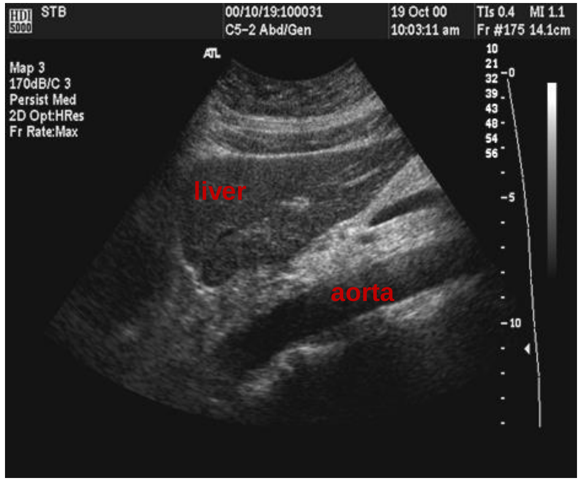


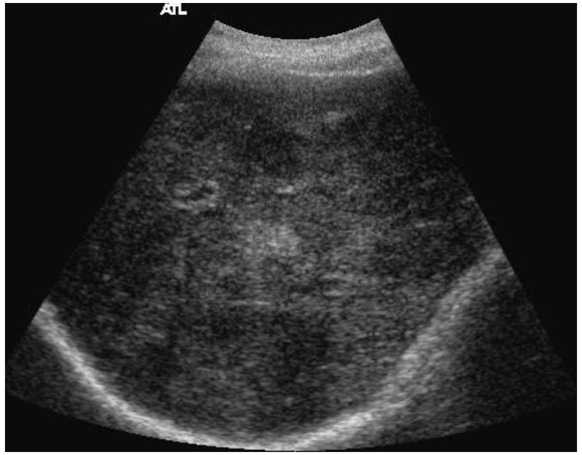


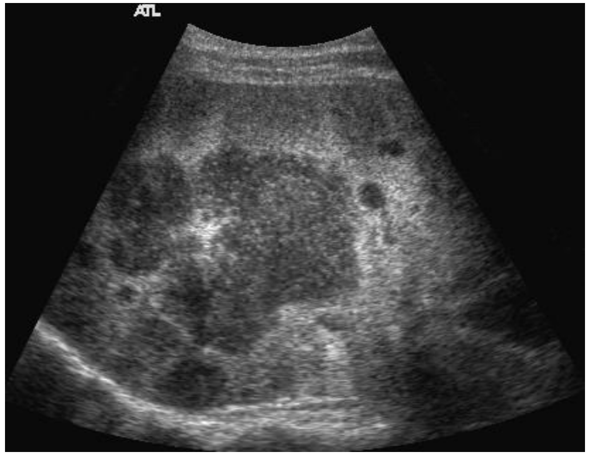
Basic principle

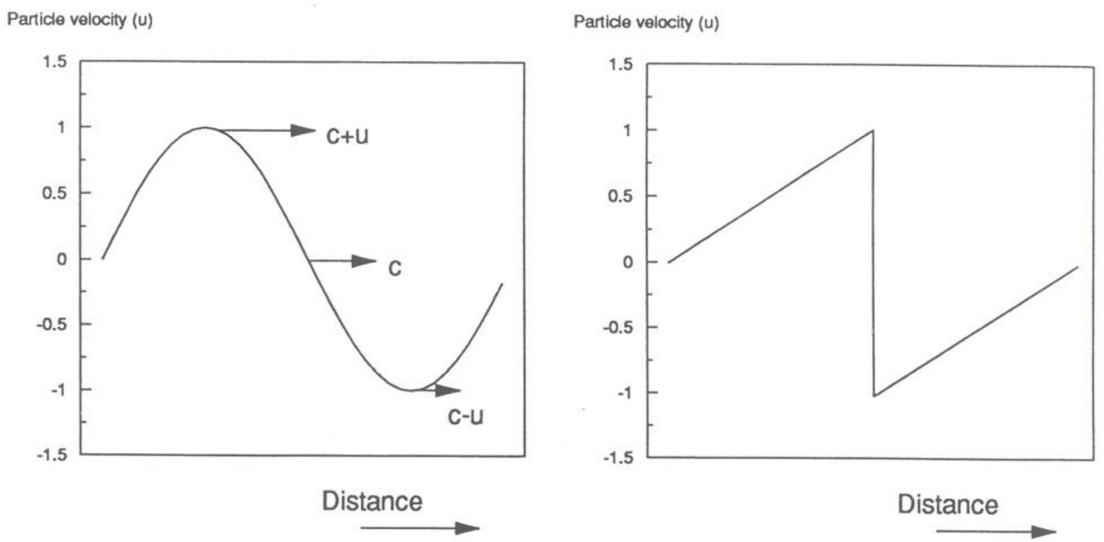
Fundamental | Harmonic (Non-linear)

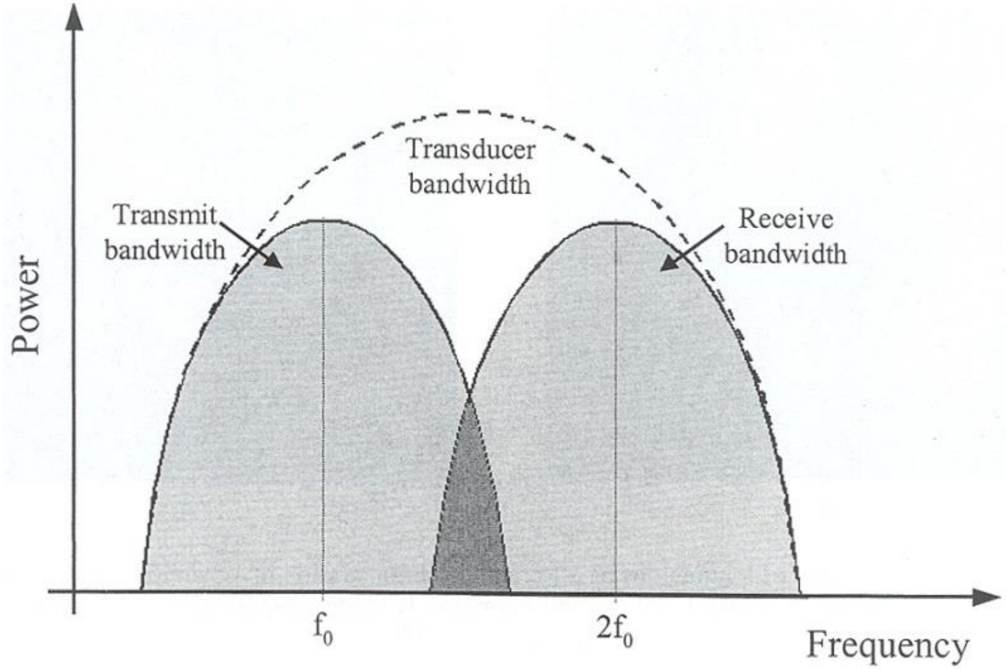












Basic principle

- Ultrasound parameters of healthy & pathological tissues
  : liver

tissue	Acoustic impedance rayls, kg/(m^2 sec)	Attenuation Coefficient (dB/cm at 4MHz)	Non-linear parameter (B/A)
Healthy liver	1,683,280 (ref)	2.09 (ref)	6.8 (ref)
Parasitic hepatitis	**1,638,000 (-3%)**	**2.20 (+5%)**	**7.4 (+9%)**
Fibrinoid hepatitis	**1,641,150 (-3%)**	**2.21 (+6%)**	**7.4 (+9%)**
Necrotic hepatitis	**1,630,650 (-3%)**	**2.19 (+5%)**	**7.6 (+12%)**
Intersitial hepatocirrhosis	**1,638,000 (-3%)**	**2.18 (+4%)**	**7.4 (+9%)**
Parasitic hepatocirrhosis	**1,603,350 (-5%)**	**2.41 (+15%)**	**7.8 (+15%)**
Biliary hepatocirrhosis	**1,609,650 (-4%)**	**2.51 (+20%)**	**8.1 (+19%)**
Fatty liver	**1,604,400 (-5%)**	**2.44 (+17%)**	**8.3 (+22%)**
Hepatonecrosis	**1,617,000 (-4%)**	**2.44 (+17%)**	**8.5 (+25%)**

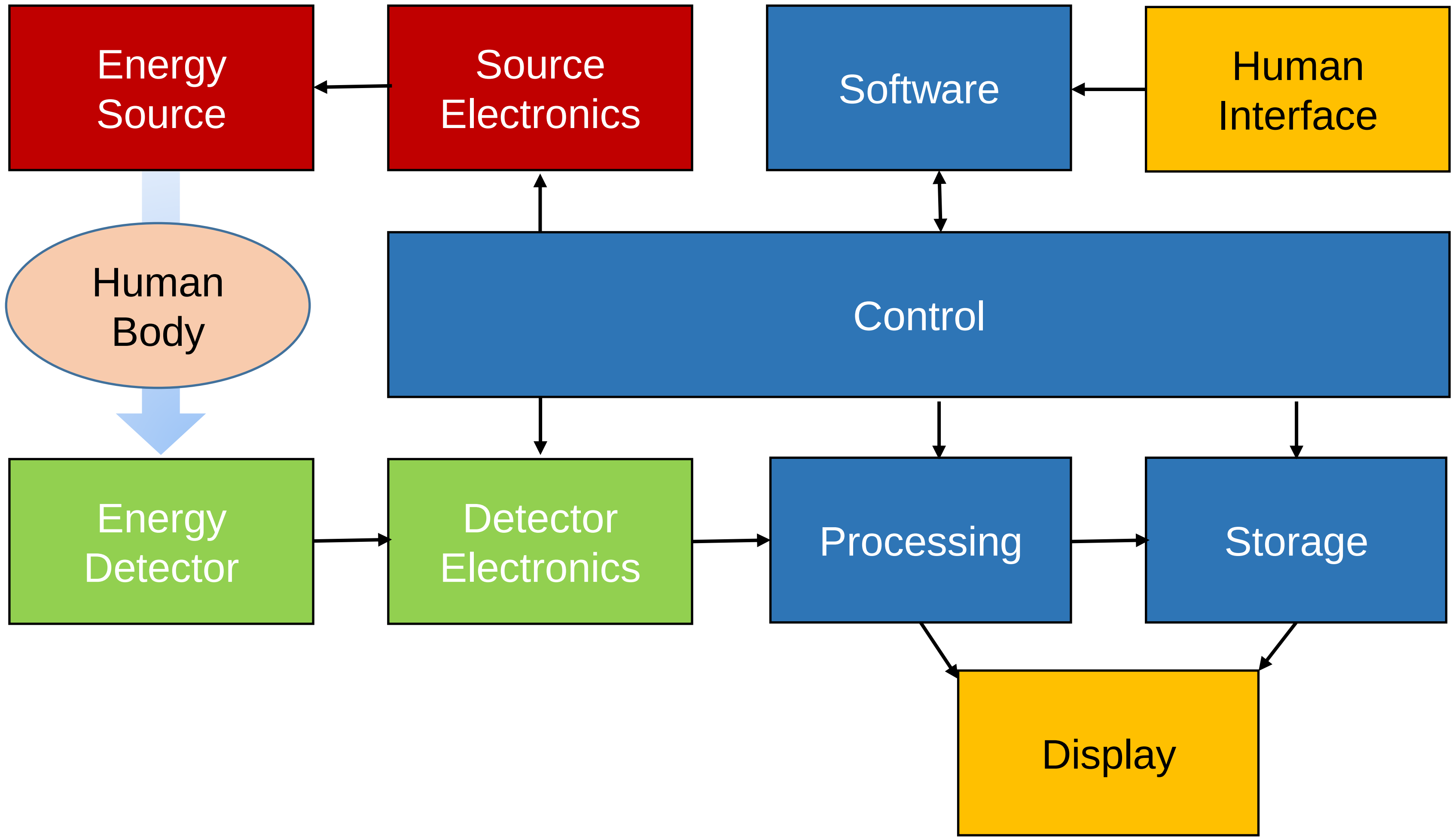
Ref: Zhang and Gong (1999). UMB, 25(4), 593-599

Parameters

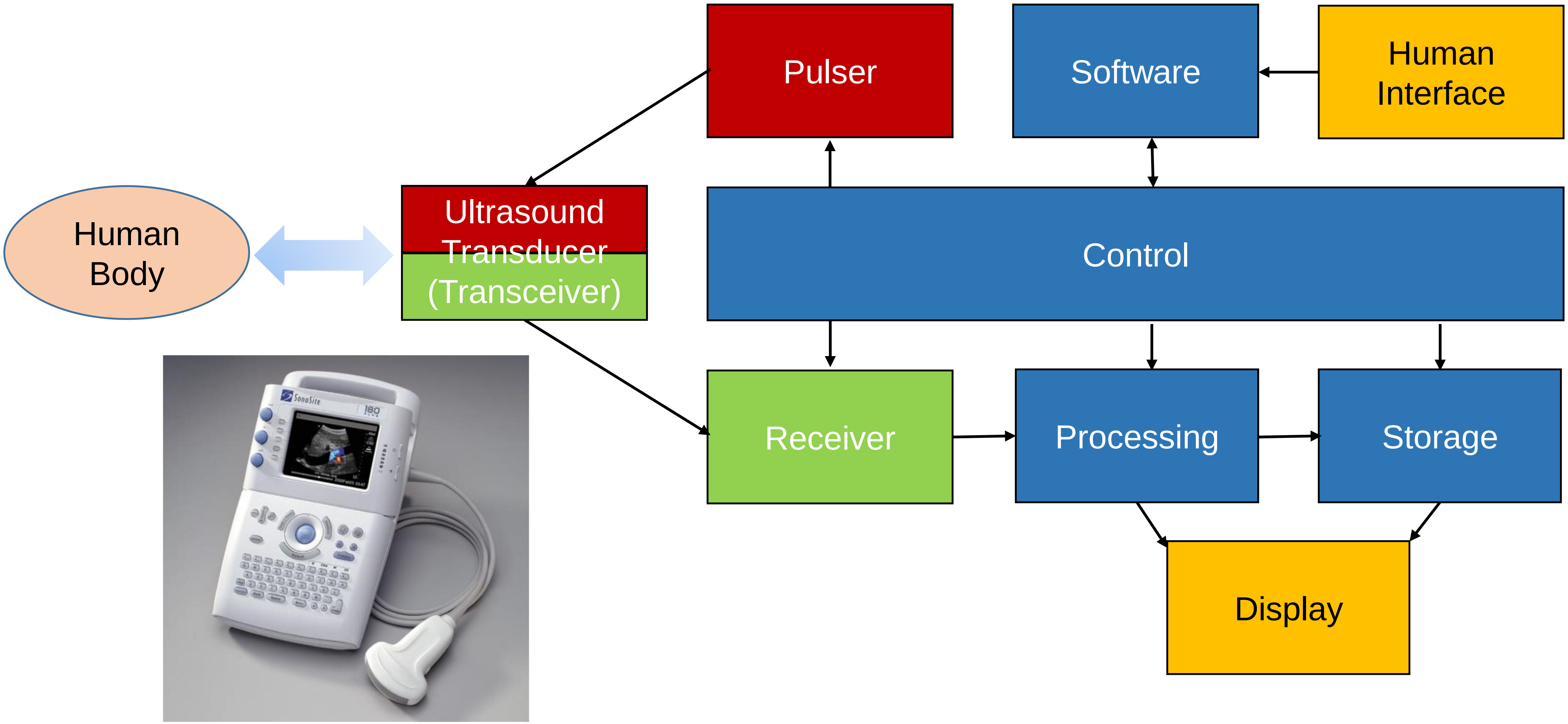
- **Wave characteristic (in Pulser)**
  - Frequency vs Attenuation
  - Duration (Bandwidth) vs Spatial resolution
  - Amplitude vs Safety
  - Pulse repetition frequency vs temporal resolution & imaging depth

- **Transducer**
  - Frequency is normally determined by the transducer
  - Duration is also primarily determined by the transducer
  - Needs to be efficient during transmit and sensitive during receive

Medical Imaging System



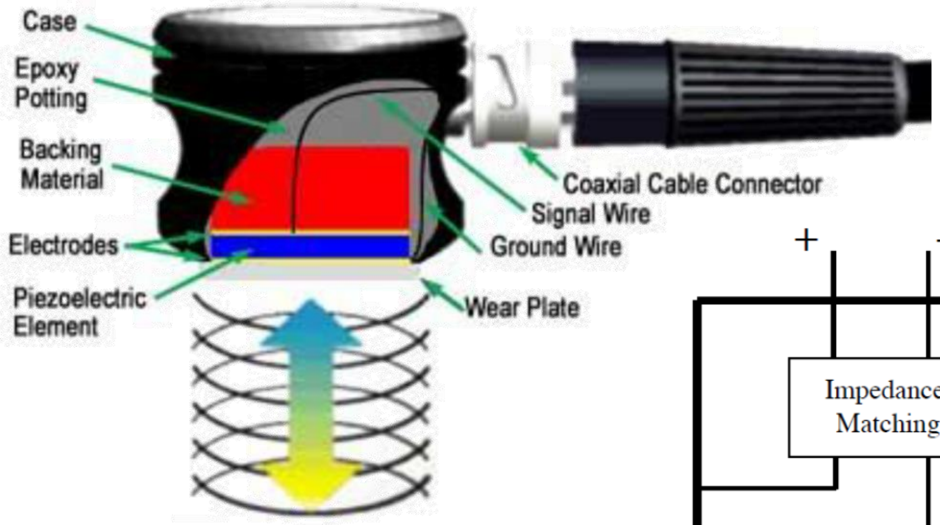
Medical Ultrasound Imaging System

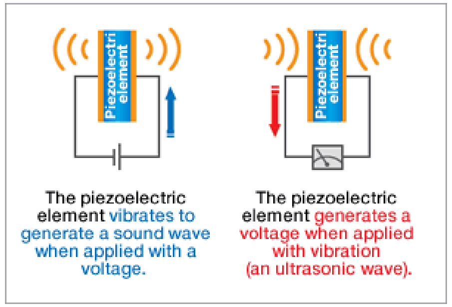


Transducers

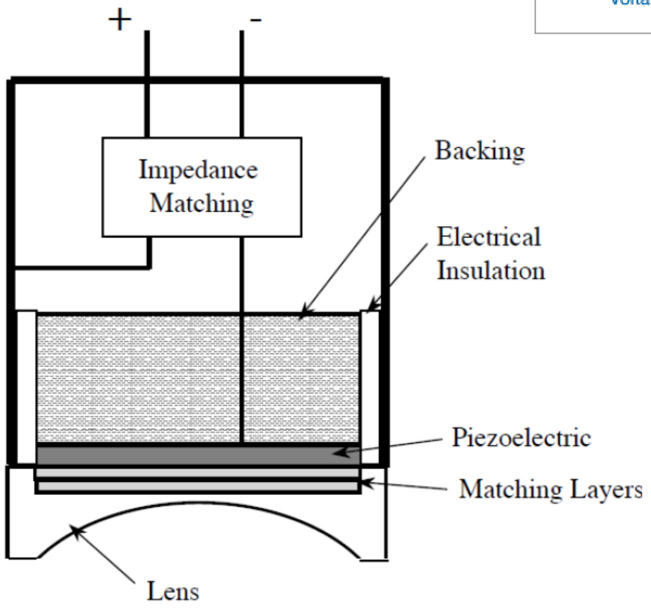
Transducer

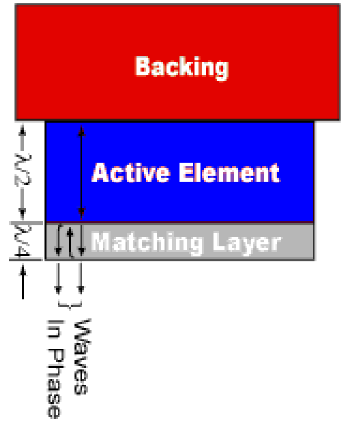
- PZT (Lead Zirconate Titanate) crystal: Electro-mechanical (Piezo-electric) element that is used both to transmit and to receive the ultrasound signal





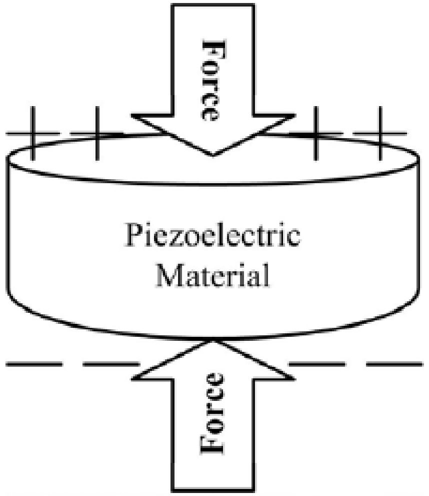
- Backing
- Matching layer
- Lens

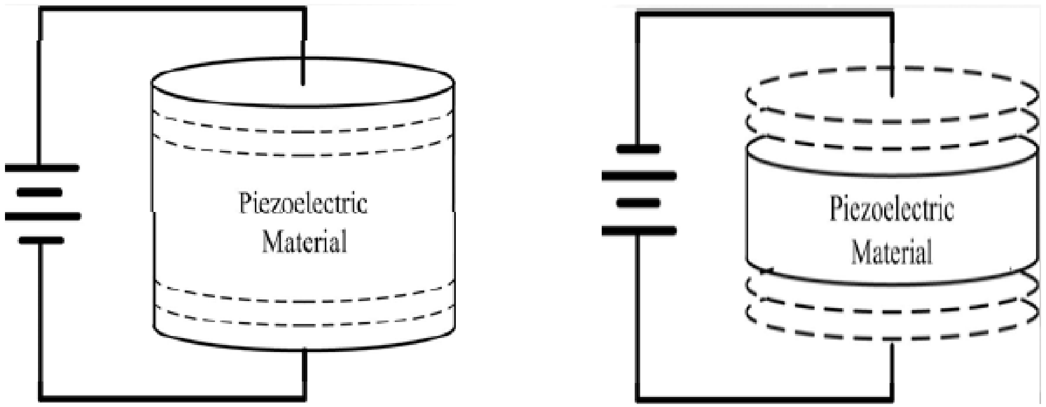




Piezoelectric Effect

Piezoelectric effect: Mechanical Energy ↔ Electrical Energy
(e.g, quartz watch!)





<u>Direct Piezoelectric effect</u>

A force applied to a piezoelectric material produces an electrical signal

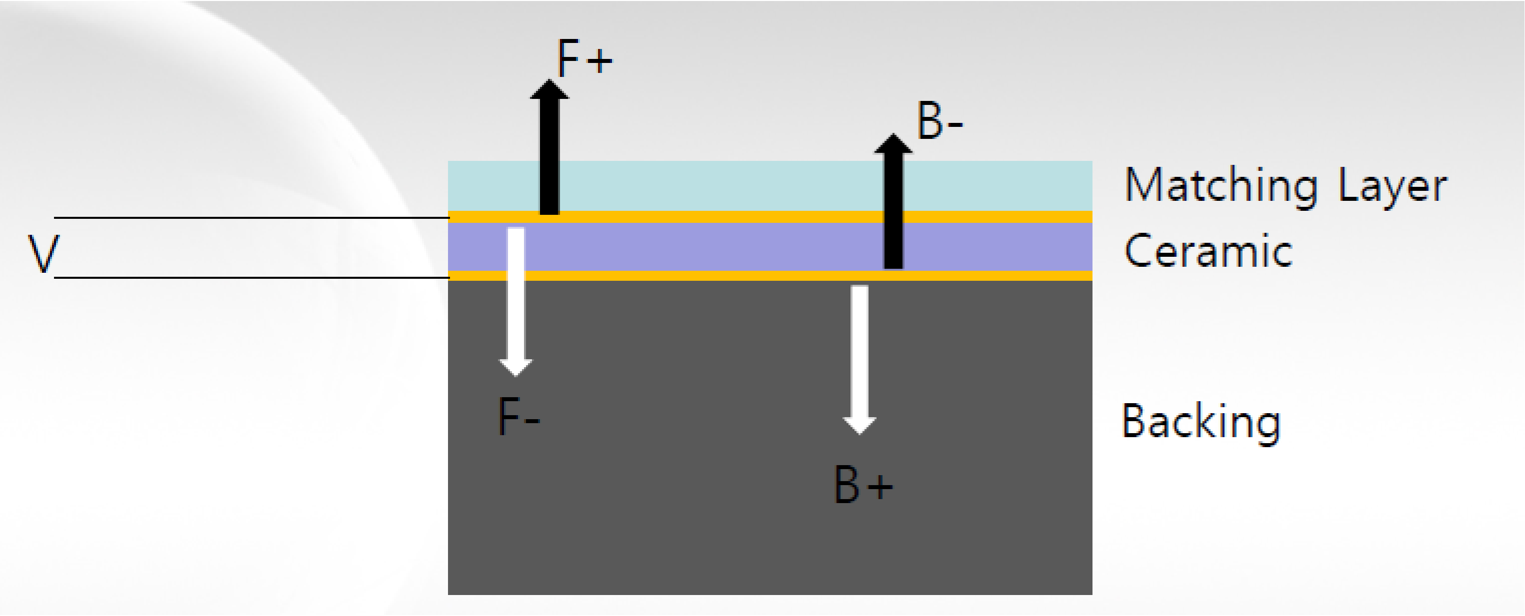
<u>Reverse Piezoelectric effect</u>

An electrical voltage applied to the piezoelectrical material causes it to vibrate

Piezoelectric Effect

Transducer

- If the ceramic is surrounded by air or water, most of the wave will be reflected at the boundaries of the ceramic ⟶ **Ringing for a very long time… (= long duration signal = narrow bandwidth)**
- Reflected pulse amplitude is proportional to the difference in acoustic impedance between the two materials on either side of boundary.
- We want to generate broadband signal (short pulse duration ultrasound, but not always.)



Transducer – Backing layer

- Attaching backing material to the back side of the ceramic with an acoustic impedance close to that of ceramic in order to damp out the ringing
  - Reducing transducer's sensitivity and frequency due to mass loading
  - *For B-mode, heavy backing (for broadband, short pulse)*
  - *For Doppler, light or air backing (for narrow, long pulse)*

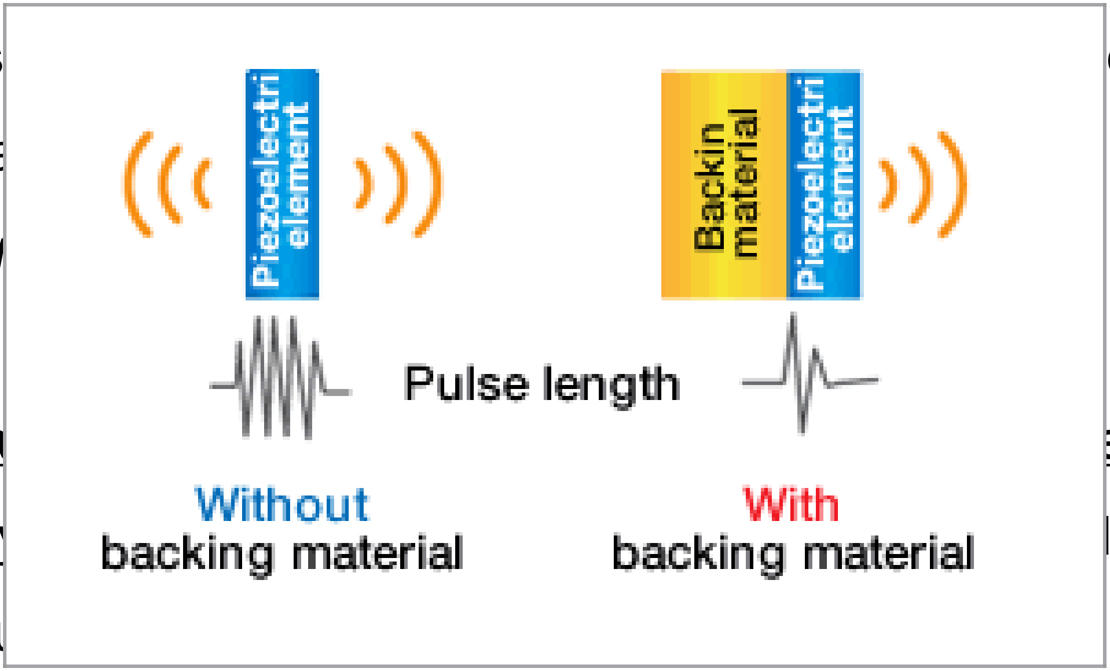
- **Backing with high attenuation** is used to **eliminate the internal reverberations** of the ceramic.
  - Attenuative polymers loaded with particles such as glass beads and gas filled spheres in order to provide maximum backing attenuation.

- The backing layer can be used to **dissipate heat and provide electrical connection** to the ceramic.
  - E-solder and Silver epoxy can be used.

Transducer – Backing layer

• Attaching backing material to the back side of the ceramic with an acoustic impedance close to that of ceramic in order to damp out the ringing

- Reducing trans … oading
- *For B-mode, he*
- *For Doppler, lig*



• **Backing with high a** … **everberations** of the ceramic.

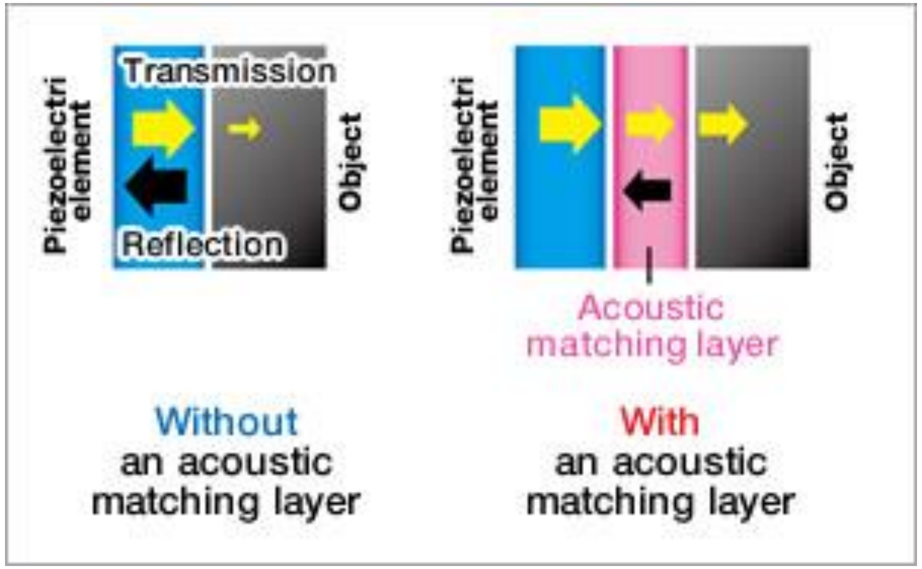
- Attenuative poly … ls and gas filled spheres in order to provide maximu

• The backing layer can be used to **dissipate heat and provide electrical connection** to the ceramic.

- E-solder and Silver epoxy can be used.

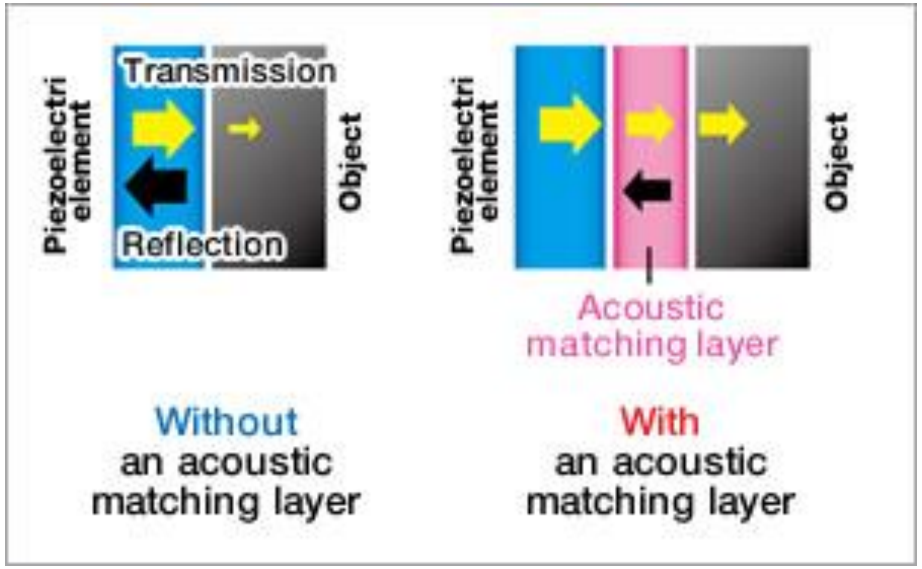
Transducer – Matching layer

- Acoustic impedance: Tissue (~1.5 Mrayls) << Most piezoelectric materials (>25 Mrayls)
  - The mismatching causes most of ultrasound energy to go into backing, instead of going forward into tissue.
  - To increase the relative amount of energy transmitted into the front medium, acoustic impedance between the ceramic and tissue should be matched. ⟶ Matching Layers are necessary (single matching layer & Two matching layers)



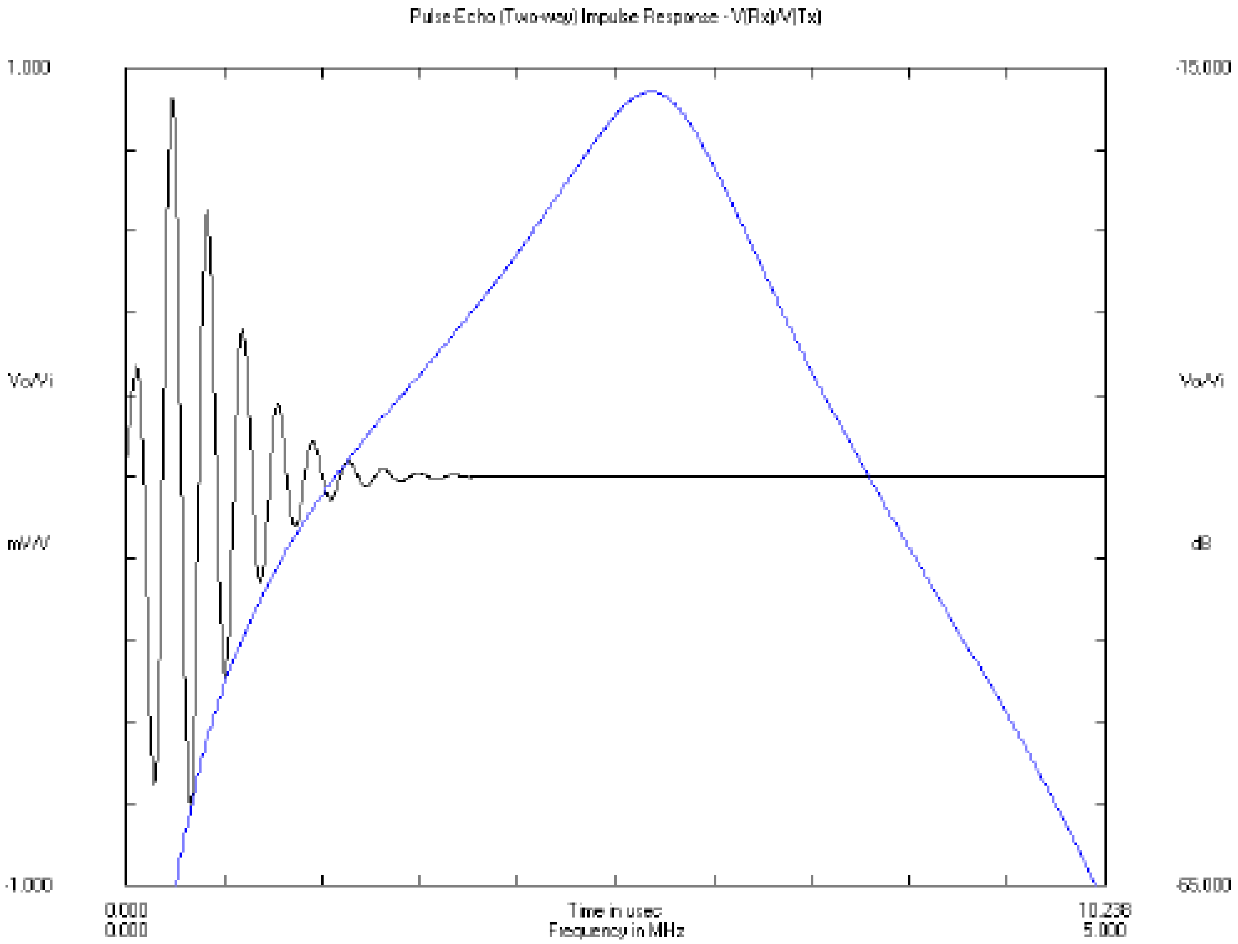
Transducer – Matching layer

- Acoustic impedance: Tissue (~1.5 Mrayls) << Most piezoelectric materials (>25 Mrayls)
  - The mismatching causes most of ultrasound energy to go into backing, instead of going forward into tissue.
  - To increase the relative amount of energy transmitted into the front medium, acoustic impedance between the ceramic and tissue should be matched. ⟶ Matching Layers are necessary (single matching layer & Two matching layers)

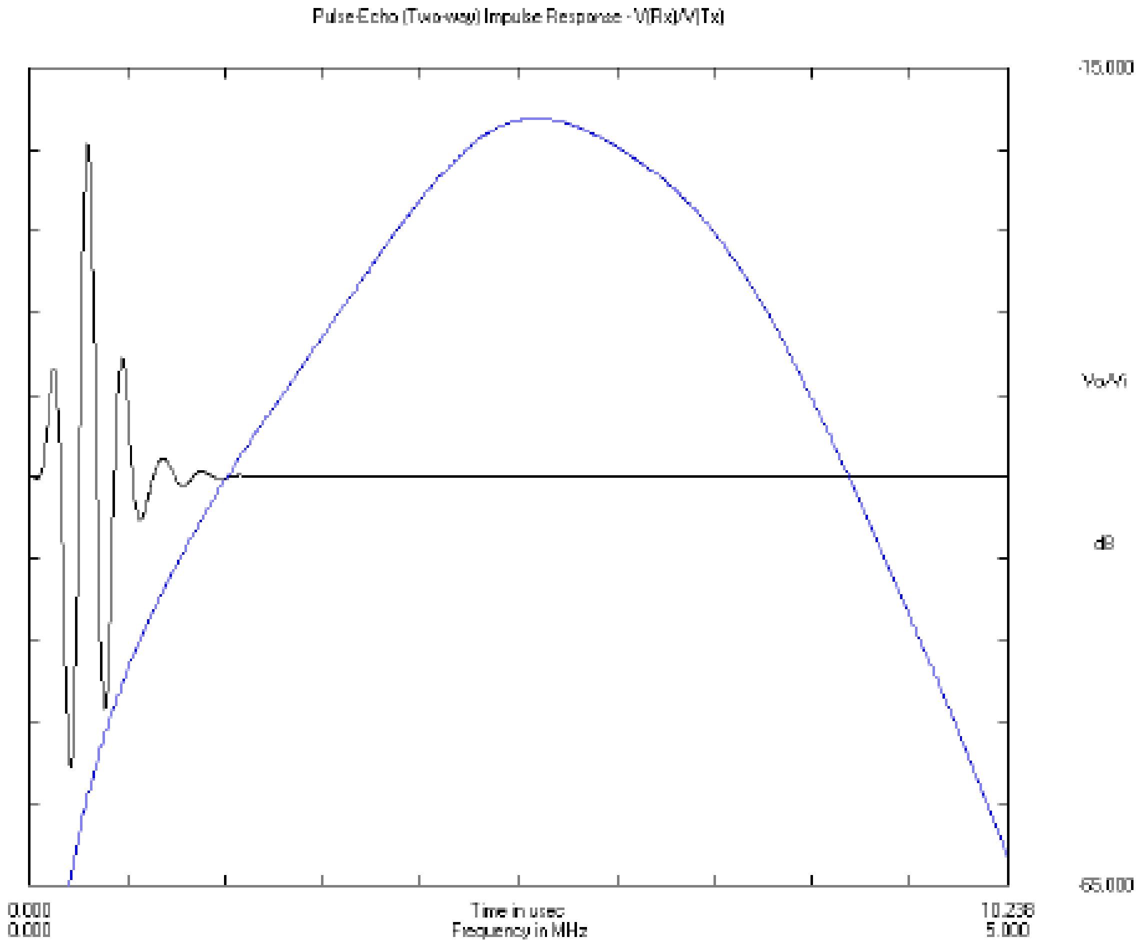


Transducer – Backing & Matching layer

w/o Backing & Matching layers



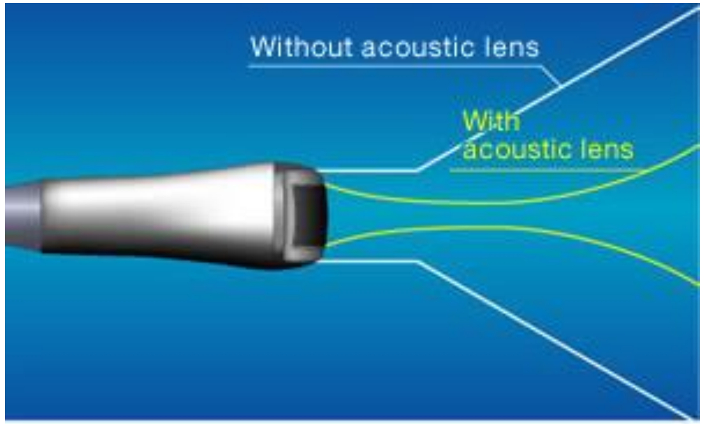
with Backing & Matching layers



Transducer – Lens

- For elevation focusing, an acoustic lens is used
  - Material for lens should have velocity lower than target medium such as water and tissue.
- Popular material for a lens

	Acoustic Impedance (Mrayl)	Sound velocity (m/s)
RTV rubber	0.99 ~ 1.46	960 ~ 1160
Polyurethane	1.38 ~ 2.36	1330 ~ 2090
Silicone rubber (Sylgard)	1.03	1050



Beam focused by the acoustic lens

- The focal length of a lens:

$$z_f = \frac{R_c}{1 - \frac{c_{medium}}{c_{lens}}}$$

R_c The radius of curvature

c The velocity

Transducers

Linear array

Curved array

Phased array

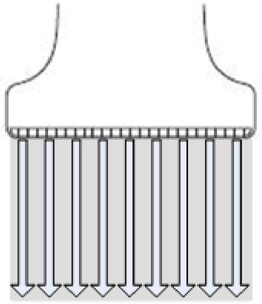
Intracavitary curved array

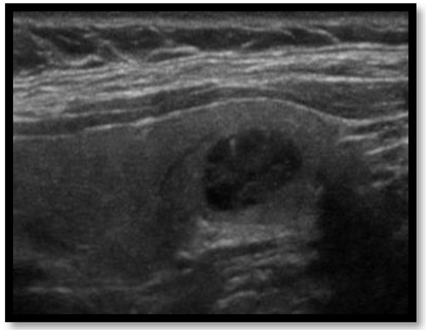
Matrix array

Intravascular probe

Transducers

Linear array





Linear array

- High frequency: 5 ~ 13MHz
- High resolution / shallow depth (low penetration)
- Superficial structures and US-guide
- Vascular
- Deep venous thrombosis
- Breast
- Skin and soft tissue for abscess, foreign body
- Musculoskeletal – tendons, bones, muscles
- Testicular, ..., etc

Transducers

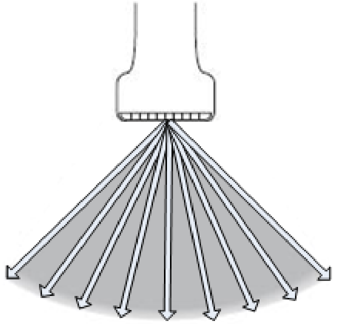
Convex array

Convex array

- Scanning deeper structures
- High frequency: 1 ~ 8 MHz
- Greater penetration, less resolution
- Abdominal, pelvic, msk, obese patients
- Abdominal aorta
- Biliary, gallbladder, liver, pancreas
- Kidney and bladder evaluation
- Transabdominal pelvic evaluation
- etc

Transducers

Phased array



Phased array

- Small linear array : element spacing ~$\lambda/2$
- Beam steering
- frequency: 2 ~ 8 MHz
- Cardiac imaging
- Imaging between ribs
- etc

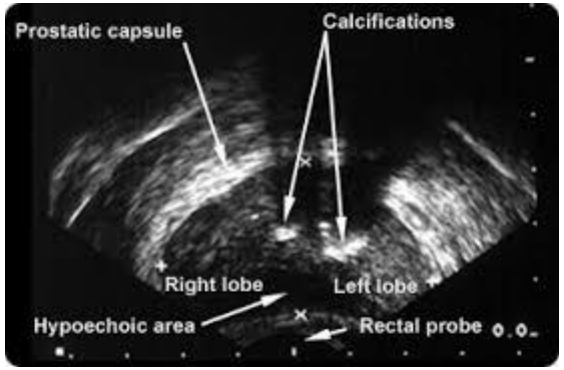
Transducers

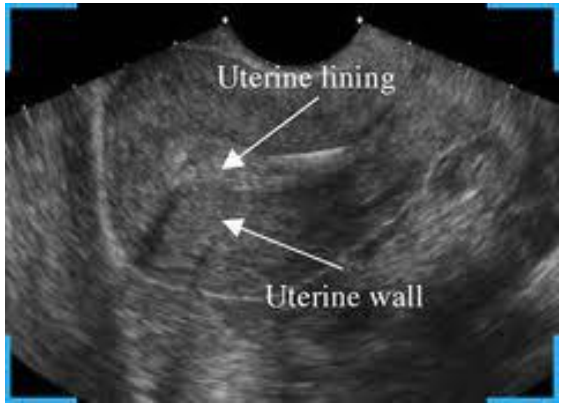
Intracavitary curved array

Intravascular probe

Endocavitary array

- Ob/Gyn: Transvaginal
- Prostate
- Introral





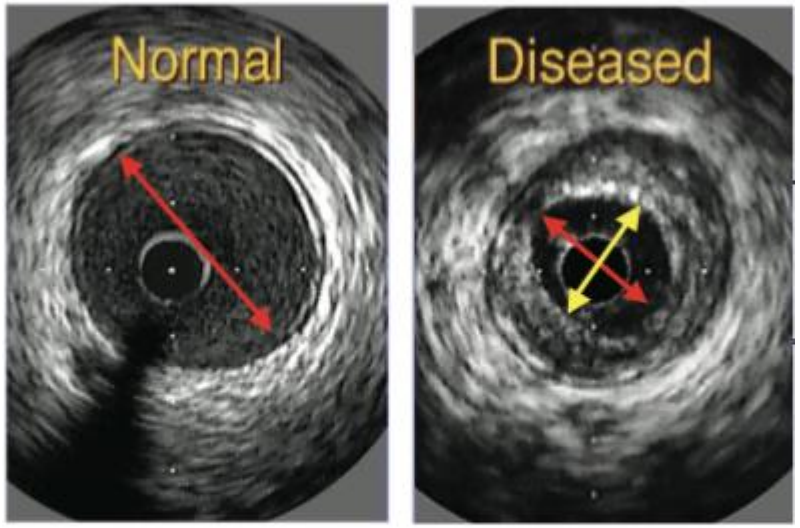
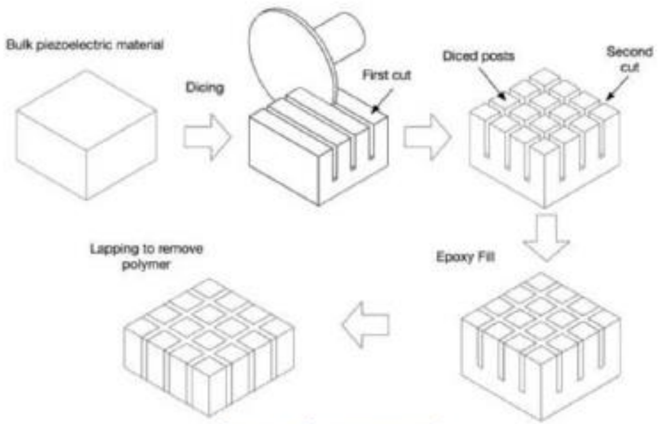


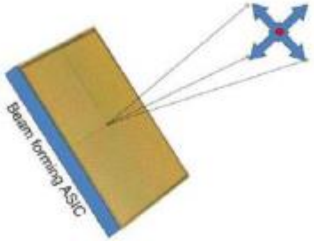
Figure 2. IVUS images with diameter markers of normal vessel (left) and lumen dimensions in diseased artery (right).

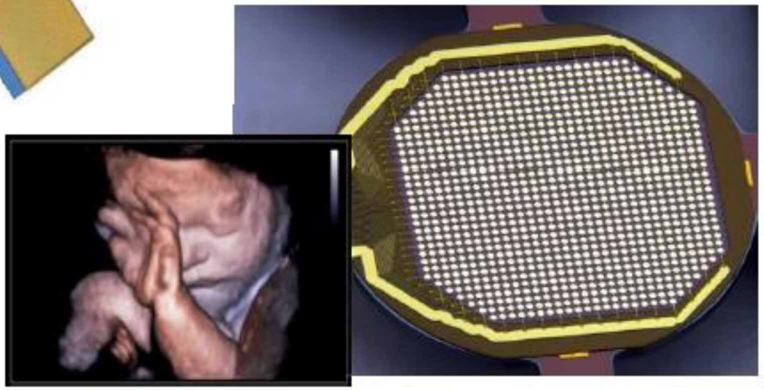
State-of-the-art: MEMS-based Transducer



Sawing of piezo material;
Source: Jet Propulsion Laboratory, California Institute of Technology

Todays ultrasound imaging:
- Based on piezo-ceramics
- Difficult to manufacture
- No volume production
- Labor intensive → expensive
- Reserved for professional use





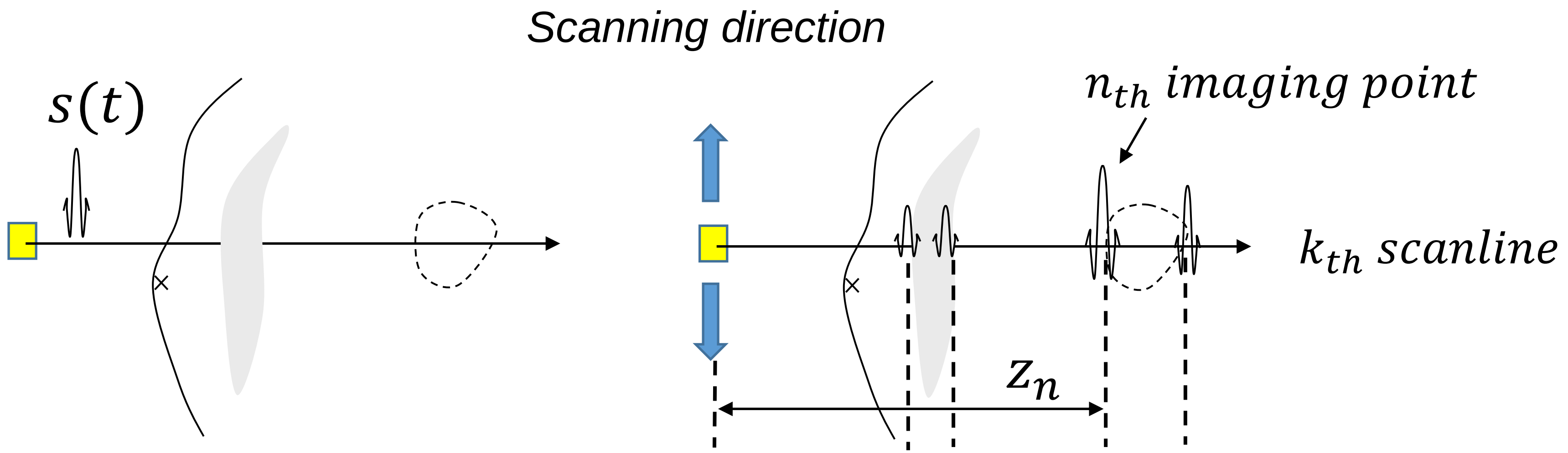
The MEMS US revolution:
- High volume production
- Eliminate (manual) assembly
- Low-cost platform → multiple applications
- Miniaturization → catheters
- Higher frequencies
- 3D imaging compatible
- Enter consumer market

pMUT and cMUT

Image formation

Pulse-echo

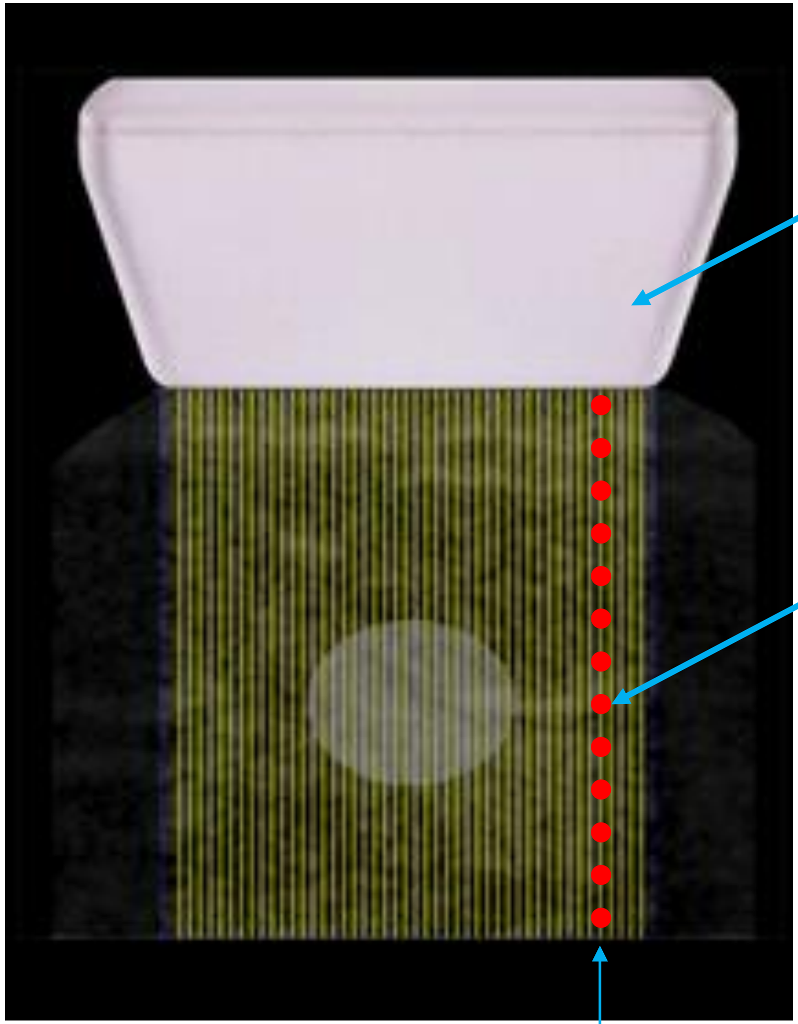
- Pulse-echo: transmit and receive
- Scan: Move directions of ultrasound transmit/receive



$$r_k(t) = \sum R_n \cdot s(t - \tau_n) \qquad \tau_n = 2 \cdot z_n / c$$

Time delay *Distance from the target*

B-mode (Brightness)



- Ultrasound transducer (probe)

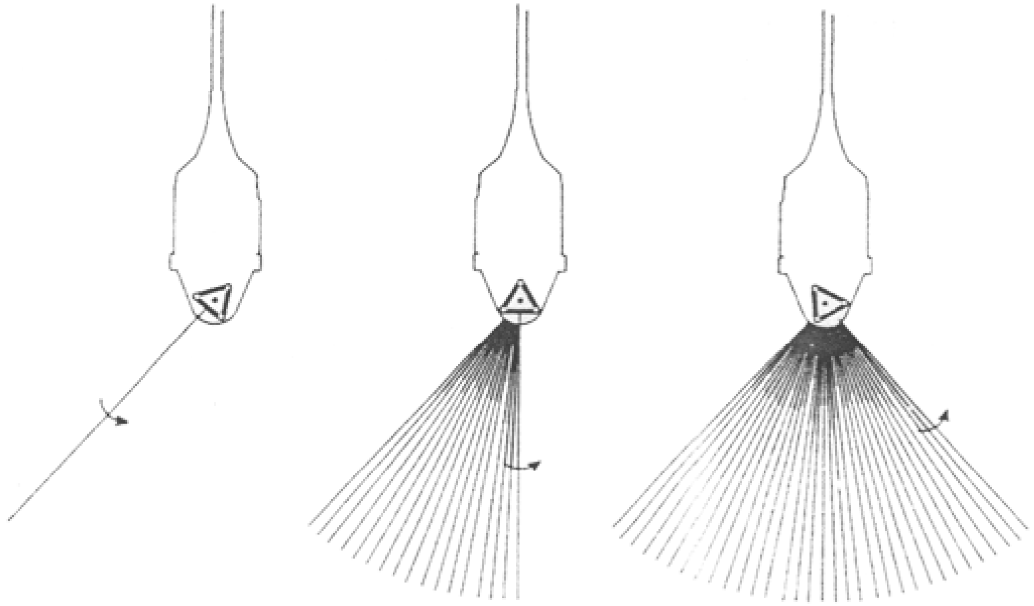
n_{th} *imaging point*

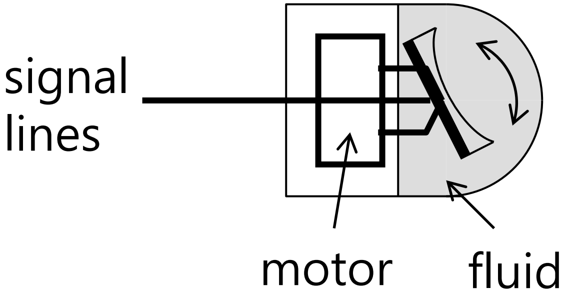
k_{th} *scanline*

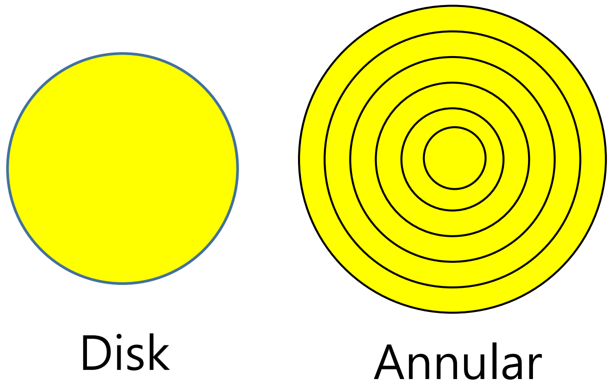
B-mode (Brightness)

Mechanical sector scan

- using a lensed disk transducer or an annular array
- moving back and forth along the azimuthal direction
- Limited scanning abilities



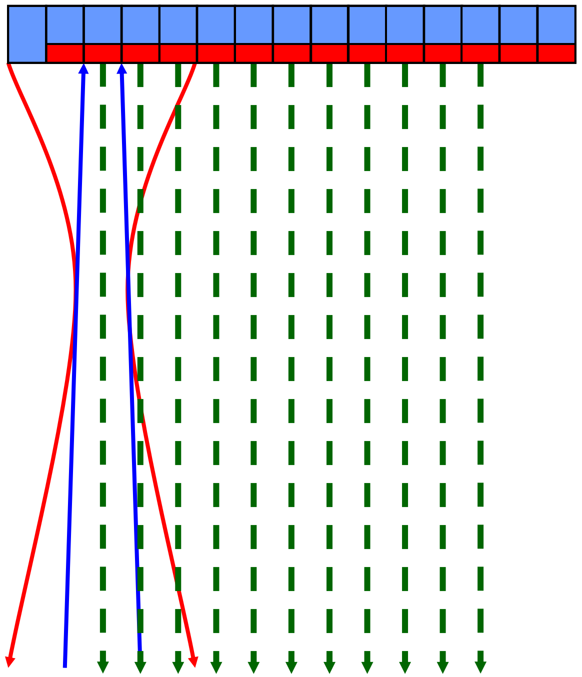




B-mode (Brightness)

Electric scan

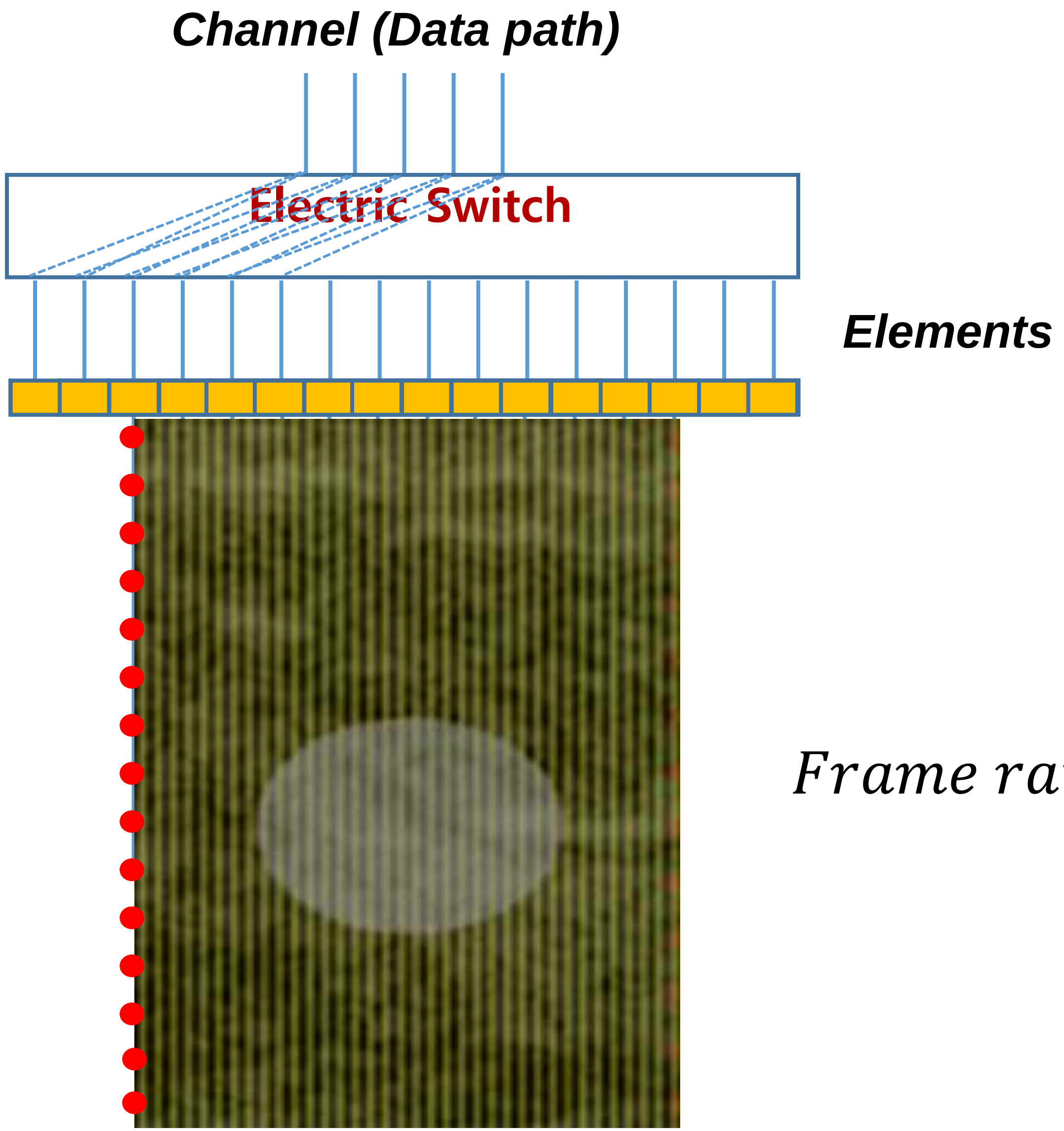
- using array transducers
- Electrical scanning and beamforming (focusing)
- Available to any scan sequence



B-mode (Brightness)

Electric scan + Multiplexer

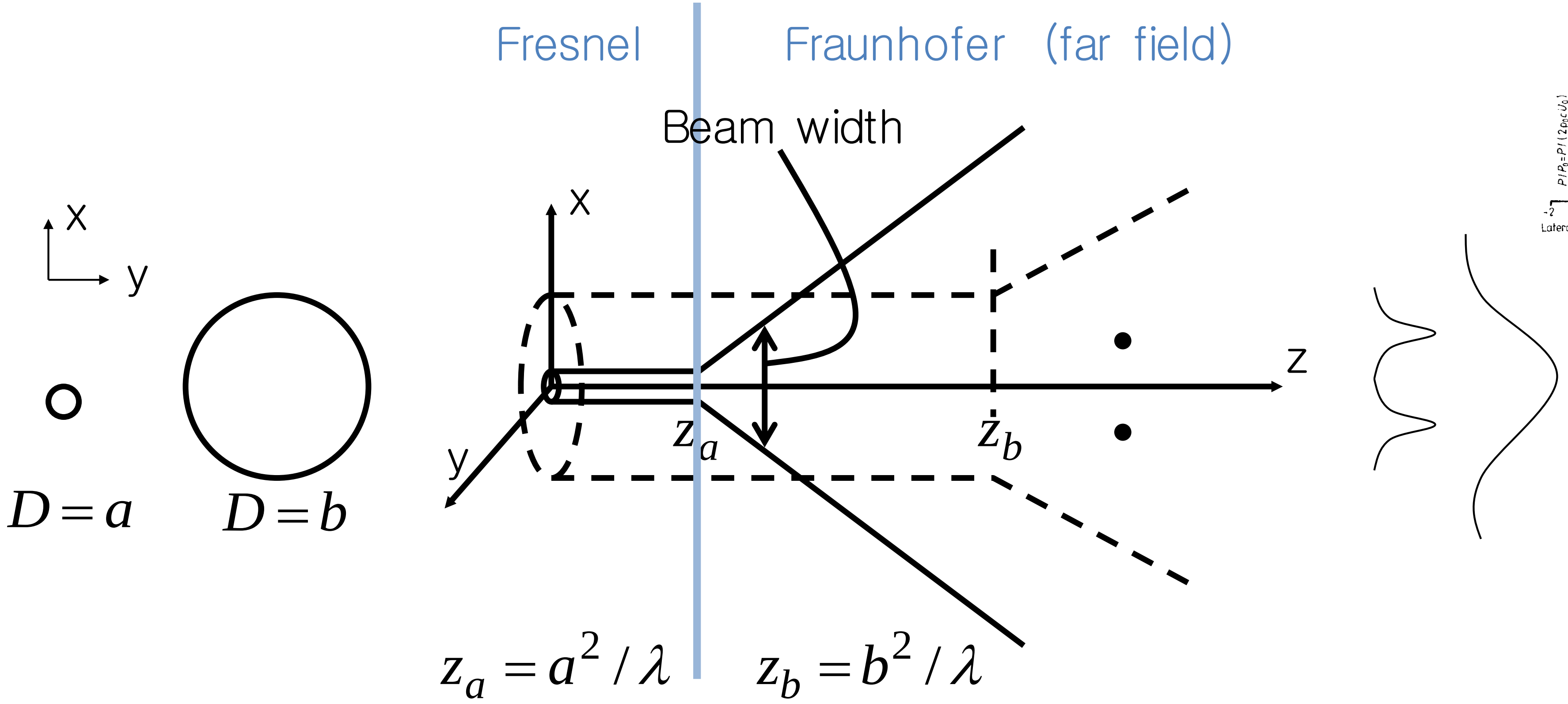
(when #'s Ch < #'s Elements)

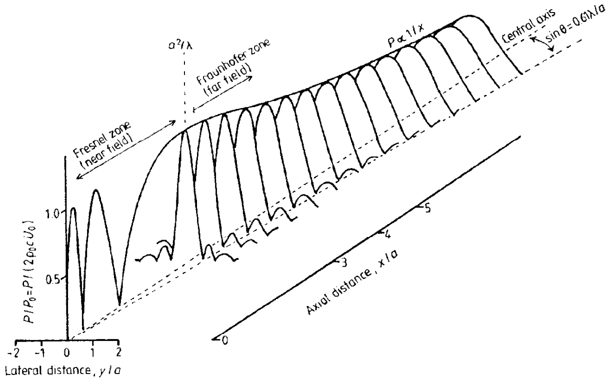


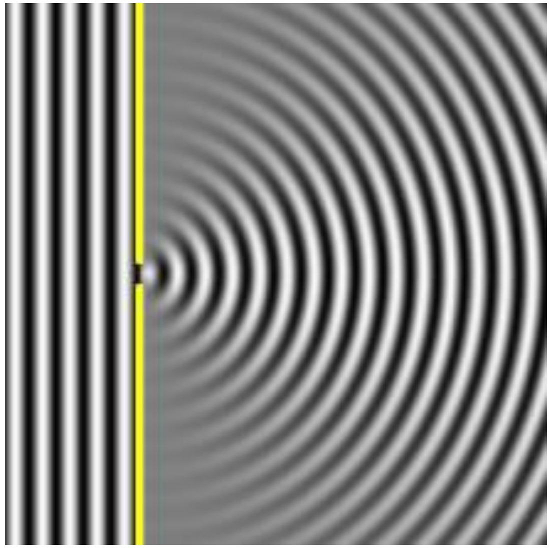
$$Frame\ rate \propto \frac{1}{\#scanlines}$$

Diffraction

- Diffraction





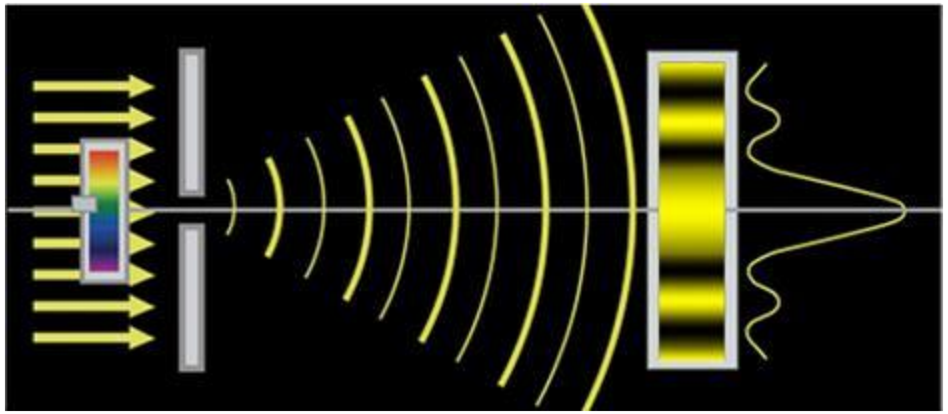


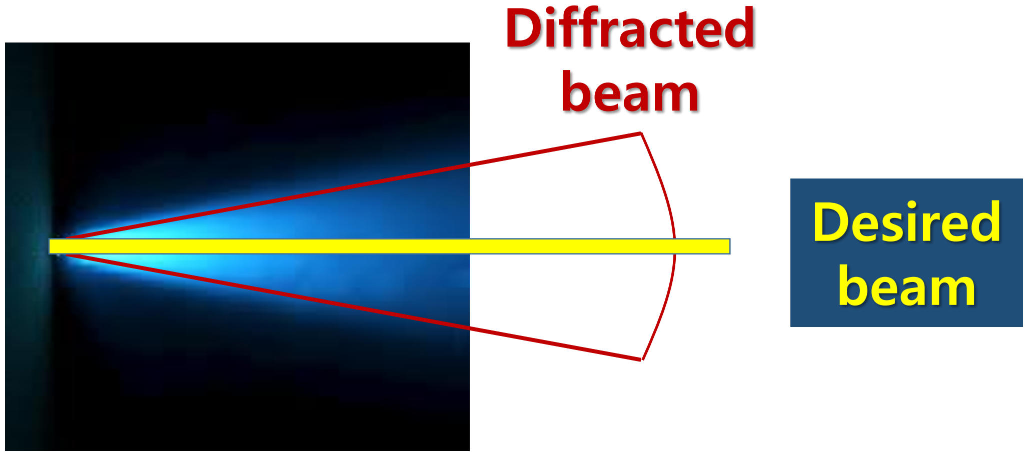
We will talk about the field analysis later.

Focusing

Wave equation

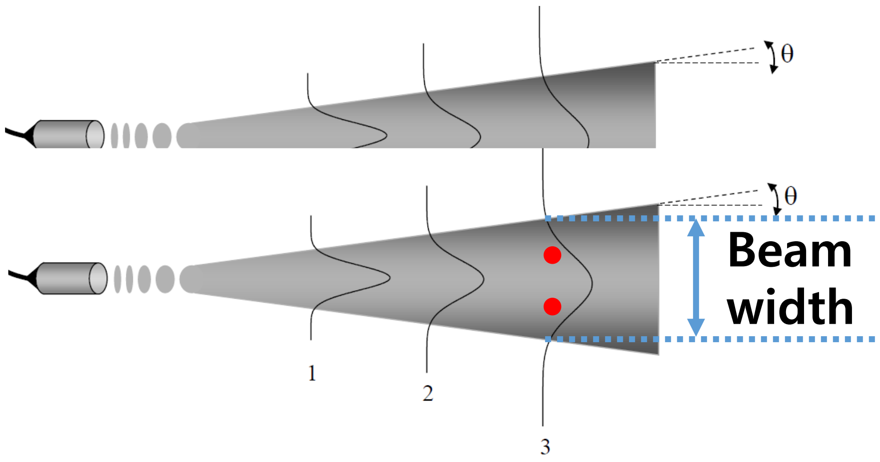
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u(\mathbf{r}, t) = 0.$$

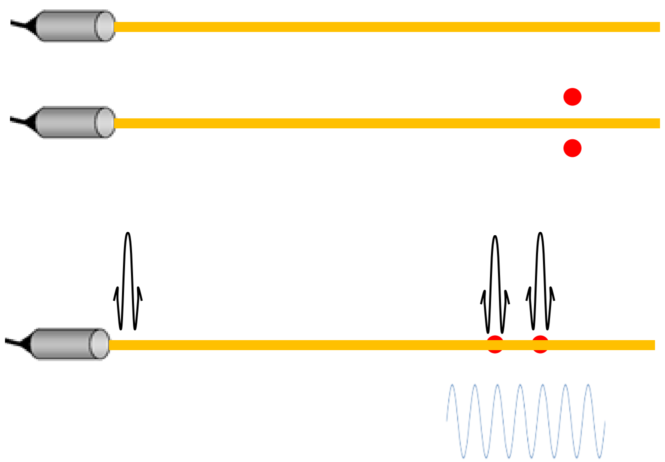




- *Waves are subject to diffraction spreading*
- *Focusing is employed to produce a “narrow beam”*

Focusing



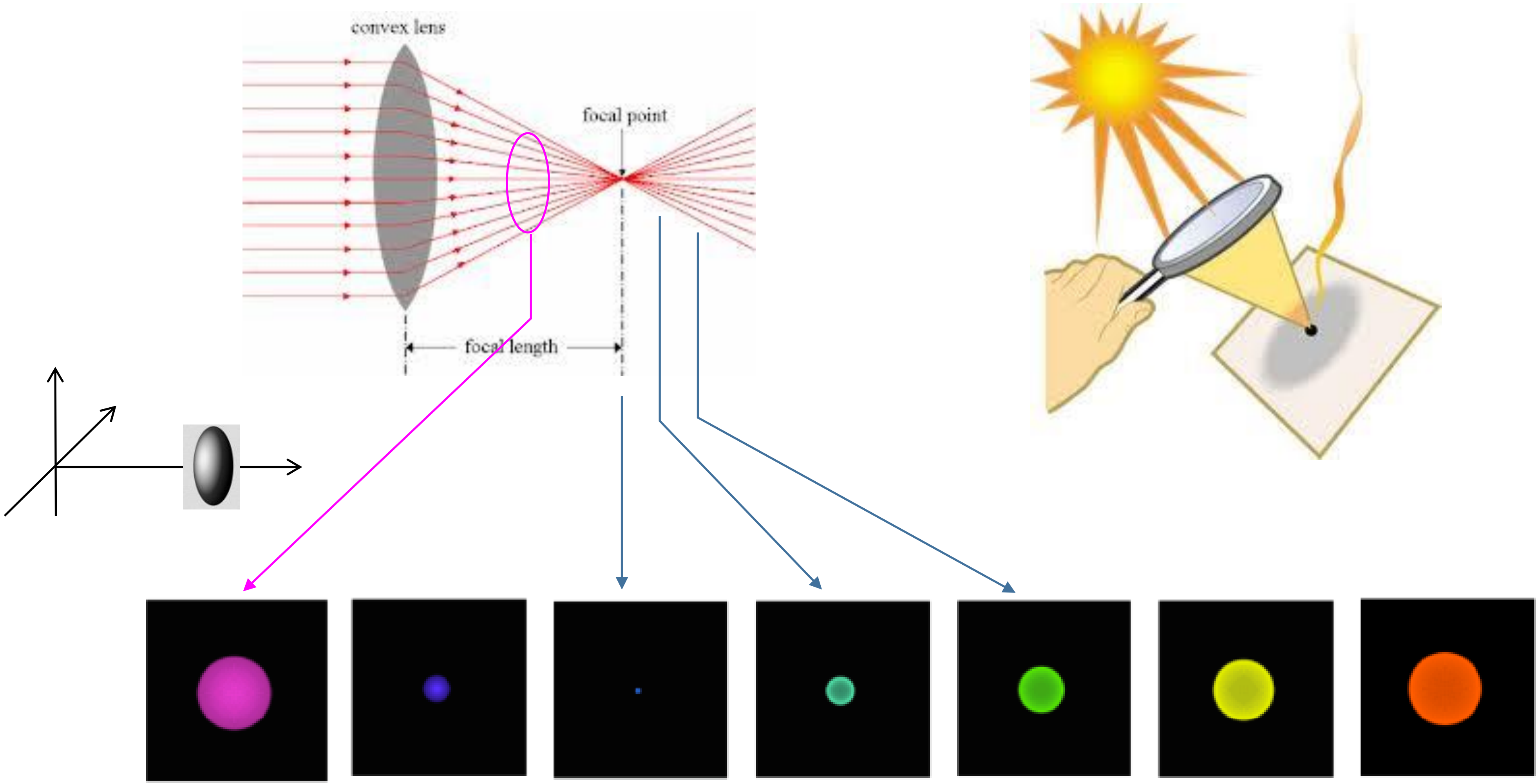


Low Spatial resolution

High Spatial resolution

Focusing

Electromagnetic wave(light): How to focus?



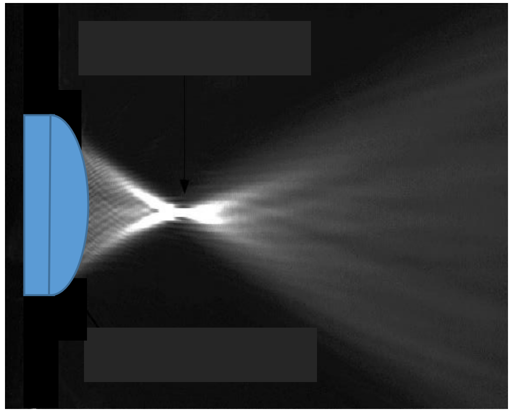
Focusing by lens

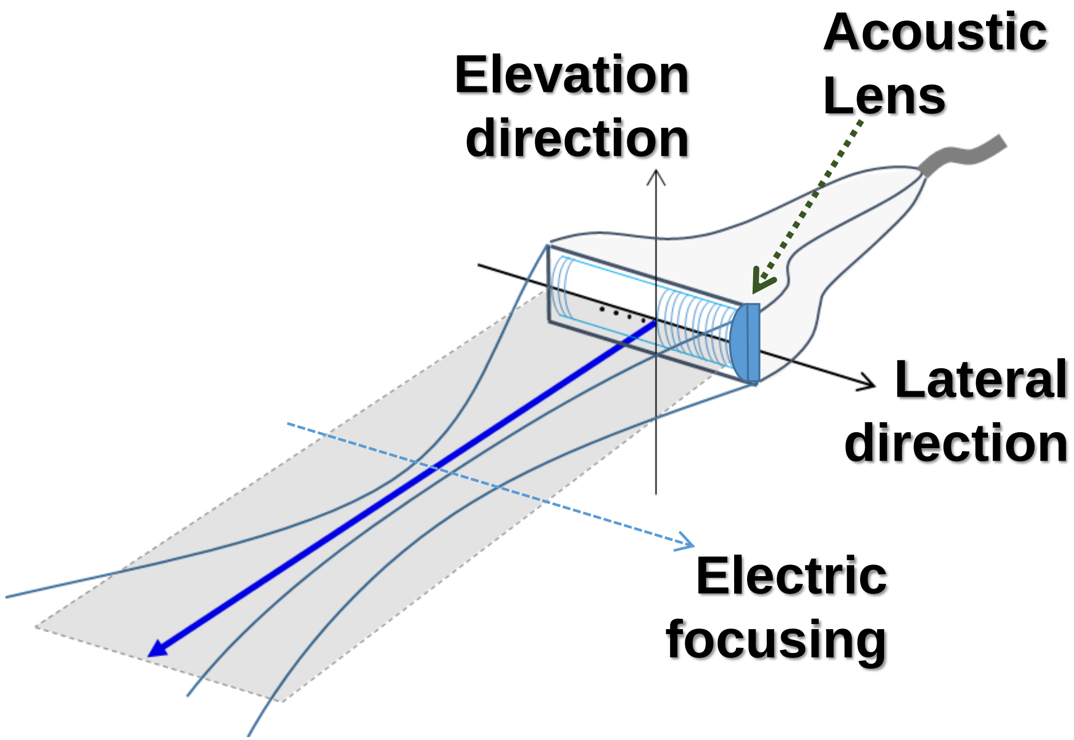
Light

Convex Lens

Ultrasound

Acoustic Lens

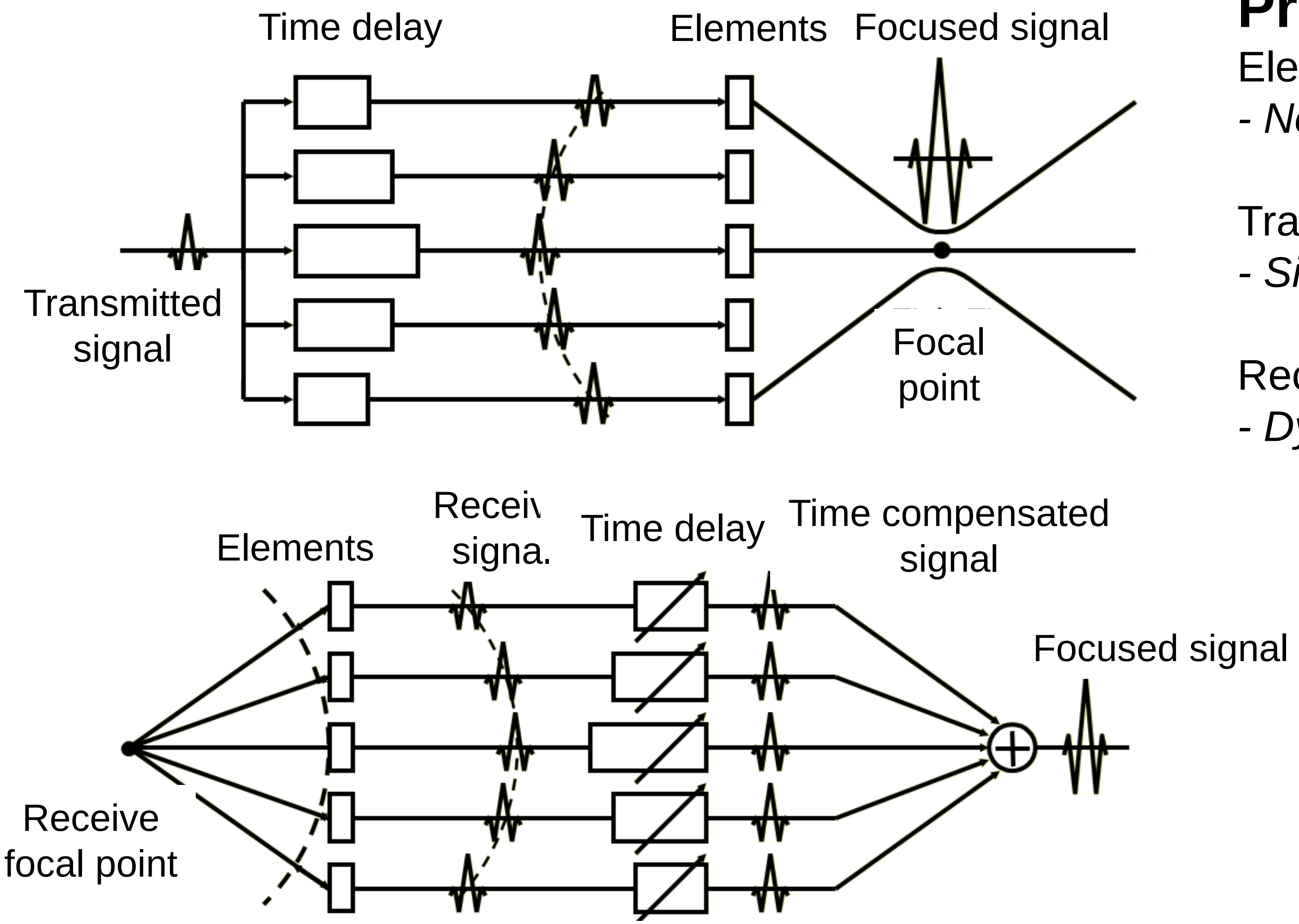




Electronic focusing (Dynamic)

Transmit focusing

Receive focusing



Pros

Electronic scan

- *No moving parts*

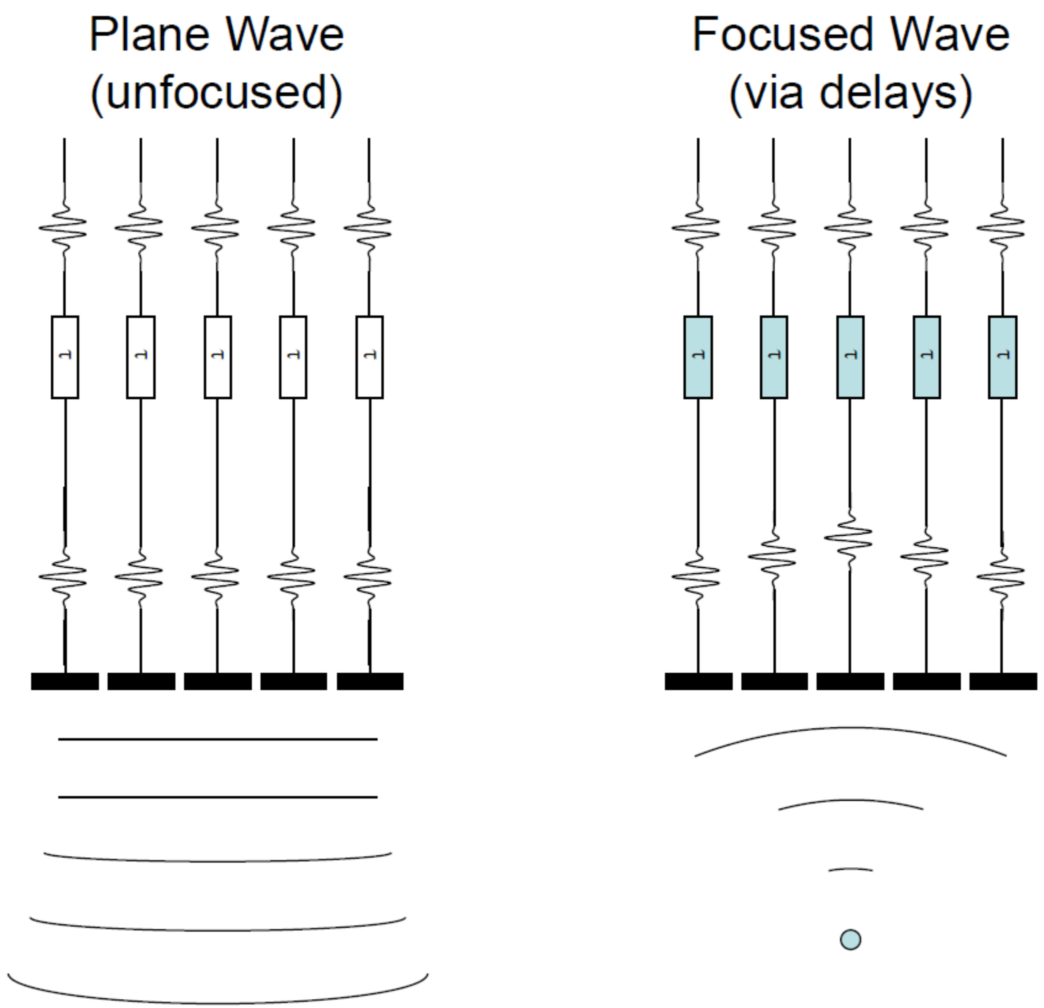
Transmit focus

- *Single or multi-zone focus*

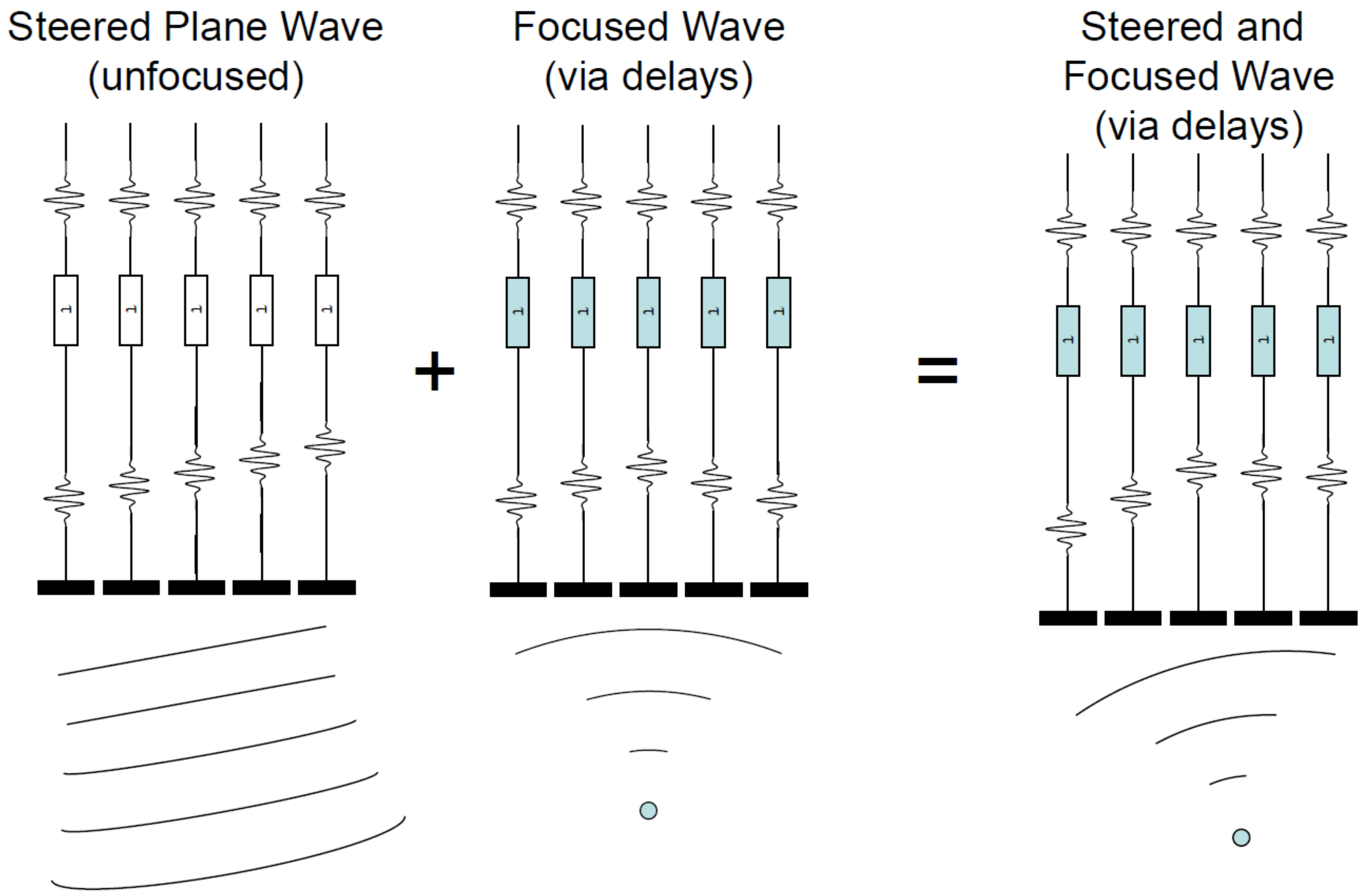
Receive focus

- *Dynamic focusing*

Array Transducer - Electronic Focusing & Steering

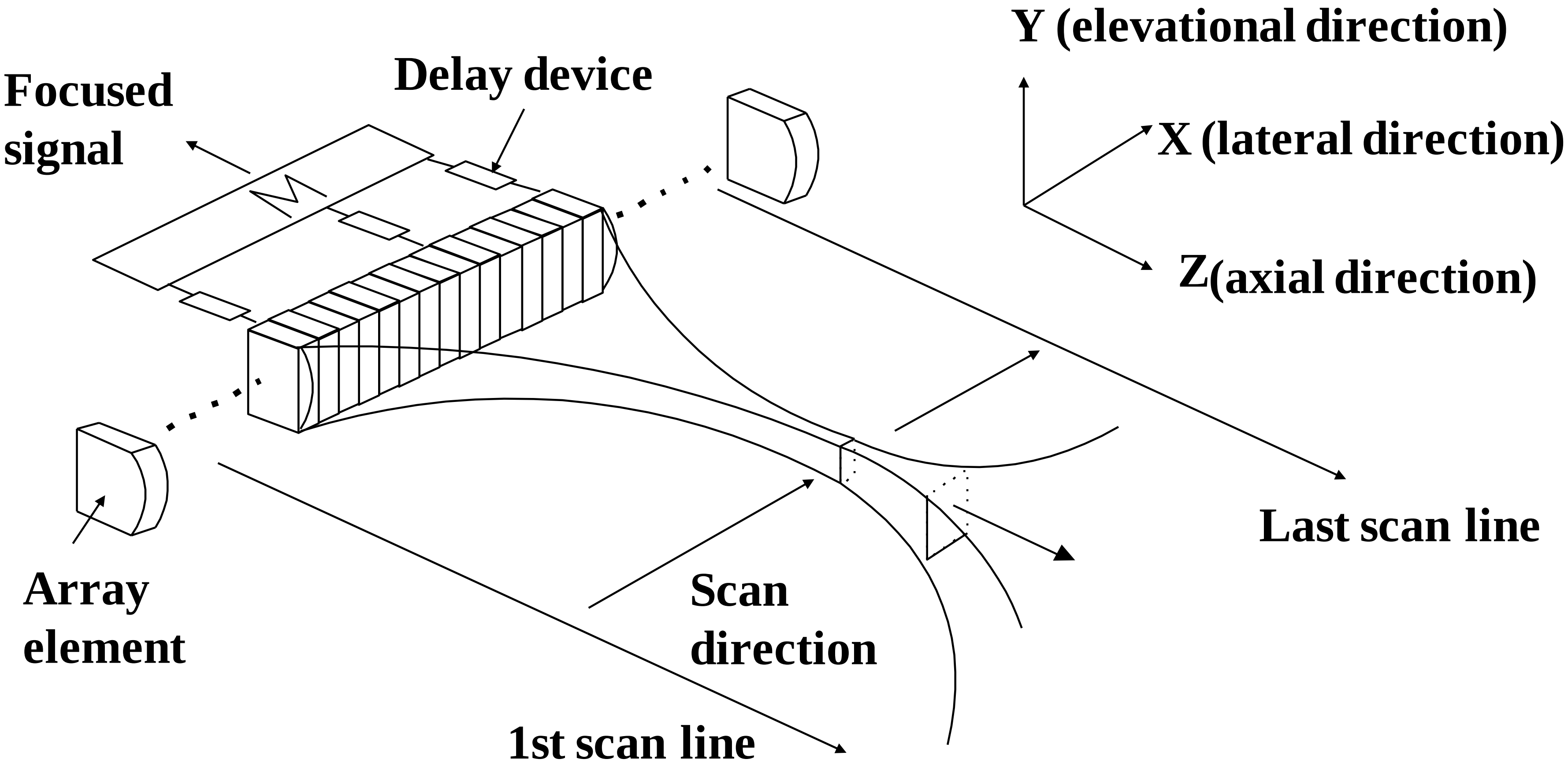


Array Transducer - Electronic Focusing & Steering



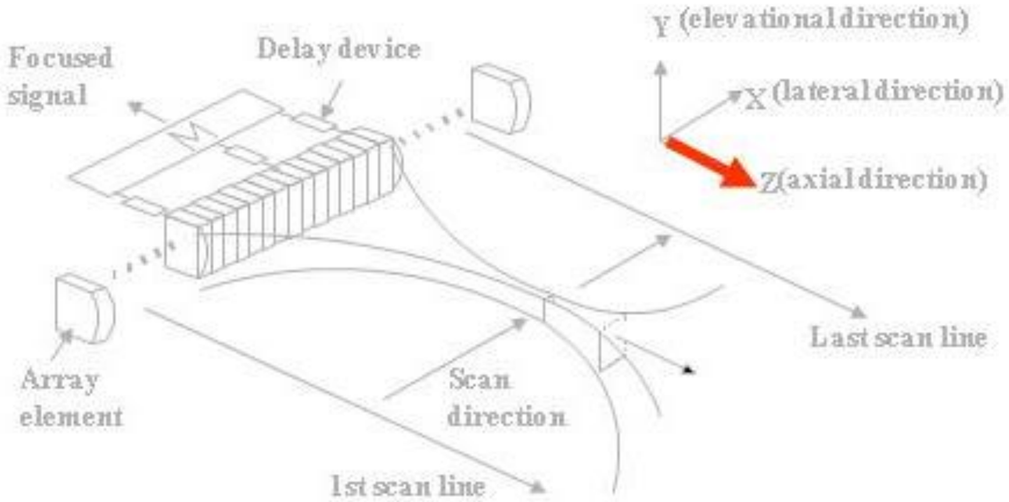
Spatial resolution

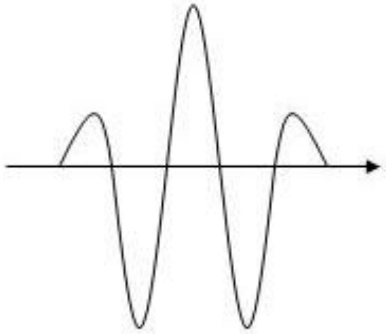
- Spatial resolution of US imaging



Spatial resolution

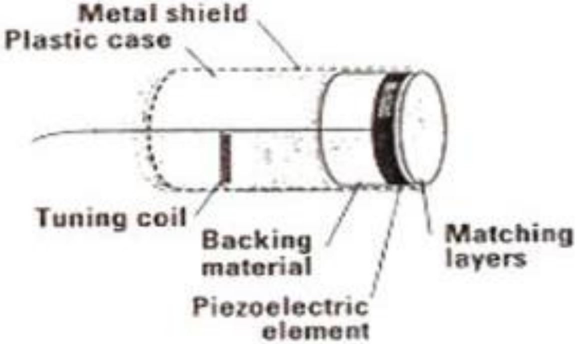
- Axial resolution
  - Axial resolution depends on pulse-width





TX pulse

*

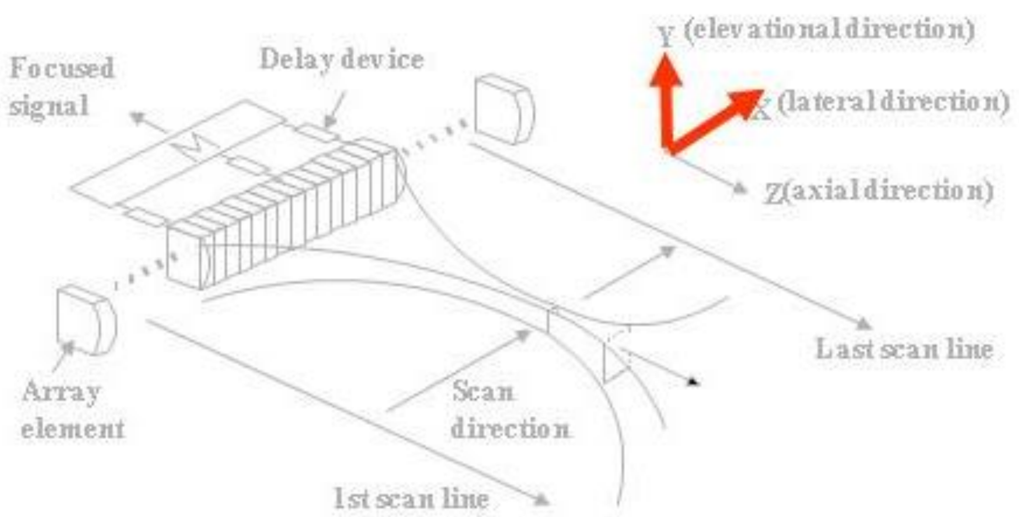


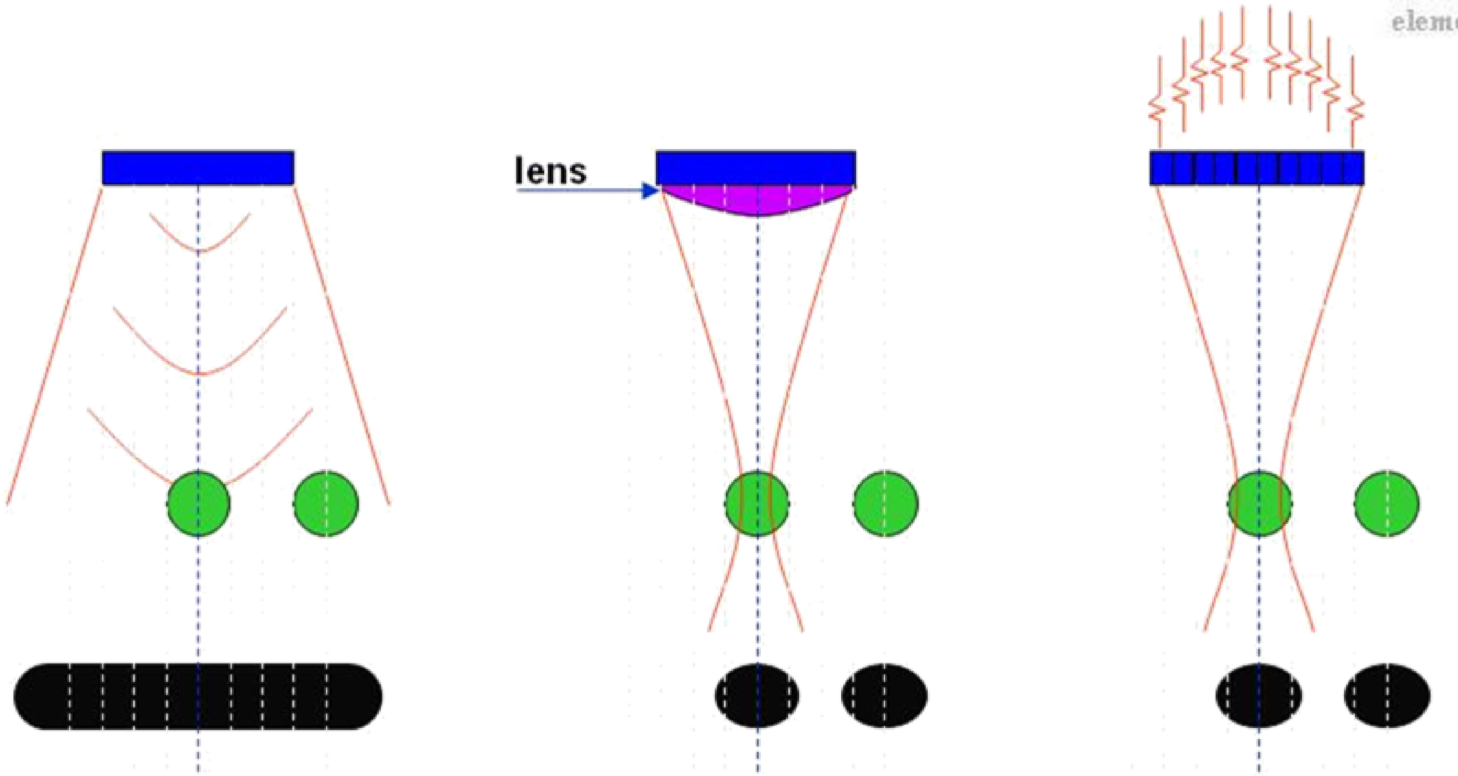
Impulse response of transducer

Pulse duration (=Bandwidth)

Spatial resolution

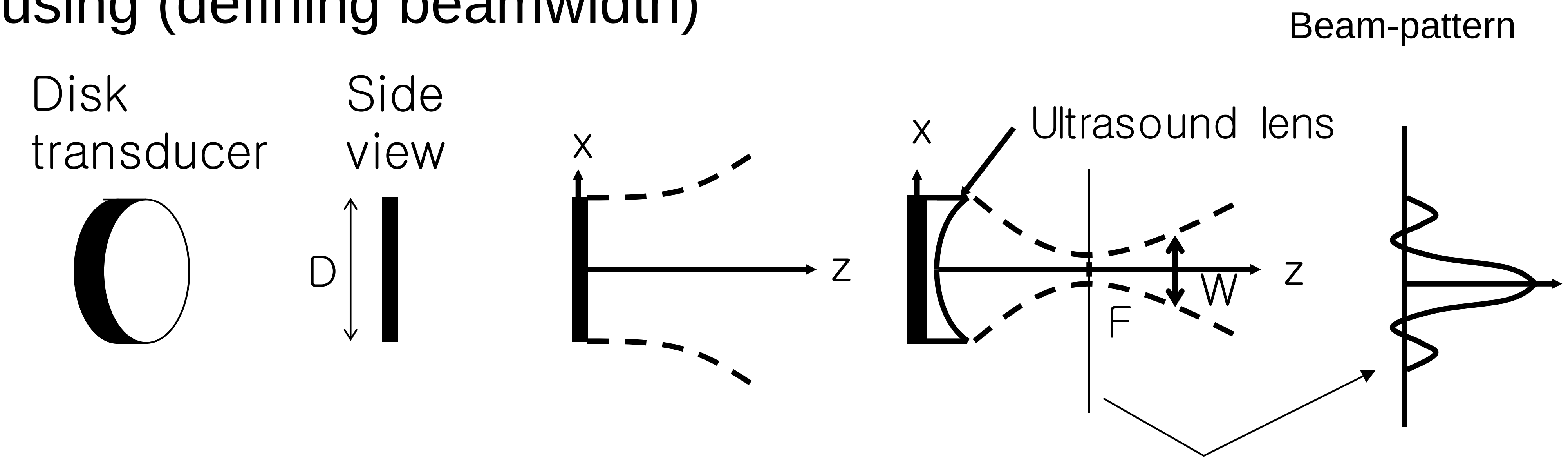
- Lateral/Elevational resolution
  - Lateral/Elevational resolution depends on beamwidth (Frequency and aperture size)





Spatial resolution

- Focusing (defining beamwidth)

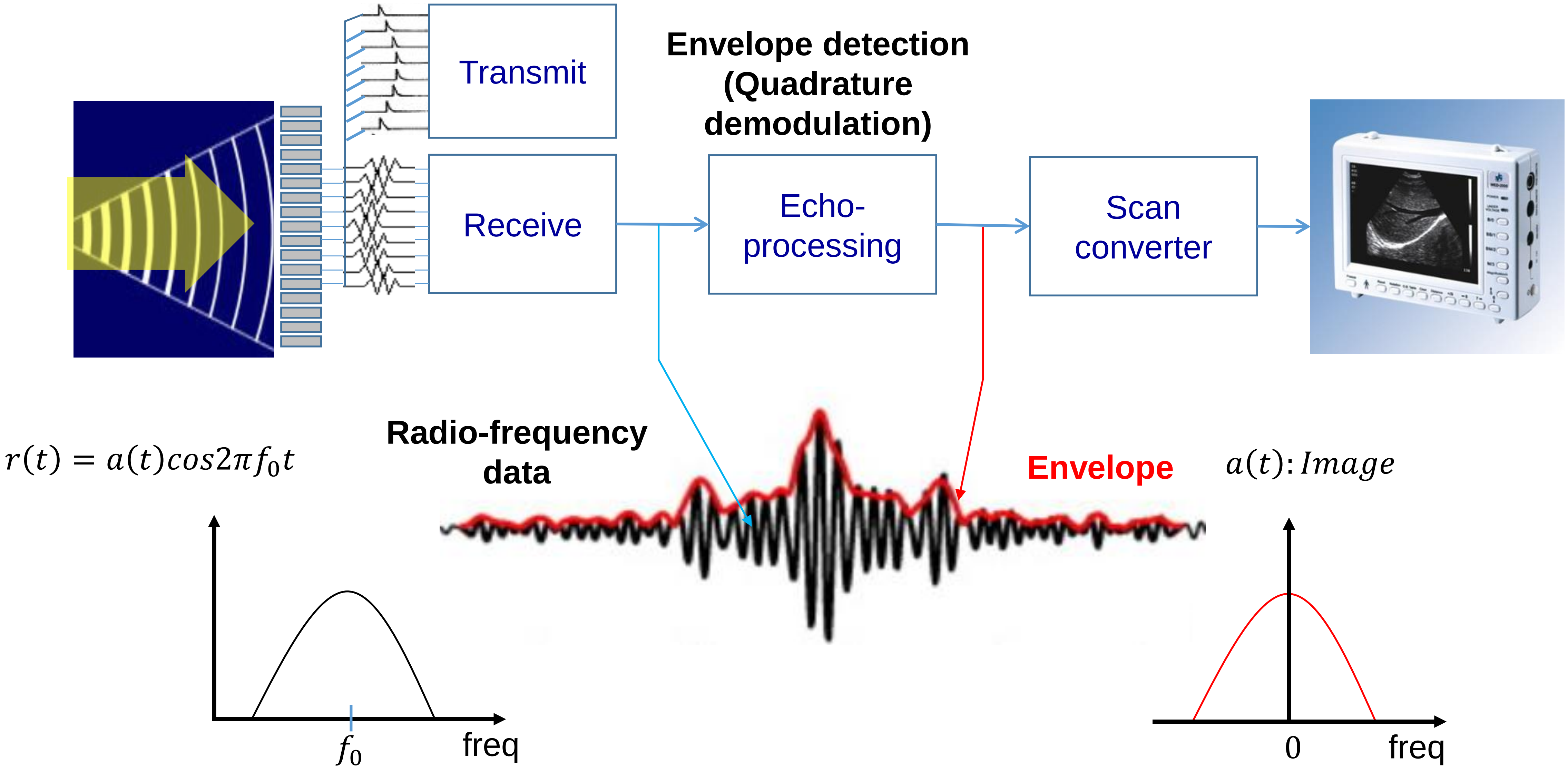


$$W \propto \frac{\lambda z}{D}$$

To obtain high resolution (narrow beam width), we need to use

- smaller wavelength, i.e., higher frequency
- larger aperture (D)

Basic ultrasound image formation

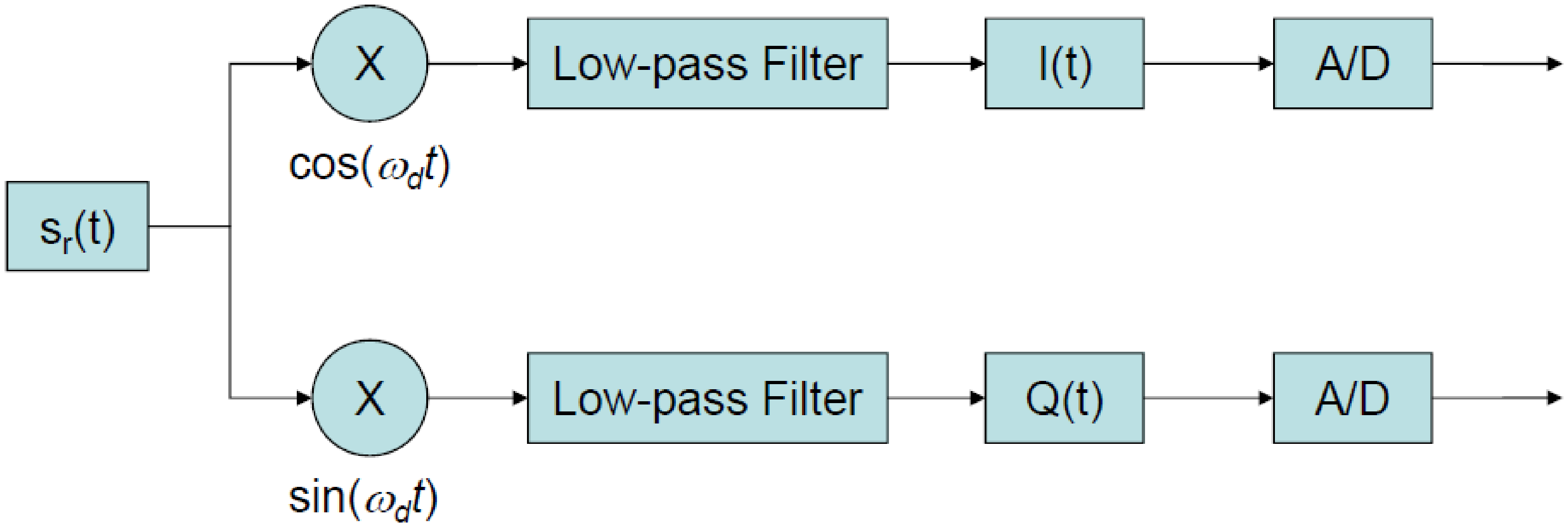


Quadrature demodulation

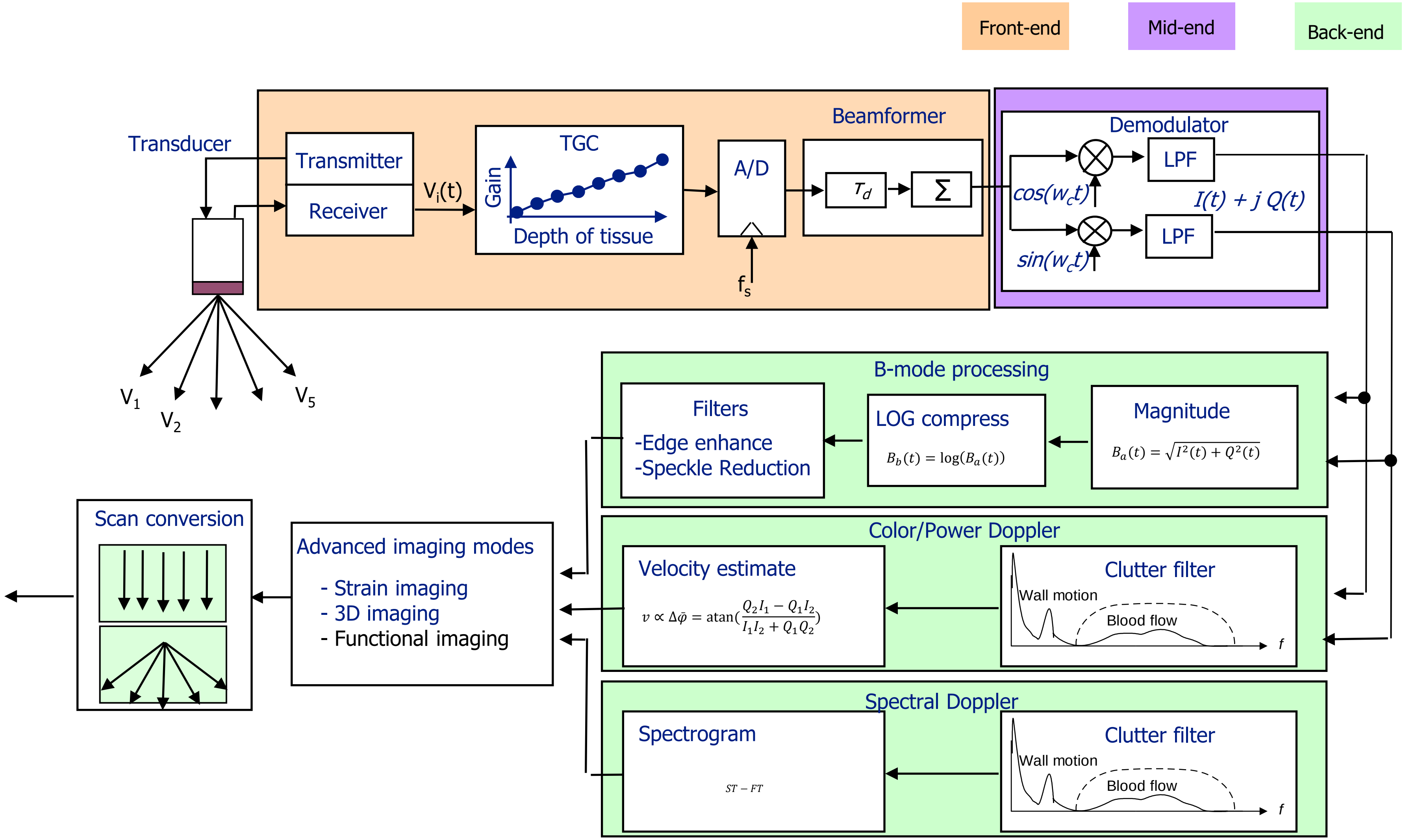
$$s_r(t)\cdot e^{+i\omega_d t} = s(t)\cdot\cos(\omega t)\cdot e^{+i\omega_d t} = \frac{1}{2}s(t)\left[e^{-i(\omega_0-\omega_d)t} + \cancel{e^{+i(\omega_0+\omega_d)t}}\right]$$

Complex signal,
needs two channels (I and Q)

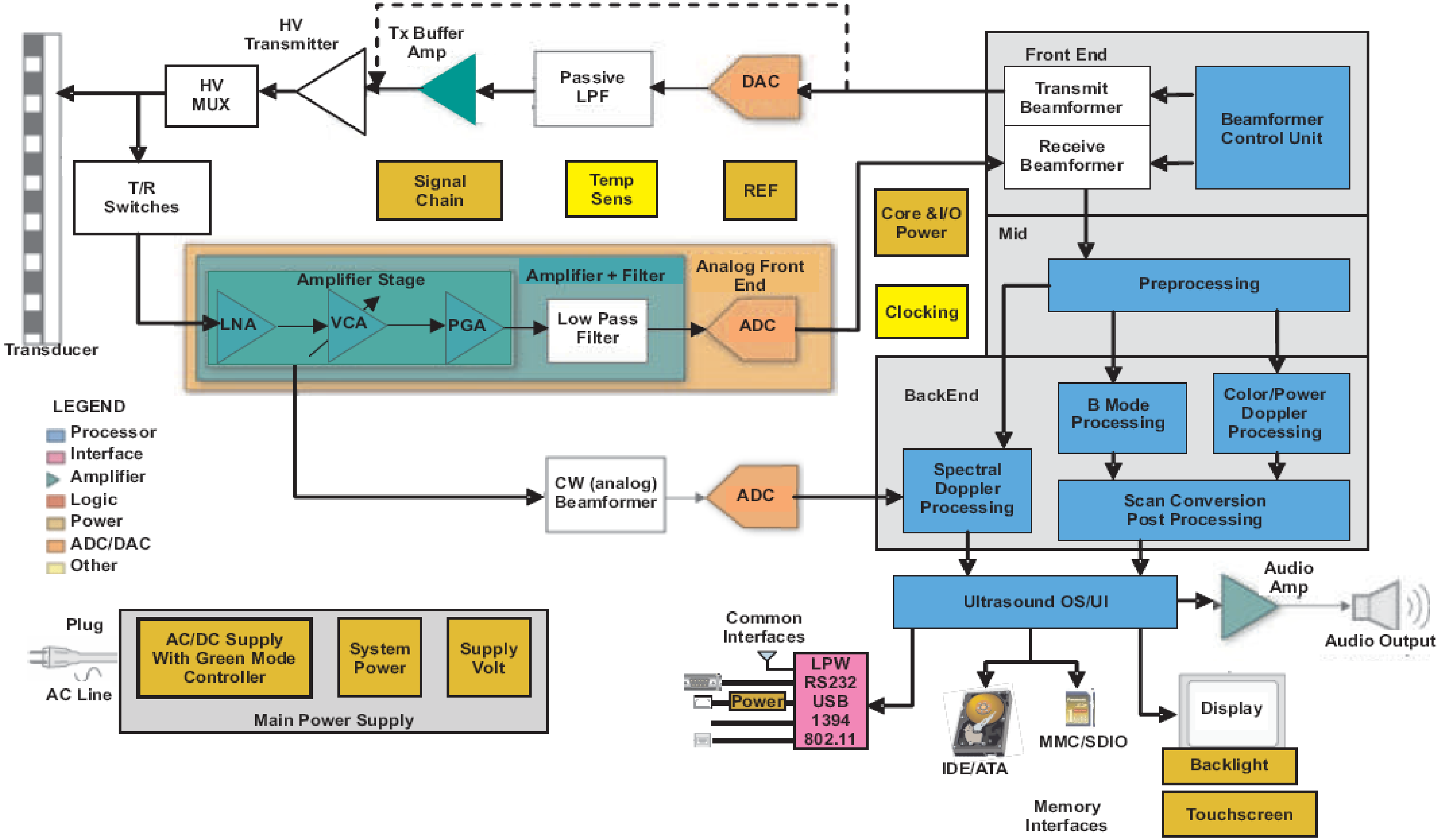
This term must be removed
(low-pass filter)



Ultrasound Imaging System: Functional blocks



Ultrasound Imaging System: System blocks

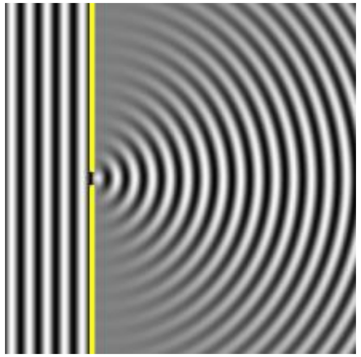


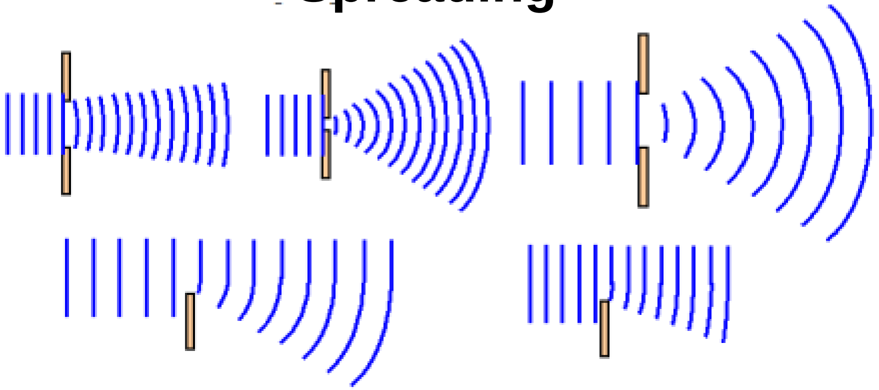
Field analysis

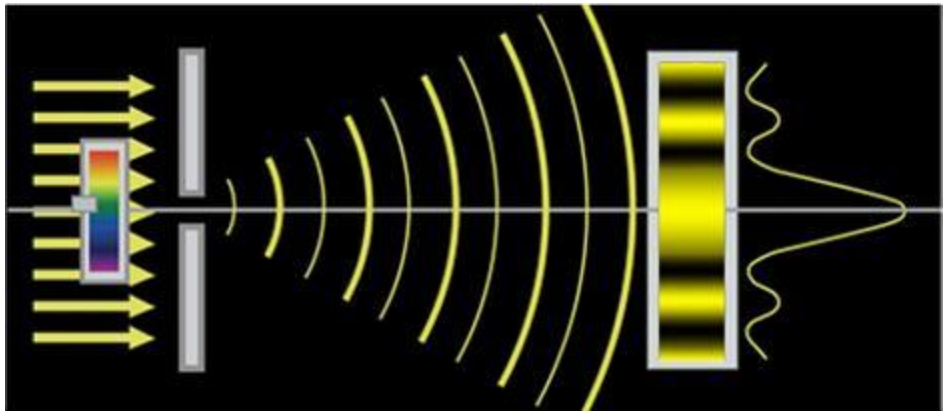
Wave diffraction

- Diffraction

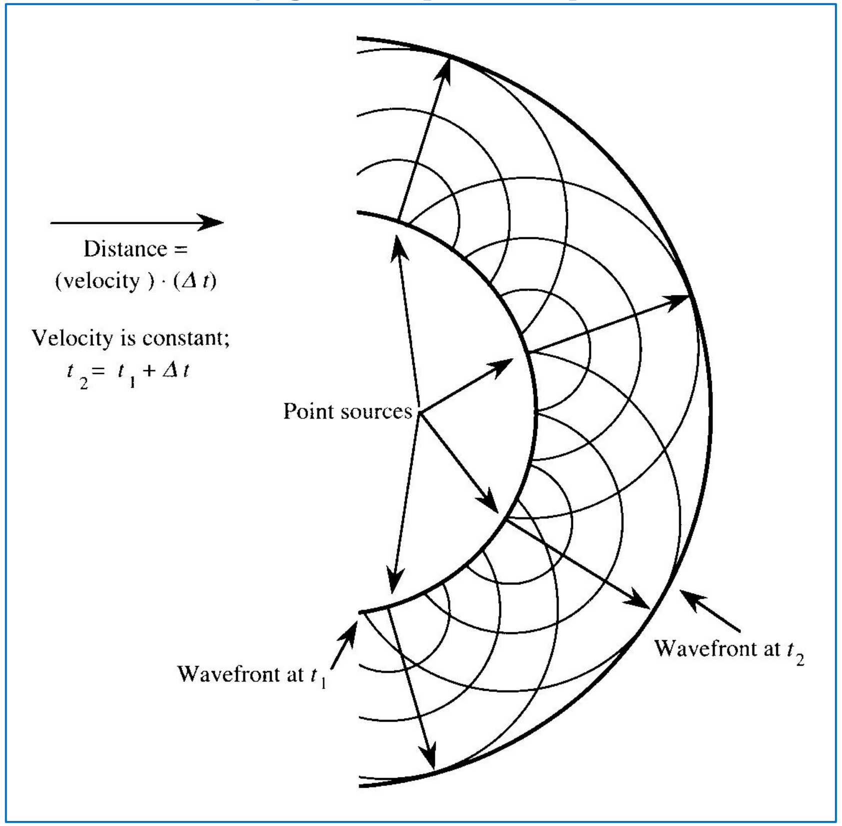
Spreading





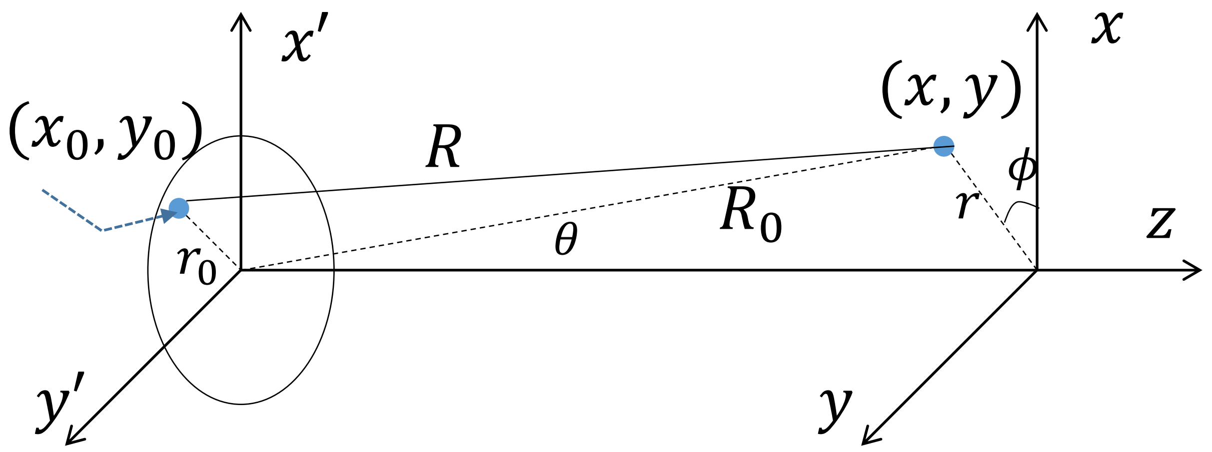


Huygens principle



Field analysis

Rayleigh-Sommerfeld Diffraction Formula



$$U(x,y,z,t) = \frac{1}{j\lambda}\int_{-\infty}^{\infty} P(x_0,y_0)\frac{e^{jkR}}{R}dx_0dy_0 \cdot \underline{e^{-j\omega t}}$$

Monochromatic wave

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$: Wave number

Field analysis

Rayleigh-Sommerfeld Diffraction Formula

$$R = \sqrt{z^2 + (x - x_0)^2 + (y - y_0)^2}$$

$$= \sqrt{x^2 + y^2 + z^2 + x_0^2 + y_0^2 - 2xx_0 - 2yy_0}$$

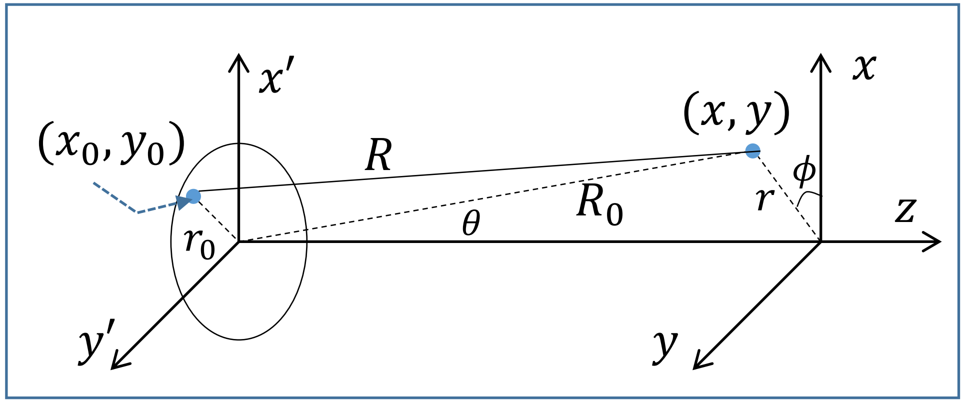
$$= \sqrt{R_0^2 \left(1 + \frac{x_0^2 + y_0^2 - 2xx_0 - 2yy_0}{R_0^2}\right)}$$

$$= R_0 \sqrt{1 + \frac{x_0^2 + y_0^2 - 2xx_0 - 2yy_0}{R_0^2}}$$

$$\approx R_0 \left(1 + \frac{x_0^2 + y_0^2 - 2xx_0 - 2yy_0}{2R_0^2}\right), \quad \sqrt{1+x} \approx 1 + \frac{x}{2} \quad if \quad |x| \ll 1$$

<u>*Fresnel approximation*</u>

$$= R_0 + \frac{x_0^2 + y_0^2}{2R_0} - \frac{xx_0 + yy_0}{R_0} = R_0 + \frac{r_0^2}{2R_0} - \frac{xx_0 + yy_0}{R_0}$$



$$\sqrt{1+b} = 1 + \frac{1}{2}b - \frac{1}{8}b^2 + \ldots$$

used to approximate r_{01} truncated to 2 terms:

$$r_{01} \approx z\left[1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right]$$

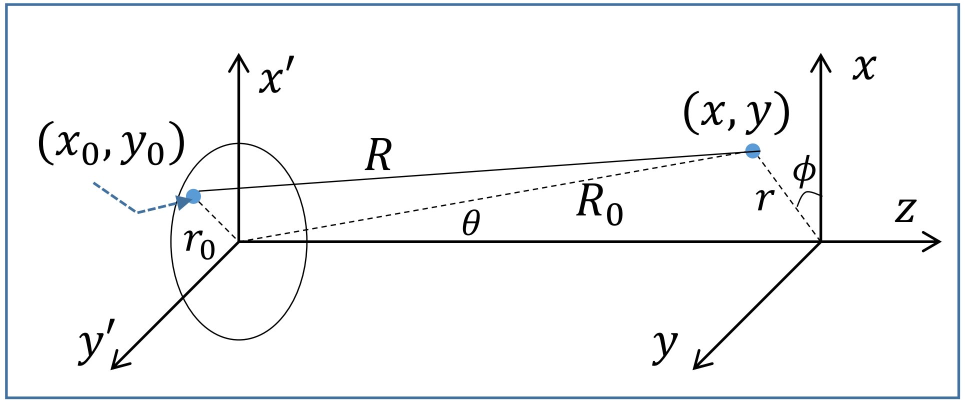
Field analysis

Rayleigh-Sommerfeld Diffraction Formula

$$r_0 = \sqrt{x_0^2 + y_0^2}, \quad r = \sqrt{x^2 + y^2}$$

$$\frac{x}{R_0} = \frac{r}{R_0} \cdot \frac{x}{r} = \sin\theta \cdot \cos\phi$$

$$\frac{y}{R_0} = \frac{r}{R_0} \cdot \frac{y}{r} = \sin\theta \cdot \sin\phi$$



$$\Phi(x,y,z) \approx \frac{e^{jR_0}}{j\lambda R_0} \int P(x_0,y_0) e^{jk(x_0^2+y_0^2)/2R_0} e^{-jk(xx_0+yy_0)/R_0} dx_0 dy_0,$$

Fourier transform

$$= \frac{e^{jR_0}}{j\lambda R_0} \int P(x_0,y_0) e^{jk(x_0^2+y_0^2)/2R_0} \boxed{e^{-j2\pi(xx_0+yy_0)/\lambda R_0}} dx_0 dy_0,$$

$$\Phi \propto FT\left[P(x_0,y_0) e^{jk(x_0^2+y_0^2)/2R_0}\right]_{f_x,f_y}$$

IMPORTANT!
The pressure amplitude measured in the field ≈ The Fourier transform of the aperture.

Field analysis

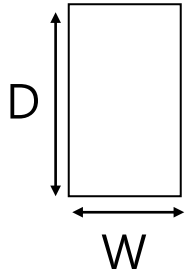
Rayleigh-Sommerfeld Diffraction Formula : Rectangle aperture

$$P(x_0,y_0) = P(x_0)\cdot P(y_0)$$

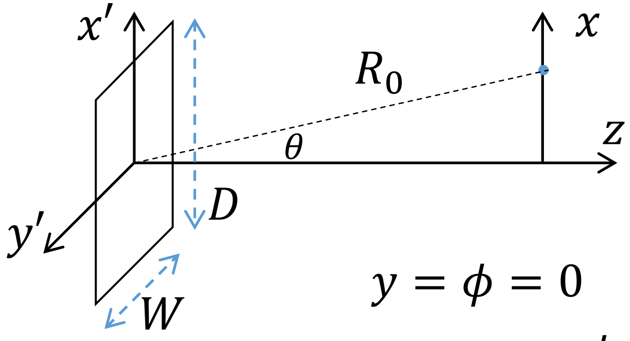
$$\Phi \propto FT\left[P(x_0,y_0)e^{jk(x_0^2+y_0^2)/2R_0}\right]_{f_x,f_y}$$

$$= FT\left[P(x_0)e^{jkx_0^2/2R_0}\right]_{f_x}\cdot FT\left[P(y_0)e^{jky_0^2/2R_0}\right]_{f_y}$$

$$\Phi_x = FT\left[P(x_0)e^{jkx_0^2/2R_0}\right]_{f_x = x/\lambda R_0}$$



$$P(x_0,\, y_0) = rect\left(\frac{x_0}{D}\right)\cdot rect\left(\frac{y_0}{W}\right)$$



$$y = \phi = 0$$

$$r = x, \cos\phi = 1$$

<u>IMPORTANT!</u>

<u>The pressure amplitude measured in the field</u>
<u>≈ The Fourier transform of the aperture.</u>

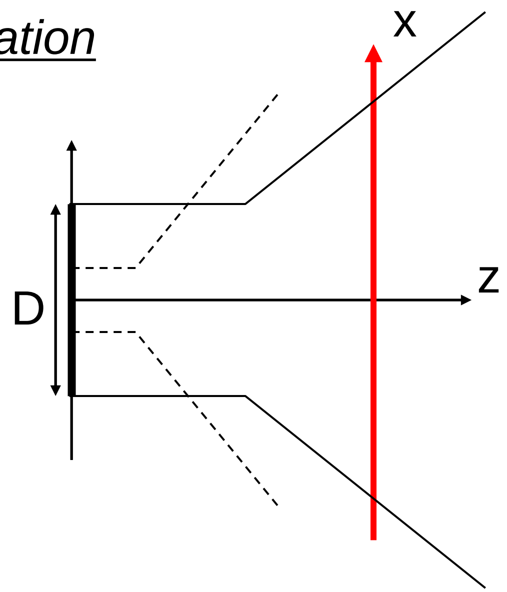
Field analysis

Rayleigh-Sommerfeld Diffraction Formula : Rectangle aperture

At far field, $z \gg x_0 \Rightarrow x_0^2/2R_0 \approx 0$ *Fraunhofer approximation*

$$\Phi_x = FT[P(x_0)]_{f_x = x/\lambda R_0} = D \cdot \text{sinc}\left(\frac{x}{\lambda R_0} D\right)$$

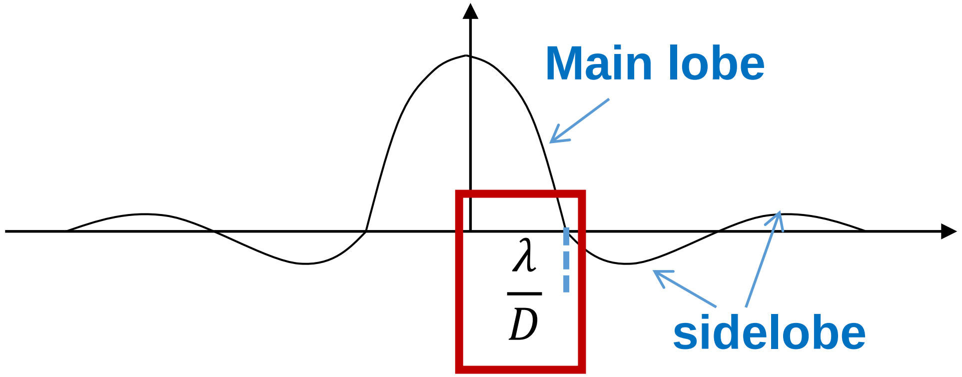
$$= D \cdot \text{sinc}\left(\frac{\sin\theta}{\lambda/D}\right), \quad when \ y=0 \ (\cos\phi = 1)$$



$$\sin\theta = \frac{x}{R_0} \approx \theta, x \ll R_0$$

Small angle approximation

Diffraction limited resolution



To obtain a good resolution

- **Use larger D, smaller wavelength (higher frequency)**
- **Make mainlobe width and sidelobe level as low as possible**

Point-spread function

Field analysis

Rayleigh-Sommerfeld Diffraction Formula : Rectangle aperture

- **Small angle approximation**
  – Far enough away that angle off axis is small

- **Fresnel approximation**
  – Neglecting higher order terms in binomial expansion (valid mid and farfield)

- **Fraunhofer approximation**
  – Aperture size (spatial extent) is small relative to distance between aperture and observation point distance (valid farfield)

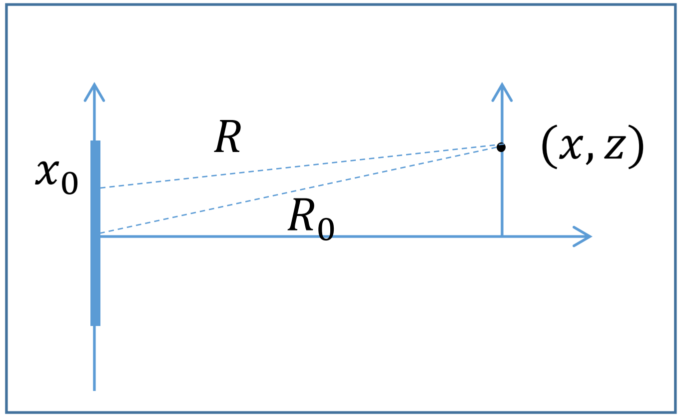
Field analysis

Rayleigh-Sommerfeld Diffraction Formula : Linear aperture – ‘1-D’ model

$$R = \sqrt{z^2 + (x - x_0)^2} = \sqrt{z^2 + x^2 + x_0^2 - 2xx_0}$$

$$= \sqrt{z^2\left(1 + \frac{x^2 + x_0^2 - 2xx_0}{z^2}\right)} = z\sqrt{1 + \frac{x^2 + x_0^2 - 2xx_0}{z^2}}$$

$$\approx z\left(1 + \frac{x^2 + x_0^2 - 2xx_0}{2z^2}\right) = z + \frac{x^2}{2z} + \frac{x_0^2}{2z} - \frac{xx_0}{z}$$



$$\Phi(x, y, z) \approx \frac{e^{j\Omega}}{j\lambda z} \int P(x_0) e^{jkx_0^2/2z} e^{-jk\frac{xx_0}{z}} dx_0 dy_0, \quad \Omega = z + \frac{x^2}{2z}$$

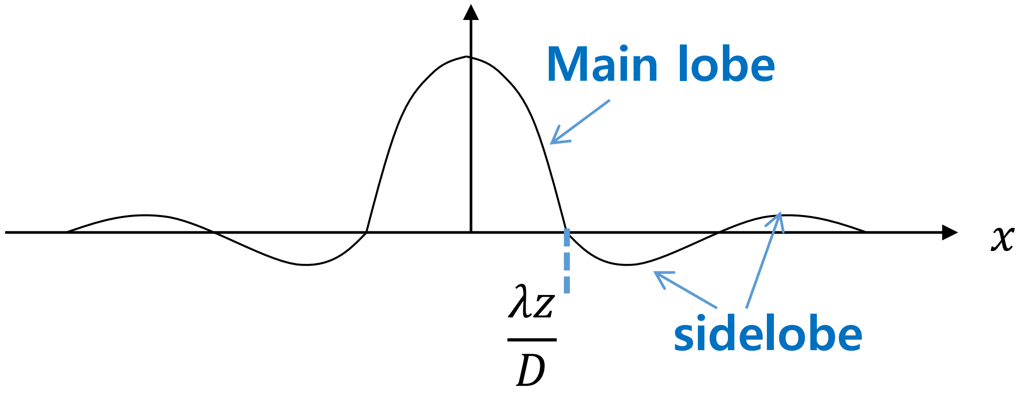
$$\propto FT\left[P(x_0) e^{jkx_0^2/2z}\right]_{f_x = x/\lambda z}$$

Field analysis

Rayleigh-Sommerfeld Diffraction Formula : Linear aperture – '1-D' model

At far field, where $z >> x_0 \Rightarrow x_0^2/2z \approx 0$

$$\varphi_x = FT[P(x_0)]_{f_x = x/\lambda z} = D \cdot \sin c\left(\frac{x}{\lambda z/D}\right) \qquad P(x_0) = rect(\frac{x_0}{D})$$



$$\boxed{\frac{x}{z} = tan\theta \approx \theta, \theta \ll 1}$$

Natural focusing is achieved at far field

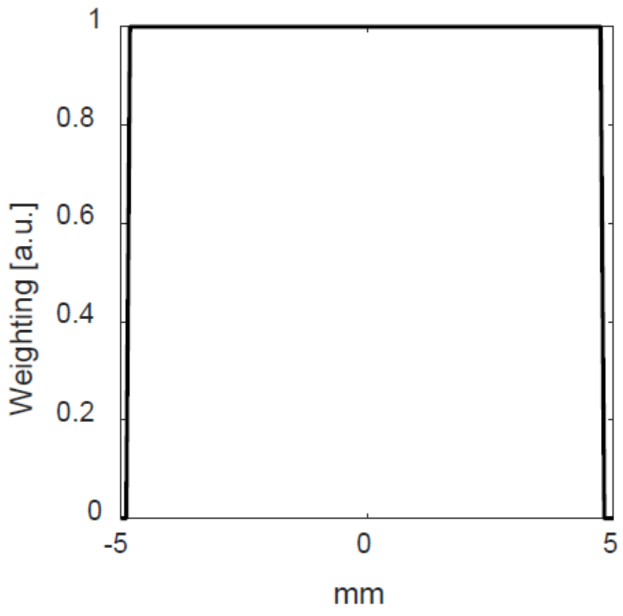
→ No focusing operation is required.

Point-spread function

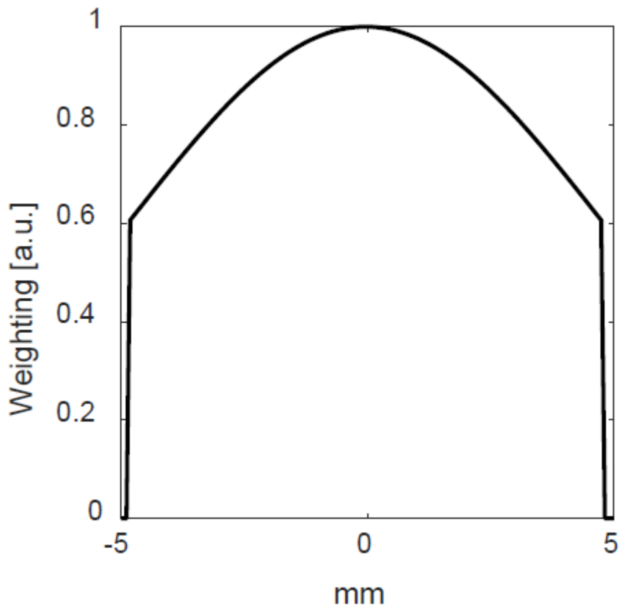
Field analysis: Apodization technique

There are several ways, but anything to adjust the aperture (rect* rect) will affect the point spread function (=beam pattern) and resulting aperture. One way is to change the relative contribution of each element in the aperture using apodization, which changes the rectangular aperture to (for example) a Gaussian-shaped aperture

All elements in array weighted to 1



Non-uniform weighting across aperture



Field analysis: Apodization technique

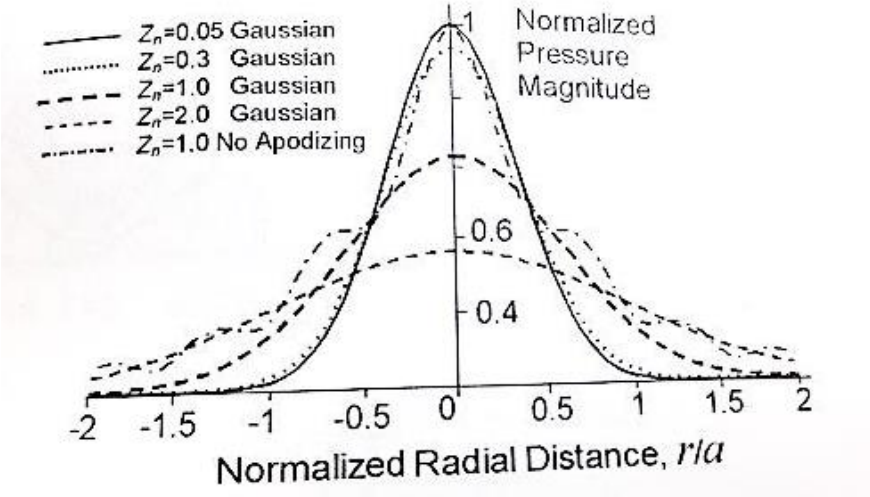
• What is the effect of apodization(also called weighting, shading)?

> Gaussian (decreased side lobes relative to main lobe but also decreased transmit pressure in main lobe)

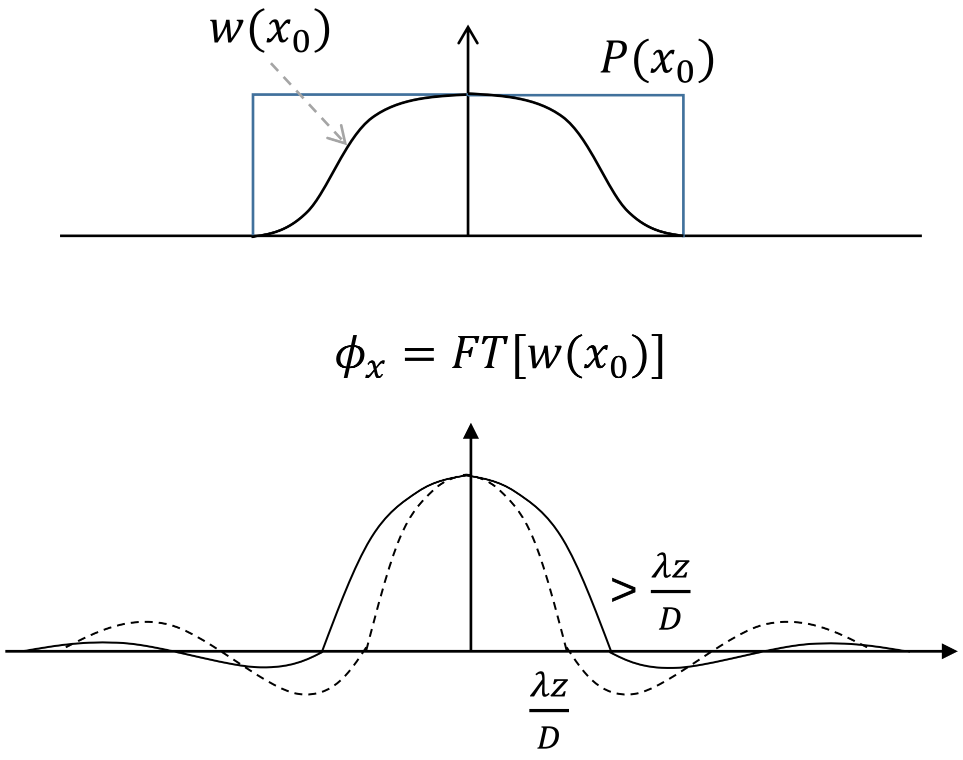
• That is, we previously assumed all elements in the aperture had equal weights (=1). What if they do not?

• Modify PSF

• Also SNR



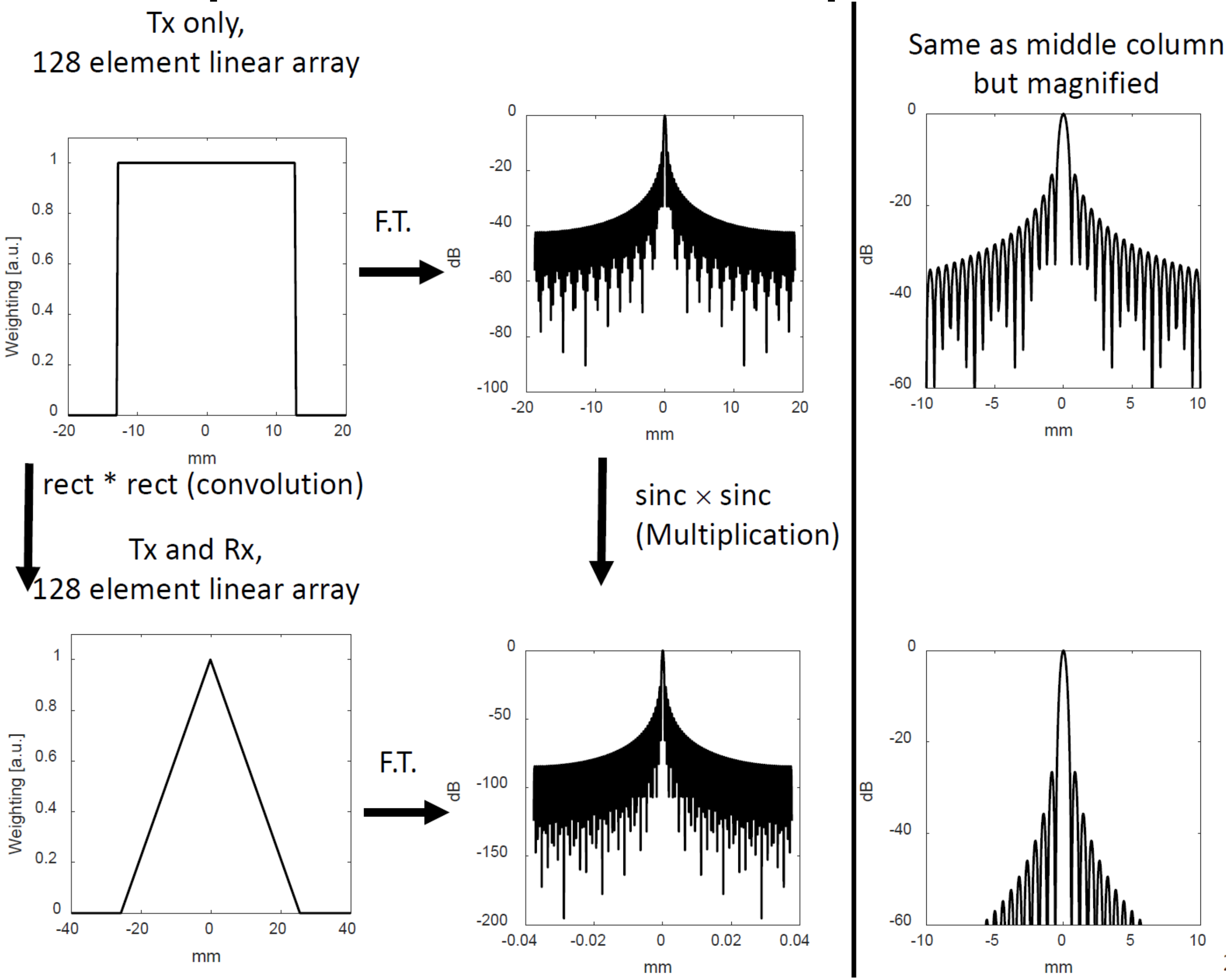
Field analysis: Apodization technique



Apodization technique

Goal: Reduced sidelobe levels
with increased main lobe width

Field analysis: Apodization technique



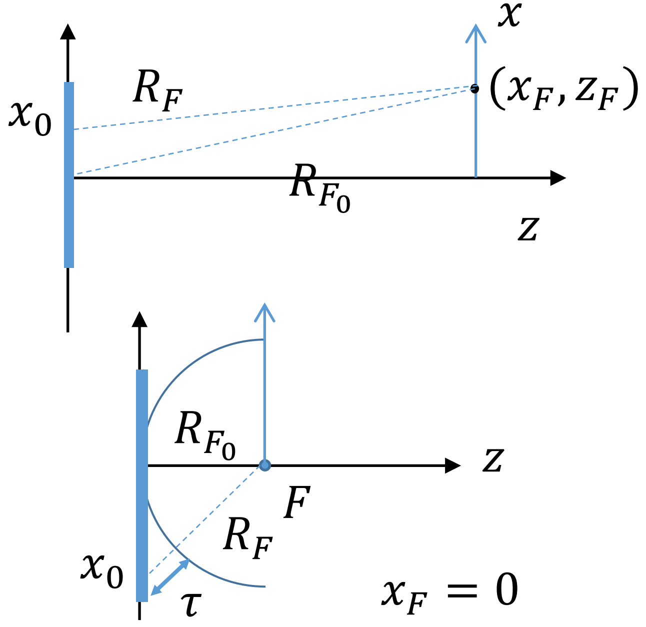
Field analysis

Rayleigh-Sommerfeld Diffraction Formula : Linear + Focus – ‘1-D’ model

$$P_l(x_0) = e^{-jk(R_F - R_0)} P(x_0) = e^{-j\omega\frac{R_f - R}{c}} P(x_0)$$

$$= e^{-j\omega\tau} P(x_0) = e^{-j\omega\left(\sqrt{F^2 + x_0^2} - F\right)/c} P(x_0)$$

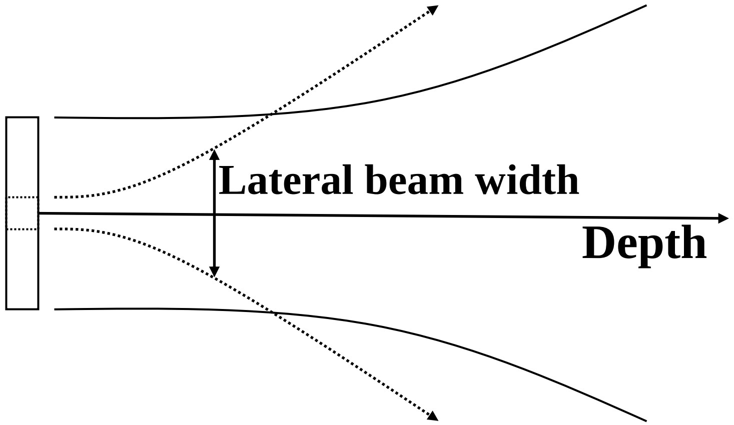
$$\boxed{\approx e^{-jkx_0^2/2F} P(x_0)}$$



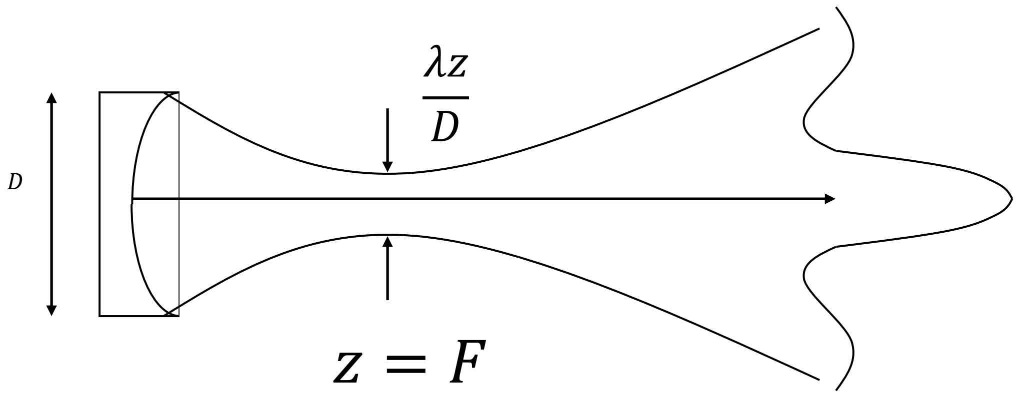
$$\varphi_x = FT[P_l'(x_0)] = \begin{cases} FT[P(x_0)] = D \cdot \sin c\left(\dfrac{x}{\lambda F/D}\right), & \mathrm{z = F} \\ FT\left[e^{ik\beta x_0^2} P(x_0)\right] \ (\beta = 1/2\mathrm{z} - 1/2\mathrm{F}), & z \neq F \end{cases}$$

Field analysis

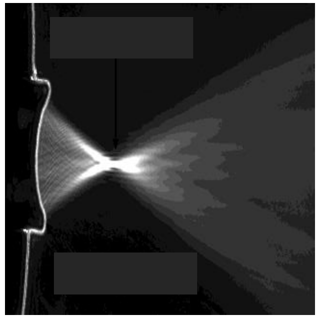
Geometric focusing with an acoustic lens



Unfocused



Focused (with lens)



- Fixed focusing: focal depth is fixed
- Focused beam width decreases as a source diameter increases, frequency gets higher, and the focal depth gets closer to source.
- Beam width gets larger as z is more away from the focal depth

Field analysis

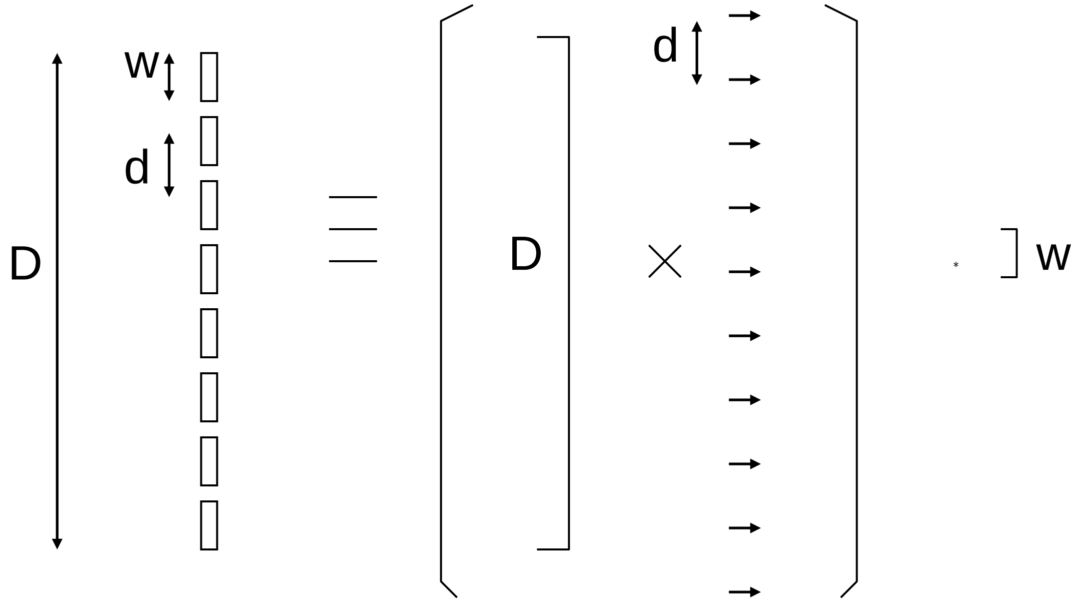
1-D Linear Array model

<u>IMPORTANT! (again)</u>

<u>The pressure amplitude measured in the field</u>

<u>≈ The Fourier transform of the aperture.</u>

Array model

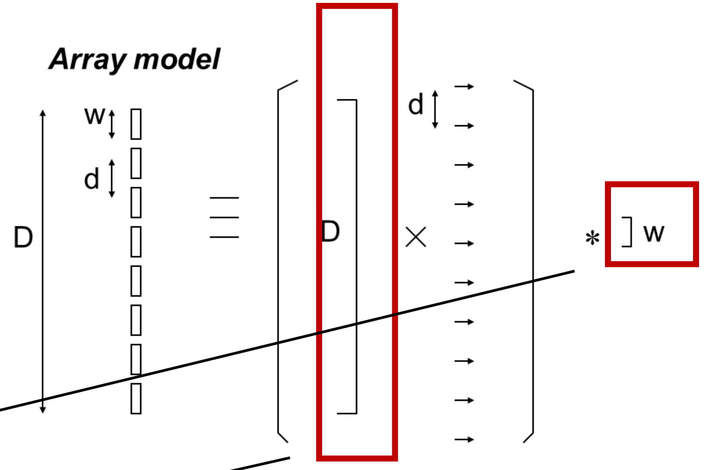


$$a(x_0) = \left\{ rect\left(\frac{x_0}{D}\right) \cdot \sum_{k=-\infty}^{\infty} \delta(x_0 - kd) \right\} * rect\left(\frac{x_0}{w}\right)$$

Field analysis

1-D Linear Array model

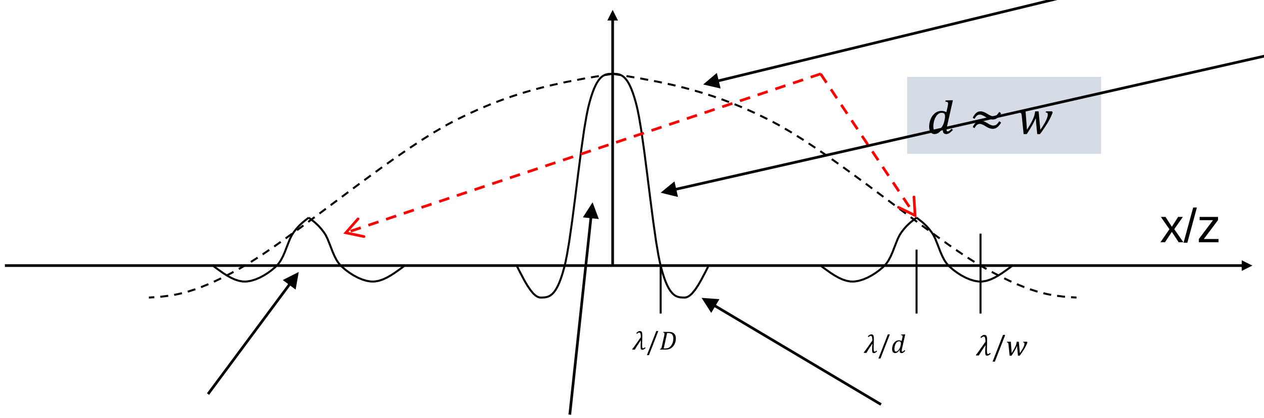
Convolution ⟷ Multiplication (FT)
Multiplication ⟷ Convolution



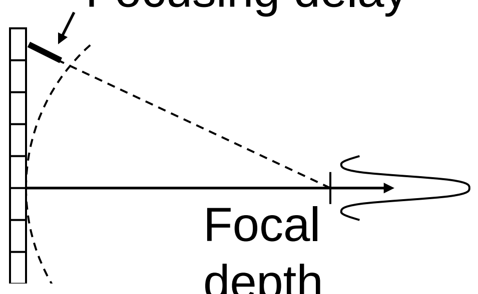
$$FT\left[\left\{rect\left(\frac{x_0}{D}\right)\cdot\sum_{k=-\infty}^{\infty}\delta(x_0-kd)\right\}* rect\left(\frac{x_0}{w}\right)\right]_{f_x=x/\lambda z}$$

$$\Rightarrow \left[D\cdot \mathrm{sinc}(f_x\cdot D) * \frac{1}{d}\sum_{k=-\infty}^{\infty}\delta\left(f_x-k\frac{1}{d}\right)\right]\times \omega\cdot \mathrm{sinc}(f_x\cdot d)$$

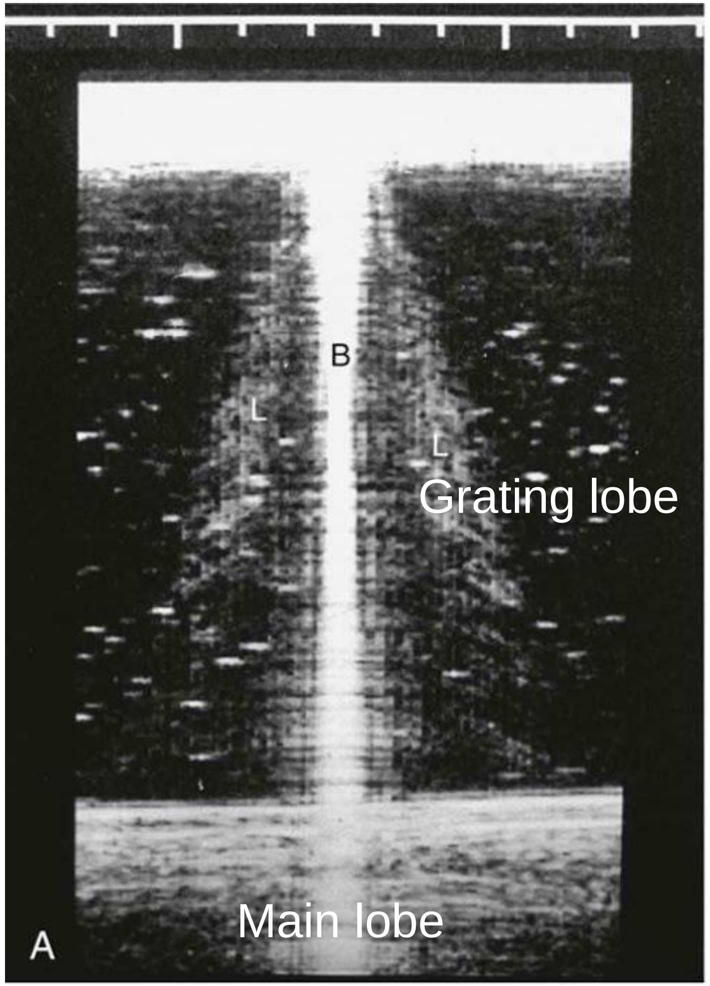
to suppress grating lobes



Grating lobe **Main lobe** **Side lobe**



Side lobe and Grating lobe artifact



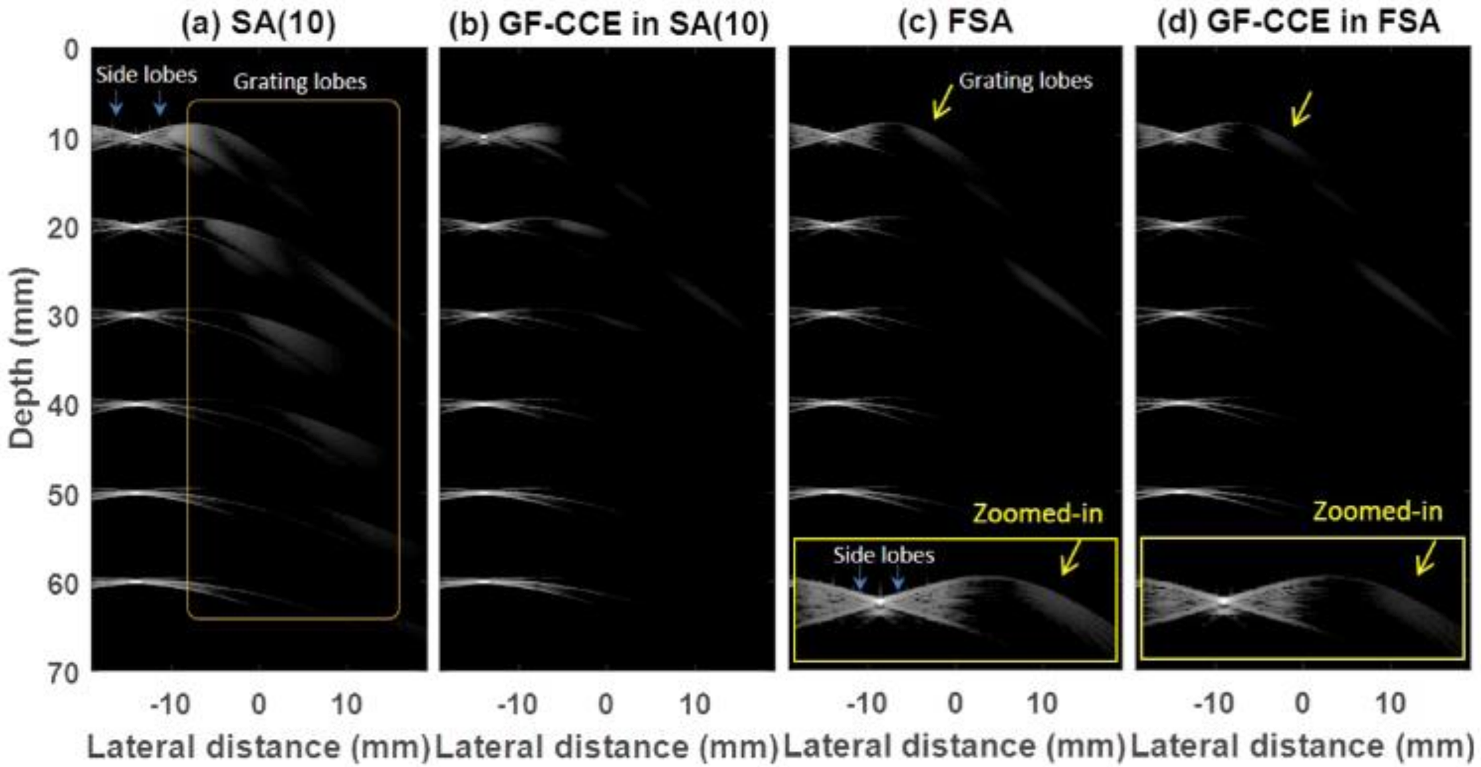


Fig. 4. (Color online) Comparison of pulse-echo point spread functions (80 dB dynamic range): (a) SA (10), (b) GF-CCE in SA (10), (c) FSA, and (d) GF-CCE in FSA. Insets are magnified images of the first wire target.

Field analysis

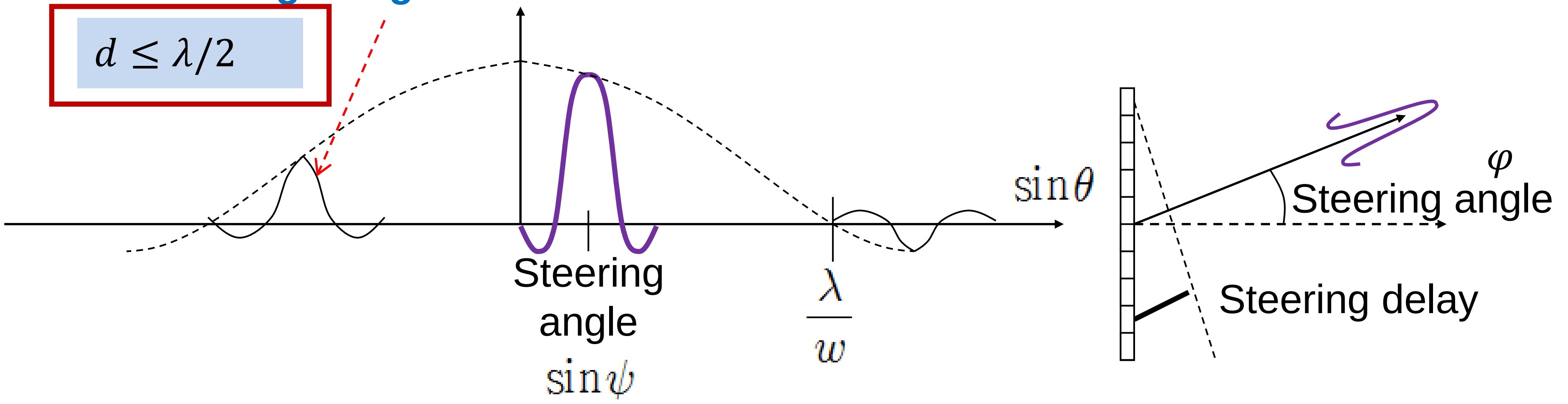
1-D Linear Array model + Steering

$$FT\left[\left\{rect\left(\frac{x_0}{D}\right)e^{jkx_0\sin\psi}\cdot\sum_{k=-\infty}^{\infty}\delta(x_0-kd)\right\}*rect\left(\frac{x_0}{w}\right)\right]_{f_x=x/\lambda R_0=\sin\theta/\lambda}$$

$$\left[D\cdot\mathrm{sinc}\left\{\left(\frac{\sin\theta}{\lambda}-\frac{\sin\psi}{\lambda}\right)\cdot D\right\}*\frac{1}{d}\sum_{k=-\infty}^{\infty}\delta\left(\frac{\sin\theta}{\lambda}-k\frac{1}{d}\right)\right]\times\omega\cdot\mathrm{sinc}\left(\frac{\sin\theta}{\lambda}\cdot d\right)$$

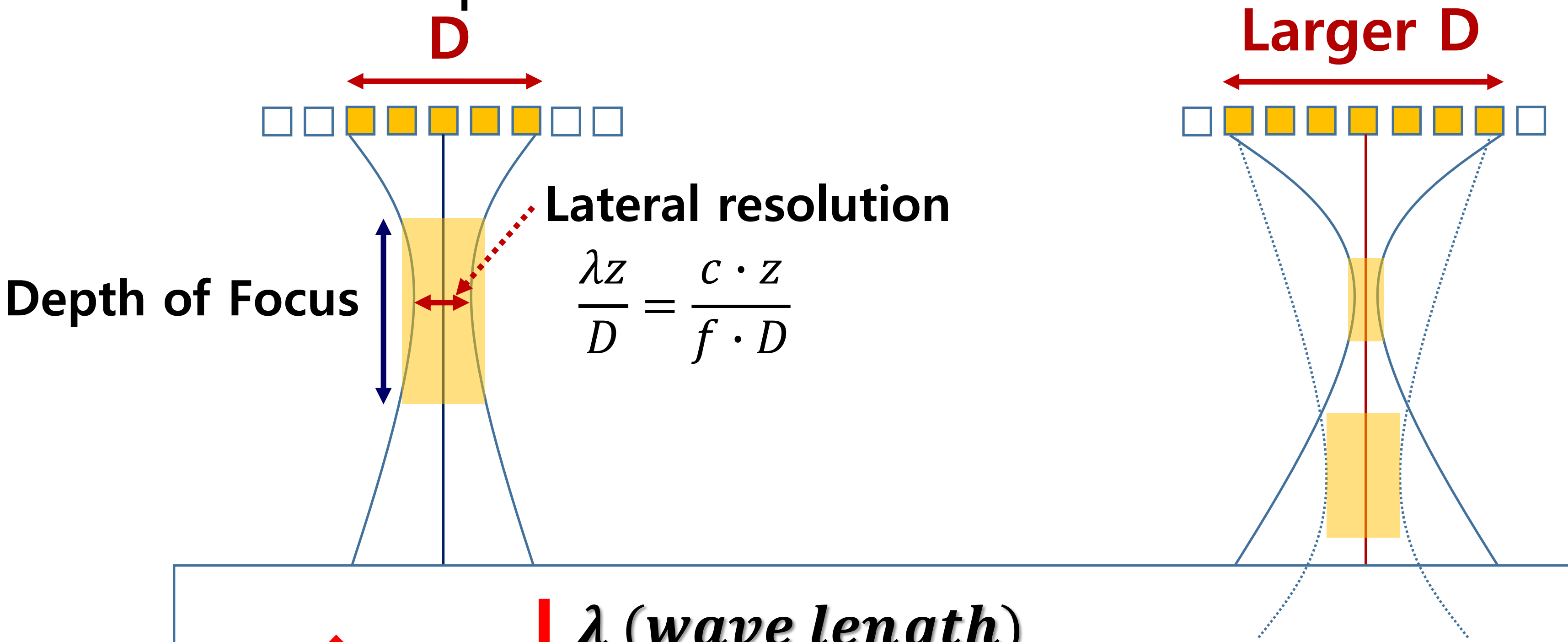
to eliminate grating lobes

$\boxed{d\leq\lambda/2}$



Focusing

Lateral resolution vs Depth of focus



$$\frac{\lambda z}{D} = \frac{c \cdot z}{f \cdot D}$$

↑ $\boldsymbol{D}$

↓ $\boldsymbol{\lambda\ (wave\ length)}$

↑ $\boldsymbol{f\ (frequency)}$

$$\boldsymbol{f \cdot \lambda = c}$$

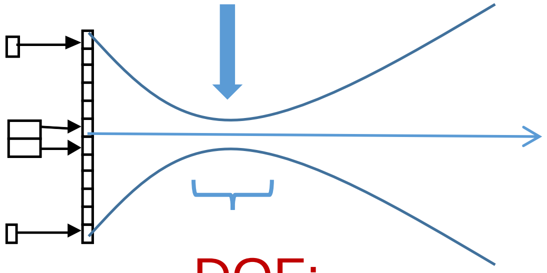
→ Smaller Beam width (Higher Spatial Resolution)

→ Reduced Depth of Focus

Multi-zone blending

Fixed focusing (Tx) – Fixed focusing (Rx)

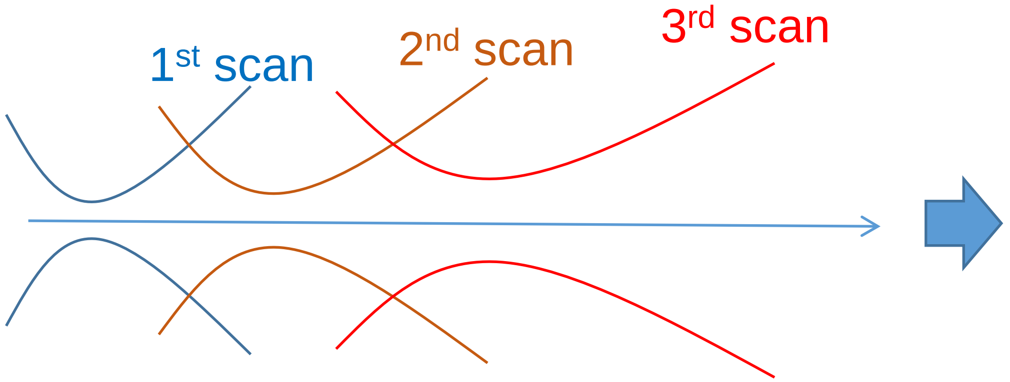
Controllable focal depth



DOF:
Depth of Focus

Lensed transducer
has a fixed focal depth
that can't be changed

Multi-zone focusing



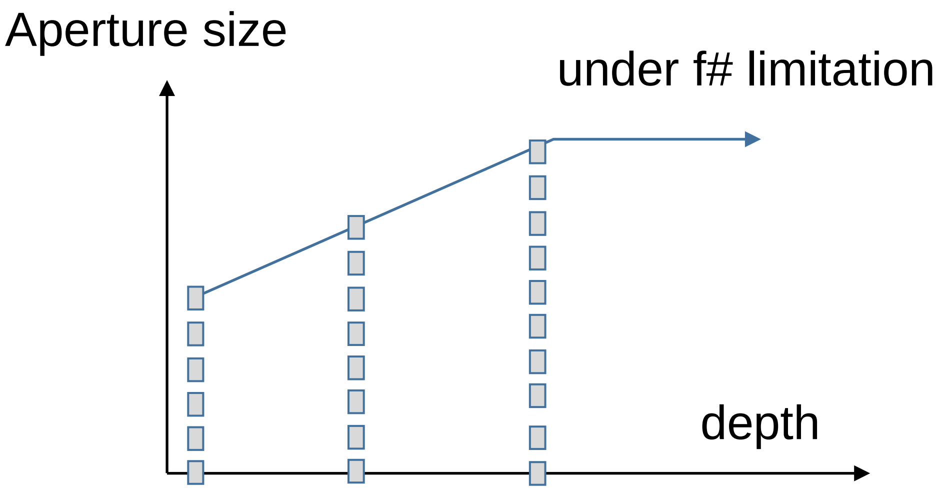
Zone blending
→ Scan line

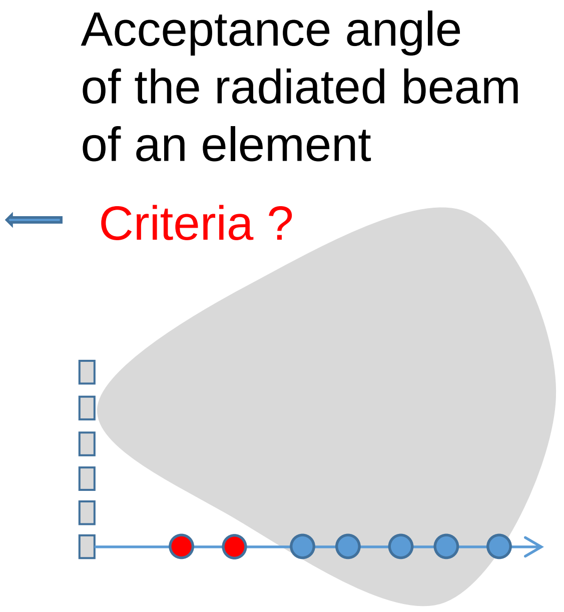
Dynamic aperture

F-number

$$f\# = \frac{F}{D} \quad or \quad \frac{z}{D}$$

Dynamic aperture





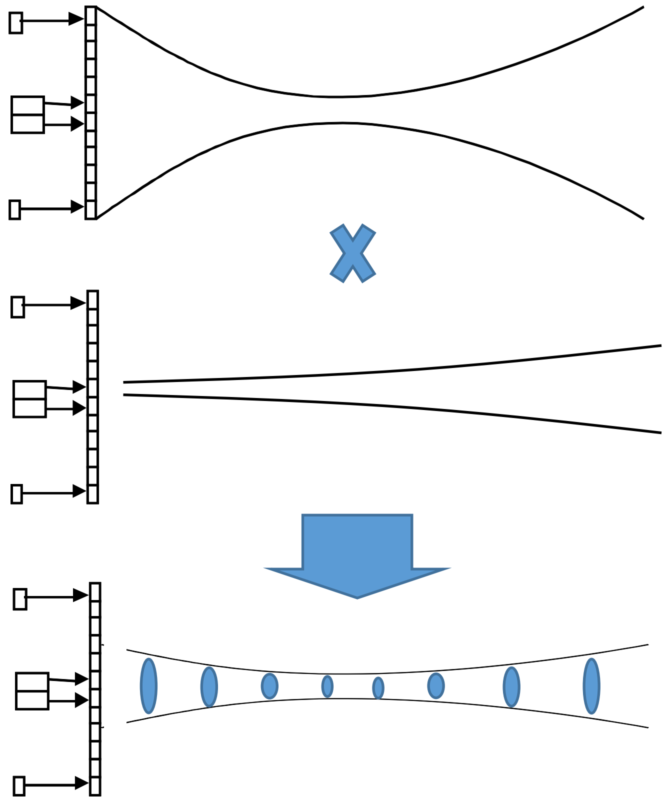
Beamforming

Receive dynamic focusing

Tx: Fixed focusing

Rx: Dynamic focusing

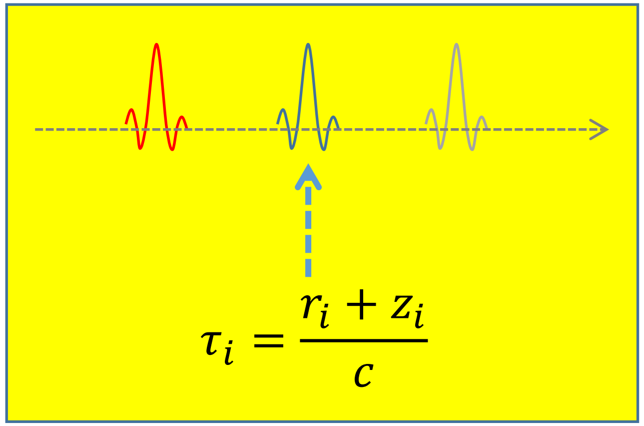
Overall (roundtrip) beam pattern

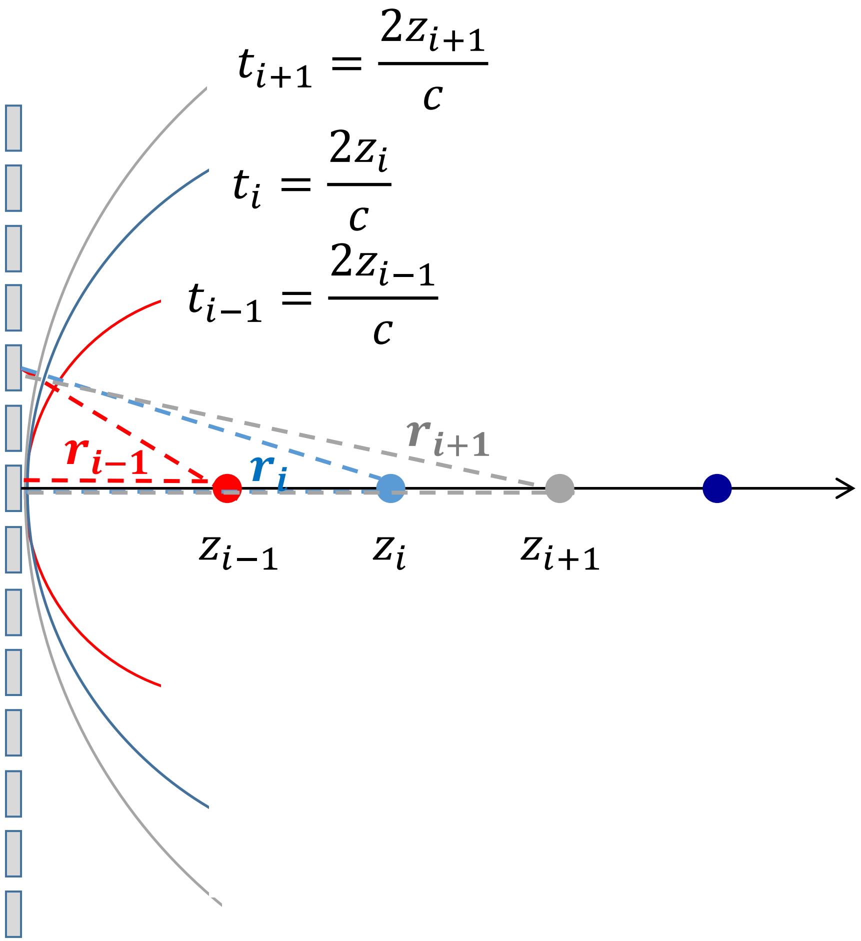


Beamforming

Receive dynamic focusing

Change the delay pattern
dynamically with depth(i.e., time)

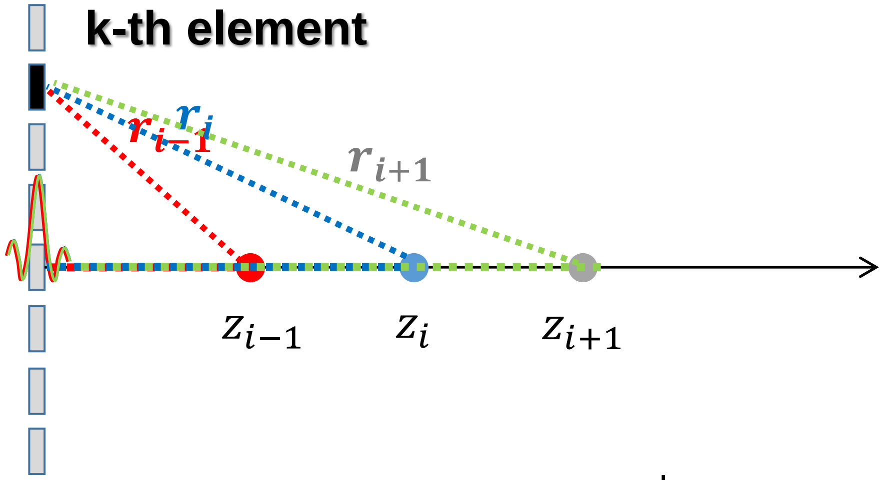




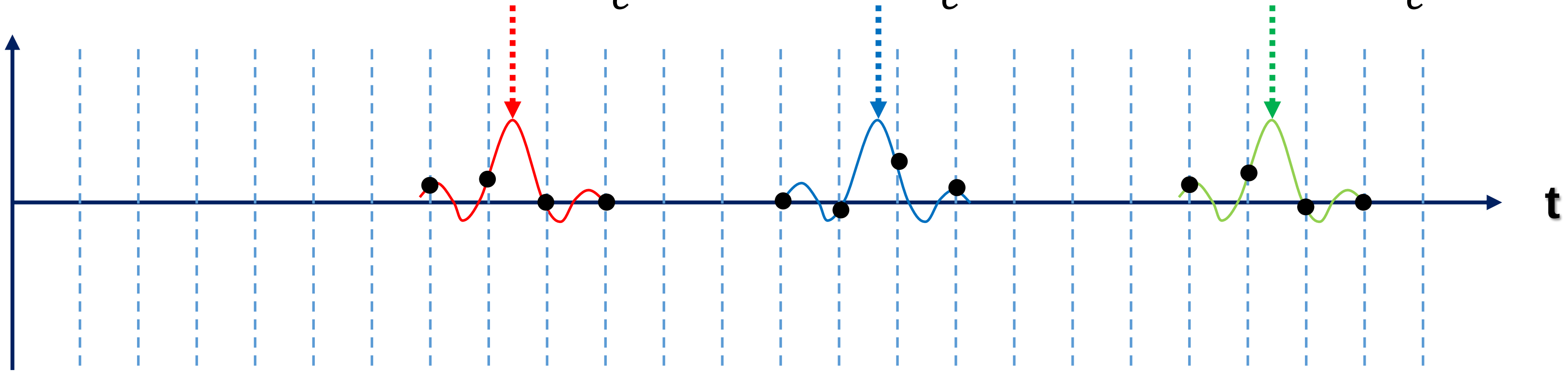
Do we have to use all the elements
for all imaging points?

Beamforming

Receive dynamic focusing



$$t^k{}_{i-1} = \frac{z_{i-1} + r_{i-1}}{c} \qquad t^k{}_{i} = \frac{z_{i} + r_{i}}{c} \qquad t^k{}_{i+1} = \frac{z_{i+1} + r_{i+1}}{c}$$



Beamforming

Receive dynamic focusing

$$Focused\_Signal\,(z_i) = \sum_{k=0}^{N-1} Received\_Signal(t - \tau^k{}_i)$$

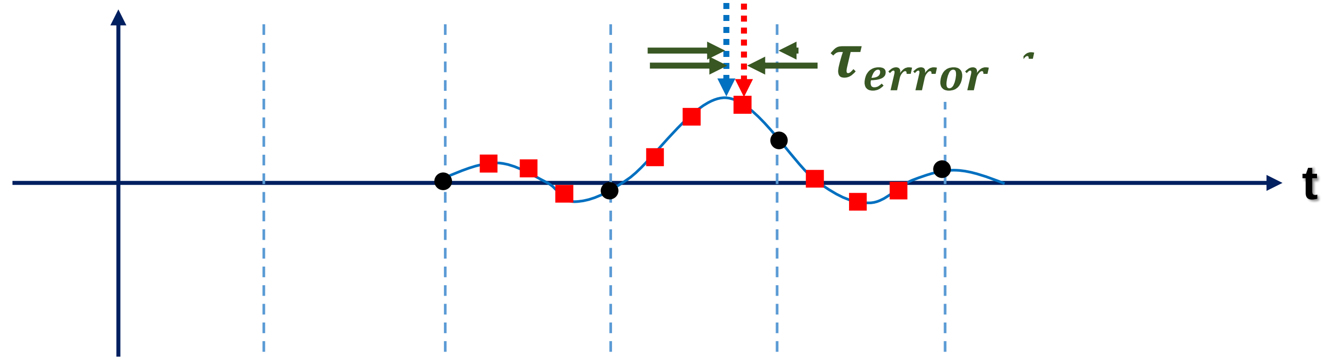
Focsuing delay

$$\tau^k{}_i = t^k{}_i + \boxed{\tau_{error}}$$ **Focusing Delay Error**

$$|\tau_{error}| \le \frac{1}{16 f_0} \quad (f_0\text{: Center frequency})$$

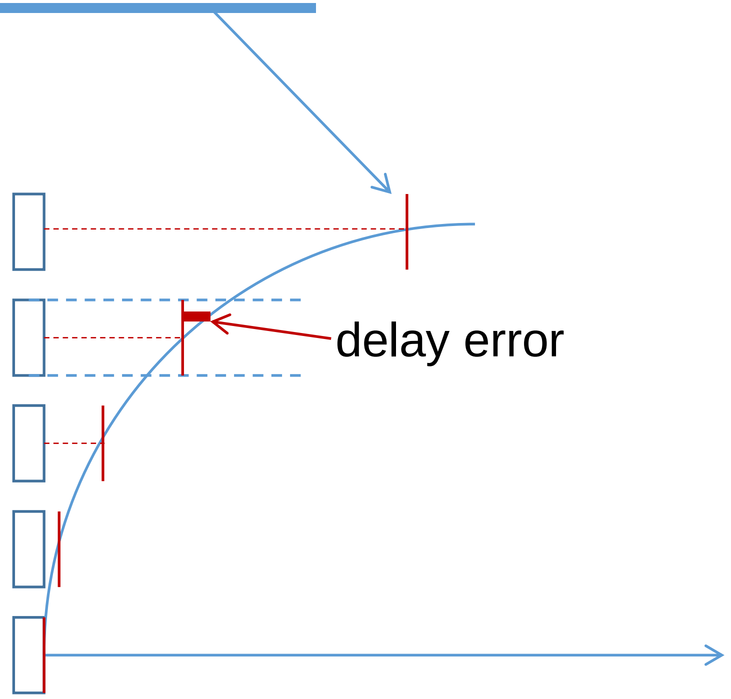
$$t^k{}_i = \frac{z_i + r_i}{c}$$

x4 interpolation



Beamforming

Array quantization & delay quantization

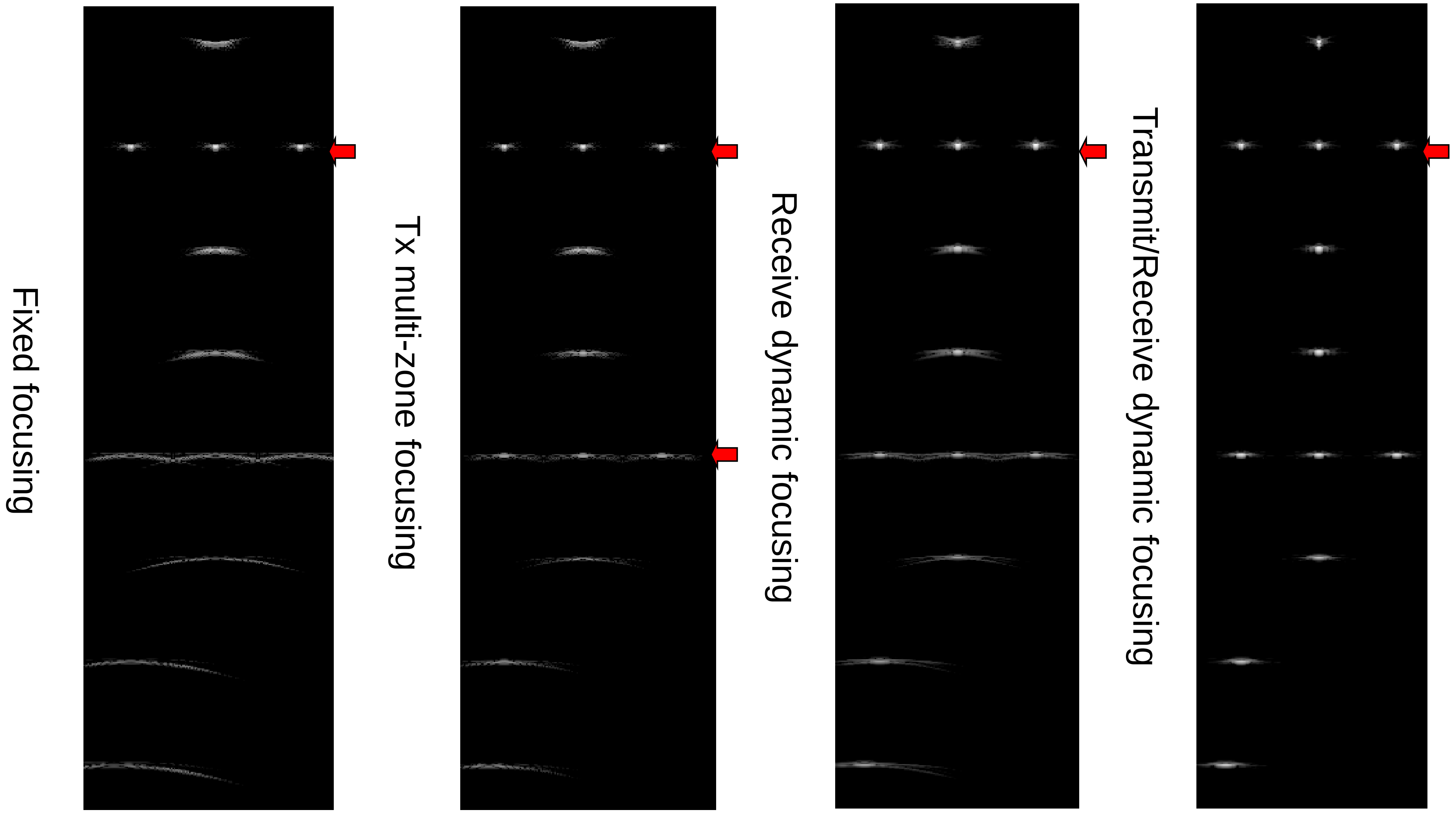


Delay error should be properly controlled.

Element size ↑

Delay error ↑

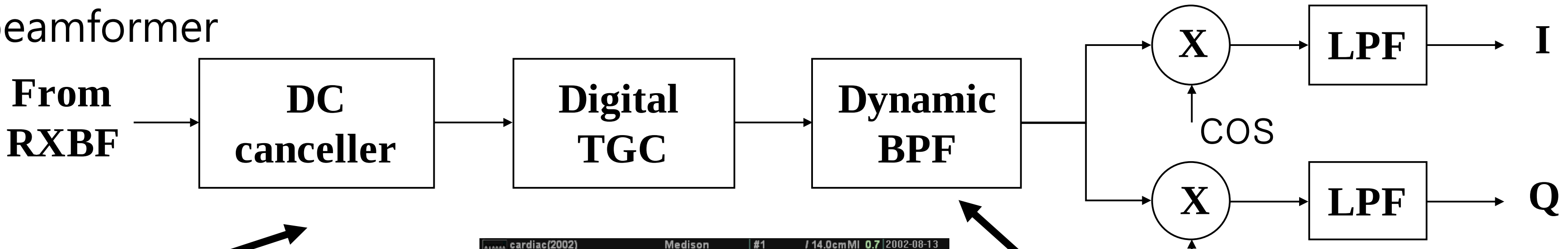
Beamforming



B-mode echo processing

❖ Echo Processor: Front-end (RF processing)

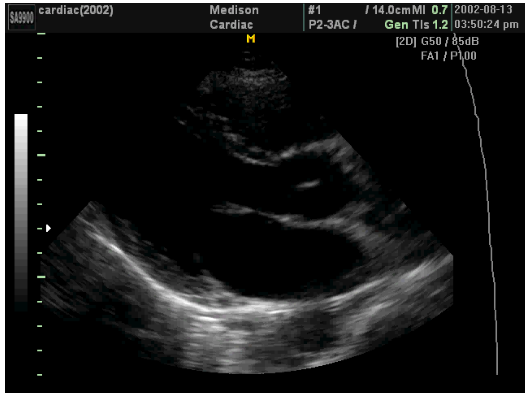
RXBF: Receive beamformer

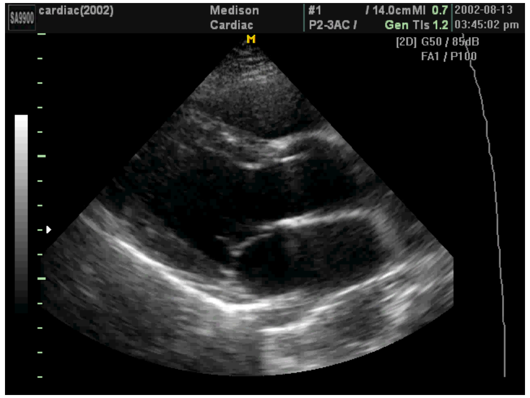


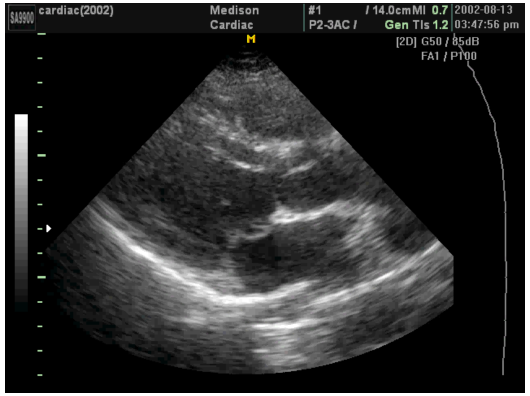
DC noise rejection

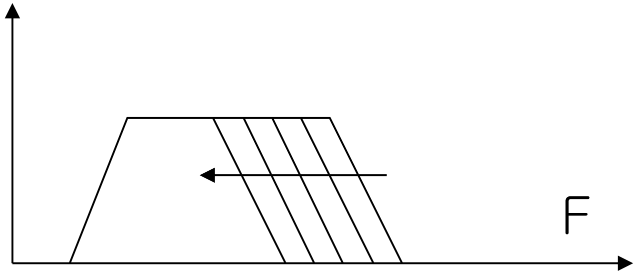
Band selection

Quadrature demodulation









B-mode echo processing – quadrature demodulator

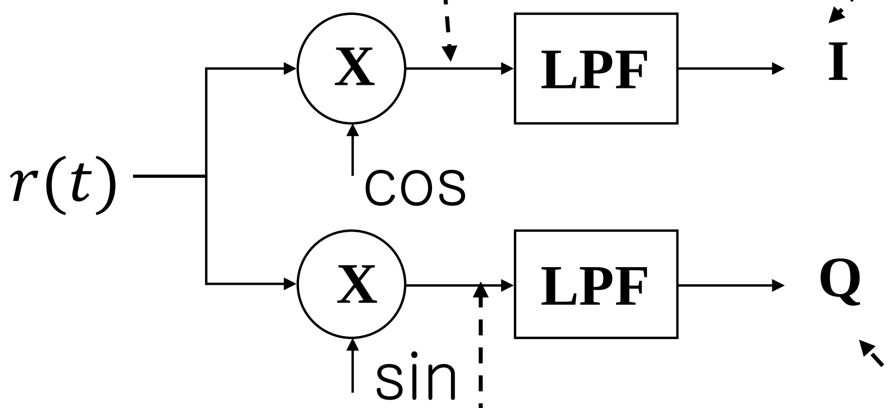
$$r(t)\cos(2\pi f_0 t)$$
$$= a(t)\cos(2\pi f_0 t + \phi)\cos(2\pi f_0 t)$$
$$= \frac{a(t)}{2}\left(\cos(\phi) + \cancel{\cos(2\pi(2f_0) + \phi)}\right)$$

$$I = \frac{a(t)}{2}(\cos(\phi))$$

$$r(t) = a(t)\cos(2\pi f_0 t + \phi)$$

r(t): Received signal

a(t): Image



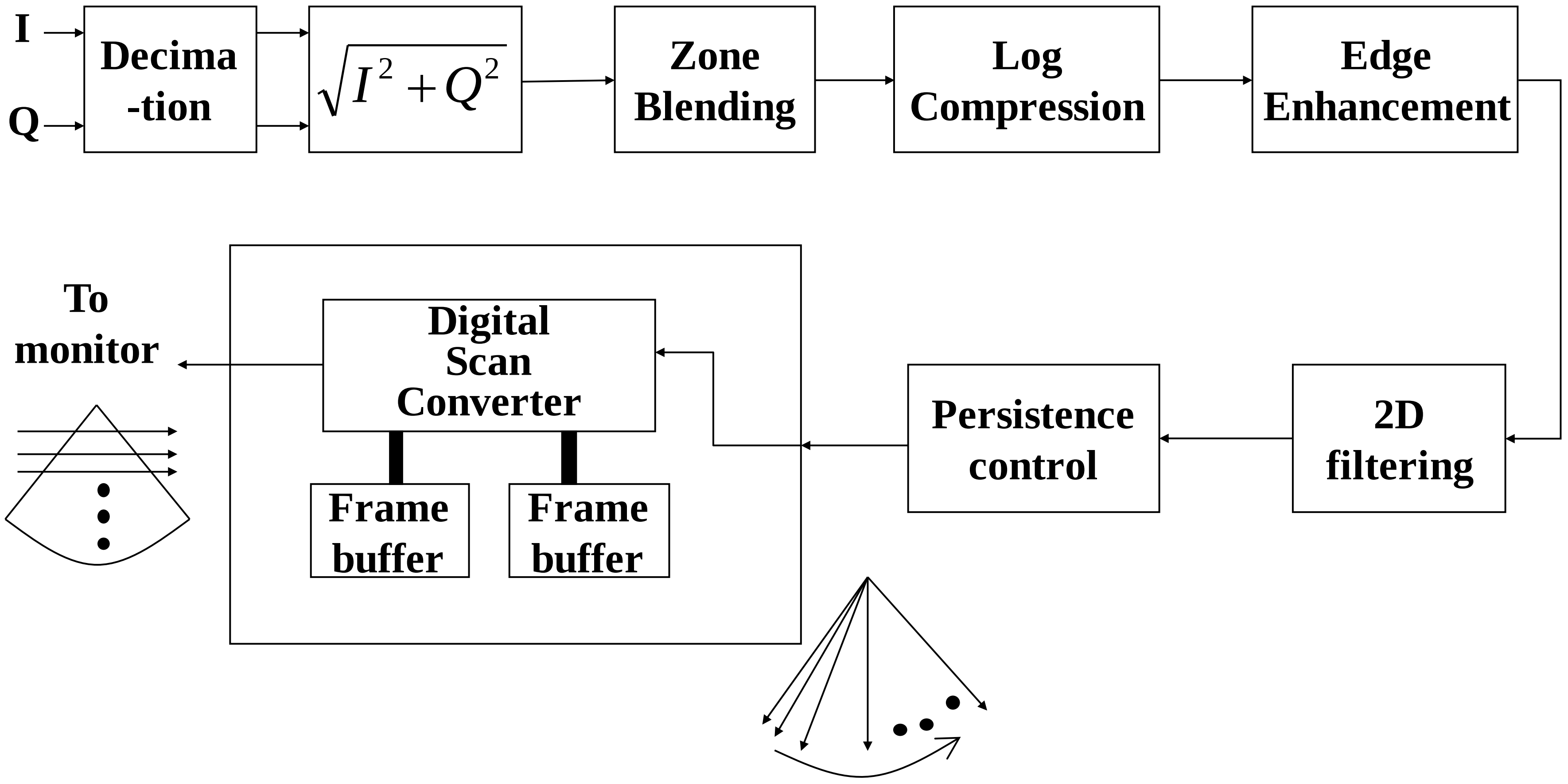
$$\sqrt{I^2 + Q^2}$$
$$= \frac{a(t)}{2}\sqrt{\cos^2(\phi) + \sin^2(\phi)}$$
$$= \frac{a(t)}{2}$$

$$r(t)\sin(2\pi f_0 t)$$
$$= a(t)\cos(2\pi f_0 t + \phi)\sin(2\pi f_0 t)$$
$$= \frac{a(t)}{2}\left(-\sin(\phi) + \cancel{\sin(2\pi(2f_0) + \phi)}\right)$$

$$Q = -\frac{a(t)}{2}(\sin(\phi))$$

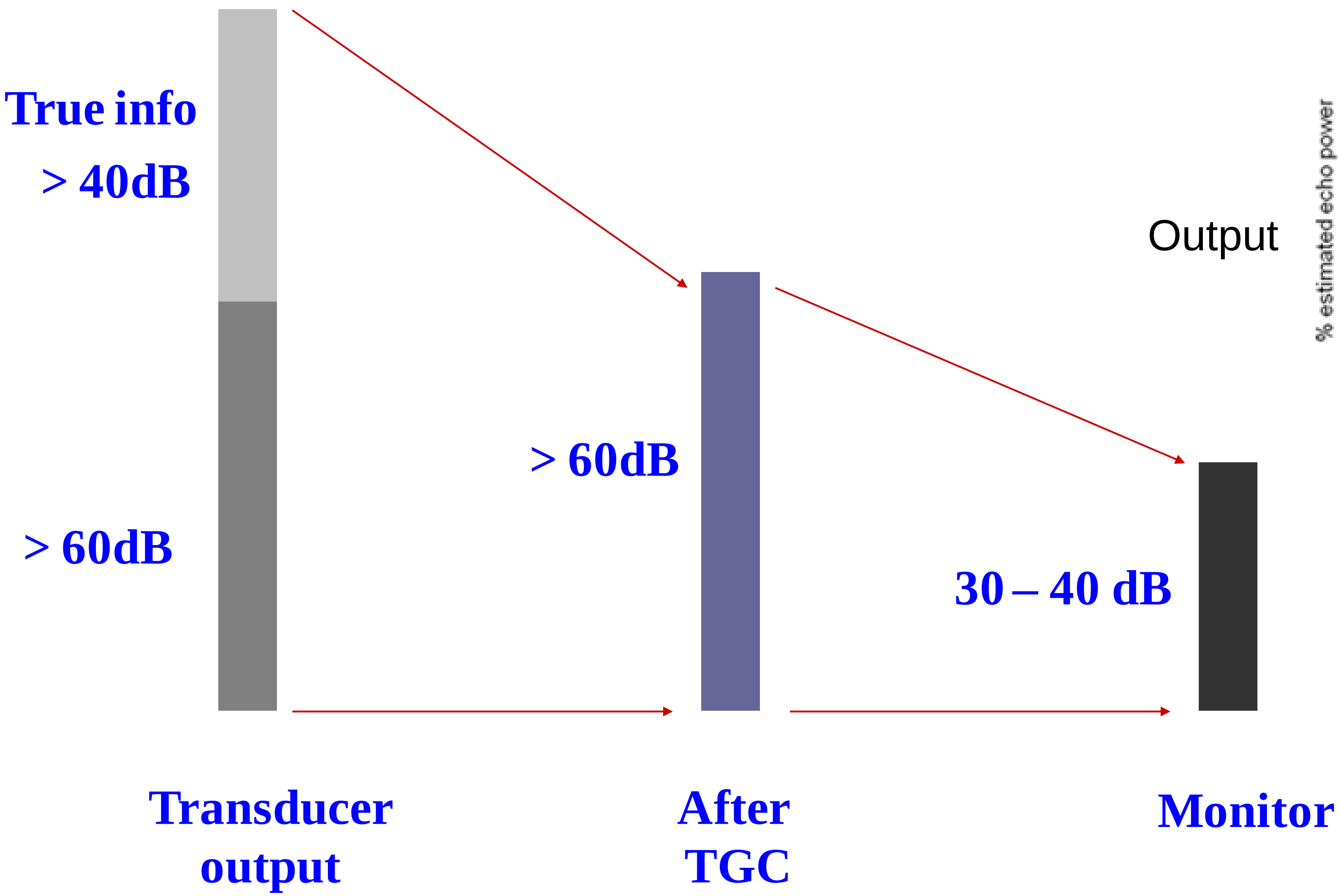
B-mode echo processing

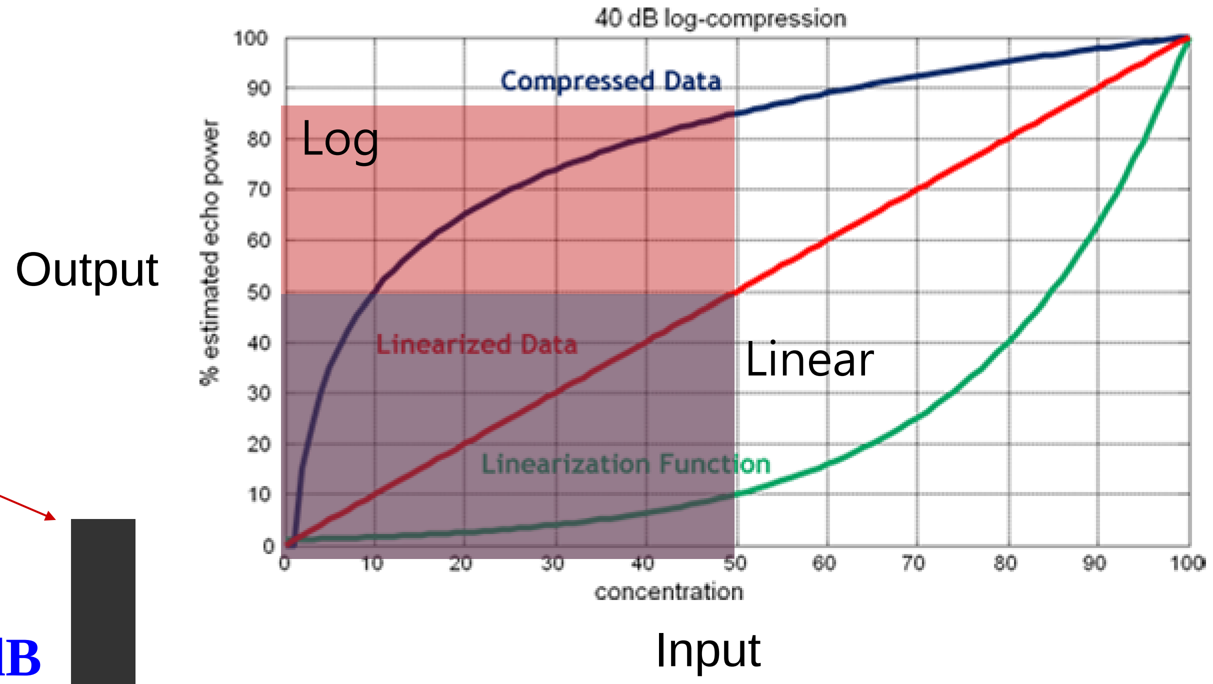
❖ Echo processor: Back-end



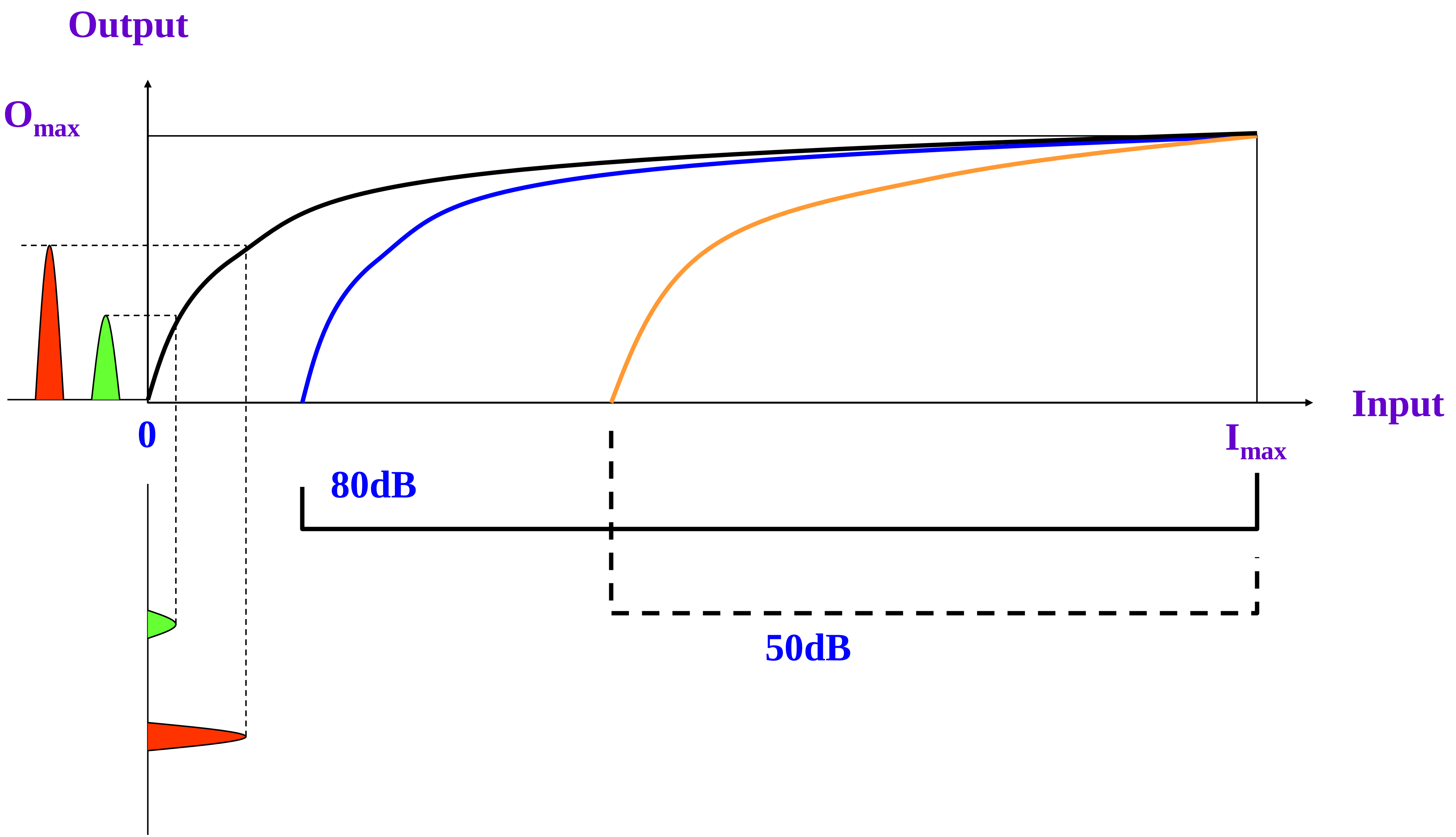
B-mode echo processing

❖ Dynamic Range Control / Log Compression





B-mode echo processing

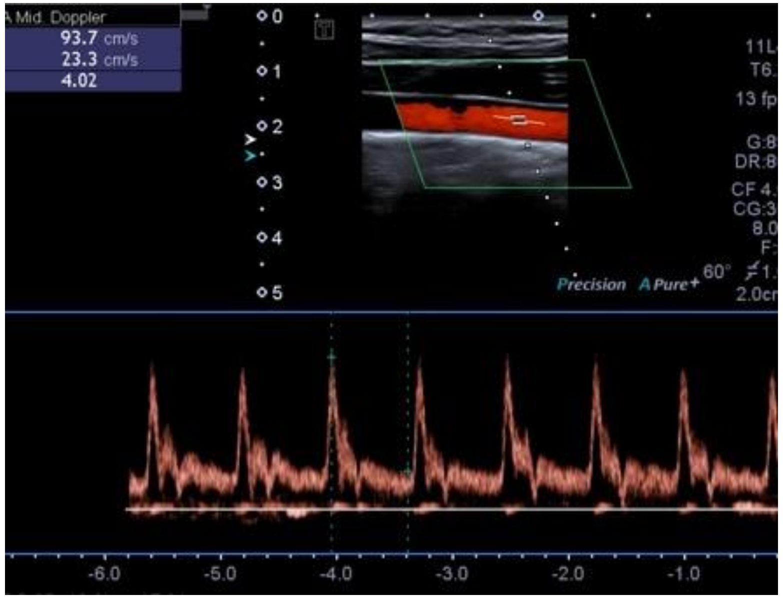


Doppler

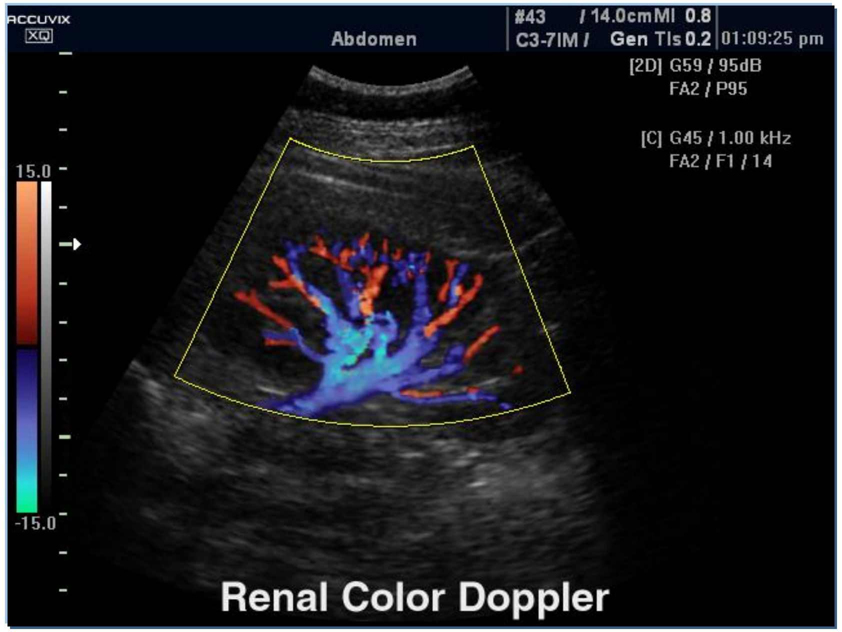
Doppler ultrasound

Doppler ultrasound

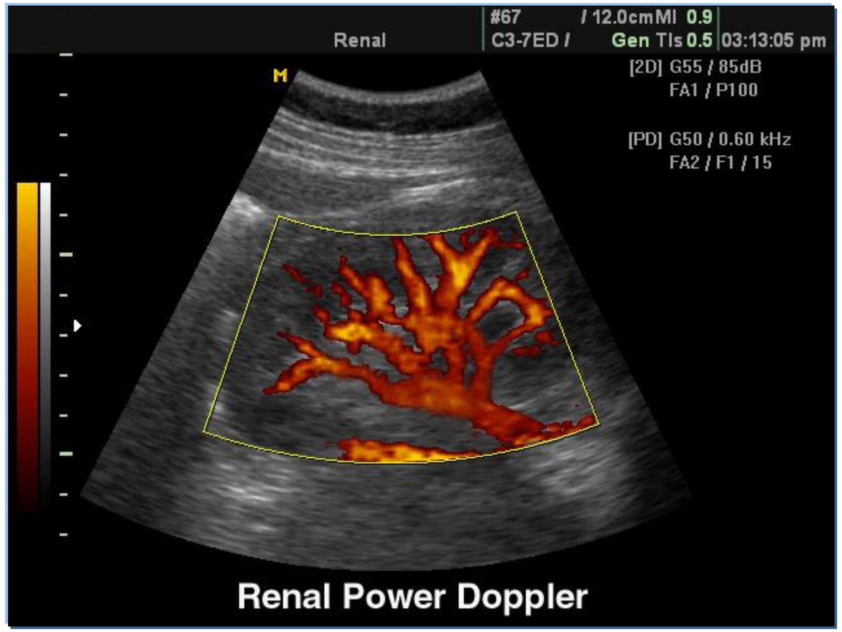
Spectral Doppler (1-D)



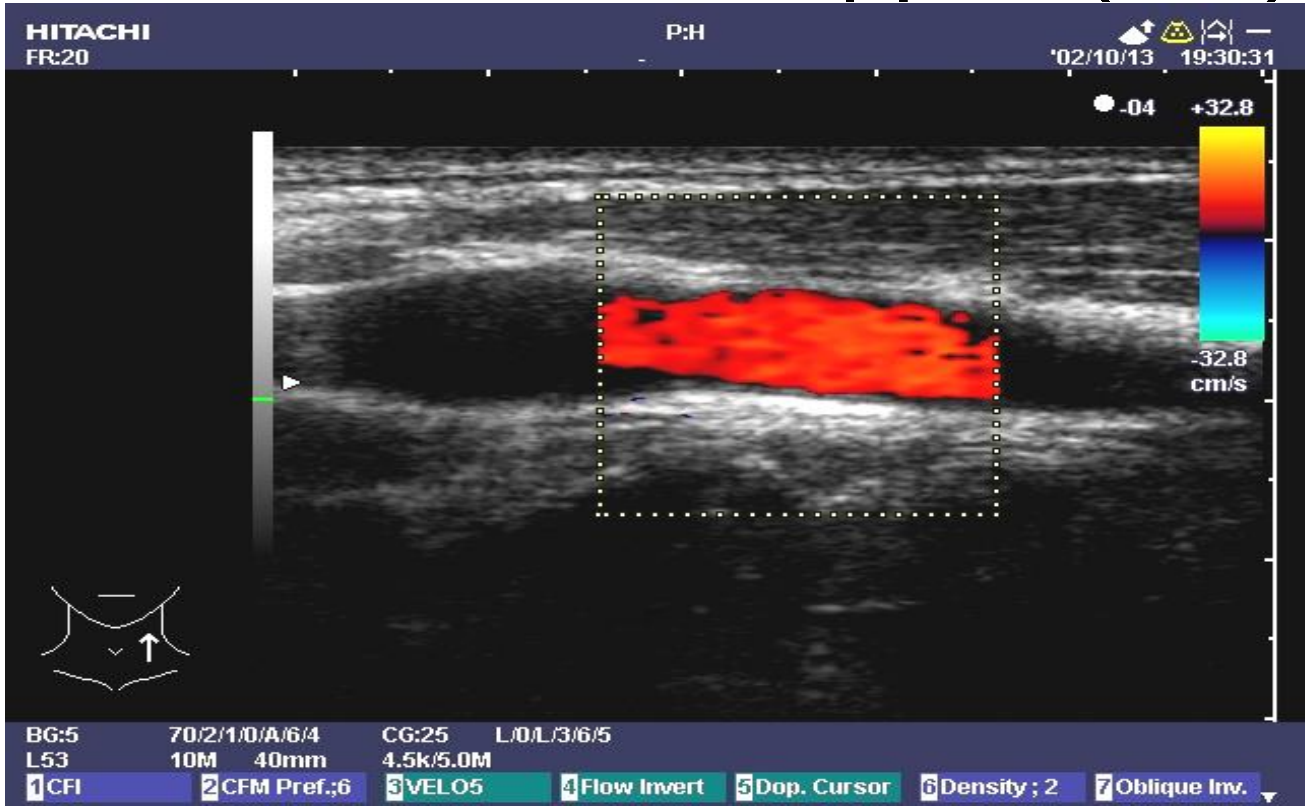
Color Doppler (2-D)
(Flow velocity)



Power Doppler (2-D)
(Flow power), More sensitive



Doppler ultrasound – Color Doppler (2-D)



Functional imaging mode that uses pseudo-color to show blood flow

- Different colors (red/blue) are used to represent **blood flow direction (towards and away)** from transducer
- Intensity of color is indication of **velocity** at which blood is flowing

Doppler ultrasound

· 1D: Spectral Doppler / **2D: Color/Power Doppler**

· Color Doppler mode displays the distribution of blood velocities in a 2D region of the body in real-time (5-100 frames/s)

· **Color Doppler images are *qualitative***

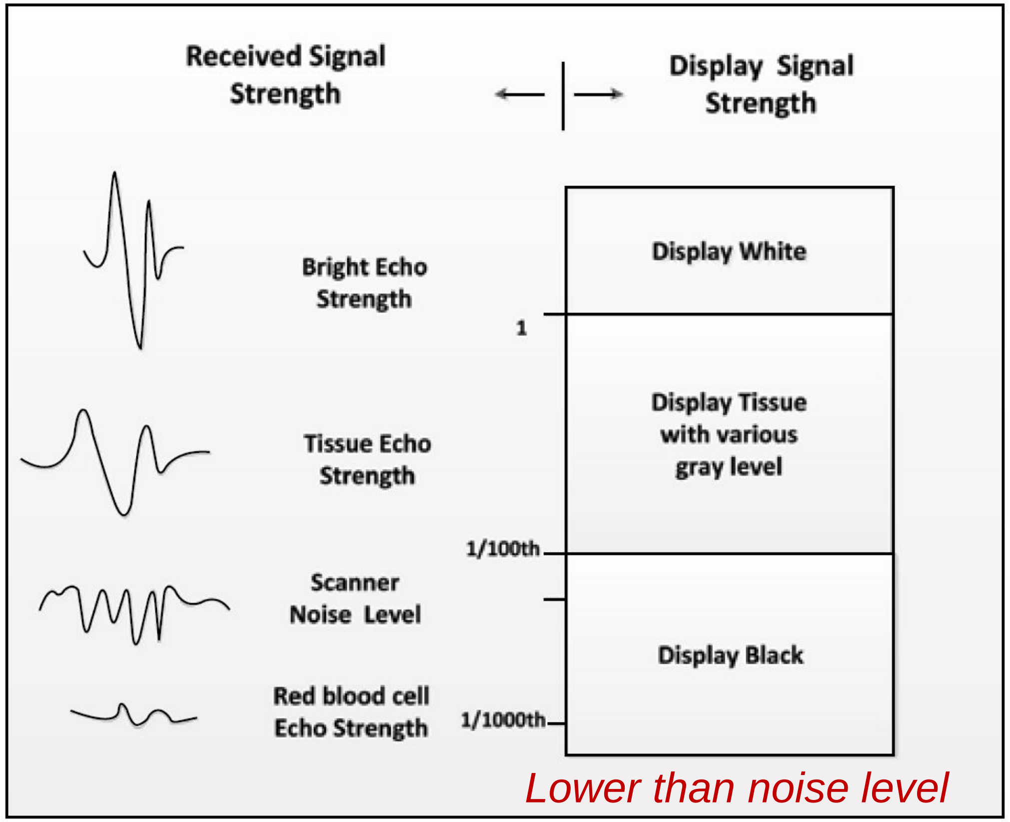
- Blood flow velocity at any point in the body cannot be accurately determined

· **Most useful information in Color Doppler images**

- Distribution of blood flow velocities in a 2D region of the body can be used to detect atherosclerotic deposits, occlusions, etc.
- Changes in flow velocity at a particular point over a period of time are used to detect arterial reflux, etc.

Doppler ultrasound

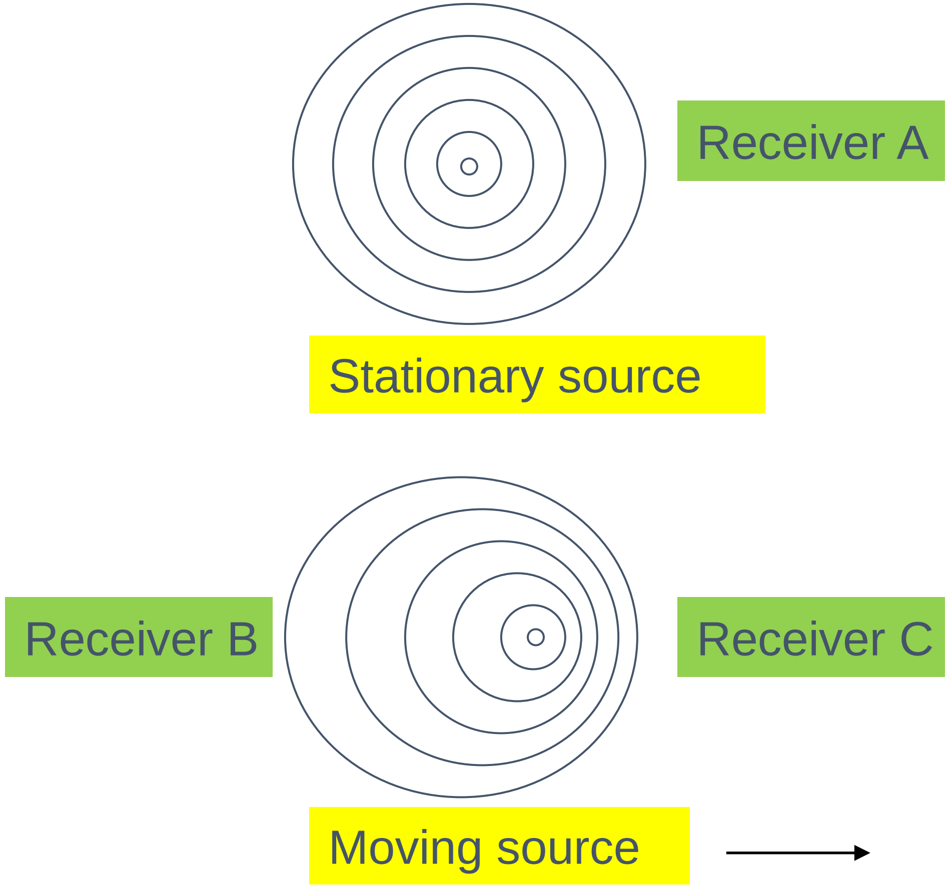
Component	Composition	Relative Abundance (%)	Interaction with ultrasound
Plasma	Proteins	3.80	Negligible
	Solutes	0.20	Negligible
	Water	51.00	Mild attenuation
Formed elements	Red blood cells	44.95	Scattering
	Leukocytes	0.03	Negligible
	Platelets	0.02	Negligible



· Diameter of red blood cells (8 μm) << wavelength of ultrasound (440 μm at 3.5 MHz)

  - Scattering is major mode of interaction with ultrasound

Doppler physics



- Frequency of transmitted wave

  $= f_0$

- Receive frequency
  - by Receiver A

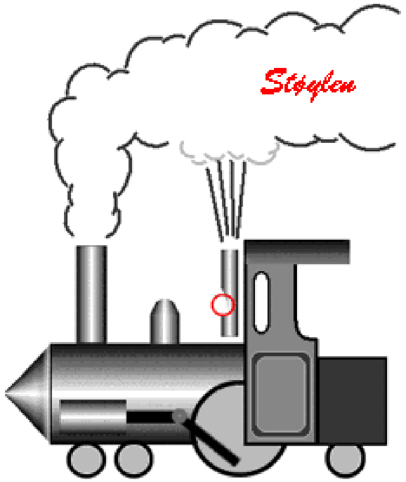
    $= f_0$
  - by Receiver B

    $< f_0$
  - by Receiver C

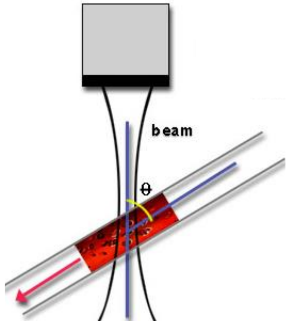
    $> f_0$

Doppler physics

Christian Andreas Doppler
(1803 - 1853)



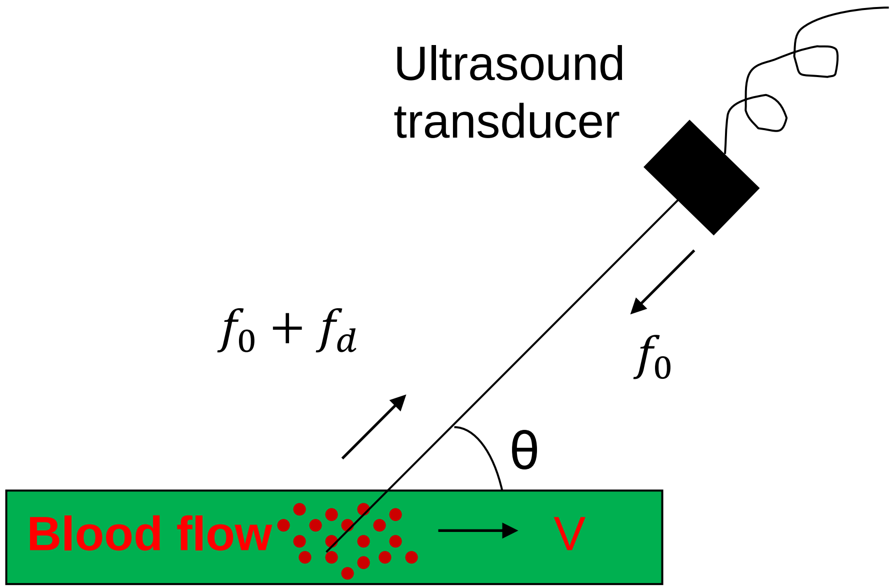
$$f_d = 2f_0 \frac{v}{c} cos\theta \qquad v = \frac{f_d c}{2f_0 cos\theta}$$



v : Blood velocity
c : Sound velocity
f_0 : Transmitted frequency
f_d : Doppler shift of reflected ultrasound
θ : Insonation angle between ultrasound beam and direction of motion

Doppler physics

Flow velocity can be derived by doppler frequency.



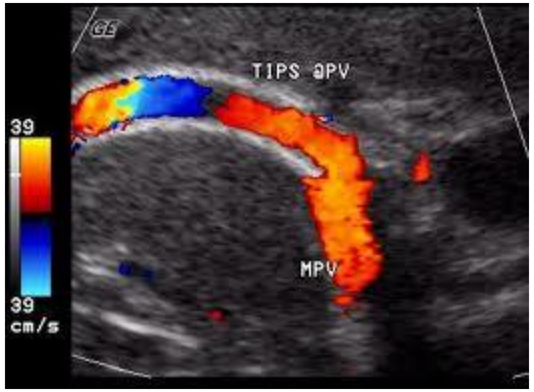
$$\boxed{f_d} = \frac{\boxed{2v} \cdot f_0}{c} \cos\theta$$

c: Ultrasound speed

v: Flow speed

f_0: Transmit Center Frequency

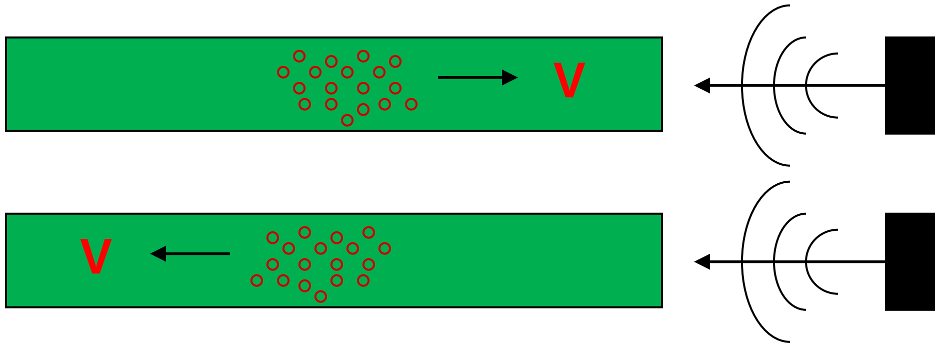
✔ $\boxed{v \Leftrightarrow f_d}$



Doppler physics

$$f_d = \frac{2v \cdot f_0}{c} \cos\theta$$

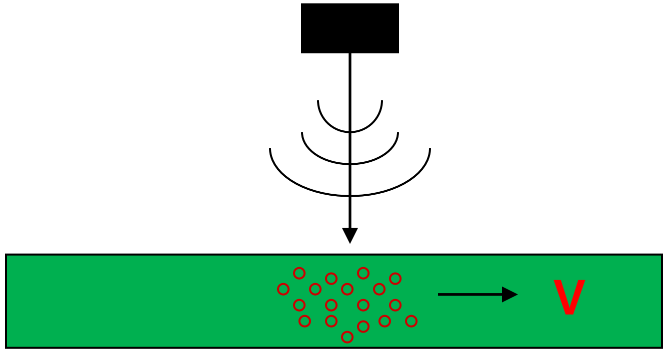
Case 1: θ = 0°



$$f_d = 2v \cdot f_0 / c > 0$$

$$f_d = 2v \cdot f_0 / c < 0$$

Case 2: θ = 90°



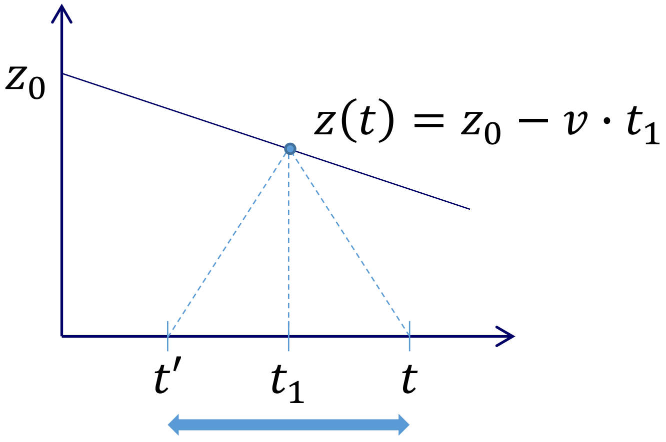
$$f_d = 2v \cdot f_0 \cos(90^o) / c = 0$$

Doppler physics: Mathematical model

Doppler signal model – Mathematical Analysis

Transmitted waveform: $f(t)$

Received waveform: $r(t) = \alpha f\big(t - \tau(t)\big),\ \tau(t) = \frac{2}{c} z(t)$



$$\tau(t) = t - t'$$

$$\frac{\tau(t)}{2} = t_1 - t' = t - t_1$$

$$\tau(t) = \frac{2}{c} z(t) = \frac{2}{c}\{z_0 - v \cdot t_1\}$$

$$\tau(t) = \frac{2}{c}\left\{z_0 - v \cdot \left[t - \frac{\tau(t)}{2}\right]\right\}$$

$$\Rightarrow \quad \tau(t) = \frac{2(z_0 - v \cdot t)}{c - v}$$

$$t - \tau(t) = \frac{c+v}{c-v} \cdot \left(t - \frac{2 \cdot z_0}{c+v}\right)$$

$$r(t) = f\left(\frac{c+v}{c-v} \cdot \left(t - \frac{2 \cdot z_0}{c+v}\right)\right)$$

$$r(t) = f\big(a(t - t_0)\big)$$

Doppler physics: Mathematical model

Received signal due to target motion – Doppler signal

$$r(t) = f\left\{\frac{c+v}{c-v}\left(t - \frac{2z_0}{c+v}\right)\right\}$$
$$\approx f\left\{\left(1+\frac{2v}{c}\right)(t-2\right.$$

$$\frac{c+v}{c-v} = \frac{(c+v)^2}{(c-v)(c+v)} = \frac{c^2+2cv+v^2}{c^2-v^2} = \frac{1+2v/c+v^2/c^2}{1-v^2/c^2} \approx 1+2v/c$$

For $f(t) = a(t)\cos 2\pi f_0 t$

$$r(t) = A(t)\cos[2\pi(f_0+f_d)t+\varphi]$$

where

$$A(t) \approx a\left\{\left(1+\frac{2v}{c}\right)(t-2z_0/c)\right\} \approx a(t-2z_0/c)$$

$$\varphi = -2\pi f_0 \frac{2z_0}{c-v}$$

$$f_d = \frac{2vf_0}{c} \left(=\frac{2vf_0}{c}\cos\theta\right)$$

$$v \rightarrow v\cdot\cos(\theta)$$

By scattering from Red Blood Cells

$$r(t) = \sum_i f\left\{\frac{c+v}{c-v}\left(t-\frac{2z_i}{c+v}\right)\right\}$$
$$\approx \sum_i f\left\{\left(1+\frac{2v}{c}\right)(t-2\right.$$

$$r(t) = \sum_i A_i(t)\cos[2\pi(f_0+f_d)t+\varphi_i]$$

Doppler physics

Simplified Doppler signal model

Doppler frequency

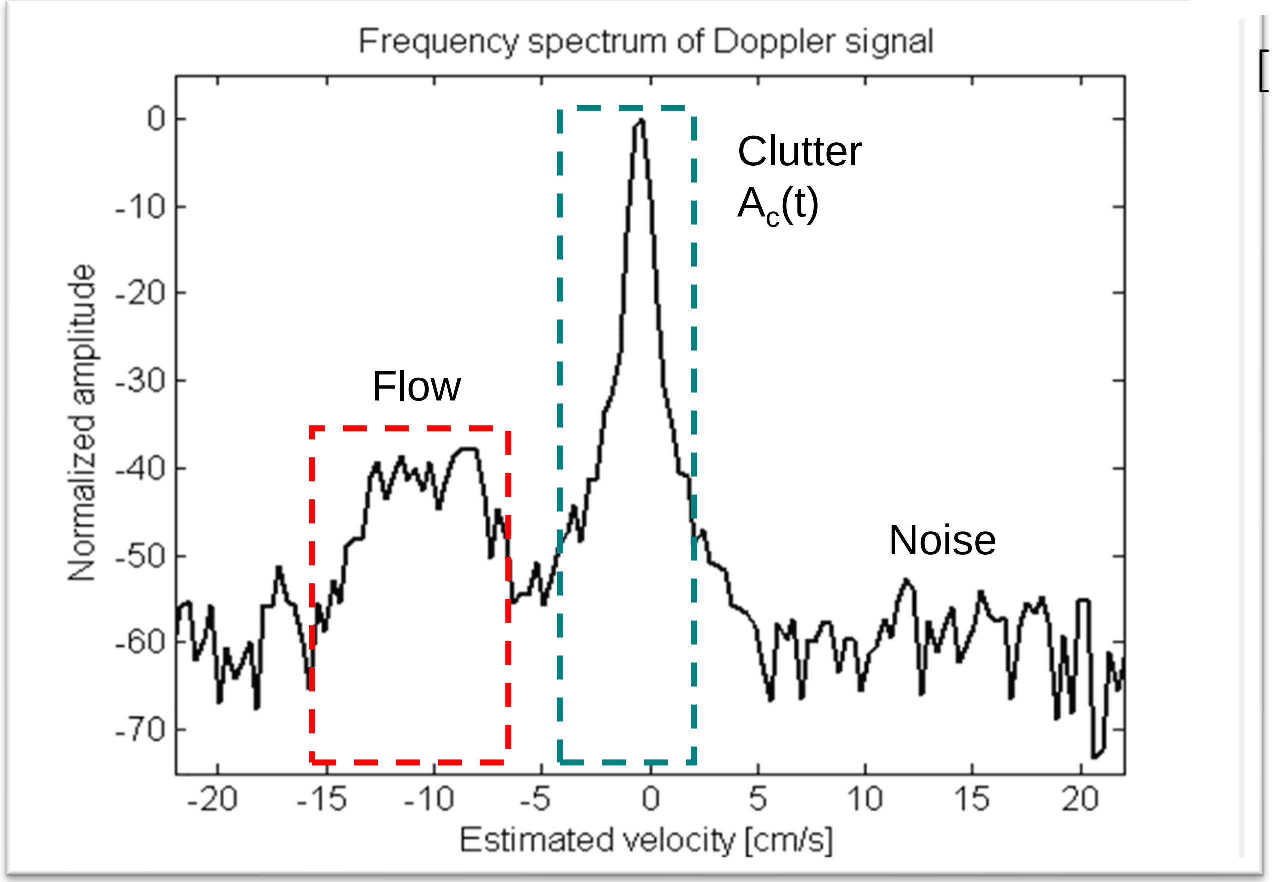
$$r(t) = A_c(t)\cos 2\pi f_0 t + A_f(t)\cos 2\pi (f_0 + f_f)t + A_r(t)\cos 2\pi (f_0 - f_r)t$$

Clutter ***Forward flow*** ***Backward flow***

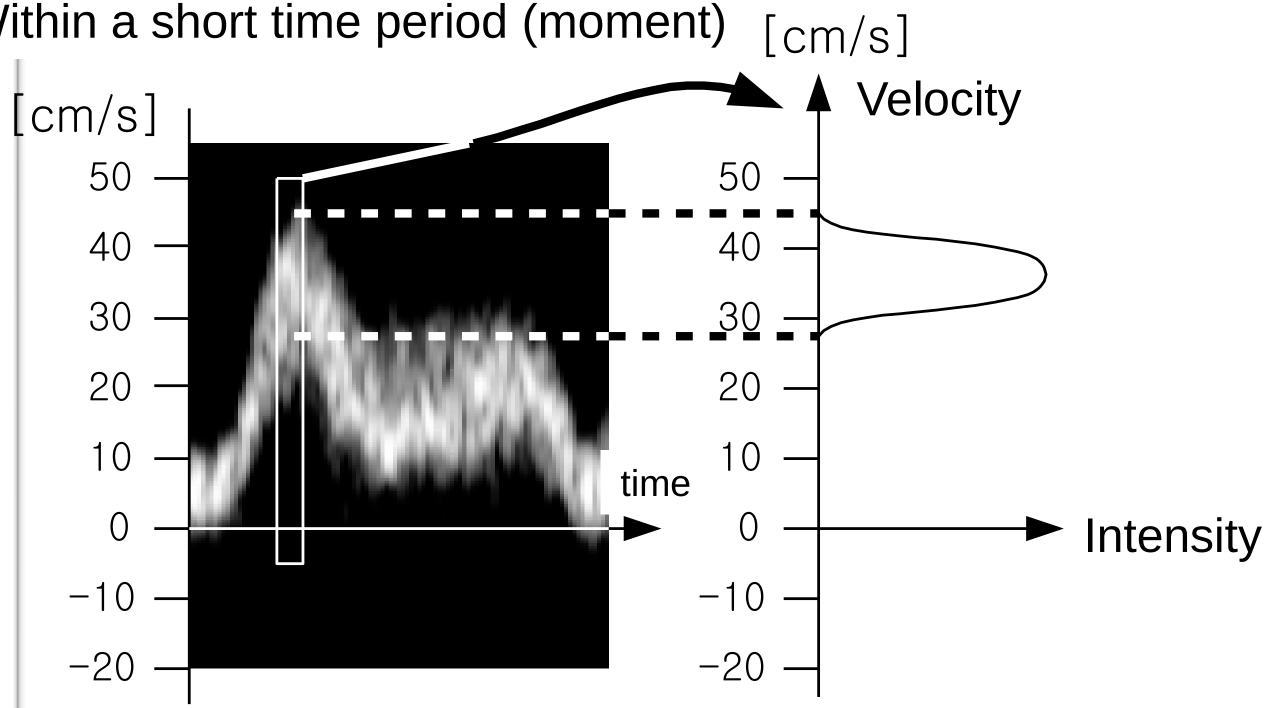
Signal processing pipeline

- Clutter elimination $\text{Remove } A_c(t) \text{ from } r(t)$
- Doppler signal detection
  - Detect separately $d_f(t) = A_f(t)\cos 2\pi f_f t$ or $d_r(t) = A_r(t)\cos 2\pi f_r t$
    – Forward / Reverse flow signal
- Blood flow estimation
  - Relation between Doppler frequency and flow velocity $f_d = \frac{2vf_0}{c}\cos\theta \Rightarrow v = \frac{c}{2f_0\cos\theta} f_d$

Doppler Spectrum



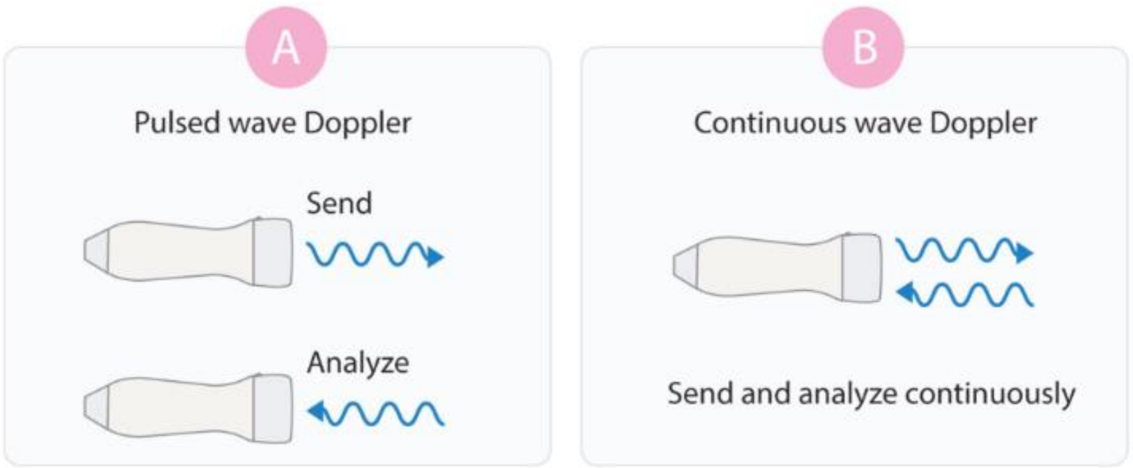
Within a short time period (moment)



Doppler spectrogram presentation over time series

Doppler spectrogram at the moment

Doppler physics: Wave selection



- CW Doppler

Pros:

- No limit in estimation frequency
- Used for detect peak flow speed.

Cons

- No range information
- All the flow information along the beam path are combined.

- PW Doppler

Pros

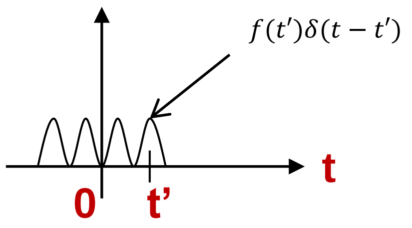
- Doppler estimation with “range” information

Cons

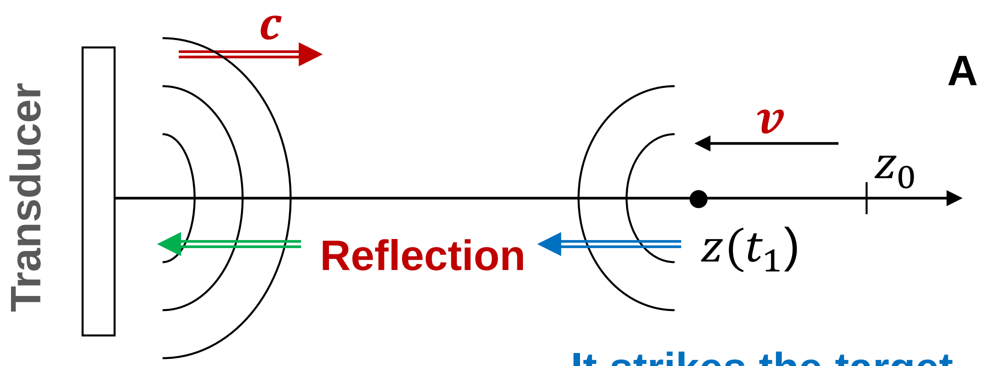
- Limit on highest detectable frequency exists.

PW Doppler

How to estimate the Doppler frequency in PW doppler?



A point on the transmitted waveform radiated at time t' propagates toward a target at a constant speed c

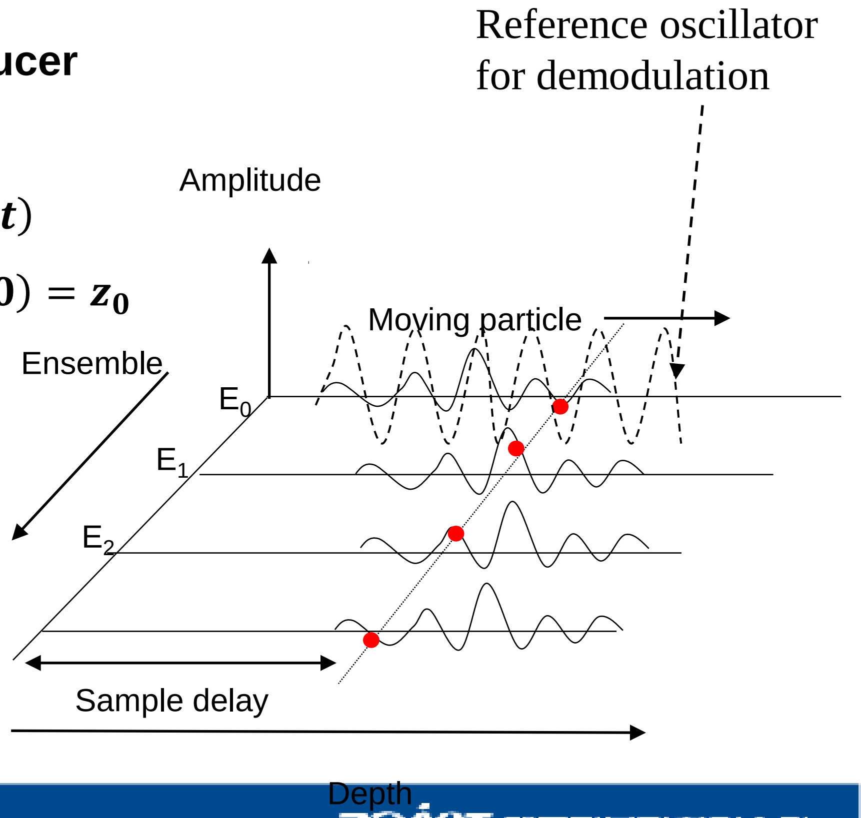


The reflected wave is received at time t

It strikes the target at time t_1

A target moving toward a transducer

- **At a constant speed v**
- **Range location at time t: $z(t)$**
- **Initial location at time 0: $z(0) = z_0$**

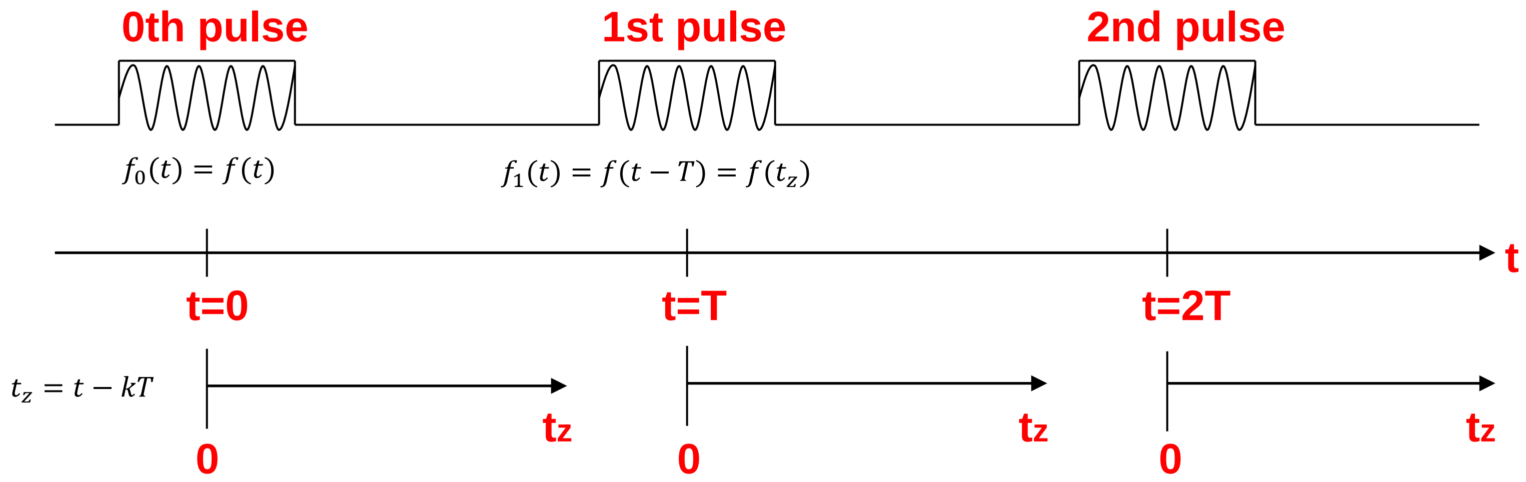


Analysis for a moving target with an acceleration speed?

PW Doppler: Principle

- Burst pulse and PRF (Pulse Repetition Frequency)

  PRF = 1/ T



PW Doppler: Principle

- Transmit burst pulse

$$f_k(t) = f(t - kT), k = 0,1,2,3, \ldots \qquad f(t) = g(t)\cos(2\pi f_0 t)$$

- Receive signal for k-th burst pulse

$$r_k(t) = f\left(\frac{c+v}{c-v} \cdot \left(t - \frac{2 \cdot (z_0 - v \cdot kT)}{c+v}\right)\right) = f\left(\underbrace{\frac{c+v}{c-v} \cdot \left(t - \frac{2 \cdot z_0}{c+v}\right)}_{\textbf{Constant phase}} + \frac{2v}{c-v} kT\right)$$

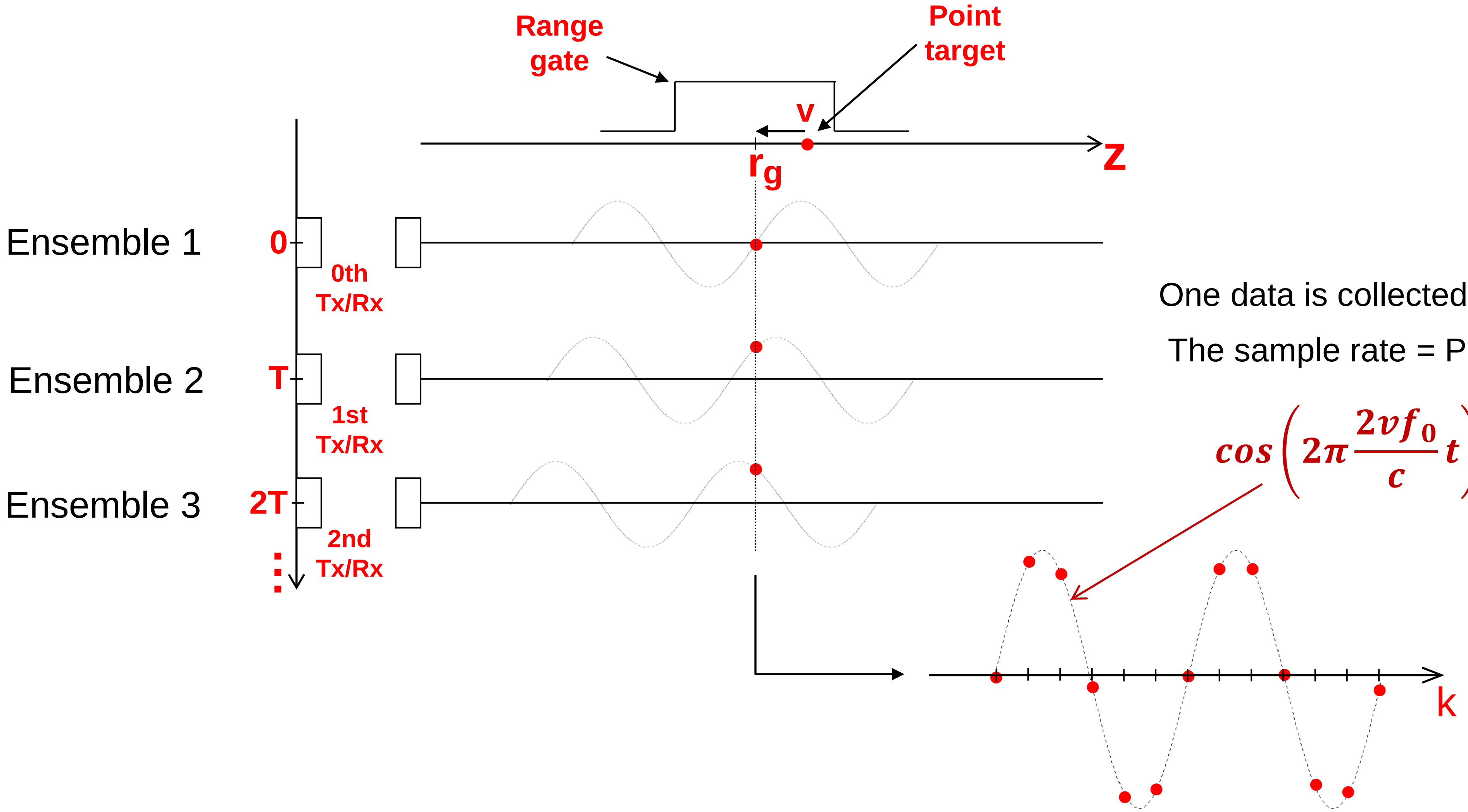
$$r_k(t) \approx \cos\left(2\pi \frac{2v}{c-v} f_0 kT\right) = \cos(2\pi f_d t)|_{t=kT}$$

$$\Delta\phi = 2\pi f_0 \tau = \frac{4\pi v}{c} f_0 T = \boldsymbol{2\pi \frac{2 v f_0}{c} T}$$

Phase difference!

PW Doppler: Principle

- Transmit / Receive Sequence



One data is collected in each Tx / Rx cycle

The sample rate = PRF = 1/ T = F_p

$$\cos\left(2\pi \frac{2vf_0}{c} t\right)\Big|_{t=kT}$$

PW Doppler: Principle

PW Doppler

- Doppler frequency detection

$$\cos(2\pi f_d t) \quad \Rightarrow \quad \cos(2\pi f_d kT)$$

$$\Delta\varphi = 2\pi \cdot f_d \cdot T = 2\pi \cdot f_d / F_p$$

- Maximum detectable frequency

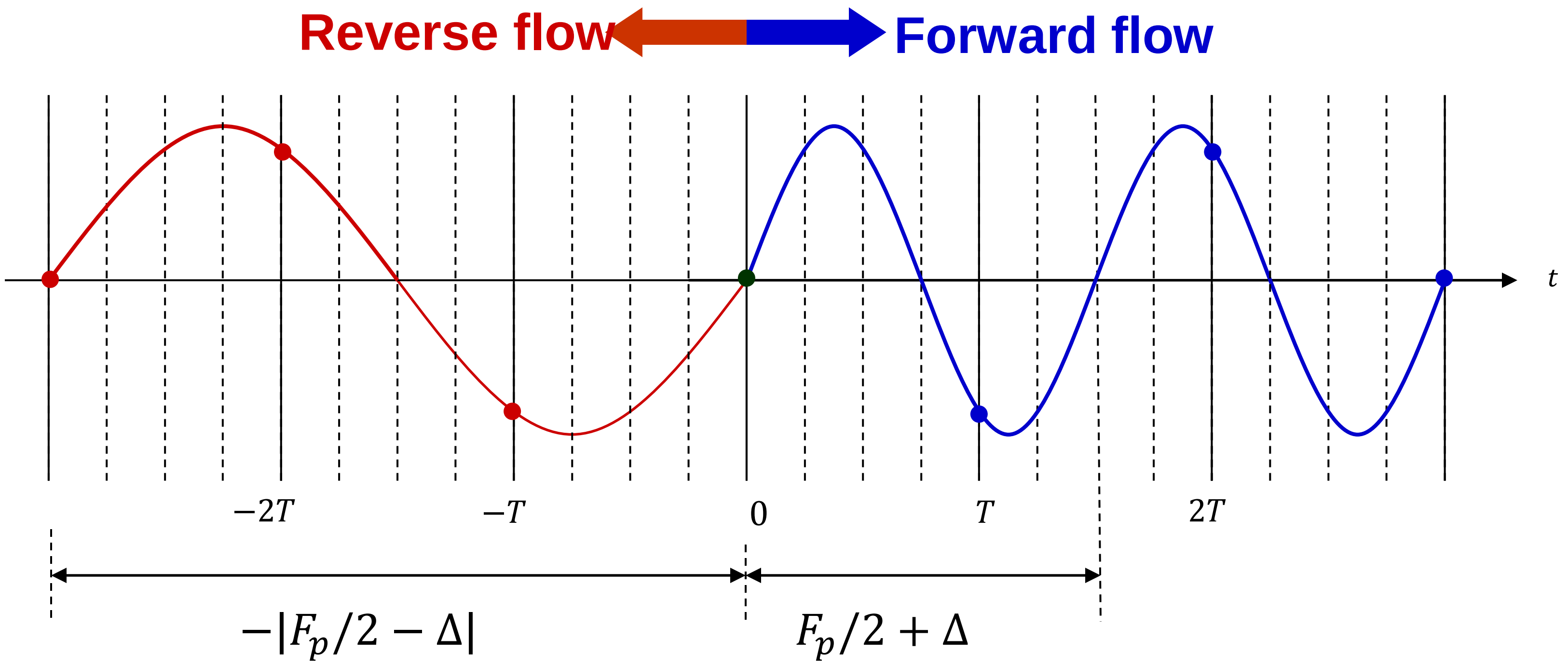
$$-\pi \le \Delta\varphi \le \pi \ \text{ for } \ -F_p/2 \le f_d \le F_p/2$$

$$\begin{aligned}\cos(2\pi f_d kT) &= \cos(2\pi k(F_p/2 + \Delta)T) \\ &= \cos(2\pi \Delta kT + \pi k) \\ &= \cos(2\pi(-F_p/2 + \Delta)kT)\end{aligned}$$

PW Doppler: Principle

PW Doppler

- Limit on detectable Doppler shift frequency

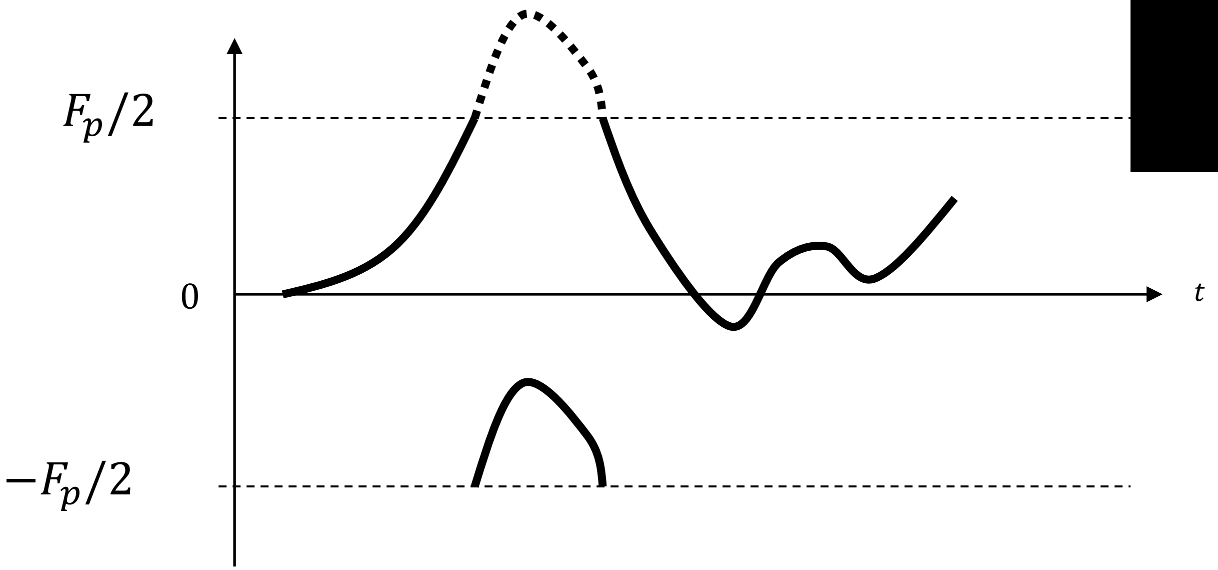


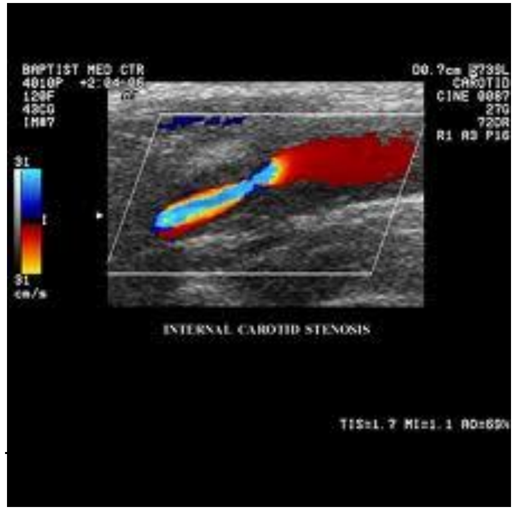
PW Doppler: Principle

PW Doppler

- Frequency aliasing

$$F_p/2 + \Delta \quad \Rightarrow \quad -\,|F_p/2 - \Delta|$$

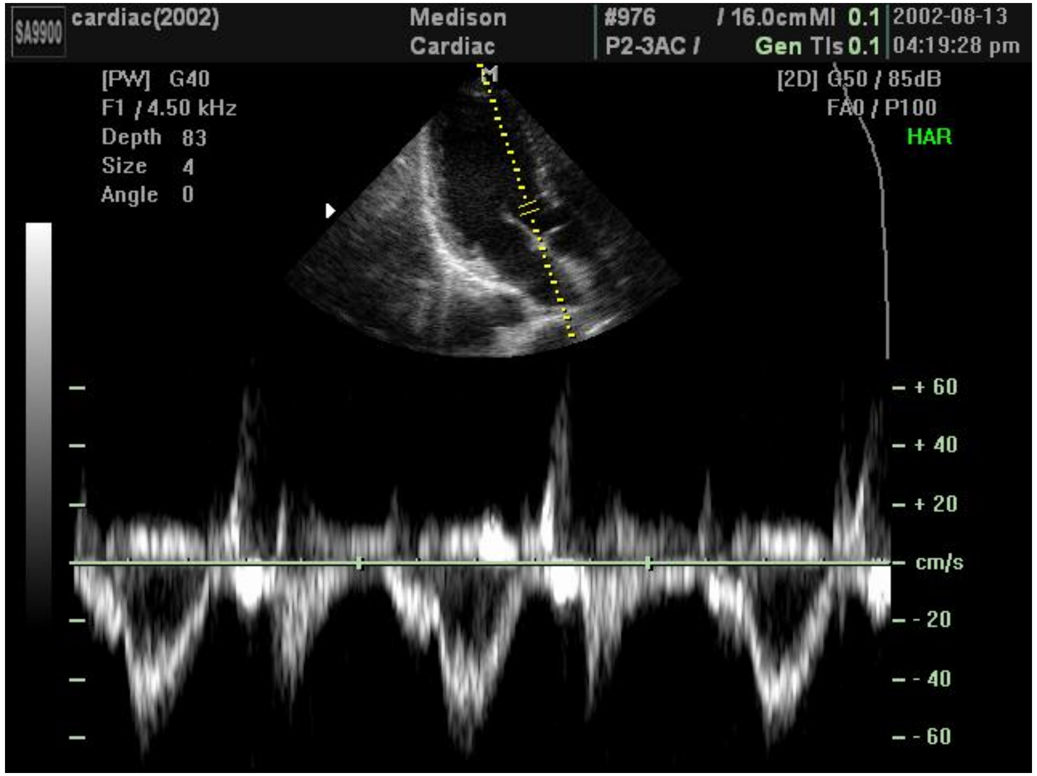




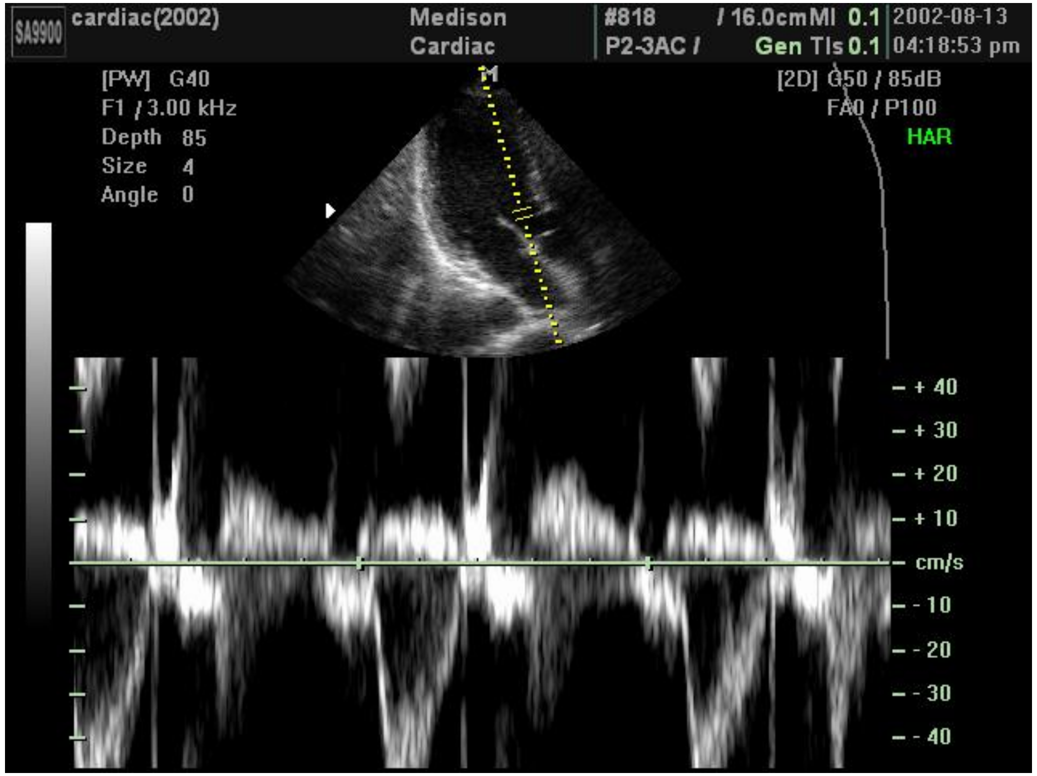
PW Doppler: Principle

PW Doppler

- Frequency aliasing

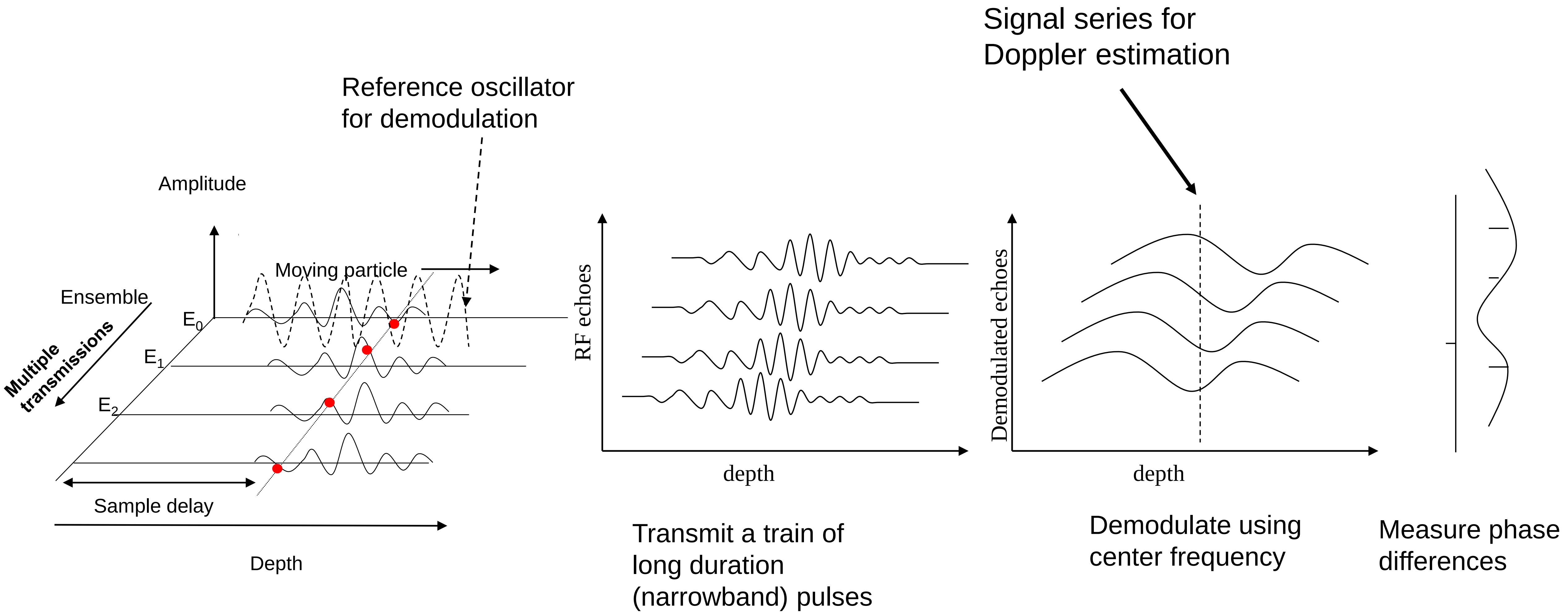


PRF=4.5KHz



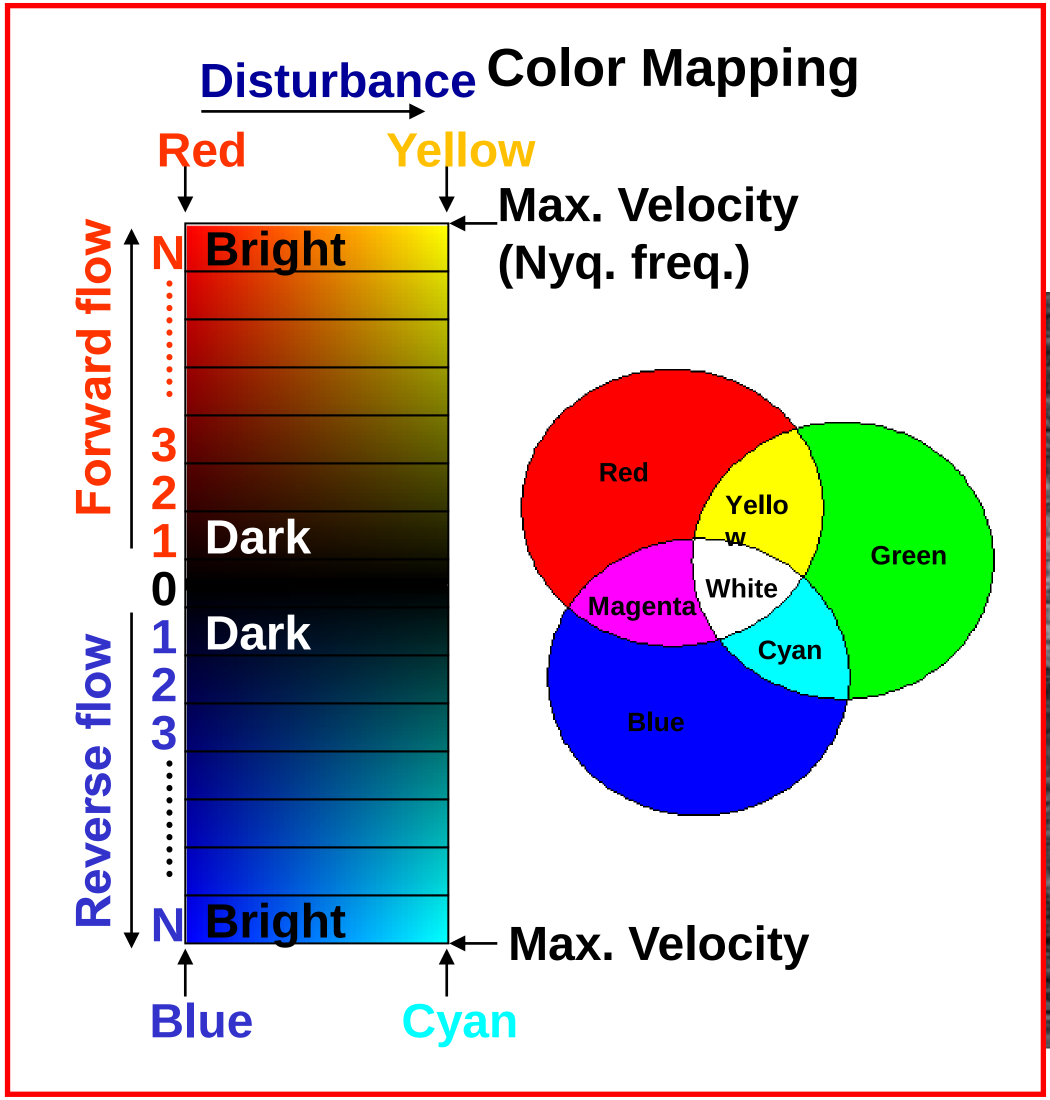
PRF=3KHz

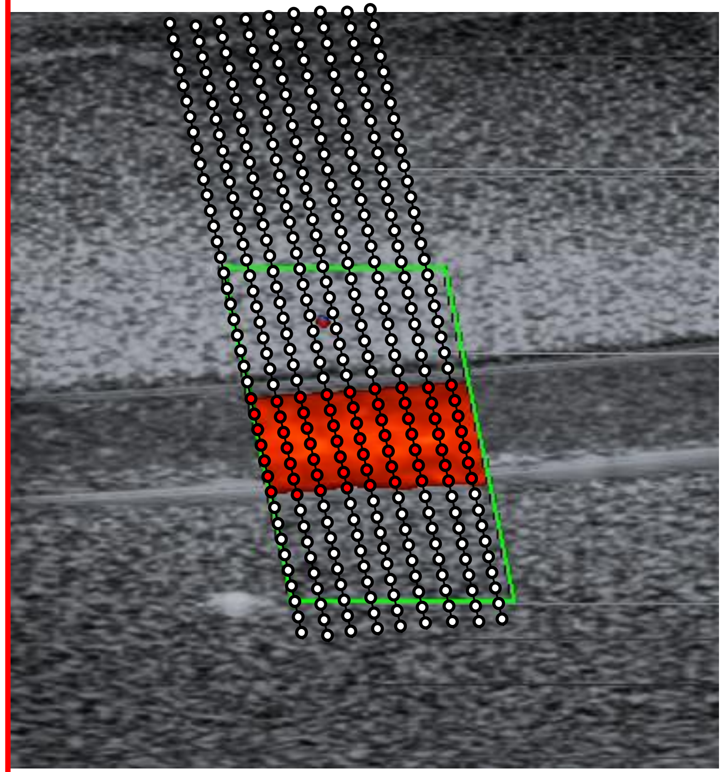
PW Doppler



#'s of Ensembles = 4~32

2-D Doppler Imaging System

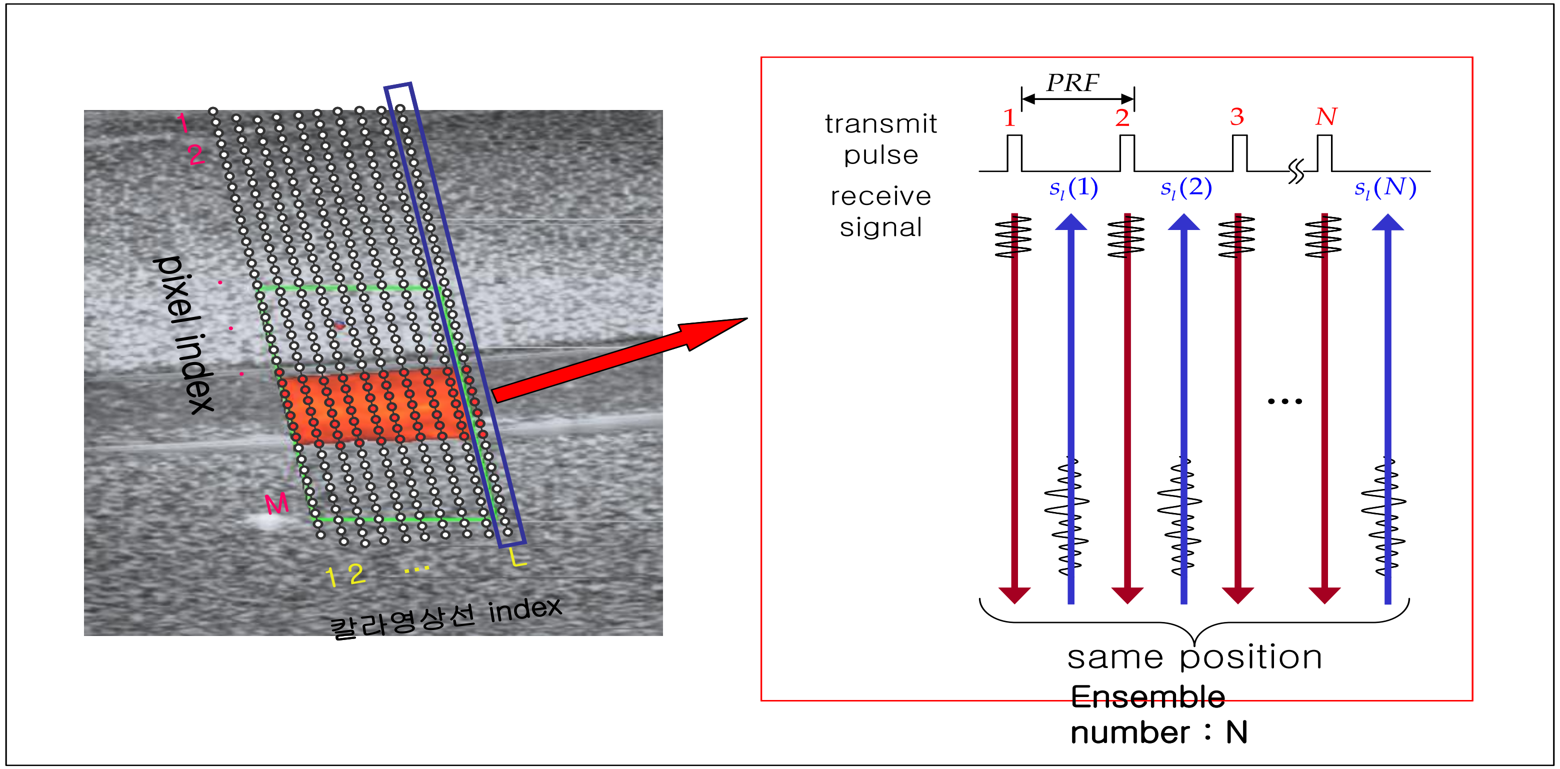




2-D Doppler Imaging System

- **Color Frame rate:** $F = \frac{PRF}{N \times L}\left(\leftarrow \frac{1}{F} = \frac{1}{F_{PRF}} \times N \times L\right)$

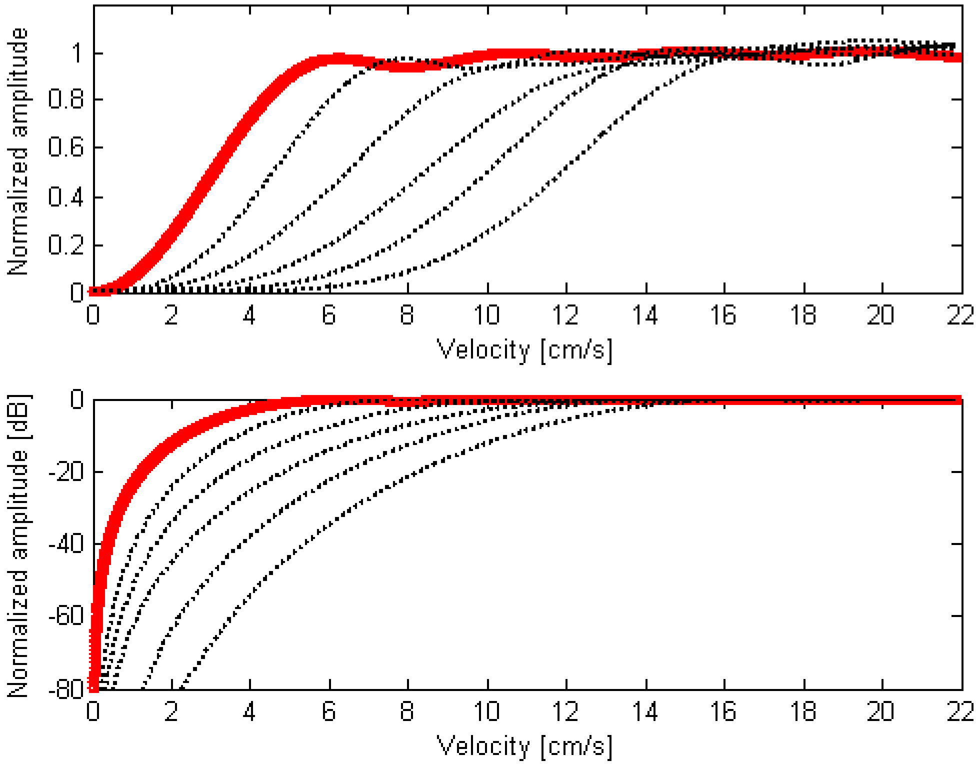
- **PRF (Pulse Repetition Frequency): Pulse-to-Pulse duration (Sampling frequency for Color flow)**
- **N : ensemble number**
- **L : #'s of scanline for color flow**

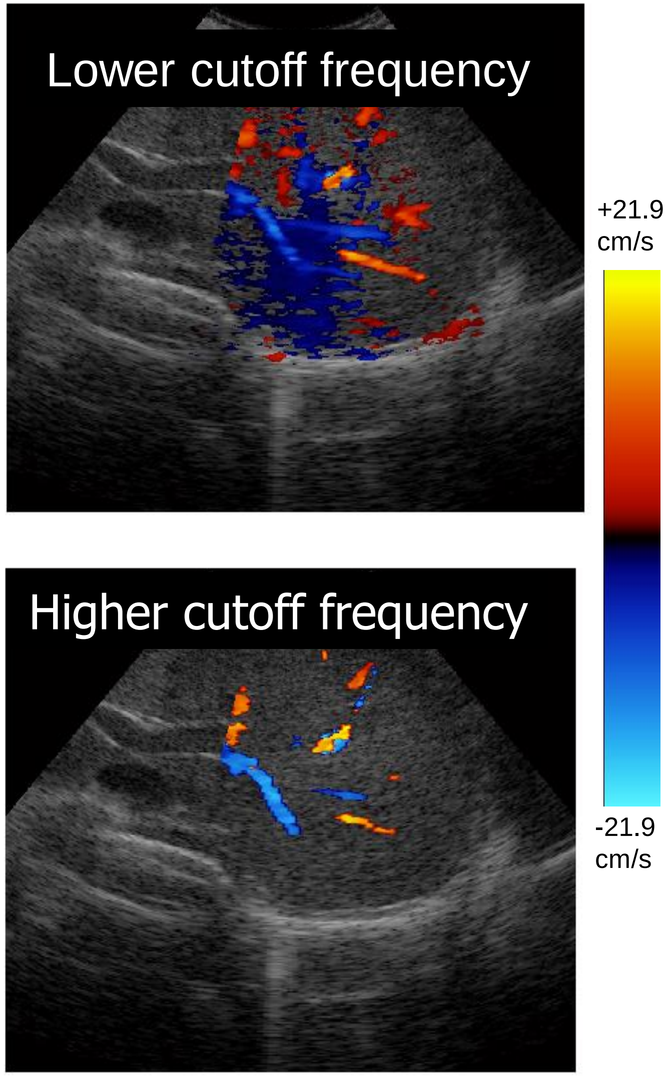


Clutter filter

- Clutter causes a bias on velocity estimation and introduces artifacts in Color Doppler images

Frequency response of clutter filters





Clutter filter

- Challenging: the limited number of ensembles (=samples)

- FIR filter
  - Filter length K < #'s of Ensemble N
  - Lower order FIR filter performance cannot satisfy the requirement (40dB suppression + sharpness)
- IIR filter ***(the most traditional clutter filter)***
  - Lower order IIR filter can satisfy the requirement
  - Transient response problem? > Step / Projection initialized technique
- Polynomial Regression Filter
  - Project the modeled clutter signal to polynomial basis

For further information,
Please look into
the Digital signal processing!

Doppler Imaging System

Mean frequency estimation technique

- **Various methods exist**

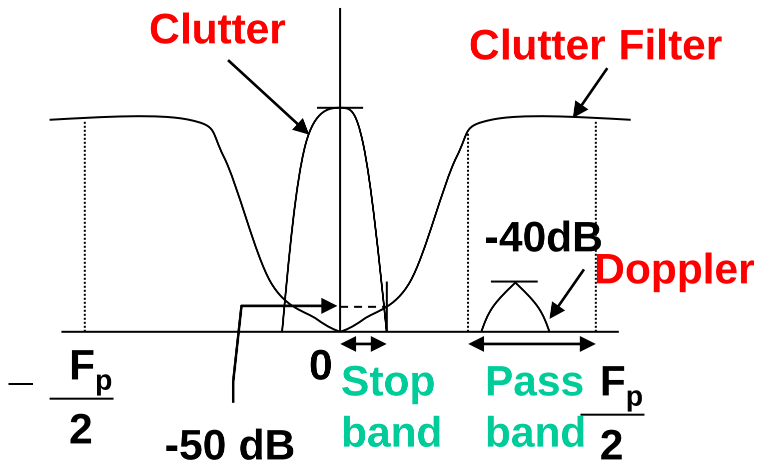
  Zero crossing detector

  FFT method

  Power spectrum centroid

  Instantaneous frequency estimator

  Cross-correlation estimator



- **Important Issues**

  Efficient clutter filtering

  Noise insensitive estimation

  Reduce biasing effect due to frequency aliasing

Limited number of samples (i.e., small ensemble number)

Doppler Imaging System

Autocorrelation-based phase estimation (finding Power spectrum centroid)

$$\Delta\phi = 2\pi f_0 \tau = \frac{4\pi v}{c} f_0 T = \frac{4\pi v}{c} \cdot \frac{f_0}{f_p}, f_p = 1/T$$

$$-\pi \le \Delta\phi \le \pi$$

$$\boxed{-\pi \le \frac{4\pi v}{c} \cdot \frac{f_0}{f_p} \le \pi \qquad -\frac{f_p}{2} \le \frac{2 v f_0}{c} \le +\frac{f_p}{2}}$$

Highest frequency detectable
→ Aliasing

Phase estimation based on autocorrelation

$$r_k(t) \rightarrow z_k(t) : analytic\ signal$$

$$R_k(n) = \sum_{k=1}^{N} z_k(n) z_{k-1}(n)$$

$$v(n) = \frac{c}{4\pi f_0 T} tan^{-1}\left[\frac{Im\{R_k(n)\}}{Re\{R_k(n)\}}\right]$$

Doppler Imaging System

Mean frequency estimation from S(f), the Power spectrum of z(t).

$$\hat{f} = \frac{\int f S(f) df}{\int S(f) df}$$

where

$$R(\tau) = \int z(t) z^*(t-\tau) dt$$

$$S(f) = \int R(\tau) e^{-j2\pi f\tau} d\tau$$

$$R(\tau) = \int S(f) e^{j2\pi f\tau} df$$

Fourier pair

Autocorrelation <> Power spectrum

$$R(\tau)|_{\tau=0} = \int S(f) df$$

$$\frac{1}{j2\pi}\frac{\partial}{\partial\tau} R(\tau)|_{\tau=0} = \int f S(f) df$$

$$\hat{f} = \frac{1}{j2\pi}\frac{(\partial/\partial\tau)R(\tau)}{R(\tau)}\bigg|_{\tau=0}$$

$$R^*(-\tau) = \int z^*(t) z(t+\tau) dt$$

$$= \int z(t') z^*(t'-\tau)\, dt' \quad (t+\tau = t')$$

$$= R(\tau)$$

$$R(\tau) = A(\tau) e^{j\varphi(\tau)}$$

$$A(-\tau) e^{-j\varphi(-\tau)} = A(\tau) e^{j\varphi(\tau)}$$

$$A(\tau) = A(-\tau) \quad \Rightarrow \quad A'(0) = 0$$

$$\varphi(\tau) = -\varphi(-\tau) \quad \Rightarrow \quad \varphi(0) = 0$$

$$\hat{f} = \frac{1}{j2\pi}\frac{A'(0)e^{j\varphi(0)} + j\varphi'(0)A(0)e^{j\varphi(0)}}{A(0)e^{j\varphi(0)}}$$

$$= \frac{j\varphi'(0)}{j2\pi} = \frac{1}{2\pi}\varphi'(0)$$

Doppler Imaging System

Mean frequency estimation from S(f), the Power spectrum of z(t).

$$\hat{f} = \frac{1}{2\pi}\varphi'(0) \approx \frac{1}{2\pi}\frac{\varphi(T)-\varphi(0)}{T} = \frac{F_p}{2\pi}\varphi(T)$$
$$= \frac{F_p}{2\pi}\tan^{-1}\frac{\mathrm{Im}[R(T)]}{\mathrm{Re}[R(T)]}$$

$$\begin{aligned} R(T) &= E[z(k)z^*(k-1)] \\ &= E[\{i(k)+jq(k)\}\{i(k-1)-jq(k \\ &= E[\{i(k)i(k-1)+q(k)q(k-1)\} \\ & j\{q(k)i(k-1)-i(k)q(k-1)\}] \end{aligned}$$

$$\boxed{\hat{f} = \frac{F_p}{2\pi}\tan^{-1}\frac{\sum_{k=1}^{J}\{q(k)i(k-1)-i(k)q(k-1)\}}{\sum_{k=1}^{J}\{i(k)i(k-1)+q(k)q(k-1)\}}}$$

- **Frequency aliasing**

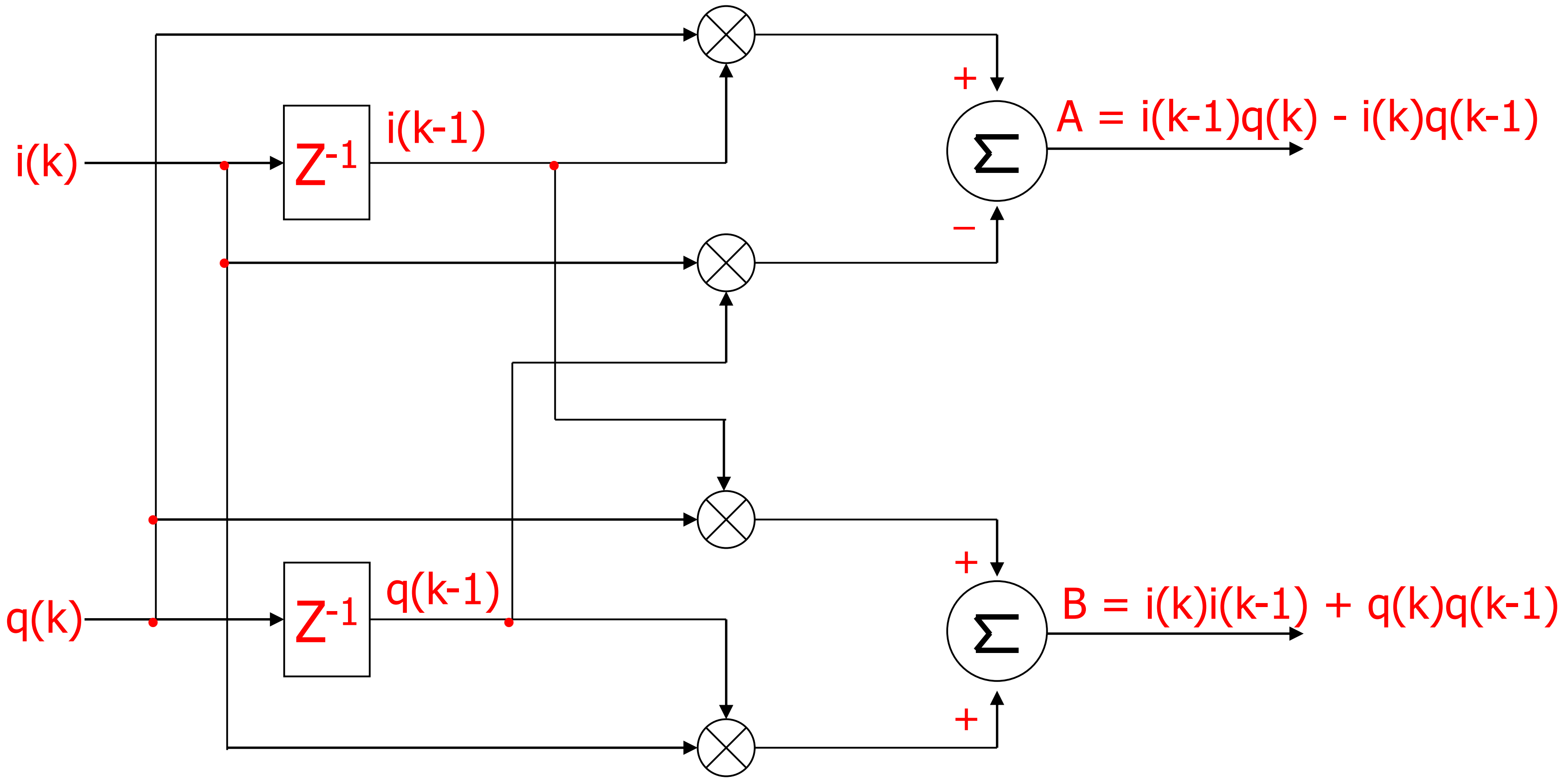
$$-\pi \le \tan^{-1}(x) \le \pi \Rightarrow -\frac{F_p}{2} \le \hat{f} \le \frac{F_p}{2}$$

- **Turbulence**

$$\mathrm{var} = \frac{\int (f-\hat{f})^2 S(f)df}{\int S(f)df} \approx \frac{2}{T^2}\{1-\frac{|R(T)|}{R(0)}\}$$

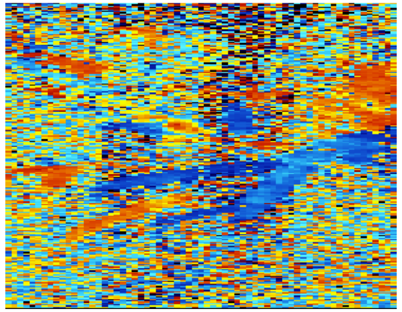
Doppler Imaging System

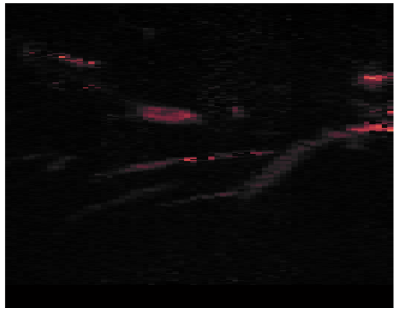
Auto-correlator - implementation



Additional processing for Doppler

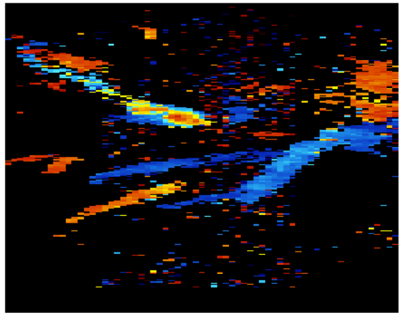
Velocity estimation



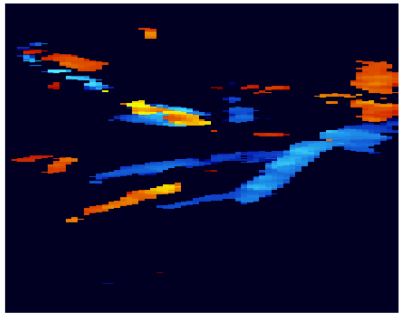


Power estimation

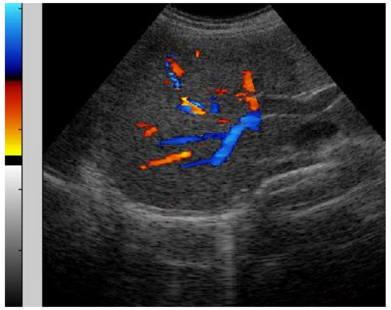
Thresholding



Post-processing



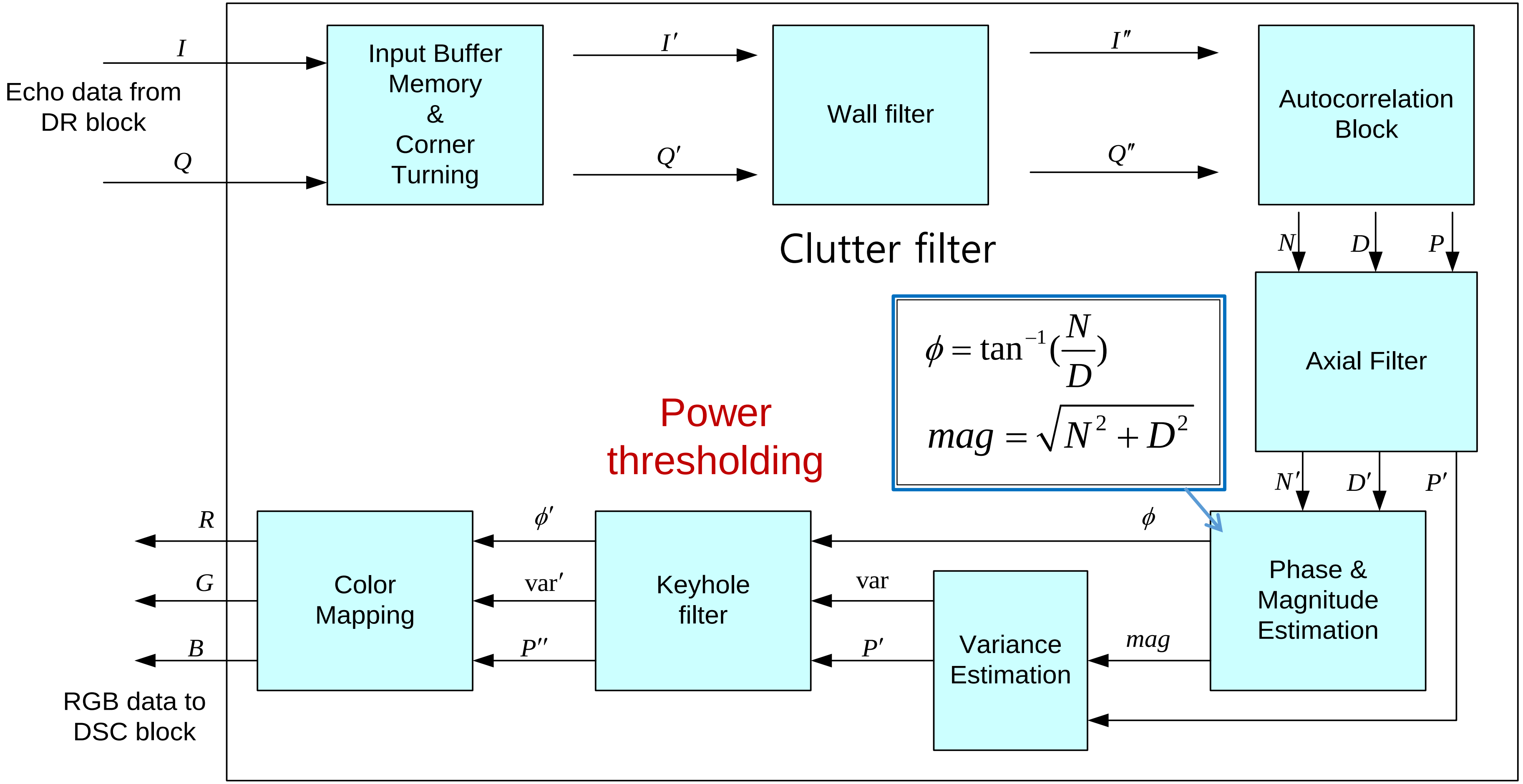
Tissue flow decision
+
Scan conversion



Post-processing filtering

- Median filtering [3 × 3]
- Axial and lateral filtering

2-D Doppler Imaging System (example)

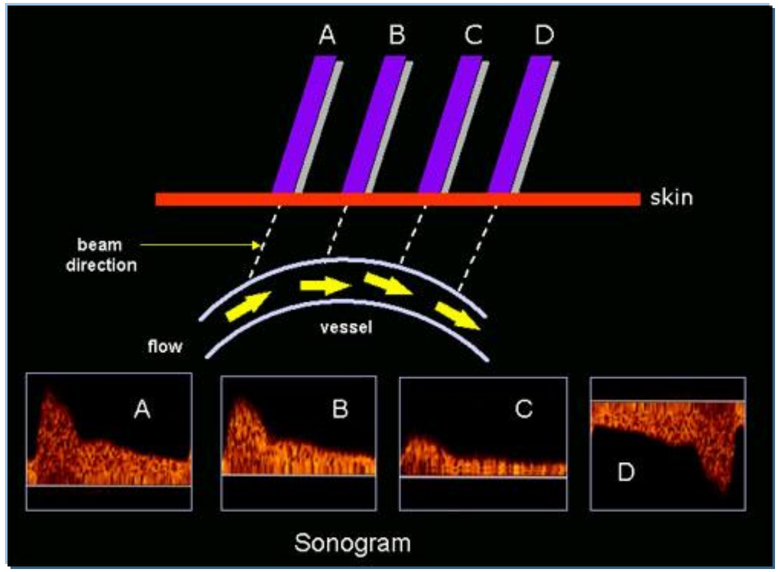


Factors Affecting Doppler Ultrasound Quality

- Transmit
  - Angle
  - Pulse repetition frequency (PRF)
  - Transmit frequency
  - Sample volume (spectral Doppler)
  - Ensemble (color Doppler)

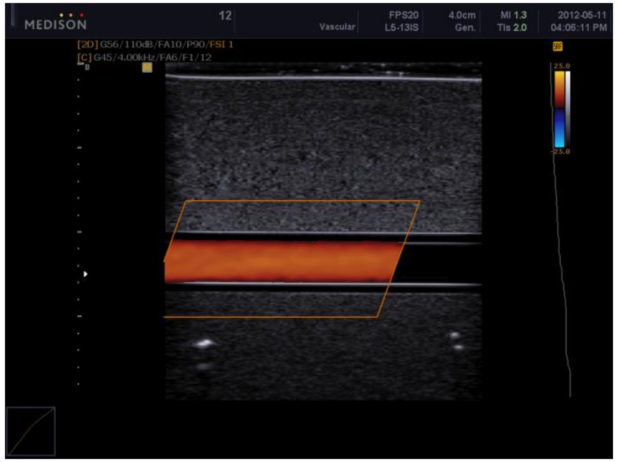
- Receive
  - Wall filtering cutoff frequency
  - Gain

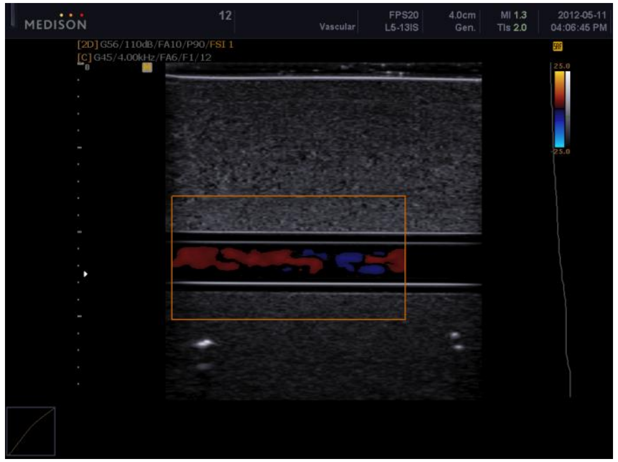
Doppler Angle

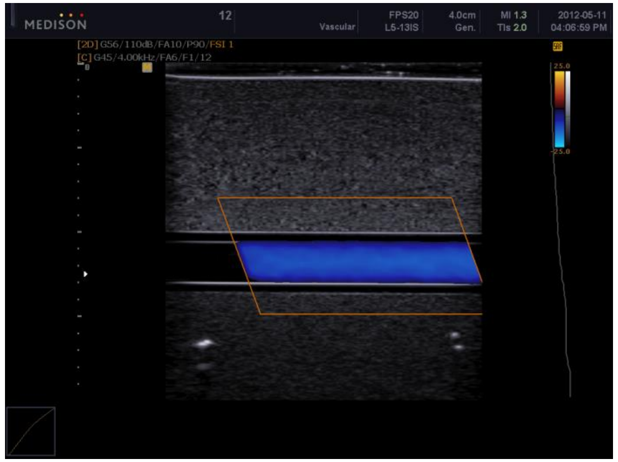


$$f_d = 2f_0\frac{v}{c}cos\theta$$

Effect of the Doppler angle in the sonogram. (A) higher-frequency Doppler signal is obtained if the beam is aligned more to the direction of flow. In the diagram, beam (A) is more aligned than (B) and produces higher-frequency Doppler signals. The beam/flow angle at (C) is almost 90° and there is a very poor Doppler signal. The flow at (D) is away from the beam and there is a negative signal.



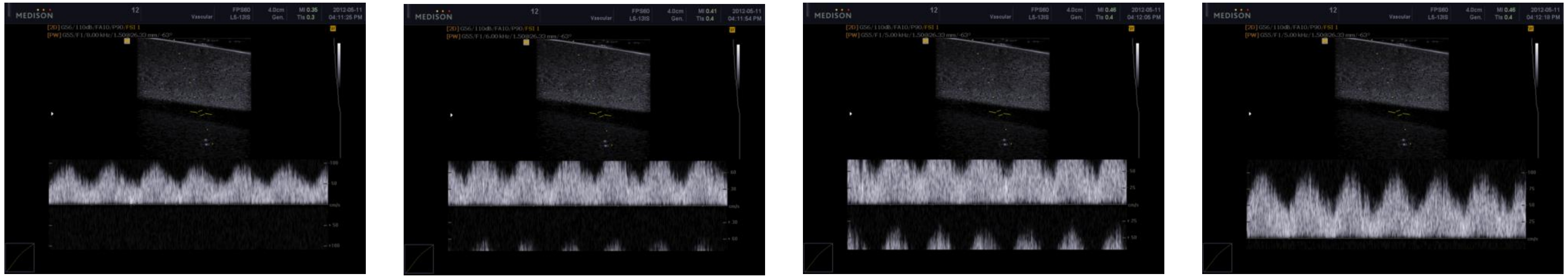


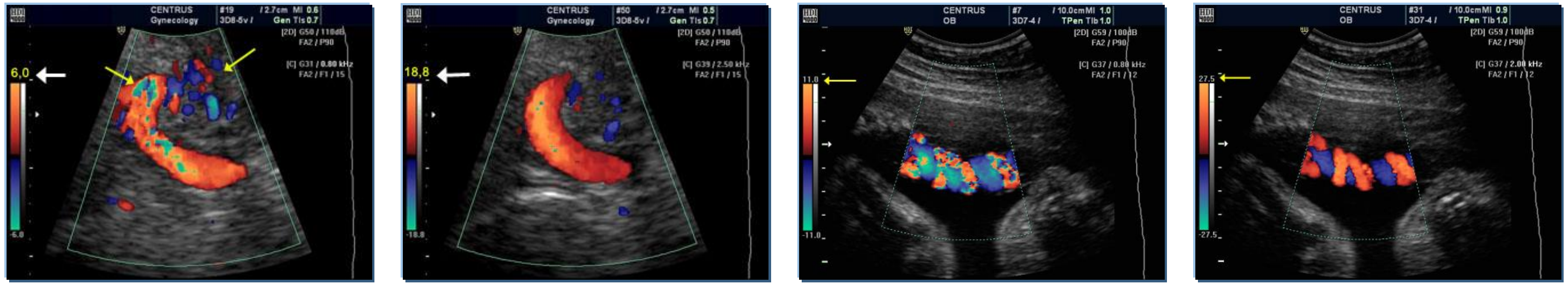


Pulse Repetition Frequency (PRF)

- Determine the maximum velocity that can be measured without aliasing

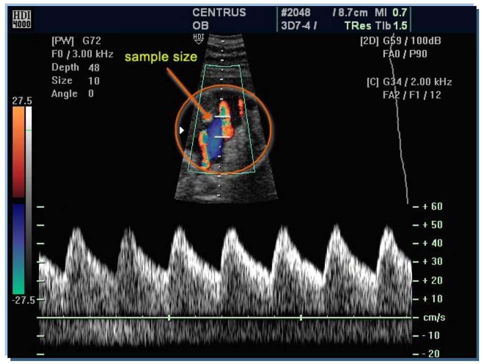
$$f_{d,max} = \frac{1}{2} PRF \qquad v = \frac{f_d c}{2 f_0 cos\theta}$$





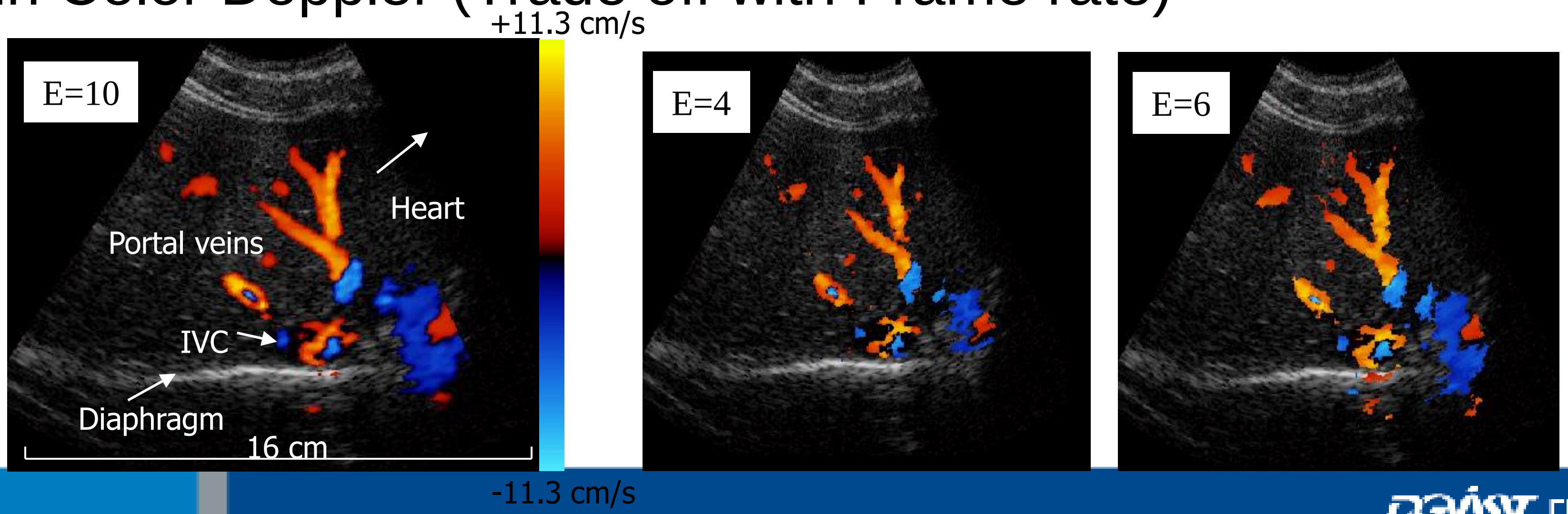
Sample Volume and Ensemble Size

- Sample Volume in Spectral Doppler
  - Determine the detectable flow



- Small SV: less variance in detected flow velocities
- Large SV: more variance in detected flow velocities

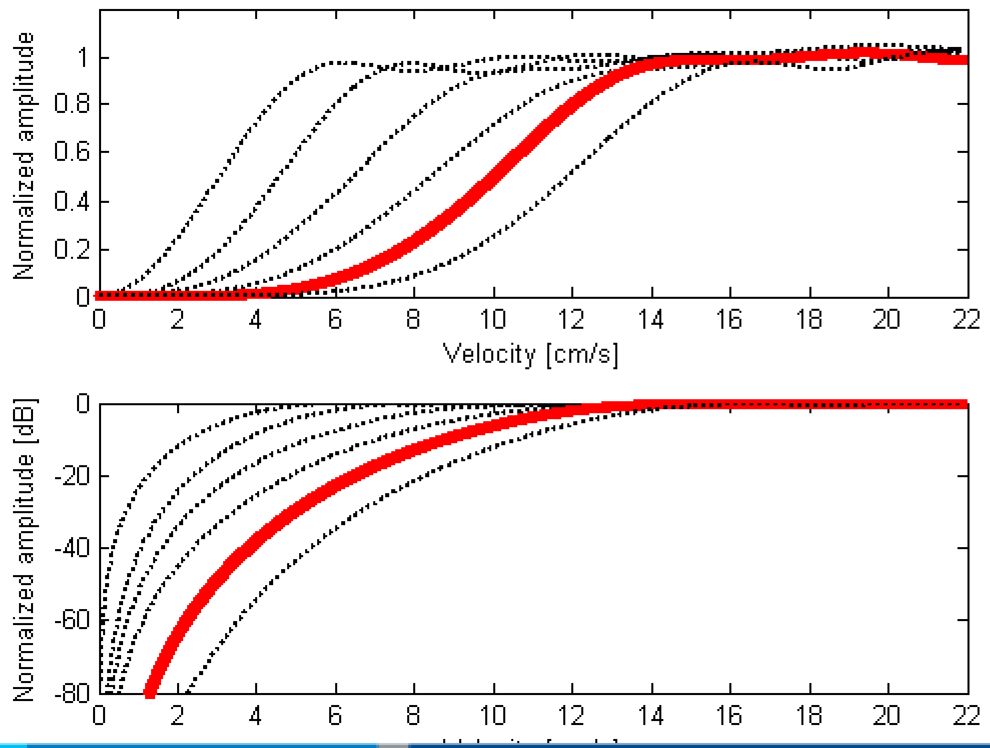
- Ensemble size in Color Doppler (Trade off with Frame rate)

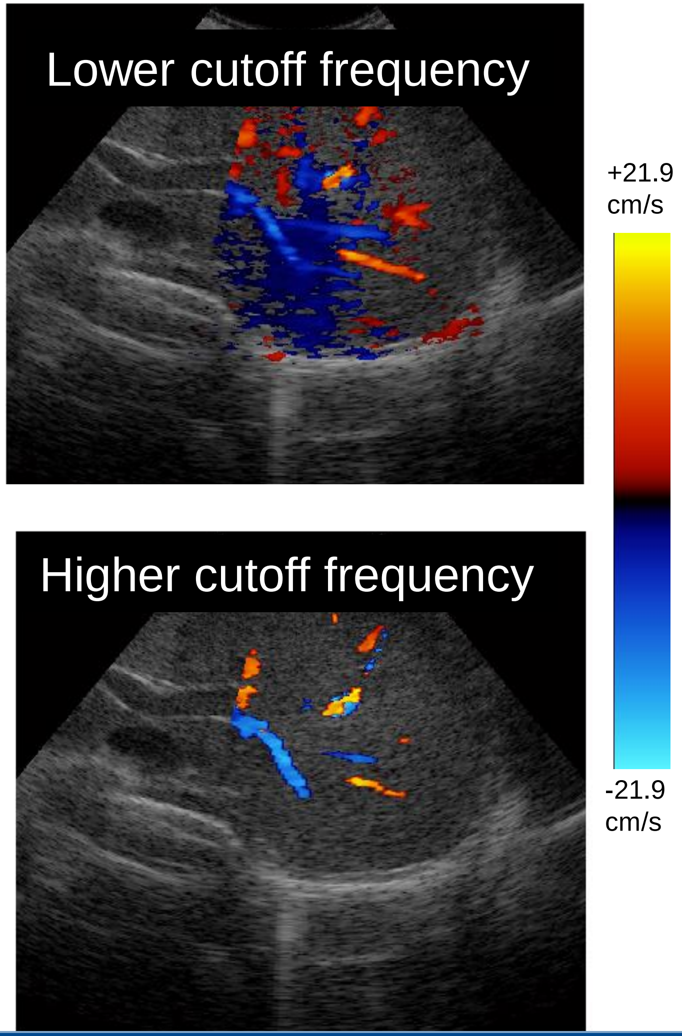


Clutter Filter design

- Clutter filter cutoff frequency

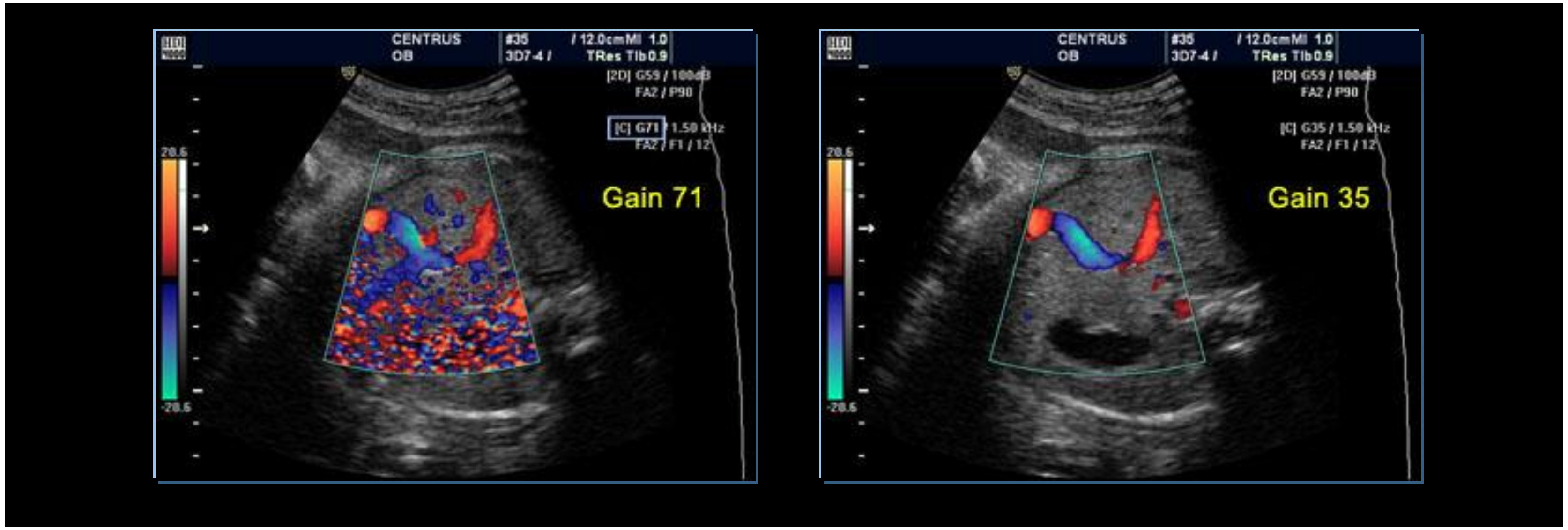
Frequency response of clutter filters

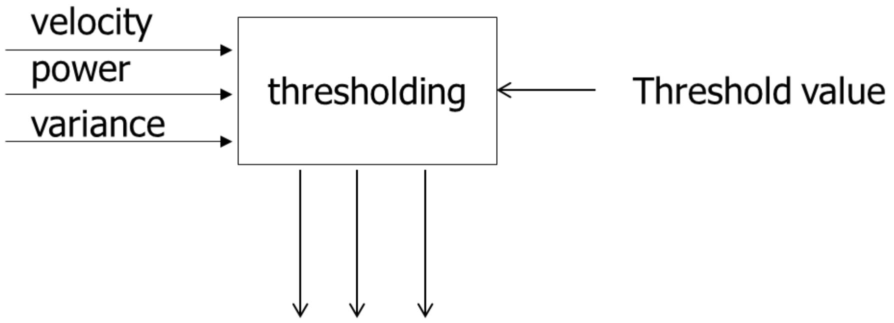




Doppler gain

- Determine overall sensitivity to flow signals (not real sensitivity improvement)





Elastography

Elastography – Glance at History

Tissue Elasticity

Biomechanics
(muscle, skin, ...)

Sarvazyan et al, 1975
Frizzell et al, 1976
Fung, 1981
Duck, 1990
Parker et al, 1990
Krouskop et al, 1998
Erkamp et al, 1998

Ultrasound

Oestreicher, 1951
Wilson and Robinson, 1982
Dickinson and Hill, 1982
Eisensher et al, 1983
Tristam et al, 1986
Krouskop, (Le)vinson, 1987
Lerner and Parker, 1988
Yamakoshi et al, 1988
Adler et al, 1989
Meunier and Bertrand, 1989
DeJong et al, 1990
<u>Ophir et al, 1991</u>
<u>Sarvazyan and Skovoroda, 1991</u>

Avalanche of papers

MRI

Fowlkes et al, 1992
Sarvazyan et al, 1995
Fowlkes et al, 1995
Muthupillai et al, 1995
Plewes et al, 1995
Chenevert et al, 1998

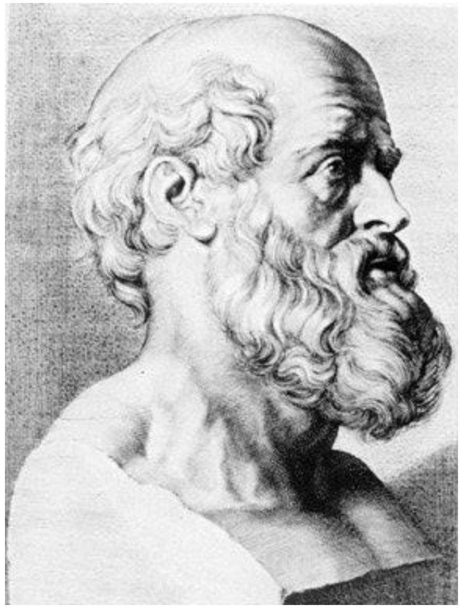
Many papers

Other methods

Frank et al, 1948
Thompson et al, 1981
Sarvazyan et al, 1984
Pereira et al, 1990

Elasticity – Glance at History

Changes in tissue elasticity are related to pathological changes



Hippocrates

… Such swellings as are soft, free from pain, and yield to the finger, … and are less dangerous than the others.
… then, as are painful, hard, and large, indicate danger of speedy death; but such as are soft, free of pain, and yield when pressed with the finger, are more chronic than these.

THE BOOK OF PROGNOSTICS, Hippocrates, 400 B.C.

It is the business of the physician to know, in the first place, things similar and things dissimilar; … which are to be seen, touched, and heard; which are to be perceived in the sight, and the touch, and the hearing, … which are to be known by all the means we know other things.

ON THE SURGERY, Hippocrates, 400 B.C.

Hippocrates, 400 B.C.

Mechanical Properties of Tissue (i.e., Why Bother)

Tissue Elasticity reflects 1) pathological changes such as fibrosis , calcification, and cancer development etc … - Static (requires external deformation)

Tissue Elasticity reflects 2) contractility of organs such as cardiac and cardiovascular system, and skeletal muscles etc… - Dynamic (use physiological pulsation or external stimulation…)

Mechanical Properties of Tissue

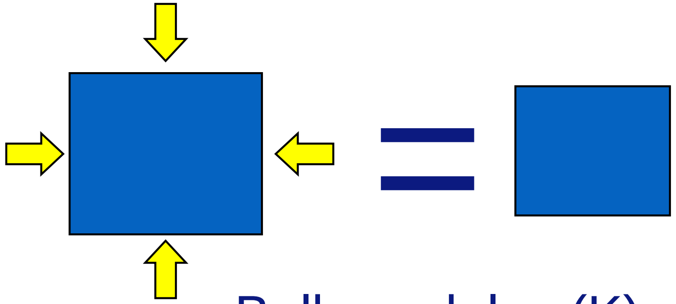
Elasticity (e.g., bulk and shear moduli)

Viscosity (e.g., bulk and shear viscosities)

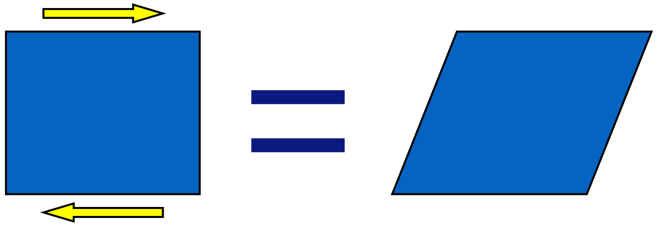
Nonlinearity (e.g., strain hardening)

Other (e.g., anisotropy, pseudoelasticity)

Elastic Moduli



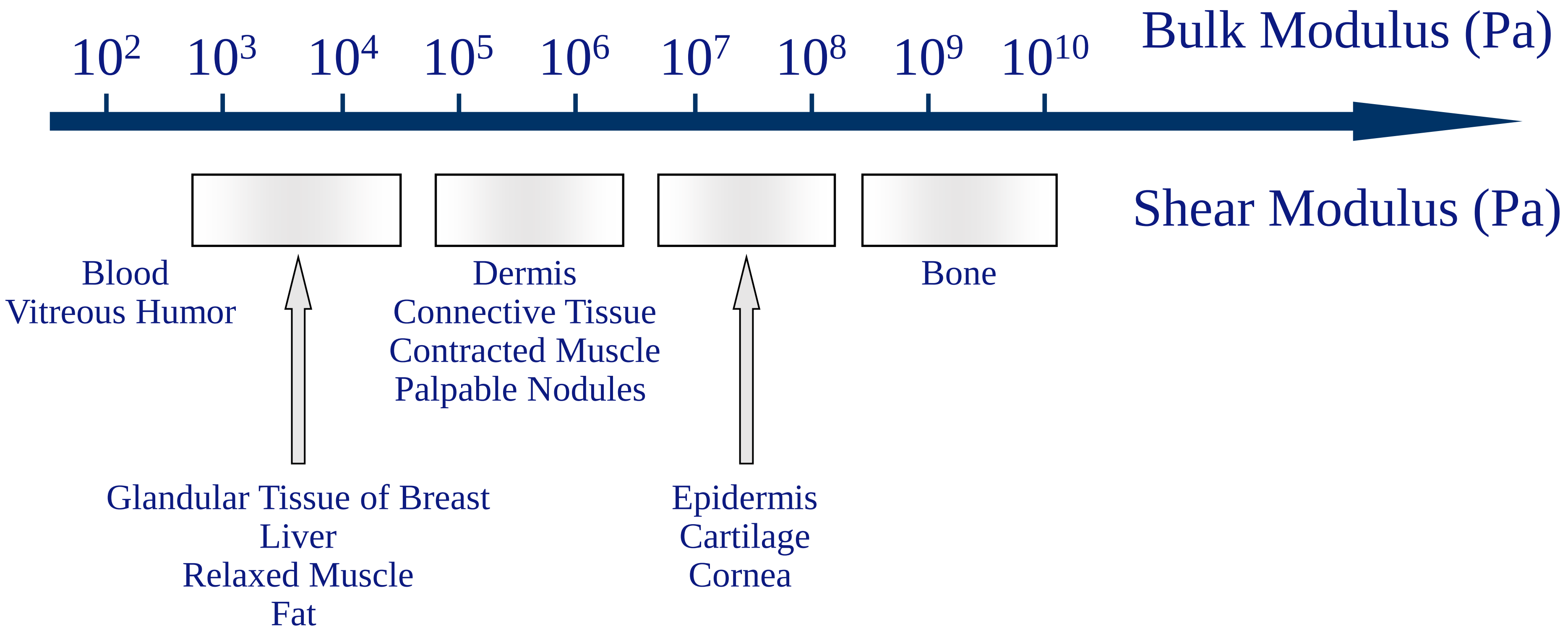
Bulk modulus (K)
Young's modulus (E)
Poisson's ratio (*n*)



Shear modulus (*m*=G)

	Volume change	Shape change
Gas	+	+
Liquid	–	+
Solid	–	–
Tissue	–	±

Contrast in Elastography



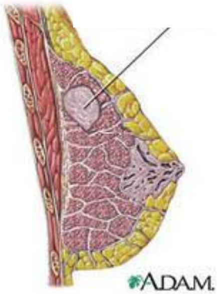
Sarvazyan et al, 1995

Breast Tissue Elasticity and Pathology

Skovoroda et al., 1995, *Biophysics*, 40(6):1359-1364.

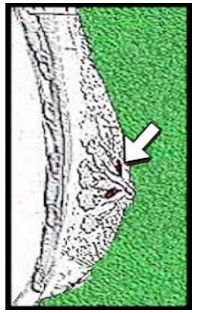
Breast Tissue Type	Normal gland	Infiltrative ductal cancer with alveolar tissue predominating.	Fibroadenomas of glandular origin	Infiltrative ductal cancer with fibrous tissue predominating.	Ductal fibroadenoma
Young's Modulus (kPa)	**0.5-1.5**	**1.0-1.5**	**1.5-2.5**	**2.0-3.0**	**8.0-12.0**

Krouskop et al., 1998, *Ultrasonic Imaging*, 20:260-274.



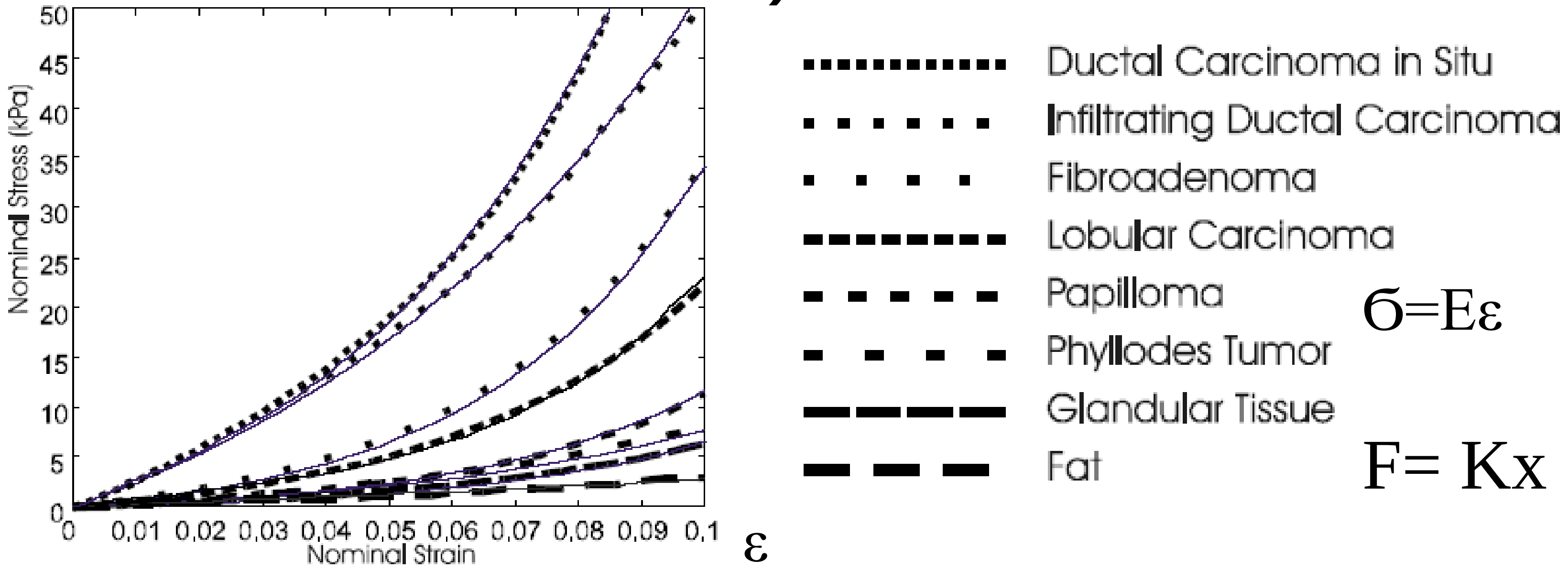
Benign tumor

Breast Tissue Type	**Tissue Elastic Modulus (kPa)**					
	5% precompression			**20% precompression**		
	Loading frequency (Hz)			**Loading frequency (Hz)**		
	0.1	1.0	4.0	0.1	1.0	4.0
Normal fat (n = 8)	18 ± 7	19 ± 7	22 ± 12	20 ± 8	20 ± 6	24 ± 6
Normal glandular tissue (n = 31)	28 ± 14	33 ± 11	35 ± 14	48 ± 15	57 ± 19	66 ± 17
Fibrous tissue (n = 18)	96 ± 34	107 ± 31	116 ± 28	218 ± 87	232 ± 60	244 ± 85
Ductal carcinoma *in situ* (n = 23)	22 ± 8	25 ± 4	26 ± 5	291 ± 67	301 ± 58	307 ± 78
Invasive and infiltrating ductal carcinoma (n = 32)	106 32	93 ± 33	112 ± 43	558 ± 180	490 ± 112	460 ± 178



Ductal carcinoma:
Malignant cancer

Strain hardening (nonlinear)



ε

$\sigma = E\varepsilon$

$F = Kx$

	Ratio to Fat at			
Tissue Type	Strain = 0.01	Strain = 0.05	Strain = 0.10	Strain = 0.15
Gland	4	5	8	16
Phyllodes Tumor	12	14	16	17
Papilloma	5	8	16	31
Lobular Carcinoma	7	12	21	36
Fibroadenoma	9	15	28	51
Infiltrating Ductal Carcinoma	10	18	37	79
Ductal Carcinoma in Situ	15	29	61	124

Table 4: The ratio of elastic modulus of each tissue type to fat at 4 different strain levels.

Wellman et al., 1999

Strain hardening (nonlinear)

This and other graphs as well as other literature data suggest that tissue strain hardening (or nonlinearity in stress-strain relations) can be used for tissue analysis including composition, differentiation, etc.

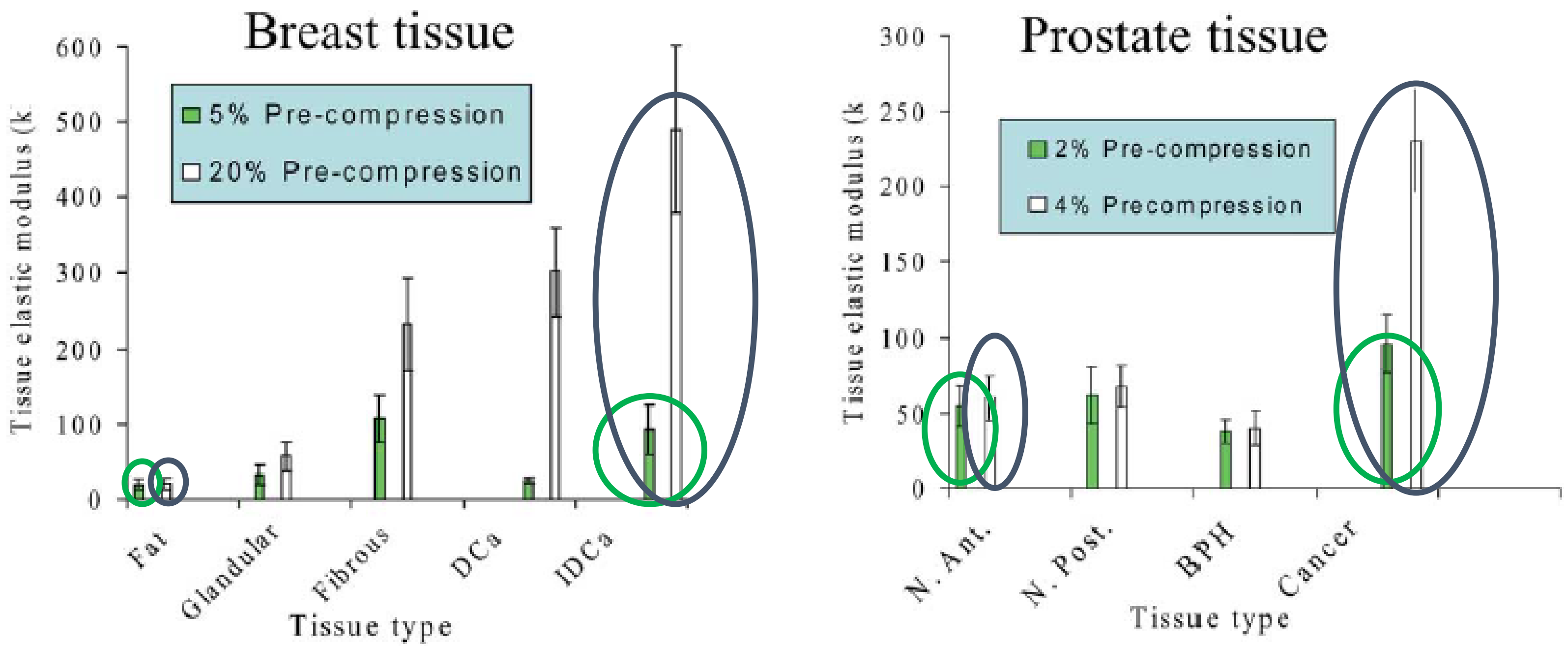
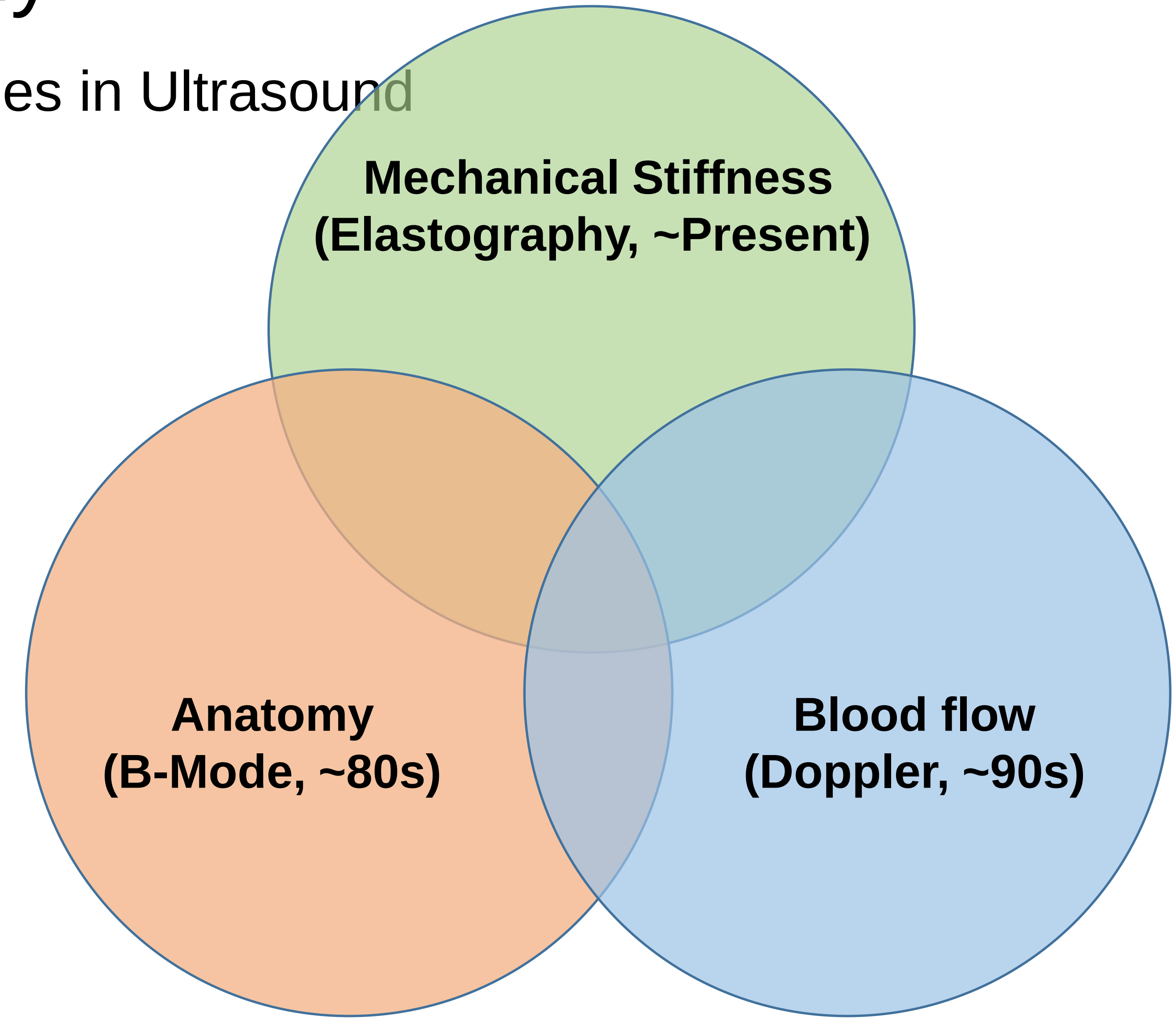


Figure 1. Tissue elastic moduli obtained from normal and abnormal breast and prostate tissues [26]. DCa = ductal carcinoma, IDCa = intraductal carcinoma, N. Ant. = anterior portion of the normal prostate, N. Post. = posterior portion of the normal prostate, BPH = benign prostatic hypertrophy.

Krouskop et al, 1998
Ophir et al., 2001

Elastography

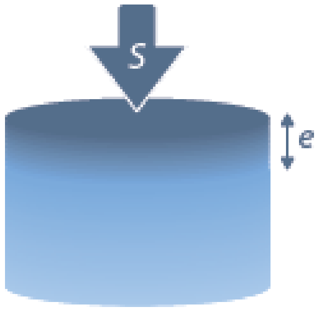
- Major 3 modalities in Ultrasound



Elastography

- Tissue Stiffness
  - Measured by Young's Modulus (in KPa, 1Pa = 1 N/m^2).
  - Definition : (Hard Tissues have higher value)

$$E = \frac{s}{e}, (\text{ E : Young's Modulus, s : stress, e : strain })$$



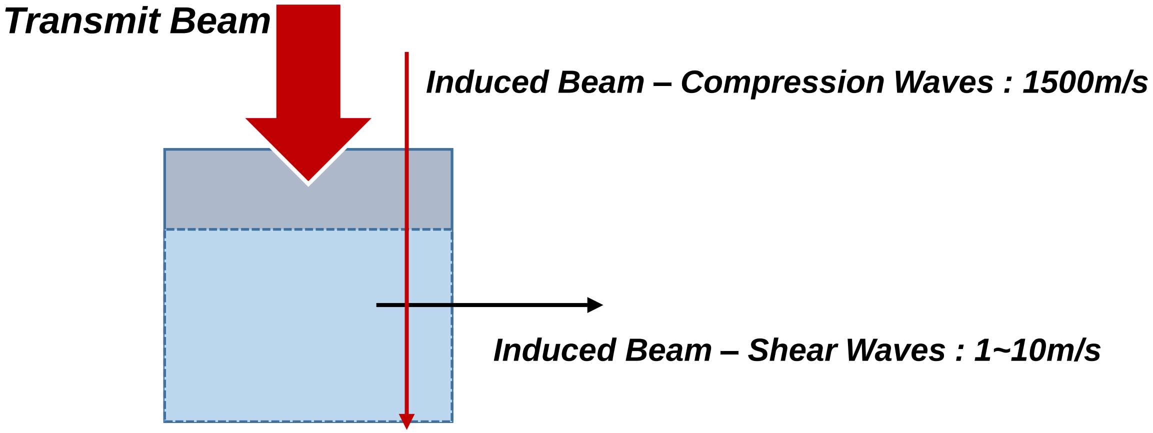
Type of soft tissue		Young's Modulus (E in kPa)	Density (kg/m3)
Breast	Normal Fat	18-24	1000 +/- 8% (~water)
	Normal glandular	28-66	
	Fibrous tissue	96-244	
	Carcinoma	**22-560**	
Prostate	Normal anterior	55-63	
	Normal posterior	62-71	
	BPH	36-41	
	Carcinoma	**96-241**	
Liver	Normal	0.4-6	
	Cirrhosis	**15-100**	

Elastography - Mechanical perturbation

- More complex than static compression.
- Two Types of mechanically induced waves
  - **Compression (Bulk) Waves : Longitudinal Wave**
    - Very fast propagating in tissues (1500m/s)
    - Echo of compressional waves on tissue scatterers => Conventional Ultrasound Imaging
  - **Shear Waves : Transverse Wave**
    - Slower than compression waves (1~10m/s)
    - Being related to Tissue stiffness

$$E = 3\rho c^2, (\text{ E : Young's Modulus, } \rho \text{ : Density kg/m}^3\text{, c : shear wave propagation speed })$$

※Like Seismic Wave.
- P Wave : Compression Wave
- S Wave : Shear Wave



Elastography

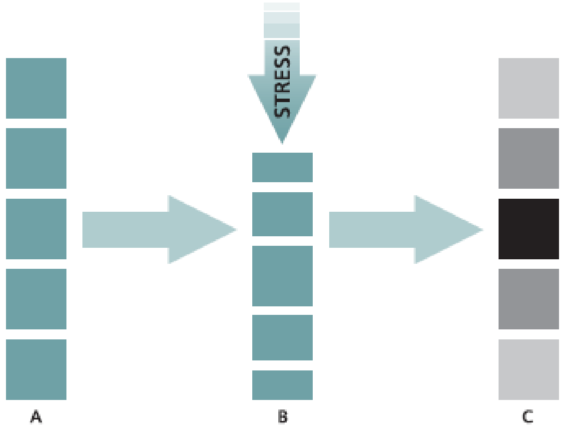
- General Methodology of elastography
  - Step 1. Generate a low frequency vibration in tissue to induce shear stress.
  - Step 2. Image the tissue with the goal of analyzing the resulting stress.
  - Step 3. Deduce from this analysis a parameter related to tissue stiffness.

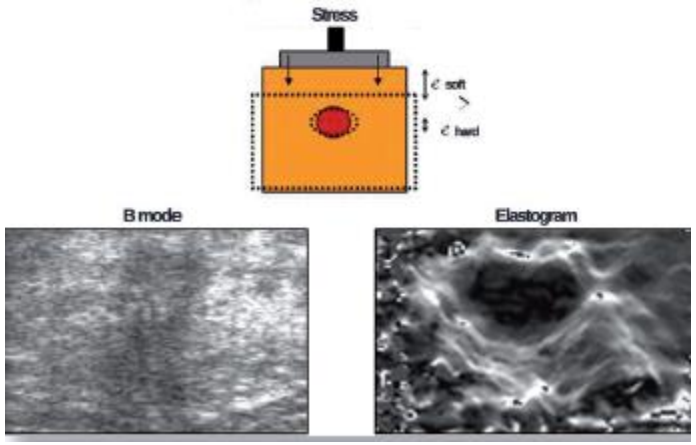
- Three different approaches of elastography

Static Elastography : Using uniform compression	Dynamic Elastography : Using continuous vibration	Shear-wave based Elastography : Using transient pulse to generate shear wave

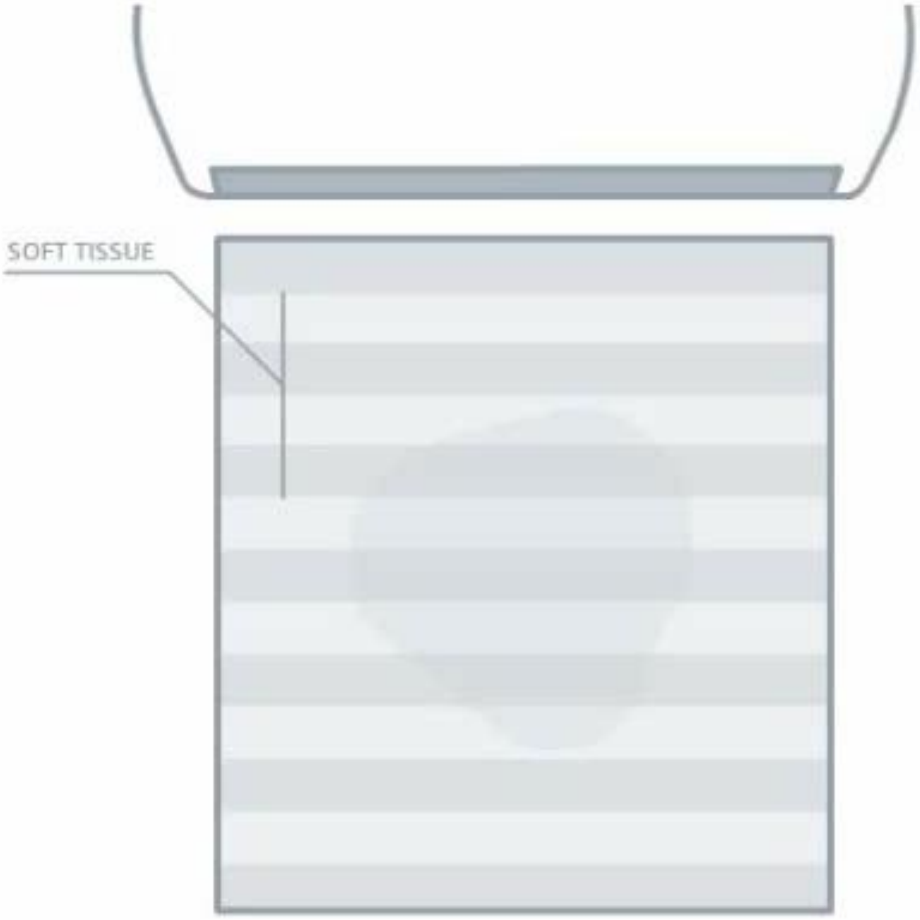
Static Elastography

- Using uniform compression at the surface of the body to cause deformation of the tissue.
- Procedure
  - Step 1. The compression is applied by the user.
  - Step 2. the ultrasound machine calculates and displays the induced deformation in the imaging plane.
- *Cons*
  - Have dependency of user’s skill.
  - Not Quantitative value <= Because, the stress within the tissues induced is unknown.



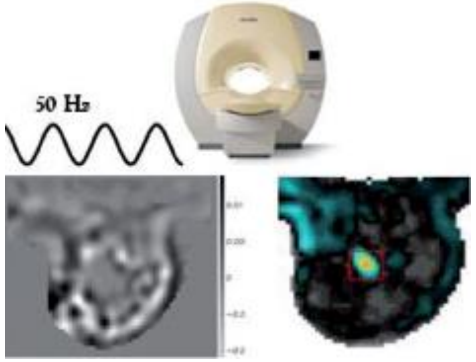


Static Elastography

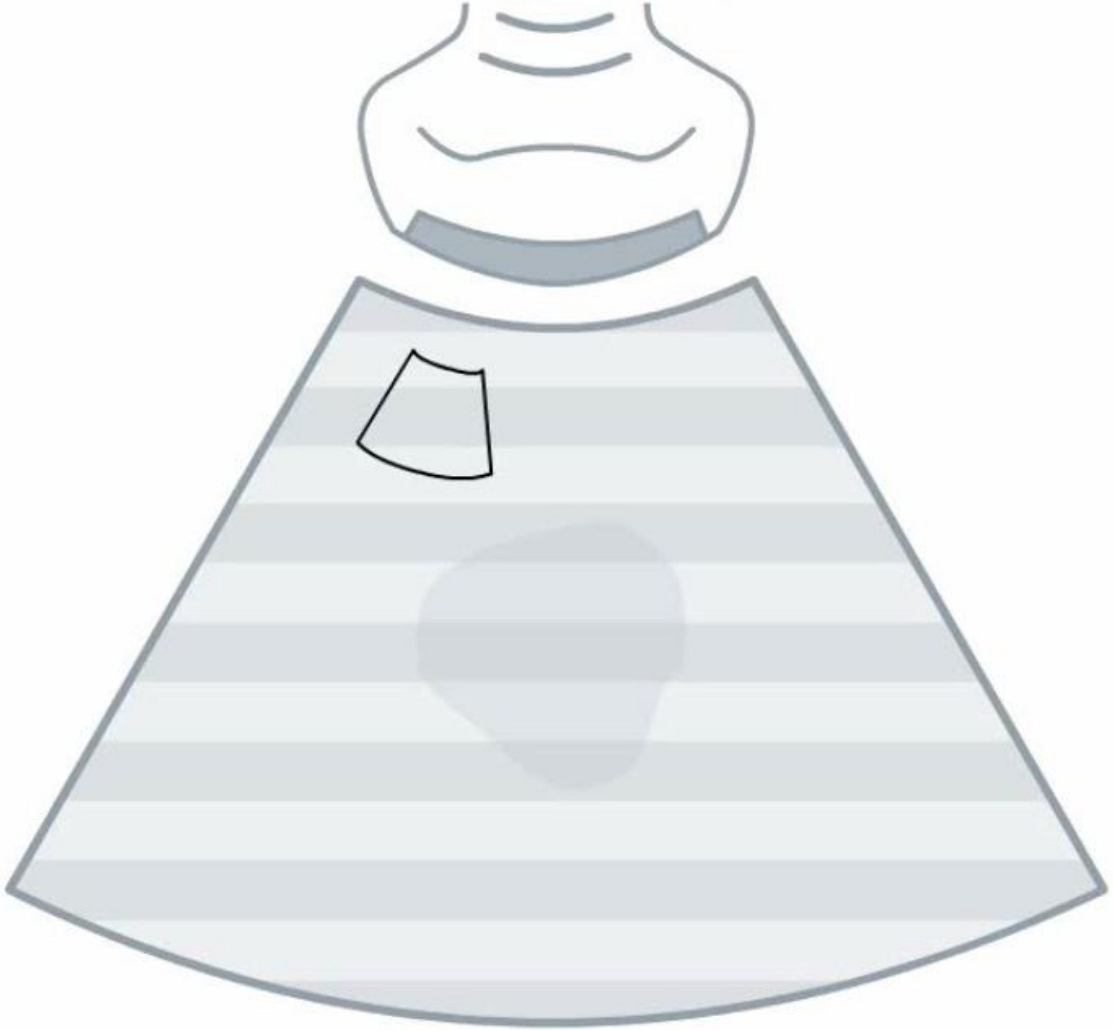


Dynamic Elastography (a.k.a. ARFI)

- Procedure
  - Step 1 : Using a continuous vibration
  - Step 2 : Stationary waves induced in the body are analyzed to estimate elasticity
- Well suited for MR systems, because the vibration pattern in not time-dependent but must be assessed in a volume
- Developed for US system (Acoustic Radiation Force Impulse: ARFI)
- *Pros*
  - Quantitative Technique
- *Cons*
  - Small imaging area



Dynamic Elastography (a.k.a. ARFI)

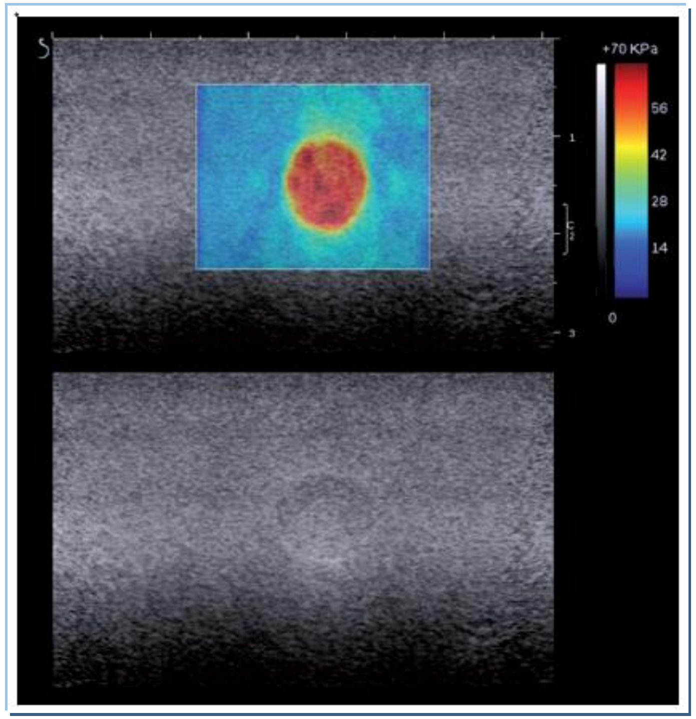


Shear-wave Elastography

- Procedure
  - Step 1 : Using a transient wave to generate shear wave.
  - Step 2 : Calculation Young’s Modulus from measuring the speed of wave propagation.

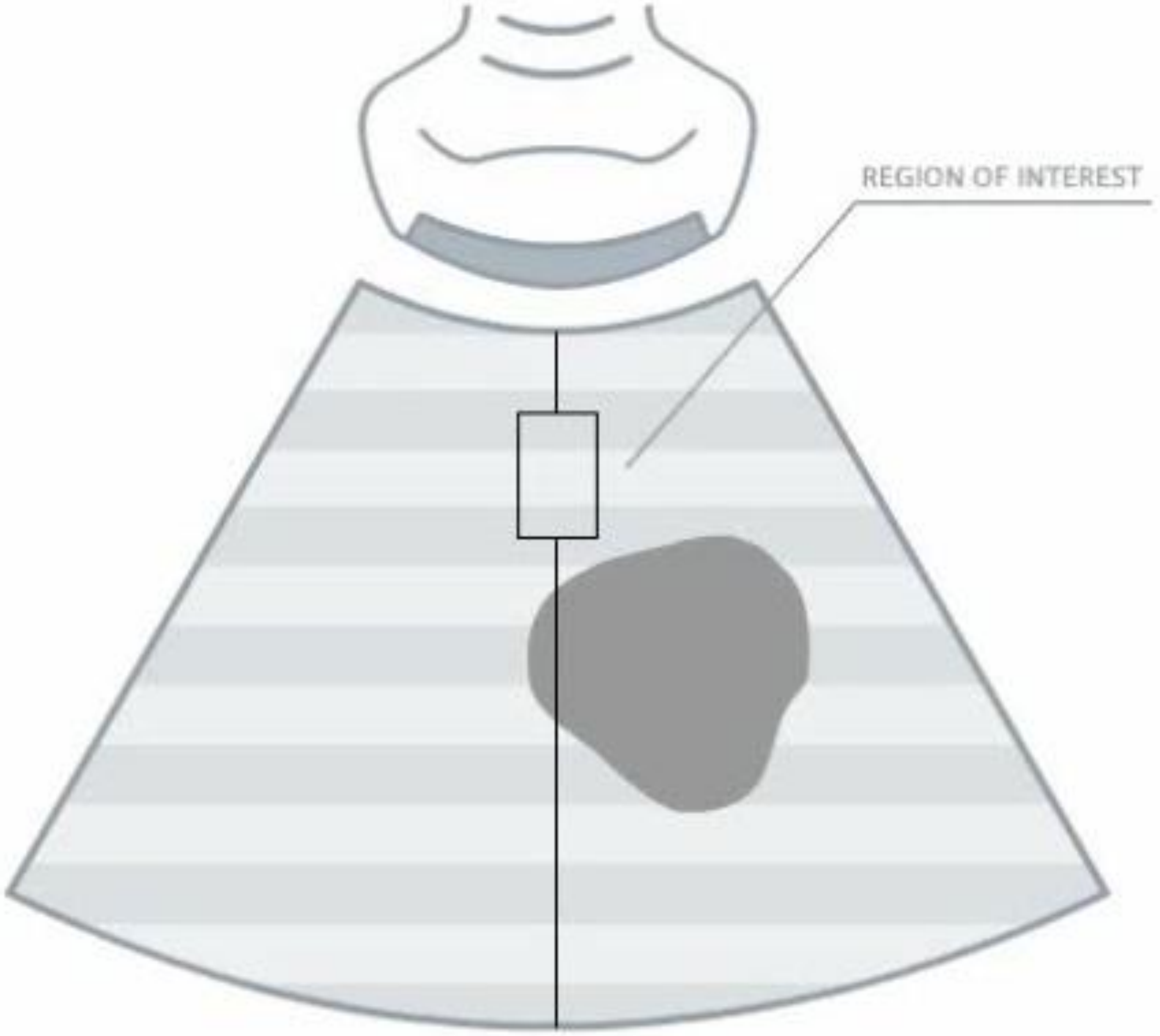
$$E = 3\rho c^2, (\text{ E : Young's Modulus, } \rho \text{ : Density kg/m}^3\text{, c : shear wave propagation speed })$$

- *Pros*
  - Quantitative Technique.
  - Real-time Imaging Methods.



Shear-wave generation

Shear-wave Elastography



Shear-wave Elastography