Medical Imaging
(RT516 – Spring 2021)
Lecture 5: Computed Tomography

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Why Computed Tomography?

- Conventional Radiography (X-ray) is great, but… limited:

- Imposition of a 3D subject onto a 2D plane (overlapping objects due to ‘projection’)
- Limited differentiation of different densities (e.g., water and soft tissue look the same)

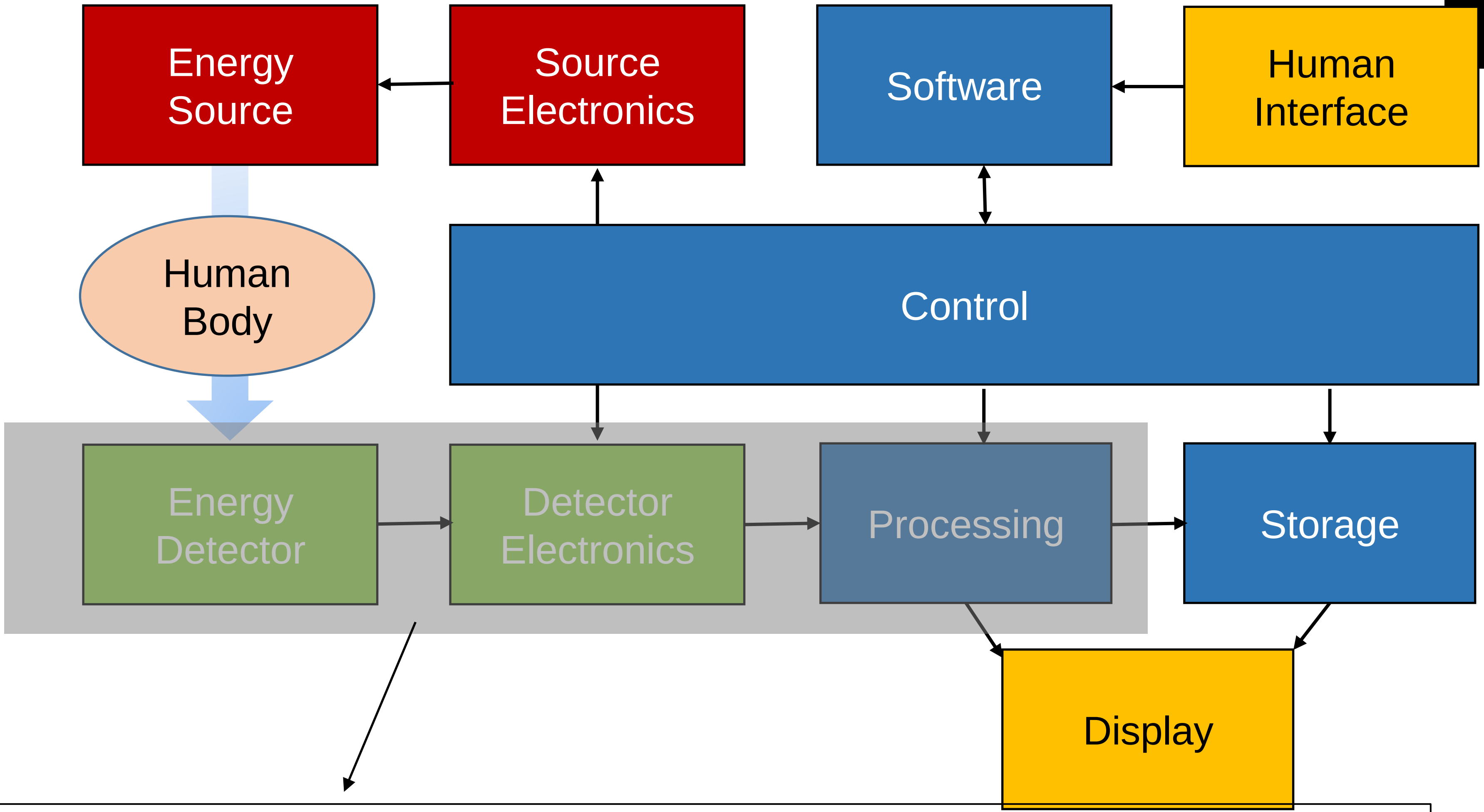
- **Goal in this lecture:** Key will be understanding how to form images using acquired data.

Computed Tomography

- Tomography refers to a ***picture*** (-graph) of a ***slice*** (tomo-)
- Truly ***"Computed"***, i.e., impossible without computers

- Introduced in 1972, Last 40+ years, Improved over 7 generations
- Godfrey Hounsfield (UK) and Allan Cormack (U.S.) Nobel prize in medicine (1979)
- First CT scanner, EMI Mark 1: 80x80 pixels, 3-mm pixel, 4.5 minutes to acquire, 1.5 minute to reconstruct. Now, 1024x1024 pixels, 0.05-mm pixel, 130 images/s
- Original development: head imaging only. The capture/reconstruction were too slow for cardiac imaging.

Computed Tomography (CT)



As CT utilized same energy source of the projection radiography, Our focus will be on
Instrumentation / Image reconstruction / Image quality / Applications

Basic principles (Let's see the video, again)

Basic principles

- Two orthogonal projections (views):

  3-D localization

- CT: 1,000 views
- Data acquisition:

  Parallel beam geometry

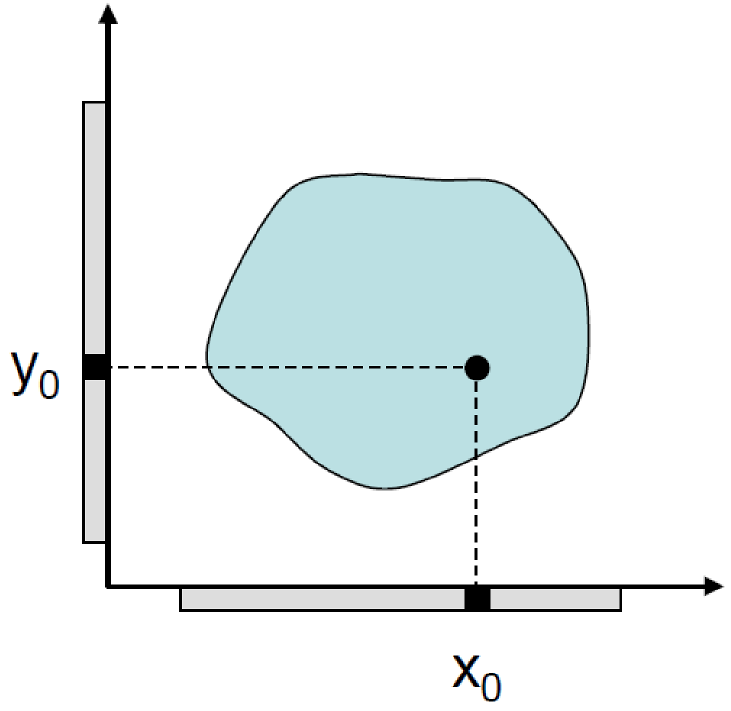
  Fan beam geometry

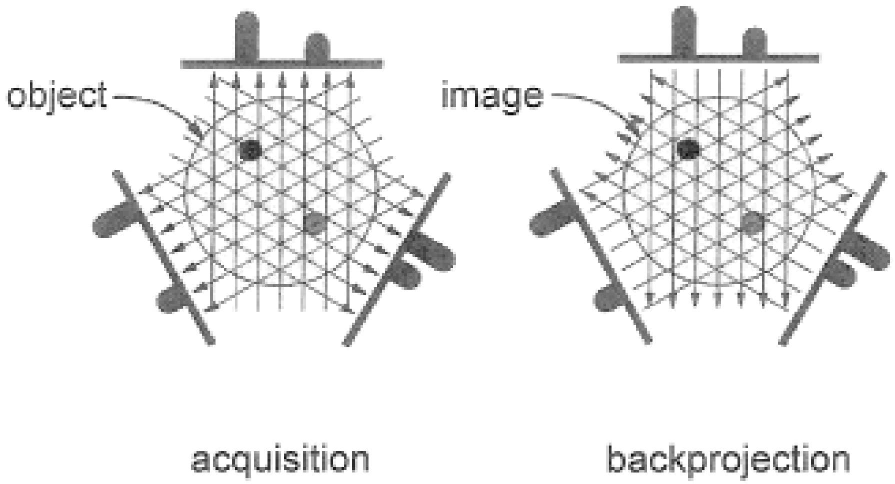
  800 rays at 1,000 angles

- Reconstruction

$$I = I_0 e^{-\mu r}$$

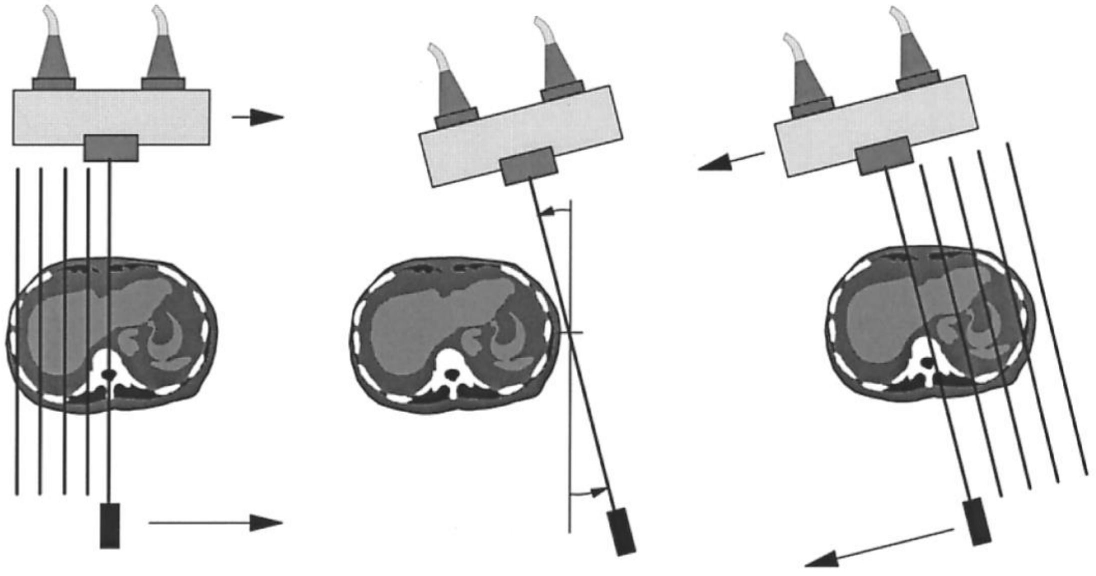
$$\ln\left(\frac{I_0}{I}\right) = \mu r$$





First generation

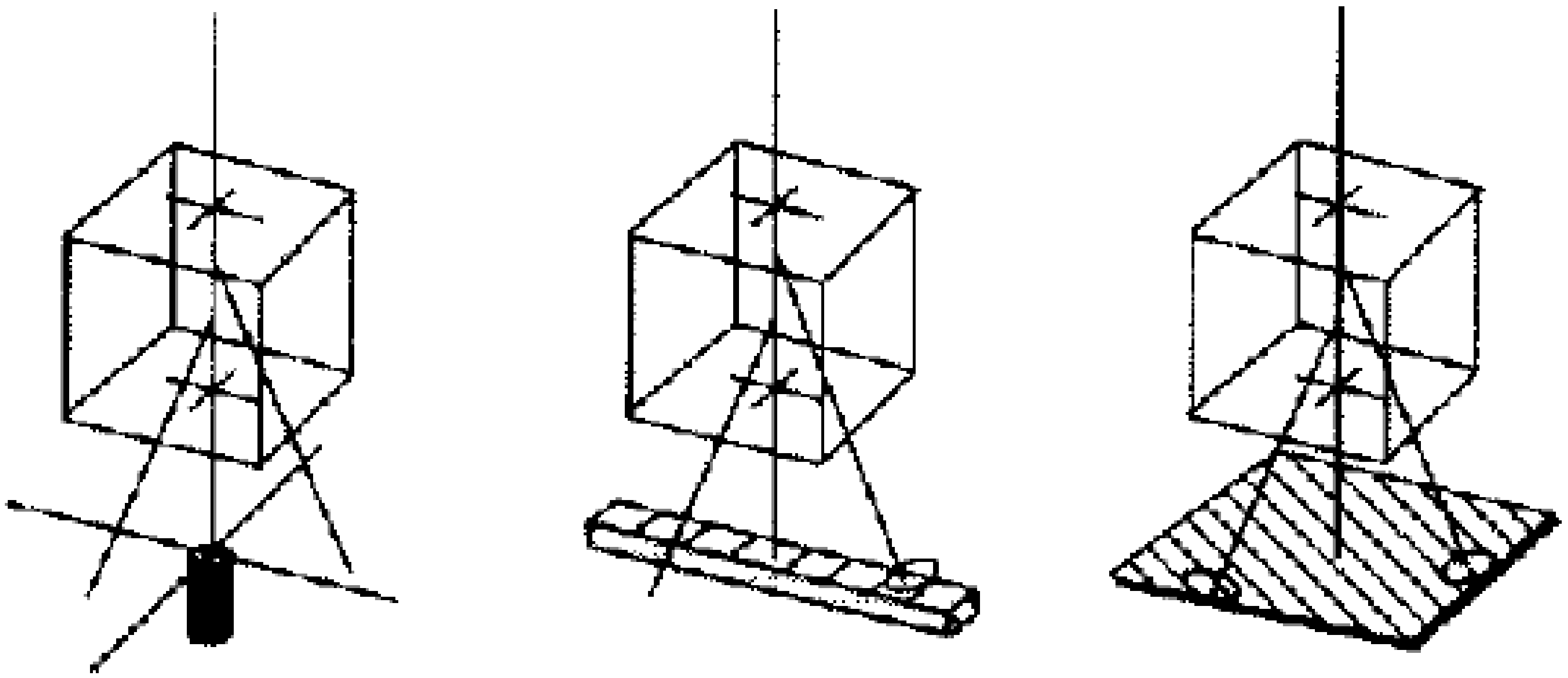
- Rotate/Translate Pencil Beam
- One detector
- 160 parallel rays, 24 cm field of view
- 180 projections (1 degree rotation)
- Total of 180x160=28,800 rays were measured



Pros. Scattering can be easily rejected

Cons. Motion artifact, Acquisition Time

FYI, Beam shape

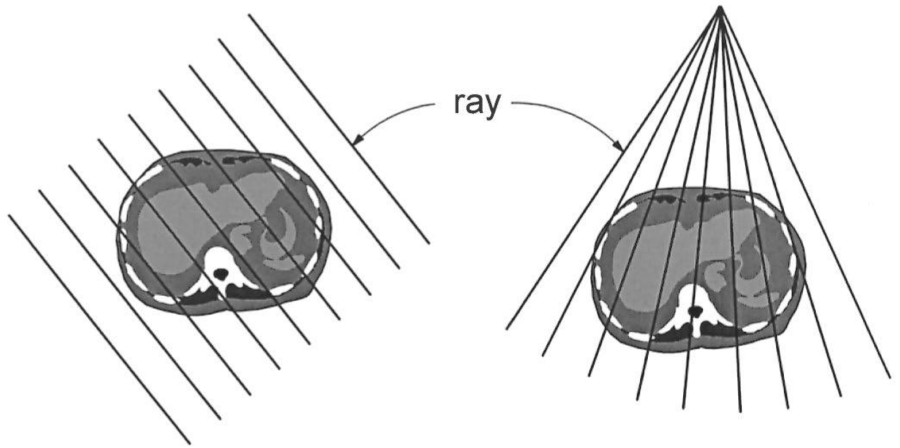


Pencil beam　Fan beam　Open beam

Second generation

- Rotate/Translate

  Narrow Fan Beam (10-degree)



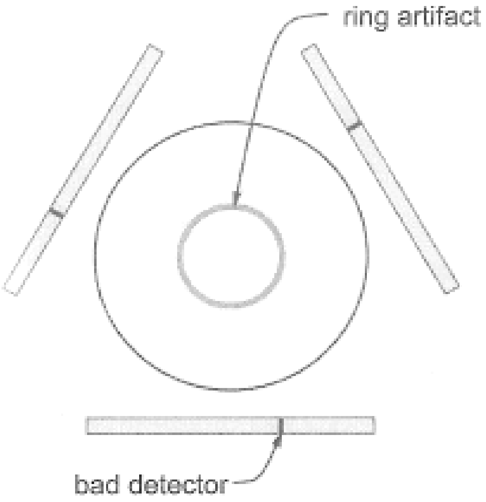
- 30 detectors (small!) / 600 rays / 540 views

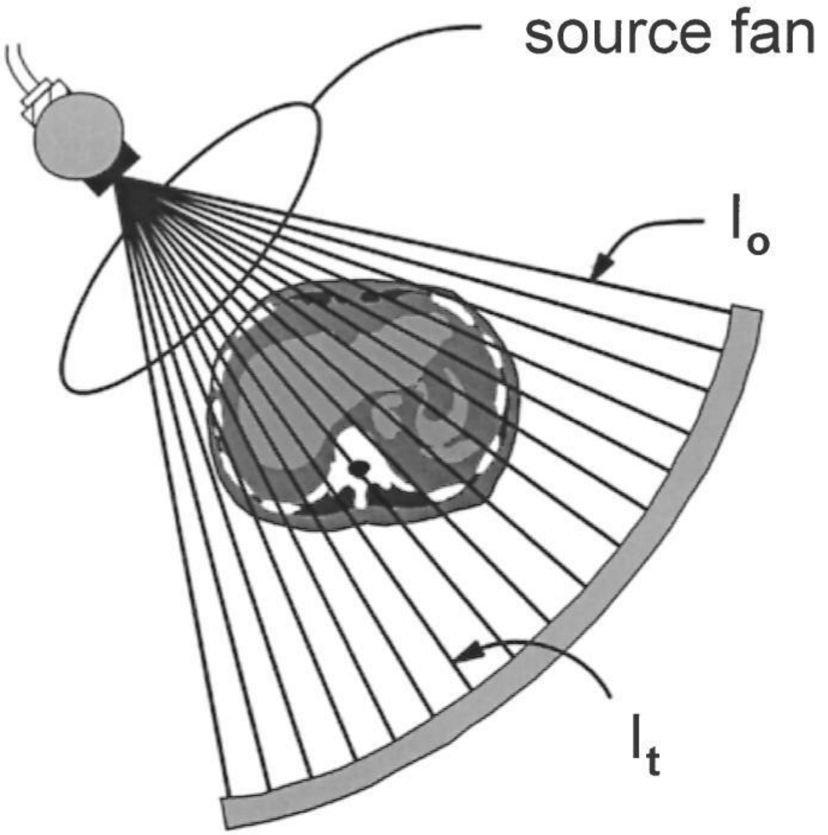
- Total of 600x540=324,000 rays (data points) were measured

Pros. 15 times faster than the 1st generation (18s / slice)
Cons. Increased scattering

Third generation

- Rotate/Rotate Wide Fan Beam
- > 800 detector
- 5 seconds per slice
- Major improvement in CT
- Artifacts (?)

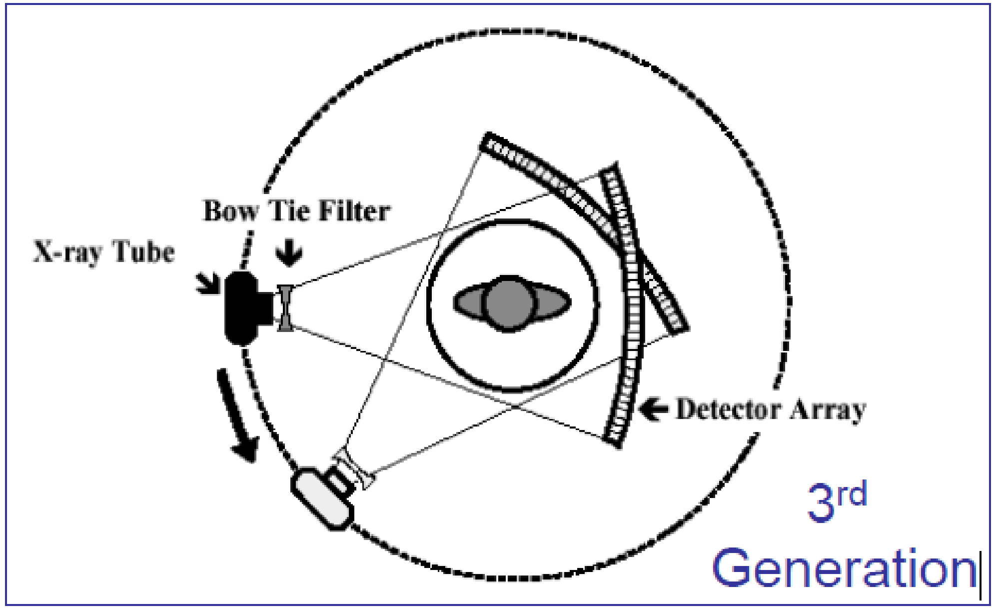


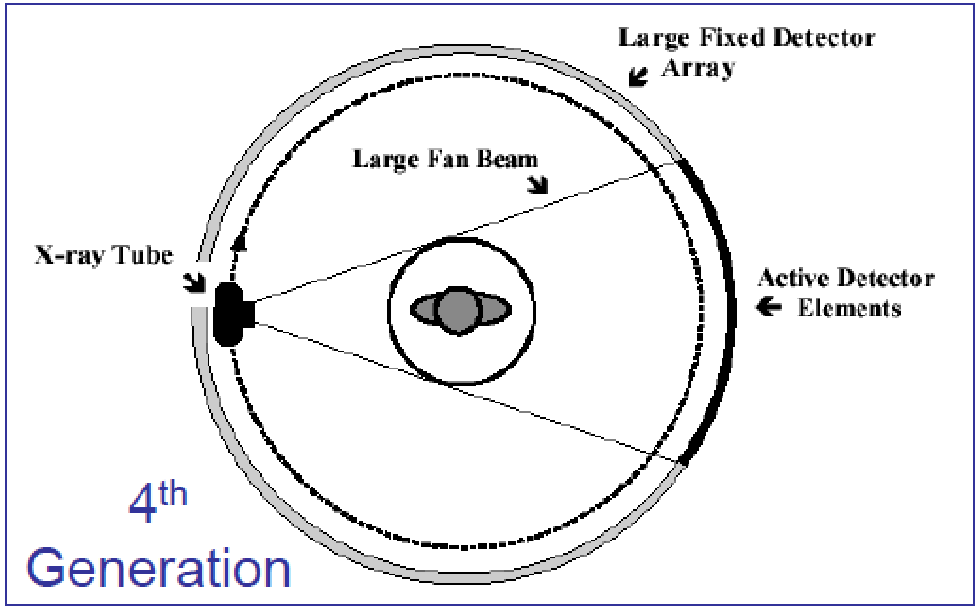


Pros. 3 times faster than the 2nd generation (5s / slice)

Cons. Scattering artifact

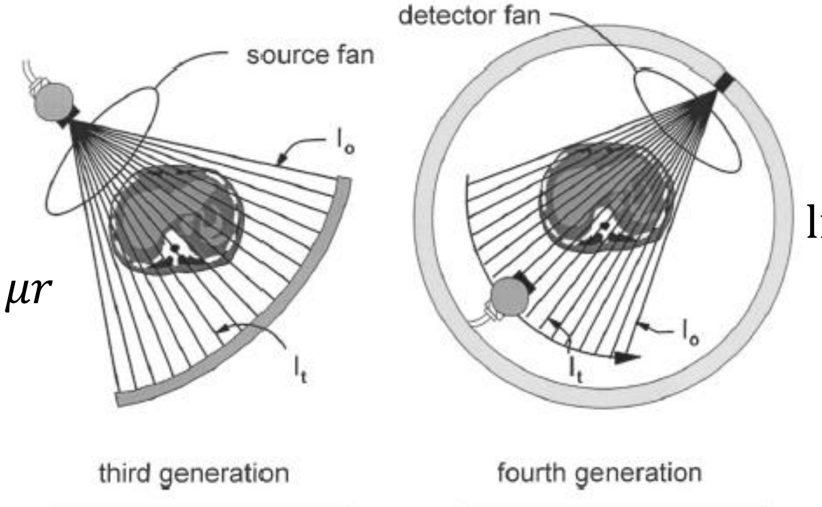
Fourth generation (Modern CT)





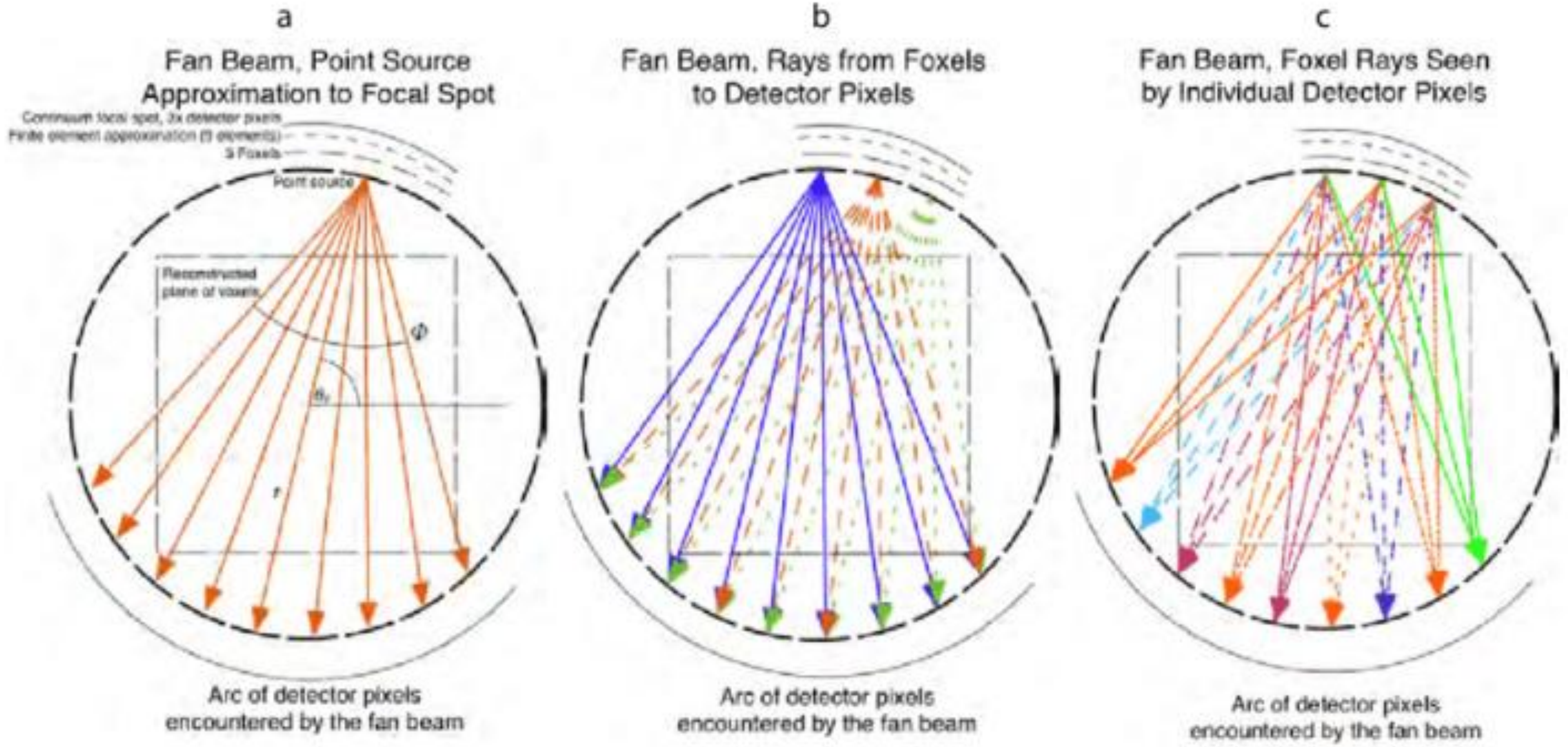
- Rotate/Stationary Wide Fan Beam
- 360° stationary detectors
- Reduction of Ring artifacts

$$\ln\left(\frac{g_k I_0}{g_n I}\right) = \mu r$$



$$\ln\left(\frac{\boxed{g_n} I_0}{\boxed{g_n} I}\right) = \mu r$$

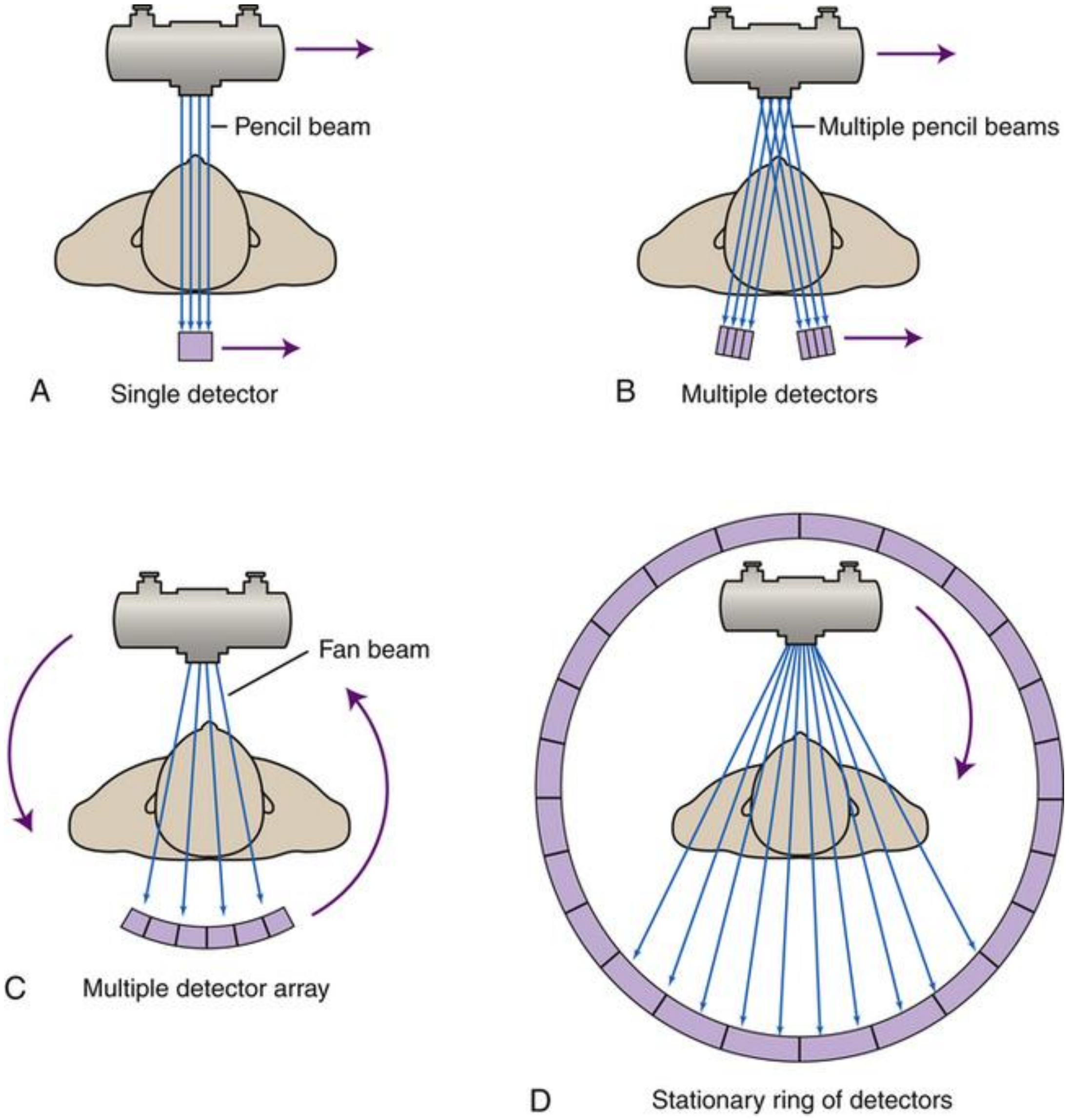
Fourth generation (Modern CT)



$$\ln\left(\frac{g_k I_0}{g_n I}\right) = \mu r$$

$$\ln\left(\frac{g_n I_0}{g_n I}\right) = \mu r$$

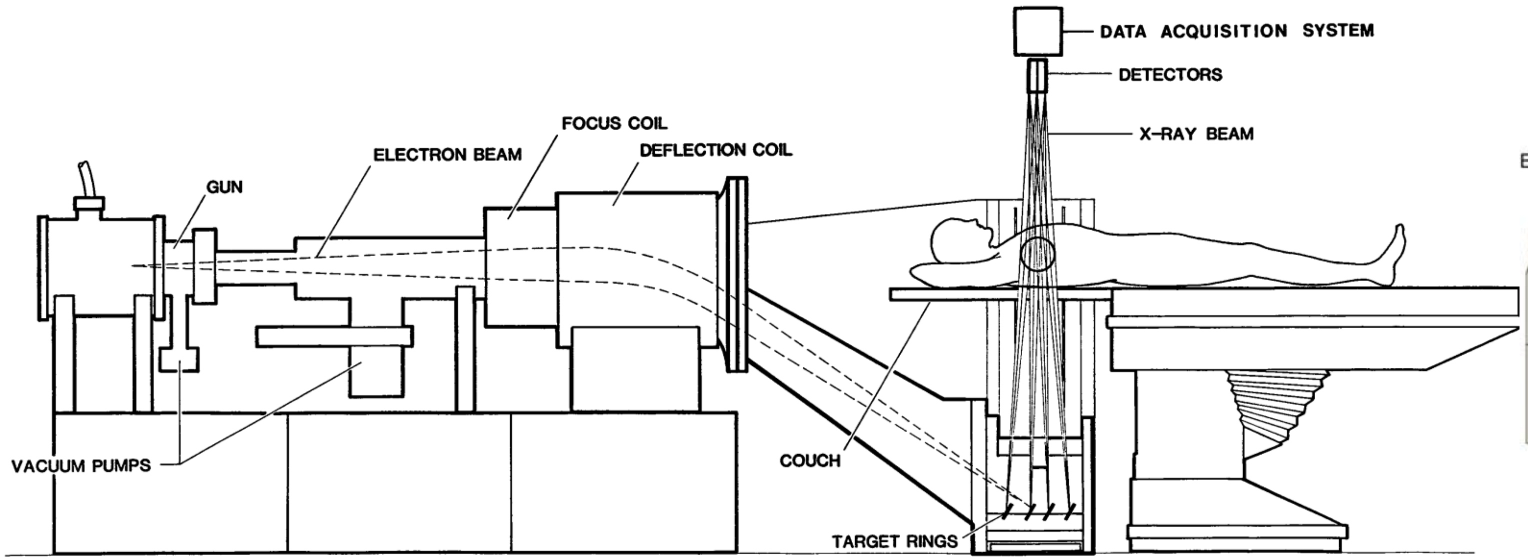
Comparison

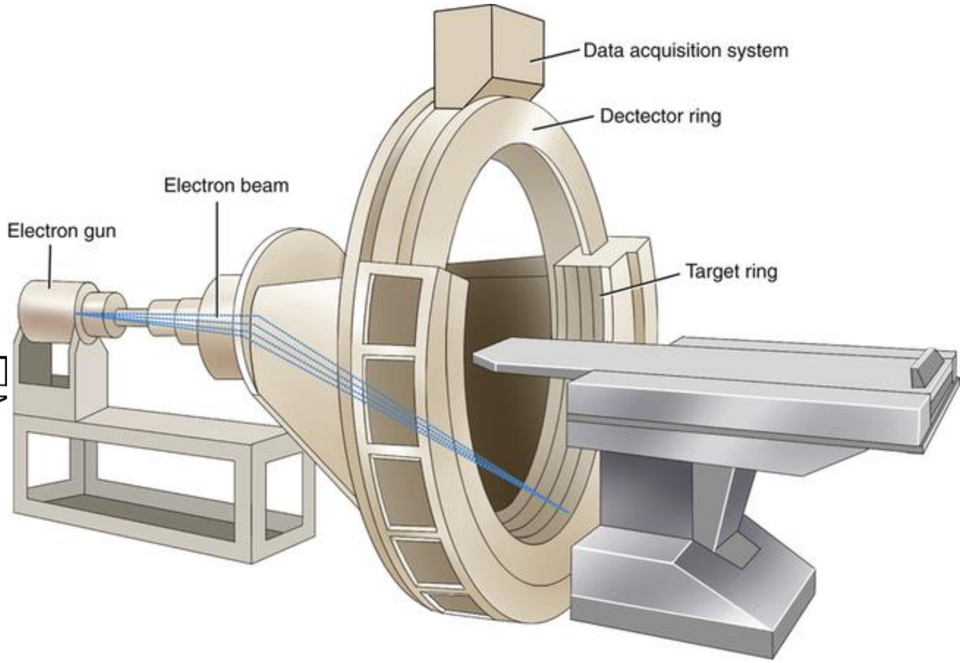


Pencil beam
A
Single detector
Multiple pencil beams
B
Multiple detectors
Fan beam
C
Multiple detector array
D
Stationary ring of detectors

Fifth generation

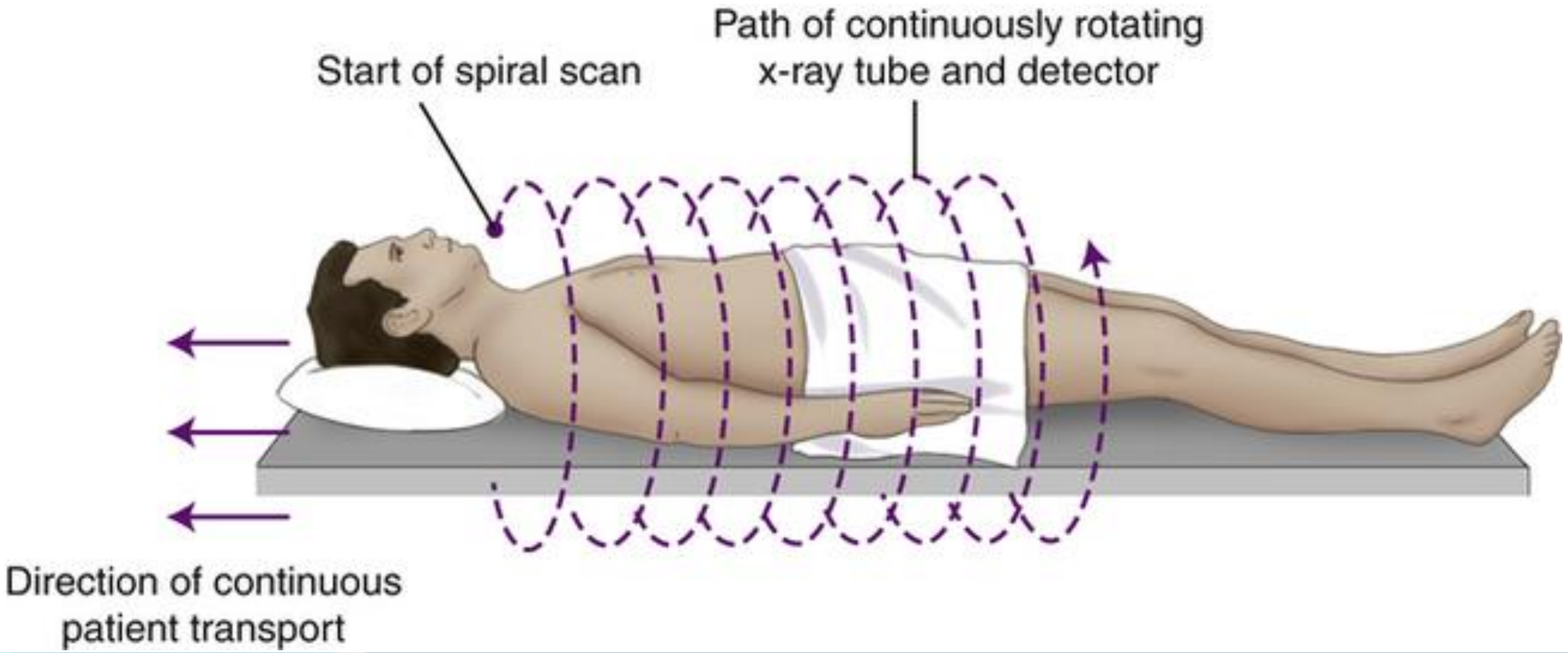
- Stationary/Stationary
- Tungsten arc source around patient, detector ring
- Faster! (50 ms scan time) – Cardiac applications available !





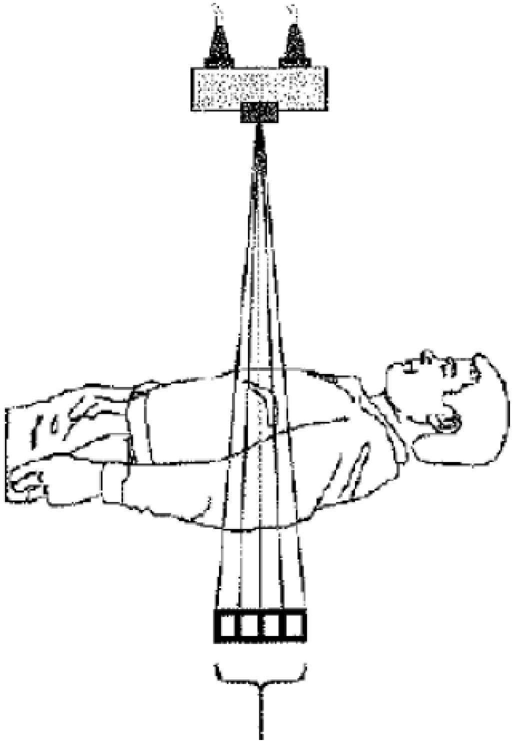
Sixth generation

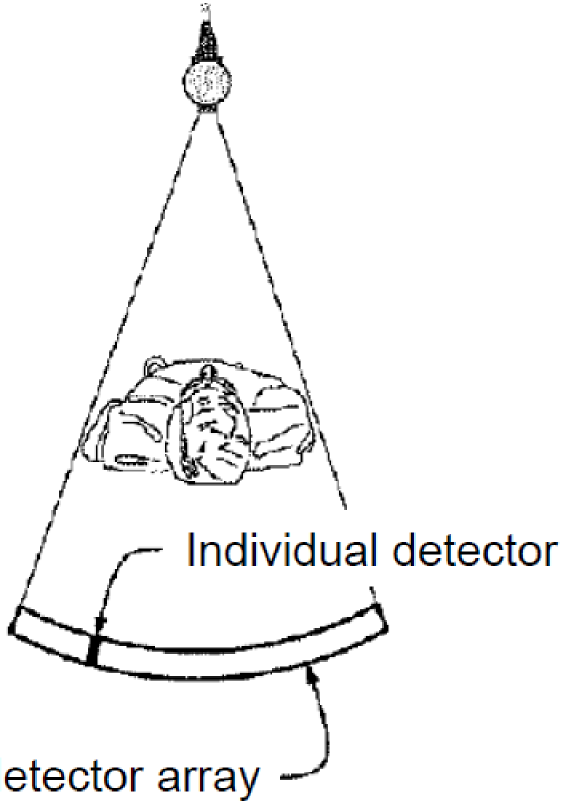
- Helical (spiral) scanning
  - Ribbon cables / Slip ring technology
- Increased scan time: (30 second scan for entire abdomen (60 cm))

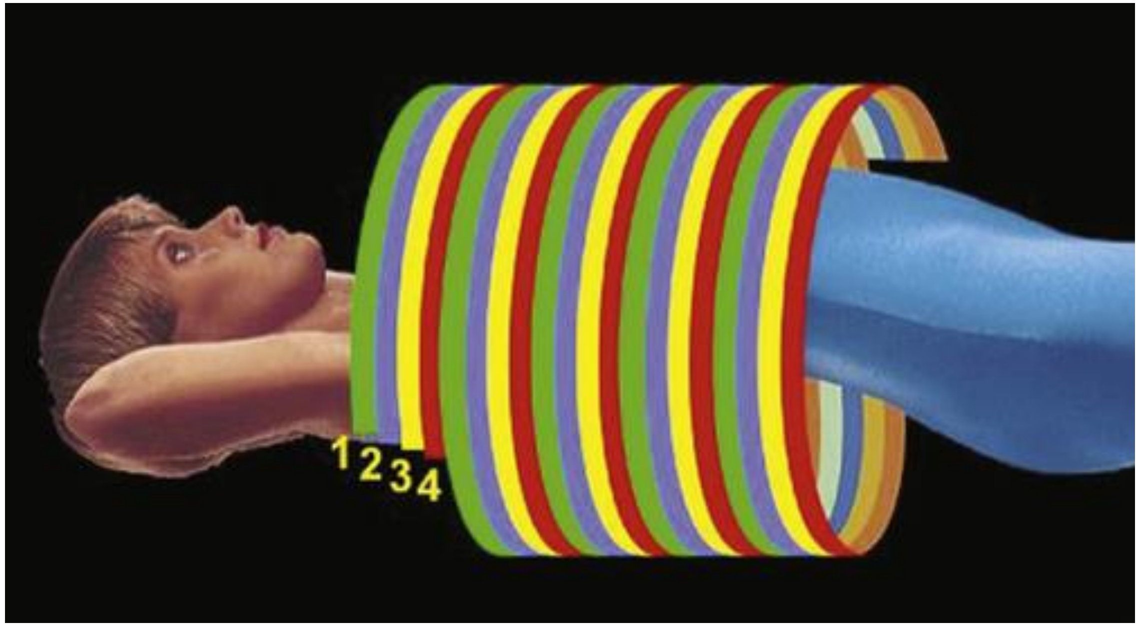


Seventh generation

- Multiple detector array
- Better use of x-rays

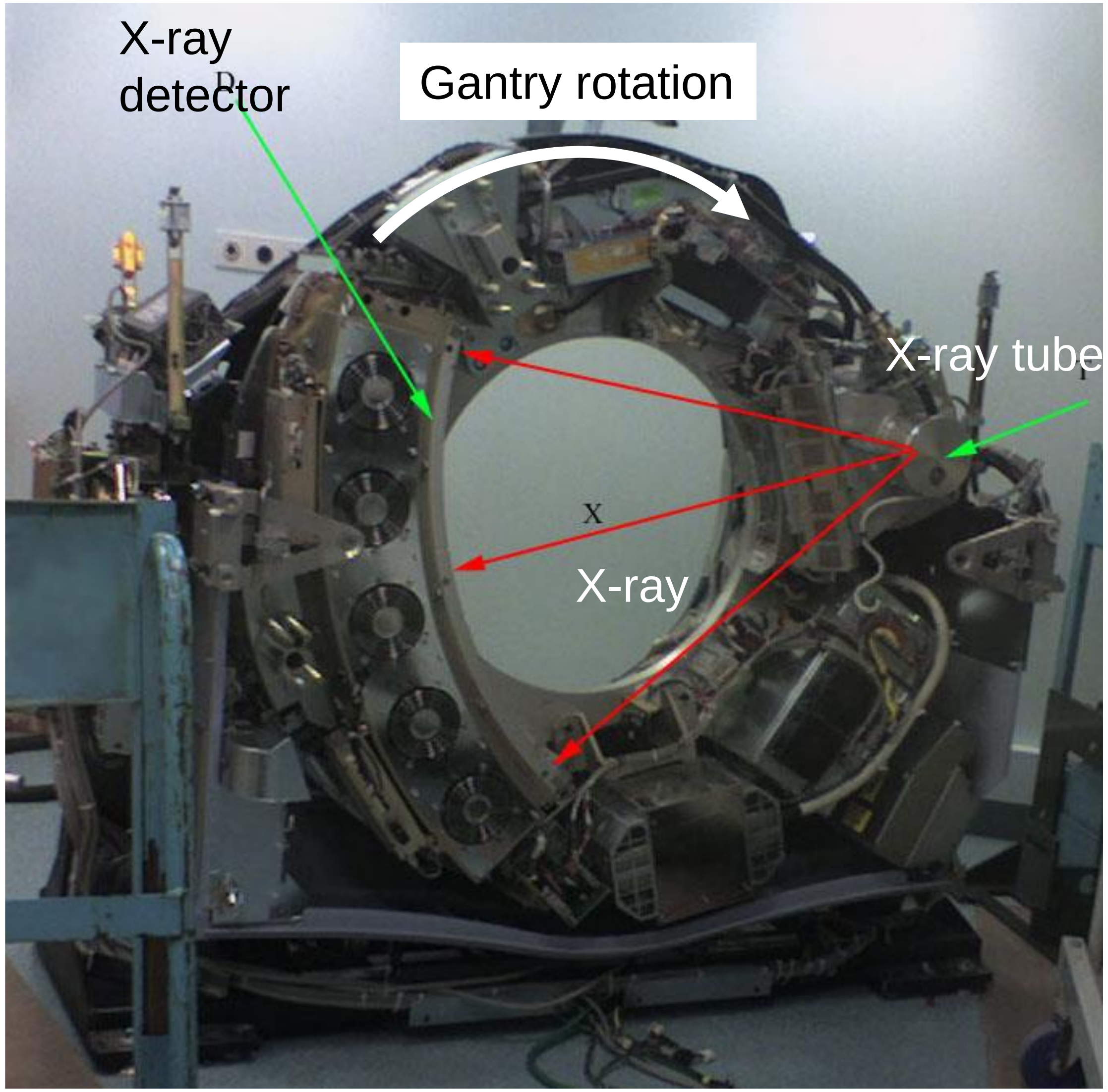






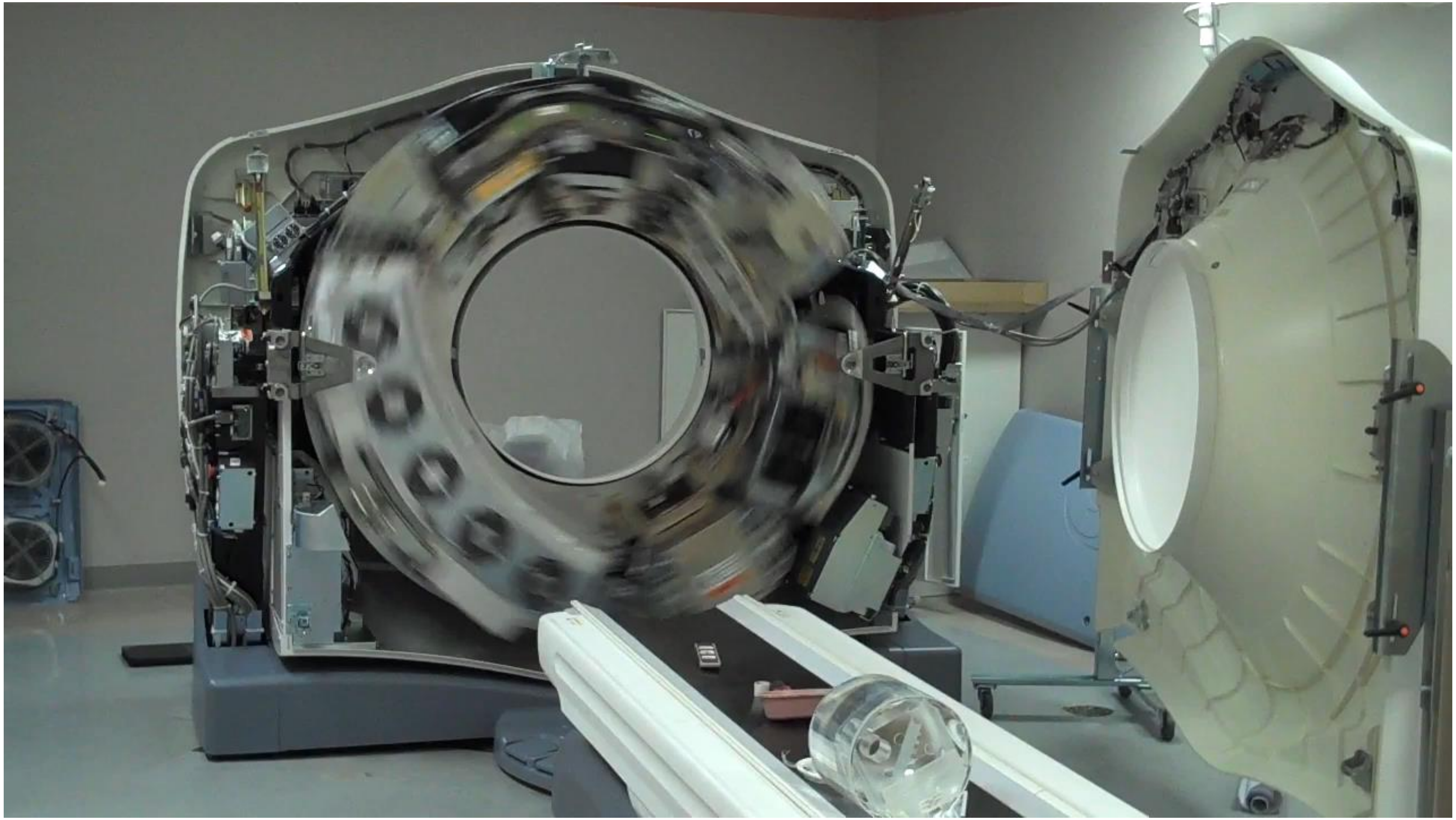
Multi-slice Spiral CT

CT scanner

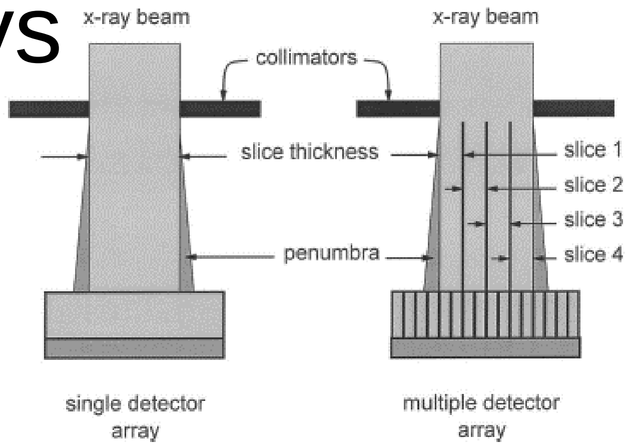


- A modern (2006) CT scanner with the cover removed, demonstrating the principle of operation.
- The X-ray tube and the detectors are mounted on a ring shaped gantry.
- The patient lies in the center of the gantry while the gantry rotates around them.
- This arrangement, a broad fan-shaped X-ray beam with rotating source and detectors, is the 'third-generation' configuration.
- This scanner is capable of helical scanning -the gantry is able to rotate freely while the patient moves continuously through the plane of the scan.
- Power (up to 150 kW) and data (up to 5 Gbps) are transferred to and from the gantry using slip rings (not shown).

CT scanner inside



Multi Slice CT: Multiple Detector Arrays

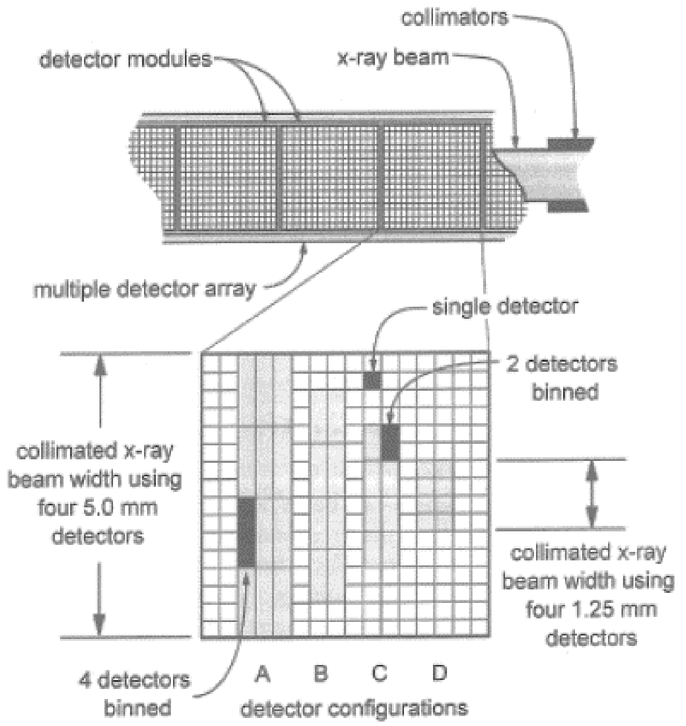


- Acquisition of multiple images per scan
- Electronic post-patient collimation:

  slice thickness -detectors vs. collimator

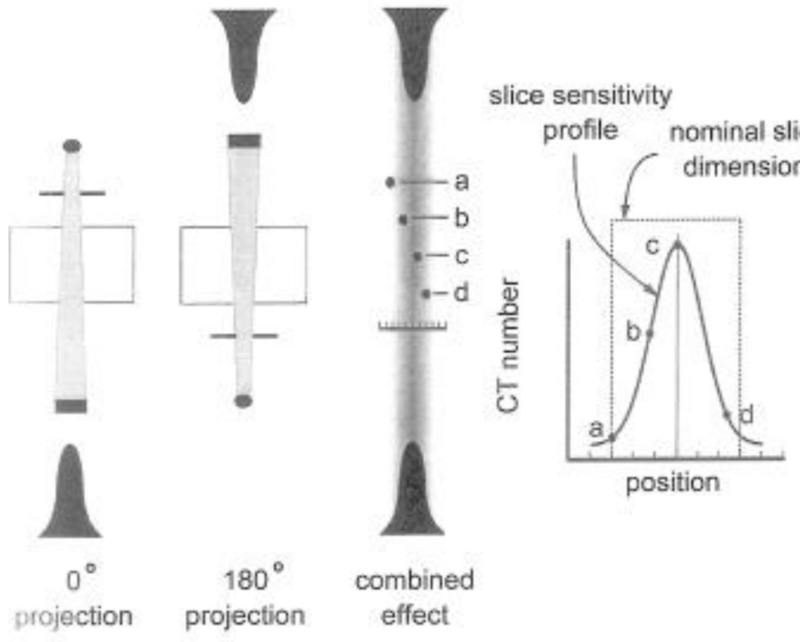
- **Faster** volume acquisition times
- Better bolus tracking and thin slices for 3D
- Multi Slice CT uses 3rd generation platform

Detector	Beam thickness (mm)	Number of rotations	Total scan time (sec)
1 × 1.25	1.3	160	128
4 × 1.25	5	40	32
8 × 1.25	10	20	16
16 × 1.25	20	10	8

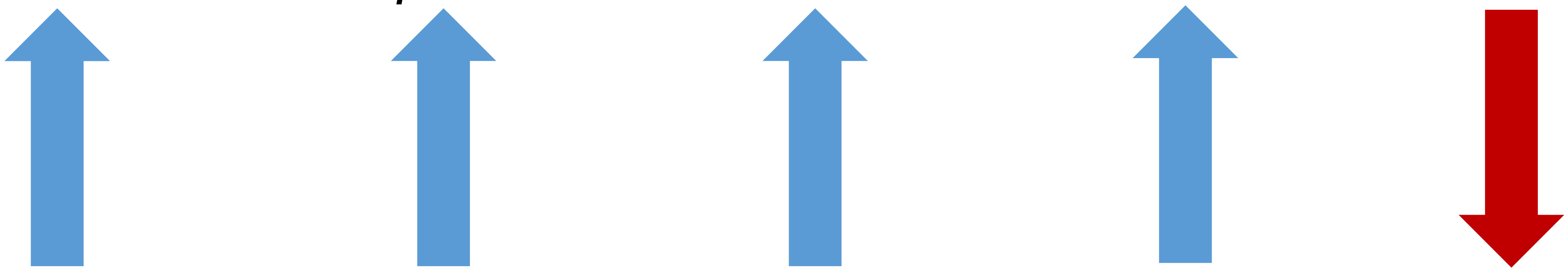


Slice thickness, Detector and Collimator

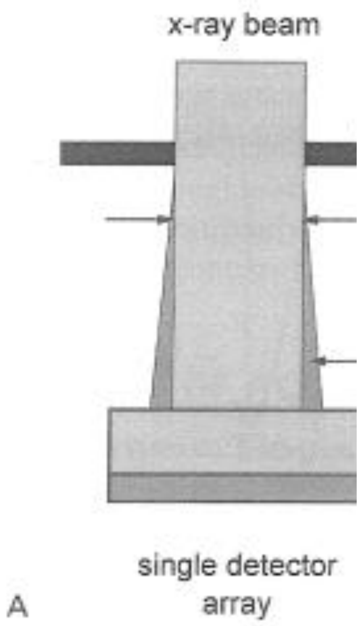
- Design parameters for CT scanner
- Slice thickness ~ number of x-ray photons
- (Number of x-ray photons)$^{1/2}$~ SNR
- SNR ~ CNR
- Slice thickness ~ 1 / Resolution

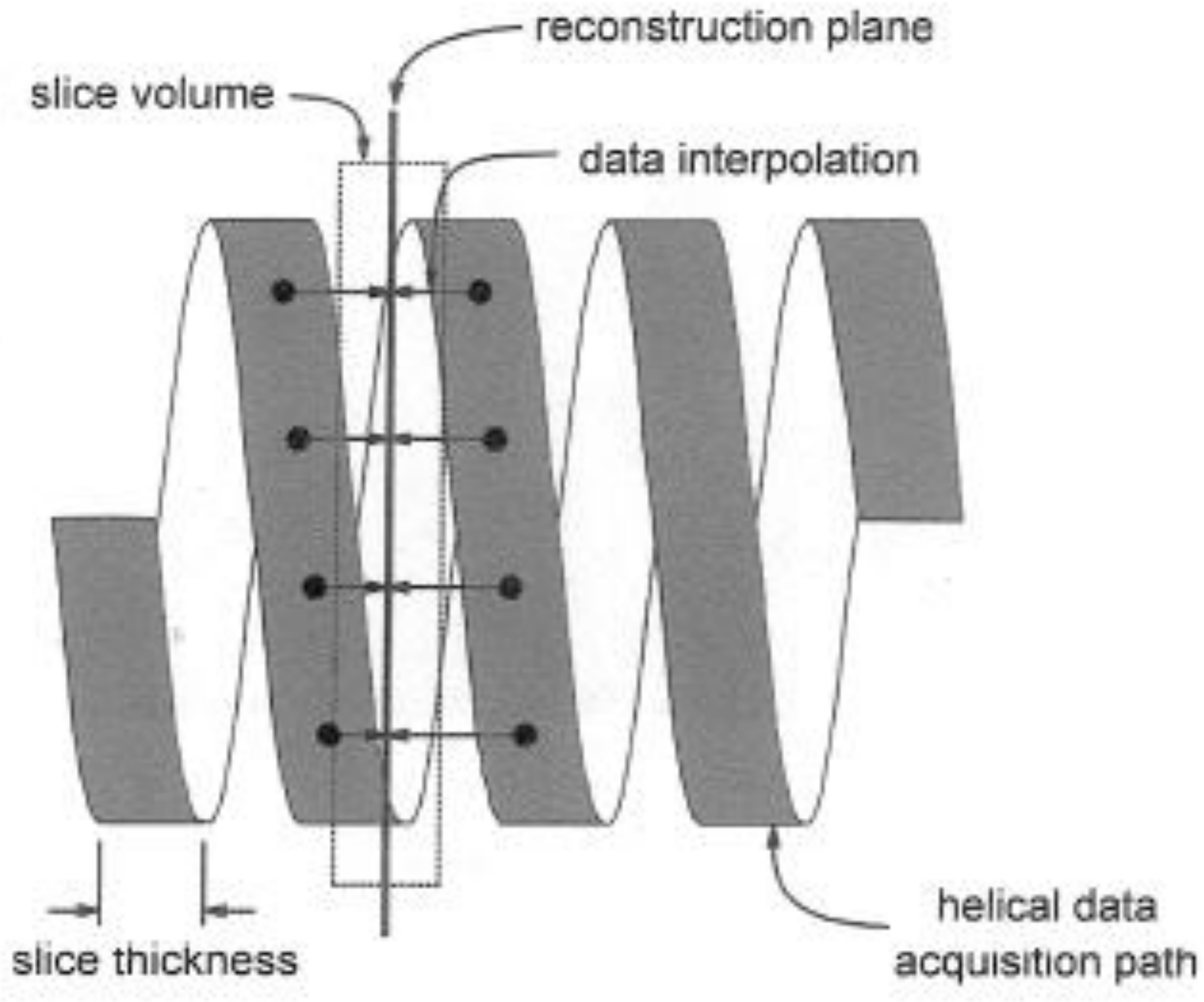


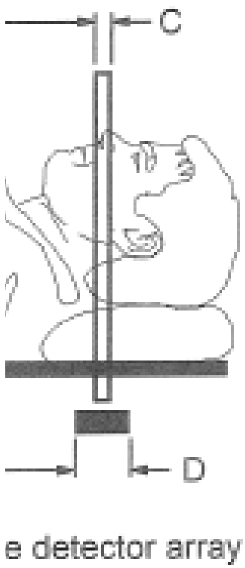
Slice thickness | ***Image acquisition speed*** | ***SNR*** | ***CNR*** | ***Resolution***



Slice thickness, Detector and Collimator







- Image q
- Radiatio
- Scan tim

ator Pitch =

movement) / (collimator width)

Image Reconstruction: Projection of Data

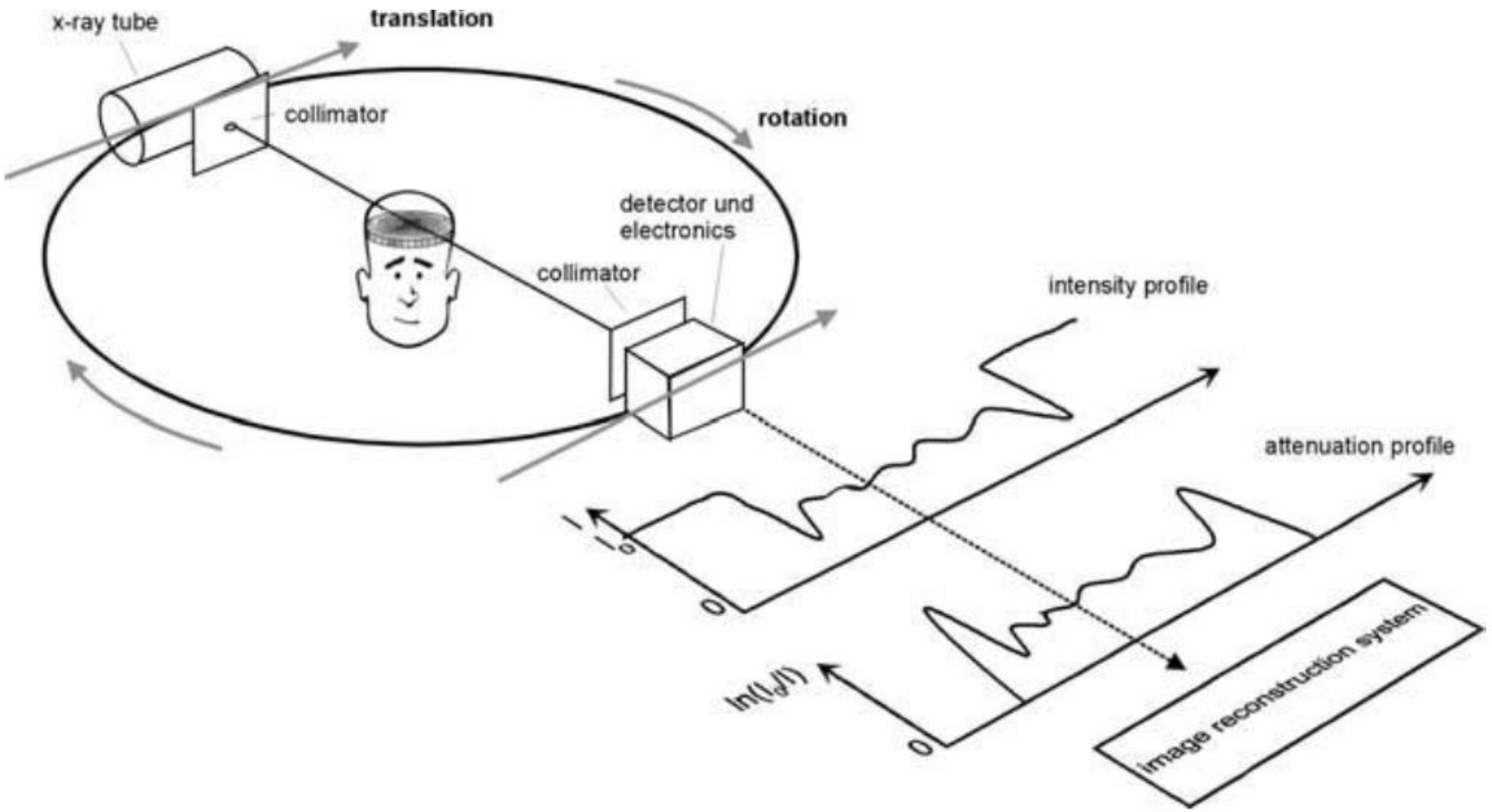


Image Reconstruction: Projection of Data

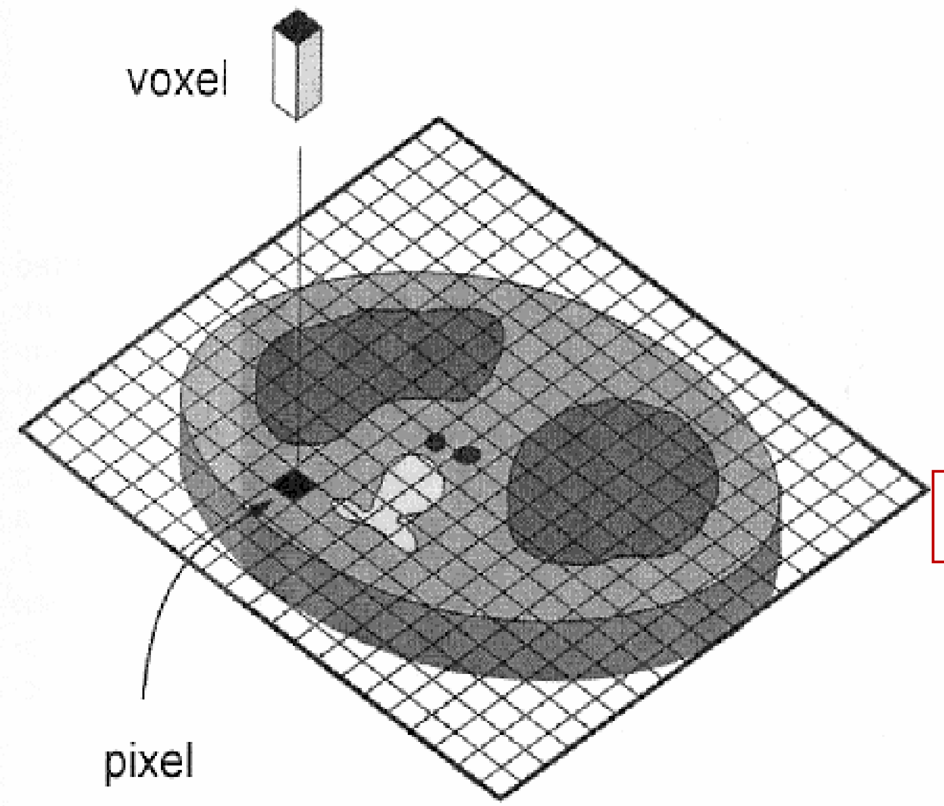
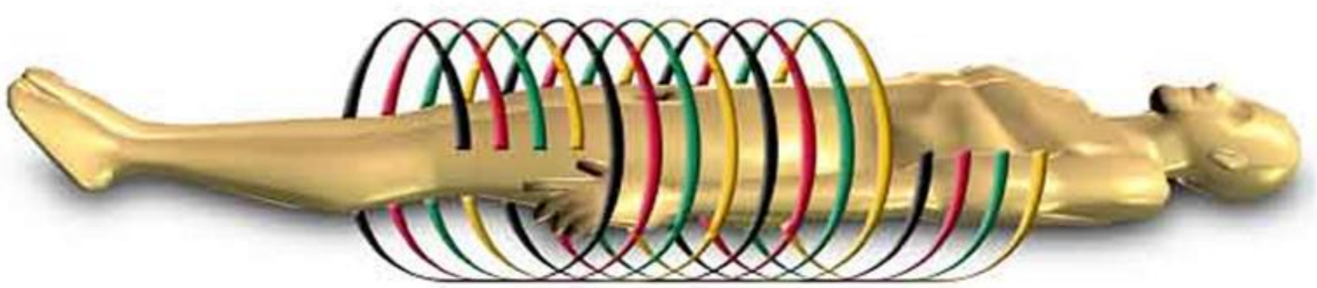


FIGURE 13-2. A pixel (picture element) is the basic two-dimensional element of a digital image. Computed tomographic (CT) images are typically square arrays containing 512 × 512 pixels, each pixel representing 4,096 possible shades of gray (12 bits). Each pixel in the CT image corresponds to a voxel (volume element) in the patient. The voxel has two dimensions equal to the pixel in the plane of the image, and the third dimension represents the slice thickness of the CT scan.

4,096 values
(12bits/voxel)



Tomographic Reconstruction

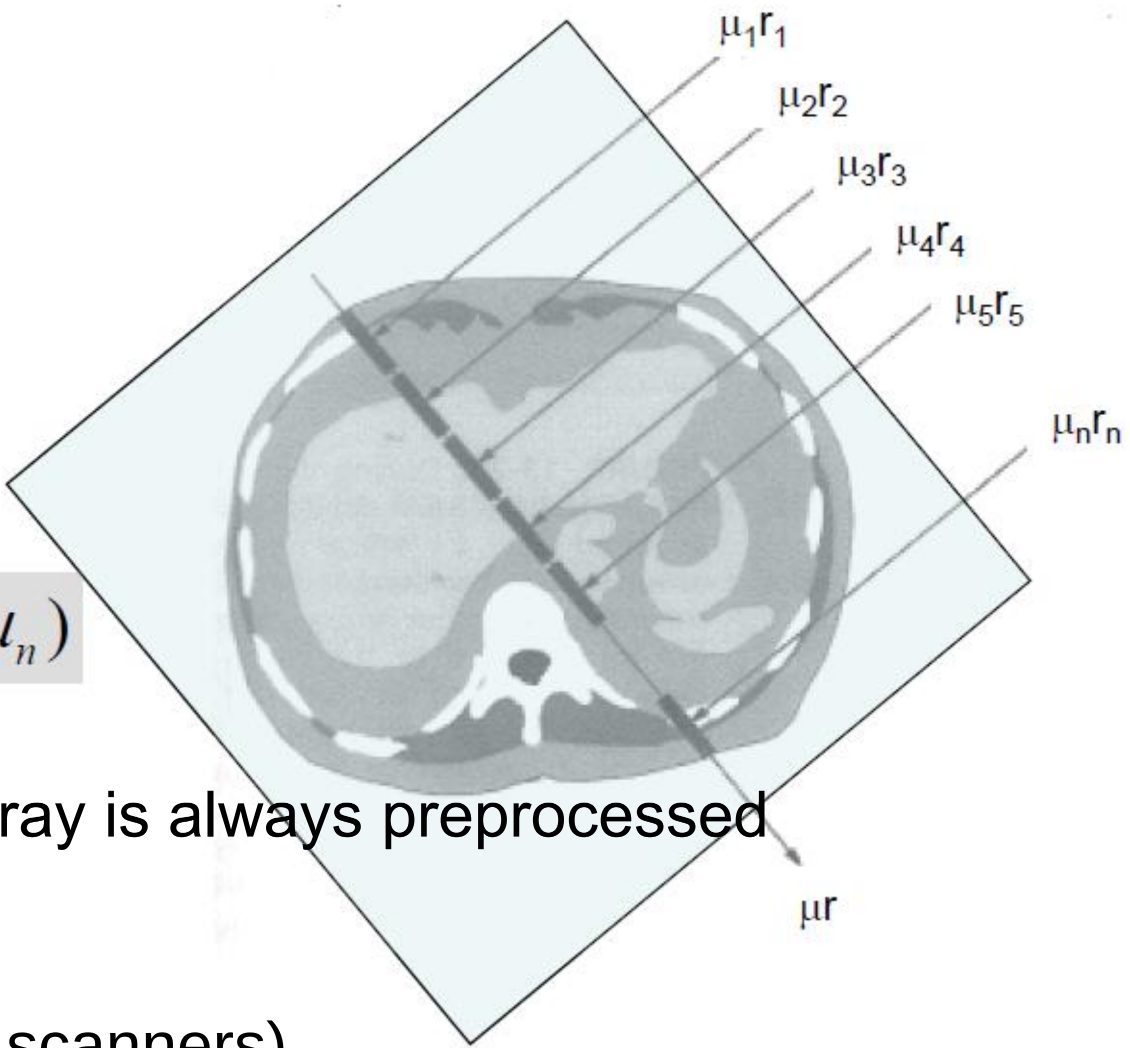
- Rays and Views: the sinogram
- Preprocessing the data
- Iterative reconstruction techniques
- Simple backprojection reconstruction
- Filtered backprojection reconstruction

Step 1: Preprocessing

$$\ln\left(\frac{I_0}{I}\right) = \mu r$$

$$\mu r = \mu_1 r_1 + \mu_2 r_2 + \mu_3 r_3 + \ldots + \mu_n r_n$$

$$\mu r = \Delta r(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \ldots + \mu_n)$$



- The data acquired by the detector array is always preprocessed
- Preprocessing requires calibration

  (compare 3rd and 4th generation of scanners)
- CT number

Reminder: Dirac Delta Function

- The delta function is defined through two integral relations

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(x-x_0,y-y_0)dxdy=1$$

$$\delta(x-x_0,y-y_0)=0\ \ for\ \ x\neq x_0,y\neq y_0 \qquad (1)$$

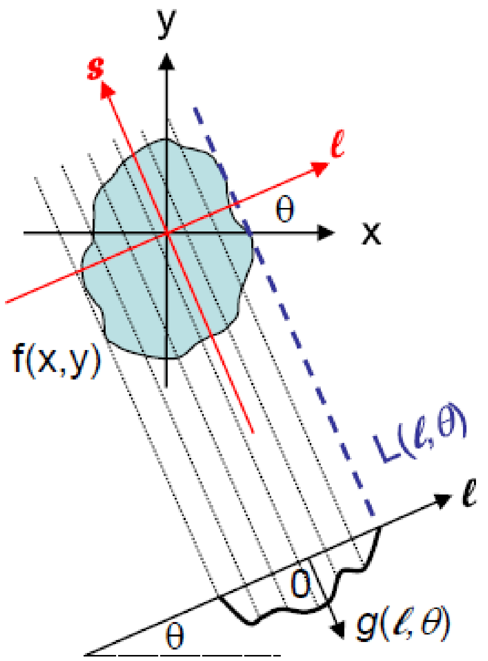
$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(\xi,\eta)\delta(x-\xi,y-\eta)d\xi d\eta \qquad (2)$$

- The second property (sifting property) of the delta function is what makes it so powerful for math and engineering in general and imaging in particular.

Step 1: Preprocessing

$$L(l,\theta) = \{(x,y) \mid x\cos\theta + y\sin\theta = l\}$$

$$L(l,\theta) = l - x\cos\theta - y\sin\theta$$
$$(\text{or } L(l,\theta) = x\cos\theta + y\sin\theta - l)$$



$$g(l,\theta) = \int_{-\infty}^{\infty} f(x(s), y(s))ds$$

$$x(s) = l\cdot\cos\theta - s\cdot\sin\theta$$
$$y(s) = l\cdot\sin\theta + s\cdot\cos\theta$$

$$f(x,y) = \mu(x,y,\overline{E})$$

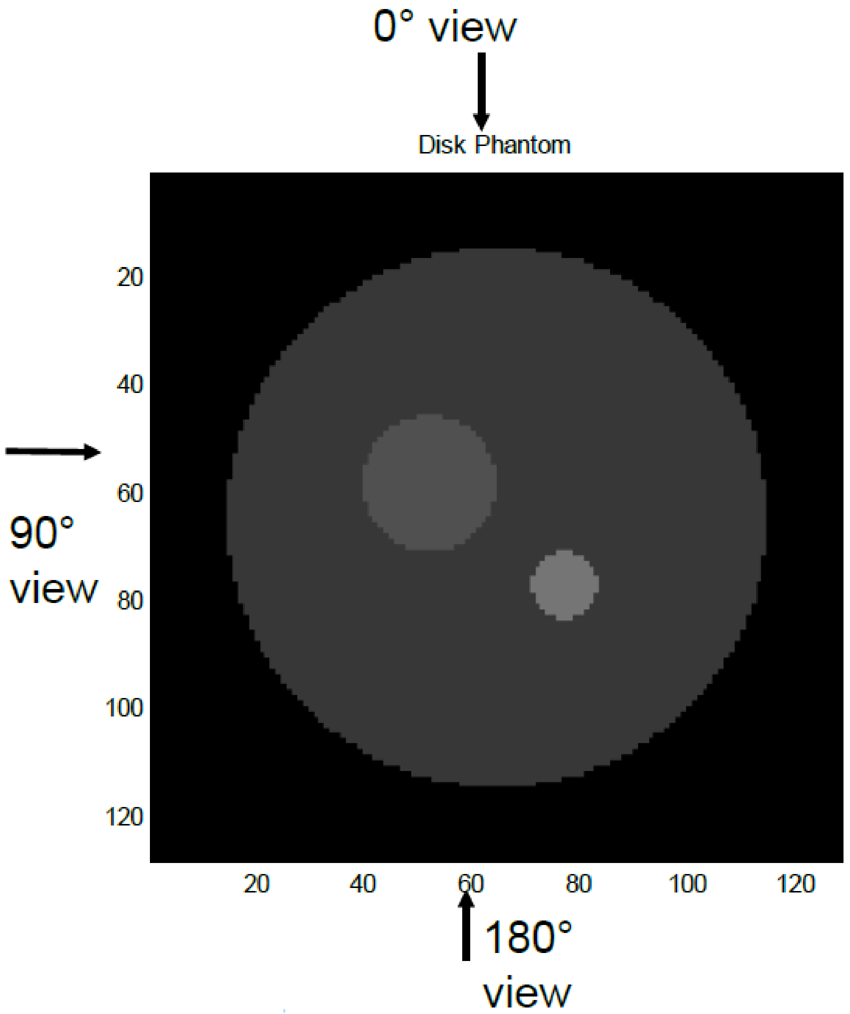
$$g(l,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - l)dxdy$$

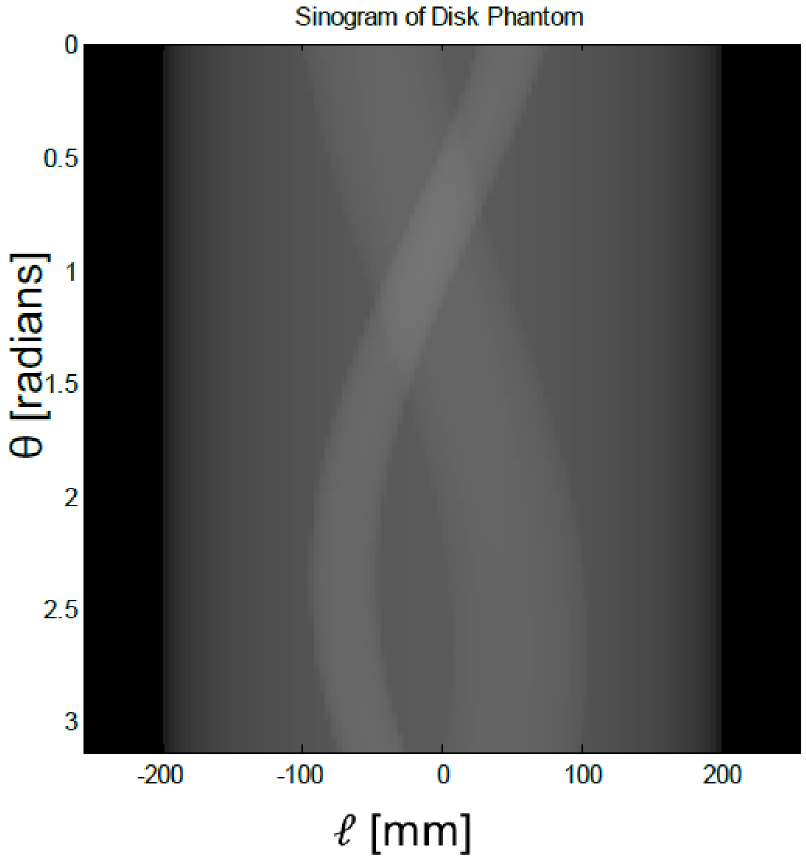
Rotate & Translate

$$g(l,\theta) = -\ln\left(\frac{I_0}{I}\right)$$

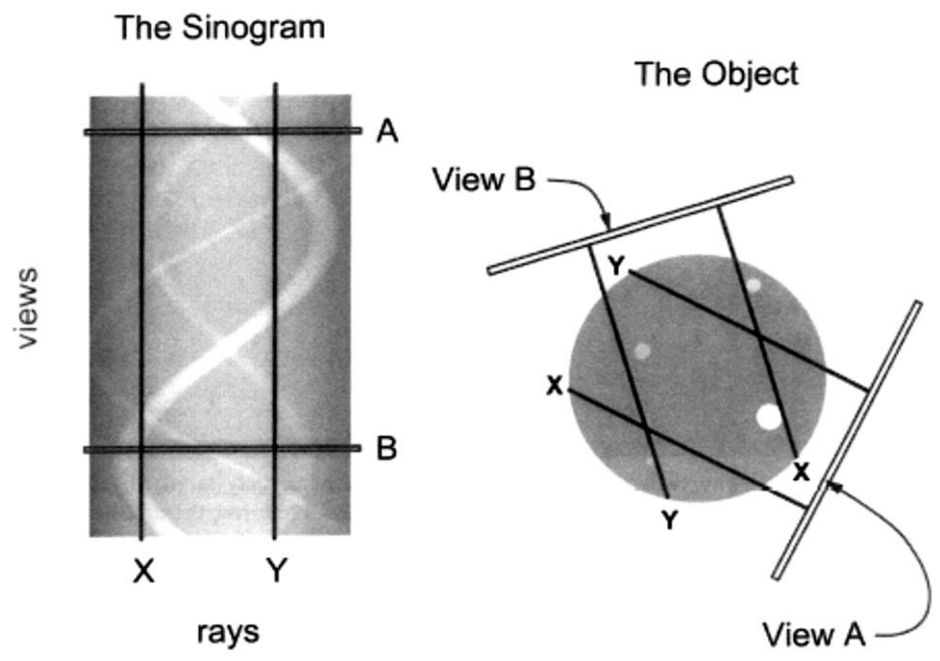
Step 2: Sinogram

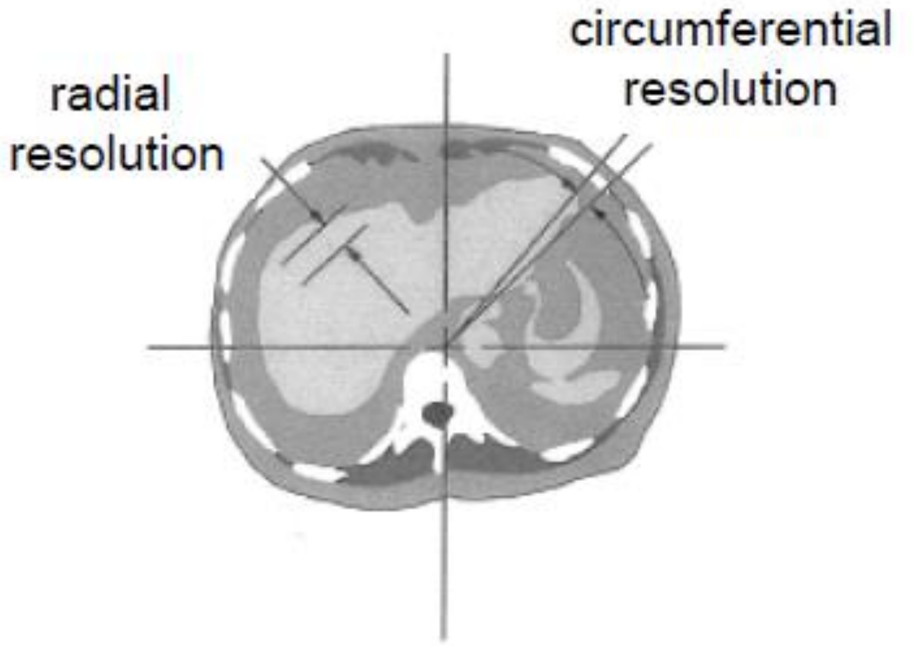
- Raw data displayed before reconstruction.





Step 2: Sinogram





- Data display before reconstruction
- Radial resolution: number of rays
  - "Ray" = 1 transmission measurement made through patient at 1 moment in time
- Circumferential resolution: number of views

Reconstruction Techniques in CT

- Algebraic Reconstruction
- Fourier Reconstruction
- Projection Reconstruction
  - Backprojection
  - Filtered backprojection
  - Convolution + backprojection
- Further flavors
  - Parallel beam geometry
  - Fan beam geometry
  - Cone beam geometry
  - Helical CT reconstruction

Reconstruction Techniques in CT

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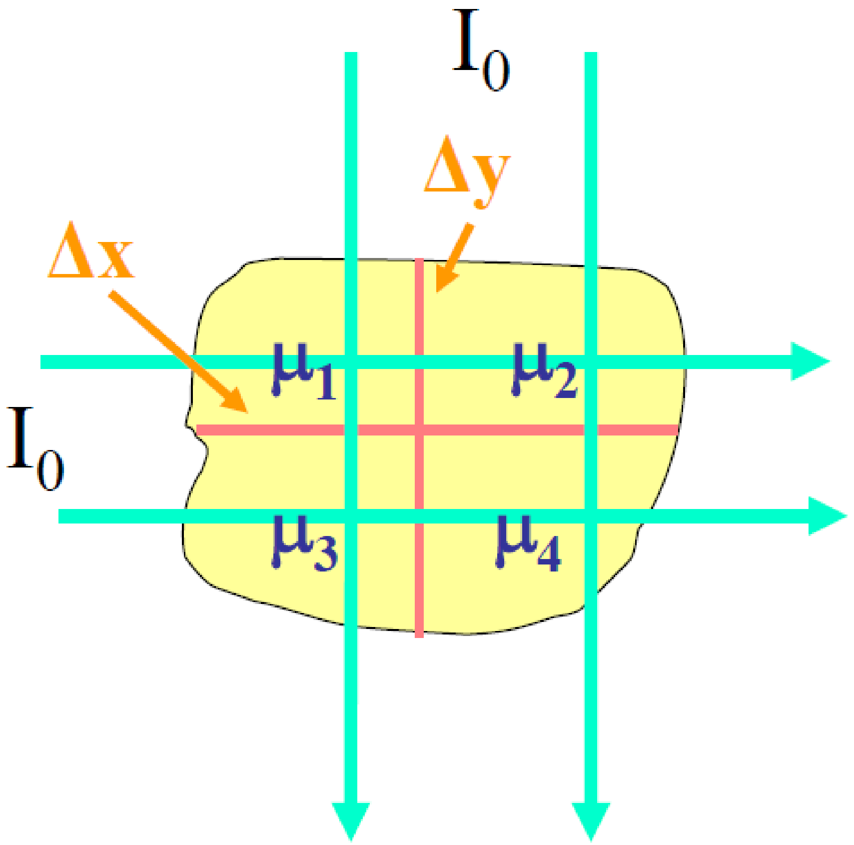
Direct Reconstruction

- Matrix Inversion
- Iterative Procedure
  - Algebraic Reconstruction Technique (ART)
- Best match of reconstruction and measured projections

Matrix Inversion



$$I_1 = I_0 e^{-(\mu_1+\mu_2)\Delta x}$$

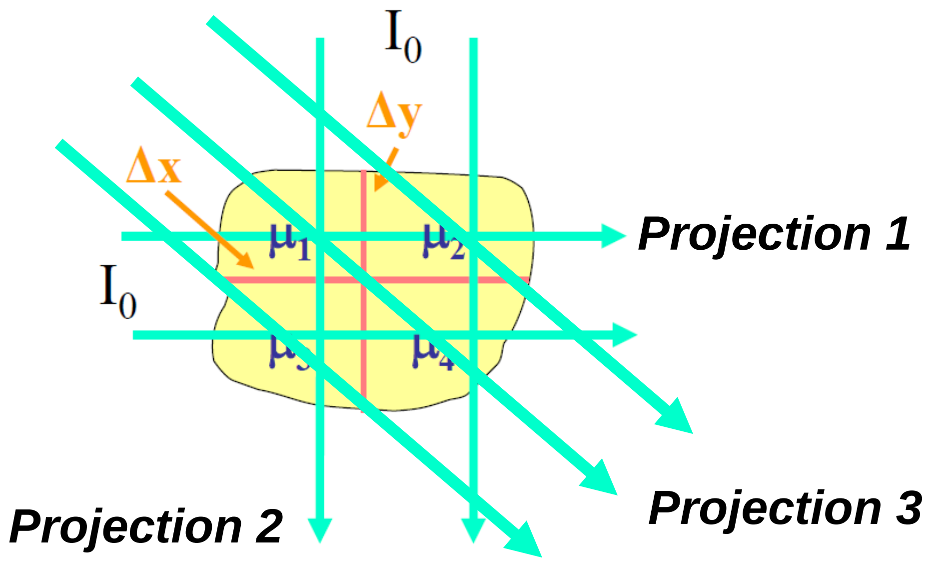
$$I_2 = I_0 e^{-(\mu_3+\mu_4)\Delta x}$$

$$I_3 = I_0 e^{-(\mu_1+\mu_3)\Delta y} \qquad I_4 = I_0 e^{-(\mu_2+\mu_4)\Delta y}$$

4 equations, 4 unknown

Overdetermined system (#'s Equations > #'s variables)

- Each projection contains Q data points, and there are P rotations.
- For each projection, the signal intensity depends upon the composite attenuation coefficient (μ) of the tissue corresponding to the particular beam path.
- For the simple case shown below, a two-by-two matrix is imaged by projection 1 (I_1 and I_2), projection 2 (I_3 and I_4) and projection 3 (I_5, I_6 and I_7).
- The intensities of the projections can be expressed in terms of the linear attenuation coefficients:



$I_1 = I_0\, e^{-(\mu_1+\mu_2)x}$ $I_2 = I_0\, e^{-(\mu_3+\mu_4)x}$	***Projection 1***
$I_3 = I_0\, e^{-(\mu_1+\mu_3)x}$ $I_4 = I_0\, e^{-(\mu_2+\mu_4)x}$	***Projection 2***
$I_5 = I_0\, e^{-(\mu_3 x\sqrt{2})}$ $I_6 = I_0\, e^{-((\mu_1+\mu_4)x\sqrt{2})}$ $I_7 = I_0\, e^{-(\mu_2 x\sqrt{2})}$	***Projection 3***

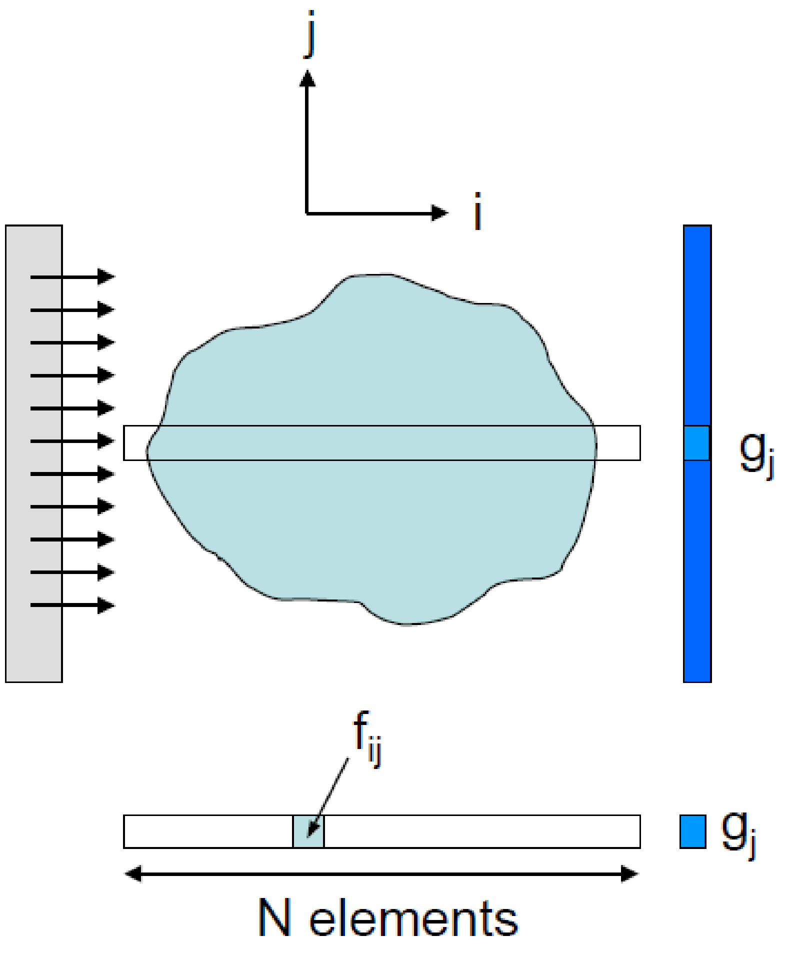
Algebraic Reconstruction Technique (ART)

- Direct matrix inversion is impractical
- Solve linear equations iteratively

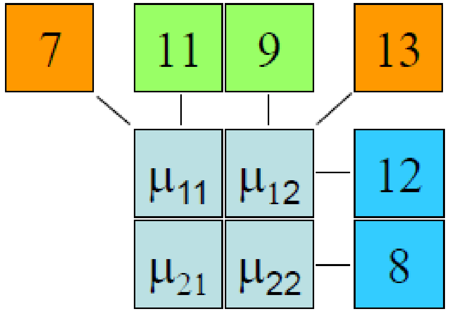
$$g_j - \sum_{i=1}^{N} f_{ij} = 0$$

$$g_j - \sum_{i=1}^{N} f_{ij}^{k} = \frac{1}{\tau}\left(f_{ij}^{k+1} - f_{ij}^{k}\right)$$

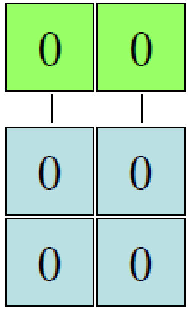
$$f_{ij}^{k+1} = f_{ij}^{k} + \frac{g_j - \sum_{i=1}^{N} f_{ij}^{k}}{N}, \quad \tau = \frac{1}{N}$$



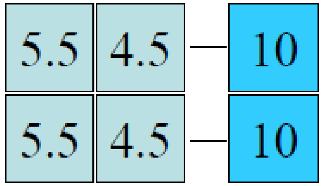
Example



$$f_{ij}^{k+1} = f_{ij}^{k} + \frac{g_j - \sum_{i=1}^{N} f_{ij}^{k}}{N}$$



Initial guess



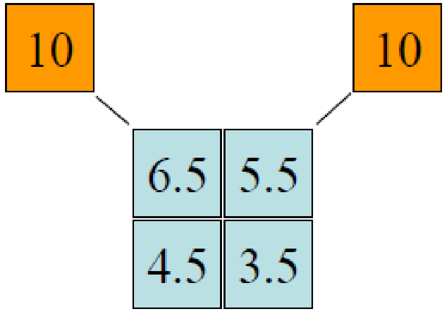
Vertical rays

$$f_{11}^{1} = 0 + \frac{11-0}{2} = 5.5$$

$$f_{21}^{1} = 0 + \frac{11-0}{2} = 5.5$$

$$f_{12}^{1} = 0 + \frac{9-0}{2} = 4.5$$

$$f_{22}^{1} = 0 + \frac{9-0}{2} = 4.5$$



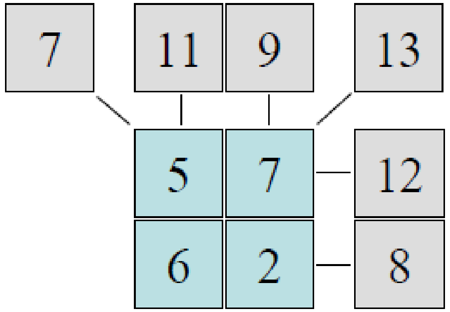
Horizontal rays

$$f_{11}^{2} = 5.5 + \frac{12-10}{2} = 6.5$$

$$f_{21}^{2} = 5.5 + \frac{8-10}{2} = 4.5$$

$$f_{12}^{2} = 4.5 + \frac{12-10}{2} = 5.5$$

$$f_{22}^{2} = 4.5 + \frac{8-10}{2} = 3.5$$



Diagonal rays

$$f_{11}^{3} = 6.5 + \frac{7-10}{2} = 5$$

$$f_{21}^{3} = 4.5 + \frac{13-10}{2} = 6$$

$$f_{12}^{3} = 5.5 + \frac{13-10}{2} = 7$$

$$f_{22}^{3} = 3.5 + \frac{7-10}{2} = 2$$

Reconstruction Techniques in CT

- Algebraic Reconstruction
- Fourier Reconstruction
- Projection Reconstruction
  - Backprojection
  - Filtered backprojection
  - Convolution + backprojection
- Further flavors
  - Parallel beam geometry
  - Fan beam geometry
  - Cone beam geometry
  - Helical CT reconstruction

Reminder: 2-D Fourier Reconstruction

- The continuous 2-D Fourier transform $G(u,v)$ of a function $g(x,y)$ is defined as

$$G(u,v) = F[g(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-i2\pi(ux+vy)}dxdy$$

where u and v are often referred to as spatial frequencies by analogy to 1-D transform

- Physically, each component $G(u,v)$ can be considered as component of the function $g(x,y)$ resulting from a plane wave of wave vector k, and propagating along direction θ, where

$$k = 2\pi\sqrt{u^2 + v^2} \qquad \theta = \tan^{-1}(v/u)$$

- The inverse transform is similarly defined as

$$g(x,y) = F^{-1}[G(u,v)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(u,v)e^{i2\pi(ux+vy)}dudv$$

Reminder: Fourier Transform Relations

- An impulse in one domain results in a constant level at all coordinates of the transform domain (**delta function response**)

$$F[\delta(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x,y) e^{-i2\pi(ux+vy)} dxdy = 1$$

- The Fourier Transform operation is linear (**linearity**)

$$F[\alpha \cdot g(x,y) + \beta \cdot h(x,y)] = \alpha \cdot F[g(x,y)] + \beta \cdot F[h(x,y)]$$

- Translation of a function in one space introduces a linear phase shift in the transform domain (**shift**)

$$F[g(x-\alpha, y-\beta)] = G(u,v)e^{-i2\pi(\alpha u+\beta v)}$$

- If g(x,y) = g_x(x)g_y(y) (**separability**), then

$$F[g(x,y)] = F\left[g_x(x)g_y(y)\right] = F_x\left[g_x(x)\right]F_y\left[g_y(y)\right]$$

Projection-Slice Theorem

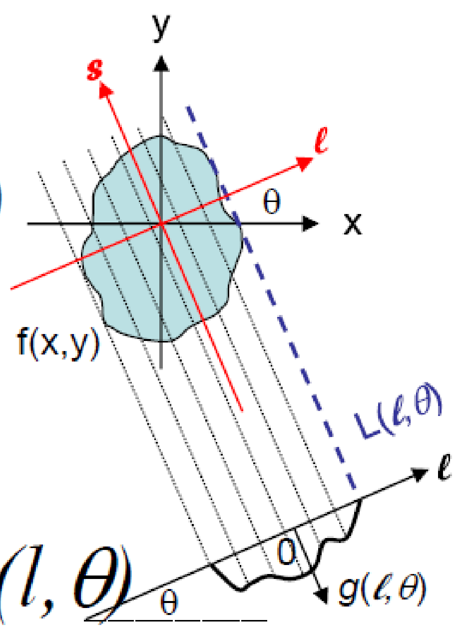
Ref. astra toolbox

Projection-Slice Theorem

- Let start with the projection $g(l,\theta)$

$$g(l,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta+y\sin\theta-l)dxdy$$

- Now let's consider a 1-D FT of $g(l,\theta)$



$$\begin{aligned}
G(\rho,\theta) &= \int g(l,\theta)e^{-i2\pi\rho l}dl = \\
&= \int\left[\iint f(x,y)\delta(x\cos\theta+y\sin\theta-l)dxdy\right]e^{-i2\pi\rho l}dl = \\
&= \iint f(x,y)\int e^{-i2\pi\rho l}\delta(x\cos\theta+y\sin\theta-l)dldxdy = \\
&= \iint f(x,y)e^{-i2\pi\rho(x\cos\theta+y\sin\theta)}dxdy
\end{aligned}$$

Projection-Slice Theorem

- From the previous slide

$$
\begin{aligned}
G(\rho,\theta) &= \int g(l,\theta) e^{-i2\pi\rho l} dl = \\
&= \int \left[\iint f(x,y)\delta(x\cos\theta + y\sin\theta - l) dxdy \right] e^{-i2\pi\rho l} dl = \\
&= \iint f(x,y) \int e^{-i2\pi\rho l} \delta(x\cos\theta + y\sin\theta - l) dxdydl = \\
&= \iint f(x,y) e^{-i2\pi\rho(x\cos\theta + y\sin\theta)} dxdy
\end{aligned}
$$

- Which is 2-D Fourier transform

$$F(u,v) = \iint f(x,y) e^{-i2\pi(ux+vy)} dxdy$$

- In polar coordinates $u = \rho\cos\theta \quad \text{and} \quad v = \rho\sin\theta$

Projection-Slice Theorem

- Therefore, the desired function *f(x,y)* can be reconstructed from *G(ρ,θ)*

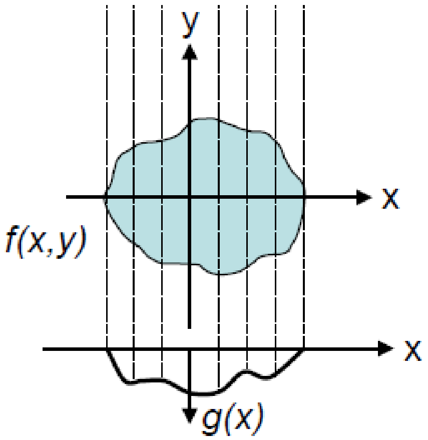
$$f(x,y)=\int\int F(u,v)e^{i2\pi(ux+vy)}dudv=$$
$$=\int_0^{2\pi}d\theta\int_0^{\infty}G(\rho,\theta)e^{i2\pi\rho(x\cos\theta+y\sin\theta)}\rho d\rho$$

- where

$$dudv=\rho d\rho d\theta$$

$$\begin{vmatrix}\cos\theta & -\rho\sin\theta\\ \sin\theta & \rho\cos\theta\end{vmatrix}=\rho(\cos^2\theta+\sin^2\theta)=\rho$$

Really, what does it all mean…

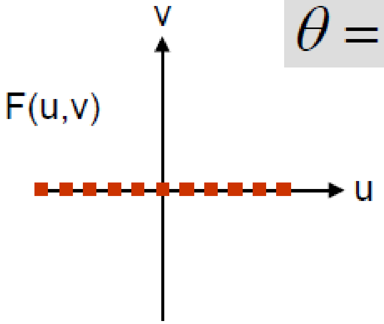


- Single projection of *f(x,y)*

$$g(x) = \int f(x,y)\,dy$$

- 2-D Fourier transform

$$F(u,v) = \int\int f(x,y)e^{-i2\pi(ux+vy)}\,dxdy$$



$$\theta = 0$$

- Along v=0 line

$$\begin{aligned} F(u,v=0) &= \int\int f(x,y)e^{-i2\pi(ux+0y)}\,dxdy = \\ &= \int\left[\int f(x,y)\,dy\right]e^{-i2\pi ux}\,dx = \int g(x)e^{-i2\pi ux}\,dx = \\ &= F_{1-D}\left[g(x)\right] \end{aligned}$$

Really, what does it all mean…

- Single projection of *f(x,y)*

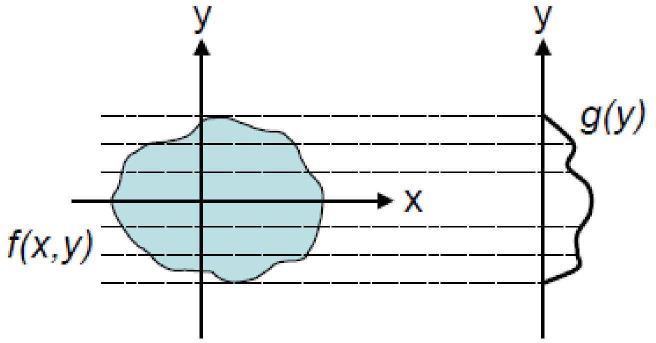
$$g(y) = \int f(x,y)dx$$

- 2-D Fourier transform

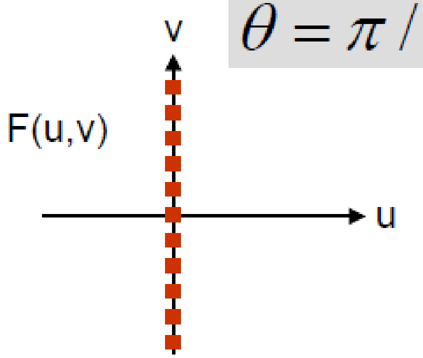
$$F(u,v) = \int\int f(x,y)e^{-i2\pi(ux+vy)}dxdy$$

- Along u=0 line

$$F(0,v) = \int\int f(x,y)e^{-i2\pi(0x+vy)}dxdy =$$
$$= \int\left[\int f(x,y)dx\right]e^{-i2\pi vy}dy = \int g(y)e^{-i2\pi vy}dy =$$
$$= F_{1-D}[g(y)]$$



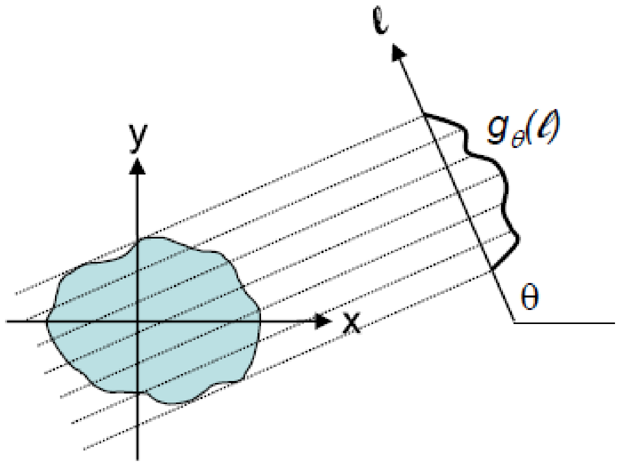
$$\theta = \pi/2 = 90^0$$



Central Selection Theorem

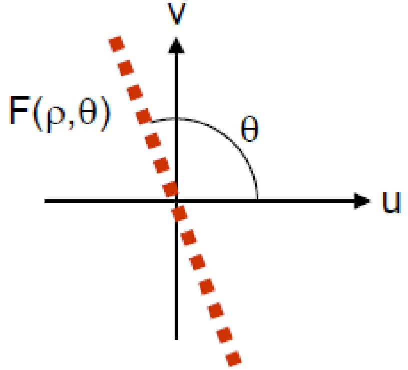
- It can be shown that

$$F(\rho,\theta) = F_{1-D}\left[g_\theta(l)\right]$$

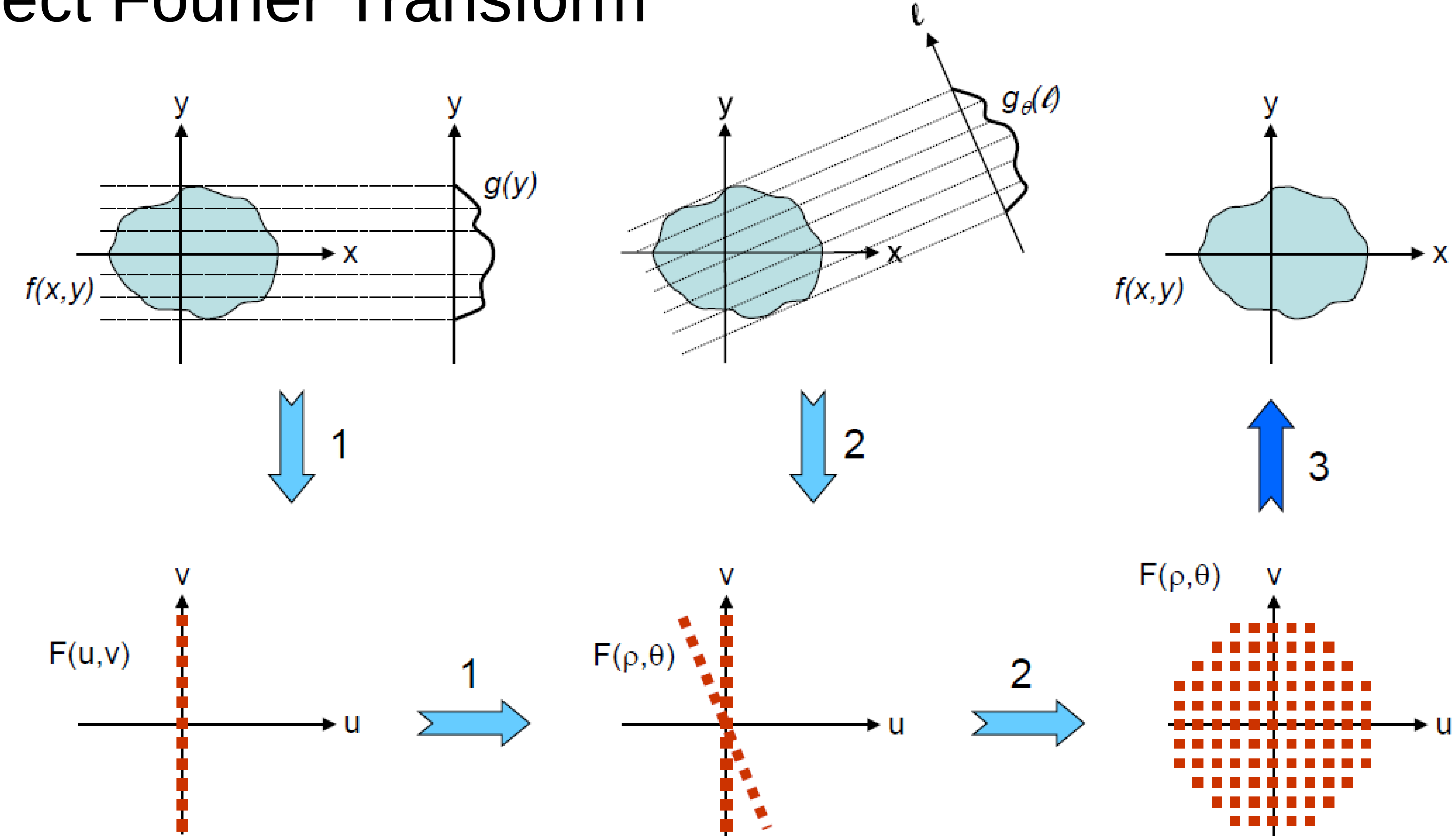


- 2-D Inverse Fourier transform

$$f(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} F(\rho,\theta) e^{i2\pi\rho(x\cos\theta + y\sin\theta)} \rho d\rho$$

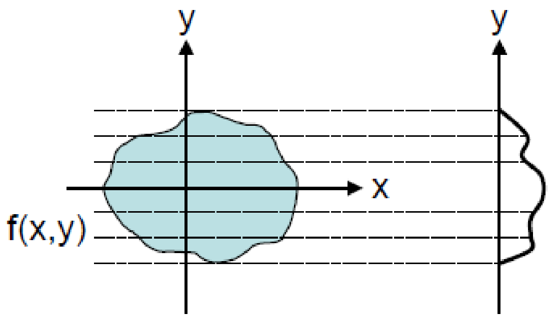


Direct Fourier Transform

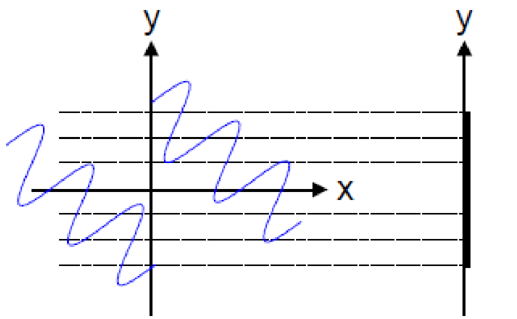


Physical Insight into Central Selection Theorem

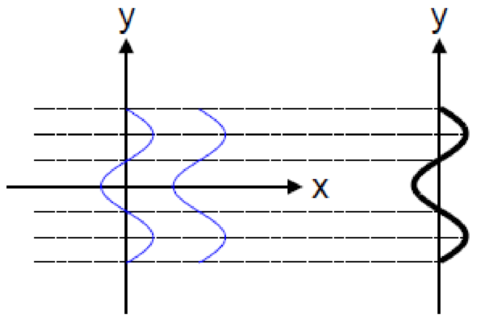
- Single projection of *f(x,y)*



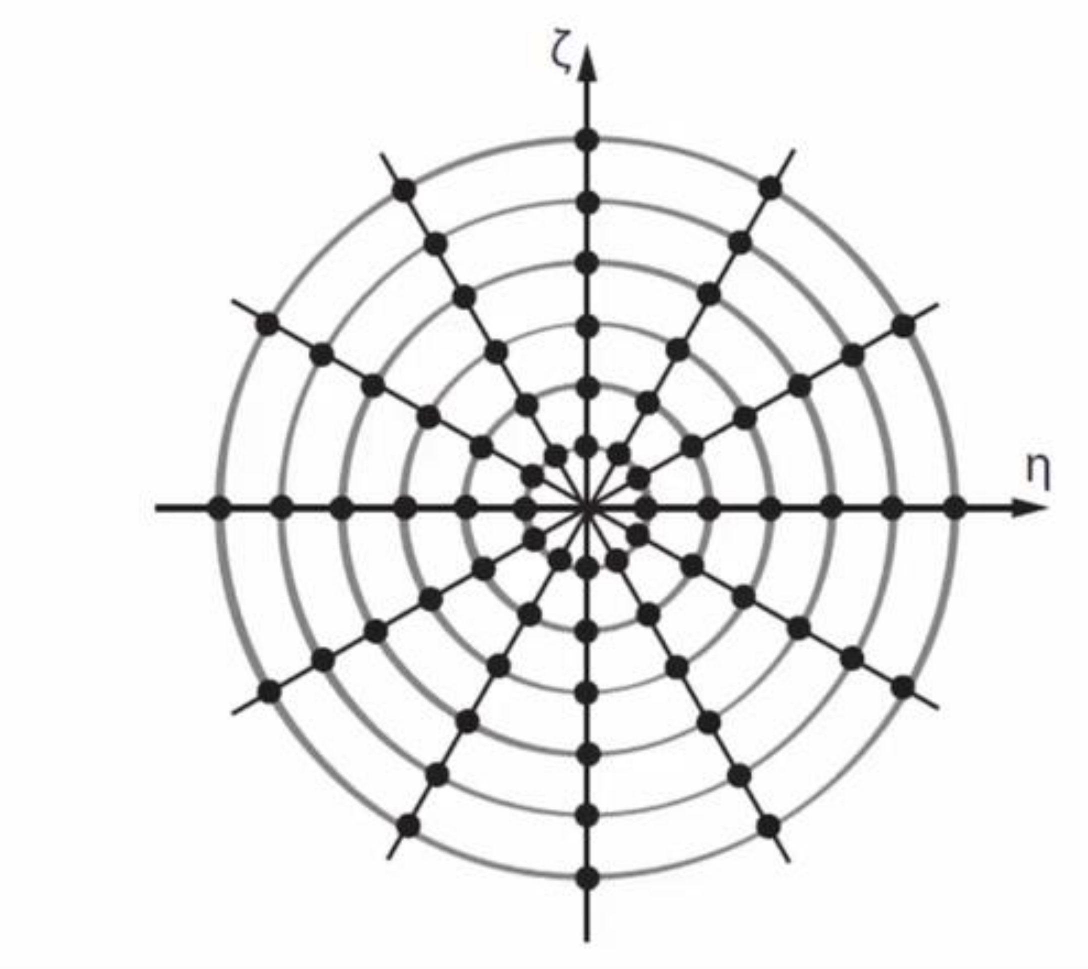
- 2-D Fourier transform
  f(x,y) is decomposed into array of 2-D sinusoids



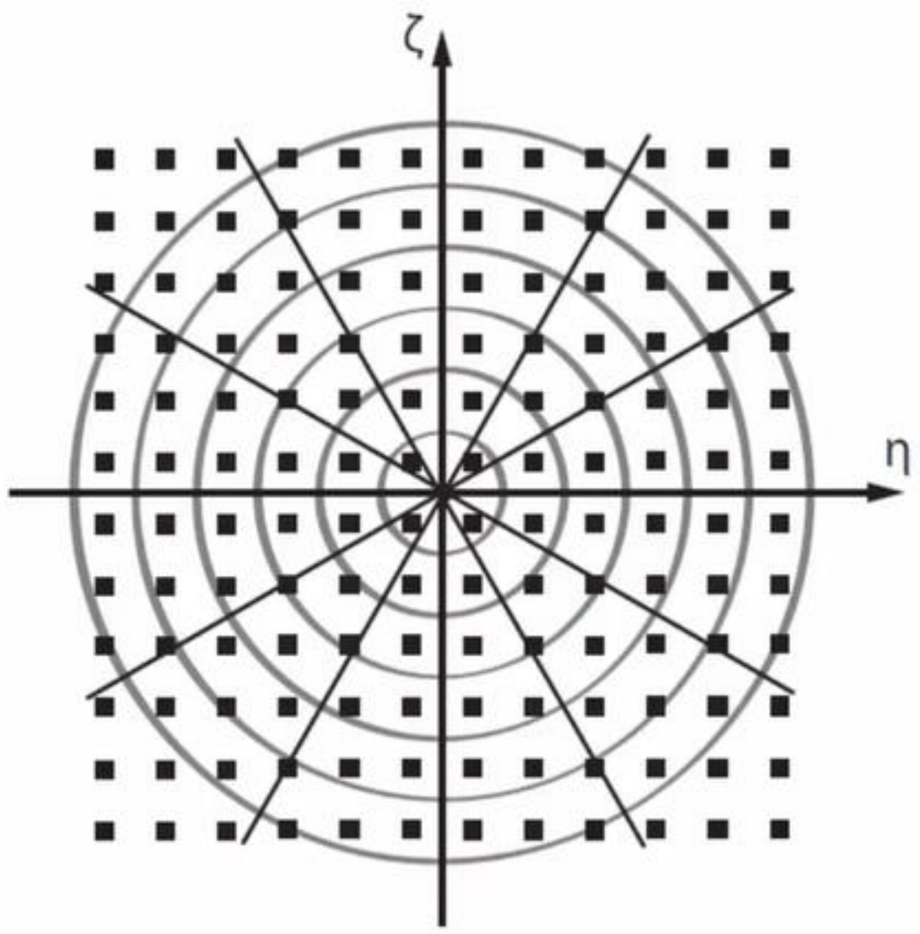
- Projection of sinusoidal signals



Different sampling approaches

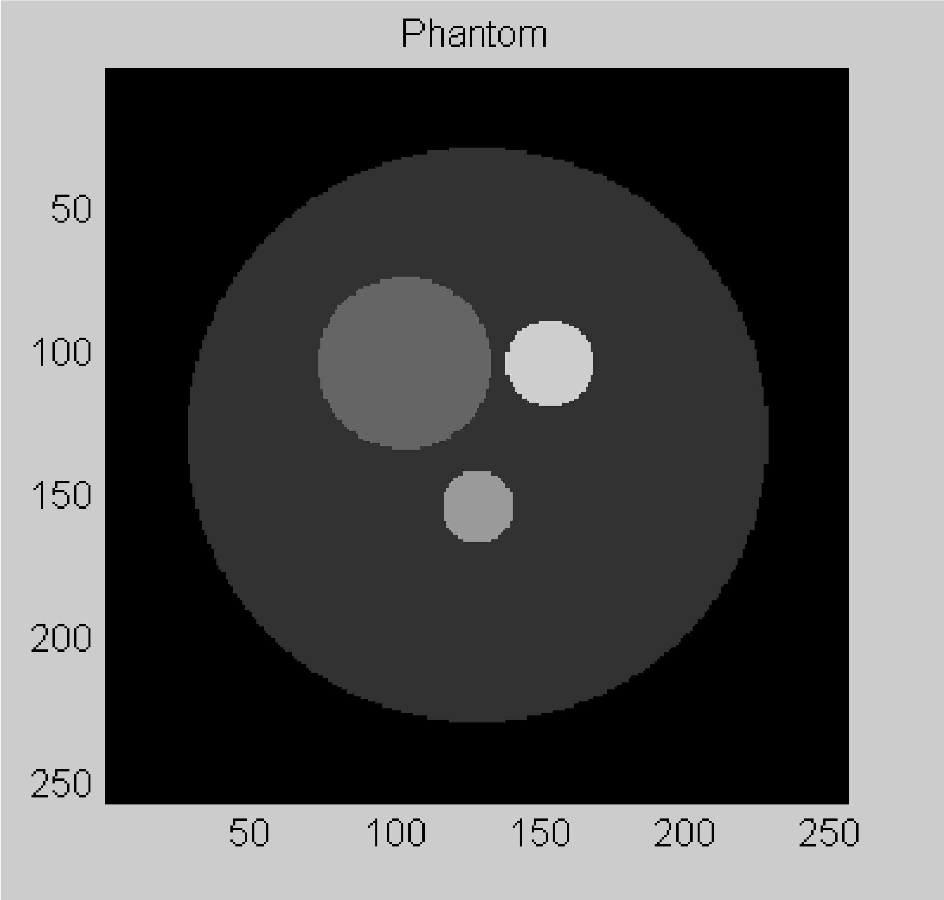


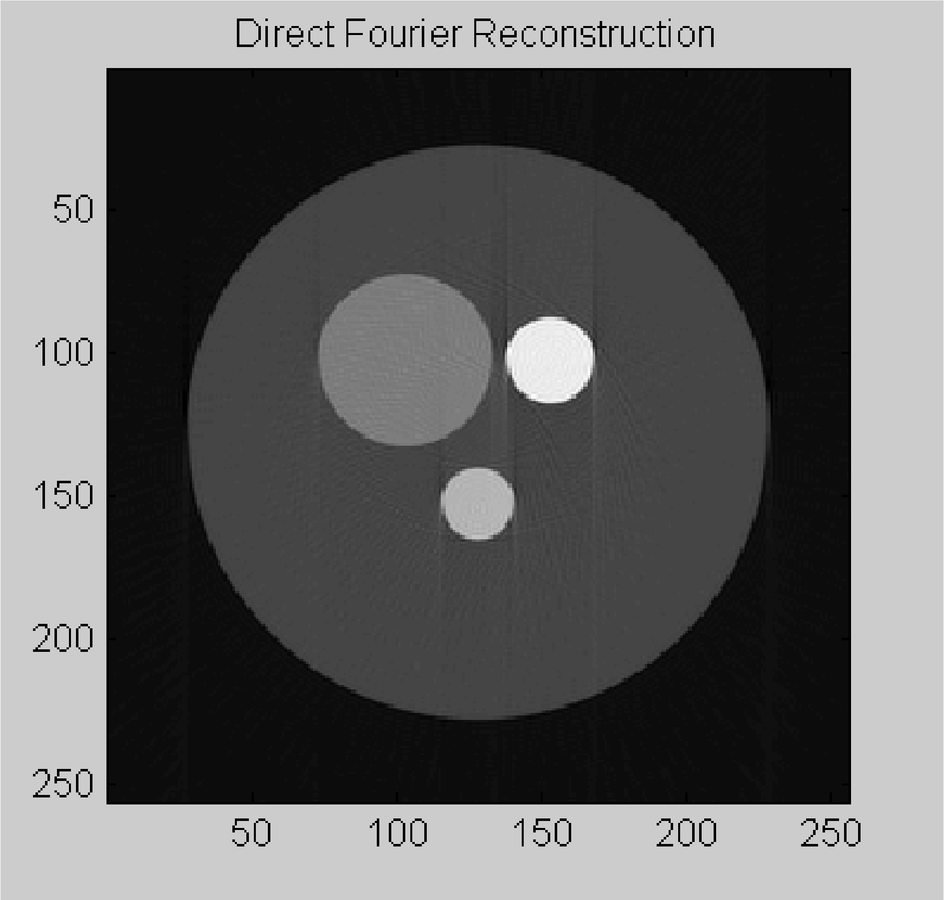
Fourier sampling with
Central selection (Fourier slice) theorem



Sampling required by
Discrete fourier transform

Fourier Reconstruction: Example





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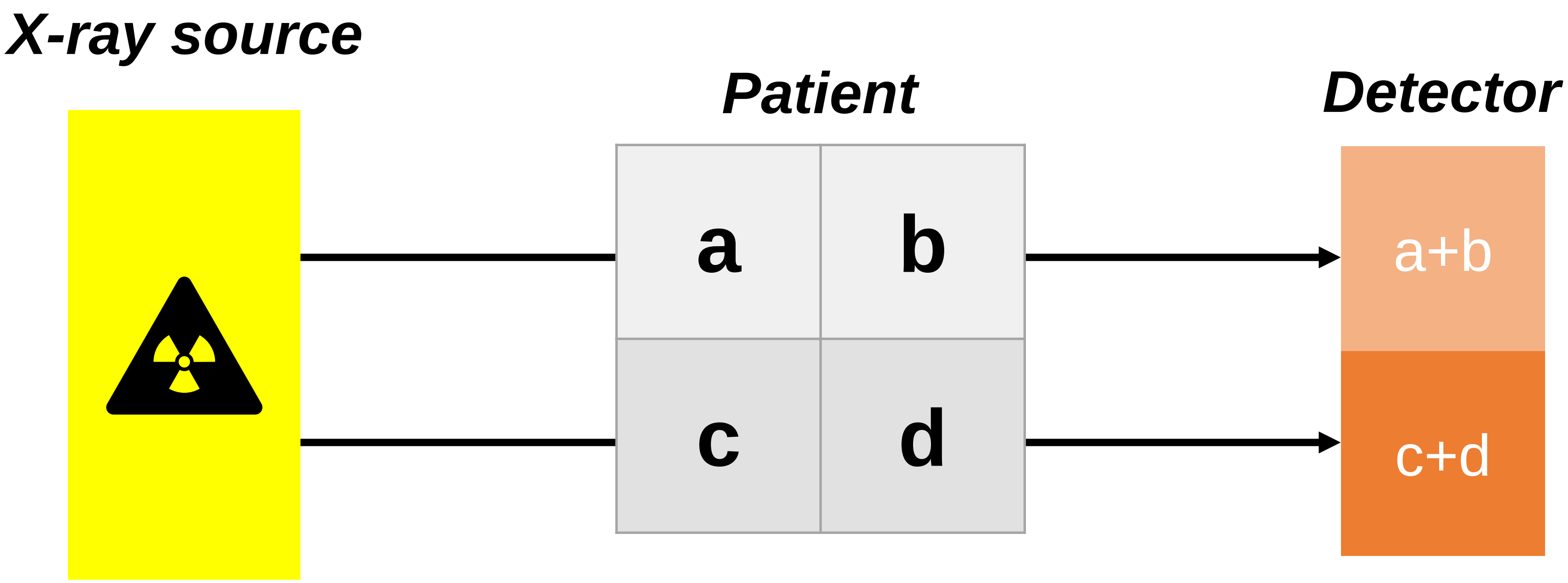
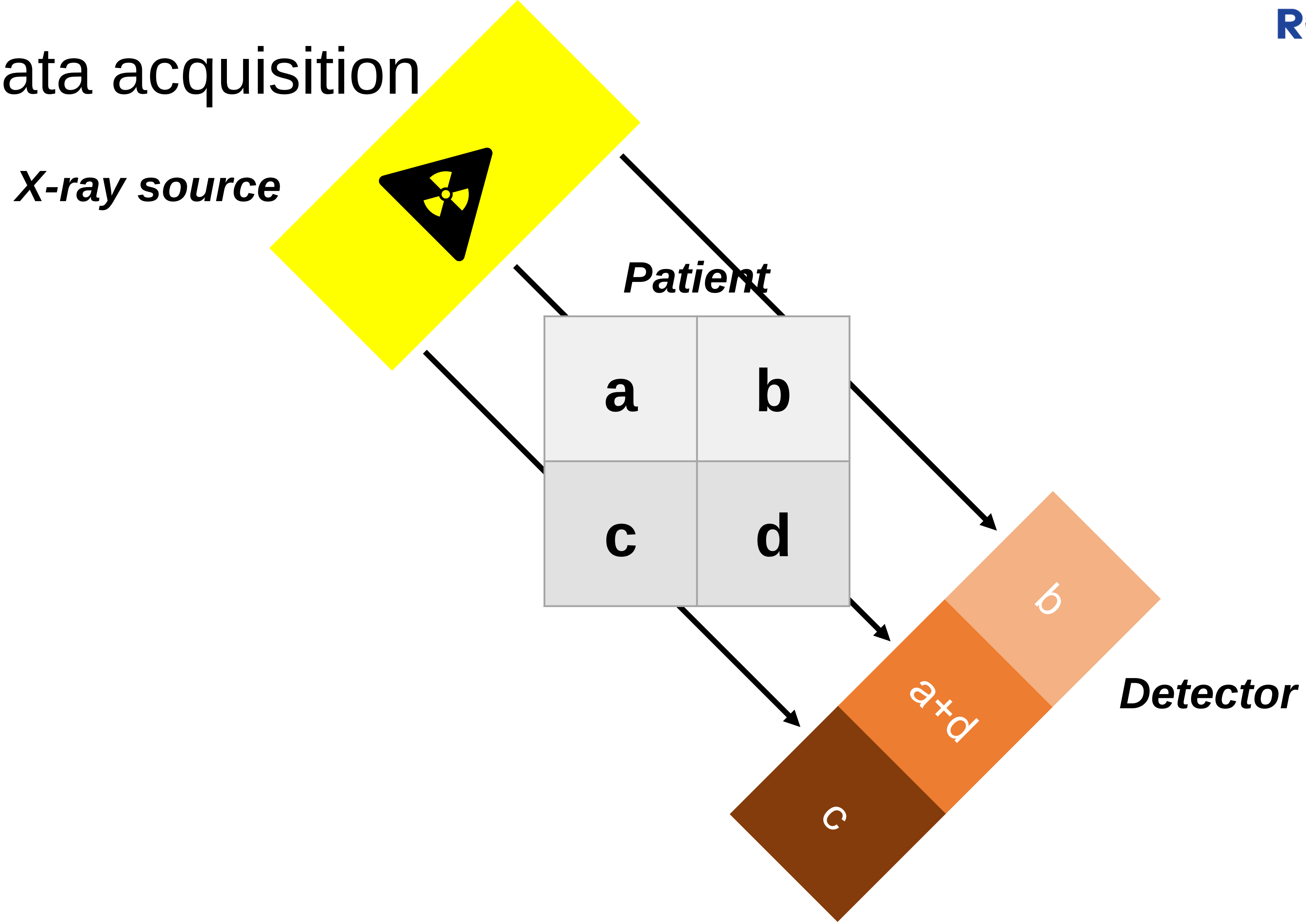
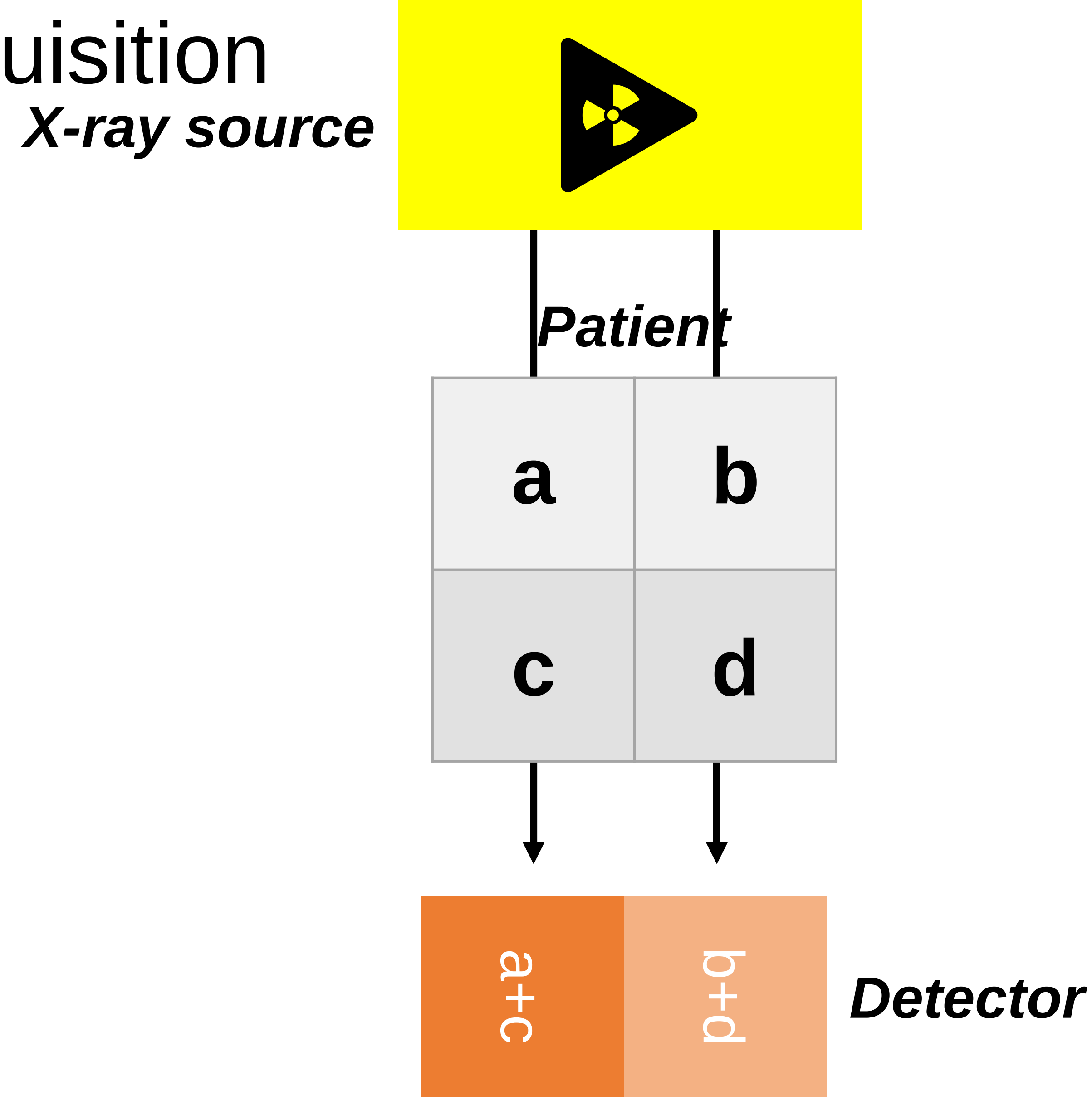
CT data acquisition



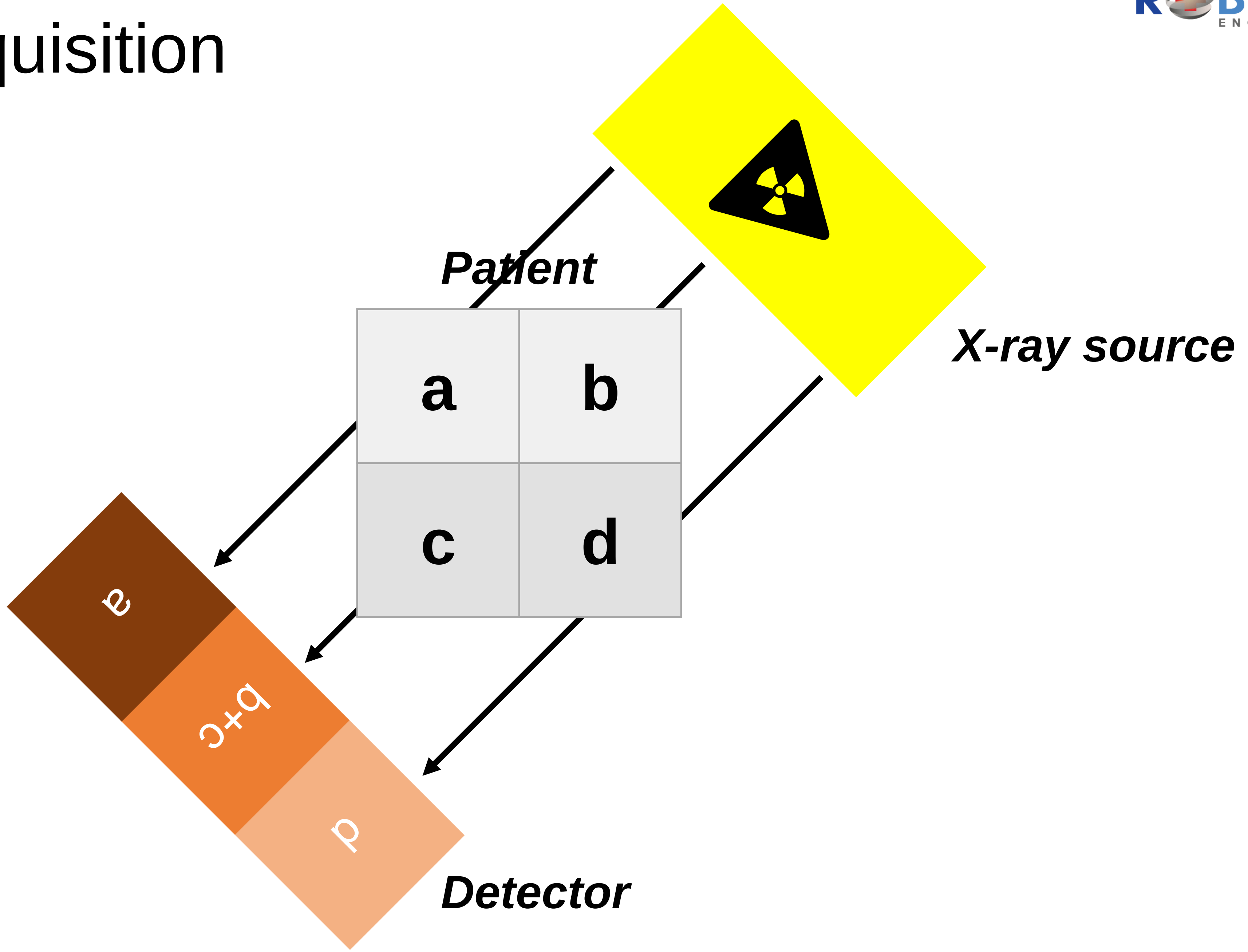
CT data acquisition



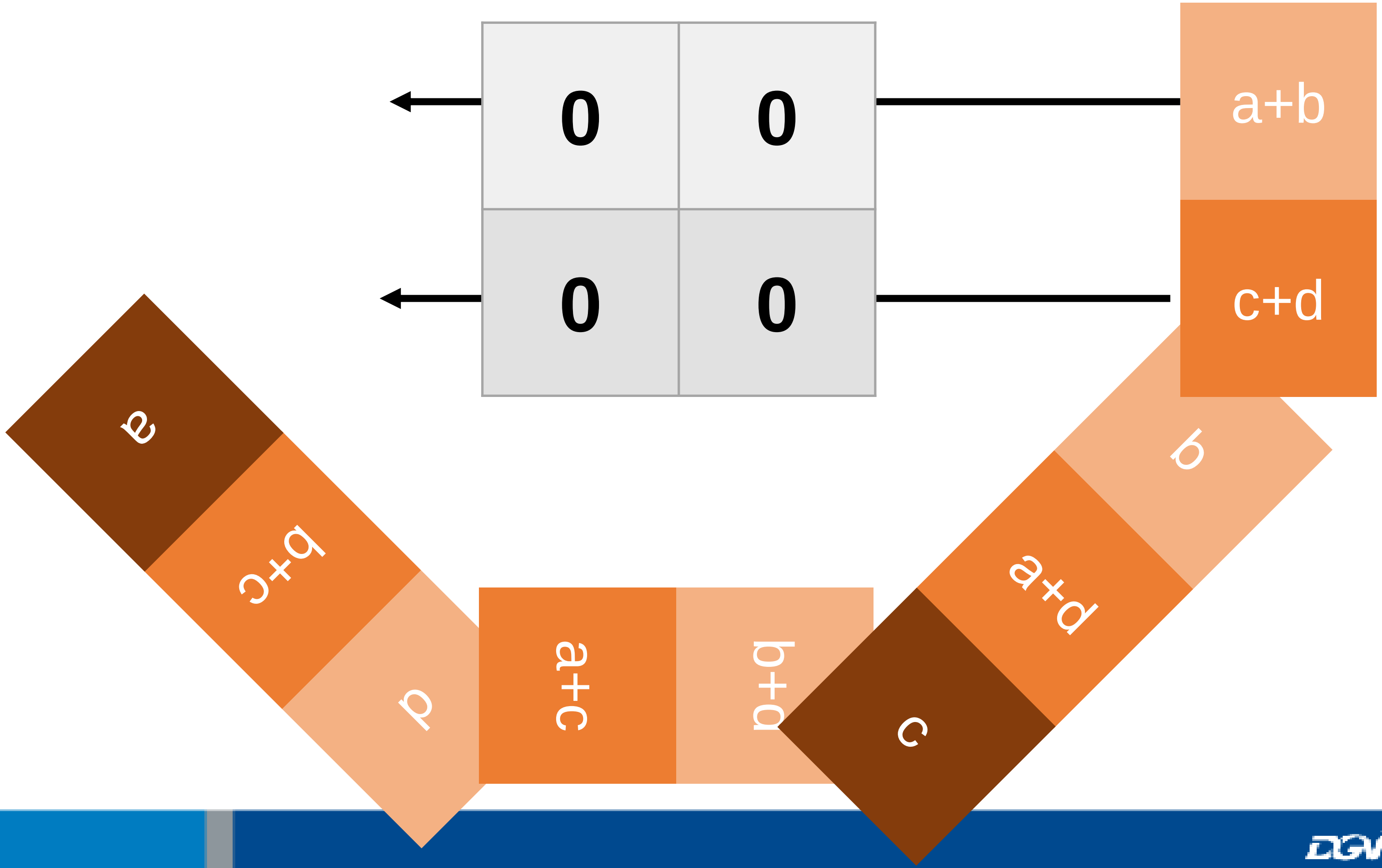
CT data acquisition



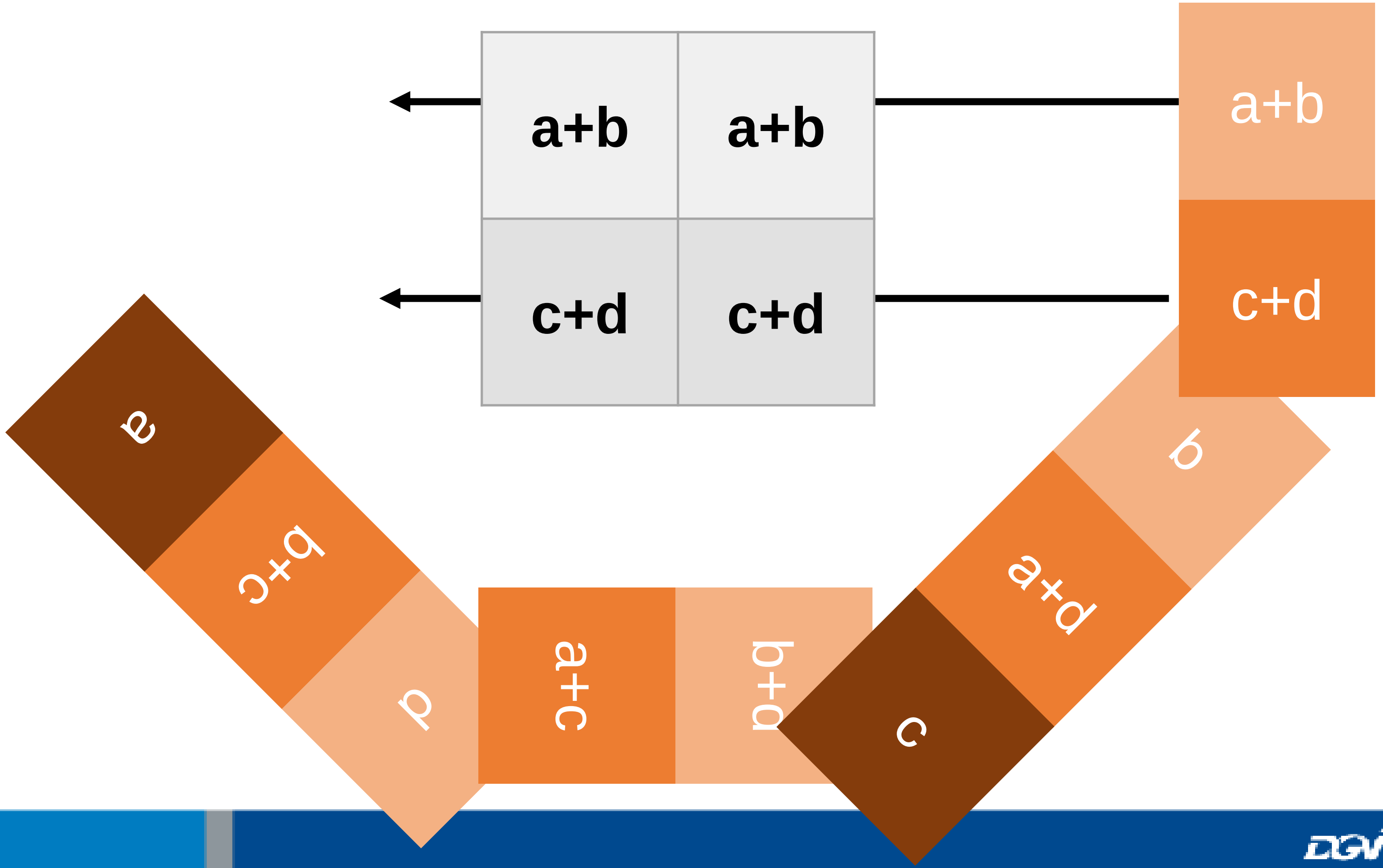
CT data acquisition



Reconstruction – Backprojection (Concept, not real)

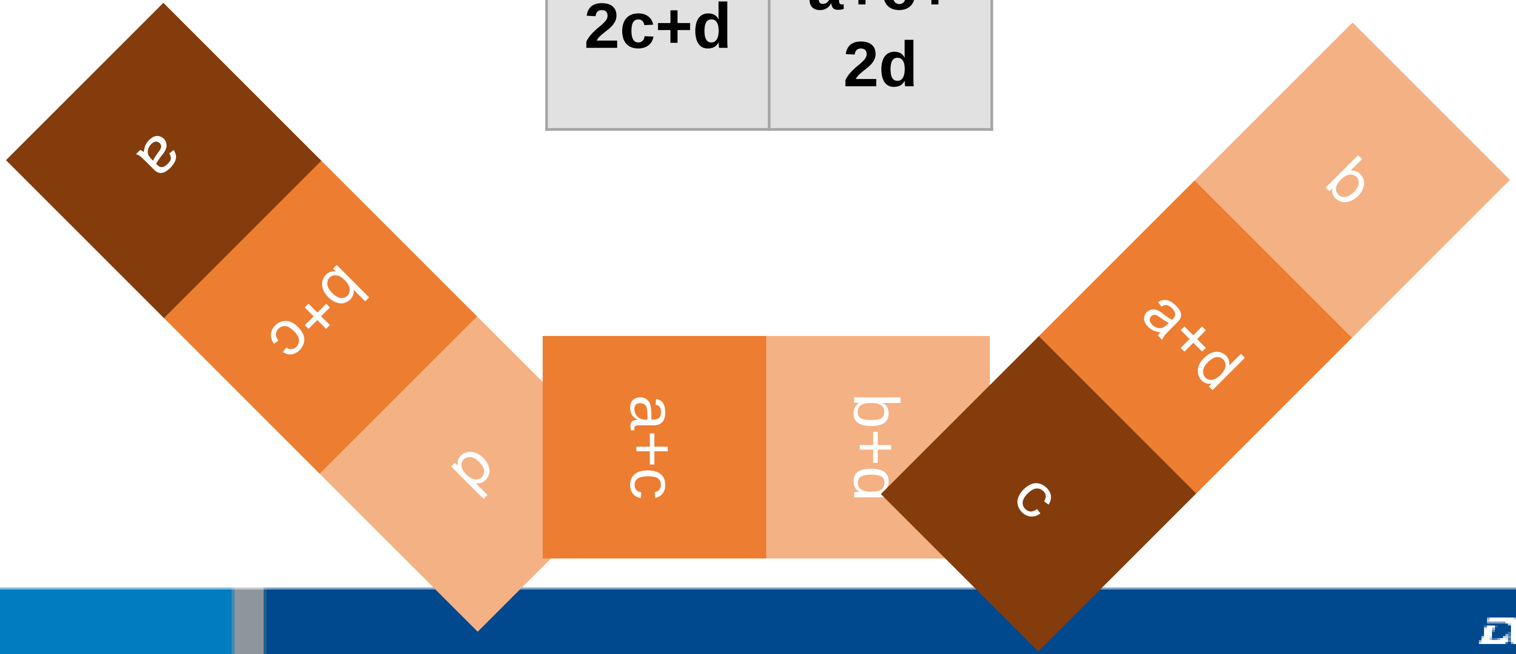


Reconstruction – Backprojection (Concept, not real)



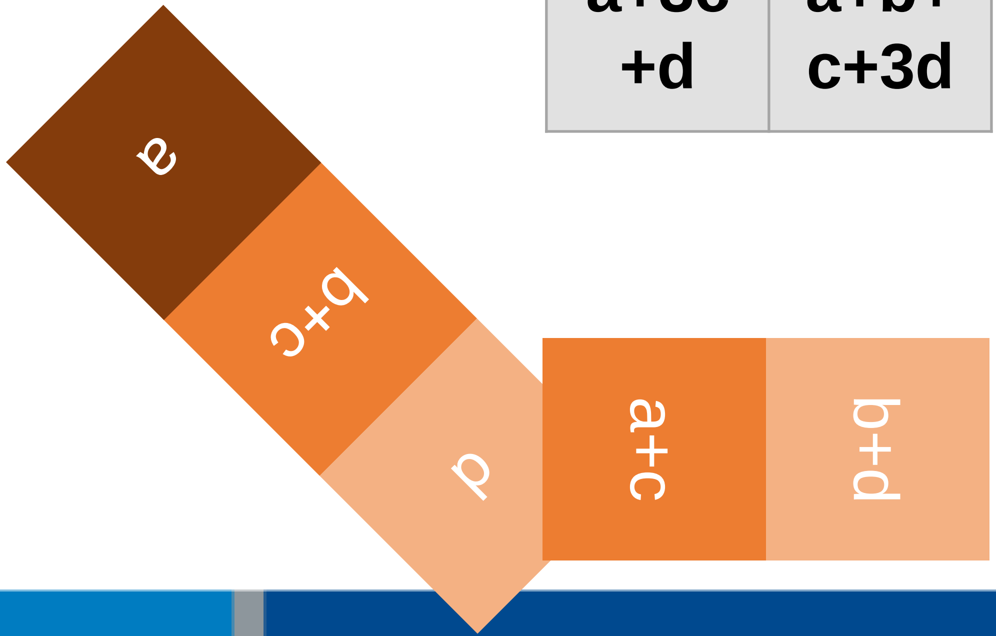
Reconstruction – Backprojection (Concept, not real)

2a+b +d	a+2b
2c+d	a+c+ 2d



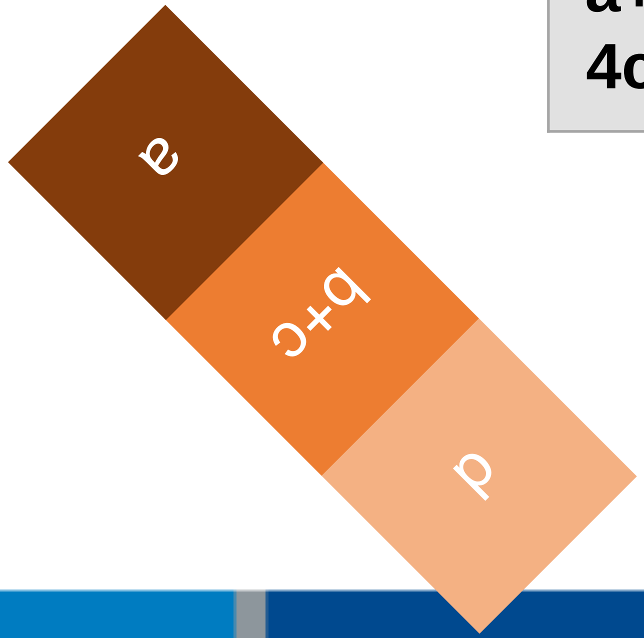
Reconstruction – Backprojection (Concept, not real)

3a+b +c+d	a+3b +d
a+3c +d	a+b+ c+3d



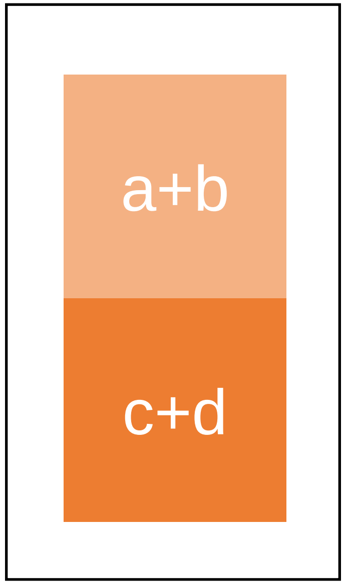
Reconstruction – Backprojection (Concept, not real)

4a+b +c+d	a+4b +c+d
a+b+ 4c+d	a+b+ c+4d



Reconstruction – Backprojection (Concept, not real)

4a+b +c+d	**a+4b +c+d**
a+b+ 4c+d	**a+b+ c+4d**



= a+b+c+d

Reconstruction – Backprojection (Concept, not real)

3a	3b
3c	3d

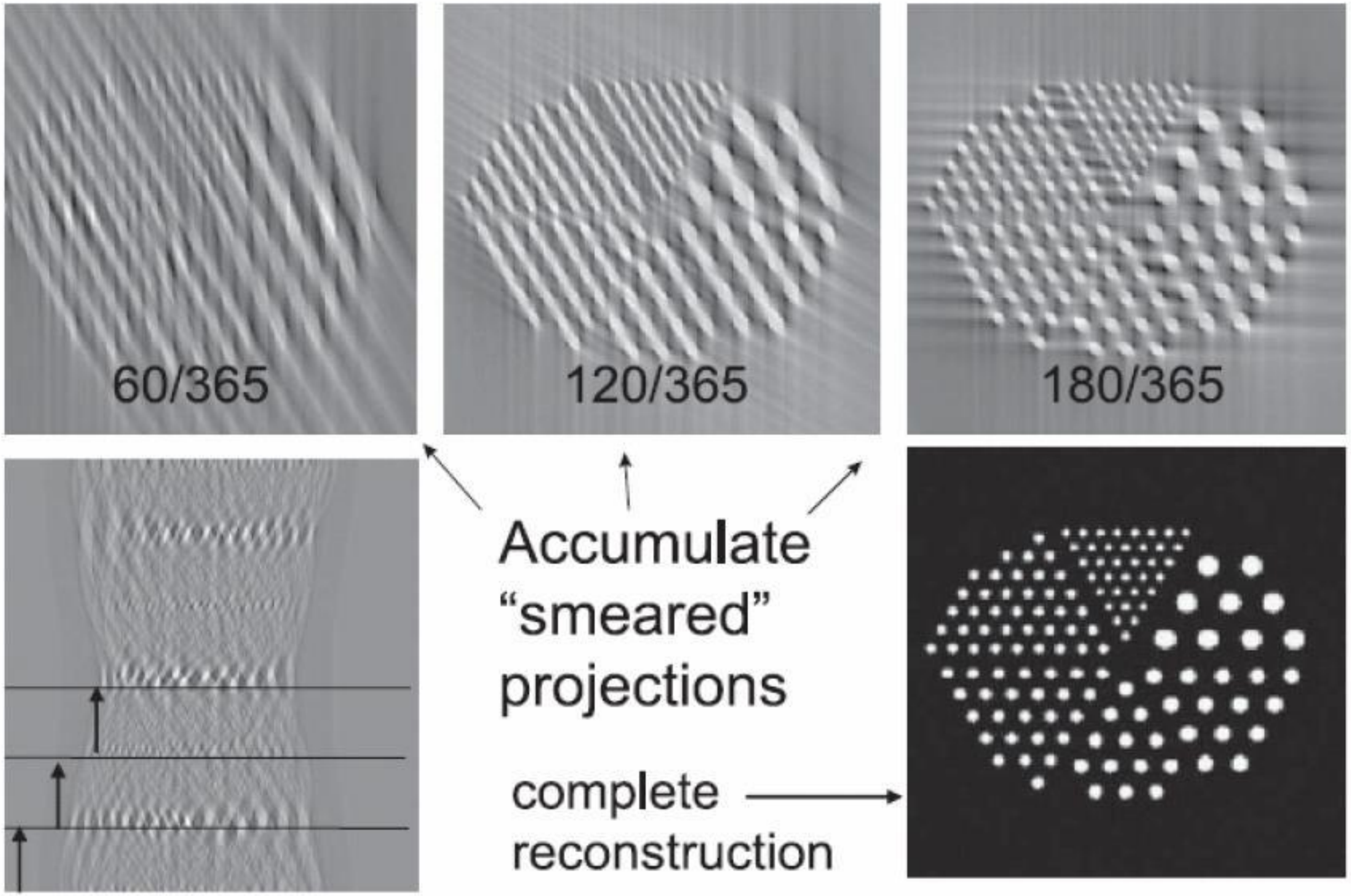
/ 3

Reconstruction – Backprojection (Concept, not real)

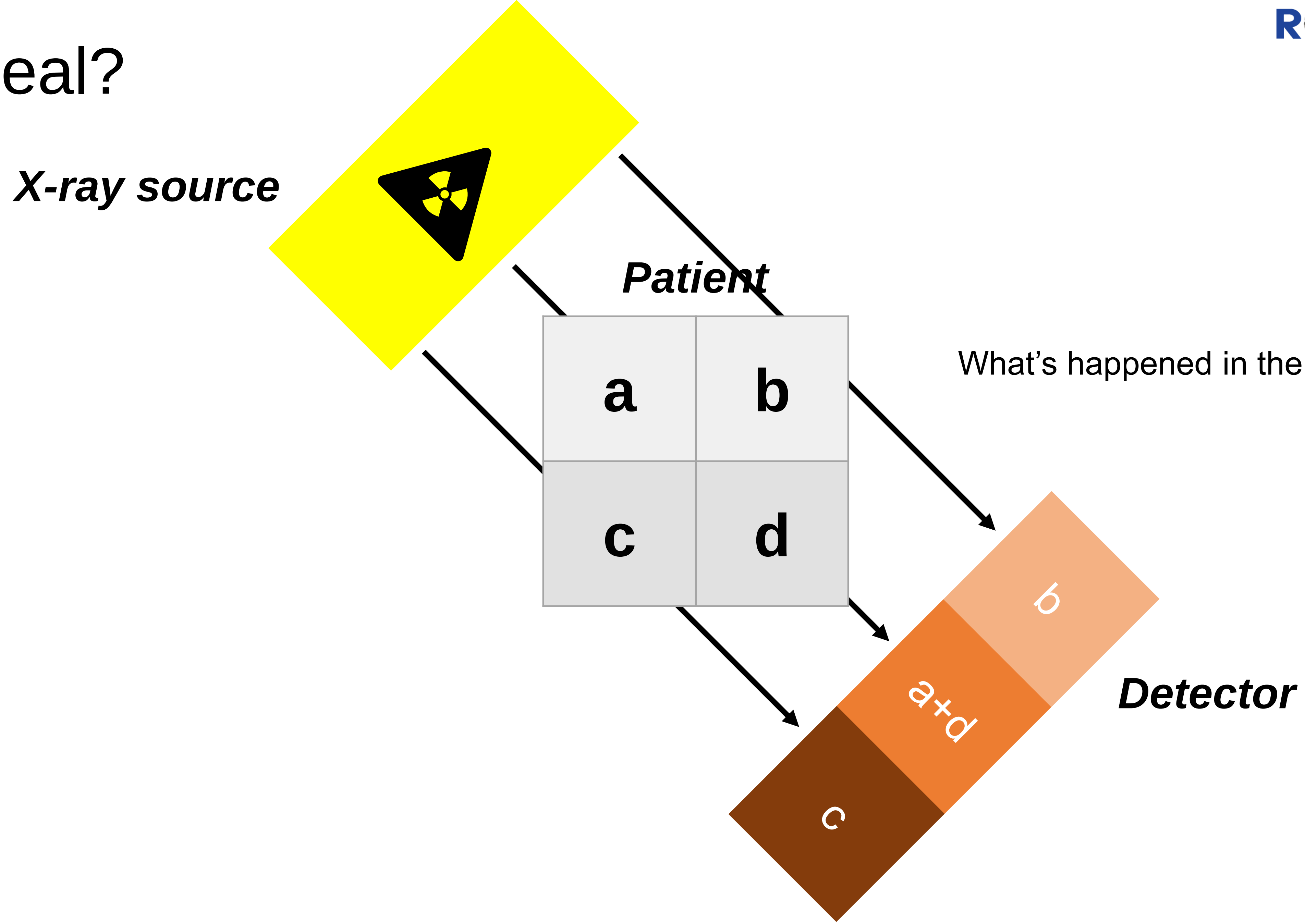
a	b
c	d

It's perfect in this case, however…

Reconstruction – Backprojection (Concept, not real)



Is it real?



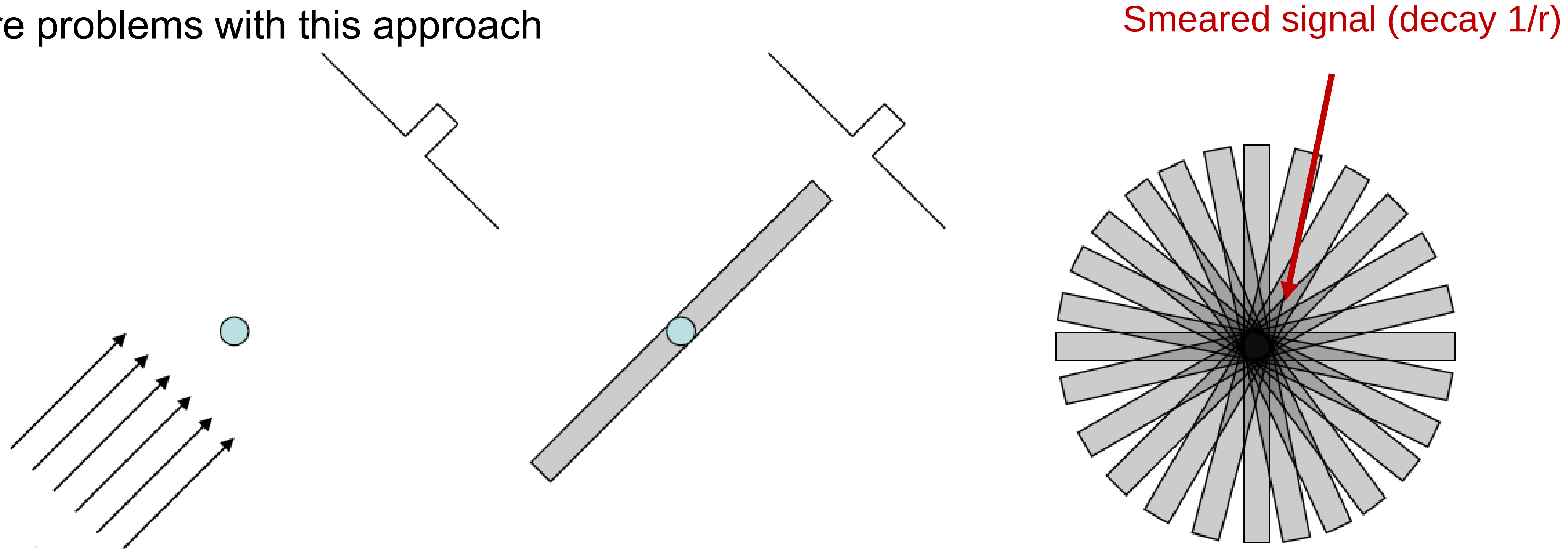
What's happened in the real world?

Backprojection

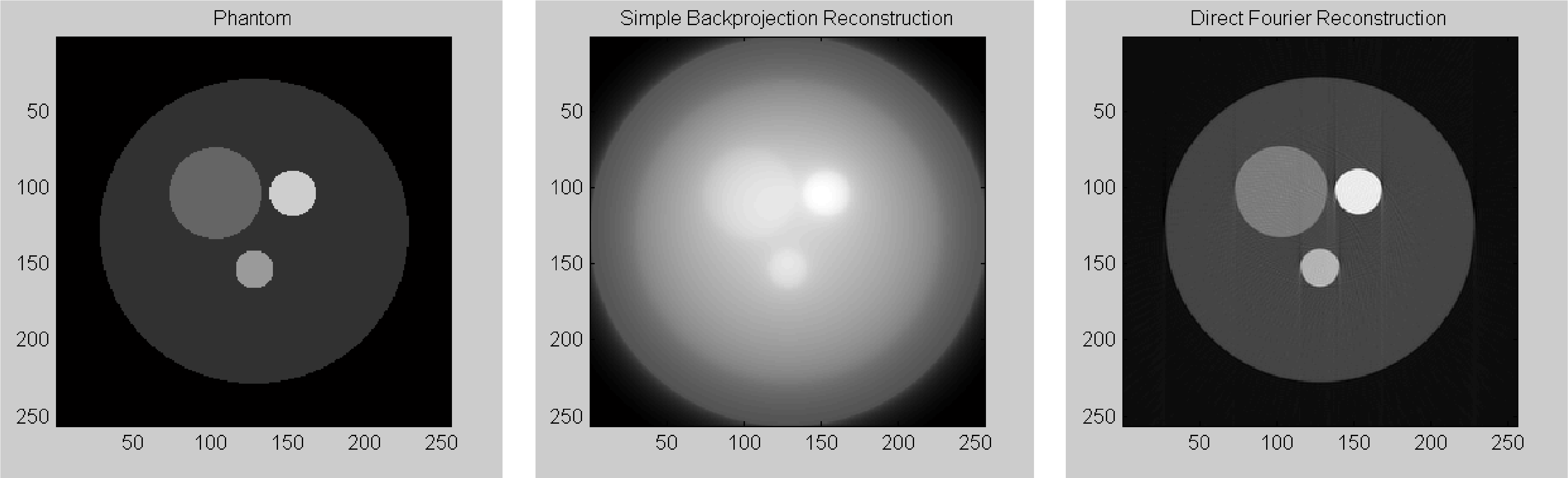
- Alternative direct reconstruction: backprojection
- The measured projections are “**smeared**” across the object: laminogram
- Crude reconstruction
- Nature of distortion: 1/r

Backprojection

- Consider an example of a simple circular absorber
- The detected profile suggests the presence of the absorber along the x-ray source and detector
- Backproject the profile back to the object, i.e., assign the same value along that line
- Repeat for every projection
- There are problems with this approach

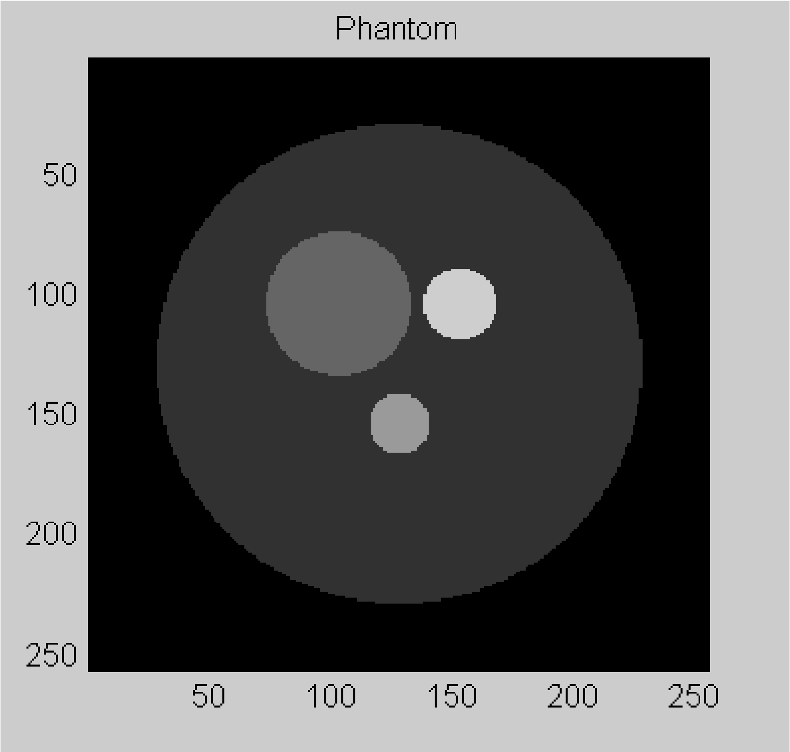


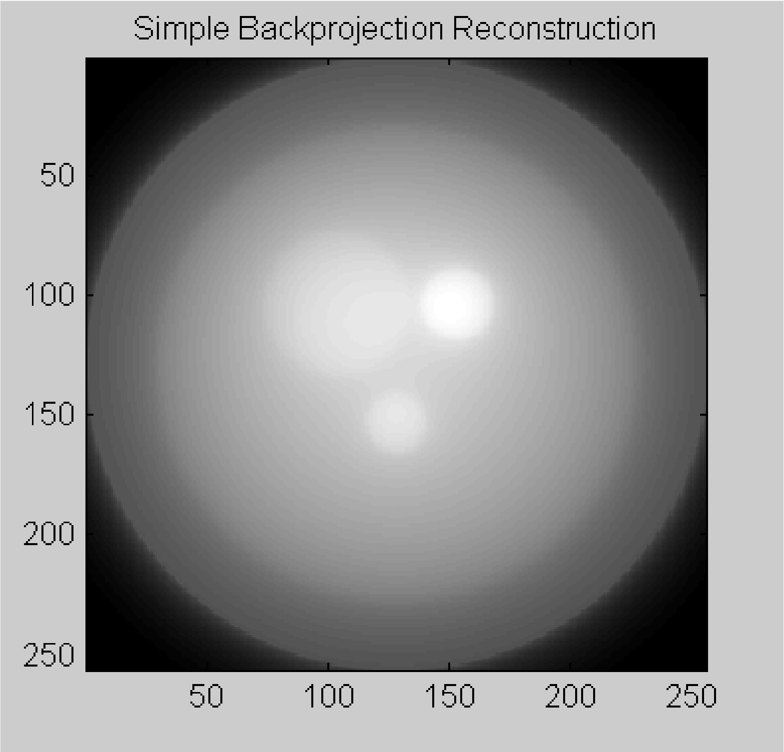
Backprojection: Example



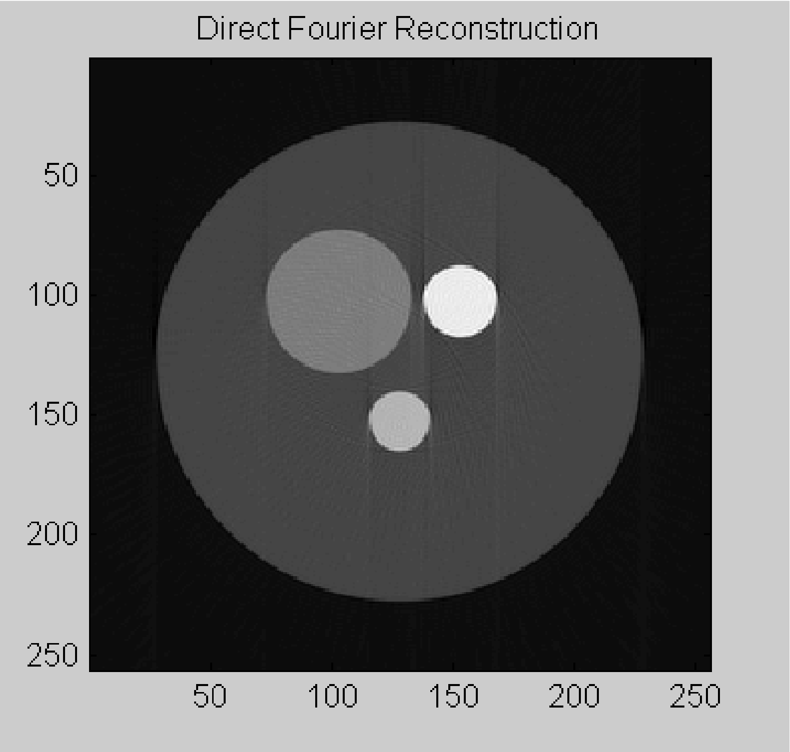
‘Smeared’

Backprojection vs Fourier Transform





Computationally simple but ***inaccurate***



Accurate but ***computationally intensive***

> *Filtered Backprojection: Computationally simple* & *Accurate*

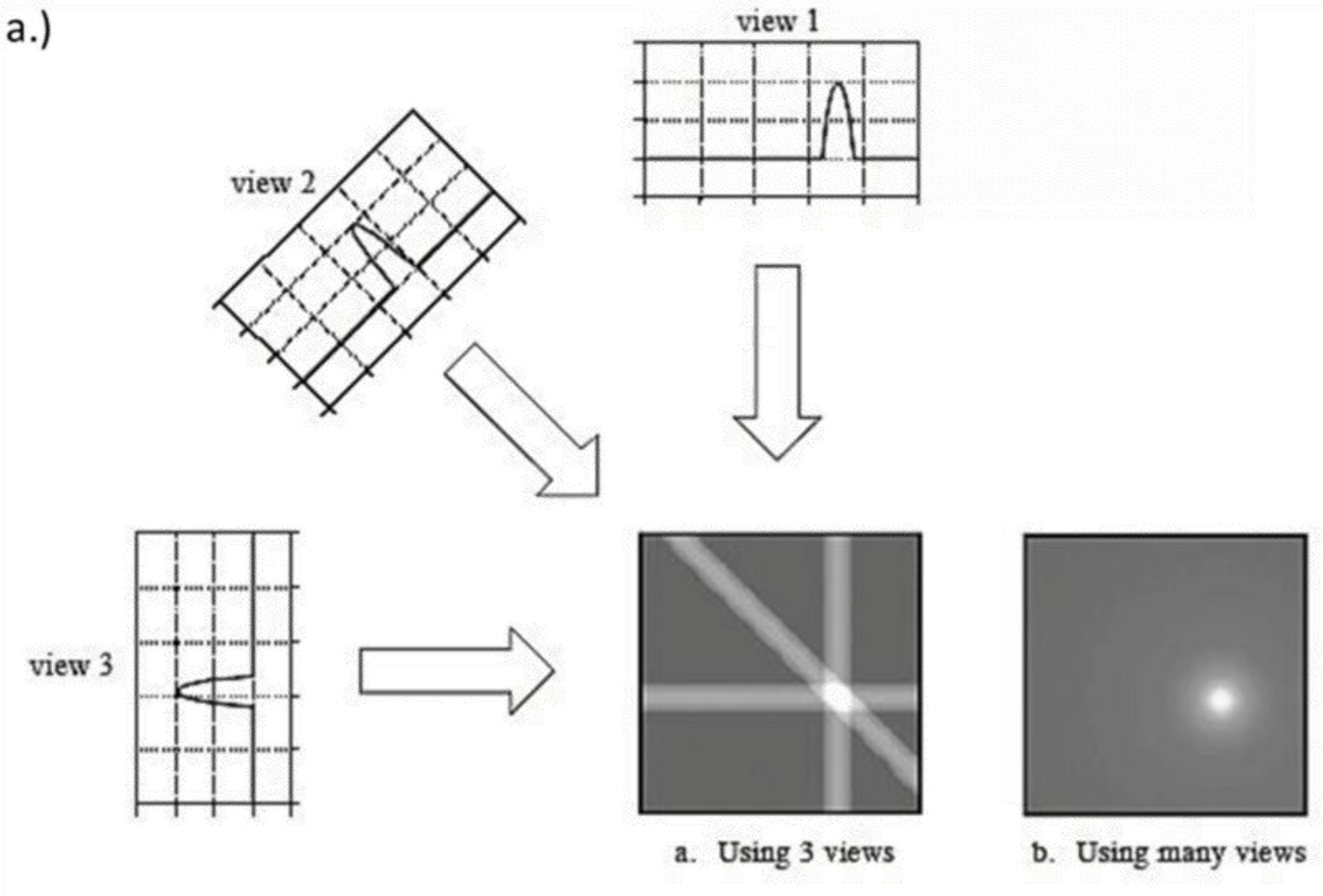
Filtered(Convolution) Backprojection

- Filtered Backprojection: corrects for **“1/r blur”**
- Convolution Backprojection: Filtered Back projection using convolution theorem of Fourier transform
- Consider *f(x,y)* as derived previously:

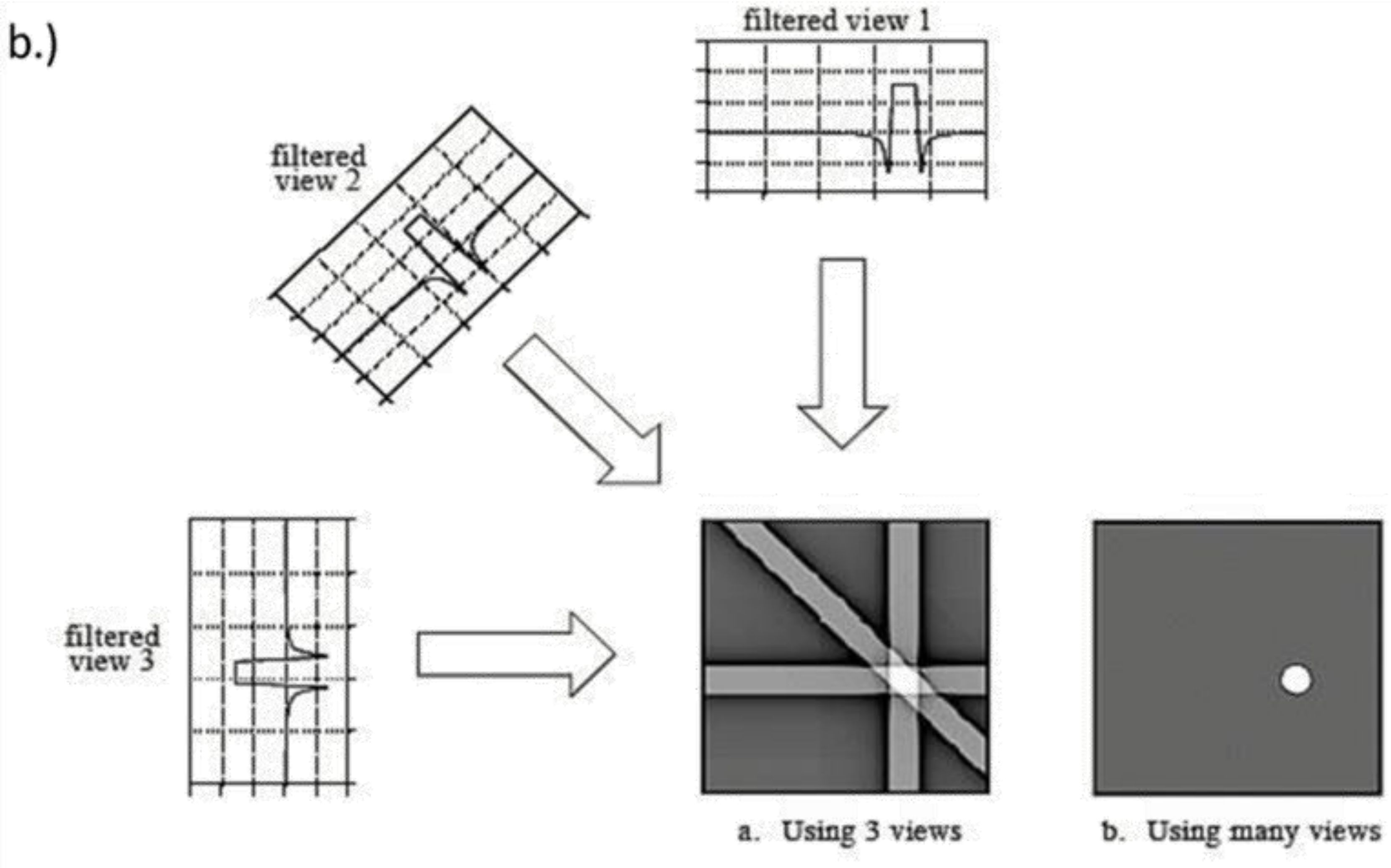
$$
\begin{aligned}
f(x,y) &= \int_0^{2\pi} d\theta \int_0^{\infty} G(\rho,\theta) e^{i2\pi\rho(x\cos\theta + y\sin\theta)} \rho d\rho = \\
&= \int_0^{\pi} \int_{-\infty}^{\infty} |\rho| G(\rho,\theta) e^{i2\pi\rho(x\cos\theta + y\sin\theta)} d\rho d\theta = \\
&= \int_0^{\pi} \left[\int_{-\infty}^{\infty} |\rho| G(\rho,\theta) e^{i2\pi\rho l} d\rho \right] d\theta
\end{aligned}
$$

- Here |ρ| is a frequency filter where FT of *g(l,θ)* is multiplied by |ρ| and then IFT, and then backprojected

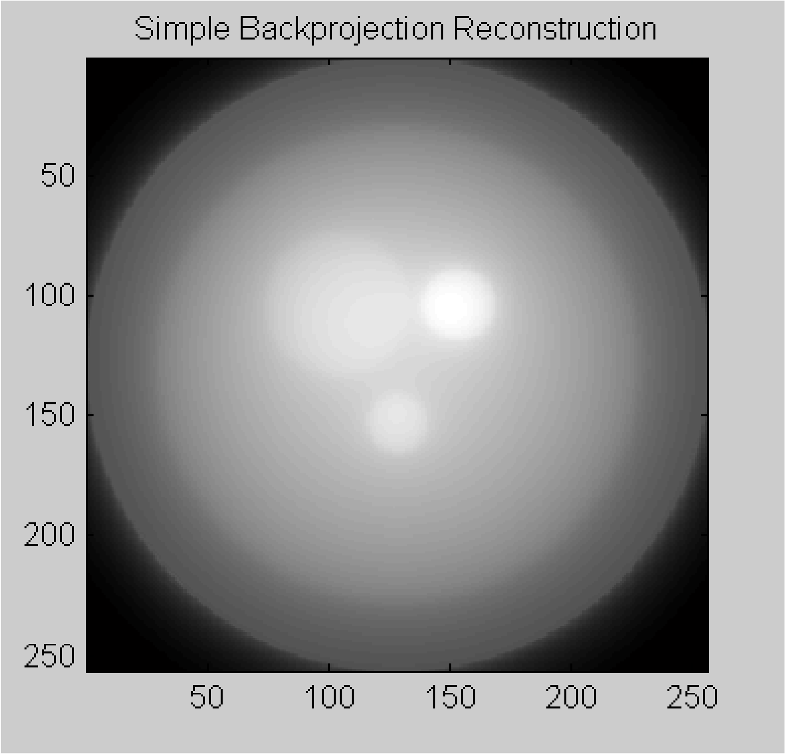
Concept of Backprojection



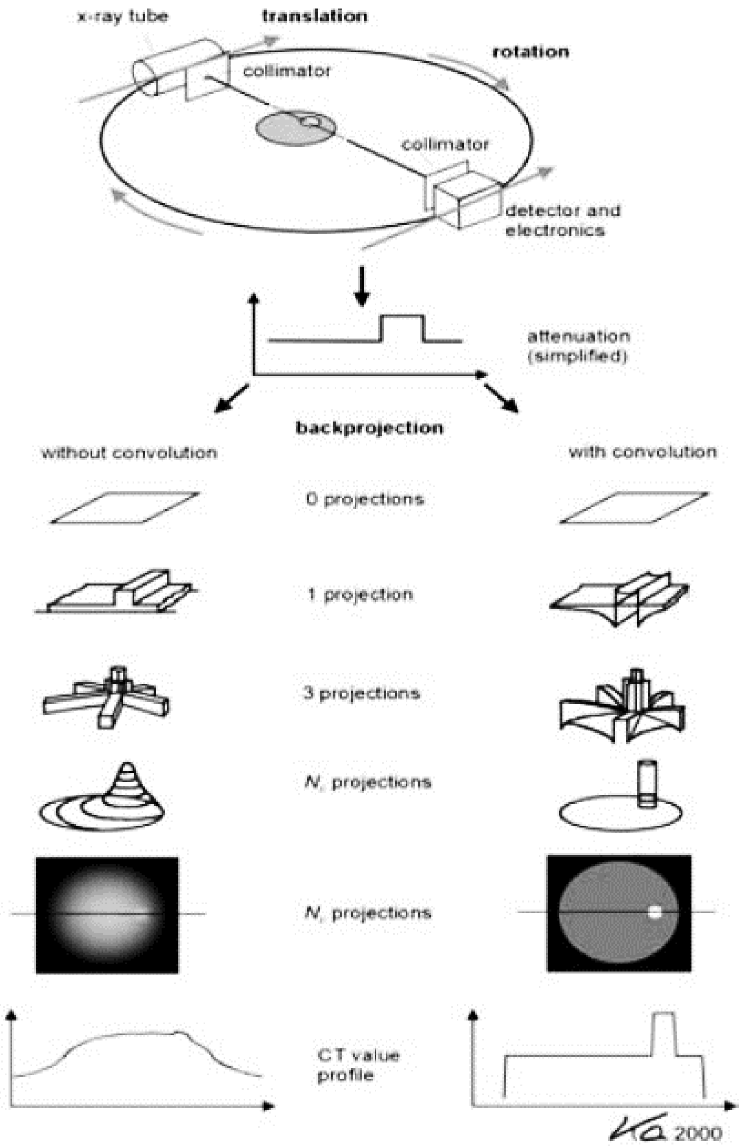
Concept of Filtered Backprojection



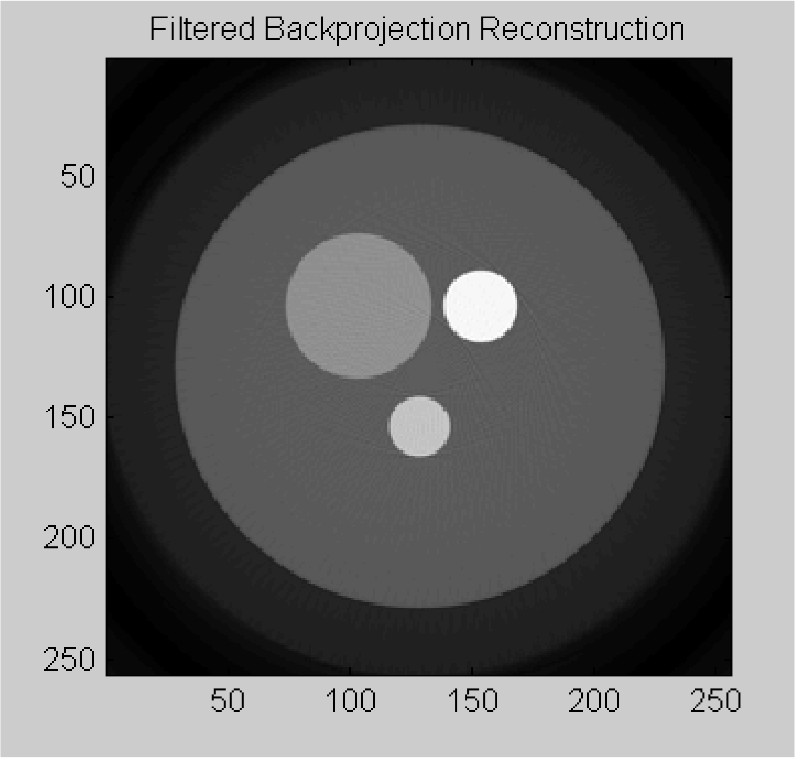
Comparison



Simple Backprojection Reconstruction
50
100
150
200
250
50
100
150
200
250

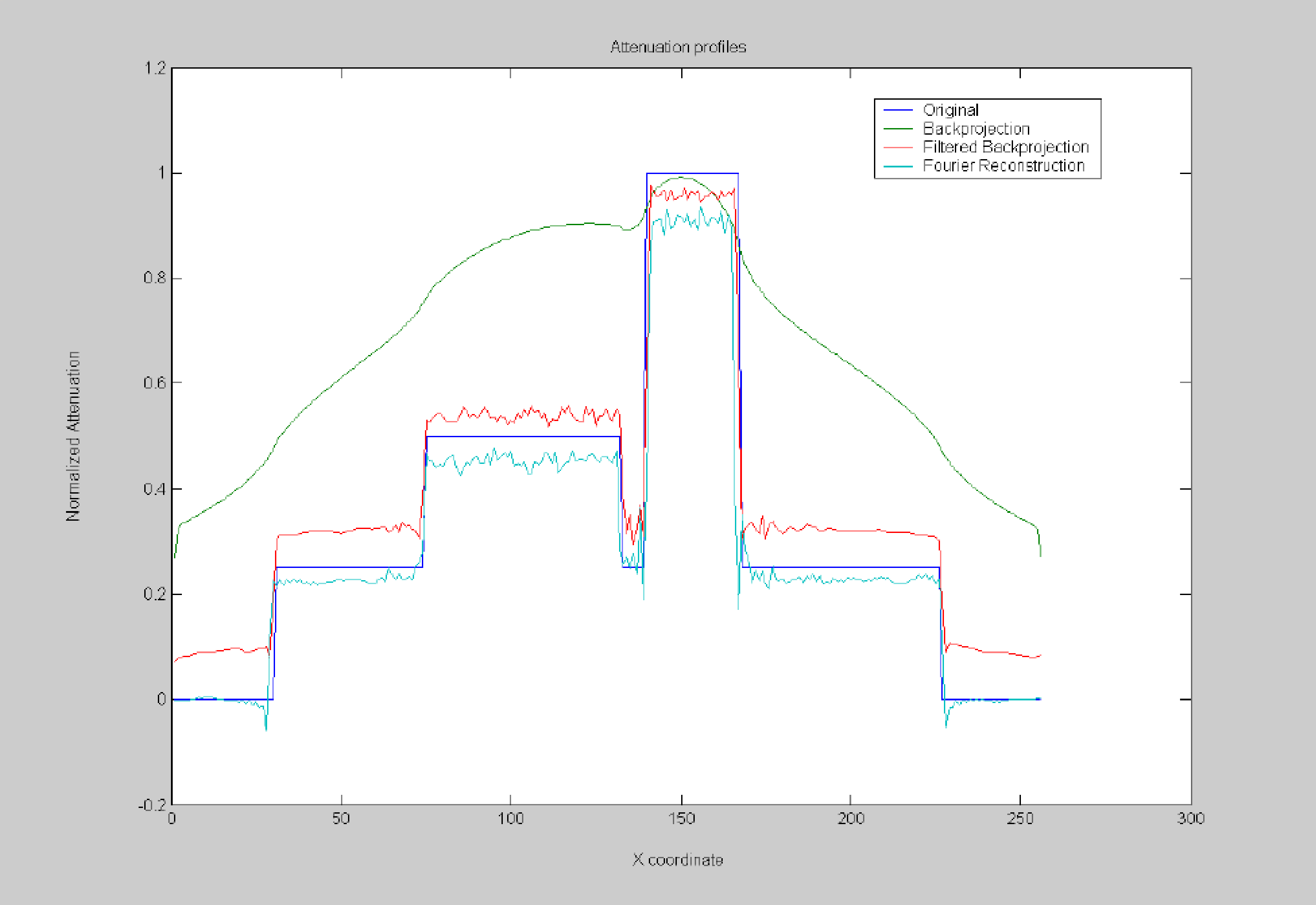


x-ray tube
translation
rotation
collimator
collimator
detector and electronics
attenuation (simplified)
backprojection
without convolution
with convolution
0 projections
1 projection
3 projections
N. projections
N. projections
CT value profile
Ka 2000



Filtered Backprojection Reconstruction
50
100
150
200
250
50
100
150
200
250

Comparison



Attenuation profiles
Original
Backprojection
Filtered Backprojection
Fourier Reconstruction
Normalized Attenuation
X coordinate
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
50
100
150
200
250
300

Reminder: Fourier Transform Relations

- The convolution of two functions in space can be represented by simply multiplying their frequency spectra (**convolution**)

$$F[g(x,y)\otimes h(x,y)] = F\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)h(x-\xi,y-\eta)d\xi d\eta\right] = G(u,v)H(u,v)$$

<u>NOTE: This is very important relationship for imaging.</u> A 2-D convolution is a common operation in imaging systems (for example, convolve the object with the point spread function of the imaging system). This same operation can be represented as the product of the spectra of the two functions [for example, product of spectra of PSF with object spectra results in image spectra).

Convolution Backprojection

- Consider filtered backprojection equation and look at it from a convolution perspective

$$f(x,y)=\int_0^{\pi}\left[\int_{-\infty}^{\infty}|\rho|G(\rho,\theta)e^{i2\pi\rho l}d\rho\right]d\theta=\int_0^{\pi}\left[F_{1D}^{-1}[|\rho|]*g(l,\theta)\right]d\theta=$$

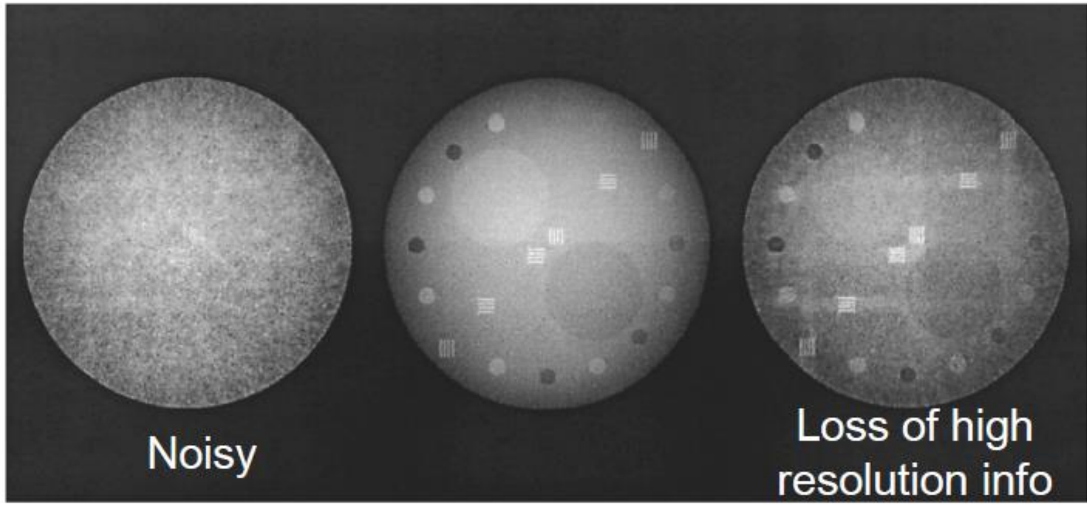
$$=\int_0^{\pi}\left[c(l)*g(l,\theta)\right]_{l=x\cos\theta+y\sin\theta}d\theta=$$

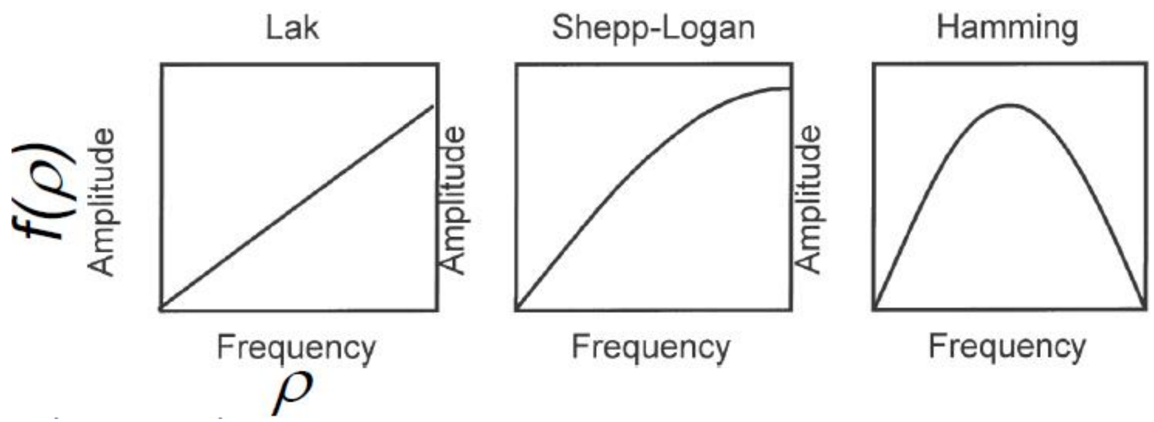
$$=\int_0^{\pi}\int_{-\infty}^{\infty}g(l,\theta)c(x\cos\theta+y\sin\theta-l)dl\,d\theta$$

- Here c(.)is IFT of |ρ| and the equation above outlines the convolution backprojection

Convolution Backprojection

- Unfortunately, IFT of f(ρ)=|ρ| does not exist, and we often limit/modify f(ρ)to have IFT possible.





Rams-Lak filter (Simple)

$$f(\rho) = \begin{cases} |\rho| & 0 \le \rho \le \rho_{\max} \\ 0 & \rho > \rho_{\max} \end{cases}$$

Shepp-Logan filter

Hamming

What does f(ρ) mean? -> Filter

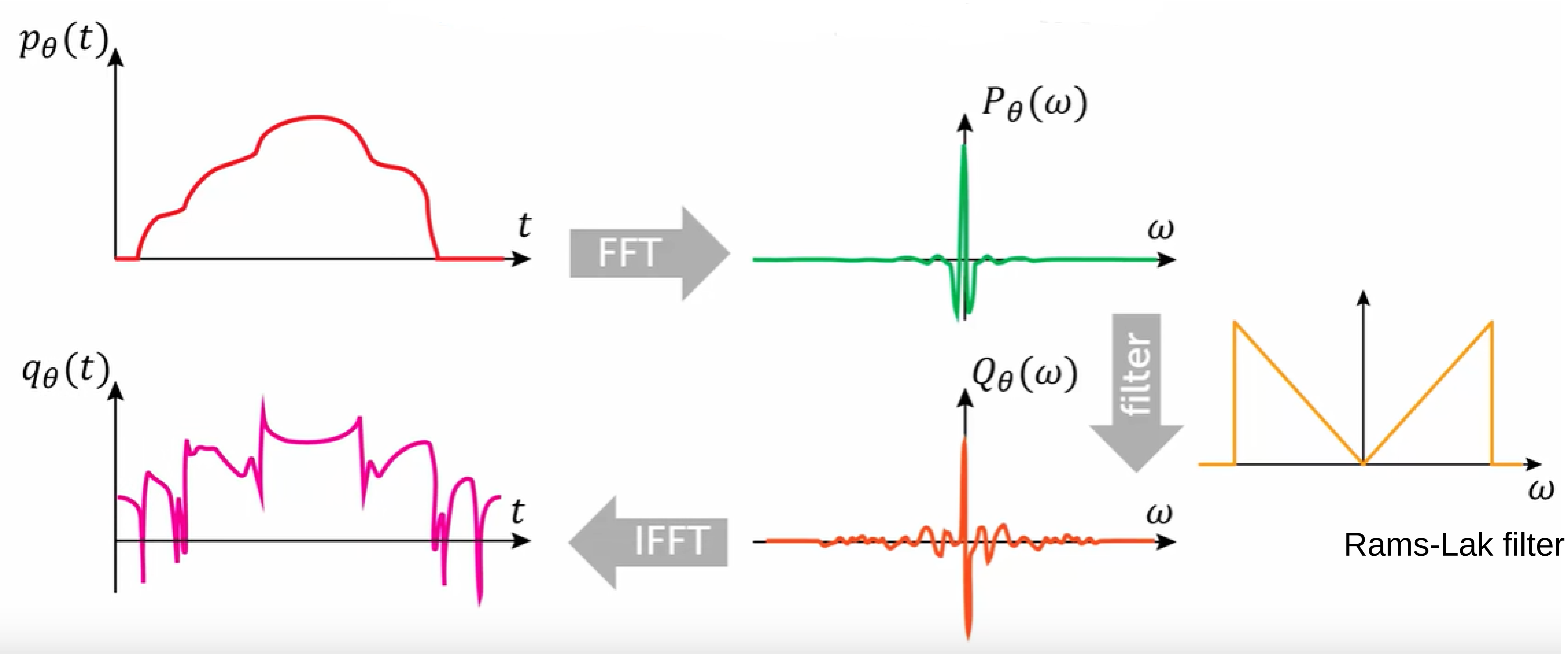
- Consider filtered backprojection equation :

$$f(x,y) = \int_0^{\pi}\left[\int_{-\infty}^{\infty}|\rho| G(\rho,\theta)\cdot e^{i2\pi\rho l}\,d\rho\right]d\theta = \int_0^{\pi}\left[\int_{-\infty}^{\infty}\left(f(\rho)\cdot G(\rho,\theta)\right)e^{i2\pi\rho l}\,d\rho\right]d\theta =$$

$$= \int_0^{\pi}\left[\int_{-\infty}^{\infty} G^{filtered}(\rho,\theta)\cdot e^{i2\pi\rho l}\,d\rho\right]d\theta = \int_0^{\pi} g^{filtered}(l,\theta)\,d\theta$$

- In Fourier domain, G(ρ,θ)is multiplied by f(ρ), i.e., the extent (and content) of G(ρ,θ)is affected by the functional form of f(ρ).In image domain, i.e., if we consider g(l,θ),such procedure represents filtering.
- Same applies to convolution-based back projection

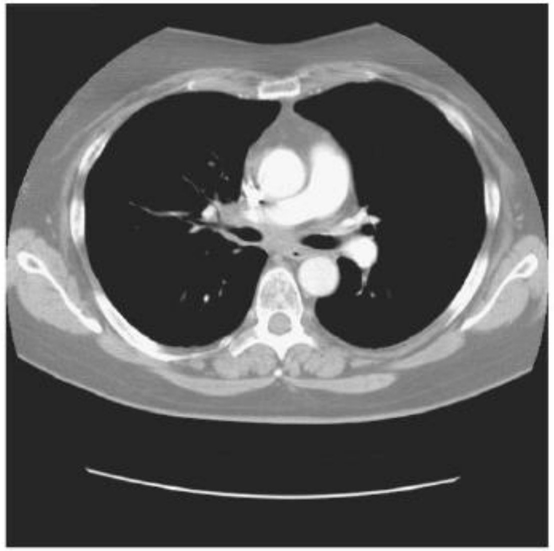
Process



$p_\theta(t)$
t
FFT
$P_\theta(\omega)$
ω
filter
$q_\theta(t)$
t
IFFT
$Q_\theta(\omega)$
ω
ω
Rams-Lak filter

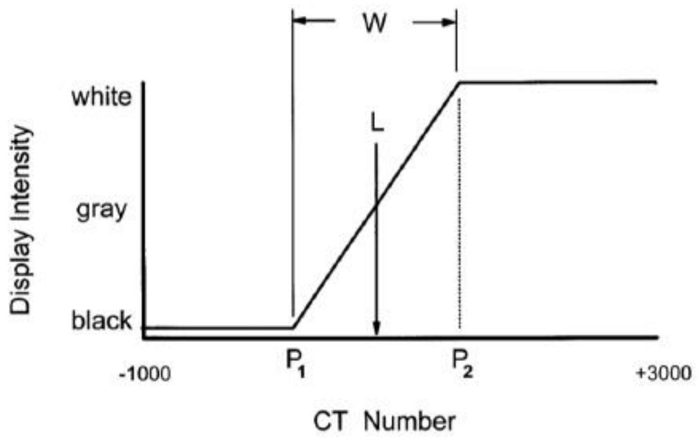
Same image, different display

W = 4095, L = 1048



W = 600, L = –100

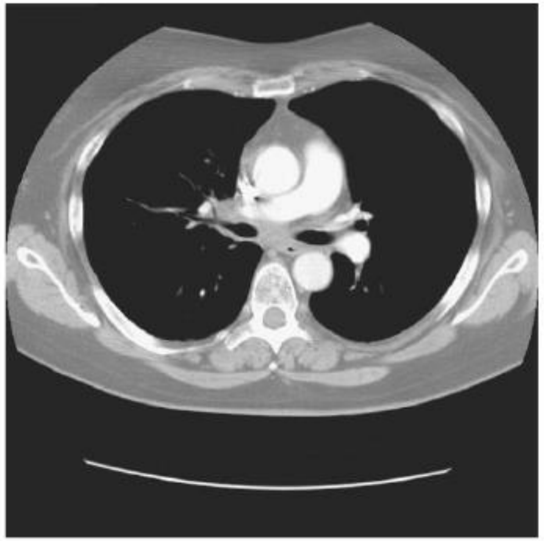
W = 700, L = –650



Same image, different display

W = 4095, L = 1048

May be too large DR, good for bone

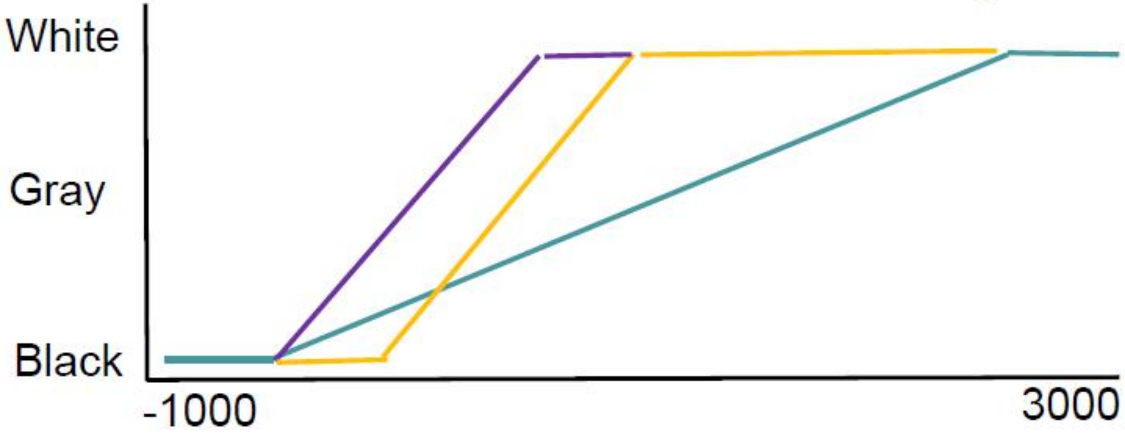


W = 600, L = –100

Optimal Dynamic Range For mediastinum

W = 700, L = –650

Saturated for bone, good for lung structure



CT number and Display

- 12 bits: 2^{12} combinations = 4096 values
- Contrast scale: Hounsfield unit (HU) scale

$$\Delta N = -\mu \cdot N \cdot \Delta z$$

$$N = N_0 e^{-\mu \cdot z}$$

$$HU = 1000 \frac{\mu_m - \mu_{water}}{\mu_{water}}$$

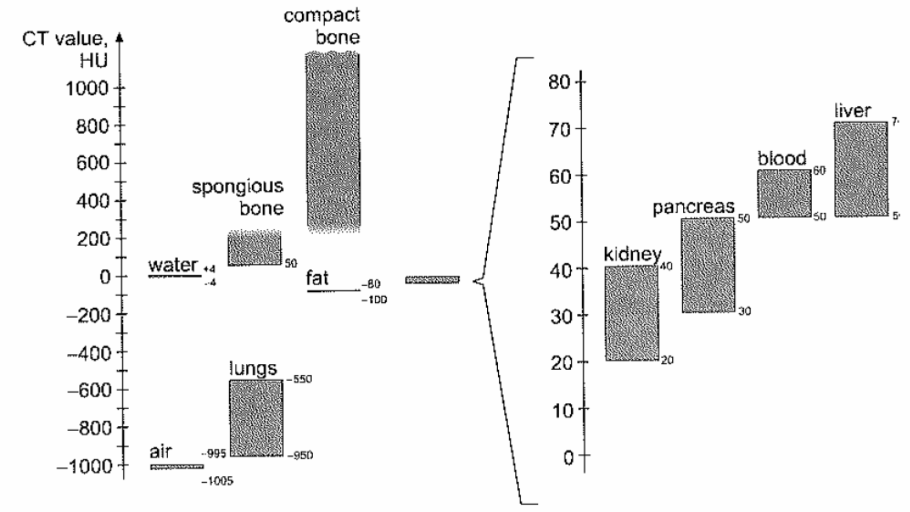
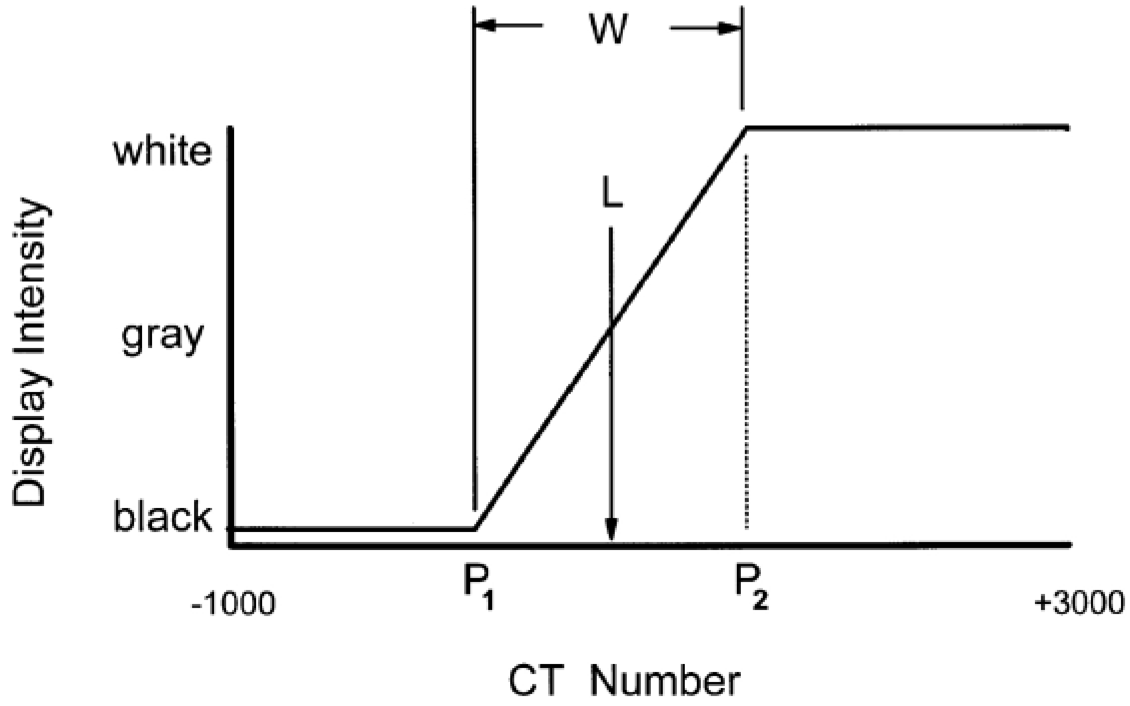


Figure 1.9
The Hounsfield scale. CT values characterize the linear attenuation coefficient of the tissue in each volume element relative to the μ-value of water. The CT values of different tissues are therefore defined to be relatively stable and to a high degree independent of the x-ray spectrum.

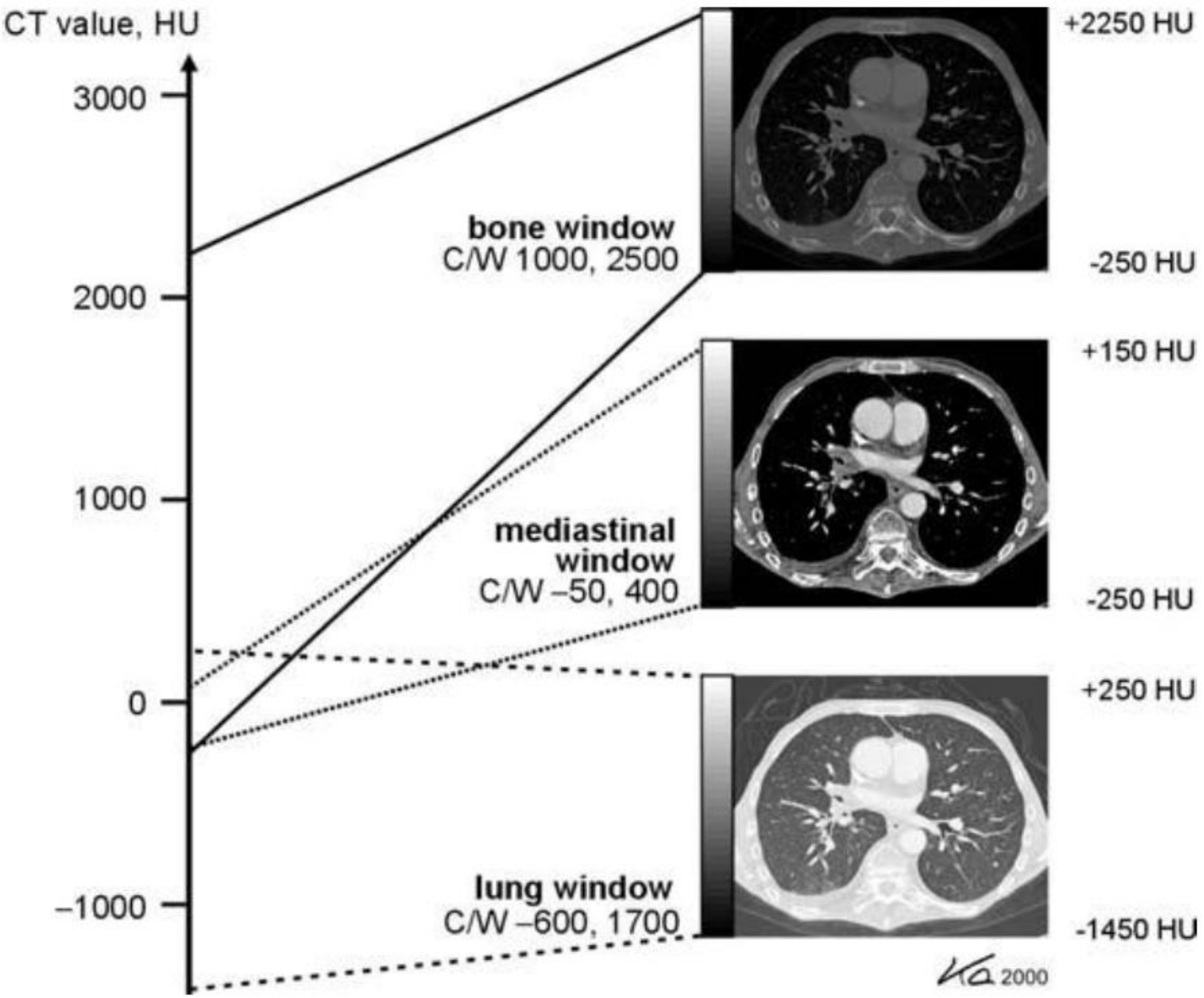


CT number and Display

Substance		HU
Air		−1000
Fat		−120 to −90[2]
Soft tissue on contrast CT		+100 to +300
Bone	Cancellous	+300 to +400[3]
	Cortical	+1800 to +1900[3]
Subdural hematoma	First hours	+75 to +100[4]
	After 3 days	+65 to +85[4]
	After 10-14 days	+35 to +40[5]
Other blood	Unclotted	+13[6] to +50[7]
	Clotted	+50[8] to +75[6][8]
Pleural effusion	Transudate	+2 to +15 [9]
	Exudate	+4 to +33[9]
Other fluids	Chyle	−30[10]
	Water	0
	Urine	-5 to +15[2]
	Bile	-5 to +15[2]
	CSF	+15
	Abscess / Pus	0[11] or +20[12], to +40[12] or +45[11]
	Mucus	0[13] - 130[14] ("high attenuating" at over 70 HU)[15][16]
Parenchyma	Lung	-700 to −600[17]
	Kidney	+20 to +45[2]
	Liver	60 ± 6[18]
	Lymph nodes	+10 to +20[19]
	Muscle	+35 to +55[2]
	Thymus	• +20 to +40 in children[20] • +20 to +120 in adolescents[20]
	White matter	+20 to +30
	Grey matter	+37 to +45
Gallstone	Cholesterol stone	+30 to +100[21]
	Bilirubin stone	+90 to +120[21]
Foreign body[22]	Windowpane glass	+500
	Aluminum, tarmac, car window glass, bottle glass, and other rocks	+2,100 to +2,300
	Limestone	+2,800
	Copper	+14,000
	Silver	+17,000
	Steel	+20,000
	Gold, steel, and brass	+30,000 (upper measurable limit)
Earwax		<0

From, wikipedia

CT number and Display



Ref. kalendar

Parameters for Spatial / Contrast Resolution

- Detector pitch
- Detector aperture
- Number of views
- Number of rays
- Focal spot size
- Object magnification

- Slice thickness
- Slice sensitivity profile
- Helical pitch
- Reconstruction kernel
- Pixel matrix
- Patient motion
- Field of view

- Number of x-ray photons
- Dose
- Pixel size
- Slice thickness
- Reconstruction filter
- Patient size
- Rotation speed

Artifacts

- Aliasing
- Patient motion
- Beam hardening
- Partial volume averaging

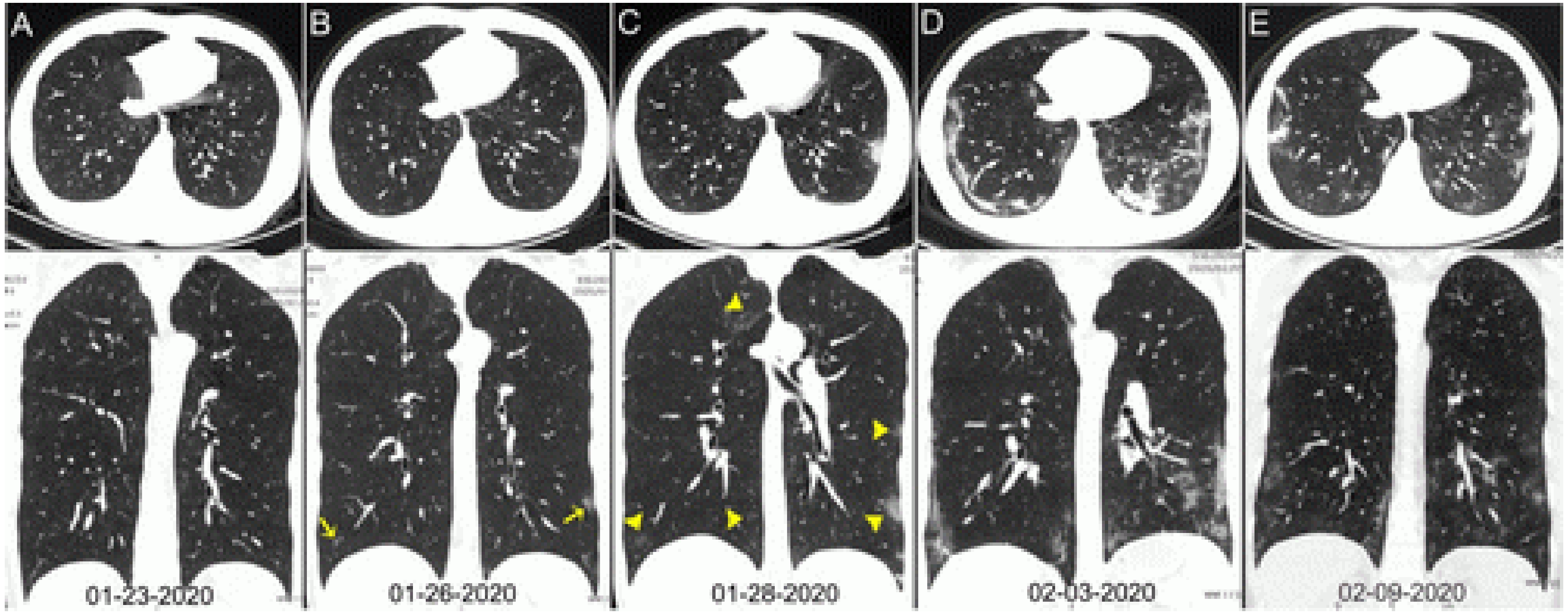
Applications: Anatomical, Functional, Interventional

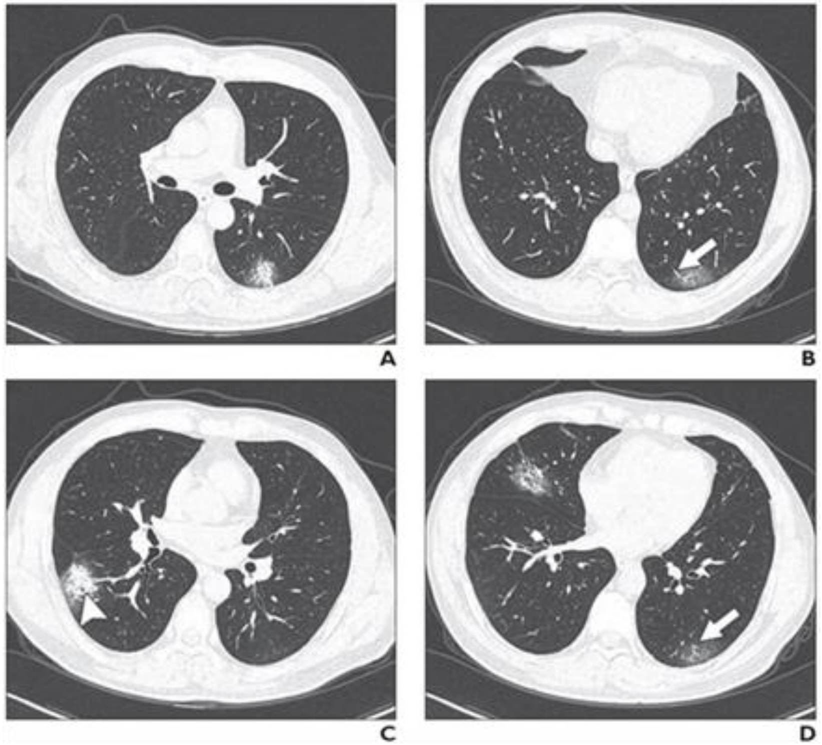
- Cardiology
- CT angiography
- Pre-, intra-, and post-operative imaging
- Image guided neurosurgery
- Orthopedic
- Trauma evaluation
- Lung CT imaging
- Pulmonary CT Imaging
- Neuro CT

Applications: Anatomical, Functional, Interventional

CT Provides Best Diagnosis for Novel Coronavirus (COVID-19)

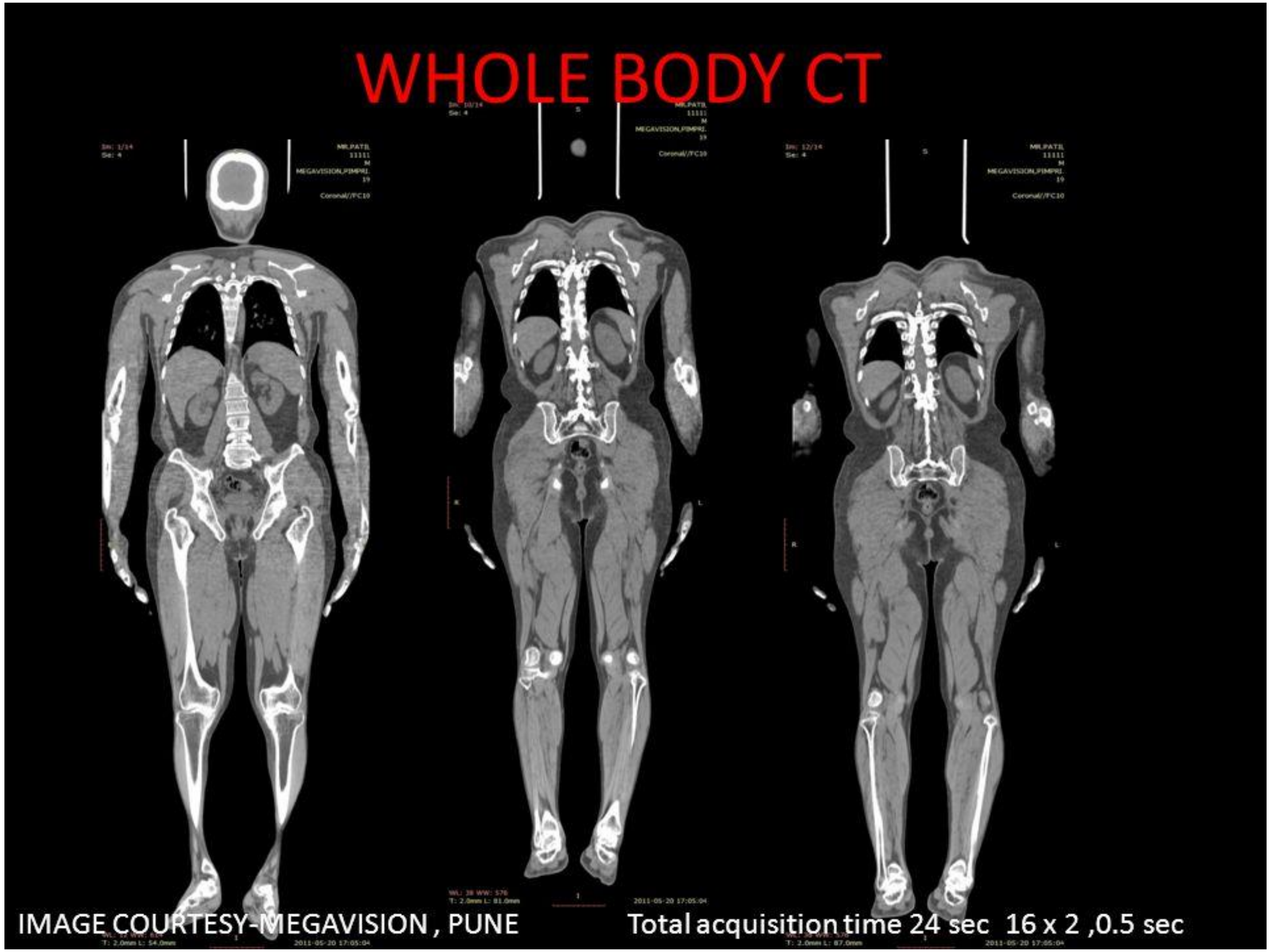
Applications: Anatomical, Functional, Interventional





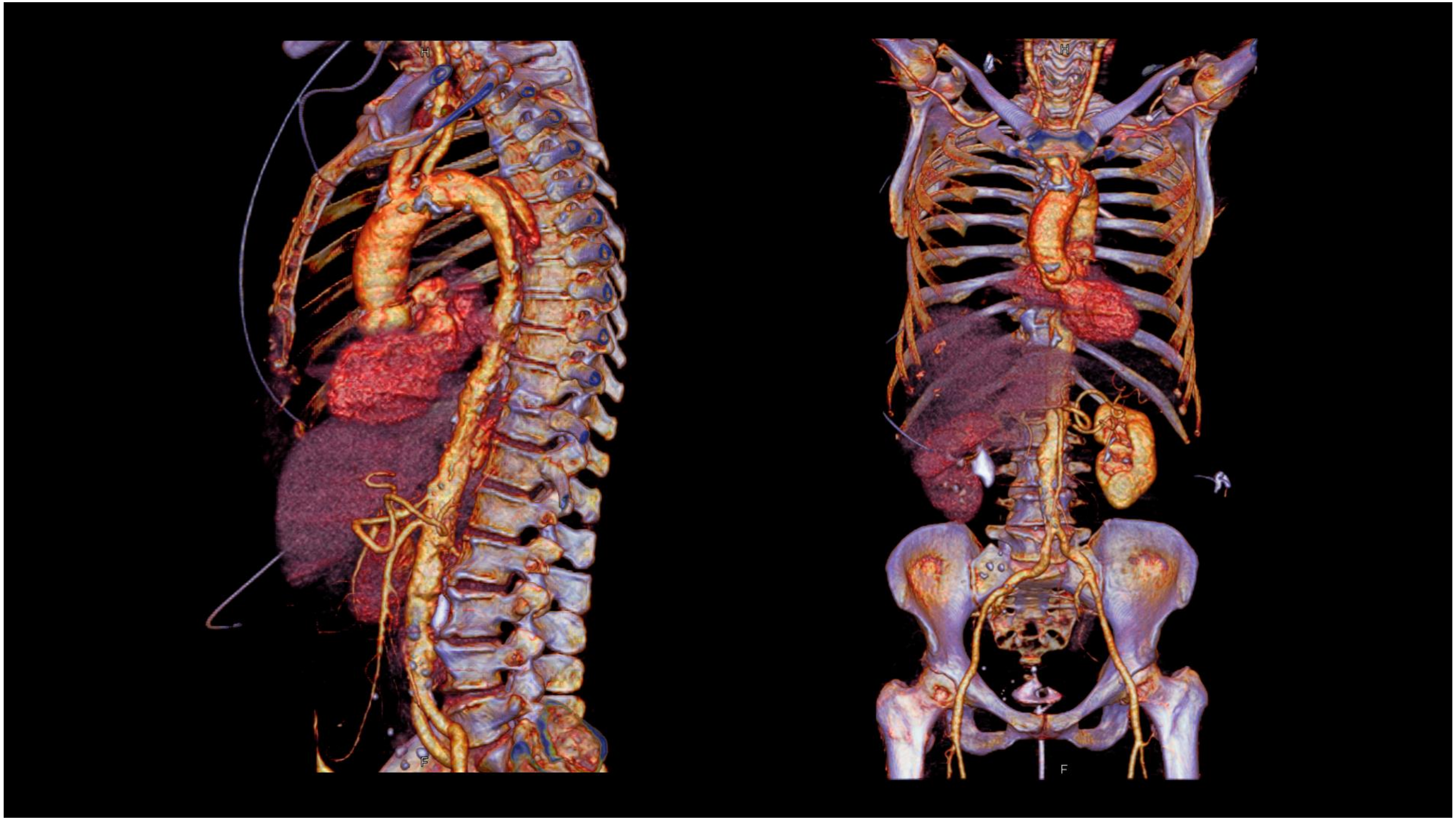
CT Provides Best Diagnosis for Novel Coronavirus (COVID-19)

Applications: Anatomical, Functional, Interventional



Whole body CT scan

CT Angiography



Limitations of Non-contrast CT

Modern CT scanner, GE healthcare, 2019

S/W revolution in 21c

- Core technology is same
- A.I. powered automation
- A.I. powered diagnosis

Past and Future of Computed Tomography

- Previous efforts: faster imaging
  - Cardiac imaging
- Current development
  - 64-slice scanners
    - 128 images acquired per second !!!
    - Pulmonary embolism (PE) exam –5 sec to cover 40 cm, 640 images
  - Spatial resolution
  - Functional Imaging
  - Contrast agent development
  - Combined/Multi-modality