수치해석 14주차 과제

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Q. 다음의 방정식을 LU 분해 법으로 풀어라.

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \cdots ①$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \cdots ②$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \cdots ③$$

$$\Rightarrow \underbrace{\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}}_{x} = \underbrace{\begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}}_{b}$$

1) 전진소거

(1) $①_2' = ① \times \frac{a_{21}}{a_{11}} = f_{21}$

$$\Rightarrow 0.1x_1 - \frac{0.01}{3}x_2 - \frac{0.02}{3}x_3 = \frac{0.1}{3} \times 7.85 \cdots ①_2'$$

(2) $② ' = ② - ①_2'$

$$\Rightarrow +\left(7+\frac{0.01}{3}\right)x_2 + \left(-0.3+\frac{0.02}{3}\right)x_3 = -19.3 - \frac{0.1}{3} \times 7.85$$

$$\Rightarrow \frac{2101}{300}x_2 - \frac{22}{75}x_3 = -\frac{11737}{600} \cdots ②'$$

(3) $①_3' = ① \times \frac{a_{31}}{a_{11}} = f_{31}$

$$\Rightarrow 0.3x_1 - 0.01x_2 - 0.02x_3 = 0.785 \cdots ①_3'$$

(4) $③' = ③ - ①_3'$

$$\Rightarrow -0.19x_2 + 10.02x_3 = 70.615 \cdots ③'$$

(5) $②_3'' = ②' \times \frac{a_{32}'}{a_{22}'} = f_{32}$

$$\Rightarrow -0.19x_2 + \frac{38}{4775}x_3 = \frac{20273}{38200} \cdots ②_3''$$

(6) $③'' = ③' - ②_3''$

$$\Rightarrow \frac{19123}{1910}x_3 = \frac{133861}{1910} \cdots ③''$$

$$\therefore [U] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & \frac{2101}{300} & -\frac{22}{75} \\ 0 & 0 & \frac{19123}{1910} \end{bmatrix},$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{0.1}{3} & 1 & 0 \\ \frac{0.3}{3} & \frac{-0.19}{\frac{2101}{300}} & 1 \end{bmatrix}$$

2) 전진 대입

$$[L]\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \{b\}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{0.1}{3} & 1 & 0 \\ \frac{0.3}{3} & \frac{-0.19}{\frac{2101}{300}} & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}$$

$d_1 = 7.85$

$\frac{0.1}{3} d_1 + d_2 = -19.3 \Rightarrow d_2 = -19.3 - \frac{0.1}{3}(7.85) = -19.5617$

$\frac{0.3}{3} d_1 - \frac{0.19}{\frac{2101}{300}} d_2 + d_3 = 71.4 \Rightarrow d_3 = 71.4 - 0.1(7.85) + \frac{-0.19}{\frac{2101}{300}} \times (-19.5617) = 70.0843$

$$\therefore \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix}$$

3) 후진 대입

$$[U] \cdot \{x\} = \{d\}$$

$$\Rightarrow \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & \frac{2101}{300} & -\frac{22}{75} \\ 0 & 0 & \frac{19123}{1910} \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix}$$

$x_3 = \frac{1910}{19123} \times 70.0843 = 7.00003$

$x_2 = \frac{300}{2101} \times \left(-19.5617 + \frac{22}{75} x_3\right) = -2.5$

$x_1 = \frac{1}{3} \times (7.85 + 0.1 x_2 + 0.2 x_3) = 3$

$$\therefore \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3 \\ -2.5 \\ 7.00003 \end{Bmatrix}$$