로봇공학 동역학 과제1

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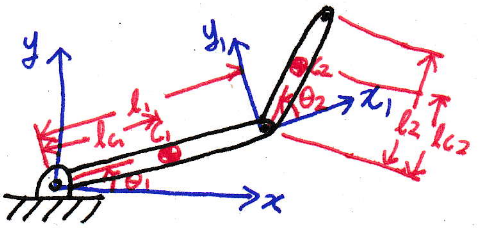
*2-link 머니퓰레이터의 Dynamics 해석을 수행하시오
(Newton-Euler 방식을 활용하고, 수업시간에 생략했던 대입과정 수행)

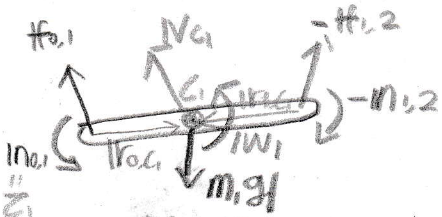


Goal: $\boxed{\tau_i = f(\theta, \dot\theta, \ddot\theta)} \leftarrow x, y, z$

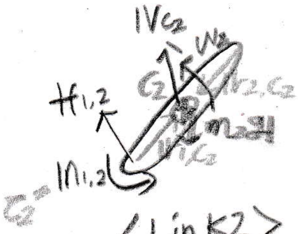
Assumption: Planar manipulator

$$\begin{cases} \mathbb{v}_{c_i} = \{v_{x_i}\ v_{y_i}\}^T \\ \mathbb{w}_i = w_i \hat{z}_{i-1} \\ I_i = \text{scalar moment of inertia } I_i \end{cases}$$

FBD



<Link1>



<Link2>

① $\Sigma \mathbb{F} = m_1 \dot{\mathbb{v}}_{c_1} = \mathbb{f}_{0,1} - \mathbb{f}_{1,2} + m_1 \mathbb{g}$

② $\Sigma \mathbb{M} = I_1 \dot{\mathbb{w}}_1 + \underbrace{\boxed{\mathbb{w}_1 \times (I_1 \mathbb{w}_1)}}_{=0} = \mathbb{n}_{0,1} - \mathbb{n}_{1,2} - \mathbb{r}_{0,c_1} \times \mathbb{f}_{0,1} + (-\mathbb{r}_{1,c_1}) \times (-\mathbb{f}_{1,2})$

③ $\Sigma \mathbb{F} = m_2 \dot{\mathbb{v}}_{c_2} = \mathbb{f}_{1,2} \underbrace{(-\mathbb{f}_{2,3})}_{=0} + m_2 \mathbb{g}$

④ $\Sigma \mathbb{M} = I_2 \dot{\mathbb{w}}_2 \underbrace{\boxed{+\mathbb{w}_2 \times (I_2 \mathbb{w}_2)}}_{=0} = \mathbb{n}_{1,2} \underbrace{(-\mathbb{n}_{2,3})}_{=0} + (-\mathbb{r}_{1,c_2}) \times \mathbb{f}_{1,2} \underbrace{\boxed{(-\mathbb{r}_{2,c_2}) \times (\mathbb{f}_{2,3})}}_{=0}$

⑤ $\mathbb{n}_{0,1} = \tau_1$ & $\mathbb{n}_{1,2} = \tau_2$

⑥ $\boxed{\mathbb{f}_{1,2}\text{를 제거 (③,⑤→④)}}$

$\Rightarrow I_2 \dot{\mathbb{w}}_2 = \tau_2 + (-\mathbb{r}_{1,c_2}) \times (m_2 \dot{\mathbb{v}}_{c_2} - m_2 \mathbb{g})$

⑦ $\boxed{\mathbb{f}_{0,1}\text{을 제거 (①③⑤→②)}}$

$$\Rightarrow I_1 \dot{\mathbb{w}}_1 = \tau_1 - \tau_2 + (-\mathbb{r}_{0,c_1}) \times (m_1 \dot{\mathbb{v}}_{c_1} - m_1 \mathbb{g} + m_2 \dot{\mathbb{v}}_{c_2} - m_2 \mathbb{g}) + (-\mathbb{r}_{1,c_1}) \times (-m_2 \dot{\mathbb{v}}_{c_2} + m_2 \mathbb{g})$$

$$\boxed{\text{Since} \begin{cases} (-\mathbb{r}_{0,c_1} + \mathbb{r}_{1,c_1}) \times m_2 \dot{\mathbb{v}}_{c_2} = -\mathbb{r}_{0,1} \times m_2 \dot{\mathbb{v}}_{c_2} \\ (+\mathbb{r}_{0,c_1} - \mathbb{r}_{1,c_1}) \times m_2 \mathbb{g} = \mathbb{r}_{0,1} \times m_2 \mathbb{g} \end{cases}}$$

$$\Rightarrow I_1 \dot{\mathbb{w}}_1 = \tau_1 - \tau_2 + (-\mathbb{r}_{0,c_1}) \times (m_1 \dot{\mathbb{v}}_{c_1} - m_1 \mathbb{g}) + (-\mathbb{r}_{0,1}) \times (m_2 \dot{\mathbb{v}}_{c_2} - m_2 \mathbb{g})$$

Expressing $\omega_1, \omega_2, {}^I v_{c_1}, {}^I v_{c_2}$ in terms of θ_1 and θ_2

8 $\omega_1 = \dot{\theta}_1, \quad \omega_2 = \dot{\theta}_1 + \dot{\theta}_2$

$\Rightarrow$ 9 $${}^I r_{O,C_1} = \begin{Bmatrix} l_{c_1} c_1 \\ l_{c_1} s_1 \end{Bmatrix}, \quad {}^I v_{c_1} = \begin{Bmatrix} -l_{c_1}\dot{\theta}_1 s_1 \\ l_{c_1}\dot{\theta}_1 c_1 \end{Bmatrix}, \quad {}^I \dot{v}_{c_1} = \begin{Bmatrix} -l_{c_1}\ddot{\theta}_1 s_1 + l_{c_1}\dot{\theta}_1^2 c_1 \\ l_{c_1}\ddot{\theta}_1 c_1 - l_{c_1}\dot{\theta}_1^2 s_1 \end{Bmatrix}$$

$$= l_{c_1}\begin{Bmatrix} -s_1 \\ c_1 \end{Bmatrix}\ddot{\theta}_1 + l_{c_1}\begin{Bmatrix} c_1 \\ -s_1 \end{Bmatrix}\dot{\theta}_1^2$$

10 $${}^I r_{O,C_2} = \begin{Bmatrix} l_1 c_1 + l_{c_2} c_{12} \\ l_1 s_1 + l_{c_2} s_{12} \end{Bmatrix}$$

$${}^I v_{c_2} = \begin{Bmatrix} -l_1\dot{\theta}_1 s_1 - l_{c_2}(\dot{\theta}_1+\dot{\theta}_2)s_{12} \\ l_1\dot{\theta}_1 c_1 + l_{c_2}(\dot{\theta}_1+\dot{\theta}_2)c_{12} \end{Bmatrix} = \begin{Bmatrix} -l_1 s_1 - l_{c_2} s_{12} \\ l_1 c_1 + l_{c_2} c_{12} \end{Bmatrix}\dot{\theta}_1 + \begin{Bmatrix} -l_{c_2} s_{12} \\ l_{c_2} c_{12} \end{Bmatrix}\dot{\theta}_2$$

$${}^I \dot{v}_{c_2} = \begin{Bmatrix} -l_1 s_1 - l_{c_2} s_{12} \\ l_1 c_1 + l_{c_2} c_{12} \end{Bmatrix}\ddot{\theta}_1 + \begin{Bmatrix} -l_{c_2} s_{12} \\ l_{c_2} c_{12} \end{Bmatrix}\ddot{\theta}_2 + \begin{Bmatrix} -l_1 c_1 - l_{c_2} c_{12} \\ -l_1 s_1 - l_{c_2} s_{12} \end{Bmatrix}\dot{\theta}_1^2 + 2\begin{Bmatrix} -l_{c_2} c_{12} \\ -l_{c_2} s_{12} \end{Bmatrix}\dot{\theta}_1\dot{\theta}_2$$

$$+ \begin{Bmatrix} -l_{c_2} c_{12} \\ -l_{c_2} s_{12} \end{Bmatrix}\dot{\theta}_2^2$$

Inverse dynamics (10, 10 → 6, 7)

6 $I_2 {}^I\dot{\omega}_2 = \tau_2 + (-{}^I r_{1,C_2}) \times (m_2 {}^I\dot{v}_{c_2} - m_2 g {}^I)$

$$\tau_2 = m_2 l_1 l_{c_2} c_2 \ddot{\theta}_1 + m_2 l_{c_2}^2 \ddot{\theta}_1 + I_2\ddot{\theta}_1 + m_2 l_{c_2}^2 \ddot{\theta}_2 + I_2\ddot{\theta}_2 + m_2 l_1 l_{c_2} s_2 \dot{\theta}_1^2$$
$$+ m_2 l_{c_2} g c_{12}$$

$$\tau_1 = m_1 l_{c_1}^2 \ddot{\theta}_1 + I_1\ddot{\theta}_1 + m_2[l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2} c_2]\ddot{\theta}_1 + I_2\ddot{\theta}_1$$
$$+ m_2 l_1 l_{c_2} c_2 \ddot{\theta}_2 + m_2 l_{c_2}^2 \ddot{\theta}_2 + I_2\ddot{\theta}_2 - 2 m_2 l_1 l_{c_2} s_2 \dot{\theta}_1\dot{\theta}_2 - m_2 l_1 l_{c_2} s_2 \dot{\theta}_2^2$$
$$+ m_1 l_{c_1} g c_1 + m_2 g[l_{c_2} c_{12} + l_1 c_1]$$

$$\therefore \quad \tau_1 = m_{11}\ddot{\theta}_1 + m_{12}\ddot{\theta}_2 - 2c\dot{\theta}_1\dot{\theta}_2 - c\dot{\theta}_2^2 + g_1$$
$$\tau_2 = m_{12}\ddot{\theta}_1 + m_{22}\ddot{\theta}_2 + c\dot{\theta}_1^2 + g_2$$

where
$$\begin{cases} m_{11} = m_1 l_{c_1}^2 + I_1 + m_2[l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2}\cos\theta_2] + I_2 \\ m_{22} = m_2 l_{c_2}^2 + I_2 \\ m_{12} = m_2 l_1 l_{c_2}\cos\theta_2 + m_2 l_{c_2}^2 + I_2 \\ c = m_2 l_1 l_{c_2}\sin\theta_2 \\ g_1 = m_1 l_{c_1} g\cos\theta_1 + m_2 g[l_{c_2}\cos(\theta_1+\theta_2) + l_1\cos\theta_1] \\ g_2 = m_2 l_{c_2} g\cos(\theta_1+\theta_2) \end{cases}$$