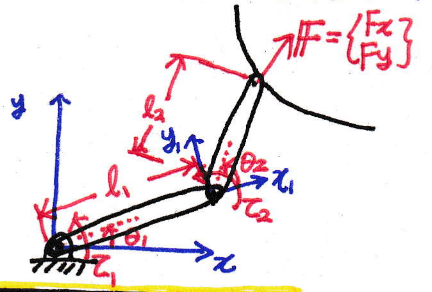
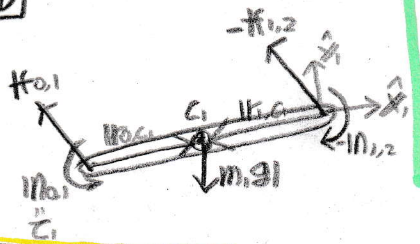
로봇공학 정역학 연습문제

*2-link 머니퓰레이터를 중력을 고려하여 statics 해석을 수행하시오. 20150339 김진민



i) Balances for link1

FBD



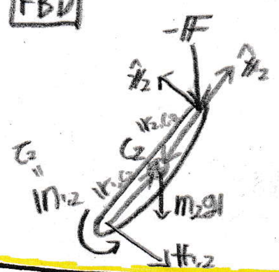
- Force : $f_{0,1} - f_{1,2} + m_1 g = 0 \rightarrow f_{0,1} = f_{1,2} - m_1 g$
- Moment : $n_{0,1} - n_{1,2} + (-r_{0,c_1}) \times f_{0,1} + (-r_{1,c_1}) \times (-f_{1,2}) = 0$

$$\Rightarrow n_{0,1} - n_{1,2} - \frac{l_1}{2}\hat{x}_1 \times f_{0,1} + \frac{l_1}{2}\hat{x}_1 \times (-f_{1,2}) = 0$$

$$\boxed{\Rightarrow n_{0,1} - n_{1,2} - l_1\hat{x}_1 \times f_{1,2} - \frac{l_1}{2}\hat{x}_1 \times (-m_1 g) = 0 \cdots ①}$$

ii) Balances for link 2

FBD

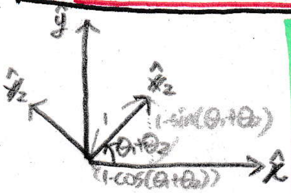


- Force : $f_{1,2} - F + m_2 g = 0 \rightarrow f_{1,2} = F - m_2 g$
- Moment : $n_{1,2} + (-r_{1,c_2}) \times f_{1,2} + (-r_{2,c_2}) \times (-F) = 0$

$$\Rightarrow n_{1,2} - \frac{l_2}{2}\hat{x}_2 \times f_{1,2} + \frac{l_2}{2}\hat{x}_2 \times (-F) = 0$$

$$\boxed{\Rightarrow n_{1,2} - l_2\hat{x}_2 \times F - \frac{l_2}{2}\hat{x}_2 \times (-m_2 g) = 0 \cdots ②}$$

iii) Computation of τ_2



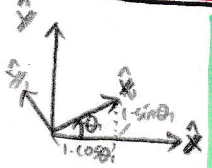
②식에서, $\tau_2 = n_{1,2} = l_2\hat{x}_2 \times F + \frac{l_2}{2}\hat{x}_2 \times (-m_2 g)$

$$= l_2 \begin{Bmatrix} \cos(\theta_1+\theta_2)\hat{x} \\ \sin(\theta_1+\theta_2)\hat{y} \\ 0\hat{z} \end{Bmatrix} \times \begin{Bmatrix} F_x\hat{x} \\ F_y\hat{y} \\ 0\hat{z} \end{Bmatrix} + \frac{l_2}{2} \begin{Bmatrix} \cos(\theta_1+\theta_2)\hat{x} \\ \sin(\theta_1+\theta_2)\hat{y} \\ 0\hat{z} \end{Bmatrix} \times \begin{Bmatrix} 0\cdot\hat{x} \\ +m_2 g\hat{y} \\ 0\hat{z} \end{Bmatrix}$$

$$= l_2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 \\ F_x & F_y & 0 \end{vmatrix} + \frac{l_2}{2} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 \\ 0 & m_2 g & 0 \end{vmatrix}$$

$$\boxed{\therefore \tau_2 = \left(l_2\cos(\theta_1+\theta_2)F_y - l_2\sin(\theta_1+\theta_2)F_x + \frac{l_2}{2}\cos(\theta_1+\theta_2)m_2 g\right)\hat{z}}$$

iv) Computation of τ_1



①식에서, $\tau_1 = n_{0,1} = \tau_2 + l_1\hat{x}_1 \times f_{1,2} + \frac{l_1}{2}\hat{x}_1 \times (-m_1 g) = 0$

$$= \tau_2 + l_1\hat{x}_1 \times (F - m_2 g) + \frac{l_1}{2}\hat{x}_1 \times (-m_1 g) = 0$$

$$= \tau_2 + l_1 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\theta_1 & \sin\theta_1 & 0 \\ F_x & F_y + m_2 g & 0 \end{vmatrix} + \frac{l_1}{2} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\theta_1 & \sin\theta_1 & 0 \\ 0 & m_1 g & 0 \end{vmatrix}$$

$$= \tau_2 + (l_1(F_y + m_2 g)\cos\theta_1 - l_1 F_x \sin\theta_1)\hat{z} + \left(\frac{l_1}{2} m_1 g \cos\theta_1\right)\hat{z}$$

$$\boxed{\therefore \tau_1 = \left[l_2\left(F_y + \frac{m_2 g}{2}\right)\cos(\theta_1+\theta_2) - l_2 F_x \sin(\theta_1+\theta_2) + l_1(F_y + m_2 g)\cos\theta_1 - l_1 F_x \sin\theta_1 + \frac{l_1 m_1 g}{2}\cos\theta_1\right]\hat{z}}$$

F_x, F_y로 정리하면,

$$\tau_1 = [-l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2)]F_x + [l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2)]F_y + \frac{l_2 m_2 g}{2}\cos(\theta_1+\theta_2) + l_1\left(m_2 + \frac{m_1}{2}\right)g\cos\theta_1$$

$$\tau_2 = [-l_2\sin(\theta_1+\theta_2)]F_x + [l_2\cos(\theta_1+\theta_2)]F_y + \frac{l_2 m_2 g}{2}\cos(\theta_1+\theta_2)$$