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What is a stellarator?*

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A stellarator is a toroidal plasma confinement concept that uses effects that arise in the absence of toroidal symmetry to maintain the magnetic configuration without the need for current drive. The largest magnetic fusion machines under construction are stellarators, and the plasma parameters achieved in stellarators are second only to those in tokamaks. Stellarators are poised for rapid progress toward showing the feasibility of fusion power. The physics and mathematical concepts that are required to understand stellarators are reviewed. © *1998 American Institute of Physics.* [S1070-664X(98)93905-2]

I. INTRODUCTION

The largest magnetic fusion machines under construction are two superconducting stellarators: the Large Helical Device (LHD) in Japan,[1] which is to begin operations in 1998, (Fig. 1) and the Wendelstein 7-X (W7-X) in Germany,[2] (Fig. 2), which is to begin operations in 2004. The plasma parameters achieved in stellarator experiments are second only to those in tokamaks: the energy confinement time is as long[3] as 43 ms, the ion temperature as high[4] as 1.6 kev, the electron temperature as high[5] as 4 kev, the plasma density as great[4] as $2\times10^{20}/\mathrm{m}^3$, and a volume-averaged plasma beta (plasma pressure over magnetic pressure) up[6] to 2.1%. The upper limit on the plasma beta is determined by the available power and not an instability.

The promise of the stellarator is the plasma performance of advanced tokamaks without the issues associated with driving the plasma current and disruptive plasma phenomena. To achieve these features, stellarators sacrifice the toroidal symmetry of the tokamak and are fully three-dimensional toroidal plasma confinement devices.

II. FORMATION OF MAGNETIC SURFACES

The gradient in the pressure of a confined plasma must be in a region in which the magnetic field lines lie in surfaces, the surfaces of constant pressure, P. This follows from the equilibrium equation, $\boldsymbol{\nabla} P=\mathbf{j}\times\mathbf{B}$, and its corollary $\mathbf{B}\cdot\boldsymbol{\nabla} P=0$. Topology limits the possible shapes of surfaces formed by field lines to just the torus. In tokamaks and stellarators the field lines wind around a torus both toroidally (the long way) and poloidally (the short way). The average angular advance of a field line poloidally each time it encircles the torus toroidally is 360° times iota, ι, which is called the rotational transform. The safety factor q, which is used in the tokamak literature, is $1/\iota$. The rotational transform, or poloidal winding of the field lines, in a tokamak is produced by a toroidal current in the central region of the plasma, Fig. 3.

A stellarator has the twist of the magnetic field lines, or rotational transform, produced in part by external coils. This is a possibility that many find counterintuitive. The difficulty arises from the absence of rotational transform due to external coils in toroidally symmetric systems. The Heliac stellarator[7] provides an example of how magnetic field lines can form surfaces in a curl-free field. First, have the toroidal current in the tokamak diagram, Fig. 3, produced by a coil, instead of a plasma current, with the toroidal current sufficiently large to form a $q=1$ surface. The field lines on a $q=1$ surface close after encircling the torus precisely once toroidally and once poloidally. Second, wind a helical coil around the outside of the torus along a path that is parallel to a field line of the $q=1$ surface, Fig. 4. The $q=1$ surface is then split by a magnetic island, the so-called $m=1$ island. The field lines in the interior of the magnetic island form toroidal surfaces, which contain no current. The stellarator that has been formed is called a Heliac.[7] A Heliac stellarator can also be produced by displacing the locations of the toroidal field coils, Fig. 5.

The rotational transform, or poloidal twist, of the magnetic field lines can be produced in three ways in toroidal devices: (1) A net plasma current flowing along the magnetic field lines, which is the principle used in the tokamak. (2) A torsion of the magnetic axis.[8] The magnetic axis is the central closed field line about which the other field lines wind. The torsion of a curve τ measures the extent to which the curve fails to lie in a plane. If the magnetic surfaces are nearly circular the transform due to the torsion is $\iota_\tau=\oint\tau dl/2\pi$. Axis torsion is the source of the transform in the Heliac. (3) A wobble of the field lines in resonance with noncircular deformation of the surfaces.[8] This type of rotational transform is produced by helical windings and is the source of the transform in LHD in the absence of a plasma. To obtain transform near the magnetic axis the magnetic surfaces must have an elliptical distortion with the major axis of the ellipse rotating as one goes around the torus by $M_p/2$ times 360°, with M_p the number of periods. In the simplest solution, the major axis of the ellipse is $1+\delta$ times the minor axis, the field lines wobble poloidally through an angle δ, and the transform is $\iota_w=M_p\delta^2/2$.

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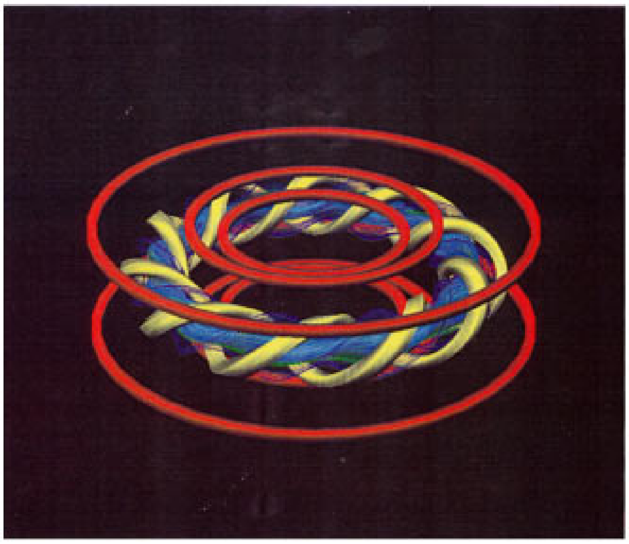

FIG. 1. *The LHD.* The coils and field lines of the LHD device are shown. The plasma will have a major radius of 3.9 m and a minor radius of 0.65 m. The field will be 3 T produced by steady-state superconducting coils.

The formation of magnetic surfaces in the absence of toroidal symmetry requires careful control of the magnetic field and is a constraint on the design of stellarators. As will be discussed in Sec. IV, the magnetic field lines in a stellarator have identical mathematical properties to the particle trajectories of Hamiltonians of one and one-half degrees of freedom.[9–12] This means a field line that encircles a torus can do the following: (1) close on itself, (2) come arbitrarily close to every point on a surface without deviating from the surface, or (3) pass arbitrarily close to every point in a nonzero volume of space. Only the first two possibilities exist in perfect toroidal symmetry. When a field line fulfills the third possibility, it is said to be chaotic or stochastic. Regions of chaotic field lines are inconsistent with the maintenance of a

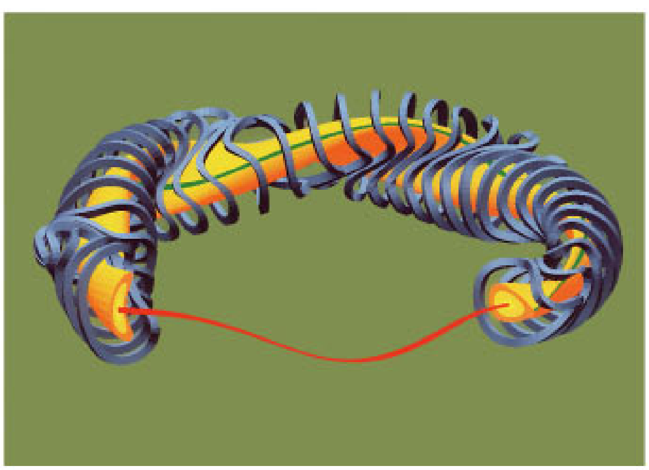

FIG. 2. *The W7-X.* The steady-state magnetic field of 3 T of W7-X will be produced by superconducting modular coils. The plasma major radius will be 5.5 m and the minor radius 0.52 m. The plasma cross section has a crescent shape where the magnetic axis curvature and the field strength are strongest and a triangular shape where the axis curvature and the field strength are weakest.

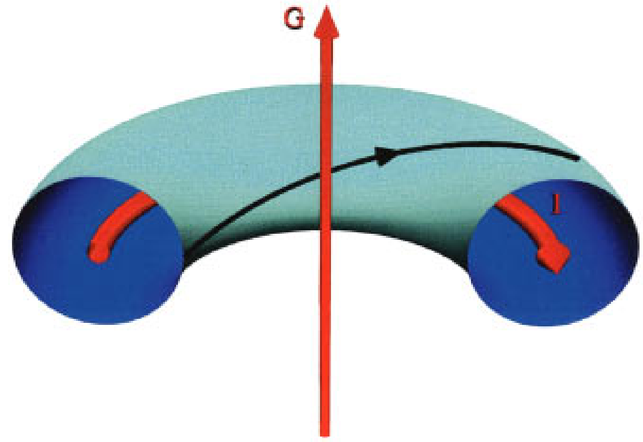

FIG. 3. *The tokamak concept.* The magnetic field of a tokamak is produced by a toroidal plasma current I in addition to the current G of the toroidal field coils. The vertical field coils, which are needed to balance the hoop stress on the toroidal current, are not shown. The black line is a closed magnetic field line on the $q=1$ surface.

pressure gradient with $\nabla P = \mathbf{j}\times\mathbf{B}$, and such regions must be small in practical stellarator designs.

III. STELLARATOR COILS

The complexity of the coils required to produce a stellarator configuration was long considered to be prohibitive for the manufacture and maintenance of stellarator power plants. However, the coils can be greatly simplified using the nonuniqueness of the coils required for a given stellarator configuration. An example of nonuniqueness is the Heliac, which can be formed by adding a helical winding to a toroidally symmetric set of coils, Fig. 4, or by displacing the locations of the toroidal field coil set, Fig. 5.

An important concept for the simplification of the coil set is modular coils,[13] which is the basis W7-X design,[2] Fig. 2. Such coils have been used successfully for many years in the Wendelstein 7-AS stellarator.[14] Modular coils are essentially toroidal field coils with helical shaping and can, in

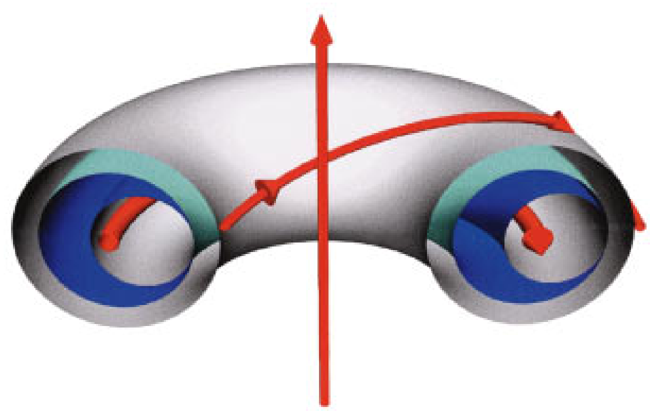

FIG. 4. *A Heliac produced by a helical coil.* If the configuration of Fig. 3 is augmented by a helical coil (red) that is parallel to a closed field line on the $q=1$ surface (the black line of Fig. 3), a magnetic island is formed. That is, the blue surface of Fig. 3 is split to form a magnetic island. The magnetic surfaces inside the island form a stellarator of a type known as a Heliac.

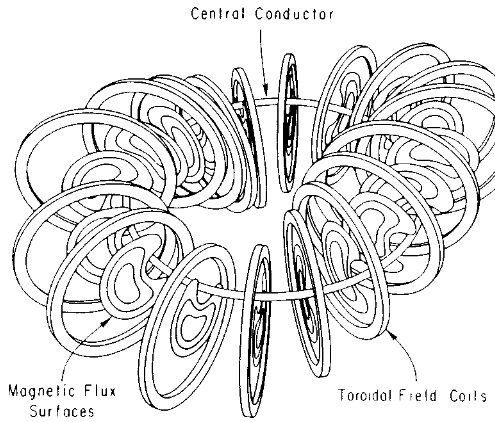

FIG. 5. *A Heliac produced by displaced coils.* A Heliac stellarator can also be produced by displacing the toroidal field coils from their toroidally symmetric positions in the configuration of Fig. 3.

principle, be used to produce any stellarator configuration. Another idea for a simplification of the coil set is symmetric toroidal field coils plus saddle coils. In the limit of small spatial dimensions, a saddle coil is a magnetic dipole. Potential advantages of saddle coils are (1) the formation of a number of stellarator configurations using a single set of coils and (2) a simplification of machine construction and maintenance. No stellarator has yet been designed based on the saddle coil concept, but work on the concept has just begun.

The magnetic field in the LHD device, Fig. 1, is produced by two large helical coils that carry current in the same direction plus circular coils that cancel the average vertical component of the field produced by the helical coils.[1] This coil design offers better access to the plasma than any other, but the two helical coils must be wound in place because of their size and because they interlock.

IV. MAGNETIC COORDINATES

Magnetic coordinates $(\psi_t, \theta, \varphi)$ are defined so they conform to the shape of the magnetic surfaces and trivialize the equations for the field lines. The radial coordinate ψ_t is the flux enclosed by a magnetic surface, θ is a poloidal angle, and φ is a toroidal angle, Fig. 6. Magnetic coordinates are required for understanding the physics of stellarators, much as cylindrical coordinates are required for understanding Laplace's equation, $\nabla^2\Phi = 0$, in cylindrical shapes. Magnetic

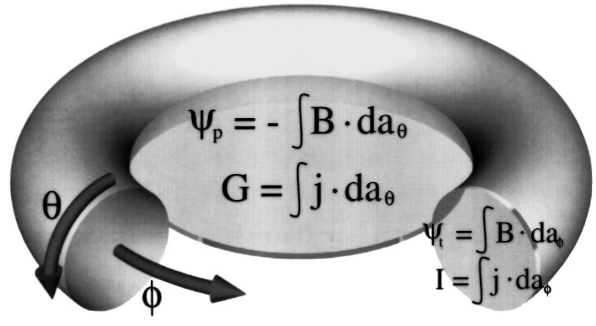

FIG. 6. Magnetic fluxes, angles, and currents.

coordinates greatly simplify calculations of the (1) trajectories of the charged particles that form the plasma, (2) properties of the plasma equilibrium, (3) stability of the plasma to distortions or breakup of the magnetic surfaces, which are called magnetohydrodynamic (MHD) instabilities, and (4) stability of the plasma to microinstabilities. Microinstabilities have a scale comparable to the ion gyroradius, are primarily perturbations in the electric potential, and are thought to lead to the enhanced plasma transport seen in experiments.

The most important form for the magnetic field is called the contravariant representation, in which the divergence-free nature of the field is explicit, and the equations for magnetic field lines are trivial. Any divergence-free vector can be written as crossed gradients, $\mathbf{B} = \nabla\psi_t \times \nabla\theta_0$, which is called the Clebsch representation. The Clebsch representation makes the divergence-free nature of $\mathbf{B}$ manifest for $\nabla\psi_t \times \nabla\theta_0 = \nabla\times(\psi_t\nabla\theta_0)$, and the divergence of a curl is zero. The two functions that appear in the Clebsch representation, $\psi_t(\mathbf{x})$ and $\theta_0(\mathbf{x})$, are constant along the magnetic field lines, $\mathbf{B}\cdot\nabla\psi_t = 0$ and $\mathbf{B}\cdot\nabla\theta_0 = 0$. The function $\psi_t(\mathbf{x})$ is defined as the magnetic flux enclosed by a surface of constant pressure and is called the toroidal flux, Fig. 6. The field lines wind poloidally and toroidally around the constant pressure surfaces, so a poloidal angle can be defined by $\theta \equiv \theta_0 + \iota(\psi_t)\varphi$, with $\iota(\psi_t)$ the rotational transform on the toroidal pressure surface and φ a toroidal angle, Fig. 6. Inserting this expression for the poloidal angle into the Clebsch representation, one obtains what is called the contravariant representation of the magnetic field,[15–17]

$$B = \nabla\psi_t \times \nabla\theta + \iota(\psi_t)\nabla\varphi \times \nabla\psi_t. \tag{1}$$

Two points should be made about the contravariant representation. First, the poloidal magnetic flux $\psi_p(\psi_t)$ is given by $d\psi_p/d\psi_t = \iota(\psi_t)$ with the integration constant set by making ψ_p the magnetic flux enclosed by the hole of the torus formed by the edge of the plasma, Fig. 6. Second, the angles of the contravariant representation are not uniquely defined. Suppose θ_1 and φ_1 are one set of angles. Let $\varphi_2 \equiv \varphi_1 + \nu(\mathbf{x})$, with $\nu(\mathbf{x})$ an arbitrary, single-valued, function of position. Then, φ_2 can be used as a toroidal angle if the poloidal angle is changed to $\theta_2 = \theta_1 + \iota\nu$. Both sets of angles satisfy the required equation, $\theta_0 = \theta - \iota\varphi$. The freedom of $\nu(\mathbf{x})$ implies essentially total freedom in the choice of toroidal angle while maintaining the contravariant representation.

The second representation of the magnetic field is called the covariant representation, and is defined to have a simple curl in the same sense that the contravariant representation is defined to have a simple divergence.

The existence of the covariant representation can be most easily demonstrated by assuming the magnetic field is curl-free in the toroidal region that would be occupied by the plasma. The magnetic field can then be written in terms of a scalar potential, $\mathbf{B} = \nabla\Phi_m$, and the scalar potential Φ_m, when properly normalized, can be used as the toroidal angle. To have a nontrivial field, a current, $G = \int \mathbf{j}\cdot d\mathbf{a}$, must be enclosed by the hole of the torus formed by the edge of the plasma, Fig. 6. Using Ampère's law and Stokes' theorem, $\mu_0 G$ equals $\oint \mathbf{B}\cdot d\mathbf{l}$, which is the change in potential Φ_m per transit of the torus. If the toroidal angle is chosen so it advances by unity each transit of the torus, instead of the conventional 2π, then one can let $\varphi=\Phi_m/\mu_0 G$ and

$$\mathbf{B}=\mu_0 G\boldsymbol{\nabla}\varphi. \tag{2}$$

The only issue is whether one can simultaneously choose the magnetic field to have a simple covariant and contravariant representation. The answer is yes because of the freedom of $\nu(\mathbf{x})$, in the definition of the toroidal angle of the contravariant representation.[18]

The magnetic field associated with a nontrivial plasma equilibrium has a curl. However, a simple covariant representation still exists,[17,18]

$$\mathbf{B}=\mu_0 G(\psi_t)\boldsymbol{\nabla}\varphi+\mu_0 I(\psi_t)\boldsymbol{\nabla}\theta+\mu_0\beta_*(\psi_t,\theta,\varphi)\boldsymbol{\nabla}\psi_t. \tag{3}$$

The poloidal current enclosed by the toroidal hole in a pressure, or magnetic, surface that contains toroidal magnetic flux ψ_t is $G(\psi_t)$. The toroidal current that is enclosed by that magnetic surface is $I(\psi_t)$, Fig. 6, and $\beta_*(\psi_t,\theta,\varphi)$ is related to the Pfirsch–Schlüter current, which will be discussed in Sec. VII.

A general coordinate system (ψ_t,θ,φ) is defined by giving the Cartesian coordinates as functions of the new coordinates: $X(\psi_t,\theta,\varphi)$, $Y(\psi_t,\theta,\varphi)$, and $Z(\psi_t,\theta,\varphi)$. For brevity one writes $\mathbf{X}(\psi_t,\theta,\varphi)$ with

$$\mathbf{X}(\psi_t,\theta,\varphi)=X(\psi_t,\theta,\varphi)\hat{\mathbf{x}}+Y(\psi_t,\theta,\varphi)\hat{\mathbf{y}}+Z(\psi_t,\theta,\varphi)\hat{\mathbf{z}}. \tag{4}$$

For example, if the trajectory of a particle is known in (ψ_t,θ,φ) coordinates, one can use $\mathbf{X}(\psi_t,\theta,\varphi)$ to plot the trajectory in ordinary Cartesian coordinates. By magnetic coordinates is meant any coordinate system (ψ_t,θ,φ) for which the simple contravariant representation of $\mathbf{B}$ exists, Eq. (1). By Boozer coordinates is meant a magnetic coordinate system for which the simple covariant representation of $\mathbf{B}$ also exists, Eq. (3).

For those not familiar with the mathematical theory of general coordinates,[17] the coordinate gradients, which appear in Eqs. (1)–(3), may appear difficult to calculate using Eq. (4). Coordinate gradients are easily calculated using the dual relations, which hold for any coordinate system,

$$\boldsymbol{\nabla}\psi_t=\frac{1}{\mathscr{J}}\frac{\partial\mathbf{X}}{\partial\theta}\times\frac{\partial\mathbf{X}}{\partial\varphi}, \tag{5}$$

with the coordinate Jacobian

$$\mathscr{J}=\frac{\partial\mathbf{X}}{\partial\psi_t}\cdot\left(\frac{\partial\mathbf{X}}{\partial\theta}\times\frac{\partial\mathbf{X}}{\partial\varphi}\right).$$

Even permutations of (ψ_t,θ,φ) in Eq. (5) also give valid relations. The dual relations follow from the orthogonality relations: $\boldsymbol{\nabla}\psi_t\cdot\partial\mathbf{X}/\partial\psi_t=1$, $\boldsymbol{\nabla}\psi_t\cdot\partial\mathbf{X}/\partial\theta=0$, $\boldsymbol{\nabla}\psi_t\cdot\partial\mathbf{X}/\partial\varphi=0$, etc. The orthogonality relations are derived using the chain rule. The dual relations can be used to show that the Jacobian is also given by the well-known expression, $1/\mathscr{J}=\boldsymbol{\nabla}\psi_t\cdot(\boldsymbol{\nabla}\theta\times\boldsymbol{\nabla}\varphi)$. If the (ψ_t,θ,φ) coordinates are consistent with both a simple covariant, Eq. (3), as well as contravariant representation, Eq. (1), then the dot product of the two representations implies $\mathscr{J}=\mu_0(G+\iota I)/B^2$.

As noted in Sec. II, magnetic field lines need not lie in surfaces throughout the volume of a toroidal plasma. Canonical coordinates (ψ_t,θ,φ), that are a generalization of magnetic coordinates, do not require that magnetic surfaces exist. The only constraint on the choice of the two angles (θ,φ) of canonical coordinates is that a third coordinate ρ exist, such that $\boldsymbol{\nabla}\rho\cdot(\boldsymbol{\nabla}\theta\times\boldsymbol{\nabla}\varphi)$ is nonzero. An arbitrary vector $\mathbf{A}(\mathbf{x})$ can then be written in the covariant form $\mathbf{A}=\psi_t\boldsymbol{\nabla}\theta-\psi_p\boldsymbol{\nabla}\varphi+\boldsymbol{\nabla}g$. Since $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$, the magnetic field has the generalized contravariant representation,

$$\mathbf{B}=\boldsymbol{\nabla}\psi_t\times\boldsymbol{\nabla}\theta+\boldsymbol{\nabla}\varphi\times\boldsymbol{\nabla}\psi_p(\psi_t,\theta,\varphi),$$

which is called the canonical representation of the magnetic field.[11] The poloidal flux ψ_p has been written using (ψ_t,θ,φ) as coordinates, which is a valid choice if $\mathbf{B}\cdot\boldsymbol{\nabla}\varphi=(\boldsymbol{\nabla}\psi_t\times\boldsymbol{\nabla}\theta)\cdot\boldsymbol{\nabla}\varphi$ is nonzero, as is the case in a stellarator. The magnetic field lines are given by the equations $d\psi_t/d\varphi=\mathbf{B}\cdot\boldsymbol{\nabla}\psi_t/\mathbf{B}\cdot\boldsymbol{\nabla}\varphi$ and $d\theta/d\varphi=\mathbf{B}\cdot\boldsymbol{\nabla}\theta/\mathbf{B}\cdot\boldsymbol{\nabla}\varphi$, which can be rewritten using the canonical representation as[11]

$$\frac{d\psi_t}{d\varphi}=-\frac{\partial\psi_p}{\partial\theta}\quad\text{and}\quad\frac{d\theta}{d\varphi}=\frac{\partial\psi_p}{\partial\psi_t}.$$

These equations are mathematically identical to the equations of Hamiltonian mechanics with a Hamiltonian $H(p,q,t)$ that is periodic in the time-like variable. In the mechanics literature, a Hamiltonian of this form is said to have one and one-half degrees of freedom and is well known as the simplest Hamiltonian system in which chaotic trajectories can arise. If the poloidal flux ψ_p is independent of the toroidal angle, as it is in a toroidally symmetric system like a tokamak, then ψ_p is a constant of the motion and the field lines must lie in perfect surfaces, the constant-ψ_p surfaces. Practical stellarator designs require that magnetic surfaces exist through the bulk of the plasma volume, so one must be able to approximate the poloidal flux of the canonical representation $\psi_p(\psi_t,\theta,\varphi)$ by a function of ψ_t alone.

V. PARTICLE TRAJECTORIES

An important corollary follows from the existence of a simple covariant, Eq. (3), as well as contravariant, Eq. (1), representation of the magnetic field. The properties of the trajectories of the charged particles that form a toroidal plasma are determined by the variation of the field strength, $B^2(\psi_t,\theta,\varphi)$, in the magnetic surfaces.[19–21]

Gibson and Taylor[22] showed in 1967 that the particle trajectories in the then-existing stellarator designs deviated unacceptably from the constant pressure surfaces. The poor confinement associated with these trajectories was long considered a fatal flaw of stellarators. The demonstration that the properties of particle trajectories are determined by the variation in the field strength was followed by two suggestions of methods for improving the trajectories: quasisymmetry[20,23] and linked mirrors.[24,25] Nührenberg and his collaborators found stellarator configurations that successfully incorporated these suggestions.[23,25] The linked mirror concept is the basis of the W7-*X* design,[2] and quasisymmetry is the basis of the Helically Symmetric Experiment (HSX) stellarator,[26] Fig. 7, being built at the University of Wisconsin, as well as

FIG. 7. *The HSX.* The HSX will have a field of 1 T of 0.1 s duration. The major radius of the plasma will be 1.2 m and the minor radius 0.15 m. The different colors on the coils indicate which coils are of each of the six distinct types.

the well-known Modular Helias-Like Heliac 2 (MHH2) design of Garabedian.[27,28] Particle trajectories that remain close to the constant pressure surfaces are now seen as a design constraint rather than a fatal flaw of stellarators.

The relation between the field strength variation and the particle trajectories will be derived for the special case of a curl-free magnetic field, and the general result will be stated. In the plasmas of interest for magnetic fusion, the gyroradii of the charged particles are small compared to the system size, so the particle trajectories are accurately given by the drift approximation. The most familiar expression for the drift motion in a curl-free magnetic field is due to Alfvén,[29]

$$\mathbf{v}_d=\frac{m(v_\parallel^2+\frac{1}{2}v_\perp^2)}{eB}\frac{\mathbf{B}}{B^2}\times\nabla\mathbf{B}.$$

The chain rule plus the contravariant representation of $\mathbf{B}$, Eq. (1), implies that $d\psi_t(\mathbf{x})/dt=\mathbf{v}_d\cdot\nabla\psi_t$ with $d\mathbf{x}/dt=\mathbf{v}_d+(v_\parallel/B)\mathbf{B}$. Using the covariant representation, Eq. (2), for the field

$$(\mathbf{B}\times\nabla B)\cdot\nabla\psi_t=-\mu_0G(\nabla\psi_t\times\nabla\varphi)\cdot\nabla B,$$

but $(\nabla\psi_t\times\nabla\varphi)\cdot\nabla B=(\nabla\psi_t\times\nabla\varphi)\cdot\nabla\theta\,\partial B/\partial\theta$. The dot product of the covariant and contravariant representations implies $(\nabla\psi_t\times\nabla\varphi)\cdot\nabla\theta=B^2/\mu_0G$. The drift motion conserves energy, $H=mv^2/2$, and magnetic moment, $\mu=mv_\perp^2/2B$. Putting the pieces together, one finds

$$\frac{d\psi_t}{dt}=-\frac{2H-\mu B}{eB}\frac{\partial B(\psi_t,\theta,\varphi)}{\partial\theta},$$

which gives $d\psi_t/dt$ in terms of constants of the motion, H and μ, and $B^2(\psi_t,\theta,\varphi)$. One can obtain analogous expressions for $d\theta/dt$ and $d\varphi/dt$.

The simplest description of particle drift trajectories in a general plasma equilibrium is the drift Hamiltonian,[21]

$$H(\theta,P_\theta,\varphi,P_\varphi)=\tfrac{1}{2}mv_\parallel^2+\mu B+e\Phi;$$

$$P_\theta=\frac{\mu_0 I}{B}mv_\parallel+e\psi_t;$$

$$P_\varphi=\frac{\mu_0 G}{B}mv_\parallel-e\psi_p.$$

The electric potential is $\Phi(\mathbf{x})$, the canonical poloidal angular momentum is P_θ, and the canonical toroidal angular momentum is P_φ. This Hamiltonian holds in time-dependent, as well as time-independent, electric and magnetic fields. For example, the loop voltage on a magnetic surface[18] is $\partial\psi_p(\psi_t,t)/\partial t$.

VI. TRAJECTORY CONFINEMENT CONCEPTS

The charged particles that form the plasma in a fusion power plant are in a paradoxical collisionality regime. The required energy confinement time for deuterium–tritium fusion is approximately 10^2 ion and 10^4 electron collision times. For this reason, the ions and electrons of a fusion plasma are sufficiently collisional to have essentially Maxwellian distributions. However, the electrons and ions move approximately 10 km between collisions, and the fusion alphas go about 1000 km before slowing down. Compared to a plasma radius of a couple of meters, the particles that form a fusion plasma are highly collisionless. The paradox of low and high collisionality can only be resolved if the ions deviate by no more than 10% and the electrons by no more than 1% of the plasma radius from a constant pressure surface between collisions. The transport associated with the deviations of particle drift trajectories from the constant pressure surfaces is called neoclassical transport. The large deviations of the trajectories[22] and the associated neoclassical transport were thought fatal flaws of stellarators, but can be solved using quasisymmetry or linked mirrors.

The simpler concept is quasisymmetry,[23] which is the basis of the HSX stellarator[26] at Wisconsin and the MHH2 design.[27,28] In a quasisymmetric stellarator the magnetic field strength has the form $B(\psi_t,\theta-N_p\varphi)$, with N_p an integer. That is, the field strength is a function of only two variables: a radial variable ψ_t and an angular variable $\Theta=\theta-N_p\varphi$. One uses the term quasisymmetric because the overall geometry of the stellarator depends on all three coordinates, Fig. 7. This is in contrast to a configuration with toroidal symmetry, like an ideal tokamak, in which both the field strength and the geometry are independent of the toroidal angle.

A distinction is made between two types of quasisymmetry. If $N_p=0$, the system is said to have quasiaxisymmetry. Examples are MHH2 and the ideal tokamak. If $N_p\neq 0$, the system is said to have quasihelical symmetry. An example is HSX with $N_p=4$.

The orbits and neoclassical transport in quasisymmetric stellarators are tokamak-like.[20] This follows from the existence of a conserved component of the canonical angular momentum, $P_h=P_\varphi+N_pP_\theta$. As in a tokamak, the conserved component of the canonical angular momentum confines the drift trajectories to within a banana orbit of the constant pressure surfaces, which is an acceptable deviation.

The second concept for obtaining good trajectory confinement is usually known as linked mirrors but in a more general context is called omnigeneity.[30–32] Examples are the[2] W7-X and the Smarth design.[33] The original concept[24] was to make the various minima of the field strength along a magnetic field line have the same value, $B_{\min}$. The reason for this condition is that the most deeply trapped particles stay near a minimum of the field strength and must drift in a way that keeps $\mu B_{\min}$ constant. If $B_{\min}$ is not constant on a ψ_t surface, the deeply trapped particles have to move in and out in ψ_t to maintain the constancy of $\mu B_{\min}$.

The condition of $B_{\min}$ constant on each of the magnetic surfaces of the plasma cross section places a constraint on the curvature of the magnetic axis. The curvature of the magnetic axis, $\kappa(\varphi)$, and its first derivative, $d\kappa/d\varphi$, should vanish at the place where the field strength has a minimum along the magnetic axis. The reason is that the variation in the field strength within the magnetic surfaces is proportional to the field line curvature. The deeply trapped particles near the magnetic axis are located where the field strength has a minimum, so if the curvature is nonzero they can $B\times\nabla B$ drift across the magnetic surfaces. The basic idea of linked mirrors is to make the magnetic axis straight near minima of the field strength and have the curvature, which is required to bend the axis to form a torus, concentrated in regions of higher field strength, Fig. 2.

Important new ideas have been put forward for flexible methods of achieving linked mirror, or more generally omnigeneous, configurations.[31–33] These configurations can have properties that are impossible to achieve in quasisymmetry. The best known of these properties, because they are utilized in the W7-X design, are the absence of a bootstrap current[2] and the location of the bulk of the trapped particles in a region of good magnetic curvature.[34] The physics of quasisymmetric configurations is too much like that of tokamaks to achieve either of these properties. In a stellarator, the bootstrap current can be beneficial through changes in the rotational transform profile or, as suggested by Reiman,[35] by working against the formation of magnetic islands. However, the existence of the pressure-driven bootstrap current makes the properties of the configuration more sensitive to the pressure profile. For this reason, the W7-X configuration was optimized to have essentially no bootstrap current. This can be done because the sign of the bootstrap current[20] in a quasisymmetric stellarator is determined by the sign of $\iota-N_p$. W7-X has strong Fourier terms in the field strength with $N_p=0$ and with $N_p=5$ and balances two opposing bootstrap currents to achieve essentially no net current. Locating the bulk of the trapped particles in a region of good magnetic curvature is thought to have a stabilizing effect on microinstabilities, but this potential benefit has not yet been quantified.

The existence of stellarator configurations that have quasisymmetry can be seen from the general expression for the magnetic field strength near a magnetic axis,[36]

$$B(\psi_t,\theta,\varphi)=B_a(\varphi)[1-\kappa(\varphi)x(\psi_t,\theta,\varphi)+\cdots];$$

$B_a(\varphi)$ is the field strength along the magnetic axis and must be constant for quasisymmetry. This can always be achieved by adjusting the currents in the toroidal field coils. The curvature of the axis, $\kappa(\varphi)$, is multiplied by $x=-\mathbf{r}(\psi_t,\theta,\varphi)\cdot\hat{\kappa}$ with $\mathbf{r}(\psi_t,\theta,\varphi)$, the distance between a (ψ_t,θ,φ) point on a magnetic surface and the axis. The magnetic field strength varies along the curvature vector $\boldsymbol{\kappa}=\kappa\hat{\boldsymbol{\kappa}}$, so $\mathbf{r}\cdot\boldsymbol{\kappa}$ measures the magnitude of this variation. The distance along the curvature, $-x(\psi_t,\theta,\varphi)$, has the form

$$x(\psi_t,\theta,\varphi)=w(\varphi)\sqrt{\psi_t}\cos[2\pi(\theta-N_\rho\varphi)],$$

with $w(\varphi)$ proportional to the width, along the curvature direction, of the magnetic surfaces near the axis. The square root of the toroidal flux, $\sqrt{\psi_t}$, gives the average radius of the surfaces. By choosing the width so $\kappa(\varphi)w(\varphi)$ is a constant, one achieves quasisymmetry. A curl-free magnetic field, $\nabla^2\Phi_m=0$, is determined, except for its magnitude, by the shape of an outermost surface formed by the field lines, so one can always choose κw to be a constant. It should be noted that the axis curvature $\kappa(\varphi)$ cannot vanish in a quasisymmetric stellarator. Also, the third method of producing rotation transform, a wobble of the field lines, is not consistent with quasisymmetry. Field line wobble carries the lines through regions of different field strengths. Quasisymmetric stellarators must have a magnetic axis with torsion, which means the magnetic axis cannot lie in a plane. For linked mirrors, one chooses $B_a(\varphi)$ to vary and makes $\kappa(\varphi)$ and $d\kappa/d\varphi$ vanish at the places where $B_a(\varphi)$ has its minima.

VII. EQUILIBRIUM PRESSURE LIMITS

If there were no variation in the magnetic field strength in the pressure surfaces of a toroidal plasma, then there would be no change in the geometric properties as the plasma pressure is changed. Unfortunately, it is impossible to make the field strength constant on a toroidal magnetic surface due to the field line curvature associated with a torus. The variations in the field strength in a magnetic surface give rise to a parallel current, called the Pfirsch–Schlüter current, which causes a distortion of the magnetic configuration. The physics is simple. The equilibrium equation $\nabla P=\mathbf{j}\times\mathbf{B}$ determines the current $\mathbf{j}_\perp$ that flows perpendicular to $\mathbf{B}$. The constraint that the current be divergence-free, $B\cdot\nabla(j_\parallel/B)=-\nabla\cdot\mathbf{j}_\perp$, determines the current along the magnetic field, $j_\parallel$.

The precise condition for the perpendicular current $\mathbf{j}_\perp$ to be divergence-free is $(\mathbf{B}\times\nabla B)\cdot\nabla P=0$, which is called the Palumbo condition.[37] The Palumbo condition, which is also the condition for particle drift trajectories not to cross the pressure surfaces, is inconsistent with toroidal equilibrium. The divergence of $\mathbf{j}_\perp$ associated with the breaking of the Palumbo condition makes $j_\parallel$ nonzero. This parallel current, the Pfirsch–Schlüter current j_{PS}, is determined by the variation in the field strength in the magnetic surfaces in the (ψ_t,θ,φ) coordinate system that has simple covariant, Eq. (3), and contravariant, Eq. (1), representations. The answer is[18]

$$\frac{j_{\mathrm{PS}}}{B}=\frac{2\mu_0}{B_{\mathrm{av}}^2}\frac{dP}{d\psi_t}\sum_{mn}\frac{mG+nI}{n-\iota m}\,\epsilon_{mn}\cos[2\pi(n\varphi-m\theta)]. \tag{6}$$

The $\epsilon_{mn}(\psi_t)$ are the Fourier coefficients of the field strength written in the form

$$\frac{1}{B^2(\psi_t,\theta,\varphi)}=\frac{1}{B^2_{\mathrm{av}}(\psi_t)}\left(1+2\sum_{mn}\ \epsilon_{mn}\cos[2\pi(n\varphi-m\theta)]\right).$$

Equilibrium pressure limits are determined by the magnetic field change associated with the Pfirsch–Schlüter current. This field change creates an unacceptable distortion of the configuration when the magnetic surfaces break up, forming magnetic islands and stochastic regions, or other properties of the stellarator become undesirable.

The Pfirsch–Schlüter current is amplified by the factor of $n-\iota m$ in the denominator of Eq. (6). Indeed, the factor $\epsilon_{mn}/(n-\iota m)$ is infinite on each rational surface (surfaces on which ι is the ratio of two integers) unless the resonant ϵ_{mn} vanishes. This feature of the Pfirsch–Schlüter current makes it important to control not only the variation in the field strength, the ϵ_{mn}, but also the range of ι. One might suppose that symmetric systems, like the tokamak, would be largely immune to problems associated with the singularity of $\epsilon_{mn}/(n-\iota m)$ on the rational surfaces. This supposition is false. The pressure-driven MHD instabilities of the tokamak are produced by these singularities.[38] The distinction between equilibrium and MHD stability is subtle when considering three-dimensional equilibria. In practice, equilibria that spontaneously break a symmetry of the underlying configuration (like toroidal symmetry for the tokamak or periodicity for a stellarator) are called instabilities.

VIII. STABILITY PRESSURE LIMITS

The efficiency of plasma confinement by a magnetic field is measured by the dimensionless parameter beta, $\langle\beta\rangle=2\mu_0\langle P/B^2\rangle$, with $\langle\cdots\rangle$ meaning an average over the plasma volume. The design for the International Tokamak Experimental Reactor (ITER) envisions[39] ignited plasma operations at $\langle\beta\rangle\approx 3\%$, but a higher beta is desirable for a fusion power plant.[40] In tokamaks, stable equilibria at high beta, $\langle\beta\rangle\approx 45\%$, have been found at very tight aspect ratio, the spherical tokamak.[41,42] While in stellarators the highest theoretical predictions for stable equilibrium betas are at infinite aspect ratio[43,44] with $\langle\beta\rangle$ greater than 30%.

Ballooning modes can limit $\langle\beta\rangle$ to less than 2% in either a tokamak or a stellarator[45,46] unless the configuration is specifically optimized to obtain a higher limit. Subtleties of the ballooning mode approximation in three-dimensional systems[47] may result in stability predictions that are more pessimistic than an exact analysis would yield. But, a ballooning mode analysis of an equilibrium is relatively easily performed and often used as a criterion for high beta configurations. Ballooning mode stability with $\langle\beta\rangle$ greater than 5% is consistent with the constraint of good orbits. Examples are the W7-X,[48,49] which is of the linked mirror type, and quasiaxisymmetric configurations that are being developed at the Princeton Plasma Physics Laboratory.[50]

Ballooning modes are obtained from an asymptotic form for the energy principle in the limit in which perturbations have a short wavelength across the magnetic field lines. The wavelength can be very long along the lines. The asymptotic expression for δW is[51]

$$\delta W=\int\left(\frac{1}{2\mu_0}b_\perp^2-(\xi\cdot\nabla P)(\xi\cdot\kappa)\right)d^3x.$$

The part of the perturbation magnetic field, $\nabla\times(\xi\times\mathbf{B})$, perpendicular to the unperturbed magnetic field is $\mathbf{b}_\perp$. The displacement of the field and plasma is $\xi=(\mathbf{B}\times\nabla\varphi_0)F/B^2$, with F a function that varies along a field line and $\nabla\varphi_0$ defined by the Clebsch representation, $\mathbf{B}=\nabla\varphi_0\times\nabla\psi_p$. The local curvature of the magnetic field lines is κ. The system is ballooning unstable if an F can be found that makes δW negative. The destabilizing term, $(\xi\cdot\nabla P)(\xi\cdot\kappa)$, is determined[52] by $B^2(\psi_t,\theta,\varphi)$ and quantities that are constant on a pressure surface, like $\iota(\psi_t)$. The stabilizing term $b_\perp^2$ is determined[52] by $|\nabla\varphi_0|^2$.

Unfortunately, there have not yet been sufficient studies to determine the limitations placed on stellarator design by a high stability value for $\langle\beta\rangle$, though the tools for such studies exist. In particular, it would be useful to understand the tradeoff between the aspect ratio and beta in stellarators.

IX. MICROSTABILITY

The enhancement of transport that is seen in toroidal devices above that predicted by the neoclassical theory is thought to be due to perturbations in the electric potential with a wavelength somewhat larger, but comparable, to the ion gyroradius. Such instabilities are called microinstabilities. The empirical scaling laws for transport in stellarators[53] are similar to those in tokamaks, but the flexibility of the stellarator configuration may give more control over anomalous transport processes.

The magnetic configuration enters the microinstability analyses[52] through $B^2(\psi_t,\theta,\varphi)$, $|\nabla\varphi_0|^2$, and $|\nabla\psi_p|^2$. These quantities are also important for ballooning mode stability. The quantities $|\nabla\varphi_0|^2$ and $\nabla\psi_p|^2$, which are defined by the Clebsch representation of the field, $\mathbf{B}=\nabla\varphi_0\times\nabla\psi_p$, arise from the divergence of the polarization drift. The field strength, or $B^2(\psi_t,\theta,\varphi)$, arises in the divergence of the $\mathbf{E}\times\mathbf{B}$ drift and in the direction of the drift precession of the trapped particles.

There has been little theoretical work on the effect of the magnetic configuration on microstability, though the basic tools for doing such studies exist. Such studies would provide fundamental checks on the theory, through comparison to stellarator experiments, and may lead to improved stellarator configurations.

X. CONCLUSIONS

The large stellarator programs in Japan and Europe are focused on integrated tests of steady-state disruption-free plasmas of sufficient size to provide scaling information for the design of a stellarator burning-plasma experiment. This leaves the United States with the opportunity to develop and demonstrate the physics of even better stellarators, configurations that have compactness, higher beta, good particle orbits, improved microstability, better divertors, and simpler coils. These issues can be addressed by theory and by short pulse (less than 5 s) moderate cost experiments. The development of improved stellarator configurations is an area in which the United States can afford to be a world leader, despite the relatively modest size of its magnetic fusion program, ≈20% of the world total. Few individuals have worked on the problem of stellarator optimization and many possibilities exist—both for finding better optimization criteria and for finding configurations that meet these criteria.

The reasons stellarators were thought unacceptable for power plants have been shown to have been unfortunate features of early designs: (1) The unacceptable drifts of particle trajectories away from the constant pressure surfaces is addressed by the quasisymmetric and linked mirror concepts. (2) The complexity of the coils is addressed by modular coils and possibly saddle coils. (3) The low beta by $\langle\beta\rangle$ greater than 5% in practical designs and $\langle\beta\rangle$ greater than 30% at an infinite aspect ratio. (4) The unit size of stellarators is large but comparable to that of advanced tokamaks, about 1 GW electric.[54]

The requirement for compact, or tight aspect ratio, stellarators is driven by the need for inexpensive physics experiments, not the unit size of power plants. The Three Gorges Dam in China is expected to produce 18 GW of electric power. The perceived limitation of approximately 1 GW for fusion power systems is more a political than a technical limitation. In the United States the development of new energy sources, such as fusion, is not presently perceived as of critical importance. If that perception were to change, the minimum size of power plants would be far less relevant than it now appears.

A relatively small financial investment has been made in the operating stellarators in comparison to that in the operating tokamaks. Nevertheless, a large database exists that demonstrates the critical physical principles of stellarators. A much larger database will exist after the LHD and the W7-X stellarators begin operations in 1998 and 2004, respectively. However, important ideas for stellarator optimization await experimental tests, and advances could be made in the theoretical basis for stellarator optimization.

Stellarators circumvent two major issues in the design of tokamak power plants: the maintenance of the magnetic field configuration (current drive) and disruptive plasma behavior,[55,56] while preserving the excellent confinement properties of the tokamak.[53] They are poised for rapid progress toward showing the feasibility of fusion power. Stellarators offer the United States a leadership role in the development of even more desirable fusion configurations, despite the budgetary limitations of the US program.

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