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Radial electric field and plasma confinement in the vicinity of a magnetic island

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A theory to determine the radial electric field in the vicinity of a magnetic island in tokamaks based on the island-induced symmetry-breaking transport flux is presented. It is shown that the radial electric field is governed by a nonlinear equation that can have multiple equilibrium solutions as plasma parameters change. This results in a bifurcated state of the radial electric field. It is possible that turbulent fluctuations can be suppressed and improved confinement develops in the vicinity of the island. The evolution of the toroidal momentum equation with the effects of the symmetry-breaking flux included is also presented. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1491533]

I. INTRODUCTION

It is difficult to determine the toroidal rotation in tokamaks because of the toroidal symmetry. The Chew–Goldberg–Low viscosity[1] vanishes in the toroidal direction. Higher-order (in the gyro-radius ordering) viscosity[2–4] is too small to account for the experimentally observed toroidal momentum confinement time, which is of the same order as the ion energy confinement time. (Some argue, though, that gyro-viscosity could explain it.[5]) Thus, an anomalous toroidal viscosity is needed to model toroidal rotation in tokamaks. With the presence of a magnetic island, at least two more mechanisms can affect the toroidal rotation. One is the electromagnetic torque induced by the interaction between the islands and the wall or error fields.[6] The other is the plasma viscosity induced by the distortion of the magnetic surface in the vicinity of the islands.[7] Here, we examine the effects of the plasma viscosity due to the distortion of the magnetic surface on the radial electric field without considering the effects of island interaction with the wall or error fields, or anomalous toroidal viscosity.

The particle fluxes we obtain here and that in Ref. 7 are indicative of the toroidal component of the plasma pressure tensor because the leading-order perturbed particle distribution function that contributes to the fluxes has no variation along the magnetic field line. Here, we use the more familiar terminology, such as particle fluxes, to describe the processes of interest. However, at the end we indicate how these results can be used in the toroidal momentum equation in the presence of a magnetic island.

The theory may also play a role in the plasma confinement in the vicinity of the lower-order rational surfaces observed in the tokamak experiments.

II. PARTICLE AND HEAT FLUX IN THE COLLISIONLESS REGIME

It can be shown that in the regime where the collision frequency is less than the bounce frequency of the toroidally trapped particles, the enhanced particle diffusion coefficient scales like $1/\nu$, where ν is the collision frequency.[7] It is obvious that this $1/\nu$ scaling cannot persist indefinitely as ν decreases. The poloidal particle drift speed will eventually limit the size of the drift orbit and the particle diffusion coefficient will scale like ν as the collision frequency decreases further.

In large aspect ratio tokamaks, the poloidal $\mathbf{E}\times\mathbf{B}$ speed is larger than the $\nabla\mathbf{B}$ drift and curvature drift for thermal particles. One can approximate the poloidal drift speed as the $\mathbf{E}\times\mathbf{B}$ speed $V_{\mathbf{E}\times\mathbf{B}}$. The size of the drift orbits Δr is approximately $rV_{\mathrm{dr}}/V_{\mathbf{E}\times\mathbf{B}}$, where r is the radius and V_{dr} is the radial drift speed due to the $\nabla\mathbf{B}$ drift and curvature drift. The physical meaning of this approximation is that the possibility of forming super-bananas is neglected. Super-bananas are formed due the cancellation of the $V_{\mathbf{E}\times\mathbf{B}}$ and the $\nabla\mathbf{B}$ drift and curvature drift in the poloidal direction.[8,9] Note that including the effect of the super-bananas does not change the dependence of the particle flux on the radial electric field.[10] Thus, we will neglect this effect in exploring the electric field dependence of the transport fluxes. The diffusion coefficient D is then phenomenologically of the order of

$$D \sim f_t \nu_{\mathrm{eff}} (\Delta r)^2 \sim \nu (rV_{\mathrm{dr}}/V_{\mathbf{E}\times\mathbf{B}})^2/\varepsilon^{1/2}, \tag{1}$$

where $f_t \sim \varepsilon^{1/2}$ is the fraction of the toroidally trapped particles, $\nu_{\mathrm{eff}} \sim \nu/\varepsilon$ is the effective collision frequency, and ε is the inverse aspect ratio. This heuristic derivation is, of course, based on a random walk argument.

We now proceed with a detailed calculation by neglecting the possibilities of the super-bananas and collisionless detrapping/retrapping. The drift kinetic equation for the trapped particles in an axisymmetric tokamak in the presence of a magnetic island in the low collisional frequency regime is[7]

$$\langle \mathbf{v}_d \cdot \nabla \alpha \rangle_b \partial f/\partial \alpha + \langle \mathbf{v}_d \cdot \nabla \Psi \rangle_b \partial f_{\mathrm{M}}/\partial \Psi = \langle C(f) \rangle_b, \tag{2}$$

where angular brackets $\langle A \rangle_b = [\oint d\theta\, B\, A/(v_\parallel \mathbf{B}_0 \cdot \nabla \theta)]/[\oint d\theta\, B/(v_\parallel \mathbf{B}_0 \cdot \nabla \theta)]$ denotes the bounce average, $v_\parallel$ is the particle speed parallel to the magnetic field $\mathbf{B}$, $\mathbf{B}_0$ is the equilibrium magnetic field, f is the perturbed par-

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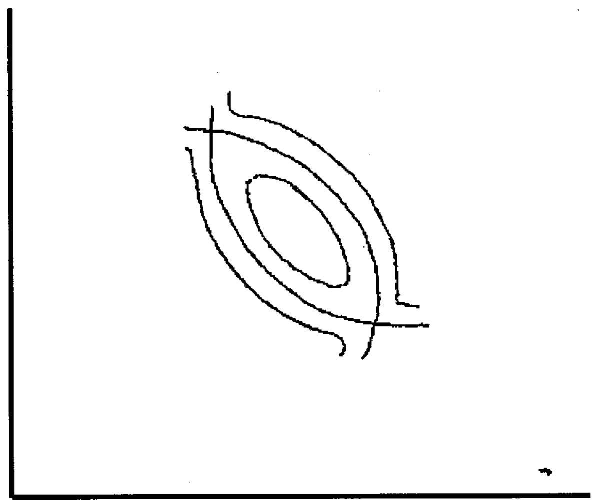

FIG. 1. A schematic diagram of a magnetic island. The constant Ψ contour is shown here. The helical angle α is similar to the polar angle in this diagram.

ticle distribution, $\mathbf{v}_d$ is the particle drift velocity, $\alpha=\theta-\zeta/q_s$ is the helical angle, q_s is the safety factor q at the rational surface where the island is centered, Ψ is the helical flux function, $C(f)$ is the Coulomb collision operator, and f_{M} is the Maxwellian distribution. A schematic diagram of a magnetic island is shown in Fig. 1. The radial surface label of constant Ψ is shown. The bounce averaged drift speed in the $\nabla\alpha$ direction is

$$\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b=-(I/M\Omega)(\partial\Psi/\partial\psi)(\mathbf{B}_0\cdot\nabla\theta/B)\times(\partial J/\partial\Psi)/(\partial J/\partial E), \tag{3}$$

where $I=RB_t$, R is the major radius, B_t is the toroidal magnetic field strength, $B=|\mathbf{B}|$, M is the mass, Ω is the gyrofrequency, ψ is the unperturbed equilibrium poloidal flux function, and E is the particle energy. The second adiabatic invariant $J=\oint d\theta\, v_\parallel$ is

$$J=16(\mu B\Delta/M)^{1/2}[\mathbf{E}(\kappa)-(1-\kappa^2)\mathbf{K}(\kappa)], \tag{4}$$

where $\Delta=\varepsilon_s+(\pm\delta_w)(\overline{\Psi}+\cos m\alpha)^{1/2}$, ε_s is the inverse aspect ratio evaluated at the rational surface, $\delta_w=[2(q_s)^2\tilde{\psi}/(q_s'Br_s)]^{1/2}/R$ is the width of the island divided by the major radius, $\tilde{\psi}$ is the perturbed poloidal flux due to the existence of the island, $q_s'=dq/dr$ evaluated at the rational surface, r_s is the radius of the rational surface, $\overline{\Psi}$ is the normalized helical flux function defined as the ratio of the helical flux function to $(-\tilde{\psi})$, m is the poloidal mode number of the island, μ is the magnetic moment, $\mathbf{E}(\kappa)$ and $\mathbf{K}(\kappa)$ are the complete elliptic integrals of the second kind and first kind, $\kappa^2=(E-\mu B_0-e\Phi+\mu B_0\Delta)/(2\mu B\Delta)$, e is the electric charge, and Φ is the electrostatic potential. The quantity J is calculated based on the magnetic field model $B=B_0(1-\Delta\cos\theta)$, valid in the vicinity of a magnetic island.[7] With J given in Eq. (4), we obtain

$$\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b=-(I/M\Omega)(\partial\Psi/\partial\psi)(\mathbf{B}_0\cdot\nabla\theta/B)[\mu B_0(2\mathbf{E}/\mathbf{K}-1)\partial\Delta/\partial\Psi-e\Phi'], \tag{5}$$

where $\Phi'=\partial\Phi/\partial\Psi$. For a large aspect tokamak where $\Delta\ll 1$, the $\mathbf{E}\times\mathbf{B}$ drift dominates the $\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b$ drift away from the island separatrix for thermal particles when $e\Phi/T\approx 1$. Here, T is the plasma temperature. For simplicity we assume that the $\mathbf{E}\times\mathbf{B}$ drift speed dominates the $\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b$ drift speed. This assumption removes the possibility of forming super-bananas which are caused by the cancellation of the ∇B drift and the $\mathbf{E}\times\mathbf{B}$ drift so that $\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b\approx 0$. Note that because κ^2 is a function of α through its dependence on δ_w, it is possible that trapped particles can be collisionlessly detrapped or the circulating particles can be trapped collisionlessly when κ^2 is close to 1. We will ignore such a possibility, i.e., we neglect the δ_w dependence *after* we obtain the radial drift speed from it. This is equivalent to expanding J in terms of δ_w. Neglecting both of these effects, namely, super-banana and collisionless trapping/detrapping, will not affect the radial electric dependence in the transport fluxes. The bounce averaged radial drift speed is

$$\langle \mathbf{v}_d\cdot\nabla\Psi\rangle_b=(I/M\Omega)(\partial\Psi/\partial\psi)(\mathbf{B}_0\cdot\nabla\theta/B)[\mu B_0(2\mathbf{E}/\mathbf{K}-1)\partial\Delta/\partial\alpha]. \tag{6}$$

It is obvious that Eq. (2) is a complex partial differential equation with complete elliptical integrals as coefficients. To make progress, we approximate $\mathbf{K}\approx(\pi/2)(1+\kappa^2/4+\cdots)$ and $\mathbf{E}\approx(\pi/2)(1-\kappa^2/4+\cdots)$ for the trapped particles where $\kappa^2<1$. With these approximations, we have $(2\mathbf{E}/\mathbf{K}-1)\approx 1-\kappa^2$. Note that this approximation is not accurate at or close to the boundary where κ^2 is unity, as expected. However, this approximation is consistent with our purpose, since we neglect the possibility of the collisionless trapping and detrapping that can occur when κ^2 is close to unity. The bounce averaged collision operator is

$$\langle C(f)\rangle_b=(\nu/B)[\partial(J\mu\partial f/\partial\mu)/\partial\mu]/(\partial J/\partial E). \tag{7}$$

Since κ^2 is a more convenient pitch angle parameter, $\partial/\partial\mu$ in Eq. (7) can be approximated by $\partial/\partial\mu\approx-(1/2\mu\Delta)\partial/\partial\kappa^2$. Because we neglect the α dependence in κ, $\langle C(f)\rangle_b$ is not a function of α either, except through the distribution f itself. With this approximation, the solubility constraint is dramatically simplified, as will be clear later.

Now we solve Eq. (2) in the regime where the collision frequency is less than the $\mathbf{E}\times\mathbf{B}$ drift frequency in the $\nabla\alpha$ direction. We expand Eq. (2) in terms of this small parameter. The lowest order equation is

$$\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b\partial f_0/\partial\alpha+\langle \mathbf{v}_d\cdot\nabla\Psi\rangle_b\partial f_{\mathrm{M}}/\partial\Psi=0, \tag{8}$$

and the next order equation is

$$\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b\partial f_1/\partial\alpha=\langle C(f_0)\rangle_b, \tag{9}$$

where f_0 and f_1 are the lowest order and the next order perturbed particle distribution functions. Because we only keep the $\mathbf{E}\times\mathbf{B}$ drift in $\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b$ and neglect the α dependence in κ, the solubility constraint in Eq. (9) is simplified to

$$\oint d\alpha f_0/(\partial\Psi/\partial\psi)=0. \tag{10}$$

Note that the extra $\partial\Psi/\partial\psi=1-q/q_s$ factor in Eq. (10) is from $\langle \mathbf{v}_d\cdot\nabla\alpha\rangle_b$. Integrating Eq. (8) we obtain

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$$f_0 = -(\mu B_0/e\Phi')(1-\kappa^2)[(\pm\delta_w)(\overline{\Psi}+\cos m\alpha)^{1/2} \pm C_1]\partial f_M/\partial\Psi, \tag{11}$$

where C_1 is an integration constant. It is obvious that C_1 is determined from Eq. (10). Substituting Eq. (11) into Eq. (10), we find

$$C_1 = -(\pi/2)(1+\overline{\Psi})^{1/2}/\mathbf{K}(\kappa_f) \tag{12}$$

where $\kappa_f^2 = 2/(1+\overline{\Psi})$ is a parameter that separates the inside and the outside region of the island. For the region outside the separatrix $\kappa_f^2 < 1$. Substituting Eq. (12) into Eq. (11), we obtain

$$f_0 = \pm(\mu B_0/e\Phi')(1-\kappa^2)\delta_w[(\overline{\Psi}+\cos m\alpha)^{1/2} - (\pi/2) \times(1+\overline{\Psi})^{1/2}/\mathbf{K}(\kappa_f)]\partial f_M/\partial\Psi. \tag{13}$$

Note that f_0 has the desirable boundary condition, namely, it vanishes at $\kappa^2 = 1$.

The particle flux can be calculated from Eq. (13). It is defined as $\Gamma = \langle N\mathbf{V}\cdot\nabla\Psi\rangle$:

$$\Gamma = \int d\theta \int d\alpha \int d^3v\, \mathbf{V}_d\cdot\nabla\Psi f/\nabla\Psi \times\nabla\alpha\cdot\nabla\theta \Bigg/ \left(\int d\theta \int d\alpha/\nabla\Psi\times\nabla\alpha\cdot\nabla\theta\right), \tag{14}$$

where N is plasma density, $\mathbf{V}$ is the plasma flow velocity, and the angular brackets denote the flux surface average. Instead of substituting Eq. (13) directly into Eq. (14), we use Eq. (2) to express Eq. (14) in terms of the collision operator and thus $\partial f_0/\partial\kappa^2$:

$$\Gamma = -(\pi/\langle g^{1/2}\rangle)\int d\alpha \int dE(\nu q_s/MB\Delta)[(\partial\Psi/\partial\psi) \times(\mathbf{B}_0\cdot\nabla\theta/B)\partial f_M/\partial\Psi]^{-1}\int_0^1 d\kappa^2 J(\partial f_0/\partial\kappa^2)^2, \tag{15}$$

where $\langle g^{1/2}\rangle = \int d\theta \int d\alpha/\nabla\Psi\times\nabla\alpha\cdot\nabla\theta$. Calculating $\partial f_0/\partial\kappa^2$ from Eq. (13) and inserting the result in Eq. (15), we find

$$\Gamma = -0.22N\nu(cT/eBr)^2(\delta_W/\omega_E)^2\varepsilon^{-1/2}G(\overline{\Psi})(P'/P + e\Phi'/T - 0.5T'/T), \tag{16}$$

where P is the plasma pressure, c is the speed of light, $\omega_E = cE_\Psi/(Br)$ is the $\mathbf{E}\times\mathbf{B}$ angular speed, $E_\Psi = -d\Phi/d\Psi$ is the radial electric field, and $G(\overline{\Psi}) = (1+\overline{\Psi}) \times\{[\mathbf{E}(\kappa_f)/\mathbf{K}(\kappa_f)] - [(\pi/2)/\mathbf{K}(\kappa_f)]^2\}$. We approximate the energy dependence in the collision frequency to be $(1/v)^3$ to obtain Eq. (16). We are interested in the exterior region of the magnetic island where $\overline{\Psi} > 1$ and $\kappa_f < 1$. Note that Eq. (16) has the same scaling as that in Eq. (1) if one realizes that V_{dr} is proportional to δ_W because the radial drift results from the finite width of the magnetic island, and $V_{\mathbf{E}\times\mathbf{B}}$ is proportional to ω_E. The particle flux Γ in Eq. (16) is applicable for both electrons and ions as long as $\nu_{\text{eff}} < \omega_E(RB_P)(q_s r_w/q_s)$ with B_P the poloidal magnetic field strength.

The heat flux Q can be defined as

$$Q/T = \left[\int d\theta \int d\alpha \int d^3v(x^2 - \tfrac{5}{2})\mathbf{V}_d\cdot\nabla\Psi f/\nabla\psi \times\nabla\alpha\cdot\nabla\theta\right]\left(\int d\theta \int d\alpha/\nabla\psi\times\nabla\alpha\cdot\nabla\theta\right)^{-1}, \tag{17}$$

where $x^2 = Mv^2/2T$ and is

$$Q/T = -0.22N\nu(cT/eBr)^2(\delta_W/\omega_E)^2\varepsilon^{-1/2}G(\overline{\Psi}) \times[-0.5(P'/P + e\Phi'/T) + 4.5T'/T]. \tag{18}$$

Note that the particle flux and the heat flux are Osanger symmetric.

Rigorously speaking, the fluxes calculated here are not applicable in the vicinity of the x-point of the island separatrix because it is possible there is a new class of particles in that region and a boundary layer around the x-point. However, because they are continuous across the x-point, they are mathematically valid in the region around the x-point.

III. RADIAL ELECTRIC FIELD, PLASMA CONFINEMENT, AND TOROIDAL MOMENTUM EVOLUTION IN THE VICINITY OF A MAGNETIC ISLAND

The particle flux in Eq. (16) should be connected to its collisonal limit,[7]

$$\Gamma = -0.5(N/\nu) \times(cT/eBr)^2(RB_P)^2(q_s r_W/q_s)^2(m\delta_W)^2\varepsilon^{3/2}H(\overline{\Psi}) \times(P'/P + e\Phi'/T + 2.5T'/T), \tag{19}$$

where $H(\overline{\Psi}) = -\{[8(2)^{1/2}/3](\overline{\Psi}^2-1)^{1/2}Q_{1/2}^1(\overline{\Psi})\}(1+\overline{\Psi})^{1/2}/\mathbf{K}(\kappa_f)$, and $Q_{1/2}^1$ is the associated Legendre function. We again approximate the energy dependence in the collision frequency to be $(1/v)^3$ to obtain Eq. (19). We perform a very simple connection. We neglect the T'/T term for simplicity. Examining Eqs. (16) and (19), we find the following formula seems to connect these two expressions well:

$$\mathcal{J} = (\nu/\varepsilon)(RB_P)^2(q_s' r_w/q_s)^2(m\delta_W)^2\varepsilon^{1/2}/\{\omega_E^2(RB_P)^2 \times(q_S' r_W/q_s)^2 m^2[0.22G(\overline{\Psi})]^{-1} + (\nu/\varepsilon)^2 \times[0.5H(\overline{\Psi})]^{-1}\}. \tag{20}$$

Note that in Eq. (20), we do not display the common factor $N(cT/eBr)^2(P'/P + e\Phi'/T)$ in the particle flux. One can see that in the collisional limit, Eq. (20) reproduces Eq. (19) and in the collisionless limit, it reproduces Eq. (16) except for the common factor.

With Eqs. (16), (19), and (20), we can determine the radial electric field in the vicinity of a magnetic island without including the anomalous plasma viscosity and the effects of a wall or error fields. In this case the radial electric field is determined by

$$\Gamma_i = \Gamma_e, \tag{21}$$

where Γ_i is the ion particle flux and Γ_e is the electron particle flux.

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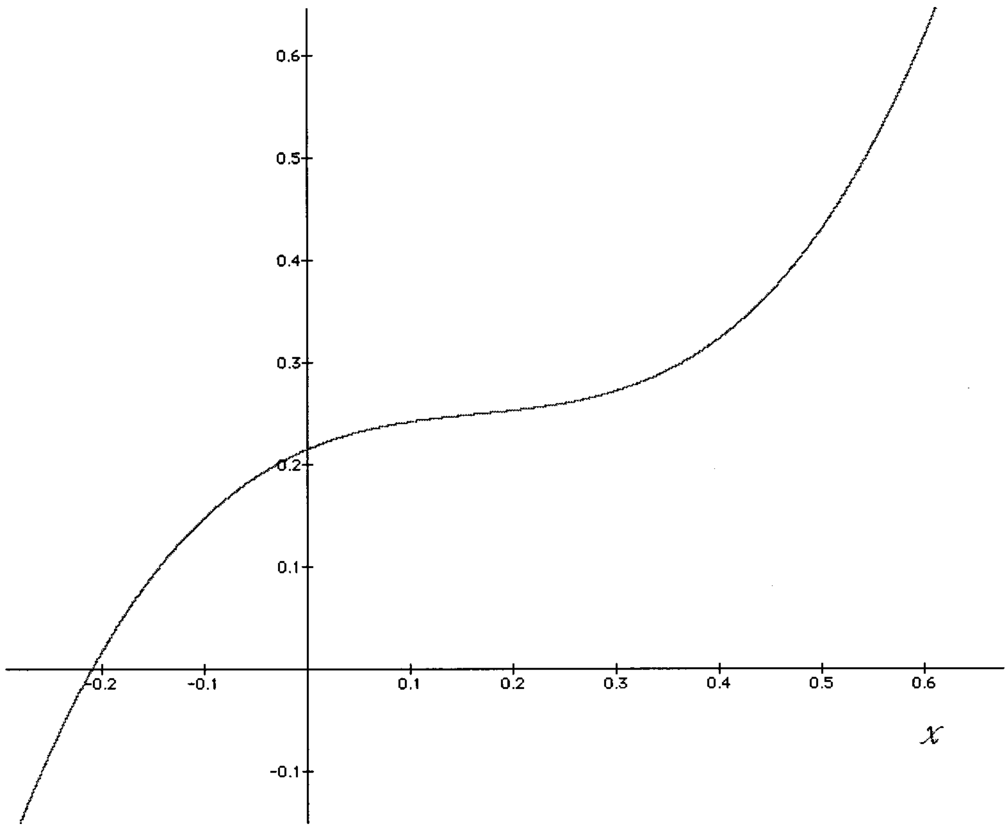

FIG. 2. The parameter $\nu_i/\varepsilon=0.1$. There is one equilibrium solution.

For simplicity, we assume that the ion temperature is the same as the electron temperature. We also assume electrons are in the $1/\nu$ regime, i.e., we only employ Eq. (19) in Γ_e. With these approximations, we can cast Eq. (21) in a simple form,

$$m^2G(\mathcal{X}/C)^3+m^2G(\mathcal{X}/C)^2+[(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2+(\nu_i/\varepsilon)^2]H(\mathcal{X}/C^3)-[(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2-(\nu_i/\varepsilon)^2]H/C^2=0, \tag{22}$$

where $\mathcal{X}=\omega_E(RB_P)(q_s' r_w/q_s)$, $C=(cT/|e|Br)(RB_P)(q_s' r_W/q_s)(N'/N)$, $G^{-1}=0.22G(\psi)$, and $H^{-1}=0.5H(\psi)$. Equation (22) is a nonlinear equation for the radial electric field variable $\mathcal{X}$. It can have multiple equilibrium solutions. To illustrate this qualitative behavior, we simplify Eq. (21) further by approximating $G\sim H\sim 1$. This approximation is appropriate for our purpose of illustrating bifurcation mechanism because both G and H are order of unity in the vicinity of the island. Of course, G and H will introduce extra radial dependence besides those from the plasma parameters if we are interested in calculating the radial electric field profile. The solutions of Eq. (22) are shown in Figs. 2–4 for the parameters $C=-0.5$, $m=2$, and $(M_i/M_e)^{1/2}=43$ with M_i the ion mass and M_e the electron mass. There is one equilibrium solution for $\nu_i/\varepsilon=0.1$ as shown in Fig. 2. When ν_i/ε decreases to 0.0316, there are three equilibrium solutions as shown in Fig. 3. The one in the middle is not stable. The new equilibrium solution has a larger value of the radial electric field and has the opposite sign. This new equilibrium solution is likely to have better plasma confinement. As ν_i/ε decreases further, the two roots on the left almost merge into one and there is one stable solution with better confinement. This is shown in Fig. 4. Note that the two left roots merged into one in the very low collision frequency limit could be a result of our approximation using only the $1/\nu$ regime result in the electron flux.

There is another bifurcation mechanism in the higher collisionality regime. This mechanism is described in a similar problem.[11] We will not illustrate this mechanism here again.

With the two possible bifurcation mechanisms, it is possible that in the vicinity of a magnetic island, the radial electric field can bifurcate to a large value. This in turn will suppress the turbulent fluctuations due to the radial gradients of the **E×B** drift and the diamagnetic drift and improve the overall plasma confinement in the vicinity of a magnetic island. This process of confinement improvement is the same as the one employed for the H (high)-mode theory.[12,13] The difference is only in the bifurcation mechanism. It has been observed in tokamak experiments that plasma confinement improves in the vicinity of lower-order rational surfaces.[14–17] The theory presented here may play a role in that phenom-

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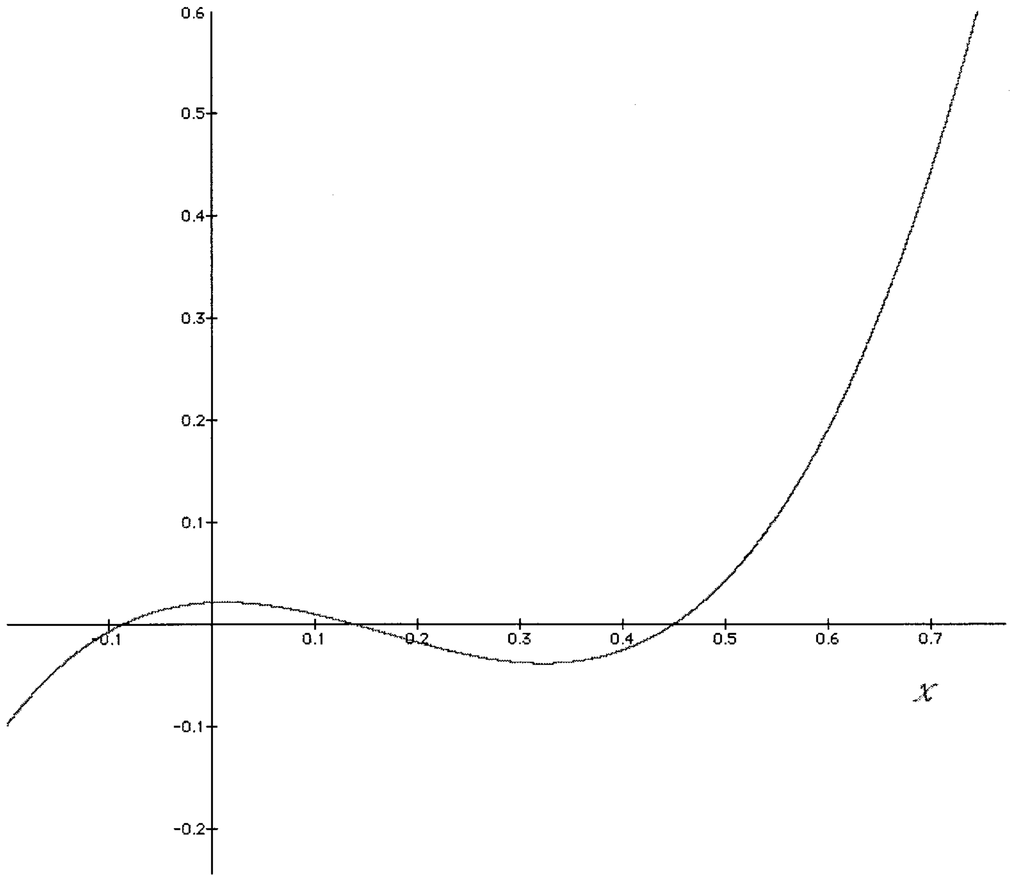

FIG. 3. The parameter $\nu_i/\varepsilon = 0.0316$. There are three equilibrium solutions. The one in the middle is unstable. The one on the right is the new solution.

enon because there can be magnetic islands centered on the lower-order rational surfaces

The symmetry-breaking-induced fluxes calculated here and in Ref. 7 are driven by the toroidal component of the pressure tensor, i.e., $\langle N\mathbf{V}\cdot\nabla\Psi\rangle = (c/e)\langle\nabla\Psi\times\nabla\theta\cdot\nabla\cdot(P\mathbf{I} + \boldsymbol{\pi})/\mathbf{B}\cdot\nabla\theta\rangle$, where $\mathbf{I}$ is the identity tensor, and $\boldsymbol{\pi}$ is the viscous tensor. The toroidal momentum damping can, thus, be affected by the presence of the island. The evolution equation without including the island–wall interaction, island–error-field interaction, and anomalous toroidal viscosity is

$$\partial\langle NM\mathbf{V}\cdot\nabla\Psi\times\nabla\theta/\mathbf{B}\cdot\nabla\theta\rangle/\partial t = -\Sigma_j e_j\langle N_j\mathbf{V}_j\cdot\nabla\Psi\rangle/c, \tag{23}$$

where the subscript j indicates the species. The condition $\Gamma_i = \Gamma_e$ in Eq. (21) can be understood as the steady solution of Eq. (23). Note that the electromagnetic field momentum is neglected in Eq. (23). The effect of the symmetry-breaking fluxes on the rotation of the island itself will be investigated separately.

We like to note that we have not included the anomalous toroidal plasma viscosity, which is known to be relevant in the toroidal force balance equation in tokamak experiments unless the fluxes calculated here dominate. Whether including such effects will change our results qualitatively remains to be investigated.

IV. CONCLUDING REMARKS

We have calculated the particle flux and heat flux in the low collisionality regime in the vicinity of a magnetic without considering the effects of super-bananas and collisionless trapping/detrapping. We find it scales linearly with respect to collision frequency and decreases as the value of the radial electric field increases. We determine the radial electric field by connecting this low collisionality regime result to the collisional result obtained earlier. We have shown that in the vicinity of a magnetic island the equilibrium radial electric field is governed by a nonlinear equation. It has multiple equilibrium solutions in the parameter space. We also illustrate that the condition $\Gamma_i = \Gamma_e$ employed to determine the radial electric field can be understood as the steady-state solution of the toroidal momentum equation.

The bifurcation mechanism discussed here could lead to a large gradient of the poloidal $\mathbf{E}\times\mathbf{B}$ and the diamagnetic angular velocity. This in turn suppresses turbulent fluctuations and improves plasma confinement in the vicinity of a magnetic island. This theory of the confinement improvement is the same as that for the H-mode. The difference is only in the bifurcation mechanism. We speculate that if in the vicinity of the lower-order rational q surfaces there are magnetic islands, the theory developed here may be employed to explain the plasma confinement improvement in the vicinity of the lower order rational q surfaces observed in the toka-

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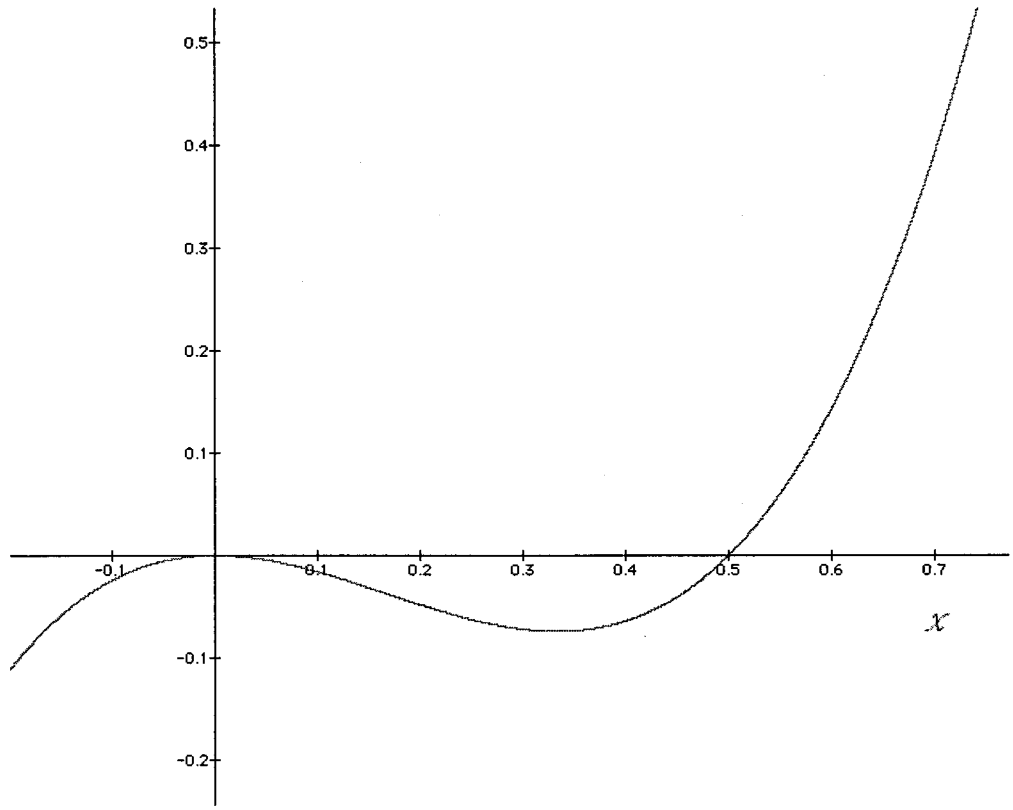

FIG. 4. The parameter $\nu_i/\varepsilon = 0.0001$. The two solutions on the left almost merge into one.

mak experiments.[14–17] Experiments like the one carried out in Ref. 18 are needed to test the theory by measuring plasma rotation and radial electric field in the vicinity of an island.

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