Energy confinement time of a plasma as a function of the discharge parameters in Tokamak-3



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# ENERGY CONFINEMENT TIME OF A PLASMA AS A FUNCTION OF THE DISCHARGE PARAMETERS IN TOKAMAK-3

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ABSTRACT. The authors investigate the thermal insulation of plasmas in Tokamak-3, taking as thermal insulation parameter the energy confinement time of the plasma $\tau_E$ and relating it to the discharge parameters: the stabilizing magnetic field $H_z$, the charged particle density $n_e$ and the discharge current I. It is found that for $n_e \geq 2 \times 10^{13}$ cm$^{-3}$ $\tau_E$ is virtually independent of $H_z$, while for $n_e = (1-5) \times 10^{13}$ cm$^{-3}$ $\tau_E$ increases somewhat with $n_e$ and in proportion to I. The experimental results fit the empirical formula $\tau_E \sim a^2 H_\varphi$.

## INTRODUCTION

The purpose of the present article is to investigate the thermal insulation of plasmas in devices of the Tokamak type. Theory predicts for such systems a fairly deep magnetic well (~ 3 - 10%) and, under certain obvious assumptions regarding the current density distribution over the plasma column cross-section, substantial shear (~ 0.05 - 0.1). We may therefore expect plasmas in such devices to be free of a number of dangerous instabilities. Although it is difficult to distinguish between the effects which different stabilizing factors have on plasma confinement in actual Tokamak experiments, comparison with systems where these factors are not very pronounced indicates the general scale of those changes which can result.

As the parameter characterizing thermal insulation we took the time of energy confinement in a plasma $\tau_E$, which is generally known as the energy lifetime of the plasma and is determined by means of the equation for the energy balance of a plasma column:

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + P \qquad (1)$$

where E is the plasma energy and P the ohmic heating power (in this paper, per length of plasma column). From measurements of E and P, $\tau_E$ was calculated as a function of t using relation (1).

The measurements were made in Tokamak-3, with a plasma column having a major diameter of 200 cm and a minor diameter of 20 - 30 cm.

The stabilizing magnetic field $H_z$ varied between 17 kOe and 30 kOe and the charged particle density $n_e$ between $8 \times 10^{12}$ cm$^{-3}$ and $5 \times 10^{13}$ cm$^{-3}$. The discharge current pulse was approximately trapezoidal with an amplitude of 20 - 90 kA and a duration of 15 - 25 ms. As working gas we used pure hydrogen, which was introduced at a constant rate into the discharge chamber. The absolute values of $\tau_E$ were found to lie in the range 0.1 - 10 ms. If we take the Bohm diffusion time $\tau_B = 3a^2 eH/ckT_e$ [1] as the thermal insulation parameter for purposes of comparison between Tokamak devices and systems with small magnetic well and shear, we find values of the ratio $\tau_E/\tau_B$ in the range 20 - 30.

## 1. METHODS OF DETERMINING THE PLASMA PARAMETERS

The method proposed by Artsimovich and co-workers [1] was used to determine $\tau_E$.

It relies essentially on the fact that, with fairly general assumptions regarding the equilibrium of a plasma column and its symmetry, it is possible to derive a system of three equations linking the measured values of the parameters with E, P and L (inductance per unit length of plasma column). Knowing E and P, it is possible to determine $\tau_E$ with the help of expression (1). This system of equations may be written in the form

$$\frac{(\Delta\phi)H_z}{2\pi(0.1\,I)^2} = 1 - \beta = 1 - \frac{4E}{3(0.1\,I)^2} \qquad (2)$$

$$\frac{I\,V\,10^7}{2\pi R} = P + \frac{d}{dt}\,\frac{L}{2}\,(0.1\,I)^2 \qquad (3)$$

$$\frac{R\,U_-}{b\,U_+} + 1 = \frac{L}{2} + \beta \qquad (4)$$

where $\Delta\phi$ is the change in the magnetic flux over the plasma cross-section, I the discharge current, V the voltage measured at the socket in the copper enclosure, $H_z$ the strength of the stabilizing magnetic field, $U_-$ the difference between and $U_+$ the sum of the signals from magnetic probes outside and inside the plasma column, R the major radius of the torus, b the radius of the copper shell and $\beta = 4E/3(0.1I)^2$ the ratio of the mean gas-kinetic pressure to the magnetic pressure of the field due to the current.

The first and third of the above equations follow from assumed equilibrium of the plasma column with respect to the minor and major radius of the torus, while the second equation results from the law of energy conservation.

The equations are analysed in detail in Appendix I. The main result of this analysis is the conclusion that, for the Tokamak-3 discharge parameters and regimes considered, all the assumptions made in deriving Eqs (2), (3) and (4) are satisfied with the required accuracy. The errors in determining $\tau_E$ are estimated.

Knowing L and assuming a parabolic current distribution over the plasma column cross-section, it is possible to estimate the minor radius a of the plasma column. Knowing a and P, it is then possible to determine the mean electric conductivity $\sigma_p$ of the plasma column.

On the other hand, knowing E and the spatial distribution of the plasma density $n_e(r)$, it is possible to calculate the mean plasma temperature $T_e + T_i = T$.

The method of determining $\sigma_p$ and $T_e + T_i$ is considered in greater detail in Appendix 2.

We shall comment on the physical meaning of the energy lifetime of a plasma.

It follows from Eq. (1) that $\tau_E$ characterizes the total power lost from the plasma, including losses due to radiation as well as losses by thermal conductivity and diffusion.

Bolometric determination of the power of the radiated energy, performed by L.L. Gorelik and V.V. Sinitsyn using a method similar to the one employed in Ref. [9], indicates that, under normal Tokamak-3 operating conditions, radiation losses amount to no more than 30% of the total energy lost by the plasma. Thus, $\tau_E$ is essentially a parameter characterizing plasma losses associated with thermal conductivity and diffusion.

## 2. PHENOMENOLOGICAL DEPENDENCE OF $\tau_E$ ON THE DISCHARGE PARAMETERS

The main parameters determining the discharge regime in Tokamak devices are: the strength of the stabilizing magnetic field $H_z$, the charged particle density $n_e$, the strength of the discharge current I and - it would seem - the shape of its pulse. If the shape of the current pulse is left unchanged (2 - 3 ms current rise, 5 - 6 ms plateau, 6 - 10 ms current decay), with the phenomenological approach the discharge regime in Tokamak-3 may be considered a function of three quantities: $H_z$, $n_e$ and I. It is therefore natural to consider $\tau_E$ in the first place as a function of these quantities.

Figure 1, for two typical discharge regimes with different values of $n_e$ and I, shows the time dependences of the combined electron and ion temperatures ($T_e + T_i$); the electron temperature calculated from the electric conductivity $T_{e\sigma}$; the total plasma energy E; the displacement of the centre of the plasma column relative to the centre of the copper shell $\Delta$; the plasma radius a; and the power lost from the plasma W. All these dependences were obtained on the basis of experimental data by means of the method described in the annexes. It is worth noting that in both

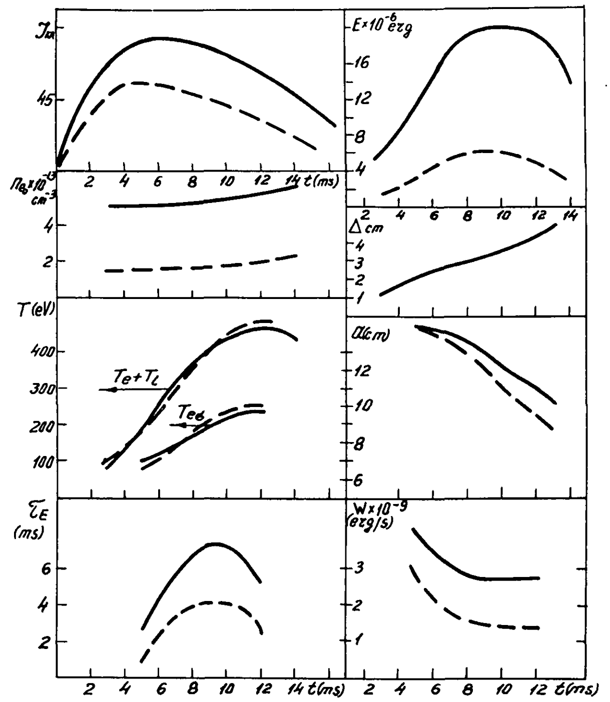

FIG.1. Two typical Tokamak-3 discharge regimes, the full curves $n_{e0}$ = 5 - 6 × $10^{13}$ cm$^{-3}$, $I_{max}$ = 76 kA, the dashed curves designate $n_e$ = 2 × $10^{13}$ cm$^{-3}$, $I_{max}$ = 48 kA.

cases the energy lifetime of the plasma changes considerably during the discharge, thereby - it would seem - making it difficult to compare plasma energy lifetimes in different discharge regimes.

It has become clear, however, that when the discharge current pulse retains its shape the functions $\tau_E(t)$ behave in approximately the same manner; this enables one to plot $\tau_E$ against $H_z$, $n_e$ and I for some moment of time $t = t_0$. The choice of $t_0$ remains somewhat arbitrary. For example, it may be taken between 8 ms and 10 ms from the beginning of the discharge, where $\tau_E$ changes only slightly and can be determined with maximum accuracy.

### 2.1. DEPENDENCE OF $\tau_E$ ON THE STRENGTH OF THE LONGITUDINAL MAGNETIC FIELD $H_z$

The function $\tau_E(H_z)$ was studied for longitudinal magnetic field strengths in the range 15 - 25 kOe and 25 - 34 kOe with fixed values of $n_e$ and I(t). It was shown - to within the accuracy of the measurements - that, as long as the plasma column retains its magnetohydrodynamic stability ($q = H_z a / H_\varphi R \geq 3$), $\tau_E$ is independent of $H_z$.

The accuracy of the determination of $\tau_E$ was such that, in the charged particle density region $n_e \geq 2 \times 10^{13}$ cm$^{-3}$, the dependence of $\tau_E$ on $H_z$ can be said to be no stronger than $H_z^{\pm\frac{1}{2}}$.

The relationship $W(H_z)$ is found to be even weaker. This is in accordance with the results of earlier works [2, 3], in which it was shown that after a plasma column has achieved magnetohydrodynamic stability its electric conductivity $\sigma$ - and accordingly the power lost $W \approx I^2/\sigma$ - ceases to depend on $H_z$.

For $n_e < 1 \times 10^{13}$ cm$^{-3}$ the accuracy with which $\tau_E$ can be measured is such that the relationship between $\tau_E$ and $H_z$ can be said to be no stronger than a first-degree dependence.

## 2.2. DEPENDENCE OF $\tau_E$ ON THE AMPLITUDE OF THE DISCHARGE CURRENT I

In Fig. 2, for $t_0 = 9$ ms, the functions $\tau_E(I)$ are plotted for three magnetic field strengths: $H_z = 17$, 25 and 34 kOe.

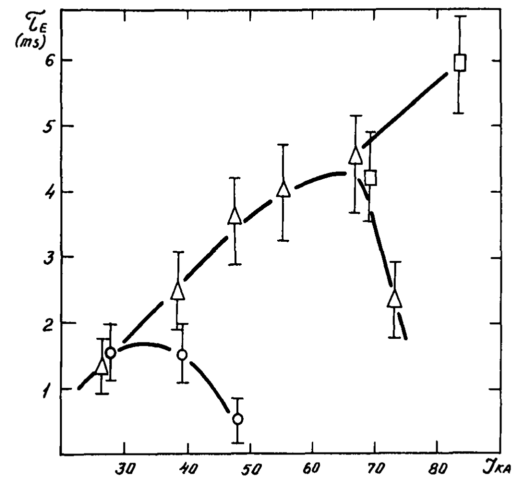


FIG. 2. The function $\tau_E(I)$ for three stabilizing field values.

| | |
|---|---|
| O - H = 17 kOe | $n_{e0} = 3 \times 10^{13}$ cm$^{-3}$ |
| Δ - H = 25 kOe | $n_{e0} = 3 \times 10^{13}$ cm$^{-3}$ |
| □ - H = 34 kOe | $n_{e0} = 2 \times 10^{13}$ cm$^{-3}$ |

It follows from the data presented that for fixed $H_z$ the energy confinement time of the plasma increases with rising current to a certain limit after which it decreases rapidly.

The accompanying oscillations on the voltage and discharge current derivative oscillograms indicate large-scale instabilities, apparently of a magnetohydrodynamic nature. Increases in $H_z$ displace the above-mentioned limit towards higher discharge currents corresponding as a rule to $q = 2.5 - 3$. Thus, it may be assumed that for a stabilized plasma column thermal insulation is determined by the magnetic field due to the discharge current.

## 2.3. DEPENDENCE OF $\tau_E$ ON THE CHARGED PARTICLE DENSITY $n_e$

Figure 3 shows $\tau_E$, $(T_e + T_i)$ and $T_{e\sigma}$ as functions of the maximum density $n_{e0}$ for $t_0 = 8$ ms. It can be seen that $\tau_E$ rises slightly with increasing $n_{e0}$.

A very important conclusion can be drawn from these data. If the thermal insulation of a plasma were determined exclusively by ion losses, then provided that the electron-ion interaction mechanism were completely Coulombic, $\tau_E$ would be approximately equal to the electron-ion energy transfer time and should increase substantially with decreasing $n_{e0}$. The broken line in Fig. 4 is the corresponding theoretical curve of $\tau_0(n_e) \approx 17\, T_e^{3/2}/n_e$, where $T_e$ is the electron temperature in °K. Since the expected increase in $\tau_E$ is not observed, it may be concluded that with decreasing $n_{e0}$ the energy losses become attributable primarily to the electron branch, or the nature of the interaction between electrons and ions becomes non-Coulombic.

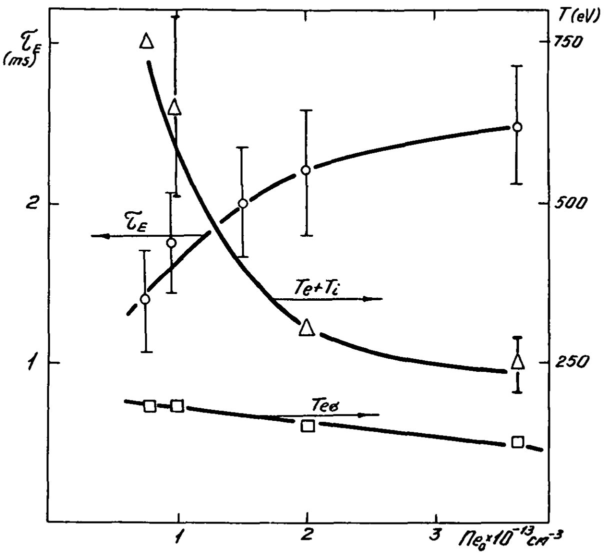


FIG. 3. The functions $\tau_E(n_{e0})$, $T(n_{e0})$ and $T_{e\sigma}(n_{e0})$; $H_z = 25$ kOe, I = 45 kA.

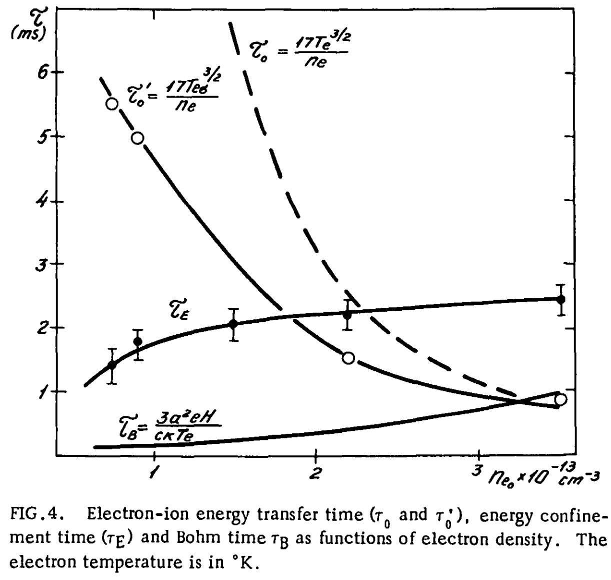


FIG. 4. Electron-ion energy transfer time ($\tau_0$ and $\tau_0'$), energy confinement time ($\tau_E$) and Bohm time $\tau_B$ as functions of electron density. The electron temperature is in °K.

The possibility of such an interaction is suggested by the fact that the gap between $T = (T_e + T_i) \approx T_e$ and $T_{e\sigma}$ increases with decreasing $n_{e0}$.

Having assumed initially that the visible manifestation of such an interaction is to be the increase in the electron-ion collision frequency, it is possible to calculate the corresponding energy transfer time $\tau_0' \approx 17T_{e\sigma}^{3/2}/n_e$. The curve of $\tau_0'(n_e)$ is shown in Fig. 4 as a continuous line. From a comparison with the experimental relationship $\tau_E(n_{e0})$ it may be concluded that at low density the observed plasma losses are attributable to the ion branch in a non-Coulombic electron-ion interaction only if five to six times more energy is transferred from the electron to the ion with each interaction event than in the case of Coulomb collisions.

If such electron-ion interactions occur at all, then it may be assumed that when $n_{e0} < 2\times10^{13}$ cm$^{-3}$ the observed plasma energy losses are attributable primarily to the electron branch.

## 2.4. PLASMA HEATING

Let us consider in greater detail the dependence of the ratio $T/T_{e\sigma}$ on the discharge parameters. We introduce the function $\eta = T^{3/2}/T_{e\sigma}^{3/2}$. In the discharge parameter region, where $T_i \ll T_e$, this function is obviously the ratio of the electric conductivity of the plasma calculated by means of Spitzer's formula $\sigma = \sigma_0\, T_e^{3/2}$ to the electric conductivity determined experimentally. If on the other hand $T_e = T_i$, the additional function remains proportional to this ratio, with the coefficient $K \simeq 2.8$.

In spite of some uncertainty regarding the intermediate case, where $T_i < T_e$, the function is of interest as an integral characteristic of the efficiency of the plasma heating.

In Tokamak-3 experiments, $\eta$ was found to lie between 2 and 6 - 8. It was noted that as a rule it changed only slightly during the discharge, and for $n_{e\sigma} > 2\times10^{13}$ cm$^{-3}$ was weakly dependent on $H_z$, $n_e$, I and T.

For $n_{e0} < 2\times10^{13}$ cm$^{-3}$ it increased substantially with decreasing $n_{e0}$, and in some cases with increasing I; this is in qualitative agreement with the results of Bobrovsky et al. [4].

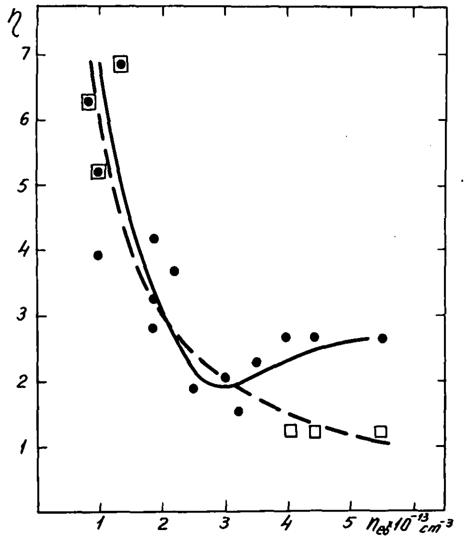


FIG. 5. The functions $\eta(n_{e0})$ - dots - and $\eta'(n_{e0})$ - squares. The broken line is the curve of $\sim 1/n_{e0}$.

The function $\eta(n_{e0})$ is shown in Fig. 5. It will be seen that there is some increase in $\eta$ for $n_{e0} = (3-5)\times10^{13}$ cm$^{-3}$. It can be attributed to heating of the ion component.

The point is that in the quasi-stationary state ($dT/dt = 0$) the electron and ion temperatures must be linked by the simple relation $T_i = T_e/(1+\tau_0/\tau_E)$, where $\tau_0$ is the electron-ion energy transfer time. Let us assume that the electron-ion interaction is Coulombic. Then, since $\tau_0/\tau_E$ decreases with increasing electron density, one may expect the electron and ion temperatures to even out at fairly high values of $n_{e0}$. Knowing $\tau_E$ and $(T_e + T_i)$, it is possible to determine these values of $n_{e0}$. In the experiments described they were $(3-5)\times10^{13}$ cm$^{-3}$, while the corresponding mean ion temperatures $T_i$ varied between 100 eV and 200 eV.

Knowing $T_i$, it is then possible to construct the function $\eta' = T_e^{3/2}/T_{e\sigma}^{3/2}$ for these discharge regimes. It is equal to unity to within the accuracy of the experiment.

## 3. DEPENDENCE OF THE ENERGY LIFETIME OF A PLASMA ON THE DISCHARGE PARAMETERS

The phenomenological relationships obtained can be used to explain the direct connection between thermal insulation and discharge parameters. Strictly speaking, such a method cannot give unique solutions. The point is that within the framework of the experiments described each change in the basic parameters ($H_z$, $n_e$, I) causes a corresponding change in q, the temperature $(T_e + T_i)$ and other discharge characteristics, which may itself be a cause of changes in the thermal insulation.

In spite of these drawbacks, however, the phenomenological approach enables one to establish certain relationships between discharge parameters which can in turn be used both for predicting temperatures in similar systems and for comparison with various consequences arising out of theoretical conceptions regarding the thermal insulation of plasmas. Let us consider one of the ways of establishing such relationships on the basis of experimental data.

We would point out that the energy lifetime increases during the discharge up to a certain steady-state value $\tau_{st}$. From this it may be concluded that thermal insulation (Figs 1 and 2) is a function of one of the plasma parameters which changes during the discharge. It is natural to assume that the plasma temperature T may be one such parameter.

Having made this assumption, let us turn to Fig. 1. The curves correspond to two Tokamak-3 regimes with different discharge current and charged particle density $n_e$. Since the temperature for both regimes is the same, it may be assumed that the observed difference between energy lifetimes is due either to the influence of the charged particle density $n_e$ or to that of the field $H_\varphi$ produced by the current.

On the other hand, the dependence of $\tau_E$ on the strength of the current at constant charged particle density (shown in Fig. 2) indicates that $\tau_E$ increases either with increasing $H_\varphi$ or with increasing temperature.

Thus, it would be possible to explain the experimental findings by assuming that $\tau_E$ increases either with $H_\varphi$ or with charged particle density and temperature.

The latter point of view appears to be more natural at first sight since it enables one to explain the plot of $\tau_E$ against time, whereas in the first case one has to make additional assumptions in order to explain the behaviour of $\tau_E$ over time.

It should be borne in mind, however, that under conditions of quasi-stationary plasma heating any change in $\tau_E$ will produce a corresponding change in plasma energy.

It is therefore impossible to determine uniquely, on the basis of phenomenological relationships alone, whether the observed change in $\tau_E$ is a direct consequence of a change in temperature or vice versa. A final solution can be obtained only in experiments involving additional methods of plasma heating.

Of course, the above points of view regarding the dependence of $\tau_E$ on $H_\varphi$ or on $n_e$ and T are in principle not mutually exclusive. Nevertheless, it is convenient to consider their implications separately.

Assuming that the thermal insulation of a plasma is determined solely by charged particle density and plasma temperature, we obtain an explicit expression for the connection between $\tau_E$, $n_e$ and T. Let us use Eq. (1). We shall consider those moments of time when the term dE/dt can be neglected. We then write

$$\frac{E}{\tau_E} = \frac{I^2}{\sigma} \tag{5}$$

where $\sigma = \sigma_0\, T_{eo}^{3/2}\, \pi a^2$.

From this we obtain

$$\frac{n_e(T_e + T_i)^{5/2}}{\tau_E} = \frac{2}{3}\,\frac{\bar{j}^2 \eta}{\sigma_0} \tag{6}$$

If we now put $\eta$ = const - which, as has been shown, is valid when $n_e \sim (2-5)\times 10^{13}$ cm$^{-3}$ - we obtain

$$\frac{a^2|\beta|(T_e + T_i)^{3/2}}{\tau_E} = \text{const} \tag{7}$$

It has been noted by Artsimovich et al. [1] that one of the characteristic features of the behaviour of plasmas in the T-3 device is the increase in plasma energy with rising charged particle concentration if all other conditions remain unchanged; i.e. $E(t_0, n_e) \sim n_e^\alpha$, and accordingly $\beta(t_0, n_e) \sim n_e^\alpha$, where $\alpha$ is an index close to unity.

Thus, within a certain range of variation in the plasma parameters $a^2 n_e(T_e + T_i)^{3/2}/\tau_E$ should remain constant.

Figure 6 shows the function $(a^2 n_e(T_e+T_i)^{3/2}/\tau_E)(t)$ during one of the discharges, while Fig. 7a shows $a^2 n_e(T_e + T_i)^{3/2}/\tau_E$ as a function of the charged particle density for the number of Tokamak-3 discharge regimes.

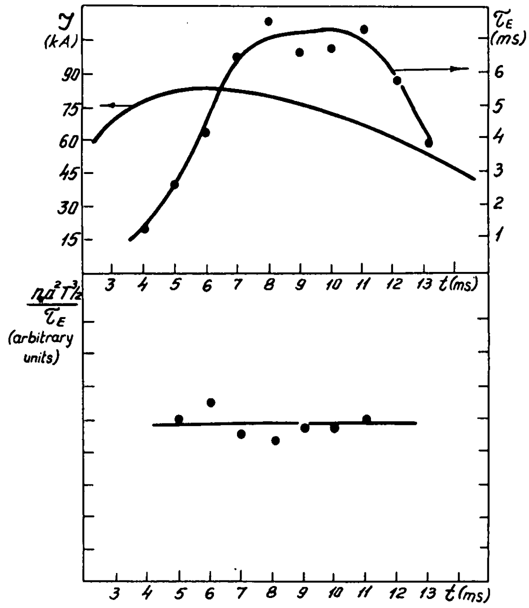


FIG. 6. The functions I(t), $\tau_E(t)$ and $(n_e a^2 T^{3/2}/\tau_E)(t)$.

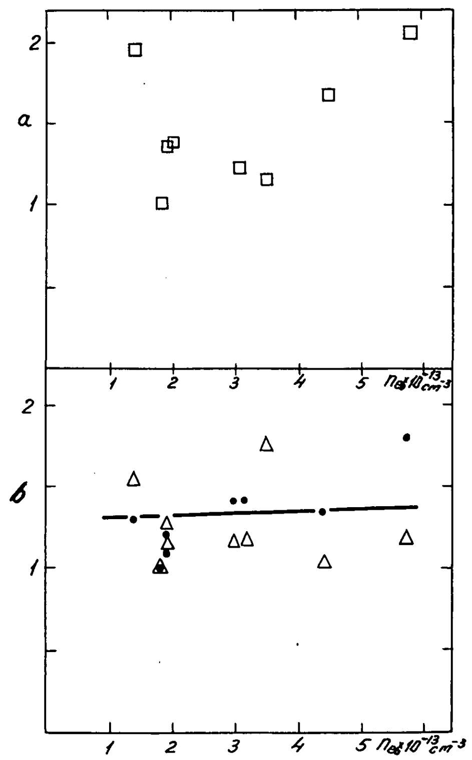


FIG. 7. (a) Values of $(n_e a^2 T^{3/2}/\tau_E)(t_0)$ at various electron densities and for a number of T-3 discharge regimes.

(b) ● - values of $(n_e^{\frac{1}{2}} a^2 H_\varphi/\tau_E)(t_0)$ and

△ - values of $(a^2 H_\varphi/\tau_E)(t_0)$ for a number of T-3 discharge regimes.

TABLE I. EXPERIMENTAL AND CALCULATED DATA FOR A NUMBER OF TOKAMAK-3 DISCHARGE REGIMES

| N | Experimental data | | | | | | Calculated data | |
|---|---|---|---|---|---|---|---|---|
| | $n_{e0} \times 10^{-13}$ cm$^{-3}$ | I(kA) | $\tau_E$ (s) | $\eta$ | a(cm) | T(eV) | $\tau_E \sim a^2 H_\varphi$, $T \sim H_\varphi^{6/5}\left(\frac{\eta}{n}\right)^{2/5}$ T(eV) | $\tau_E \sim a^2 n_e T^{3/2}$, $T \sim \eta H_\varphi^2$ T(eV) |
| 0 | 1.8 | 45 | $4 \times 10^{-3}$ | 2.8 | 11 | 400 | 400 | 400 |
| 1 | 5.5 | 76 | $7 \times 10^{-3}$ | 2.8 | 12 | 400 | 420 | 1350 |
| 2 | 4.4 | 60 | $5.8 \times 10^{-3}$ | 2.8 | 11 | 400 | 400 | 710 |
| 3 | 3.5 | 45 | $2.5 \times 10^{-3}$ | 3.7 | 11 | 260 | 350 | 530 |
| 4 | 3.0 | 50 | $4 \times 10^{-3}$ | 2.0 | 10.5 | 340 | 360 | 380 |
| 5 | 3.0 | 36 | $3 \times 10^{-3}$ | 1.5 | 11.3 | 180 | 190 | 138 |
| 6 | 1.9 | 77 | $6 \times 10^{-3}$ | 4.3 | 11 | 700 | 900 | 1830 |
| 7 | 1.9 | 60 | $5 \times 10^{-3}$ | 3.25 | 11 | 560 | 590 | 750 |
| 8 | 1.4 | 62 | $3.5 \times 10^{-3}$ | 6.8 | 10 | 800 | 1000 | 2200 |

It follows from the data presented that the quantity in question remains constant to within 30 - 40%. V.D. Kirillov has found that it also remains constant in the case of the Tokamak device TM-3. Accordingly, it should be possible to write one of the probable expressions for $\tau_E$ in the form $\tau_E \sim a^2 n_e (T_e + T_i)^{3/2}$.

Let us consider a variant of the dependence of the energy confinement time on the discharge current. Using the data presented in Fig. 2 and assuming that the energy losses are of the diffusion type, it is possible to write for $\tau_E$ an expression in the form $\tau_E \sim a^2 H_\varphi$.

In Fig. 7b are presented values of $a^2 H_\varphi/\tau_E$ for a number of Tokamak-3 discharge regimes and for different electron densities. The value of $\tau_E$ was read off at the moment when it reached a maximum (8 - 10 ms from the beginning of the discharge).

It follows from this figure that $a^2 H_\varphi/\tau_E$ remains constant to within 20 - 30%.

On the basis of direct measurements of $\tau_E$ it is therefore impossible to support any hypothesis definitely since, while the discharge parameters change, the functions $a^2 n_e(T_e + T_i)^{3/2}/\tau_E$ and $a^2 H_\varphi/\tau_E$ remain constant to within the accuracy of the determination of $\tau_E$.

However, these hypotheses can be subjected to more serious verification. If one substitutes into Eq. (6) assumed expressions for $\tau_E$, it should be possible to obtain corresponding expressions for the dependence of the temperature $T_e + T_i = T$ on the discharge parameters and, normalizing their absolute value for some initial regime, to make a comparison with the results obtained experimentally. Such a comparison for a number of Tokamak-3 discharge regimes is shown in Table I. In the far right-hand column are presented numerical values of $T = T_e + T_i \sim \eta H_\varphi^2$ for $\tau_E \sim a^2 n_e T^{3/2}$; in the first column from the right are presented values of $T \sim H_\varphi^{6/5} (\eta/n)^{2/5}$ for $\tau_E \sim a^2 H_\varphi$; in the second column from the right are presented experimental values of T.

Comparison of these columns justifies fairly definite support for the second variant.

Thus, it may be concluded that the true relationship between $\tau_E$ on one hand and $H_\varphi$, T, $n_e$ and $\eta$ on the other should not go very much beyond the limits determined by the relations

$$\tau_E \sim a^2 H_\varphi \tag{8}$$

$$T_e + T_i \sim H_\varphi^{6/5}\left(\frac{\eta}{n_e}\right)^{2/5} \tag{9}$$

We would make two additional remarks concerning the agreement between the expression found for $\tau_E$ and the results of direct experimental determination of the energy lifetime.

It has already been pointed out (Fig. 3) that, with fixed values of $H_z$ and I, $\tau_E$ increases weakly as the charged particle density ($n_{e0}$) increases. This increase becomes most noticeable in the region of low $n_{e0}$ - i.e. $(0.8 - 2) \times 10^{13}$ cm$^{-3}$. In order to take this into account it is possible to introduce into the expression for $\tau_E$ a factor $n_e^{\frac{1}{2} \pm \frac{1}{4}}$. The corresponding function $(n_e^{\frac{1}{2}} a^2 H_\varphi/\tau_E)(t_0)$ for a number of Tokamak-3 regimes is shown in Fig. 7b.

One of the main objections to presenting $\tau_E$ in the form of expression (8) is that the assumption that the thermal insulation of a plasma depends only on the field due to the current does not explain the behaviour of $\tau_E$ during the discharge. An answer to this objection may be provided to some extent by experiments which entail pulsed current changes during the discharge and which apparently enable one to link the degree of thermal insulation of the plasma with the distribution of the current density j over the plasma column cross-section.

As an example of how one uses the dependences obtained, it is possible to calculate the ratio of the energy lifetime $\tau_E$ of a plasma in Tokamak-3 to the Bohm diffusion time $\tau_B$:

$$\frac{\tau_E}{\tau_B} \sim \frac{T H_\varphi}{H_z} \sim \frac{T^2}{H_z}\left(\frac{n_e}{\eta}\right)^{\frac{1}{3}} \tag{10}$$

From this, we obtain for high values of $n_e$

$$\frac{\tau_E}{\tau_B} \sim \frac{T^2 n_e^{\frac{1}{3}}}{H_z} \tag{11}$$

and for low densities (where $\eta \sim 1/n_e$)

$$\frac{\tau_E}{\tau_B} \sim \frac{T^2 n_e^{\frac{2}{3}}}{H_z} \tag{12}$$

Thus, the function $\tau_E/\tau_B$ should in all cases increase fairly rapidly with temperature; as a rule this has been observed in experiments.

In conclusion, the authors wish to thank L.A. Artsimovich for posing the problem and for his scientific guidance, and V.S. Mukhovatov, K.A. Razumova, L.V. Maiorov and V.D. Kirillov for useful discussions and valuable advice.

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## APPENDIX I

### 1. METHOD OF DETERMINING THE ENERGY CONFINEMENT TIME OF A PLASMA

It follows from Eq.(1) that one needs to know E and P in order to calculate $\tau_E$.

The determination of E is based on measurements of the diamagnetic effect of the plasma [5]. Strictly speaking, this method enables one to determine only the transverse plasma energy component $E_\perp$. It has been shown [5] that in the operating range of Tokamak-3 parameter values the energy distribution is sufficiently isotropic - i.e. one may assume that $E = (3/2)E_\perp$.

Taking into account this fact and the paramagnetism of the plasma column, it is not difficult to obtain an expression linking the measured magnetic flux change over the plasma cross-section $\Delta\phi$ with the plasma energy E:

$$\Delta\phi = \frac{2\pi}{H_z}\left[(0.1\,I)^2 - \frac{4}{3}E\right] = \frac{2\pi}{H_z}(0.1\,I)^2(1-\beta) \tag{A.1}$$

where $H_z$ is the strength of the stabilizing magnetic field (in oersteds), I is the discharge current (in amperes) and $\beta = 4E/3(0.1\,I)^2$.

Let us consider the applicability of the above expression. Generally speaking, the expression is valid only for a plasma cylinder with current density (j) and plasma pressure (p) distributions that are uniform with respect to $\varphi$. However, it has been shown [6] that it is also valid for toroidal plasmas whose radius of curvature is not too small. From magnetic probe measurements of the non-uniformity with respect to $\varphi$ of the current-induced magnetic field $H_z$ it may be concluded that under typical discharge conditions the non-uniformity with respect to $\varphi$ in the distribution of j is negligible. Since the plasma pressure distribution with respect to $\varphi$ is unlikely to differ significantly from the distribution of j, it may be concluded that expression (A.1) is valid in our case without any additional assumptions.

The ohmic heating power P can be determined from the equation of energy conservation:

$$10^7 \frac{IV}{2\pi R} = P + \frac{d}{dt}\frac{L(0.1\,I)^2}{2} \tag{A.2}$$

where V is the voltage applied to the plasma column (measured at the gap in the copper shell), R is the major radius of the plasma column, $L(0.1\,I)^2/2$ is the magnetic energy of the plasma column per unit length, and L is the inductance per unit length of the current-carrying loop.

To measure L one can use the readings from two magnetic probes which are located on different sides of the plasma column in the symmetry plane of the torus and which measure the current-induced magnetic field of the plasma. It has been shown [7] that the difference $U_-$ between the signals from two such probes is proportional to the displacement $\Delta$ relative to the centre of the discharge chamber.

Knowing the difference between ($U_-$) and the sum of ($U_+$) the probe signals, it is possible to determine the absolute displacement.

On the other hand, in Tokamak systems the equilibrium displacement of the plasma column is a function of its inductance L, the plasma energy E, the vertical component of the transverse magnetic field $H_\perp$, and the current $I_\perp$ flowing across the plasma column.

The expression linking these parameters and the measured quantities ($U_-$, $U_+$) can be written in the form:

$$\begin{gathered}\frac{RU_-}{bU_+} + 1 = \frac{L}{2} + \beta' + \frac{RH_\perp}{0.2\,I}\\ \beta' = \beta + \frac{aH_z}{0.4\,I^2}I_\perp\end{gathered} \tag{A.3}$$

Thus, in order to determine L one needs to know $H_\perp$ and $I_\perp$ in addition to $U_-$, $U_+$ and $\beta$. Since precise determination of $H_\perp$ and $I_\perp$ is usually difficult, the principal measurements were - with the exception of certain special cases - made in those discharge regimes where their influence could be neglected. Expression (A.3) was then written in the following form:

$$\frac{RU_-}{bU_+} + 1 = \frac{L}{2} + \beta \qquad (A.4)$$

The limits of its applicability are determined primarily by the extent to which one can ignore the vertical component of the transverse magnetic field $H_\perp$ and the current $I_\perp$ flowing across the plasma column. The appearance of transverse magnetic field components may be due to the following factors: non-ideality of the windings producing the stabilizing magnetic field; stray fields of the system for exciting the longitudinal current; damping of Foucault (eddy) currents in the conducting shell.

Using a system of compensating loops [8], it was possible to reduce to ~1 - 2 Oe the averaged value of $H_\perp$ resulting from the non-ideality of the stabilizing field windings, so that it did not have to be taken into account.

It was demonstrated by special measurements that the transverse magnetic field components associated with stray fields of the system for exciting the longitudinal current are also insignificant.

It has been shown [10] that the effect produced by damping of Foucault currents in the conducting shell can be taken into account by introducing an equivalent transverse magnetic field which displaces the plasma column towards the outer wall of the discharge chamber. For the processes under consideration, with a duration of ~ 10 ms, it is also insignificant.

It should be noted that in the scheme used for determining $\tau_E$ a decline in the strength of the transverse fields which shifts the plasma column outwards led only to a slight decline in the value of $\tau_E$.

It follows from Eq. (A.3) that the contribution to the measurement of L made by the current flowing across the plasma column may be considered negligible if the additional term $(H_z a/0.4 I^2) I_\perp$ becomes comparable with the absolute error in the measurement of $\beta$.

In estimating $(H_z a/0.4 I^2) I_\perp$ it was assumed that almost the entire electric current flowing across the plasma column shorts through the conducting diaphragm, so that measurement of the corresponding current flowing across the diaphragm gives a value close to $I_\perp$.

Measurements have shown that in the initial stage of the discharge (1 - 3 ms) this current reaches 100 - 200 A and then decays rapidly to 20 - 30 A, whereafter it changes only slightly until the end of the discharge. Accordingly, the term $(H_z a/0.4 I^2) I_\perp$ can introduce an error into the determination of $\tau_E$ only in the initial stage of the discharge.

We thus have a system of four equations - (1), (A.1), (A.2) and (A.4) - linking the results of the measurements with the four unknown plasma column parameters: $\beta$, L, P and $\tau_E$. In principle, it is possible to determine these parameters by solving the system of equations.

Let us estimate the errors involved in doing this. It used to be considered [5] that the error in the determination of $\beta$ was $\approx (5/\beta)\%$. The error in the determination of L varies during the discharge, but is not more than 10% in the intermediate stage.

With regard to the calculation of P and $\tau_E$ the matter is more complicated. It follows from expression (1) that the error in the determination of $\tau_E$ consists in fact of the error in the determination of $\beta$ $(E = (3/4)\beta\,(0.1\,I)^2)$ and the error in the determination of the quantity

$$P - \frac{d}{dt} E = \frac{E}{\tau_E} = W$$

The above expression represents the total power lost from the plasma. Using expressions (A.2) and (A.4) it is easy to obtain the formula

$$W \approx 10^7 \frac{IV}{2\pi R} - \frac{d}{dt}(0.1\,I)^2 \left(\frac{R}{b}\frac{U_-}{U_+} + 1\right) \qquad (A.5)$$

for the condition $\beta \ll 4$.

Since this condition is always satisfied, it follows from the above expression that one does not need to know the plasma energy (i.e. the parameter which is measured with the poorest accuracy) in order to determine the power loss W.

Thus, each of the terms on the right-hand side of Eq. (A.5) can in principle be measured fairly precisely (5 - 10%).

The quantity W represents the difference between them. It is reasonable to assume that, if this difference is comparable in magnitude with one of the terms (and this is the case in the central part of the discharge), then the determination of W will remain fairly precise.

In a typical case one may expect a precision of ~10 - 15%. From this it may be concluded that the final precision of the calculation of $\tau_E$ was determined mainly by the precision of the measurement of E and was 20 - 40% (depending on $\beta$) in the central part of the discharge. The initial stage of the discharge, where $\beta$ is usually small and finite and where the precision of the determination of W declines, was usually not considered. The data on the basis of which the dependences of $\tau_E$ on $H_z$ and I were constructed relate to the intermediate part of the discharge and to the charged particle density region $(2 - 5) \times 10^{13}$ cm$^{-3}$ - i.e. to that region where $\beta$ is comparatively large (0.2 - 1).

All the practical calculations of $\tau_E$ were performed under the guidance of A.E. Bazhanova on an M-20 computer.

## APPENDIX II

### DETERMINATION OF THE PLASMA TEMPERATURE

The method for determining the temperature of a plasma has been described in detail [1]. Essentially it is as follows: on the basis of results (obtained from the multi-channel microwave probing of a plasma column) indicating the parabolic nature of the charged particle density distribution over the plasma column cross-section $n_e(r) = n_{e0}[1-(r/a\ )^2]$ and assuming a radial distribution of the charged particle temperature in the form $(T_e + T_i)(r) = (T_e + T_i)_{max}[1-(r/a_T)^2]$, where $T_e$ and $T_i$ are the electron and ion temperatures respectively, it is possible to calculate the total plasma energy as a function of $n_{e0}$, $T_e + T_i = T = \frac{1}{2}(T_e + T_i)_{max}$, $a_T$ and $a_n$.

Given the values of $a_T$ and $a_n$ it is possible, on the basis of direct measurements of the plasma energy E and of the phase shift $\psi = \int_{-a_n}^{+a_n} n_e(r)\,dr$ measured with a microwave interferometer, to determine $T_e + T_i$, which represents the mean temperature of the plasma.

On the other hand, using P and assuming that the T-distribution over the plasma column cross-section is parabolic, it is possible to determine $T_{e\sigma}$ - the mean electron temperature determined by electric conductivity measurements.

In the experiments described, the plasma radius $\alpha = a_T$ was obtained from the inductance of the plasma column $L = 2\ln(b/a_T) + 1$.

The radius $a_n$ was determined as $a_{ch} - \Delta$, where $a_{ch}$ is the discharge chamber radius.

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