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1957 Proc. Phys. Soc. B 70 6

(http://iopscience.iop.org/0370-1301/70/1/303)

Some Criteria for a Power Producing Thermonuclear Reactor

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Communicated by D. W. Fry; MS. received 2nd November 1956

Abstract. Calculations of the power balance in thermonuclear reactors operating under various idealized conditions are given. Two classes of reactor are considered: first, self-sustaining systems in which the charged reaction products are trapped and, secondly, pulsed systems in which all the reaction products escape so that energy must be supplied continuously during the pulse. It is found that not only must the temperature be sufficiently high, but also the reaction must be sustained long enough for a definite fraction of the fuel to be burnt.

§ 1. Introduction

It has been widely conjectured that some form of controlled thermonuclear reactor, capable of producing a useful amount of power, will some day be constructed. In this paper the power balance in such a reactor is considered, and some criteria which have to be satisfied in a power producing system are derived.

Some of the difficulties of realizing a controlled fusion reaction have been discussed by Thirring (1955) and Thonemann (1956), and a broad survey of fundamentals has been given by Post (1956). The present treatment differs from that of Thirring in the assumptions about the radiation from a hot gas, and it covers in rather more detail some of the points discussed by Thonemann and Post. The analysis is based on simple assumptions; it is designed to illustrate the essential features of the problem, and is neither rigorous nor complete. The assumptions made are in all cases optimistic, so that the criteria established are certainly necessary, though by no means sufficient, for the successful operation of a thermonuclear reactor.

§ 2. Basic Principles

Of the exoergic reactions involving light nuclei those between the hydrogen isotopes (the so-called D–D and T–D reactions) are by far the most probable at low energies. Of these the T–D reaction has the higher cross section, but since tritium does not occur naturally it is necessary to use a system in which it can be bred. This may be done by capturing the neutrons emitted in the T–D reaction in ^{6}Li, which then decays into T and ^{4}He.

The reactions of interest are shown in table 1.

Table 1

Reaction	Q (Mev)	σ_{max} (barns)	σ_{max} energy
^{2}H(d, n)^{3}He	3·3	0·09	2 Mev
^{2}H(d, p)^{3}H	4·0	0·16	2 Mev
^{2}H(t, n)^{4}He	17·6	5·0	150 kev
^{6}Li(n, α)^{3}H	4·8	$1/v$ law	

The energy released per unit time and volume by thermonuclear reactions in a hot gas is given by

$$P_{\mathrm{R}} = n_1 n_2 \overline{v\sigma}(T) E \qquad \ldots\ldots(1)$$

where n_1 and n_2 are the number densities of the nuclei of the first and second kinds, and $\overline{v\sigma}(T)$ is the product of the relative velocities of the nuclei and the reaction cross section averaged over the Maxwellian velocity distribution corresponding to a temperature T, and E is the energy released by one reaction. If the ions are of the same kind (as in the D–D reaction) $n_1 n_2$ is replaced by $2(n/2)^2 = \frac{1}{2}n^2$. Values of $\overline{v\sigma}(T)$ calculated from published values of the cross sections for the D–D and T–D reactions are given in the companion paper by Thompson (1957).

Energy can be lost from the hot gas in two ways, by radiation and by conduction. At temperatures above about 10^6 degrees hydrogen is completely ionized and radiation occurs principally as bremsstrahlung (free–free transitions). The mean free path of such radiation is large (several g cm^{-2}) and consequently in a reactor of controllable size virtually all of it would escape. The Stefan–Boltzmann T^4 law does not hold under these circumstances; the variation of intensity with temperature can only be found by a detailed study of the radiation process. The power radiated per unit volume in hydrogen is given by (Spitzer 1956)

$$P_{\mathrm{B}} = 1{\cdot}4 \times 10^{-34} n^2 T^{1/2} \text{ watts cm}^{-3}. \qquad \ldots\ldots(2)$$

If the hot gas is in a magnetic field the electrons will move in spiral orbits, and additional radiation due to the acceleration towards the axis of the spiral will occur. This radiation is similar to that obtained from electrons in a betatron, and it may be important in very intense fields. It will, however, be neglected in this paper.

Conduction loss is difficult to treat in a general way, since it depends on the geometry of the system, its density and temperature distribution, and also the wall material. In the analysis which follows it is optimistically assumed that the conduction loss is zero.

It is of interest to see at what temperature the nuclear power release is equal to the radiated power. This may be called the 'critical temperature' and is the hypothetical temperature which would be needed for a self-sustaining system if all the radiation escaped but the reaction products were retained. The critical temperature is about 150 million degrees for the D–D reaction (assuming that the tritium is burnt as soon as it is formed, but that the ^{3}He is not burnt), and 30 million degrees for the T–D reaction.

The critical temperature is a somewhat artificial concept; it does not mean that if a thermonuclear fuel is heated to this temperature a reaction will be set off in the way that a chemical explosion is set off, or that the fuel can be ignited as in a gas jet. This would only be true if the energy of the reaction were deposited close to where it was released, i.e. if the range of the reaction products were short compared with the dimensions of the apparatus. In fact, the range of the particles will almost certainly be large compared with the dimensions of the apparatus if the system is to be of controllable size, so that unless the tracks are somehow coiled up it will be the walls of the apparatus which are heated rather than the gas, and energy must be fed in continuously to sustain the reaction.

Various types of system will now be considered in a general way. No suggestions of how to realize them will be given.

§ 3. Systems in which the Reaction Products are Retained

It is not inconceivable that the charged reaction products could be contained in the hot gas by a suitable combination of electric and magnetic fields, though it seems unlikely that the escape of neutrons can be prevented. The temperature at which such a system would be self-sustaining in the absence of conduction loss can be calculated by equating the radiation loss to the energy carried by the charged disintegration products. This temperature is about 3×10^8 degrees for the D–D reaction, and 5×10^7 degrees for the T–D reaction. In the D–D system it is only just possible in principle to sustain the reaction, since above about 10^8 degrees the reaction rate increases with temperature only slightly faster than the radiation loss. At 10^9 degrees for example a conduction loss equal to the radiation loss is sufficient to quench the reaction.

As an example of the orders of magnitude involved, the slowing down range of the charged reaction products in a gas at 10^8 degrees and 10^4 atmospheres pressure ($n = 3 \times 10^{17}$ nuclei/cm^3) is of the order of a kilometre. The range of the neutrons is hundreds of kilometres.

§ 4. Systems in which the Reaction Products Escape

An alternative type of system in which the reaction products are not retained in the gas will now be considered. Since some specific proposals are for pulsed systems we shall consider the following idealized cycle: the gas is heated instantaneously to a temperature T, this temperature is maintained for a time t, after which the gas is allowed to cool. Conduction loss is neglected entirely, and it is assumed that the energy used to heat the gas and supply the radiation loss is regained as useful heat.

An important parameter R will now be introduced; this is the ratio of the energy released in the hot gas to the energy supplied. Now the energy released by the reaction appears as heat generated in the walls of the apparatus, and this has to be converted to electrical, mechanical or chemical energy before it can be fed back into the gas. If η is the efficiency with which this can be done, then the condition for a system with a net power gain is

$$\eta(R+1) > 1. \qquad \ldots\ldots(3)$$

The maximum value of η is about $\frac{1}{3}$, so that R must be greater than 2.

For the pulsed cycle described above we have

$$R = \frac{tP_{\mathrm{R}}}{tP_{\mathrm{B}} + 3n\boldsymbol{k}T} = \frac{P_{\mathrm{R}}/3n^2\boldsymbol{k}T}{P_{\mathrm{B}}/3n^2\boldsymbol{k}T + 1/nt} \qquad \ldots\ldots(4)$$

where P_{R} and P_{B} are respectively the reaction power and radiated power per unit volume. The $3n\boldsymbol{k}T$ term represents the energy required to heat the gas to a temperature T. Electron binding energies are neglected, but the contribution from electrons is included (this accounts for the factor 3 rather than $\frac{3}{2}$).

Since P_{R} and P_{B} are both proportional to n^2, R is a function of T and nt. In figure 1 curves of R against T for various values of nt are shown for the D–D reaction assuming that the tritium formed is also burnt. (In practice the tritium would have to be collected and fed back into the system with the deuterium.) The line $R = 2$ is shown dotted in the figure, and it is seen that for a useful reactor T must exceed 2×10^8 degrees and nt must exceed about 10^{16}. Thus, for a pulse of 1 microsecond duration, n must be greater than 10^{22}; this corresponds to

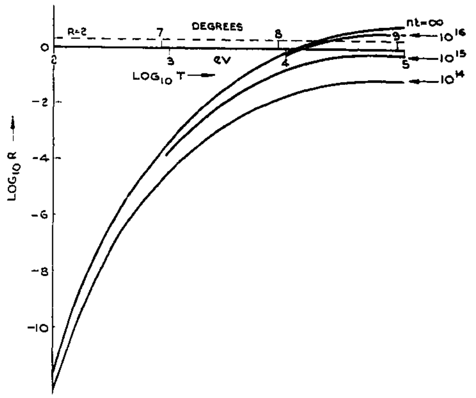

Figure 1. Variation of R with T for various values of nt for D-D reaction.

a pressure of 6×10^8 atmospheres at a temperature of 2×10^8 degrees. Particles move several metres during this time, and the mean free path for momentum transfer is several centimetres even at this high density. These distances are, of course, measured along the track, which may be spiralled or oscillatory.

Figure 2 shows similar curves for the T–D reaction. Conditions are easier, but still severe; nt must exceed 10^{14}, and the minimum temperature is 3×10^7 degrees.

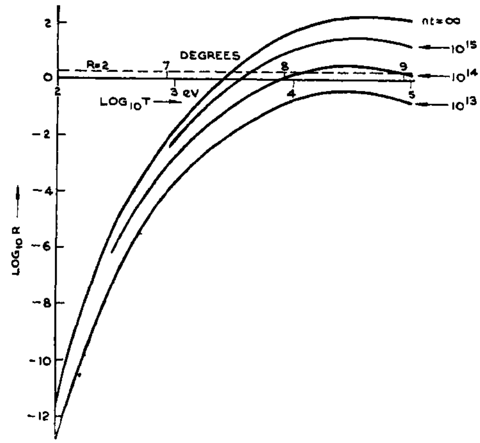

Figure 2. Variation of R with T for various values of nt for T–D reaction.

The curve marked $nt = \infty$ merely shows the ratio of the thermonuclear power release to the radiation loss, and crosses the $\log R = 0$ axis at the critical temperature. The curves are only accurate so long as t is sufficiently short that only a small fraction of the fuel is burnt. The values $nt = 10^{16}$ for a D–D system and 10^{14} for a T–D system both correspond to a burning of about 1% of the fuel.

Although these calculations refer specifically to a system in which the reaction products escape, it may easily be verified that the '1% burn-up' criterion is almost unaltered in a system in which the reaction products are retained. In any practical system, where conduction loss is present, and where a large circulating power in the system is undesirable, the fraction of fuel burnt would need to be much greater.

§ 5. Conclusion

For a successful thermonuclear reactor not only has the temperature to be sufficiently high, but also the reaction has to be sustained for a sufficient time. The reason for this is that the organized energy used to heat the gas is ultimately degraded to the temperature of the walls of the apparatus and, consequently, sufficient thermonuclear energy must be released during each heating cycle to compensate for this degradation.

No claim that the above treatment is complete or applies to all possible types of system is made, but it does give some idea of the order of magnitude of the problems involved.

Systems which depart substantially from the electrically neutral Maxwellian gas assumed here have been carefully considered, but none looks promising. Some reasons for this are discussed by Thonemann (1956).

Acknowledgments

This paper is an amplified version of the more important topics in A.E.R.E. report GP/R 1807. It owes much to discussions with several colleagues, in particular Mr. R. S. Pease, Dr. W. B. Thompson and Dr. P. C. Thonemann.

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