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Comparison of Mars Entry Performance Using Bank-Angle and Alpha-Beta Steering

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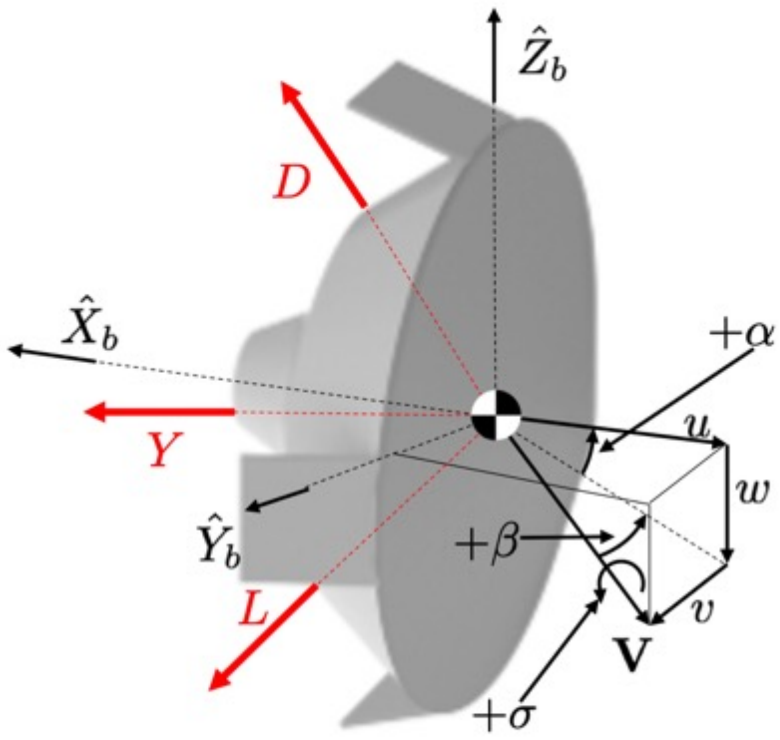
21st IPPW-June 10th, 2024

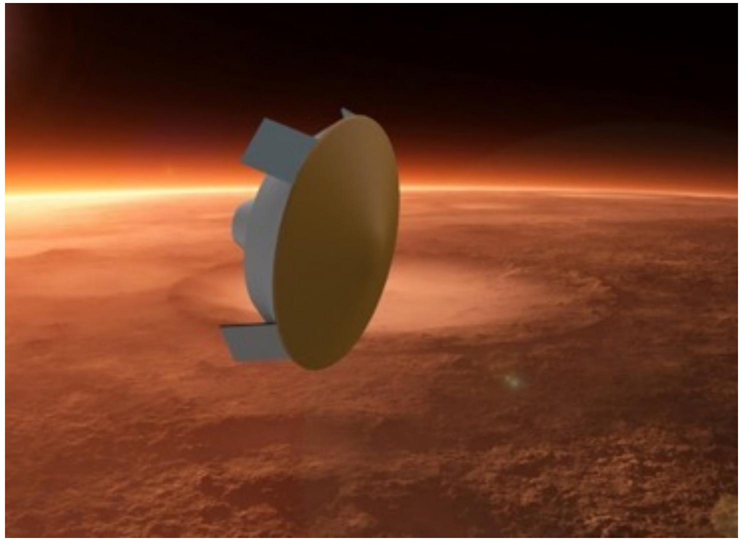
Hypersonic Steering for Mars Entry

- Bank-angle steering is state-of-the-art for hypersonic trajectory control
- Future Mars entry missions may demand higher entry performance (accuracy, altitude, mass)
- Alpha-beta (α-β) steering or direct force control (DFC) is a proposed alternative hypersonic steering scheme
  - Morphing structure
  - Center-of-gravity movement system
  - Aerodynamic flaps



Motivation

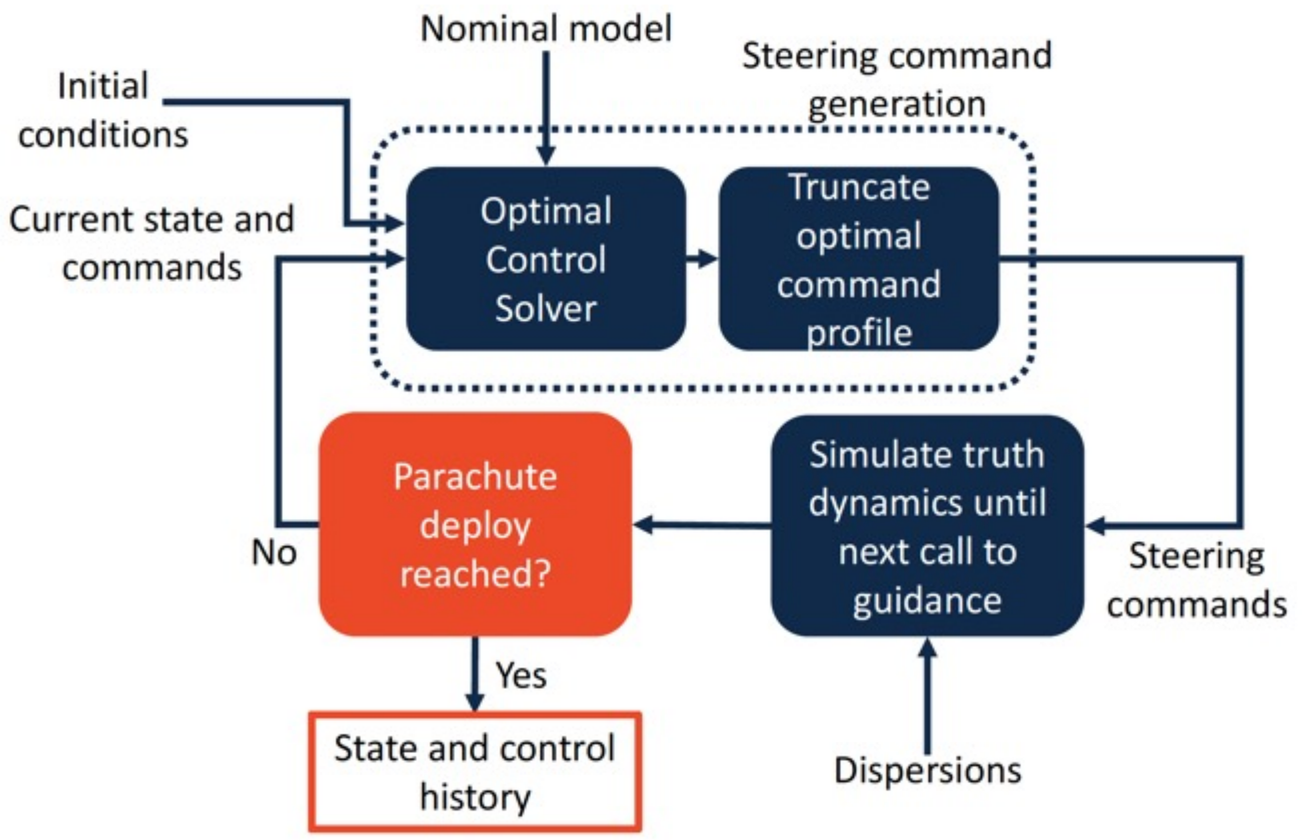
- α-β steering may improve flight performance over bank-angle steering, but magnitude of performance differences not fully understood
- Design of guidance/steering commands may influence performance comparison
  - Tuning parameters
  - Guidance structure
  - Use of bank reversals
- Optimal control can be used to provide more equal comparison over several trajectory objectives



Optimal Control Solutions

Maximize altitude at final time

- **Trajectory planning optimal control problems**
- **Use of optimal control to generate steering commands**
- All problems solved using the commercial pseudospectral solver GPOPS-II



Optimal Control Models

- Three-degree-of-freedom (3DOF) motion over spherical, non-rotating Mars
- **Atmosphere model**: exponential
- **Aerodynamic model**: fit to Mars Science Laboratory aerodynamics database
  - Drag: quadratic in α and β
  - Lift: linear in α
  - Side force: linear in β

$$\dot{z} = V \sin\gamma$$

$$\dot{\theta} = \frac{V \cos\gamma \sin\psi}{r \cos\phi}$$

$$\dot{\phi} = \frac{V \cos\gamma \cos\psi}{r}$$

$$\dot{V} = -D - g \sin\gamma$$

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma + \left(\frac{V^2}{r} - g\right)\cos\gamma\right]$$

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma + Y}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi\right]$$

Bank
$$\begin{cases} \mathbf{x}_\sigma = \begin{bmatrix} z & \theta & \phi & V & \gamma & \psi & \sigma & \dot{\sigma} \end{bmatrix}^T \\ u_\sigma = \ddot{\sigma} \end{cases}$$

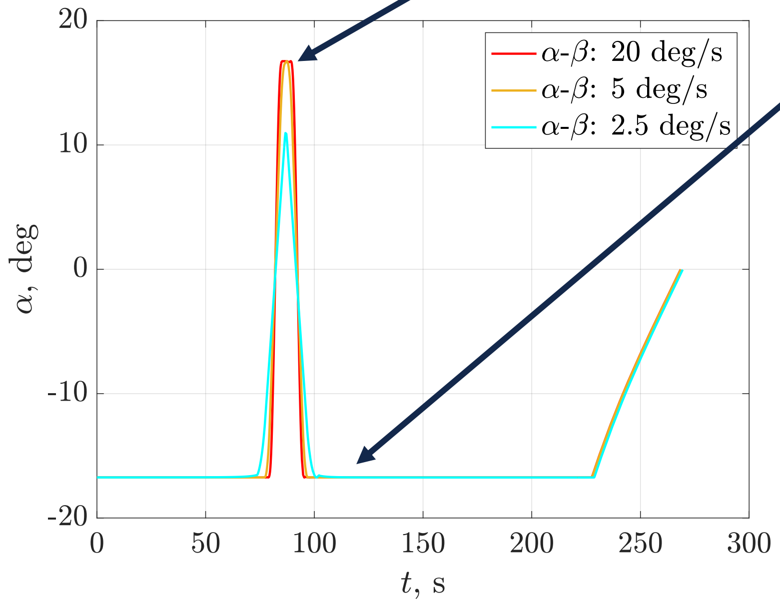
α-β
$$\begin{cases} \mathbf{x}_{\alpha\text{-}\beta} = \begin{bmatrix} z & \theta & \phi & V & \gamma & \psi & \alpha & \beta & \dot{\alpha} & \dot{\beta} \end{bmatrix}^T \\ \mathbf{u}_{\alpha\text{-}\beta} = \begin{bmatrix} \ddot{\alpha} & \ddot{\beta} \end{bmatrix}^T \end{cases}$$

Maximum Terminal Altitude

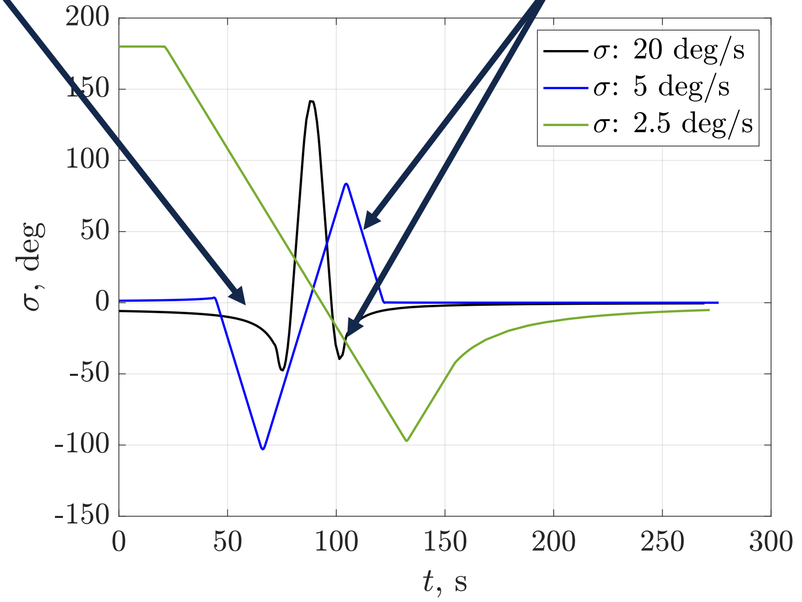
α-β steering h
through

Both steering op
near ban

Significant coupling to maximize altitude
and hit zero deg latitude target



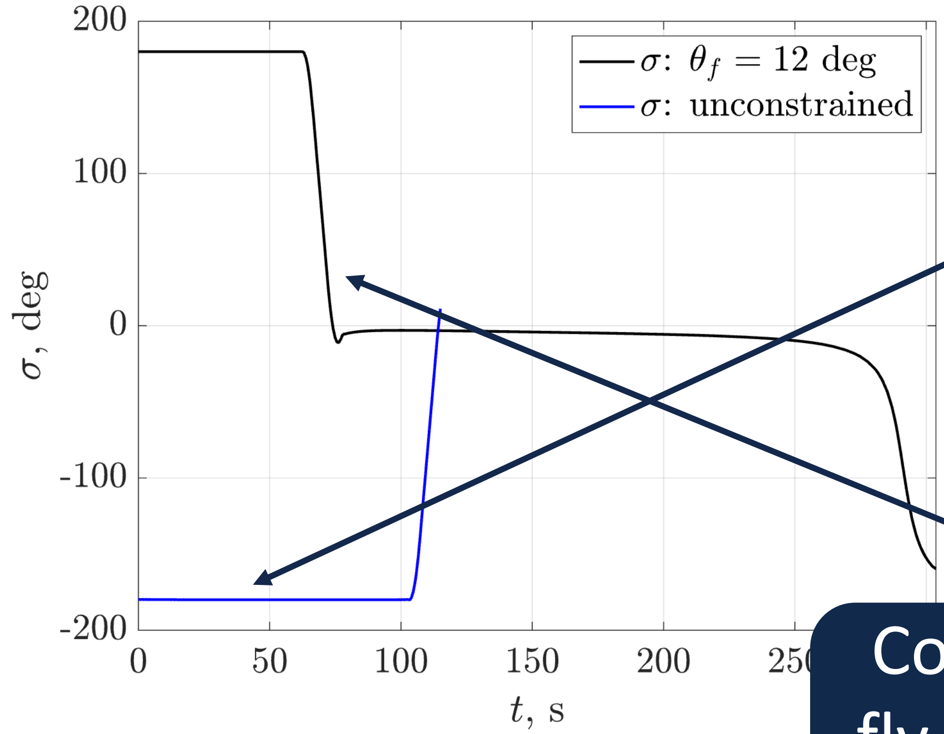
α-β



Bank

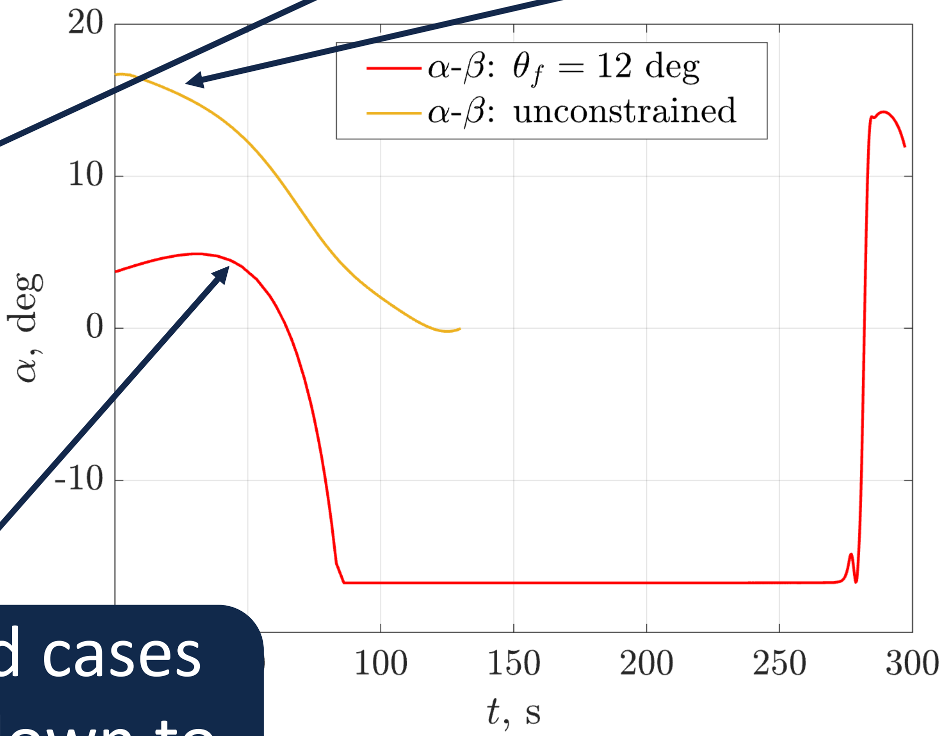
Minimum Heat Load

α-β uses lift-down flight with higher drag to reduce heat load



Bank

Constrained cases fly less lift-down to extend range

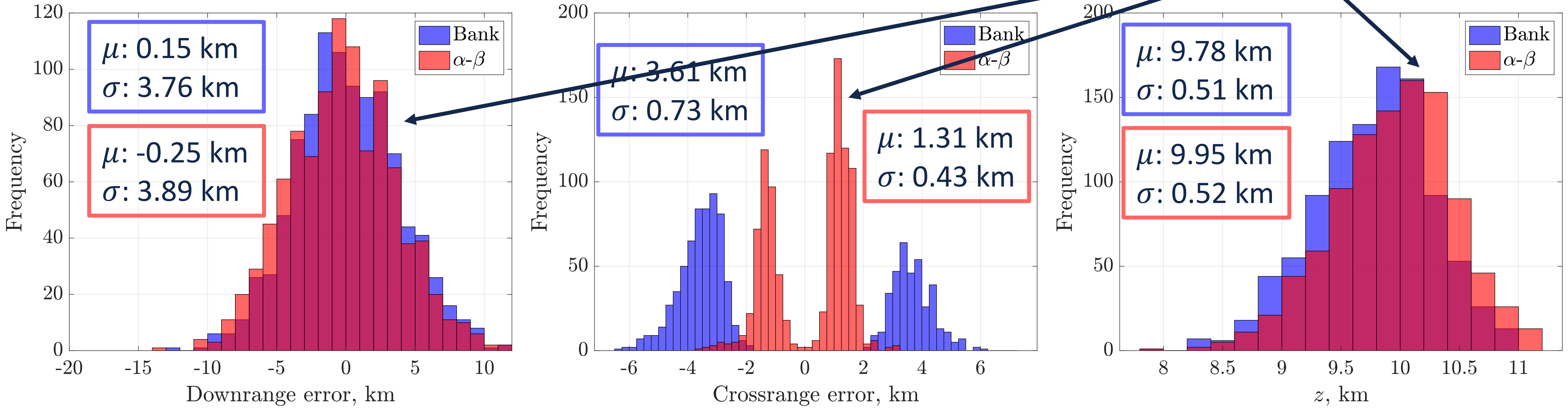


α-β

Monte Carlo: Maximum Altitude

Goal: hit desired longitude, latitude while maximizing terminal altitude

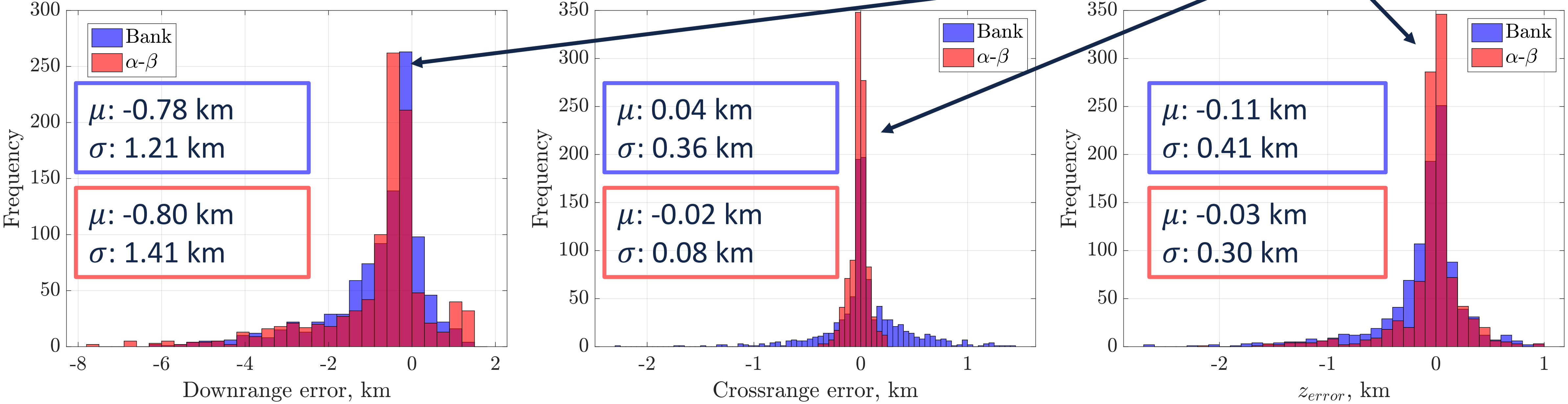
Both options have similar downrange targeting



Monte Carlo: Minimum Control Effort

Goal: hit desired longitude, latitude, and altitude target while minimizing vehicle rates

α-β has lower crossrange error



Conclusions

- Optimal control problems solved for bank-angle and α-β steering
- α-β steering results in improved performance over bank-angle steering
  - Maximum altitude (especially as rates are lowered), minimum heat load, and lateral targeting
  - Differences are due to coupling, response time, and control over drag
- α-β steering may be useful when bank-angle steering vehicle cannot move quickly, if marginal performance gains are needed, or if using simple guidance algorithms

Acknowledgements

This work was supported by a NASA Space Technology Graduate Research Opportunity.

Backup

Selected References

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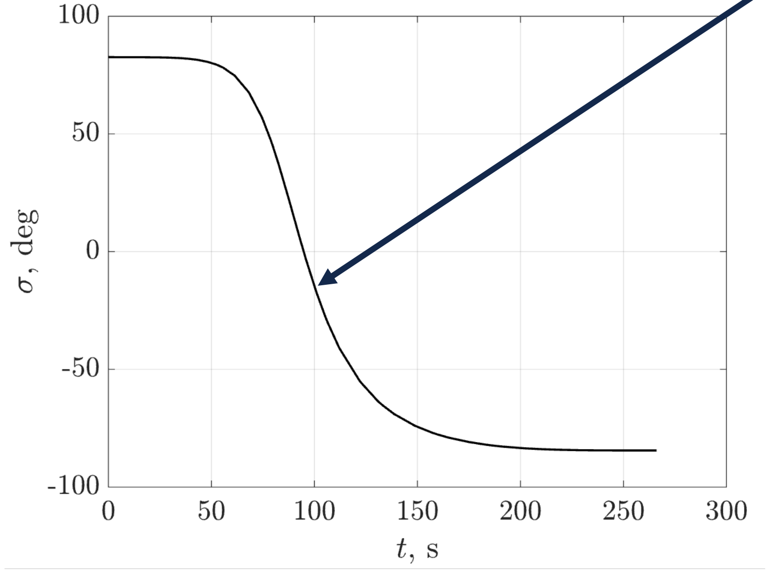
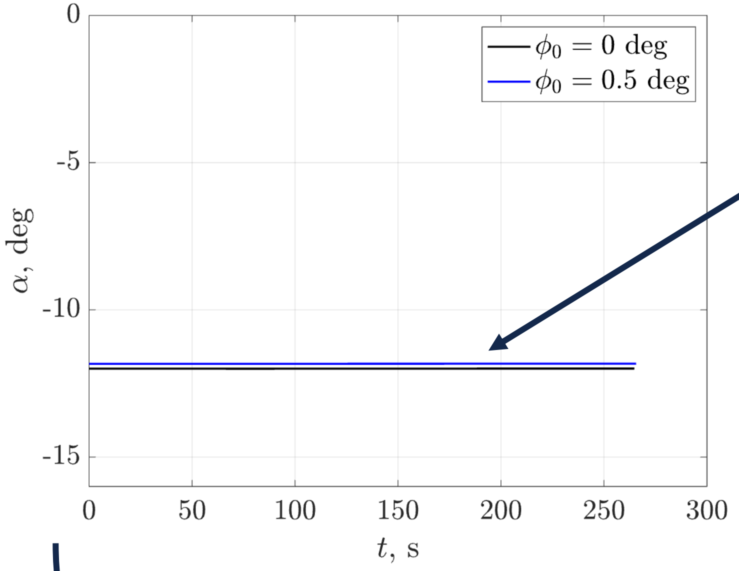
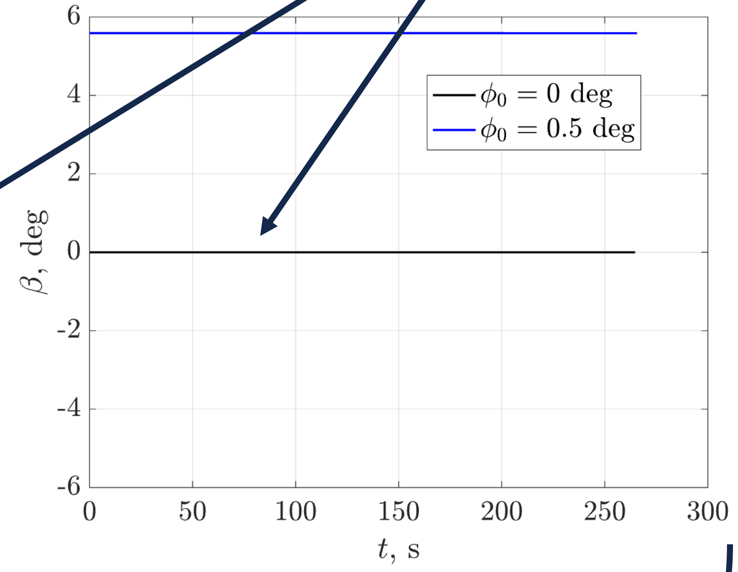
Minimum Control Effort

Single bank reversal

Constant profiles





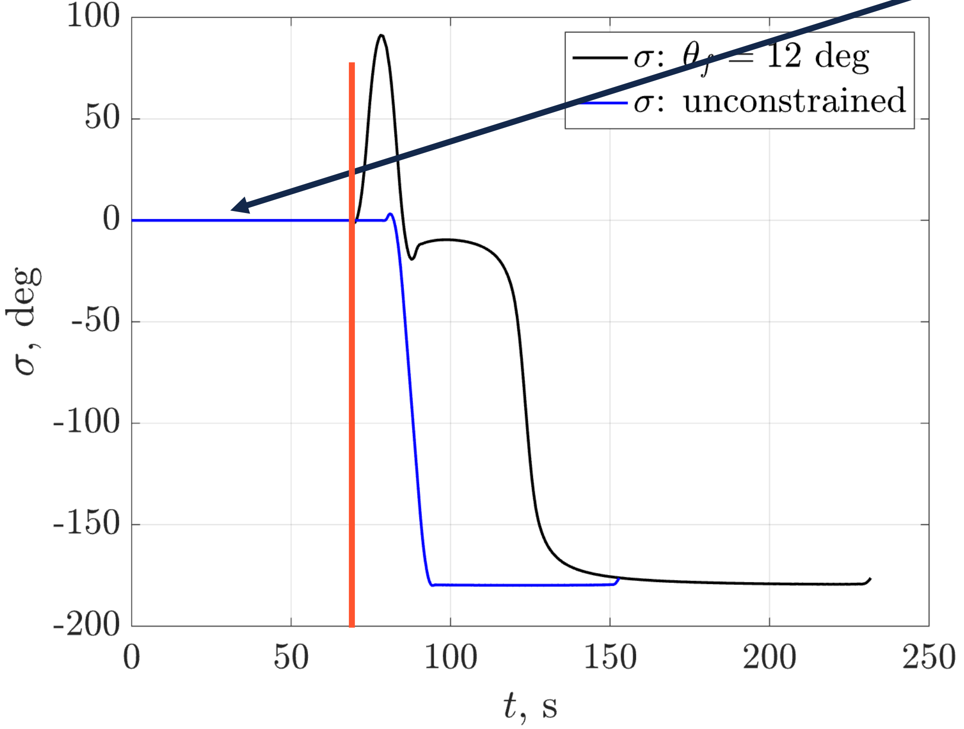


Bank

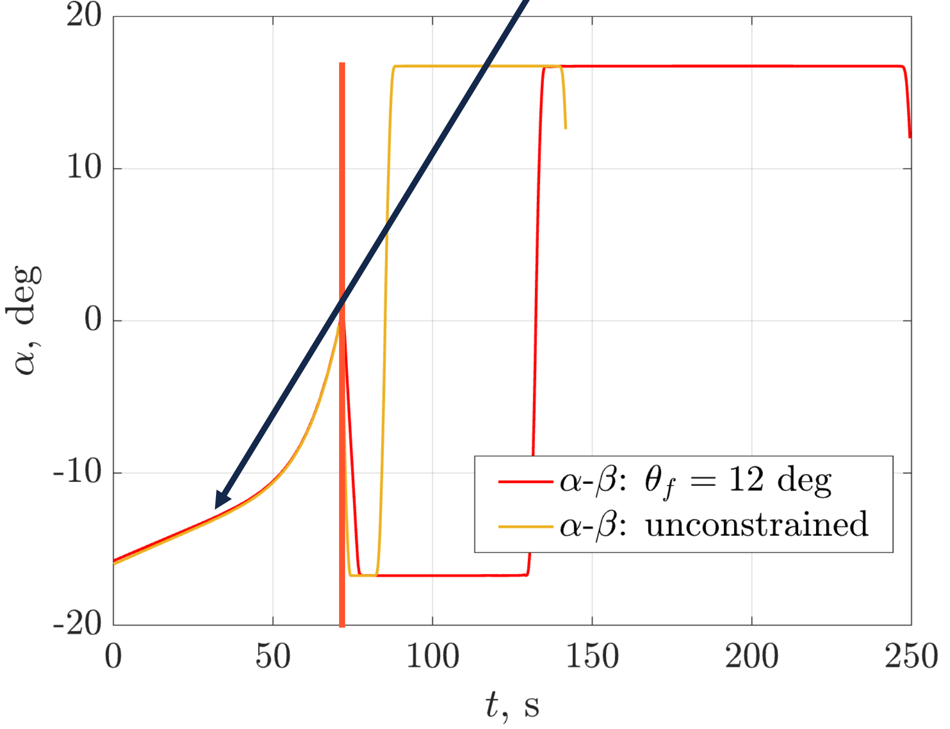
α-β

Minimum Peak Heat Rate

Bank-angle steering vehicle flies lift up



Bank



α-β

Models: Trajectory Planning

Parameter	Value
m	3300 kg
S_{ref}	15.90 m^2
$C_{D,0}$	1.67
$C_{D,\alpha}$	-2.86/rad^2
$C_{D,\beta}$	-2.86/rad^2
$C_{L,\alpha}$	-1.31/rad
$C_{Y,\beta}$	1.29/rad

Parameter	Value
Surface Density	0.02 kg/m^3
Atmospheric Scale Height	11.10 km
Surface Gravity	3.71 m/s^2
Mars Equatorial Radius	3396.20 km

$$C_D = C_{D,0} + C_{D,\alpha}\alpha^2 + C_{D,\beta}\beta^2$$

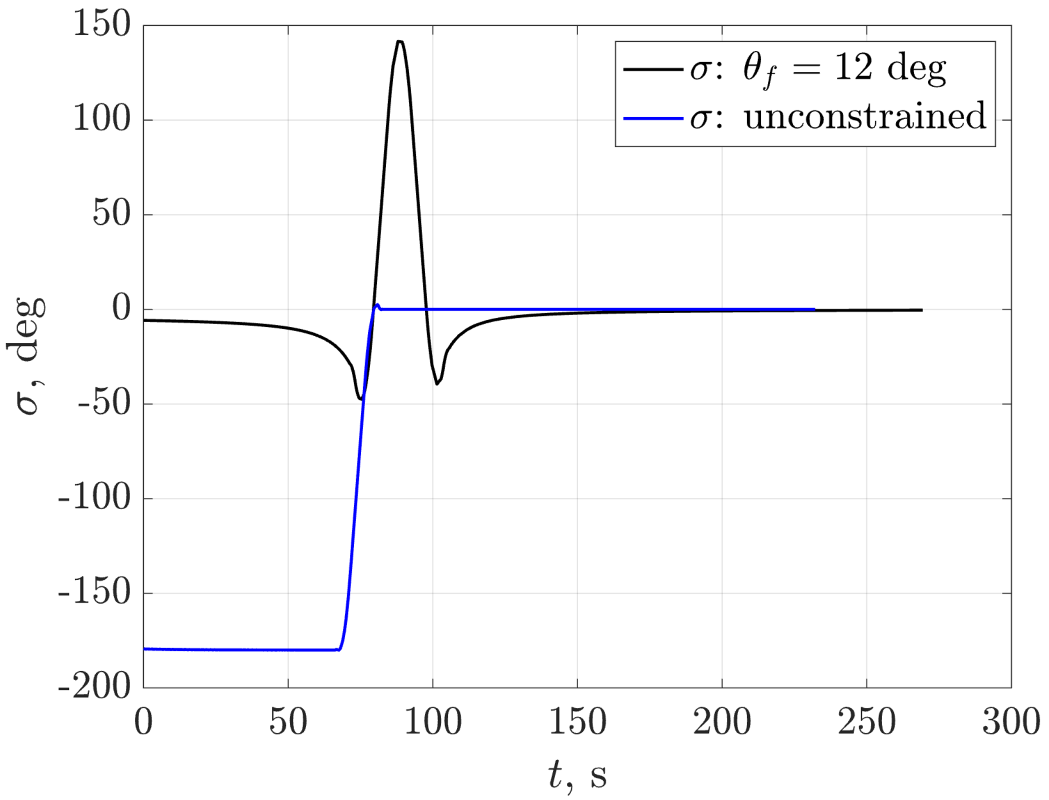
$$C_L = C_{L,\alpha}\alpha$$

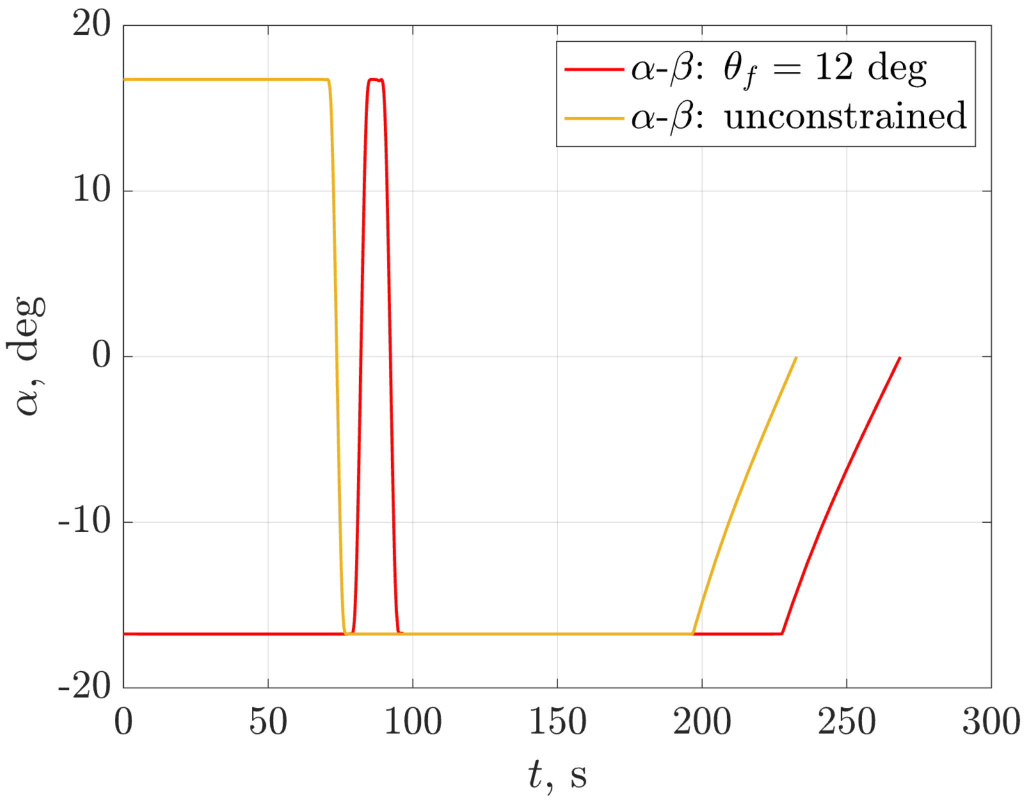
$$C_Y = C_{Y,\beta}\beta$$

Control Constraints-Trajectory Planning

Parameter	Value
α_{max}	16.74 deg
β_{max}	16.74 deg
$\dot{\alpha}_{max}$	20 deg/s
$\dot{\beta}_{max}$	20 deg/s
$\ddot{\alpha}_{max}$	5 deg/s^2
$\ddot{\beta}_{max}$	5 deg/s^2
$\dot{\sigma}_{max}$	20 deg/s
$\ddot{\sigma}_{max}$	5 deg/s^2
α_{bank}	-16.74 deg

Maximum Altitude-Constrained and Unconstrained

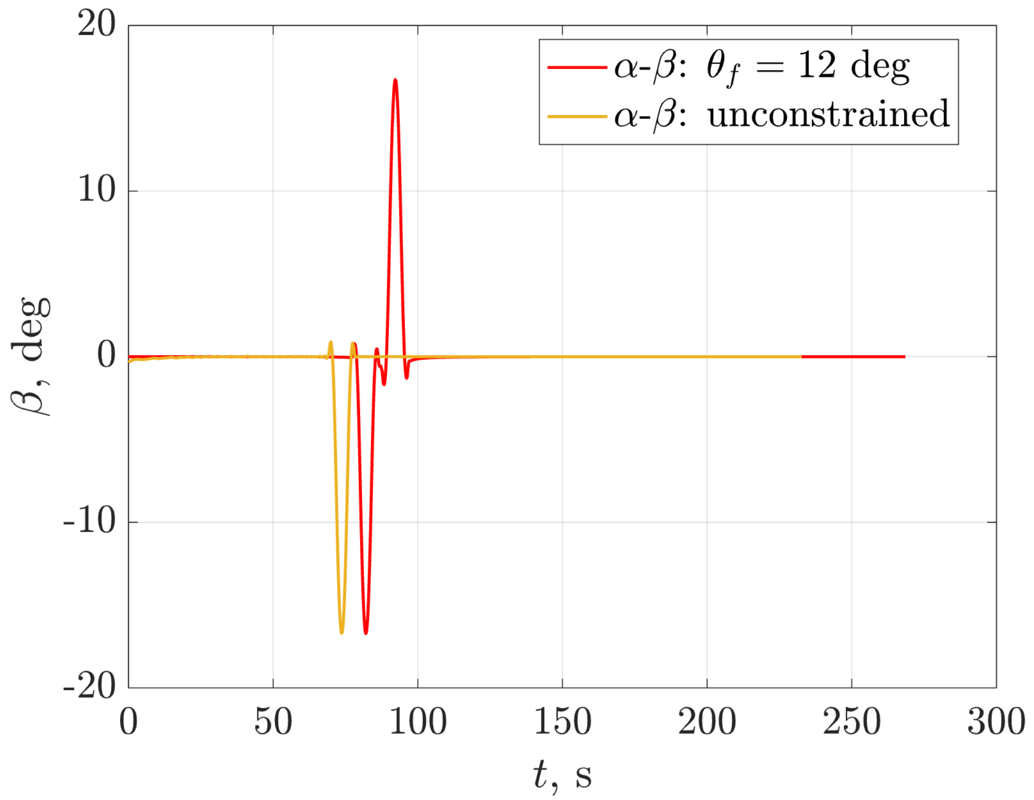


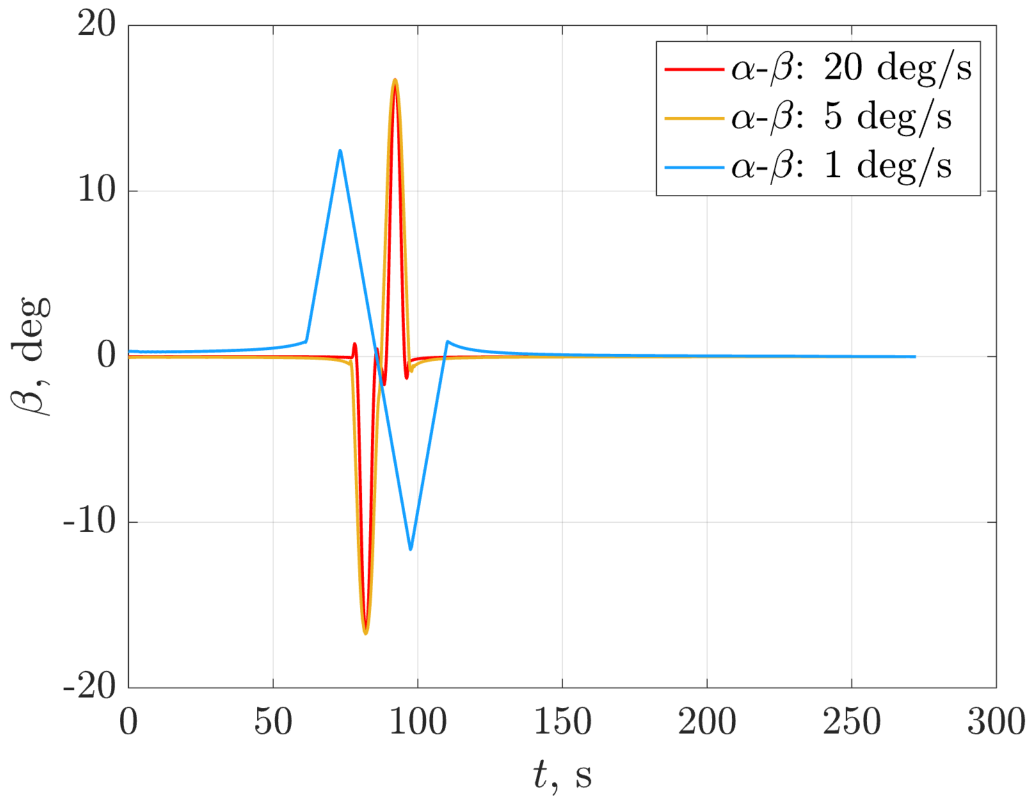


Maximum Altitude-Constrained and Unconstrained

Case	Terminal Altitude, km
$\sigma: \ \theta_f = 10$ deg	18.05
$\alpha\text{-}\beta: \ \theta_f = 10$ deg	18.35
$\sigma: \ \theta_f = 12$ deg	18.33
$\alpha\text{-}\beta: \ \theta_f = 12$ deg	18.81
$\sigma: \ \theta_f = 14$ deg	14.76
$\alpha\text{-}\beta: \ \theta_f = 14$ deg	15.22
$\sigma:$ unconstrained	19.75
$\alpha\text{-}\beta:$ unconstrained	20.04

Sideslip Angle Impact on Altitude Performance



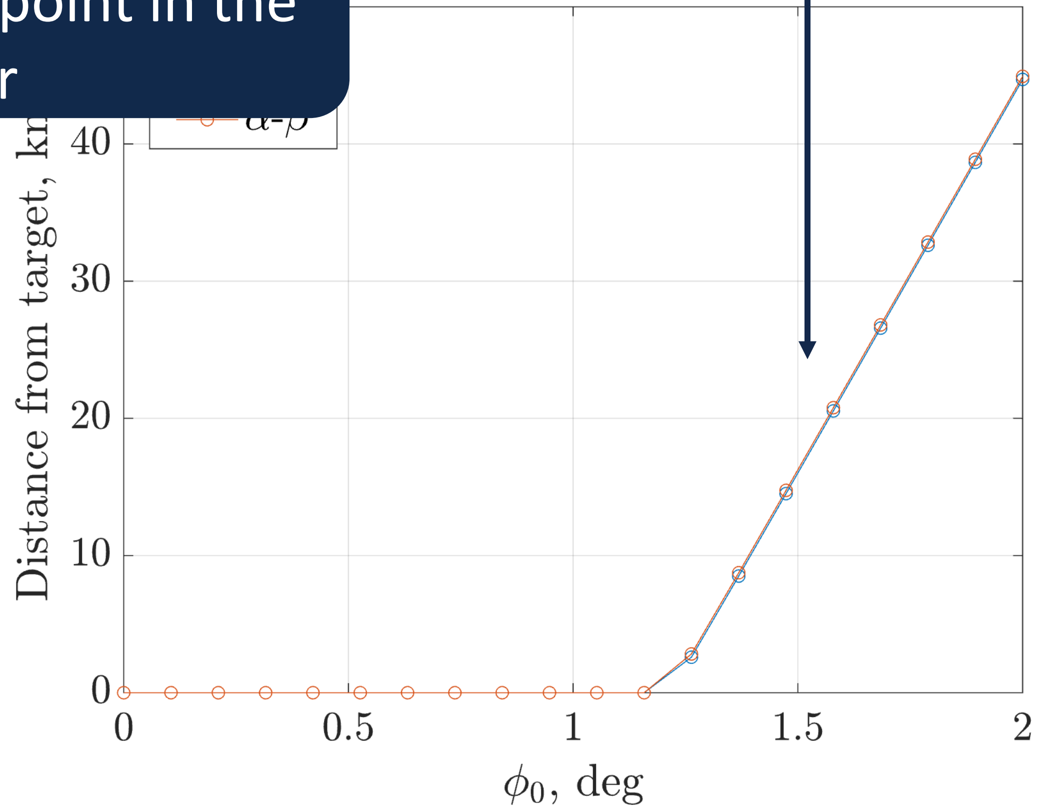


Minimum Terminal Error

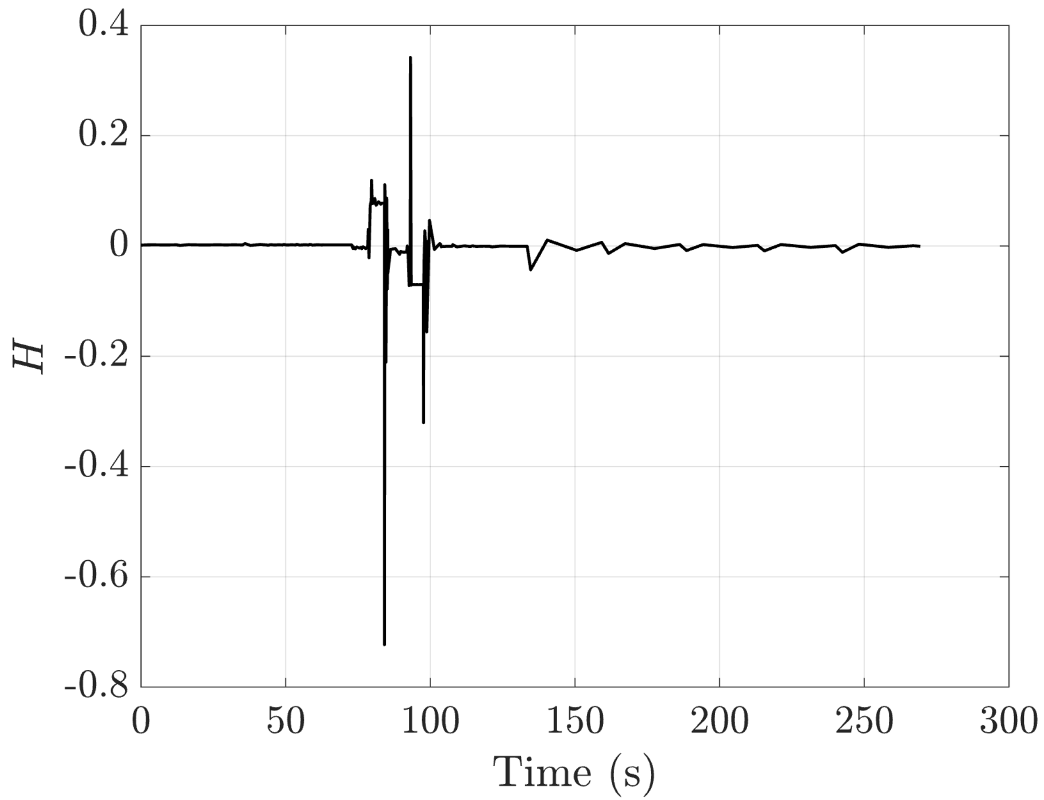
$$
\begin{aligned}
\min_{\mathbf{u}} \quad & J = \mathcal{X}_e^2 + \mathcal{Y}_e^2 + \mathcal{Z}_e^2 \\
\text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
& V(t_f) = V_f \\
\alpha\text{-}\beta : \; & \alpha \leq \alpha_{max} \qquad -\alpha \leq \alpha_{max} \\
& \beta \leq \beta_{max} \qquad -\beta \leq \beta_{max} \\
& \alpha_{total} \leq |\alpha_{bank}| \\
& \dot{\alpha} \leq \dot{\alpha}_{max} \qquad -\dot{\alpha} \leq \dot{\alpha}_{max} \\
& \ddot{\alpha} \leq \ddot{\alpha}_{max} \qquad -\ddot{\alpha} \leq \ddot{\alpha}_{max} \\
& \dot{\beta} \leq \dot{\beta}_{max} \qquad -\dot{\beta} \leq \dot{\beta}_{max} \\
& \ddot{\beta} \leq \ddot{\beta}_{max} \qquad -\ddot{\beta} \leq \ddot{\beta}_{max} \\
\sigma : \; & \dot{\sigma} \leq \dot{\sigma}_{max} \qquad -\dot{\sigma} \leq \dot{\sigma}_{max} \\
& \ddot{\sigma} \leq \ddot{\sigma}_{max} \qquad -\ddot{\sigma} \leq \ddot{\sigma}_{max}
\end{aligned}
$$

Minimize miss distance to Cartesian point in the air

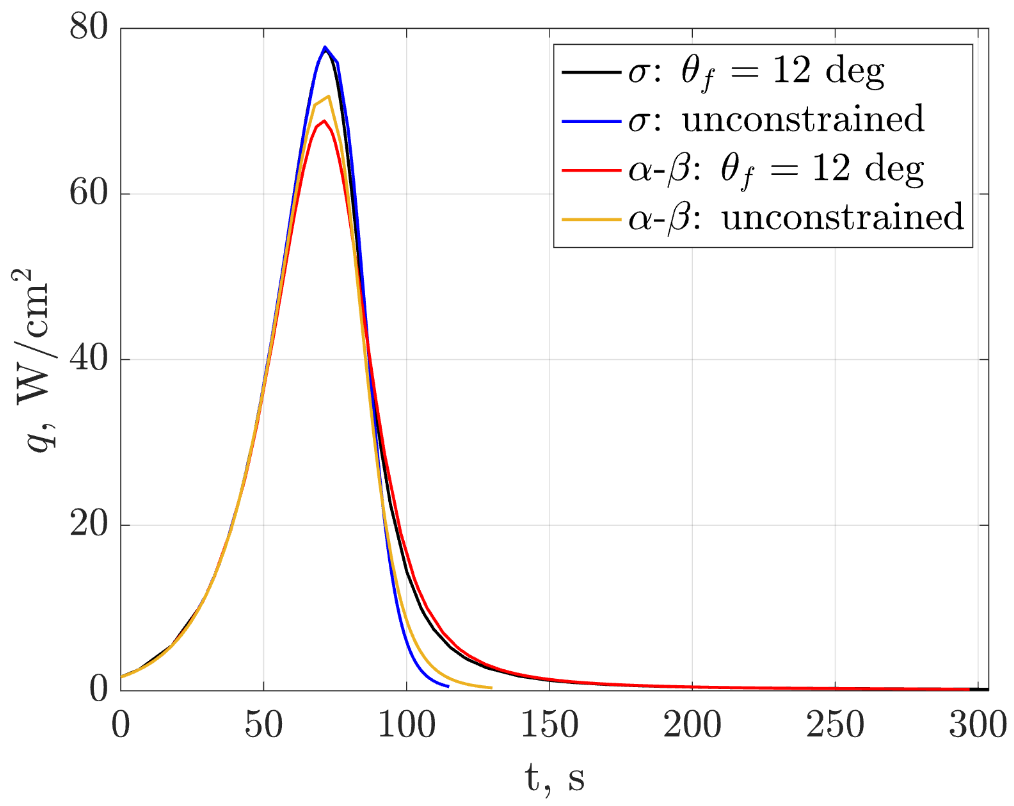
Steering options perform essentially identically as control authorities are the same



Hamiltonian

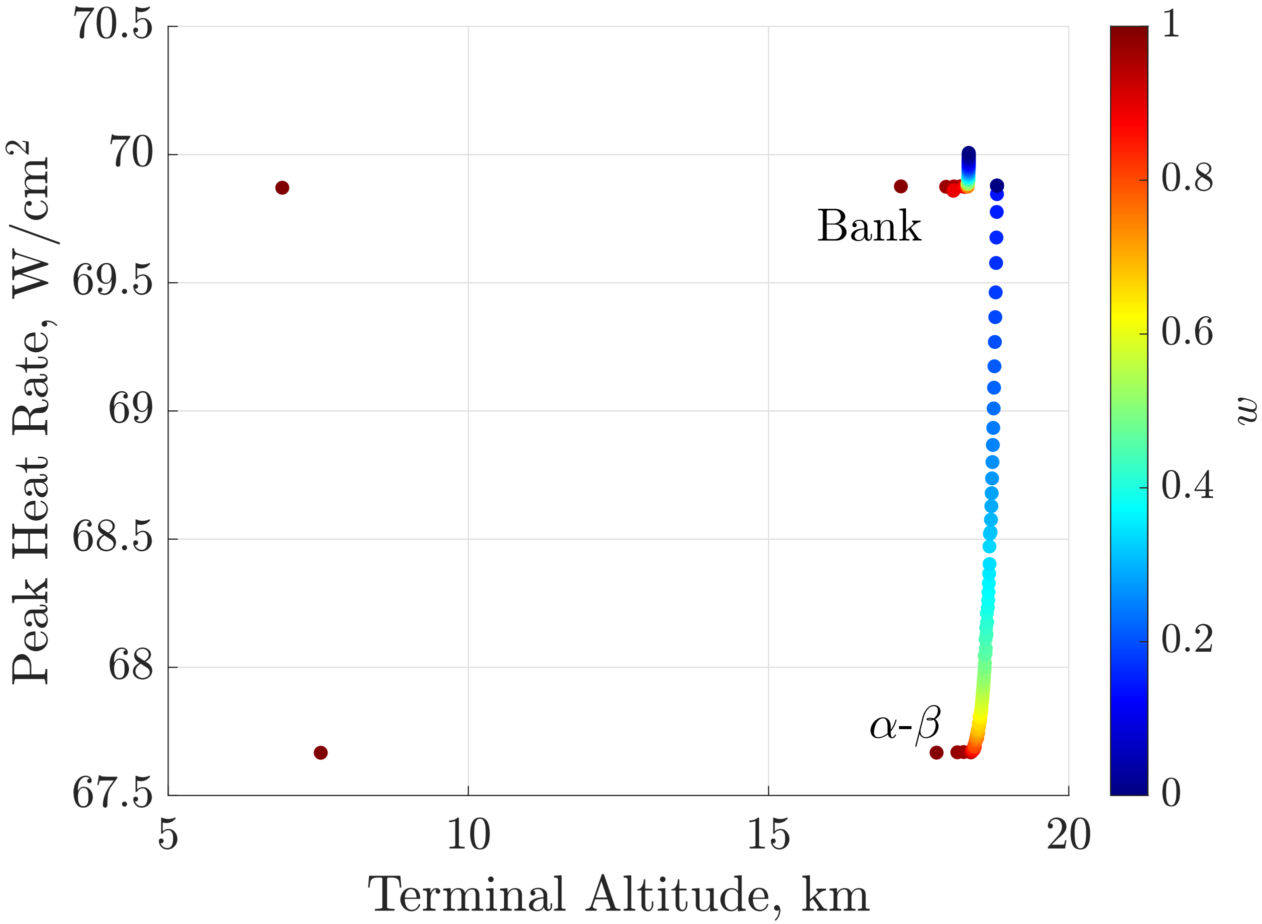


Minimum Heat Load Curves



Heat Rate Pareto Fronts

$$J = w\frac{g(t_f)}{q_{scale}} - (1-w)\frac{z(t_f)}{z_{scale}}$$

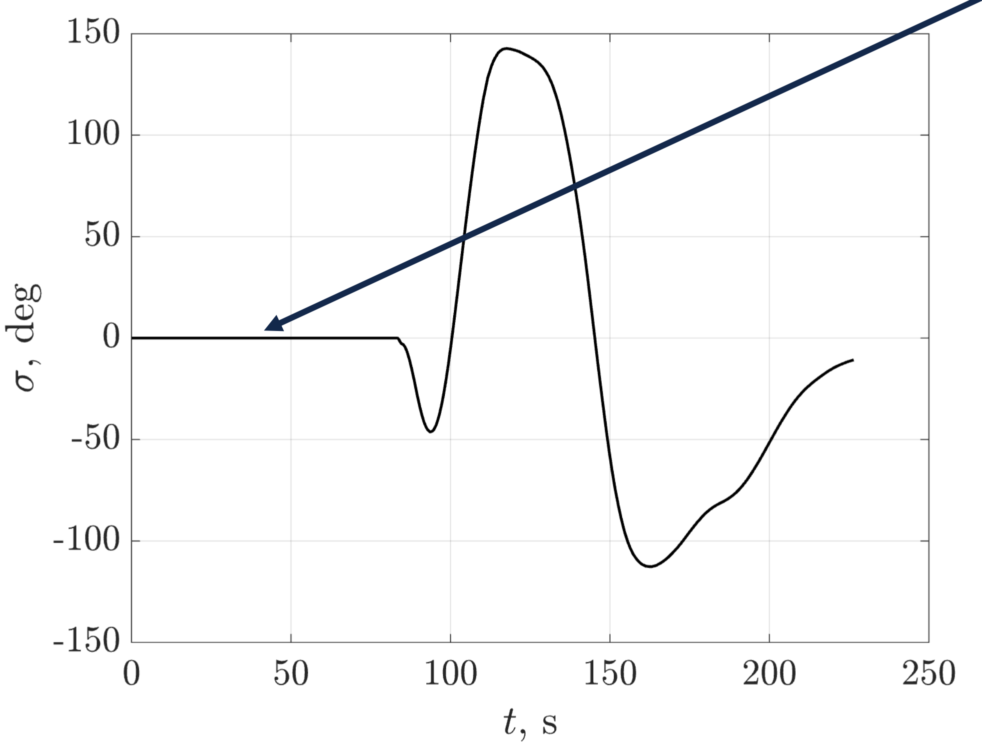


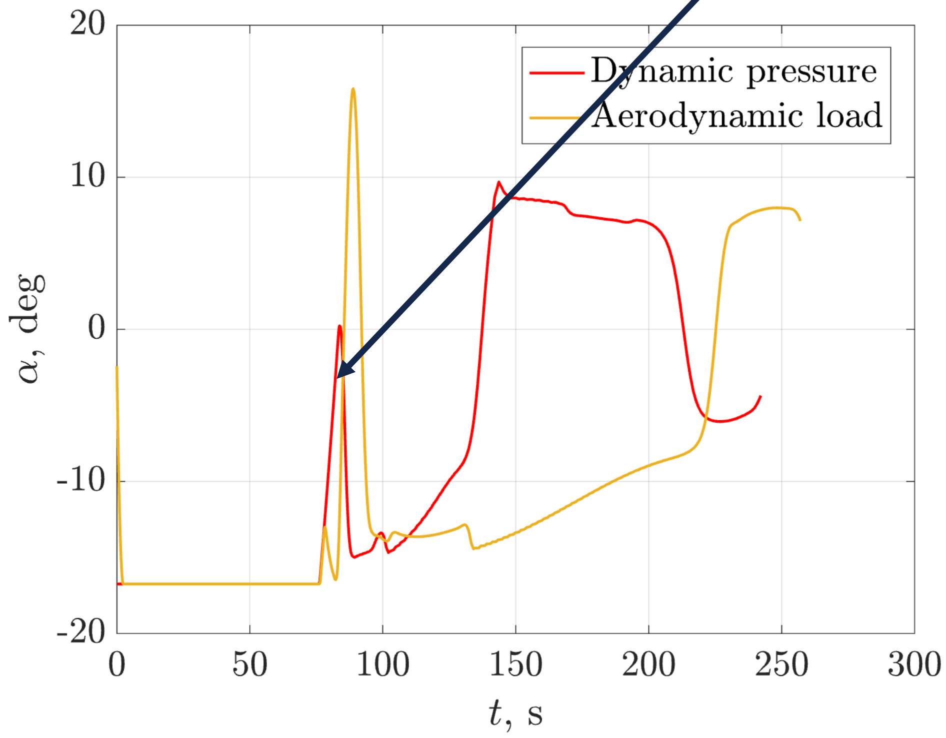
Minimum Peak Dynamic Pressure and Aerodynamic Load

Bank-angle steering
vehicle flies lift up

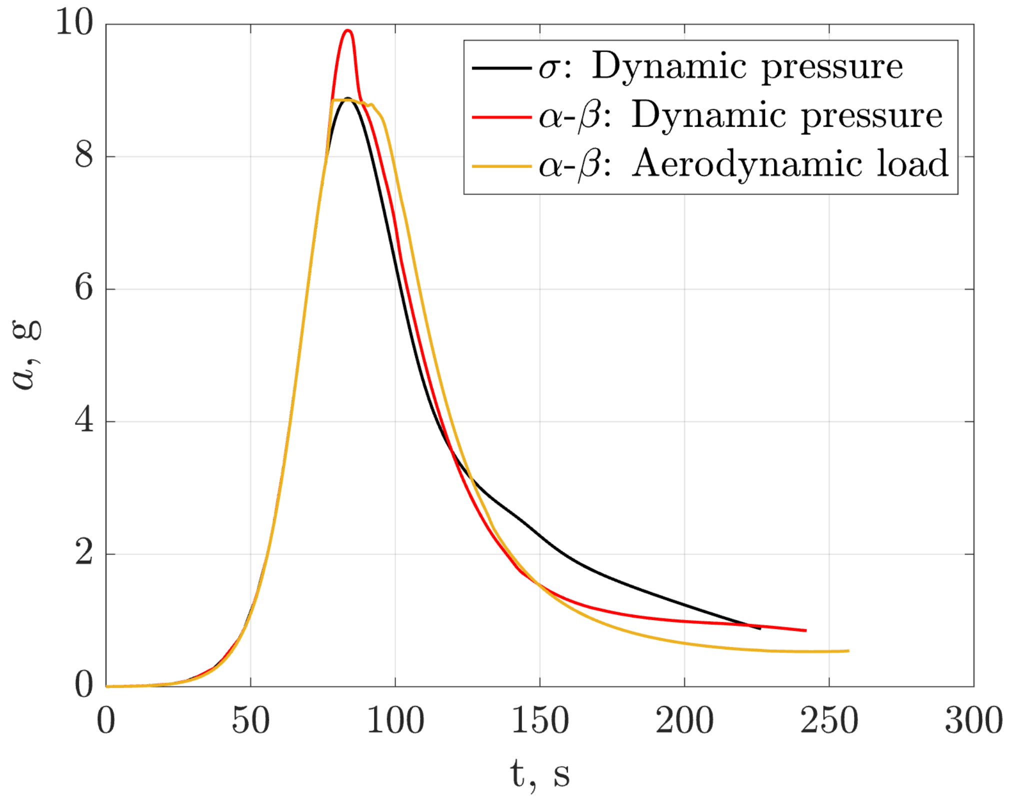
ith
eak

dynamic pressure





Minimum Peak Aerodynamic Load



Models and Equations of Motion: Closed-Loop Steering Commands

Truth dynamics consider 3DOF translational motion of a future large entry vehicle

- **Atmosphere**: Mars GRAM
- **Planet**: oblate, rotating
- **Aerodynamics**: Mars Science Laboratory aerodynamics database

Parameter	Value
m	5600 kg
S_{ref}	15.90 m^2
α_{max}	18 deg
β_{max}	18 deg
α_{bank}	-18 deg

Models Used in Optimal Control Solver

Parameter	Value
Surface Density	$0.012\ \mathrm{kg/m^3}$
Atmospheric Scale Height	11.888 km
Surface Gravity	$3.71\ \mathrm{m/s^2}$
Mars Equatorial Radius	3396.20 km

$$C_D = C_{D,0} + C_{D,\alpha}\alpha^2 + C_{D,\beta}\beta^2$$

$$C_L = C_{L,\alpha}\alpha$$

$$C_Y = C_{Y,\beta}\beta$$

Parameter	Value
$C_{D,0}$	1.67
$C_{D,\alpha}$	$-2.86/\mathrm{rad}^2$
$C_{D,\beta}$	$-2.86/\mathrm{rad}^2$
$C_{L,\alpha}$	-1.31/rad
$C_{Y,\beta}$	1.29/rad

Optimal Control Problems

$$\begin{cases} L = \dot{\sigma}^2 \\ L = \dot{\alpha}^2 + \dot{\beta}^2 \end{cases}$$

Problem 1: Maximum Altitude

$$\begin{aligned}
\min_{\mathbf{u}} \quad & J = -w_1 \frac{z(t_f)}{R_m} + \theta_{error}^2 + \phi_{error}^2 \\
\text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
& V(t_f) = V_f \\
\alpha\text{-}\beta : \; & \alpha \leq \alpha_{max} \qquad -\alpha \leq \alpha_{max} \\
& \beta \leq \beta_{max} \qquad -\beta \leq \beta_{max} \\
& \alpha_{total} \leq |\alpha_{bank}| \\
& \dot{\alpha} \leq \dot{\alpha}_{max} \qquad -\dot{\alpha} \leq \dot{\alpha}_{max} \\
& \ddot{\alpha} \leq \ddot{\alpha}_{max} \qquad -\ddot{\alpha} \leq \ddot{\alpha}_{max} \\
& \dot{\beta} \leq \dot{\beta}_{max} \qquad -\dot{\beta} \leq \dot{\beta}_{max} \\
& \ddot{\beta} \leq \ddot{\beta}_{max} \qquad -\ddot{\beta} \leq \ddot{\beta}_{max} \\
\sigma : \; & \dot{\sigma} \leq \dot{\sigma}_{max} \qquad -\dot{\sigma} \leq \dot{\sigma}_{max} \\
& \ddot{\sigma} \leq \ddot{\sigma}_{max} \qquad -\ddot{\sigma} \leq \ddot{\sigma}_{max}
\end{aligned}$$

Problem 2: Minimum Control Effort

$$\begin{aligned}
& \ldots + z_e^2) \\
& \int_{t_0} \ldots \\
& \ldots \mathbf{x}, \mathbf{u}) \\
& \ldots = V_f \\
\alpha\text{-}\beta : \; & \alpha \leq \alpha_{max} \qquad -\alpha \leq \alpha_{max} \\
& \ldots_{max} \qquad -\beta \leq \beta_{max} \\
& \ldots \leq |\alpha_{bank}| \\
& \dot{\alpha} \leq \dot{\alpha}_{max} \qquad -\dot{\alpha} \leq \dot{\alpha}_{max} \\
& \ldots_{max} \qquad -\ddot{\alpha} \leq \ddot{\alpha}_{max} \\
& \ldots_{max} \qquad -\dot{\beta} \leq \dot{\beta}_{max} \\
& \ddot{\beta} \leq \ddot{\beta}_{max} \qquad -\ddot{\beta} \leq \ddot{\beta}_{max} \\
\sigma : \; & \dot{\sigma} \leq \dot{\sigma}_{max} \qquad -\dot{\sigma} \leq \dot{\sigma}_{max} \\
& \ddot{\sigma} \leq \ddot{\sigma}_{max} \qquad -\ddot{\sigma} \leq \ddot{\sigma}_{max}
\end{aligned}$$

"Hit target latitude/longitude while maximizing altitude"

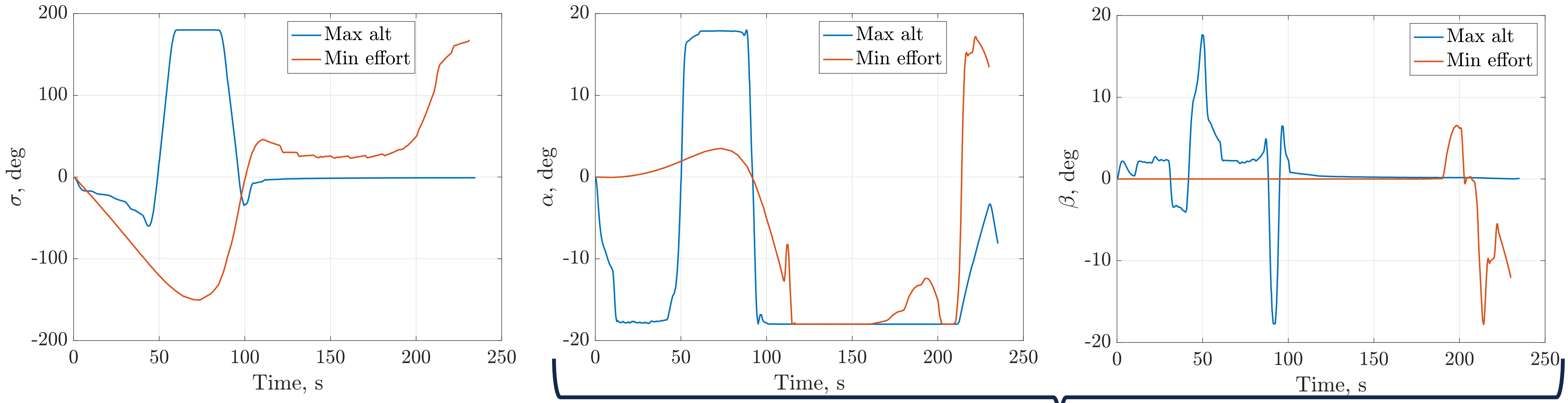
Dynamics

Terminal velocity

Control constraints

"Hit target point while minimizing control effort"

Nominal Results: Closed-Loop Steering Commands



Bank

α-β

Monte Carlo Simulation

Uncertainty analysis performed using Monte Carlo simulations with 1000 samples

Variable	Dispersion	Distribution
Initial altitude	± 2.8 km 3σ	Gaussian
Initial longitude	± 0.017 deg 3σ	Gaussian
Initial latitude	± 0.013 deg 3σ	Gaussian
Initial velocity	± 2 m/s 3σ	Gaussian
Initial flight-path angle	± 0.2 deg 3σ	Gaussian
Initial azimuth	± 0.006 deg 3σ	Gaussian
Atmosphere profile	1000 Mars GRAM atmospheres	-
Hypersonic C_A	$\pm 3\%$ 3σ	Gaussian
Hypersonic C_N	$\pm 5\%$ 3σ	Gaussian
Supersonic C_A	$\pm 10\%$ 3σ	Gaussian
Supersonic C_N	$\pm 8\%$ 3σ	Gaussian
Mass	± 2 kg 3σ	Gaussian

Impact of Vehicle Rate Limits

20 deg/s

Parameter	Downrange, km	Crossrange, km	Altitude, km
Mean: bank	0.15	3.61	9.78
SD: bank	3.76	0.73	0.51
Mean: α-β	-0.25	1.31	9.94
SD: α-β	3.89	0.43	0.52

α-β has slightly improved performance

5 deg/s

Parameter	Downrange, km	Crossrange, km	Altitude, km
Mean: bank	4.22	19.09	9.02
SD: bank	3.58	1.17	0.54
Mean: α-β	0.09	2.90	9.89
SD: α-β	3.51	0.58	0.52

Reduced performance for bank-angle steering

1 deg/s

Parameter	Downrange, km	Crossrange, km	Altitude, km
Mean: bank	3.45	56.44	4.72
SD: bank	5.74	0.91	0.52
Mean: α-β	-1.90	1.01	8.76
SD: α-β	4.07	0.37	0.57

Severe performance loss for bank-angle steering