Vector Analysis

$\vec{r} = \vec{AB} = \vec{OB} - \vec{OA}$ is the position vector between 2 points.

2 vectors are **colinear** if they lie in the same axis, i.e. $\vec{A} \times \vec{B} = \vec{O}$. If they are non colinear, then their intersection point determine a plane with the equation $\vec{r} = x\vec{a} + y\vec{b}$.
Similarly, 3 vectors are **coplanar** if they lie in the same plane, i.e. $[\vec{A}\vec{B}\vec{C} = \vec{O}]$. If they are non coplanar, then their intersection point determines a space, $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$.

Direction Vectors: A vector, $\vec{A}$, in space forms angles with the axis s.t. $\cos(\alpha) = \frac{A_1}{A}$, $\cos(\beta) = \frac{A_2}{A}$, and $\cos(\gamma) = \frac{A_3}{A}$ s.t. $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Vector Algebra:
$\vec{A} = \vec{B} \iff A = B$ and $\hat{A} = \hat{B}$
for $\vec{A}, \exists -\vec{A}$ s.t. $|\vec{A}| = |-\vec{A}|$ and $\hat{A} = -(\hat{-A})$
$\exists \vec{O}$ s.t. $\vec{A} + \vec{O} = \vec{A}$ and $\vec{A} + (-\vec{A}) = \vec{O}$
$m\vec{A} = \vec{B}$ s.t. $B = mA$ and $\hat{B} = m\hat{A}$
$\vec{A} + \vec{B} = \vec{B} + \vec{A}$
$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
$m(n\vec{A}) = (mn)\vec{A}$
$(m+n)\vec{A} = m\vec{A} + n\vec{A}$
$m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$

Dot Product: $\vec{A} \cdot \vec{B} = A_1B_1 + A_2B_2 + A_3B_3 + \cdots = AB\cos(\theta)$
Algebra: commutative, associative and distributive.
Unit vectors: $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$.
if $\vec{A} \cdot \vec{B} = 0$ and $\vec{A}, \vec{B} \neq \vec{O}$, then $\vec{A} \perp \vec{B}$.
Projection of $\vec{A}$ on $\vec{B} = \vec{A} \cdot \hat{B}$.

Cross Product, $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k & \dots \\ A_1 & A_2 & A_3 & \dots \\ B_1 & B_2 & B_3 & \dots \\ \vdots & \vdots & \vdots & \end{vmatrix} = AB\hat{u}\sin(\theta)$; $\hat{u} = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

Algebra: associative and distributive, but not commutative, i.e. $\vec{A} \times \vec{B} = -(\vec{A} \times \vec{B})$.
Unit vectors: $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{1}$ and $\hat{k} \times \hat{i} = \hat{j}$.
if $\vec{A} \times \vec{B} = \vec{O}$ and $\vec{A}, \vec{B} \neq \vec{O}$, then $\vec{A} || \vec{B}$.
Area of a parallelogram $= |\vec{A} \times \vec{B}|$.

Triple Product:
Scalar: $(\vec{A} \cdot \vec{B})\vec{C}$.
Box: $\vec{A} \cdot (\vec{B} \times \vec{C}) = [\vec{A}\vec{B}\vec{C}]$.
Cross: $\vec{A} \times \vec{B} \times \vec{C}$.
Equation of a space is $[\vec{A}\vec{B}\vec{C}] = \vec{O}$, where $\vec{A} = P\vec{P_1}$, $\vec{B} = P_2\vec{P_1}$ and $\vec{C} = P_3\vec{P_1}$
where $\vec{P} =< x, y, z >$ and P_1, P_2, P_3 are points in that space.

Differentiation

If $\vec{r}(u) = x(u)\hat{i} + y(u)\hat{j} + z(u)\hat{k}$, then $\frac{d\hat{r}}{du} = \frac{dx}{du}\hat{i} + \frac{dy}{du}\hat{j} + \frac{dz}{du}\hat{k}$.

Del operator: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} + \dots$.

Gradient of a scalar function, $\vec{\nabla}\phi$.

Divergence of a vector function, $\vec{\nabla}\vec{V}$.

Curl of a vector function, $\vec{\nabla} \times \vec{V}$.

Integration

If $\vec{r}(u) = x(u)\hat{i} + y(u)\hat{j} + z(u)\hat{k}$, then $\int_a^b \vec{r}(u)du = \hat{i}\int_a^b x(u)du + \hat{j}\int_a^b y(u)du + \hat{k}\int_a^b z(u)du$.

Line Integral of $\vec{V}(x, y, z)$ along a curve c joining a, b is $\vec{V} \cdot d\vec{r} = \int_a^b A_1dx + \int_a^b A_2dy + \int_a^b A_3dz$.

If c is a piecewise smooth curve, i.e. $c = c_1 \cup c_2 \cup c_3 \cup \dots$, then $\int\limits_c \vec{A} \cdot d\vec{r} = \int\limits_{c_1} \vec{A} \cdot d\vec{r} + \int\limits_{c_2} \vec{A} \cdot d\vec{r} + \dots$.

Surface integral of S (a 2 sided surface) is $\int\limits_S\int \vec{A} \cdot \hat{n}\frac{dydx}{|\hat{n}\cdot\hat{k}|} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{\vec{A}\cdot\hat{n}}{|\hat{n}\cdot\hat{k}|}dydx$ where $\hat{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}$
where ϕ is the equation of S.

This assumes the surface forms a projection on the xy plane and $y_2 = f(x)$. Depending on the surface, the specifics of the integral can change.

Vector is a quantity having magnitude and direction. $\vec{A} = |A|\hat{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k} + \cdots = \langle A_1, A_2, A_3, \dots \rangle$