

# Regression Line

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$$Y = a + bX + e$$

$$\hat{Y} = a + bX$$

$$Y = \hat{Y} + e$$

# Residuals

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$$e = Y - \hat{Y}$$

# OLS Estimation

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- Using calculus, we obtain

$$b = \frac{\sum[(X - M_X)(Y - M_Y)]}{\sum(X - M_X)^2} = \frac{SS_{XY}}{SS_X}$$

$$a = M_Y - bM_X$$

# OLS Estimation

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$$b = r \left( \frac{s_Y}{s_X} \right)$$

# The Standardized Model

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$$\hat{z}_Y = \beta z_X$$

# Standard Error of the Estimate

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$$s_e = \sqrt{\frac{SSE}{df}}$$

- where  $df = n - 2$  for simple linear regression
- Recall from ANOVA:  $MSE = \frac{SSE}{df}$ 
  - Thus,  $s_e$  is sometimes called root MSE

# Coefficient of Determination

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- Coefficient of determination:  $r^2$  or  $R^2$ 
  - Proportion of variance in  $Y$  that is accounted for by  $X$

# Hypothesis Test for $\rho^2$

F-statistic:

**TABLE 16.4** The  $F$  Table for an Analysis of Regression

Formulas for Completing the Analysis of Regression				
Source of Variation	$SS$	$df$	$MS$	$F_{\text{obt}}$
Regression	$r^2 SS_Y$	1	$\frac{SS_{\text{regression}}}{df_{\text{regression}}}$	$\frac{MS_{\text{regression}}}{MS_{\text{residual}}}$
Residual (error)	$(1 - r^2) SS_Y$	$n - 2$	$\frac{SS_{\text{residual}}}{df_{\text{residual}}}$	
Total	$SS_{\text{regression}} + SS_{\text{residual}}$	$n - 1$		



# Hypothesis Test for $\alpha^*$ and $\beta^*$

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t-statistic

$$t = \frac{a}{s_a} \qquad t = \frac{b}{s_b}$$

- where
  - $a$  is the parameter estimate for the intercept and  $s_a$  is the estimated standard error for the slope
  - $b$  is the parameter estimate for the intercept and  $s_b$  is the estimated standard error for the slope

# Confidence Intervals

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$$a \pm t_{cv}(s_a)$$

$$b \pm t_{cv}(s_b)$$

- If 0 is contained in the CI, then fail to reject the null hypothesis

# Hypothesis Testing

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$$F = t^2$$

\*For SLR only