Wavelet Image Threshold Denoising Based on Edge Detection

Wei Liu [a], Zhengming Ma [b]
[a] Computer School, South China Normal University, Guangzhou, 510631, China
Phone: 862031772014, E-mail: buzzingbee@126.com
[b]Information Processing Lab., Electronics Dept., Zhongshan Univ., Guangzhou,510275, China

Abstract - Most commonly used denoising methods use low pass filters to get rid of noise. However, both edge and noise information is high-frequency information, so the loss of edge information is evident and inevitable in the denoising process. Edge information is the most important high-frequency information of an image, so we should try to maintain more edge information while denoising. From this comes the thesis of this paper. In it, we present a new image denoising method: wavelet image threshold denoising based on edge detection. Before denoising, those wavelet coefficients of an image that correspond to an image's edges are first detected by wavelet edge detection. The detected wavelet coefficients will then be protected from denoising, and we can therefore set the denoising thresholds based solely on the noise variances, without damaging the image's edges. The theoretical analyses and experimental results presented in this paper show that, compared to commonly-used wavelet threshold denoising methods, our method can keep an image's edges from damage and can increase the PSNR up to 1~2dB. Finally, we can draw the conclusion that edge detection and denoising are two important branches of image processing. If we combine edge detection with denoising, we can overcome the shortcomings of commonly-used denoising methods and do denoising without notably blurring the edge.

Key Words: image processing, wavelet threshold denoising, wavelet edge detection

I. INTRODUCTION

Image denoising is a fundamental task for image processing researchers. At the present time, the basic idea of denoising is to reduce high frequencies, because noise is always presented by high frequencies. As a result, most denoising techniques tend to suppress high frequencies, especially when removing the additive white Gaussian noise (AWGN). However, these methods can cause obvious problems: as we know, both noise and edges are characterized by high frequencies. Thus, it is difficult to remove noise from images without blurring the edges. At the present time, much work is being focused on solving this problem.

In the wavelet domain, the denoising algorithm based on the threshold filter [1] is widely used, because it's comparatively efficient and easy to realize. Wavelet decomposition transforms a signal from the time domain to the time-scale domain, and describes the local feature well in both time domains. We can select a threshold according to the characteristic of the image, modifying all of the discrete detail coefficients so as to reduce the noise. However, we are still in the dilemma of determining the level of the threshold. The higher the threshold is, the better the effect of denoising will be, and, at the same time, the blurrier the edge will be.

The edges of an image mostly reflect the information of the image, and contain its basic character. According to research on human eyes, the characteristic of the edges is one of several characteristics that can strongly impress the visual system[2]. Thus, when we process denoising, the first thing that we should care about is trying to retain edge information.

This paper presents a new image denoising method: wavelet image threshold denoising based on edge detection. Before denoising, those wavelet coefficients of an image that correspond to an image's edges are first detected by wavelet edge detection. The detected wavelet coefficients will be protected from the ensuing denoising process, and therefore we can set the denoising thresholds based only on the noise variances, without worrying about damaging the image's edges. The theoretical analysis and experimental results presented in this paper shows that, compared with commonly used wavelet threshold denoising methods, the proposed denoising method is more effective. The idea of combining edge detection with denoising is doable.

The rest of this paper is organized as follows. We present the method of wavelet image edge detection in Section 2, and our image denoising method in Section 3 Experimental results to demonstrate the performance of the proposed method are given in Section 4, and conclusions and comments are given in Section 5.

II. THE MODEL OF NOISE

Images are, in many cases, degraded by noise inevitably. Noise is often taken as Poisson (related to the arrival of photons) and/or Gaussian. [3]

For example, emission and transmission tomography images are usually corrupted by quantum noise, which is Poisson distributed. Generally, images formed at low light levels are also corrupted by this kind of noise. Quantum noise is due to the quantization of energy into photons, each having energy hv where h is Planck's constant and v is the frequency of the radiation. At low light levels, the number of photons recorded is small, and the statistical variability of the number of counts about the mean limits the quality of the images [4]. When light levels are high, noise can be taken as Gaussian noise.

Another example occurs when images recorded on photographic film are scanned and recorded for storage or transmission. The resulting image is proportional to the film density and is thus corrupted by film-grain noise [4]. In most cases, film-grain noise can be modeled as white Gaussian noise with a zero mean.

This paper discusses how to remove the additive white Gaussian noise (AWGN) with a zero mean. For other kinds of noise modeling, the idea of this paper is also applicable.

III. THE METHOD OF WAVELET IMAGE EDGE DETECTION

The denoising method we present needs to detect the image's edges before denoising, so as to protect the image's edge information from damage in the following denoising process. In our method, finding out the precise location of the edges is pivotal. Many classical edge detectors are already available. Edges can be determined from the image by processing directly in the spatial domain, or by transformation to a different domain. In the spatial domain, there are Sobel edge operators, Prewitt edge operators, Kirsch edge operators, and so on. In the transforming field, wavelet transformation is adapted to the wildly-changed edges better than with the normal Fourier transformation. Wavelet transformation, which is called the "mathematical microscope," has a resolution in both the time field and the frequency field. It can focus onto any detail of the analyzed object by taking more and more fine steps of the space field. Owing to these characteristics, wavelet transform is very suitable for use in edge detection. This paper presents an image edge detection method based on wavelet transformation.

A. Principle

According to the theory of wavelet edge detection [5][6][7], a two-dimensioned dyadic wavelet is adopted. This 2-D wavelet transformation requires two wavelets, namely, $\phi_{2^j}^x(x,y)$ and $\phi_{2^j}^y(x,y)$, which are partial derivatives of a smoothing function $\theta_{2^j}(x,y)$. The wavelets are defined as: $\phi_{2^j}^x(x,y)=\dfrac{\partial\theta_{2^j}(x,y)}{\partial x}$, $\phi_{2^j}^y(x,y)=\dfrac{\partial\theta_{2^j}(x,y)}{\partial y}$. So the wavelet coefficient of digital image $f(x,y)$ at a scale 2^j, which can be written as:

$$\begin{bmatrix} W_{2^j}^x f(x,y) \\ W_{2^j}^y f(x,y) \end{bmatrix} = \begin{bmatrix} f*\phi_{2^j}^x(x,y) \\ f*\phi_{2^j}^y(x,y) \end{bmatrix} = 2^j \begin{bmatrix} \dfrac{\partial}{\partial x} f*\theta_{2^j}(x,y) \\ \dfrac{\partial}{\partial y} f*\theta_{2^j}(x,y) \end{bmatrix} = 2^j \nabla\left(f*\theta_{2^j}\right)(x,y) \tag{1}$$

From (1), we can see that the wavelet coefficients in fact correspond to the gradient of the smoothed version of $f(x,y)$ at the scale 2^j. Noticing that an edge can be defined as a local maximum of the gradient modulus along the gradient direction, we can detect the edges at the scale 2^j from $W_{2^j}^x f(x,y)$ and $W_{2^j}^y f(x,y)$.

However, when images are corrupted by AWGN due to noise, some pixels of the homogeneous regions may also have a local maximum of the gradient modulus, so we should distinguish the coefficients corresponding to noise from those corresponding to the potential edges. We know that the Lipschitz exponent values of AWGN are always negative, so the value of its corresponding local maximum of the gradient modulus will diminish at higher scales. This is different from the edges of the image, which always have positive Lipschitz exponent values. As a result, we can wipe off some coefficients corresponding to noise by using these different attributions. Furthermore, we can connect the remaining coefficients along the edge orientation, which is vertical to the gradient direction. Those that cannot be connected will be considered as coefficients corresponding to noise, and then will be wiped off.

B. The edge detection approach

In practice, we should pay attention to the following：

1. The length of the filter used in DWT should not be too long; otherwise, it will affect the effect of edge detection.

2. The boundary should be treated properly. In our experiment, we use a mirror-symmetrical extension.

The edge detecting procedure is composed of the following stages:

1. apply pretreatment to the image, using the average filter and denoting the resulting image $f(x, y)$。

2. let $S_{2^0} f(x, y) = f(x, y)$,and apply the redundant wavelet transformation to each row of $S_{2^j} f(x, y)$:

$$\begin{cases} W_{2^{j+1}}^{y} f = \left(S_{2^j} f\right) * G_j \\ S_{2^{j+1}} f = \left(S_{2^j} f\right) * H_j \qquad j = 0,1,2,\cdots \end{cases} \tag{2}$$

In the equation, 2^j denotes the scale, G_j 、and H_j represents the filter used in the DWT. The coefficients of the wavelet filter are tabulated in Table I.

TABLE I
THE COEFFICIENTS OF WAVELET FILTER

	-1	0	1	2
$H_0(x)$	0.025	0.475	0.475	0.025
$G_0(x)$	0.025	-0.475	0.475	0.025

3. Find the local maximum coefficients of every row. Record these coefficients $W_{2^j}^{y} f(x, y)$.

4. Remove the coefficients with low Lipschitz exponent values from the recorded coefficients, because they correspond to noise. Thus, we can get the coefficients $W_{2^j}^{y} f(x, y)$ corresponding to the potential edges of each row at different scales.

5. Applying stage 1,2,3, and 4 to every column, we can get the coefficients $W_{2^j}^{x} f(x, y)$ corresponding to the potential edges of each column at different scales.

6. Note that the wavelet coefficients $W_{2^j}^{x} f(x, y)$, $W_{2^j}^{y} f(x, y)$ in fact correspond to the gradient of the smoothed version of $f(x, y)$ at the scale 2^j. The edge magnitudes and orientation can be calculated from the image gradient as follows:

$$M_{2^j} f(x, y) = \sqrt{\left|W_{2^j}^{x} f(x, y)\right|^2 + \left|W_{2^j}^{y} f(x, y)\right|^2} \tag{3}$$

$$A_{2^j} f(x, y) = \arg\tan\left(\frac{W_{2^j}^{y} f(x, y)}{W_{2^j}^{x} f(x, y)}\right) \tag{4}$$

7. Join the recorded coefficients of similar edge magnitudes along the edge orientation in a chain. Those isolated coefficients are wiped off. When the length of the chain reaches the threshold T, the pixels corresponding to the wavelet coefficients in the chain are considered to be edge pixels.

C. Experimental Results

We applied our edge detecting technique to a 256*256 Lena image corrupted by AWGN. Fig.1 is the original image, while Fig.2 (a)~(d) are images corrupted by AWGN with zero mean and different variances. Fig.3 (a)~(d) shows the results of edge detection.

Lena image is an image with relatively complex edges. It is difficult for normal edge detection to completely detect the different types of edges. With a noise-corrupted Lena image, the edge detection task is even more difficult. The method we present uses the advantages of wavelet transformation, which can focus onto any detail of the analyzed object by taking more and more fine steps of the space field. At the low scale, many details of the edges, such as the girl's pupils, are detected; at a high scale, smooth longer edges, such as the pole on the left, are seen. The experimental results shown in Fig.3 (a)~(d) prove that our edge detecting method is effective.

IV. OUR IMAGE DENOISING METHOD

A. Principle

After wavelet transformation, most signal energy is supposed to be clustered in a few wavelet coefficients, whereas noises are not. The thresholding, or shrinkage on the wavelet coefficients with a proper threshold, can then significantly reduce noise. The key point of wavelet threshold denoising is selecting a proper threshold---- the higher the threshold is, the better effect of denoising will be, and, at the same time, the blurrier the edge will be. Thus, one dilemma to determine the level of the threshold.

Our denoising method is focused on solving this problem. Before denoising, those wavelet coefficients of an image that correspond to an image's edges are first detected by the method of wavelet edge detection. The detected wavelet coefficients will be protected from the ensuing denoising process, and, therefore, we can set the denoising thresholds based solely on the noise variances, without worrying about damaging the image's edges. In our experiment, we choose the VisuShink threshold, indicated in [1]: $T = \sigma\sqrt{2\ln N}$, where σ denotes the strength of the noise and N presents the number of pixels of the sub-image.

B. Our image denoising approach

The procedure is composed of the following six stages:

1. Detect the wavelet coefficients corresponding to the image's edges by the method of wavelet edge detection.

2. Preserve the coefficients corresponding to the edges.

3. Apply wavelet transform to the original noise-corrupted image.

4. Do the normal wavelet image threshold noising process.

$$\tilde{w} = \begin{cases} w & |w| \geq T \\ 0 & |w| < T \end{cases} \quad (5)$$

In (5), T presents VisuShink threshold: T= $\sigma\sqrt{2InN}$. Here, σ denotes the strength of the noise, and N represents the number of pixels of the sub-image.

5. Replace the coefficients corresponding to the edges with the preserved coefficients. The detected edges also contain noise, so they must be denoised too. Here we again use wavelet denoising based on the threshold filter, but a much lower threshold, T, is applied in order to maintain more edge information: T= $\beta\sigma\sqrt{2InN}$ (**0.2<**β <0.3) [8].

6. By applying the reverse wavelet transformation, we can get the denoised image.

C. Experimental Results

We applied three denoising methods to images that had been corrupted by white Gaussian noise with a zero mean and different variances(see Fig.2). The three methods are: the method we present, the classical image wavelet threshold denoising, $T = \sigma\sqrt{2\ln N}$ (VisuShink threshold), and the classical image wavelet threshold denoising, $T = \beta\sigma\sqrt{2\ln N}$.(β is the adjusting factor; here, β =0.3) [8], Table II shows the experimental results. Fig. 4-7 gives the resulting denoising images. From the table and the figures, we can see that, with the classical denoising method, it is difficult to decide the value of the threshold. When we use the VisuShink threshold, the denoised image is smoother, but, at the same time, more edge information is lost, so the edges are notably blurred. When we lower the threshold and multiply it by a factor β , more edge information is maintained, but the PSNR value is also lowered. Thus, with the classical denoising method, it is a dilemma to determine the level of the threshold. The higher the threshold is, the better effect of denoising will be, and, at the same time, the blurrier the edge will be.

With denoising method we presented, those wavelet coefficients of an image that correspond to an image's edges are first detected by the method of wavelet edge detection before denoising. The detected wavelet coefficients will then be protected from denoising, and we can therefore set the denoising thresholds based only on the noise variances and without damaging the image's edges. The theoretical analysis and experimental results presented in this paper show that, compared with the commonly-used wavelet threshold denoising methods, our method can keep an image's edges from damage and increase the PSNR up to 1~2dB.

TABLE II
PSNR RESULTS FOR LENA IMAGE

Variance of the noise	Our method	Wavelet Image Threshold Denoising ($T = \sigma\sqrt{2\ln N}$)		Wavelet Image Threshold Denoising ($T = 0.3\sigma\sqrt{2\ln N}$)	
	PSNR (dB)	PSNR (dB)	Number of affected edge pixels	PSNR (dB)	Number of affected edge pixels
$\sigma = 5$	34.18	32.71	3132	34.09	2712
σ=10	30.64	29.10	3157	28.53	2897
σ =15	28.72	27.18	3137	25.24	3009
σ= 20	27.43	25.97	3143	22.99	3095

V. CONCLUSIONS

Edge detection and denoising are two important branches of image processing. If we combine edge detection with denoising, we can overcome the shortcoming of commonly used denoising methods and do denoising without notably blurring the edge.

Furthermore, there are many denoising and edge detection methods now in use. Different methods are suitable for different type of images and for different noise models. We can do further research on how to combine these different denoising and edge detection methods, according to the content of the images and the nature of the noise.

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Fig.1.original 256*256 Lena image

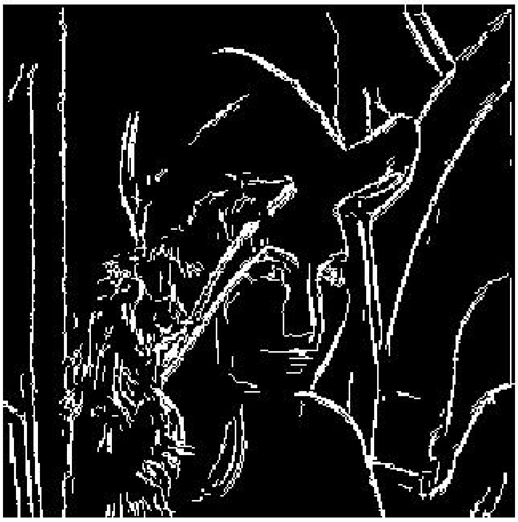
(a) $\sigma=5$

(b) $\sigma=10$

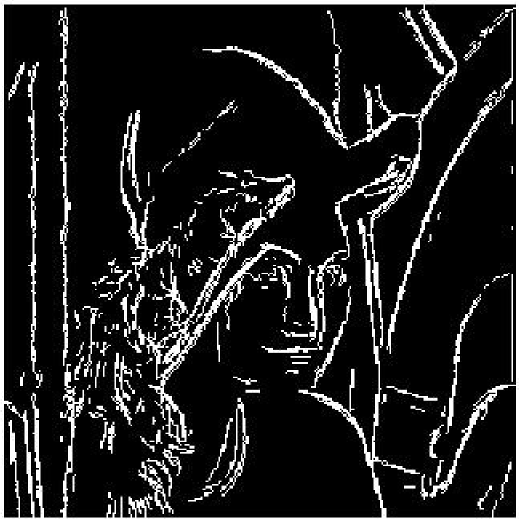
(c) $\sigma=15$

(d) $\sigma=20$

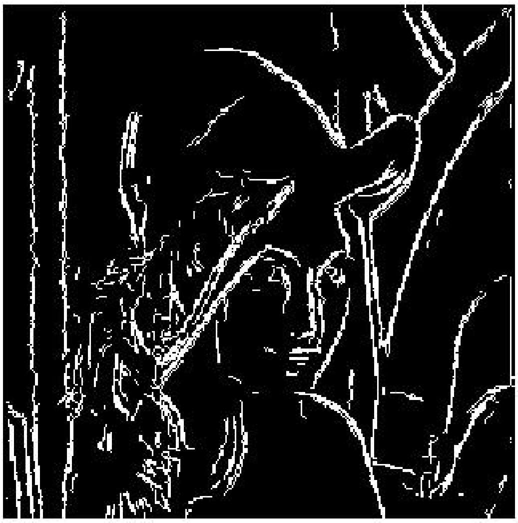
Fig.2. AWGN corrupted images



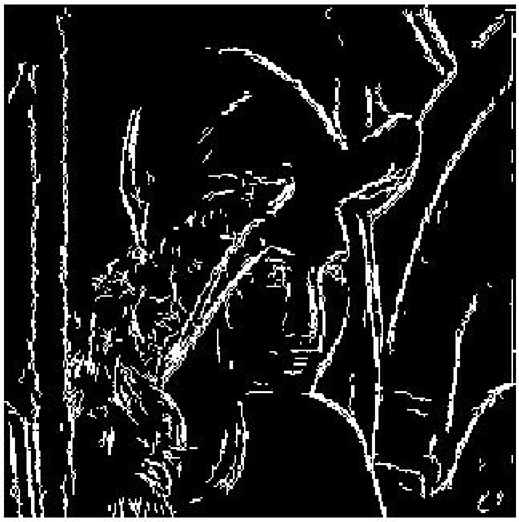
(a) $\sigma = 5$



(b) $\sigma = 10$



(c) $\sigma = 15$



(d) $\sigma = 20$

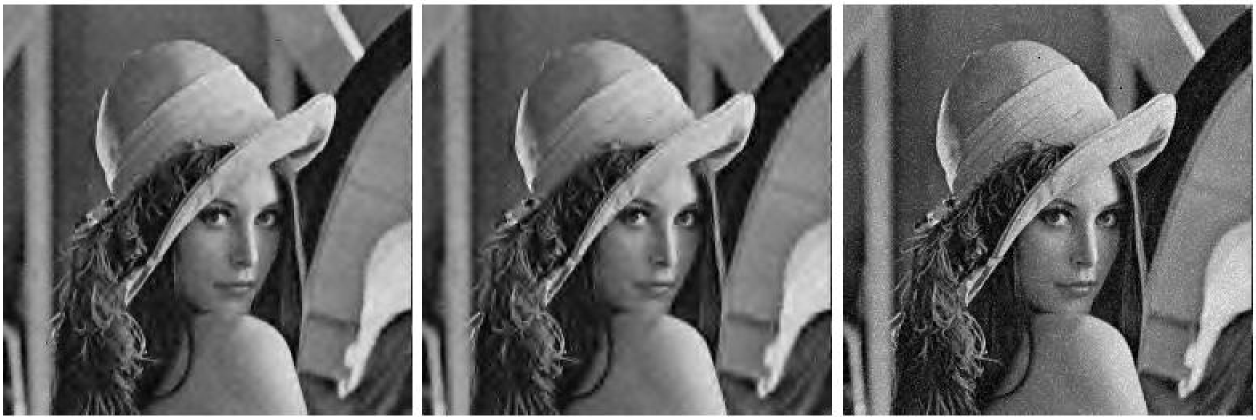
Fig.3. The detected edges at different noise level

(a) Our method

(b) Wavelet Image Threshold Denoising ($T = \sigma\sqrt{2\ln N}$)

(c) Wavelet Image Threshold Denoising ($T = 0.3\sigma\sqrt{2\ln N}$)

Fig.4. Denoising results ($\sigma = 5$)

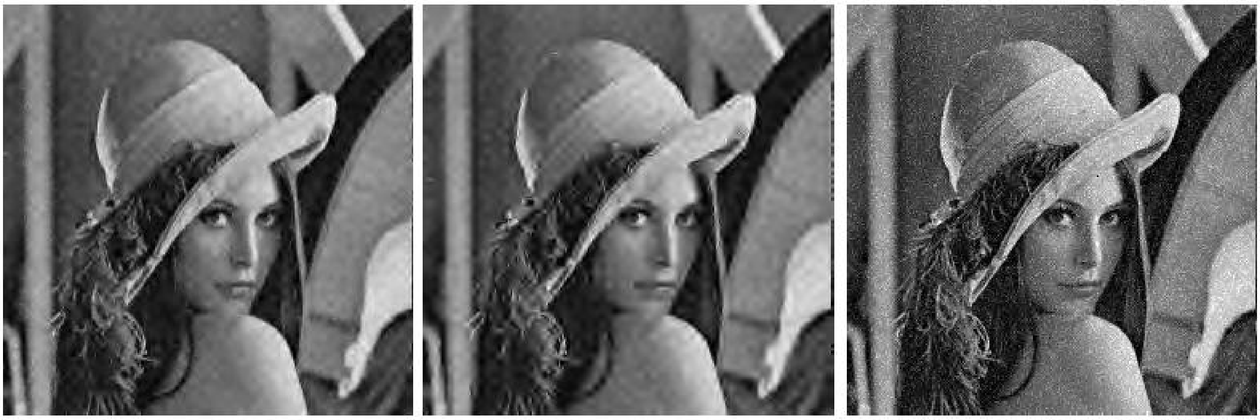


(a) Our method

(b) Wavelet Image Threshold Denoising ($T = \sigma\sqrt{2\ln N}$)

(c) Wavelet Image Threshold Denoising ($T = 0.3\sigma\sqrt{2\ln N}$)

Fig.5. Denoising results ($\sigma = 10$)

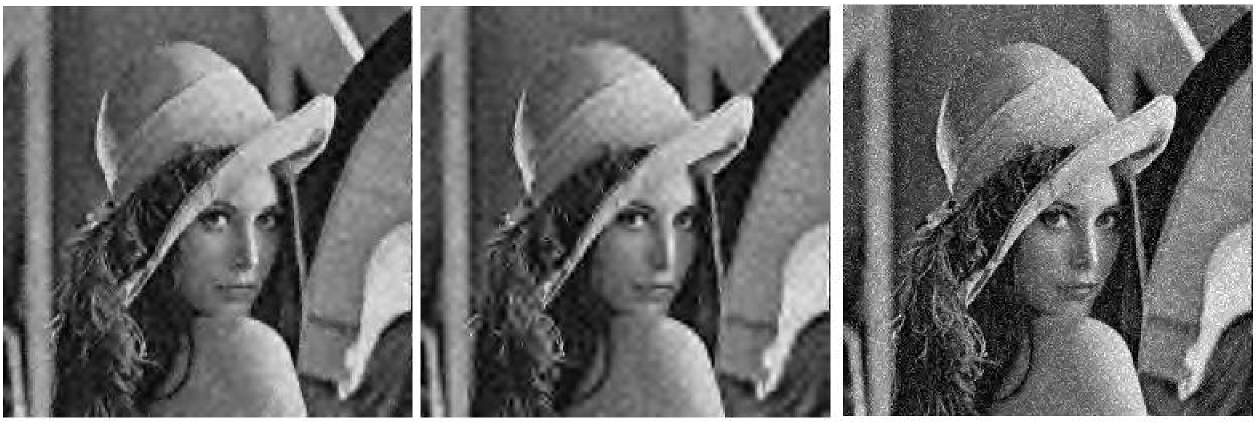


(a) Our method

(b) Wavelet Image Threshold Denoising ($T = \sigma\sqrt{2\ln N}$)

(c) Wavelet Image Threshold Denoising ($T = 0.3\sigma\sqrt{2\ln N}$)

Fig.6. Denoising results ($\sigma = 15$)



(a) Our method

(b) Wavelet Image Threshold Denoising ($T = \sigma\sqrt{2\ln N}$)

(c) Wavelet Image Threshold Denoising ($T = 0.3\sigma\sqrt{2\ln N}$)

Fig.7. Denoising results ($\sigma = 20$)