The cylinder at spatial infinity and asymptotic charges

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September 19, 2023

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Table of Contents

Introduction

- The Newman-Penrose (NP) constants serve as conserved quantities at null infinity in asymptotically flat gravitational fields.
- These constants present a comprehensive conservation system for various spins: spin-1 fields and spin-2 fields, with our research focusing on spin-0 fields linked to wave equation solutions.
- In the detailed context, while an infinite series of conserved quantities is identified in the linear theory, the non-linear General Relativity theory conserves only ten.

Conservation laws

- These charges are computed as 2-surface integrals at cuts $C \approx \mathbb{S}^2$ of null infinity $\mathscr{I}$.

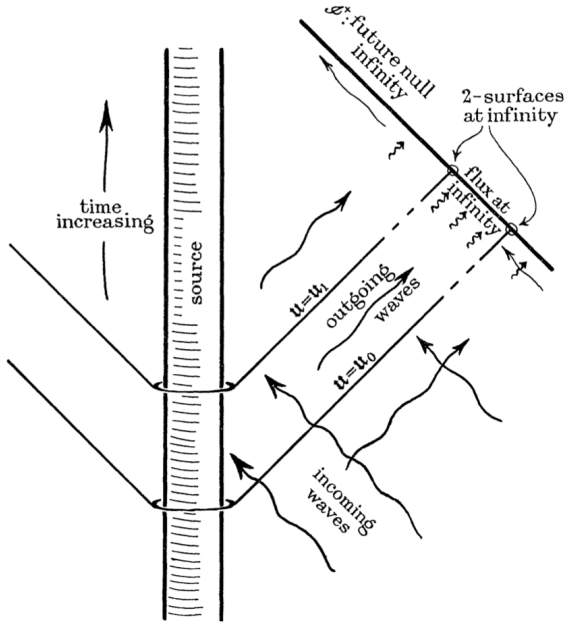

Figure: Visual representation of the behavior of the Newman-Penrose constants at null infinity.

Spin-1 (EM) Field

- A complex tetrad can be selected as follows:

$$l^\mu = \delta^\mu_1, \quad n^\mu = \delta^\mu_0 - \delta^\mu_1,$$
$$m^\mu = \frac{1}{\tau}\left(\delta^\mu_2 + \frac{1}{\sin\theta}\,\delta^\mu_3\right), \qquad \overline{m}^\mu = \frac{1}{\tau}\left(\delta^\mu_2 - \frac{1}{\sin\theta}\,\delta^\mu_3\right). \tag{1}$$

- To describe the electromagnetic (EM) field, we make use of three complex tetrad components of the Maxwell field tensor denoted as $F_{\mu\nu}$:

$$\Phi_0 = F_{\mu\nu} l^\mu n^\nu,$$
$$\Phi_1 = \frac{1}{2} F_{\mu\nu}(l^\mu n^\nu + \overline{m}^\mu m^\nu),$$
$$\Phi_2 = F_{\mu\nu}\overline{m}^\mu n^\nu. \tag{2}$$

NP Constants Calculation

- The NP constants are calculated using the following formula:

$$F_m^{n,k} = \int {}_1\overline{Y}_{n-k+1,m}\Phi_0^{n+1}d\omega. \tag{3}$$

- The interpretation of charges, such as the Newman-Penrose constants, remains an open debate, yet their conservation is evident in general asymptotically flat spacetimes, even in events like black hole collisions.

The i^0 cylinder representation in Minkowski spacetime

Methodology

Results

Conclusion

References

Thank You