The Newman-Penrose constants for spin-0 fields close to spatial and null-infinity in Minkowski spacetime

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Hairiness of GR

- Black holes are characterized by three quantities: mass, charge and spin
- The information paradox arises from the fact that objects falling into a black hole have their information erased

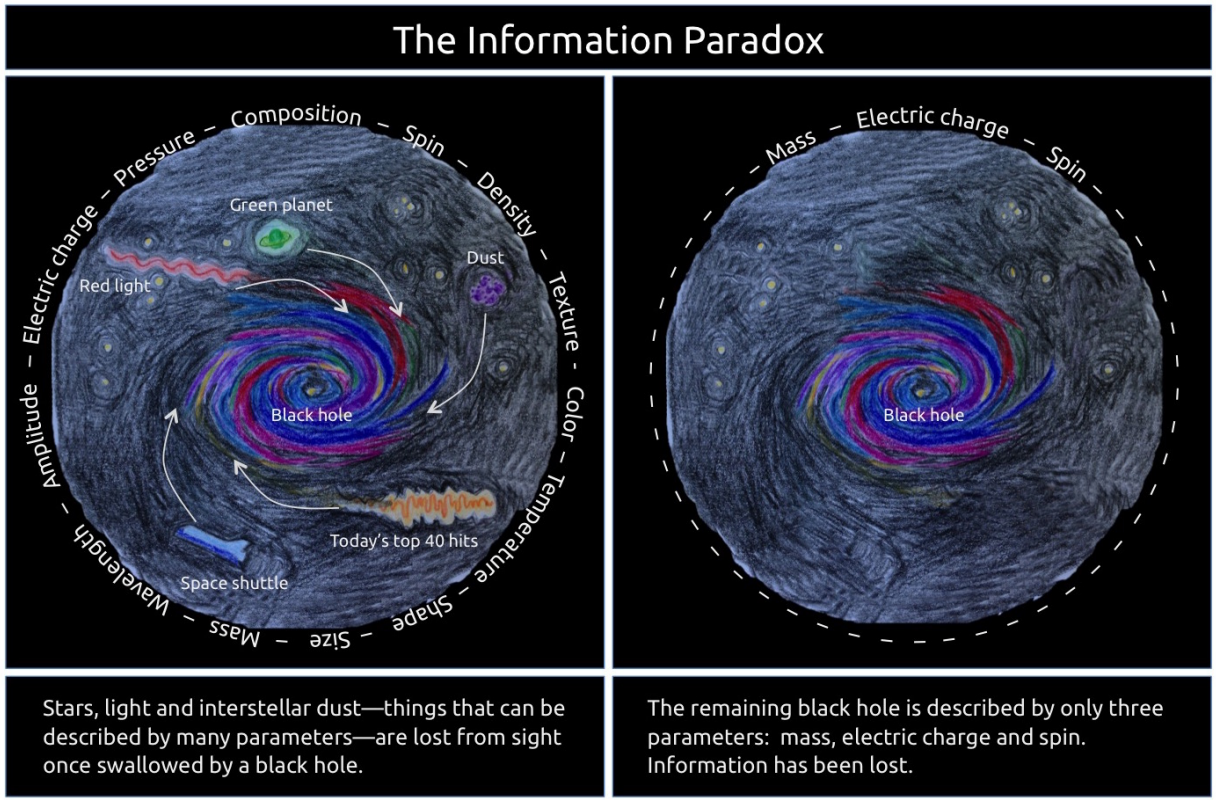
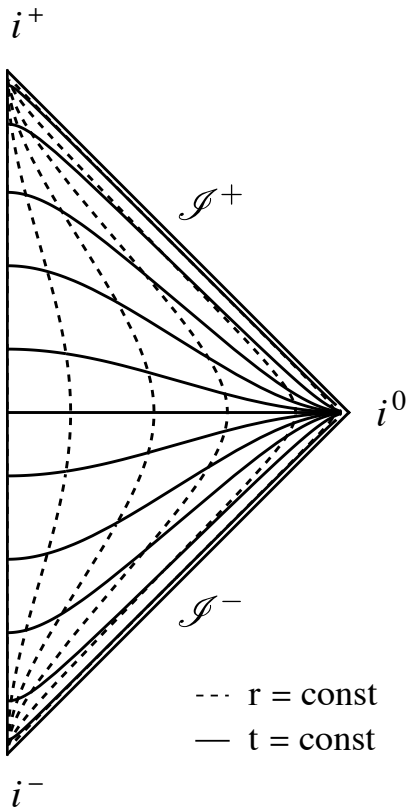
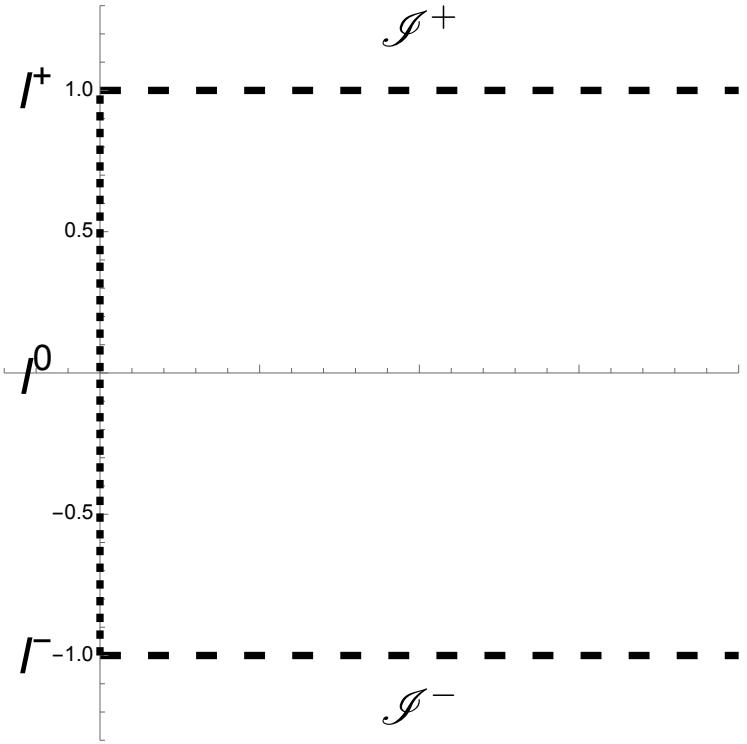


Figure: taken from Harvard Edu

NP constants and Friedrich Cylinder at i^0





- Conformal transformation
- $\tilde{\eta}$: physical spacetime
- γ: unphysical spacetime

- $\gamma = \Theta^2 \tilde{\eta}, \;\; \Theta = \rho(1 - \tau^2)$
- $\mathscr{I}^+ \equiv \{\tau = 1\}, \;\; \mathscr{I}^- \equiv \{\tau = -1\}$
- Critical sets: $\mathcal{I}^{\pm}$

NP constants for spin-0 fields

- Solve $\tilde{\Box}\tilde{\phi} = 0 \implies \Box\phi = 0$
- $\phi = \Theta^{-1}\tilde{\phi}$
- $\phi = \sum_{p,lm,} \frac{1}{p!} a_{p;l,m}(\tau)\rho^p Y_{lm}$
- $a_{plm}(\tau) = A_{p,l,m}P(\tau) + B_{p,l,m}Q(\tau)$
- $p < l$ poly, $p = l$ log (reg condition).

- Null derivative $\hat{L} = \tilde{\rho}^2\partial_{\tilde{u}} = \Lambda^2 e$
- $\Lambda(\tau, \rho)$, $e = (1+\tau)\partial_\tau - \rho\partial_\rho$

The formula to compute the NP constants is,

$$\mathcal{I}^+_{lm} = \lim_{\substack{\rho\to 0 \\ \tau\to 1}} \langle \hat{L}^{l+1}\phi, Y_{lm}\rangle \tag{1}$$

NP constants for spin-0 fields

For $l = 1$ we have

$$\mathcal{I}^{+}_{1m} = 2^{-4}\frac{1}{2!}6A_{211}$$

Lemma (Gasperín & Pinto 2023)

$$\hat{L}^n\phi = \Lambda^{2n}\sum_{p=0}^{\infty}\sum_{l=0}^{p}\sum_{m=-l}^{m=l}\frac{1}{p!}\rho^p A^{n-1}_{p,l,m}(\tau)Y_{0,l,m}, \tag{2}$$

$$A^{n-1}_{p,l,m}(\tau) = \sum_{q=0}^{n}(-1)^q k_q\binom{n}{q}(1+\tau)^{n-q}a^{(n-q)}. \tag{3}$$

NP constants for spin-0 fields

The previous lemma was proven by induction.

Lemma (Gasperín & Pinto 2023)

If the regularity condition is satisfied and the NP constants are finite then they are determined in terms of the initial data by

$$\mathcal{I}^{+}_{nm} = Q(n) A_{(n+1),n,m} \tag{4}$$

where $Q(n)$ *is a numerical coefficient &* $A_{(n+1),n,m}$ *is determined by the initial data, for* ϕ *on* $\tau = 0$.

Conclusions

- When an object falls into a black hole, it is reduced to just three numbers, leading to the loss of a large amount of information, a problem known as the "information paradox."
- Recently the concept of "soft hair" has been proposed to explain this paradox.
- The Newman-Penrose (NP) constants are quantities defined on null-infinity in general relativity.
- In the future, we want to examine the NP constants of the critical set $\mathcal{I}^-$ to see if there is a connection between the NP constants of the past and those of the future.