The wave equation in flat space close to spatial infinity

13 Dec 2022

Edgar Gasperin

Written in Mathematica version 12.3 .

Notebook based on https://arxiv.org/pdf/2209.12247.pdf : only good equation discussed here.

Load packages and general set up

In[1]:=
```
<< xAct`xTensor`
```

```
------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.

Connecting to external mac executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.

------------------------------------------------------------

These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

In[2]:=

In[3]:=
```
DefManifold[M4, 4, {a, b, c, d, f, p, h, i, j, k, l, n, v, w}]
ApplyRule[expr_] :=
 MakeRule[Evaluate[List @@ expr], MetricOn → All, ContractMetrics → True]
          calcula   lista                 tudo                  verdade
ApplyRuleN[expr_] := MakeRule[Evaluate[List @@ expr]]
                              calcula   lista
$MixedDers = False;
             falso
Off[ToCanonical::noident]
apaga mensagens
$PrePrint = ScreenDollarIndices;
pré impressão
```

```
** DefManifold: Defining manifold M4.

** DefVBundle: Defining vbundle TangentM4.
```

In[9]:=
```
<< xAct`xCoba`
```

```
------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

CopyRight (C) 2005-2021, David Yllanes and
  Jose M. Martin-Garcia, under the General Public License.

------------------------------------------------------------

These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

In[10]:=
```
<<xAct`xTerior`
```

```
------------------------------------------------------------

Package xAct`xTerior`  version 0.9.1, {2019, 5, 17}

Copyright (C) 2013-2019, Alfonso Garcia-Parrado
  Gomez-Lobo and Leo C. Stein, under the General Public License.

------------------------------------------------------------

These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

In[11]:=
```
<< xAct`TexAct`
```

```
------------------------------------------------------------

Package xAct`TexAct`  version 0.4.3, {2021, 10, 28}

CopyRight (C) 2008-2021, Thomas Bäckdahl, Jose M.
  Martin-Garcia and Barry Wardell, under the General Public License.

------------------------------------------------------------

These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

Minkowski i^0 - cylinder geometry

Metric, coordinates and null frame

Metric and coordinates

Unphysical metric

```
In[12]:= DefMetric[-1, g[-a, -b], CD, {";", "∇"}]
** DefTensor: Defining symmetric metric tensor g[-a, -b].
** DefTensor: Defining antisymmetric tensor epsilong[-a, -b, -c, -d].
** DefTensor: Defining tetrametric Tetrag[-a, -b, -c, -d].
** DefTensor: Defining tetrametric Tetrag†[-a, -b, -c, -d].
** DefCovD: Defining covariant derivative CD[-a].
** DefTensor: Defining vanishing torsion tensor TorsionCD[a, -b, -c].
** DefTensor: Defining symmetric Christoffel tensor ChristoffelCD[a, -b, -c].
** DefTensor: Defining Riemann tensor RiemannCD[-a, -b, -c, -d].
** DefTensor: Defining symmetric Ricci tensor RicciCD[-a, -b].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarCD[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor EinsteinCD[-a, -b].
** DefTensor: Defining Weyl tensor WeylCD[-a, -b, -c, -d].
** DefTensor: Defining symmetric TFRicci tensor TFRicciCD[-a, -b].
** DefTensor: Defining Kretschmann scalar KretschmannCD[].
** DefCovD:  Computing RiemannToWeylRules for dim 4
** DefCovD:  Computing RicciToTFRicci for dim 4
** DefCovD:  Computing RicciToEinsteinRules for dim 4
** DefInertHead: Defining inert head CovDCD.
** DefTensor: Defining weight +2 density Detg[]. Determinant.
```

Physical metric

In[13]:=
```
DefMetric[-1, physicalg[-a, -b], CDtilde, {";", "∇̃"}, PrintAs → "g̃"]
```

DefMetric : There are already metrics {g} in vbundle TM4.

```
** DefTensor: Defining symmetric metric tensor physicalg[-a, -b].
** DefTensor: Defining inverse metric tensor Invphysicalg[a, b]. Metric is frozen!
** DefTensor: Defining antisymmetric tensor epsilonphysicalg[-a, -b, -c, -d].
** DefTensor: Defining tetrametric Tetraphysicalg[-a, -b, -c, -d].
** DefTensor: Defining tetrametric Tetraphysicalg†[-a, -b, -c, -d].
** DefCovD: Defining covariant derivative CDtilde[-a].
** DefTensor: Defining vanishing torsion tensor TorsionCDtilde[a, -b, -c].
** DefTensor: Defining symmetric Christoffel tensor ChristoffelCDtilde[a, -b, -c].
** DefTensor: Defining Riemann tensor RiemannDownCDtilde[-a, -b, -c, -d].
** DefTensor: Defining Riemann tensor
 RiemannCDtilde[-a, -b, -c, d]. Antisymmetric only in the first pair.
** DefTensor: Defining symmetric Ricci tensor RicciCDtilde[-a, -b].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarCDtilde[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor EinsteinCDtilde[-a, -b].
** DefTensor: Defining Weyl tensor WeylCDtilde[-a, -b, -c, -d].
** DefTensor: Defining symmetric TFRicci tensor TFRicciCDtilde[-a, -b].
** DefTensor: Defining Kretschmann scalar KretschmannCDtilde[].
** DefCovD:  Computing RiemannToWeylRules for dim 4
** DefCovD:  Computing RicciToEinsteinRules for dim 4
** DefInertHead: Defining inert head CovDCDtilde.
** DefTensor: Defining weight +2 density Detphysicalg[]. Determinant.
```

In[14]:=
```
(*DefMetric[{1,3,0},g[-a,-b],CD,{";","∇"}]*)
```

Conformal transformation

In[15]:=
```
DefTensor[CF[], M4, PrintAs → "Θ"]
```

```
** DefTensor: Defining tensor CF[].
```

In[16]:=
```
UnphysicalToPhysical = g[-a, -b] == CF[]^2 × physicalg[-a, -b]
```

Out[16]=

$$g_{ab} == \Theta^2 \, \tilde{g}_{ab}$$

The inverse transformation is given by:

In[17]:=
```
PhysicalToUnPhysical =
  Equal @@ (Solve[UnphysicalToPhysical, physicalg[-a, -b]] // Flatten // First)
  igual       resolve                                         achatar    primeiro
```

Out[17]=

$$\tilde{g}_{ab} == \frac{g_{ab}}{\Theta^2}$$

Compute now the contravariant physical metric:

In[18]:=
```
Times[#, g[c, a]] & /@ UnphysicalToPhysical
multiplicação
```

Out[18]=

$$\delta_b{}^c == \Theta^2\ g^{ca}\ \tilde{g}_{ab}$$

In[19]:=
```
Invphysicalg[a, b] == g[a, b] × CF[]^2
InvPhysicalToPhysical = %;
```

Out[19]=

$$i\ \tilde{g}^{ab} == \Theta^2\ g^{ab}$$

F-coordinates

In[21]:=
```
DefChart[CoordBasis, M4, {0, 1, 2, 3}, {τ[], ρ[], θ[], ϕ[]},
 ChartColor → RGBColor[0, 0, 1], FormatBasis → {"Partials", "Differentials"}]
```
cores do sistema RGB

```
** DefChart: Defining chart CoordBasis.
** DefTensor: Defining coordinate scalar τ[].
** DefTensor: Defining coordinate scalar ρ[].
** DefTensor: Defining coordinate scalar θ[].
** DefTensor: Defining coordinate scalar ϕ[].
** DefMapping: Defining mapping CoordBasis.
** DefMapping: Defining inverse mapping iCoordBasis.
** DefTensor: Defining mapping differential tensor diCoordBasis[-a, iCoordBasisa].
** DefTensor: Defining mapping differential tensor dCoordBasis[-a, CoordBasisa].
** DefBasis: Defining basis CoordBasis. Coordinated basis.
** DefCovD: Defining parallel derivative PDCoordBasis[-a].
** DefTensor: Defining vanishing torsion tensor TorsionPDCoordBasis[a, -b, -c].
** DefTensor: Defining symmetric Christoffel tensor
 ChristoffelPDCoordBasis[a, -b, -c].
** DefTensor: Defining vanishing Riemann tensor RiemannPDCoordBasis[-a, -b, -c, d].
** DefTensor: Defining vanishing Ricci tensor RicciPDCoordBasis[-a, -b].
** DefInertHead: Defining inert head CovDPDCoordBasis.
** DefTensor: Defining antisymmetric +1 density etaUpCoordBasis[a, b, c, d].
** DefTensor: Defining antisymmetric -1 density etaDownCoordBasis[-a, -b, -c, -d].
** DefTensor: Defining tensor CoordBasis0[].
** DefTensor: Defining tensor CoordBasis1[].
** DefTensor: Defining tensor CoordBasis2[].
** DefTensor: Defining tensor CoordBasis3[].
```

Added independent rule $dx^{0} \to d[\tau]$ for tensor dx[M4]

Added independent rule $dx^{1} \to d[\rho]$ for tensor dx[M4]

Added independent rule $dx^{2} \to d[\theta]$ for tensor dx[M4]

Added independent rule $dx^{3} \to d[\phi]$ for tensor dx[M4]

Added independent rule $\frac{\partial}{\partial x}{}_{0} \to \frac{\partial}{\partial \tau}$ for tensor PDFrame[M4]

Added independent rule $\frac{\partial}{\partial x}{}_{1} \to \frac{\partial}{\partial \rho}$ for tensor PDFrame[M4]

Added independent rule $\frac{\partial}{\partial x}{}_{2} \to \frac{\partial}{\partial \theta}$ for tensor PDFrame[M4]

Added independent rule $\frac{\partial}{\partial x}{}_{3} \to \frac{\partial}{\partial \phi}$ for tensor PDFrame[M4]

Conformal factor in F - coordinates

In[22]:=
```
CFDef = CF[] == ρ[] * (1 - τ[] ^ 2)
```

Out[22]=

$$\Theta == \rho \left(1 - \tau^2\right)$$

The unphysical (cylinder) metric in the F-coordinate basis

In[23]:=
```
MatrixForm[MetricInCoordBasis = {
forma de matriz
    {-1, -τ[] / ρ[], 0, 0},
    {-τ[] / ρ[], ((1 - τ[] ^ 2) / ρ[] ^ 2), 0, 0},
    {0, 0, 1, 0},
    {0, 0, 0, Sin[θ[]] ^ 2}
              seno
  }]
```

Out[23]//MatrixForm=

$$\begin{pmatrix} -1 & -\frac{\tau}{\rho} & 0 & 0 \\ -\frac{\tau}{\rho} & \frac{1-\tau^2}{\rho^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{Sin}[\theta]^2 \end{pmatrix}$$

In[24]:=
```
MetricInBasis[g, -CoordBasis, MetricInCoordBasis]
```

Added independent rule $g_{00} \to -1$ for tensor g

Added independent rule $g_{01} \to -\frac{\tau}{\rho}$ for tensor g

Added independent rule $g_{02} \to 0$ for tensor g

Added independent rule $g_{03} \to 0$ for tensor g

Added dependent rule $g_{10} \to g_{01}$ for tensor g

Added independent rule $g_{11} \to \frac{1-\tau^2}{\rho^2}$ for tensor g

Added independent rule $g_{12} \to 0$ for tensor g

Added independent rule $g_{13} \to 0$ for tensor g

Added dependent rule $g_{20} \to g_{02}$ for tensor g

Added dependent rule $g_{21} \to g_{12}$ for tensor g

Added independent rule $g_{22} \to 1$ for tensor g

Added independent rule $g_{23} \to 0$ for tensor g

Added dependent rule $g_{30} \to g_{03}$ for tensor g

Added dependent rule $g_{31} \to g_{13}$ for tensor g

Added dependent rule $g_{32} \to g_{23}$ for tensor g

Added independent rule $g_{33} \to \mathrm{Sin}[\theta]^2$ for tensor g

Out[24]=

$$\left\{\left\{g_{00} \to -1,\ g_{01} \to -\frac{\tau}{\rho},\ g_{02} \to 0,\ g_{03} \to 0\right\},\right.$$
$$\left\{g_{10} \to -\frac{\tau}{\rho},\ g_{11} \to \frac{1-\tau^2}{\rho^2},\ g_{12} \to 0,\ g_{13} \to 0\right\},$$
$$\left.\left\{g_{20} \to 0,\ g_{21} \to 0,\ g_{22} \to 1,\ g_{23} \to 0\right\},\ \left\{g_{30} \to 0,\ g_{31} \to 0,\ g_{32} \to 0,\ g_{33} \to \mathrm{Sin}[\theta]^2\right\}\right\}$$

Compute the Christoffel symbols in the coordinate basis

In[25]:=
```
? MetricCompute
```
Out[25]=

Symbol

MetricCompute [g, ch, T] computes the components of the curvature tensor T associated to the metric g in the chart ch, where g and ch are symbols already known to xTensor and xCoba, respectively. The metric g is assummed to have been assigned values as explicit functions of the coordinate scalars. The notation for T is special and currently allows the 15 possibilities: "Metric" [−1, −1], "Metric" [1, 1], "DetMetric" [], "DMetric" [−1, −1, −1], "DDMetric" [−1, −1, −1, −1], "Christoffel" [−1, −1, −1], "Christoffel" [1, −1, −1], "Riemann" [−1, −1, −1, −1], "Riemann" [−1, −1, −1, 1], "Riemann" [−1, −1, 1, 1], "Ricci" [−1, −1], "RicciScalar" [], "Weyl" [−1, −1, −1, −1], "Einstein" [−1, −1], "Kretschmann" [], where −1 denotes a covariant component and 1 a contravariant component. This function computes in advance everything needed to know the required tensor T. It is possible to say All instead of a tensor T, and then those 14 tensors will be computed. There are options CVSimplify, to specify a function which is applied to each component after each tensor is computed (default is Together), and Verbose, to get info messages during the computation (default is True).

The following two lines has to be modified if the metric is too complicated -- - if the associated matrix cannot be inverted easily-- -
for the moment, for the simple example treated here, we use xcoba's Metric Compute which computes this automatically.

In[26]:=
```
MetricCompute[g, CoordBasis, "Metric"[1, 1], CVSimplify → Simplify]
```
simplifica

```
Last@TensorValues[g]
```
último

Out[27]=

$$\left\{g_{00}\to -1,\ g_{01}\to -\frac{\tau}{\rho},\ g_{02}\to 0,\ g_{03}\to 0,\ g_{11}\to \frac{1-\tau^2}{\rho^2},\ g_{12}\to 0,\ g_{13}\to 0,\ g_{22}\to 1,\ g_{23}\to 0,\ g_{33}\to \mathrm{Sin}[\theta]^2\right\}$$

In[28]:=
```
TensorValues[g]
```
Out[28]=

$$\mathrm{FoldedRule}\Big[\left\{g^{10}\to g^{01},\ g^{20}\to g^{02},\ g^{21}\to g^{12},\ g^{30}\to g^{03},\ g^{31}\to g^{13},\ g^{32}\to g^{23}\right\},$$
$$\left\{g^{00}\to -1+\tau^2,\ g^{01}\to -\rho\,\tau,\ g^{02}\to 0,\ g^{03}\to 0,\ g^{11}\to \rho^2,\ g^{12}\to 0,\ g^{13}\to 0,\ g^{22}\to 1,\ g^{23}\to 0,\ g^{33}\to \mathrm{Csc}[\theta]^2\right\},$$
$$\left\{g_{10}\to g_{01},\ g_{20}\to g_{02},\ g_{21}\to g_{12},\ g_{30}\to g_{03},\ g_{31}\to g_{13},\ g_{32}\to g_{23}\right\},$$
$$\left\{g_{00}\to -1,\ g_{01}\to -\frac{\tau}{\rho},\ g_{02}\to 0,\ g_{03}\to 0,\ g_{11}\to \frac{1-\tau^2}{\rho^2},\ g_{12}\to 0,\ g_{13}\to 0,\ g_{22}\to 1,\ g_{23}\to 0,\ g_{33}\to \mathrm{Sin}[\theta]^2\right\}\Big]$$

In[29]:=
```
MetricCompute[g, CoordBasis, "Christoffel"[1, -1, -1], CVSimplify → Simplify]
                                                                  simplifica
Last@TensorValues[ChristoffelCDPDCoordBasis]
último
```

** DefTensor: Defining tensor ChristoffelCDPDCoordBasis[a, -b, -c].

Out[30]=

$$\left\{\Gamma[\nabla,\mathcal{D}]_{000}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{001}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{002}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{003}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{010}\to 0,\right.$$
$$\Gamma[\nabla,\mathcal{D}]_{011}\to \frac{2\,\tau}{\rho^2},\ \Gamma[\nabla,\mathcal{D}]_{012}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{013}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{020}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{021}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{022}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{023}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{030}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{031}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{032}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{033}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{100}\to -\frac{1}{\rho},\ \Gamma[\nabla,\mathcal{D}]_{101}\to -\frac{\tau}{\rho^2},$$
$$\Gamma[\nabla,\mathcal{D}]_{102}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{103}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{110}\to -\frac{\tau}{\rho^2},\ \Gamma[\nabla,\mathcal{D}]_{111}\to \frac{-1+\tau^2}{\rho^3},$$
$$\Gamma[\nabla,\mathcal{D}]_{112}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{113}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{120}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{121}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{122}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{123}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{130}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{131}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{132}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{133}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{200}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{201}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{202}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{203}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{210}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{211}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{212}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{213}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{220}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{221}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{222}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{223}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{230}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{231}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{232}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{233}\to -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta],\ \Gamma[\nabla,\mathcal{D}]_{300}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{301}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{302}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{303}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{310}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{311}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{312}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{313}\to 0,$$
$$\Gamma[\nabla,\mathcal{D}]_{320}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{321}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{322}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{323}\to \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta],$$
$$\left.\Gamma[\nabla,\mathcal{D}]_{330}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{331}\to 0,\ \Gamma[\nabla,\mathcal{D}]_{332}\to \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta],\ \Gamma[\nabla,\mathcal{D}]_{333}\to 0\right\}$$

Compute the curvature in the coordinate basis

In[31]:=
```
MetricCompute[g, CoordBasis, "Riemann"[-1, -1, -1, -1], CVSimplify → Simplify]
                                                                    simplifica
Last@TensorValues[RiemannCD]
último
```

Out[32]=

$$\left\{R[\nabla]_{0101}\to \frac{1}{\rho^2},\ R[\nabla]_{0102}\to 0,\ R[\nabla]_{0103}\to 0,\ R[\nabla]_{0112}\to 0,\ R[\nabla]_{0113}\to 0,\right.$$
$$R[\nabla]_{0123}\to 0,\ R[\nabla]_{0202}\to 0,\ R[\nabla]_{0203}\to 0,\ R[\nabla]_{0212}\to 0,\ R[\nabla]_{0213}\to 0,\ R[\nabla]_{0223}\to 0,$$
$$R[\nabla]_{0303}\to 0,\ R[\nabla]_{0312}\to 0,\ R[\nabla]_{0313}\to 0,\ R[\nabla]_{0323}\to 0,\ R[\nabla]_{1212}\to 0,$$
$$\left.R[\nabla]_{1213}\to 0,\ R[\nabla]_{1223}\to 0,\ R[\nabla]_{1313}\to 0,\ R[\nabla]_{1323}\to 0,\ R[\nabla]_{2323}\to \mathrm{Sin}[\theta]^2\right\}$$

In[33]:=
```
MetricCompute[g, CoordBasis, "Ricci"[-1, -1], CVSimplify → Simplify]
                                                          simplifica
Last@TensorValues[RicciCD]
último
```

Out[34]=

$$\left\{R[\nabla]_{00} \to 1,\ R[\nabla]_{01} \to \frac{\tau}{\rho},\ R[\nabla]_{02} \to 0,\ R[\nabla]_{03} \to 0,\ R[\nabla]_{11} \to \frac{-1+\tau^2}{\rho^2},\ R[\nabla]_{12} \to 0,\ R[\nabla]_{13} \to 0,\ R[\nabla]_{22} \to 1,\ R[\nabla]_{23} \to 0,\ R[\nabla]_{33} \to \mathrm{Sin}[\theta]^2\right\}$$

In[35]:=
```
MetricCompute[g, CoordBasis, "RicciScalar"[], CVSimplify → Simplify]
                                                          simplifica
Last@TensorValues[RicciScalarCD]
último
```

Out[36]=

$$\{R[\nabla] \to 0\}$$

F-Null tetrad

Translation to the physical coordinates and physical metric

Wave equation

Wave equation

Friedrich’s canonical Ansatz

The good (Wave equation)

Translation of the solution to the physical spacetime set up

NP - Constants and conserved quantities

Identities and conservation laws in the physical picture

Definitions

Conservation Laws in the physical picture

Identities check to finite order in unphysical picture

NP - constants with F[R] = R^n

l = 0 order

In[792]:=

```
R[]^n * PhyRetTimeVector[a] × PD[-a]@Phi[]
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@Normal@PhiCanonicalAnsatzOrderN[3];
                  normal
% /. ApplyRuleGeneralConstantsRule;
% // Expand;
     expande fatores
% /. ApplyRule@CFDef // Simplify // Expand;
                       simplifica   expande fatores
% /. ApplyRule@DefBoost // Simplify // Expand;
                          simplifica   expande fatores
Coefficient[%, Ysph[-LI[0], -LI[0]]]
coeficiente
% /. ApplyRule@DefBoost // Simplify // Expand;
                          simplifica   expande fatores
% /. ApplyRule@CFDef // Simplify // Expand;
                       simplifica   expande fatores
% /. asol → SolutionGeneralPL // Simplify // Expand;
                                simplifica   expande fatores
% /. ApplyRuleGeneralConstantsRule // Simplify // Expand
                                     simplifica   expande f
Collect[%, ρ[], Simplify];
agrupa coeficientes  simplifica
% /. ApplyRulePhysicalToUnphysicalCoords // Expand;
                                           expande fatores
Collect[%, ρ[], Simplify]
agrupa coeficientes  simplifica
```

Out[792]=

$L^a\ R^n\ \partial_a\Phi$

Out[802]=

$$-a[1,0,0,\tau]\,R^n\rho^2 - a[2,0,0,\tau]\,R^n\rho^3 - \frac{1}{2}a[3,0,0,\tau]\,R^n\rho^4 +$$
$$2\,a[1,0,0,\tau]\,R^n\rho^2\tau + 2\,a[2,0,0,\tau]\,R^n\rho^3\tau + a[3,0,0,\tau]\,R^n\rho^4\tau -$$
$$a[1,0,0,\tau]\,R^n\rho^2\tau^2 - a[2,0,0,\tau]\,R^n\rho^3\tau^2 - \frac{1}{2}a[3,0,0,\tau]\,R^n\rho^4\tau^2 +$$
$$R^n\rho\,\partial_0 a[0,0,0,\tau] - R^n\rho\,\tau\,\partial_0 a[0,0,0,\tau] - R^n\rho\,\tau^2\,\partial_0 a[0,0,0,\tau] + R^n\rho\,\tau^3\,\partial_0 a[0,0,0,\tau] +$$
$$R^n\rho^2\,\partial_0 a[1,0,0,\tau] - R^n\rho^2\tau\,\partial_0 a[1,0,0,\tau] - R^n\rho^2\tau^2\,\partial_0 a[1,0,0,\tau] +$$
$$R^n\rho^2\tau^3\,\partial_0 a[1,0,0,\tau] + \frac{1}{2}R^n\rho^3\,\partial_0 a[2,0,0,\tau] - \frac{1}{2}R^n\rho^3\tau\,\partial_0 a[2,0,0,\tau] -$$
$$\frac{1}{2}R^n\rho^3\tau^2\,\partial_0 a[2,0,0,\tau] + \frac{1}{2}R^n\rho^3\tau^3\,\partial_0 a[2,0,0,\tau] + \frac{1}{6}R^n\rho^4\,\partial_0 a[3,0,0,\tau] -$$
$$\frac{1}{6}R^n\rho^4\tau\,\partial_0 a[3,0,0,\tau] - \frac{1}{6}R^n\rho^4\tau^2\,\partial_0 a[3,0,0,\tau] + \frac{1}{6}R^n\rho^4\tau^3\,\partial_0 a[3,0,0,\tau] -$$
$$R^n\rho^2\,\partial_1 a[0,0,0,\tau] + 2\,R^n\rho^2\tau\,\partial_1 a[0,0,0,\tau] - R^n\rho^2\tau^2\,\partial_1 a[0,0,0,\tau] -$$
$$R^n\rho^3\,\partial_1 a[1,0,0,\tau] + 2\,R^n\rho^3\tau\,\partial_1 a[1,0,0,\tau] - R^n\rho^3\tau^2\,\partial_1 a[1,0,0,\tau] -$$
$$\frac{1}{2}R^n\rho^4\,\partial_1 a[2,0,0,\tau] + R^n\rho^4\tau\,\partial_1 a[2,0,0,\tau] - \frac{1}{2}R^n\rho^4\tau^2\,\partial_1 a[2,0,0,\tau] -$$
$$\frac{1}{6}R^n\rho^5\,\partial_1 a[3,0,0,\tau] + \frac{1}{3}R^n\rho^5\tau\,\partial_1 a[3,0,0,\tau] - \frac{1}{6}R^n\rho^5\tau^2\,\partial_1 a[3,0,0,\tau]$$

Out[806]=

$$D_{000}\,R^n\rho - A_{100}\,R^n\rho^2 - \frac{1}{2}A_{200}\,R^n\rho^3 - \frac{1}{8}A_{300}\,R^n\rho^4 - D_{000}\,R^n\rho\,\tau +$$
$$2\,A_{100}\,R^n\rho^2\tau + \frac{3}{2}A_{200}\,R^n\rho^3\tau + \frac{1}{2}A_{300}\,R^n\rho^4\tau - A_{100}\,R^n\rho^2\tau^2 - \frac{3}{2}A_{200}\,R^n\rho^3\tau^2 -$$
$$\frac{3}{4}A_{300}\,R^n\rho^4\tau^2 + \frac{1}{2}A_{200}\,R^n\rho^3\tau^3 + \frac{1}{2}A_{300}\,R^n\rho^4\tau^3 - \frac{1}{8}A_{300}\,R^n\rho^4\tau^4$$

Out[809]=

$$-D_{000}\,\rho\,(-1+\tau)\left(\frac{1}{\rho-\rho\,\tau^2}\right)^n - A_{100}\,\rho^2\,(-1+\tau)^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^n +$$
$$\frac{1}{2}A_{200}\,\rho^3\,(-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^n - \frac{1}{8}A_{300}\,\rho^4\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^n$$

$F[R] = R^2$ at order $l = 0$ is not defined unless the regularity (no − log) condition is imposed to this order $D_{000} = 0$.
Once the regularity condition is imposed, evaluating at future null infinity $\tau = 1$ the associated NP constant is A_{100}.
$F[R] = R^1$ at order $l = 0$ we recover D_{000}, evaluating at future null infinity $\tau = 1$ the associated NP constant is D_{000}.

In[810]:=

l = 1 order

In[984]:=

```
R[]^(n) × HoldForm[
                 ⌊forma sem avaliação
   PhyRetTimeVector[b] × PD[-b]@(R[]^n * PhyRetTimeVector[a] × PD[-a]@Phi[])]
ReleaseHold[%]
⌊desbloqueia cálculo
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand
                                                   ⌊expande fatores
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
                                                  ⌊expande fatores
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@Normal@PhiCanonicalAnsatzOrderN[4];
                ⌊normal
% /. ApplyRuleGeneralConstantsRule;
% // Expand;
     ⌊expande fatores
Coefficient[%, Ysph[-LI[1], -LI[-1]]];
⌊coeficiente
% /. ApplyRule@DefBoost // Simplify // Expand;
                           ⌊simplifica  ⌊expande fatores
% /. ApplyRule@CFDef // Simplify // Expand;
                        ⌊simplifica  ⌊expande fatores
% /. asol → SolutionGeneralPL // Simplify // Expand;
                                 ⌊simplifica  ⌊expande fatores
% /. ApplyRuleGeneralConstantsRule // Simplify // Expand;
                                      ⌊simplifica  ⌊expande fatores
Collect[%, ρ[], Simplify]
⌊agrupa coeficientes  ⌊simplifica
```

Out[984]=

$$\left(L^{b}\ \partial_{b}\left[R^{n}\ L^{a}\ \partial_{a}\Phi\right]\right) R^{n}$$

Out[985]=

$$L^{b}\ R^{n}\ \left(R^{n}\,\partial_{a}\Phi\,\partial_{b}L^{a}+L^{a}\ \left(\partial_{a}\Phi\left(n\ dR_{b}\ R^{-1+n}+\mathrm{Log}[R]\ R^{n}\,\partial_{b}n\right)+R^{n}\,\partial_{b}\partial_{a}\Phi\right)\right)$$

Out[986]=

$$-\frac{n\ d\rho_{b}\ L^{a}\ L^{b}\ R^{-1+2n}\,\partial_{a}\Phi}{\rho^{2}\left(-1+\tau^{2}\right)^{2}}+\frac{2\,n\ d\tau_{b}\ L^{a}\ L^{b}\ R^{-1+2n}\,\tau\,\partial_{a}\Phi}{\rho\left(-1+\tau^{2}\right)^{2}}+\frac{n\ d\rho_{b}\ L^{a}\ L^{b}\ R^{-1+2n}\,\tau^{2}\,\partial_{a}\Phi}{\rho^{2}\left(-1+\tau^{2}\right)^{2}}+$$
$$\mathrm{Log}[R]\ L^{a}\ L^{b}\ R^{2n}\,\partial_{a}\Phi\,\partial_{b}n+L^{b}\ R^{2n}\,\partial_{a}\Phi\,\partial_{b}L^{a}+L^{a}\ L^{b}\ R^{2n}\,\partial_{b}\partial_{a}\Phi$$

Out[1000]=

$$\frac{1}{16}\,\rho^{3}\,(-1+\tau)^{3}\left(\frac{1}{\rho-\rho\,\tau^{2}}\right)^{2n}\left(4\ C_{11-1}\ (1+\tau)^{3}\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^{2}}\right]\ (-1+\tau)\ \partial_{0}n\right)+\right.$$
$$D_{11-1}\left(16-4\,n+2\,\mathrm{Log}[1-\tau]-n\,\mathrm{Log}[1-\tau]-2\,\mathrm{Log}[1+\tau]+n\,\mathrm{Log}[1+\tau]+12\,\tau-\right.$$
$$6\,n\,\tau+6\,\mathrm{Log}[1-\tau]\,\tau-3\,n\,\mathrm{Log}[1-\tau]\,\tau-6\,\mathrm{Log}[1+\tau]\,\tau+3\,n\,\mathrm{Log}[1+\tau]\,\tau+4\,\tau^{2}-$$
$$2\,n\,\tau^{2}+6\,\mathrm{Log}[1-\tau]\,\tau^{2}-3\,n\,\mathrm{Log}[1-\tau]\,\tau^{2}-6\,\mathrm{Log}[1+\tau]\,\tau^{2}+3\,n\,\mathrm{Log}[1+\tau]\,\tau^{2}+$$

$$2\,\mathrm{Log}[1-\tau]\,\tau^3 - n\,\mathrm{Log}[1-\tau]\,\tau^3 - 2\,\mathrm{Log}[1+\tau]\,\tau^3 + n\,\mathrm{Log}[1+\tau]\,\tau^3 +$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-1+\tau^2\right)\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2 - \mathrm{Log}[1+\tau]\,(1+\tau)^2 + 2\,(2+\tau)\right)\partial_0 n\Big)\Big) -$$

$$\frac{1}{384}\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^7\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}$$

$$\left(B_{41-1}\,(1+\tau)^5 + A_{41-1}\,(-1+\tau)^3\left(31+8\,\tau+\tau^2\right)\right)$$

$$\partial_1 n + \frac{1}{384}$$

$$\rho^6$$

$$(-1+\tau)^3$$

$$\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}$$

$$\left(B_{41-1}\,(1+\tau)^6\left(2-n+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right) +\right.$$

$$A_{41-1}\,(-1+\tau)^3\left(-n\left(31+39\,\tau+9\,\tau^2+\tau^3\right) + 2\left(151+39\,\tau+9\,\tau^2+\tau^3\right) +\right.$$

$$\left.\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-31-8\,\tau+30\,\tau^2+8\,\tau^3+\tau^4\right)\partial_0 n\right) +$$

$$\left.8\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\left(-B_{31-1}\,(1+\tau)^4 + A_{31-1}\,(-1+\tau)^2\left(17+6\,\tau+\tau^2\right)\right)\partial_1 n\right) -$$

$$\frac{1}{48}\,\rho^5\,(-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\left(B_{31-1}\,(1+\tau)^5\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right) +\right.$$

$$A_{31-1}\,(-1+\tau)^2\left(-n\left(17+23\,\tau+7\,\tau^2+\tau^3\right) + 2\left(65+23\,\tau+7\,\tau^2+\tau^3\right) +\right.$$

$$\left.\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-17-6\,\tau+16\,\tau^2+6\,\tau^3+\tau^4\right)\partial_0 n\right) +$$

$$\left.6\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\left(B_{21-1}\,(1+\tau)^3 + A_{21-1}\left(-7+3\,\tau+3\,\tau^2+\tau^3\right)\right)\partial_1 n\right) +$$

$$\frac{1}{16}\,\rho^4\,(-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\left(-2\,B_{21-1}\,(1+\tau)^4\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right) +\right.$$

$$2\,A_{21-1}\,(-1+\tau)\left(-n\left(7+11\,\tau+5\,\tau^2+\tau^3\right) + 2\left(19+11\,\tau+5\,\tau^2+\tau^3\right) +\right.$$

$$\left.\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-7-4\,\tau+6\,\tau^2+4\,\tau^3+\tau^4\right)\partial_0 n\right) + \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)$$

$$\left.\left(4\,C_{11-1}\,(1+\tau)^2 - D_{11-1}\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2 - \mathrm{Log}[1+\tau]\,(1+\tau)^2 + 2\,(2+\tau)\right)\right)\partial_1 n\right)$$

In[828]:=

In[1001]:=

```
R[]^(n) × HoldForm[
        PhyRetTimeVector[b] × PD[-b]@(R[]^n * PhyRetTimeVector[a] × PD[-a]@Phi[])]
ReleaseHold[%]
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@Normal@PhiCanonicalAnsatzOrderN[4];
% /. ApplyRuleGeneralConstantsRule;
% // Expand;
Coefficient[%, Ysph[-LI[1], -LI[0]]];
% /. ApplyRule@DefBoost // Simplify // Expand;
% /. ApplyRule@CFDef // Simplify // Expand;
% /. asol → SolutionGeneralPL // Simplify // Expand;
% /. ApplyRuleGeneralConstantsRule // Simplify // Expand;
Collect[%, ρ[], Simplify]
```

forma sem avaliação; desbloqueia cálculo; expande fatores; expande fatores; normal; expande fatores; coeficiente; simplifica; expande fatores; simplifica; expande fatores; simplifica; expande fatores; simplifica; expande fatores; agrupa coeficientes; simplifica

Out[1001]=

$$\left(L^b\ \partial_b\left[R^n\ L^a\ \partial_a\Phi\right]\right) R^n$$

Out[1002]=

$$L^b\ R^n\ \left(R^n\,\partial_a\Phi\,\partial_b L^a + L^a\ \left(\partial_a\Phi\ \left(n\ dR_b\ R^{-1+n} + \mathrm{Log}[R]\ R^n\,\partial_b n\right) + R^n\,\partial_b\partial_a\Phi\right)\right)$$

Out[1003]=

$$-\frac{n\ d\rho_b\ L^a\ L^b\ R^{-1+2n}\,\partial_a\Phi}{\rho^2\ \left(-1+\tau^2\right)^2} + \frac{2\ n\ d\tau_b\ L^a\ L^b\ R^{-1+2n}\,\tau\,\partial_a\Phi}{\rho\ \left(-1+\tau^2\right)^2} + \frac{n\ d\rho_b\ L^a\ L^b\ R^{-1+2n}\,\tau^2\,\partial_a\Phi}{\rho^2\ \left(-1+\tau^2\right)^2} +$$
$$\mathrm{Log}[R]\ L^a\ L^b\ R^{2n}\,\partial_a\Phi\,\partial_b n + L^b\ R^{2n}\,\partial_a\Phi\,\partial_b L^a + L^a\ L^b\ R^{2n}\,\partial_b\partial_a\Phi$$

Out[1017]=

$$\frac{1}{16}\,\rho^3\ (-1+\tau)^3\ \left(\frac{1}{\rho-\rho\ \tau^2}\right)^{2n}\left(4\ C_{110}\ (1+\tau)^3\ \left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\ \tau^2}\right]\ (-1+\tau)\ \partial_0 n\right) +\right.$$
$$D_{110}\ \Big(16-4\,n+2\,\mathrm{Log}[1-\tau]-n\,\mathrm{Log}[1-\tau]-2\,\mathrm{Log}[1+\tau]+n\,\mathrm{Log}[1+\tau]+12\,\tau-6\,n\,\tau+$$
$$6\,\mathrm{Log}[1-\tau]\ \tau-3\,n\,\mathrm{Log}[1-\tau]\ \tau-6\,\mathrm{Log}[1+\tau]\ \tau+3\,n\,\mathrm{Log}[1+\tau]\ \tau+4\,\tau^2-$$
$$2\,n\,\tau^2+6\,\mathrm{Log}[1-\tau]\ \tau^2-3\,n\,\mathrm{Log}[1-\tau]\ \tau^2-6\,\mathrm{Log}[1+\tau]\ \tau^2+3\,n\,\mathrm{Log}[1+\tau]\ \tau^2+$$
$$2\,\mathrm{Log}[1-\tau]\ \tau^3-n\,\mathrm{Log}[1-\tau]\ \tau^3-2\,\mathrm{Log}[1+\tau]\ \tau^3+n\,\mathrm{Log}[1+\tau]\ \tau^3+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\left(-1+\tau^2\right)\,\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2-\mathrm{Log}[1+\tau]\,(1+\tau)^2+2\,(2+\tau)\right)\,\partial_0 n\Bigg)\Bigg)-$$

$$\frac{1}{384}\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\rho^7\,(-1+\tau)^4\,\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}$$

$$\left(B_{410}\,(1+\tau)^5+A_{410}\,(-1+\tau)^3\,\left(31+8\,\tau+\tau^2\right)\right)$$

$$\partial_1 n+\frac{1}{384}$$

$$\rho^6$$

$$(-1+\tau)^3$$

$$\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}$$

$$\Bigg(B_{410}\,(1+\tau)^6\,\left(2-n+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)\,\partial_0 n\right)+$$

$$A_{410}\,(-1+\tau)^3\,\Bigg(-n\,\left(31+39\,\tau+9\,\tau^2+\tau^3\right)+2\,\left(151+39\,\tau+9\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\left(-31-8\,\tau+30\,\tau^2+8\,\tau^3+\tau^4\right)\,\partial_0 n\Bigg)+$$

$$8\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)\,\left(-B_{310}\,(1+\tau)^4+A_{310}\,(-1+\tau)^2\,\left(17+6\,\tau+\tau^2\right)\right)\,\partial_1 n\Bigg)-$$

$$\frac{1}{48}\,\rho^5\,(-1+\tau)^3\,\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\,\Bigg(B_{310}\,(1+\tau)^5\,\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)\,\partial_0 n\right)+$$

$$A_{310}\,(-1+\tau)^2\,\Bigg(-n\,\left(17+23\,\tau+7\,\tau^2+\tau^3\right)+2\,\left(65+23\,\tau+7\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\left(-17-6\,\tau+16\,\tau^2+6\,\tau^3+\tau^4\right)\,\partial_0 n\Bigg)+$$

$$6\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)\,\left(B_{210}\,(1+\tau)^3+A_{210}\,\left(-7+3\,\tau+3\,\tau^2+\tau^3\right)\right)\,\partial_1 n\Bigg)+$$

$$\frac{1}{16}\,\rho^4\,(-1+\tau)^3\,\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\,\Bigg(-2\,B_{210}\,(1+\tau)^4\,\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)\,\partial_0 n\right)+$$

$$2\,A_{210}\,(-1+\tau)\,\Bigg(-n\,\left(7+11\,\tau+5\,\tau^2+\tau^3\right)+2\,\left(19+11\,\tau+5\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\left(-7-4\,\tau+6\,\tau^2+4\,\tau^3+\tau^4\right)\,\partial_0 n\Bigg)+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-1+\tau)$$

$$\left(4\,C_{110}\,(1+\tau)^2-D_{110}\,\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2-\mathrm{Log}[1+\tau]\,(1+\tau)^2+2\,(2+\tau)\right)\right)\,\partial_1 n\Bigg)$$

In[1018]:=

```
R[]^(n) × HoldForm[
  PhyRetTimeVector[b] × PD[-b]@(R[]^n * PhyRetTimeVector[a] × PD[-a]@Phi[])]
ReleaseHold[%]
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@Normal@PhiCanonicalAnsatzOrderN[4];
% /. ApplyRuleGeneralConstantsRule;
% // Expand;
Coefficient[%, Ysph[-LI[1], -LI[1]]];
% /. ApplyRule@DefBoost // Simplify // Expand;
% /. ApplyRule@CFDef // Simplify // Expand;
% /. asol → SolutionGeneralPL // Simplify // Expand;
% /. ApplyRuleGeneralConstantsRule // Simplify // Expand;
Collect[%, ρ[], Simplify]
```

forma sem avaliação; desbloqueia cálculo; expande fatores; expande fatores; normal; expande fatores; coeficiente; simplifica; expande fatores; simplifica; expande fatores; simplifica; expande fatores; simplifica; expande fatores; agrupa coeficientes; simplifica

Out[1018]=

$$\left(L^b\ \partial_b\left[R^n\ L^a\ \partial_a\Phi\right]\right)\ R^n$$

Out[1019]=

$$L^b\ R^n\ \left(R^n\,\partial_a\Phi\,\partial_b L^a + L^a\ \left(\partial_a\Phi\ \left(n\ dR_b\ R^{-1+n} + \mathrm{Log}[R]\ R^n\,\partial_b n\right) + R^n\,\partial_b\partial_a\Phi\right)\right)$$

Out[1020]=

$$-\frac{n\ d\rho_b\ L^a\ L^b\ R^{-1+2n}\,\partial_a\Phi}{\rho^2\ \left(-1+\tau^2\right)^2}+\frac{2\ n\ d\tau_b\ L^a\ L^b\ R^{-1+2n}\,\tau\,\partial_a\Phi}{\rho\ \left(-1+\tau^2\right)^2}+\frac{n\ d\rho_b\ L^a\ L^b\ R^{-1+2n}\,\tau^2\,\partial_a\Phi}{\rho^2\ \left(-1+\tau^2\right)^2}+$$
$$\mathrm{Log}[R]\ L^a\ L^b\ R^{2n}\,\partial_a\Phi\,\partial_b n + L^b\ R^{2n}\,\partial_a\Phi\,\partial_b L^a + L^a\ L^b\ R^{2n}\,\partial_b\partial_a\Phi$$

Out[1034]=

$$\frac{1}{16}\,\rho^3\ (-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\left(4\ C_{111}\ (1+\tau)^3\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\ (-1+\tau)\ \partial_0 n\right)+\right.$$
$$D_{111}\ \left(16-4\,n+2\,\mathrm{Log}[1-\tau]-n\,\mathrm{Log}[1-\tau]-2\,\mathrm{Log}[1+\tau]+n\,\mathrm{Log}[1+\tau]+12\,\tau-6\,n\,\tau+\right.$$
$$6\,\mathrm{Log}[1-\tau]\ \tau-3\,n\,\mathrm{Log}[1-\tau]\ \tau-6\,\mathrm{Log}[1+\tau]\ \tau+3\,n\,\mathrm{Log}[1+\tau]\ \tau+4\,\tau^2-$$
$$2\,n\,\tau^2+6\,\mathrm{Log}[1-\tau]\ \tau^2-3\,n\,\mathrm{Log}[1-\tau]\ \tau^2-6\,\mathrm{Log}[1+\tau]\ \tau^2+3\,n\,\mathrm{Log}[1+\tau]\ \tau^2+$$
$$2\,\mathrm{Log}[1-\tau]\ \tau^3-n\,\mathrm{Log}[1-\tau]\ \tau^3-2\,\mathrm{Log}[1+\tau]\ \tau^3+n\,\mathrm{Log}[1+\tau]\ \tau^3+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-1+\tau^2\right)\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2-\mathrm{Log}[1+\tau]\,(1+\tau)^2+2\,(2+\tau)\right)\partial_0 n\Bigg)\Bigg)-$$

$$\frac{1}{384}\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^7\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}$$

$$\left(B_{411}\,(1+\tau)^5+A_{411}\,(-1+\tau)^3\left(31+8\,\tau+\tau^2\right)\right)$$

$$\partial_1 n+\frac{1}{384}$$

$$\rho^6$$

$$(-1+\tau)^3$$

$$\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}$$

$$\Bigg(B_{411}\,(1+\tau)^6\left(2-n+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right)+$$

$$A_{411}\,(-1+\tau)^3\Bigg(-n\left(31+39\,\tau+9\,\tau^2+\tau^3\right)+2\left(151+39\,\tau+9\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-31-8\,\tau+30\,\tau^2+8\,\tau^3+\tau^4\right)\partial_0 n\Bigg)+$$

$$8\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\left(-B_{311}\,(1+\tau)^4+A_{311}\,(-1+\tau)^2\left(17+6\,\tau+\tau^2\right)\right)\partial_1 n\Bigg)-$$

$$\frac{1}{48}\,\rho^5\,(-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\Bigg(B_{311}\,(1+\tau)^5\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right)+$$

$$A_{311}\,(-1+\tau)^2\Bigg(-n\left(17+23\,\tau+7\,\tau^2+\tau^3\right)+2\left(65+23\,\tau+7\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-17-6\,\tau+16\,\tau^2+6\,\tau^3+\tau^4\right)\partial_0 n\Bigg)+$$

$$6\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\left(B_{211}\,(1+\tau)^3+A_{211}\left(-7+3\,\tau+3\,\tau^2+\tau^3\right)\right)\partial_1 n\Bigg)+$$

$$\frac{1}{16}\,\rho^4\,(-1+\tau)^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\Bigg(-2\,B_{211}\,(1+\tau)^4\left(-2+n-\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)\,\partial_0 n\right)+$$

$$2\,A_{211}\,(-1+\tau)\Bigg(-n\left(7+11\,\tau+5\,\tau^2+\tau^3\right)+2\left(19+11\,\tau+5\,\tau^2+\tau^3\right)+$$

$$\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-7-4\,\tau+6\,\tau^2+4\,\tau^3+\tau^4\right)\partial_0 n\Bigg)+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)$$

$$\left(4\,C_{111}\,(1+\tau)^2-D_{111}\left(\mathrm{Log}[1-\tau]\,(1+\tau)^2-\mathrm{Log}[1+\tau]\,(1+\tau)^2+2\,(2+\tau)\right)\right)\partial_1 n\Bigg)$$

$F[R] = R^2$ at order $l = 1$ is not defined unless the regularity (no − log) condition is imposed to this order $D_{111} = 0$.

Once the regularity condition is imposed,

evaluating at future null infinity $\tau =$

1 the associated NP constant are A_{21-1} , A_{210} , A_{211}

l = 2 order

In[863]:=

```
R[]^(n) × PhyRetTimeVector[b] × PD[-b]@Phi[]
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand;
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@CFDef // Simplify // Expand;
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRule@DefBoost // Simplify // Expand
(******)
R[]^(n) × PhyRetTimeVector[b] × PD[-b]@%;
% // Expand;
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand;
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@DefBoost // Simplify // Expand;
% /. ApplyRule@CFDef // Simplify // Expand
(***)
R[]^(n) × PhyRetTimeVector[b] × PD[-b]@%;
% // Expand;
% /. ApplyRuleDifferentialsPhysicalToUnphysical // Expand;
% /. ApplyRulePhysRetAdvTimeDerToFframeCFboost // Expand;
% /. ApplyRulePhysicalToUnphysicalCoords;
DummyToBasis[CoordBasis][%];
TraceBasisDummy[%];
% // ToValues;
% /. ApplyRule@DefBoost // Simplify // Expand;
% /. ApplyRule@CFDef // Simplify // Expand
```

```
(****)
% /. ApplyRule@Normal@PhiCanonicalAnsatzOrderN[4];
                    normal
% /. ApplyRuleGeneralConstantsRule;
% // Expand;
      expande fatores
Coefficient[%, Ysph[-LI[2], -LI[-2]]];
coeficiente
% /. ApplyRule@DefBoost // Simplify // Expand;
                           simplifica   expande fatores
% /. ApplyRule@CFDef // Simplify // Expand;
                        simplifica   expande fatores
% /. asol → SolutionGeneralPL // Simplify // Expand;
                                 simplifica   expande fatores
% /. ApplyRuleGeneralConstantsRule // Simplify // Expand;
                                      simplifica   expande fatore
Collect[%, ρ[], Simplify]
agrupa coeficientes  simplifica
```

Out[863]=

$$L^{b}\ R^{n}\ \partial_{b}\Phi$$

Out[872]=

$$\rho\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{0}\Phi-\rho\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{0}\Phi-\rho\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{0}\Phi+\rho\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{0}\Phi-\rho^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{1}\Phi+2\,\rho^2\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{1}\Phi-\rho^2\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{n}\partial_{1}\Phi$$

Out[882]=

$$-\frac{2\,\rho^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}+\frac{n\,\rho^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}+\frac{6\,\rho^2\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}-\frac{3\,n\,\rho^2\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}-$$
$$\frac{6\,\rho^2\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}+\frac{3\,n\,\rho^2\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}+\frac{2\,\rho^2\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}-$$
$$\frac{n\,\rho^2\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\Phi}{1+\tau}+\frac{\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}-\frac{\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}-$$
$$\frac{3\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}+\frac{3\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}+$$
$$\frac{3\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}-\frac{3\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}-$$
$$\frac{\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}+\frac{\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^2\,\tau^7\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}n\,\partial_{0}\Phi}{1+\tau}+$$
$$\frac{\rho^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}-\frac{\rho^2\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}-\frac{3\,\rho^2\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}+$$
$$\frac{3\,\rho^2\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}+\frac{3\,\rho^2\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}-\frac{3\,\rho^2\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_{0}\partial_{0}\Phi}{1+\tau}-$$

$$\frac{\rho^2\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_0\Phi}{1+\tau}+\frac{\rho^2\,\tau^7\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_0\Phi}{1+\tau}-\frac{\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}+\frac{2\,\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}+$$

$$\frac{\rho^3\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}-\frac{4\,\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}+\frac{\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}+$$

$$\frac{2\,\rho^3\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}-\frac{\rho^3\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\partial_1\Phi}{1+\tau}-\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}+$$

$$\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}+\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}-$$

$$\frac{4\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}+\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}+$$

$$\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}-\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0\Phi\,\partial_1 n}{1+\tau}+$$

$$\frac{4\,\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-\frac{n\,\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-\frac{8\,\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}+\frac{n\,\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}+$$

$$\frac{2\,n\,\rho^3\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}+\frac{8\,\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-\frac{2\,n\,\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-$$

$$\frac{4\,\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-\frac{n\,\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}+\frac{n\,\rho^3\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\Phi}{1+\tau}-$$

$$\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}+\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}+$$

$$\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}-\frac{4\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}+$$

$$\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}+\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}-$$

$$\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^3\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_0 n\,\partial_1\Phi}{1+\tau}+\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}-$$

$$\frac{3\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}+\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}+$$

$$\frac{2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}-\frac{3\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}+$$

$$\frac{\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^4\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1 n\,\partial_1\Phi}{1+\tau}-\frac{\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\partial_0\Phi}{1+\tau}+$$

$$\frac{2\,\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\partial_0\Phi}{1+\tau}+\frac{\rho^3\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\partial_0\Phi}{1+\tau}-\frac{4\,\rho^3\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2\,n}\partial_1\partial_0\Phi}{1+\tau}+$$

$$\frac{\rho^3\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_0\Phi}{1+\tau}+\frac{2\,\rho^3\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_0\Phi}{1+\tau}-\frac{\rho^3\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_0\Phi}{1+\tau}+$$

$$\frac{\rho^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}-\frac{3\,\rho^4\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}+\frac{2\,\rho^4\,\tau^2\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}+$$

$$\frac{2\,\rho^4\,\tau^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}-\frac{3\,\rho^4\,\tau^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}+\frac{\rho^4\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{2n}\partial_1\partial_1\Phi}{1+\tau}$$

Out[892]=

$$6\,\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_0\Phi-7\,n\,\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_0\Phi+2\,n^2\,\rho^3\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_0\Phi-$$
$$6\,\rho^3\,\tau\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_0\Phi+\dots 666\dots+6\,\rho^6\,\tau^5\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_1\partial_1\partial_1\Phi-\rho^6\,\tau^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\partial_1\partial_1\partial_1\Phi$$

Size in memory: 0.8 MB | Show more | Show all | Iconize | Store full expression in notebook

Out[901]=

$$\frac{1}{256}\,\rho^5\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}$$

$$\Bigg(16\,C_{22-2}\,(-1+\tau)\,(1+\tau)^5\left(2\left(6-5\,n+n^2\right)-\left(2+(-11+4\,n)\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\right)(-1+\tau)\,\partial_0 n+\right.$$
$$\left.2\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2(-1+\tau)^2\,(\partial_0 n)^2+\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)^2\,\partial_0\partial_0 n\right)-$$

$$D_{22-2}\Big(-336+152\,n-16\,n^2-36\,\mathrm{Log}[1-\tau]+30\,n\,\mathrm{Log}[1-\tau]-6\,n^2\,\mathrm{Log}[1-\tau]+$$
$$36\,\mathrm{Log}[1+\tau]-30\,n\,\mathrm{Log}[1+\tau]+6\,n^2\,\mathrm{Log}[1+\tau]-216\,\tau+212\,n\,\tau-28\,n^2\,\tau-$$
$$144\,\mathrm{Log}[1-\tau]\,\tau+120\,n\,\mathrm{Log}[1-\tau]\,\tau-24\,n^2\,\mathrm{Log}[1-\tau]\,\tau+144\,\mathrm{Log}[1+\tau]\,\tau-$$
$$120\,n\,\mathrm{Log}[1+\tau]\,\tau+24\,n^2\,\mathrm{Log}[1+\tau]\,\tau+96\,\tau^2-80\,n\,\tau^2+16\,n^2\,\tau^2-180\,\mathrm{Log}[1-\tau]\,\tau^2+$$
$$150\,n\,\mathrm{Log}[1-\tau]\,\tau^2-30\,n^2\,\mathrm{Log}[1-\tau]\,\tau^2+180\,\mathrm{Log}[1+\tau]\,\tau^2-150\,n\,\mathrm{Log}[1+\tau]\,\tau^2+$$
$$30\,n^2\,\mathrm{Log}[1+\tau]\,\tau^2+384\,\tau^3-320\,n\,\tau^3+64\,n^2\,\tau^3+288\,\tau^4-240\,n\,\tau^4+48\,n^2\,\tau^4+$$
$$180\,\mathrm{Log}[1-\tau]\,\tau^4-150\,n\,\mathrm{Log}[1-\tau]\,\tau^4+30\,n^2\,\mathrm{Log}[1-\tau]\,\tau^4-180\,\mathrm{Log}[1+\tau]\,\tau^4+$$
$$150\,n\,\mathrm{Log}[1+\tau]\,\tau^4-30\,n^2\,\mathrm{Log}[1+\tau]\,\tau^4+72\,\tau^5-60\,n\,\tau^5+12\,n^2\,\tau^5+$$
$$144\,\mathrm{Log}[1-\tau]\,\tau^5-120\,n\,\mathrm{Log}[1-\tau]\,\tau^5+24\,n^2\,\mathrm{Log}[1-\tau]\,\tau^5-144\,\mathrm{Log}[1+\tau]\,\tau^5+$$
$$120\,n\,\mathrm{Log}[1+\tau]\,\tau^5-24\,n^2\,\mathrm{Log}[1+\tau]\,\tau^5+36\,\mathrm{Log}[1-\tau]\,\tau^6-30\,n\,\mathrm{Log}[1-\tau]\,\tau^6+$$
$$6\,n^2\,\mathrm{Log}[1-\tau]\,\tau^6-36\,\mathrm{Log}[1+\tau]\,\tau^6+30\,n\,\mathrm{Log}[1+\tau]\,\tau^6-6\,n^2\,\mathrm{Log}[1+\tau]\,\tau^6-$$
$$\left(-1+\tau^2\right)\left(3\,\mathrm{Log}[1-\tau]\left(2+(-11+4\,n)\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\right)(-1+\tau)\,(1+\tau)^4-\right.$$
$$3\,\mathrm{Log}[1+\tau]\left(2+(-11+4\,n)\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\right)(-1+\tau)\,(1+\tau)^4+$$
$$4\left(-4-3\,\tau+7\,\tau^2+9\,\tau^3+3\,\tau^4\right)+2\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]$$
$$\left.\left(80+33\,\tau-77\,\tau^2-99\,\tau^3-33\,\tau^4+4\,n\left(-4-3\,\tau+7\,\tau^2+9\,\tau^3+3\,\tau^4\right)\right)\right)\partial_0 n+$$
$$2\,\mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2\left(-1+\tau^2\right)^2\left(3\,\mathrm{Log}[1-\tau]\,(-1+\tau)\,(1+\tau)^3-\right.$$
$$\left.3\,\mathrm{Log}[1+\tau]\,(-1+\tau)\,(1+\tau)^3+2\left(-4+\tau+6\,\tau^2+3\,\tau^3\right)\right)(\partial_0 n)^2-$$

$$8\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_0\partial_0 n - 3\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_0\partial_0 n +$$
$$3\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_0\partial_0 n + 2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau\,\partial_0\partial_0 n -$$
$$6\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau\,\partial_0\partial_0 n + 6\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau\,\partial_0\partial_0 n +$$
$$28\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_0\partial_0 n + 6\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_0\partial_0 n -$$
$$6\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_0\partial_0 n + 2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^3\,\partial_0\partial_0 n +$$
$$18\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^3\,\partial_0\partial_0 n - 18\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^3\,\partial_0\partial_0 n -$$
$$32\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^4\,\partial_0\partial_0 n - 10\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^5\,\partial_0\partial_0 n -$$
$$18\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^5\,\partial_0\partial_0 n + 18\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^5\,\partial_0\partial_0 n +$$
$$12\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_0\partial_0 n - 6\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_0\partial_0 n +$$
$$6\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_0\partial_0 n + 6\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^7\,\partial_0\partial_0 n +$$
$$6\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^7\,\partial_0\partial_0 n - 6\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^7\,\partial_0\partial_0 n +$$
$$3\,\text{Log}[1-\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^8\,\partial_0\partial_0 n - 3\,\text{Log}[1+\tau]\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^8\,\partial_0\partial_0 n\Bigg)\Bigg) -$$
$$\frac{1}{256}\rho^6\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3n}\Bigg(-8\,B_{32-2}\,(1+\tau)^6\Bigg((-2+n)\,(15-12\,\tau+n\,(-6+4\,\tau)) -$$
$$2\,(-1+\tau)\left(-3+2\,\tau+\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](15-11\,\tau+n\,(-6+4\,\tau))\right)\partial_0 n + 2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2$$
$$(-1+\tau)^2\,(-3+2\,\tau)\,(\partial_0 n)^2 + \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right](-1+\tau)^2\,(-3+2\,\tau)\,\partial_0\partial_0 n\Bigg) +$$
$$8\,A_{32-2}\,(-1+\tau)^2\Bigg(770-277\,n+26\,n^2+936\,\tau-636\,n\,\tau+84\,n^2\,\tau+828\,\tau^2 -$$
$$630\,n\,\tau^2+108\,n^2\,\tau^2+480\,\tau^3-384\,n\,\tau^3+72\,n^2\,\tau^3+162\,\tau^4-133\,n\,\tau^4+26\,n^2\,\tau^4 +$$
$$24\,\tau^5-20\,n\,\tau^5+4\,n^2\,\tau^5-2\,\left(-1+\tau^2\right)\Bigg(13+29\,\tau+25\,\tau^2+11\,\tau^3+2\,\tau^4+\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]$$
$$\left(-145-194\,\tau-148\,\tau^2-62\,\tau^3-11\,\tau^4+n\,\left(26+58\,\tau+50\,\tau^2+22\,\tau^3+4\,\tau^4\right)\right)\Bigg)\partial_0 n +$$
$$2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2\left(-1+\tau^2\right)^2\left(13+16\,\tau+9\,\tau^2+2\,\tau^3\right)(\partial_0 n)^2 +$$
$$\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\left(-1+\tau^2\right)^2\left(13+16\,\tau+9\,\tau^2+2\,\tau^3\right)\partial_0\partial_0 n\Bigg) +$$

$$
\begin{aligned}
&(-1+\tau)\Bigg(16\,C_{22\text{-}2}\,(-1+\tau)\,(1+\tau)^3\Bigg(\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(-1+\tau^2)\,\partial_0\partial_1 n+\\
&\quad\Bigg(-2\,(1+\tau)+\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(13+9\,\tau-4\,n\,(1+\tau))+\\
&\qquad 4\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]^2\,(-1+\tau^2)\,\partial_0 n\Bigg)\,\partial_1 n+\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(-1+\tau^2)\,\partial_1\partial_0 n\Bigg)+\\
&\quad D_{22\text{-}2}\,\Bigg(-\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(-1+\tau^2)\,\big(3\,\mathrm{Log}[1-\tau]\,(-1+\tau)\,(1+\tau)^3-\\
&\qquad 3\,\mathrm{Log}[1+\tau]\,(-1+\tau)\,(1+\tau)^3+2\,(-4+\tau+6\,\tau^2+3\,\tau^3)\big)\,\partial_0\partial_1 n+\\
&\quad\Bigg(-16+6\,\mathrm{Log}[1+\tau]+176\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]-32\,n\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]-\\
&\qquad 39\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]+12\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]-\\
&\qquad 12\,\tau+18\,\mathrm{Log}[1+\tau]\,\tau+46\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau-24\,n\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau-\\
&\qquad 105\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau+36\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau+\\
&\qquad 28\,\tau^2+12\,\mathrm{Log}[1+\tau]\,\tau^2-174\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^2+56\,n\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^2-\\
&\qquad 54\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^2+24\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^2+\\
&\qquad 36\,\tau^3-12\,\mathrm{Log}[1+\tau]\,\tau^3-186\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^3+72\,n\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^3+\\
&\qquad 78\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^3-24\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^3+12\,\tau^4-\\
&\qquad 18\,\mathrm{Log}[1+\tau]\,\tau^4-54\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^4+24\,n\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^4+93\,\mathrm{Log}[1+\tau]\\
&\qquad\ \mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^4-36\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^4-6\,\mathrm{Log}[1+\tau]\,\tau^5+27\\
&\qquad\ \mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^5-12\,n\,\mathrm{Log}[1+\tau]\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,\tau^5+3\,\mathrm{Log}[1-\tau]\\
&\qquad\ (-1+\tau)\,(1+\tau)^3\,\Bigg(2\,(1+\tau)+\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(-13-9\,\tau+4\,n\,(1+\tau))\Bigg)-\\
&\qquad 4\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]^2\,(-1+\tau^2)\,\big(3\,\mathrm{Log}[1-\tau]\,(-1+\tau)\,(1+\tau)^3-\\
&\qquad\quad 3\,\mathrm{Log}[1+\tau]\,(-1+\tau)\,(1+\tau)^3+2\,(-4+\tau+6\,\tau^2+3\,\tau^3)\big)\,\partial_0 n\Bigg)\,\partial_1 n-\\
&\quad \mathrm{Log}\Big[\frac{1}{\rho-\rho\,\tau^2}\Big]\,(-1+\tau^2)\,\big(3\,\mathrm{Log}[1-\tau]\,(-1+\tau)\,(1+\tau)^3-\\
&\qquad 3\,\mathrm{Log}[1+\tau]\,(-1+\tau)\,(1+\tau)^3+2\,(-4+\tau+6\,\tau^2+3\,\tau^3)\big)\,\partial_1\partial_0 n\Bigg)\Bigg)\Bigg)+
\end{aligned}
$$

$$\frac{1}{128}\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\rho^9\,(-1+\tau)^6\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3\,n}$$

$$\left(B_{42-2}\,(-2+\tau)\,(1+\tau)^5+A_{42-2}\,(-1+\tau)^3\,\left(18+15\,\tau+6\,\tau^2+\tau^3\right)\right)$$

$$\left(2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(\partial_1 n)^2+\partial_1\partial_1 n\right)+$$

$$\frac{1}{256}\,\rho^7\,(-1+\tau)^4\left(\frac{1}{\rho-\rho\,\tau^2}\right)^{3\,n}$$

$$\Bigg(2\,B_{42-2}\,(1+\tau)^7\,\Bigg((-2+n)\,(9+2\,n\,(-2+\tau)-6\,\tau)-$$

$$\left(\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(19+4\,n\,(-2+\tau)-11\,\tau)+2\,(-2+\tau)\right)\,(-1+\tau)\,\partial_0 n+$$

$$2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2\,(-2+\tau)\,(-1+\tau)^2\,(\partial_0 n)^2+\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,(-2+\tau)\,(-1+\tau)^2\,\partial_0\partial_0 n\Bigg)+$$

$$2\,A_{42-2}\,(-1+\tau)^3\,\Bigg(1842-513\,n+36\,n^2+1512\,\tau-960\,n\,\tau+102\,n^2\,\tau+936\,\tau^2-$$

$$684\,n\,\tau^2+108\,n^2\,\tau^2+396\,\tau^3-310\,n\,\tau^3+56\,n^2\,\tau^3+102\,\tau^4-83\,n\,\tau^4+16\,n^2\,\tau^4+$$

$$12\,\tau^5-10\,n\,\tau^5+2\,n^2\,\tau^5-\left(-1+\tau^2\right)\,\Bigg(2\,\left(18+33\,\tau+21\,\tau^2+7\,\tau^3+\tau^4\right)+\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]$$

$$\left(-531-480\,\tau-258\,\tau^2-80\,\tau^3-11\,\tau^4+4\,n\,\left(18+33\,\tau+21\,\tau^2+7\,\tau^3+\tau^4\right)\right)\Bigg)\,\partial_0 n+$$

$$2\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2\,\left(-1+\tau^2\right)^2\,\left(18+15\,\tau+6\,\tau^2+\tau^3\right)\,(\partial_0 n)^2+18\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\partial_0\partial_0 n+$$

$$15\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau\,\partial_0\partial_0 n-30\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^2\,\partial_0\partial_0 n-$$

$$29\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^3\,\partial_0\partial_0 n+6\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^4\,\partial_0\partial_0 n+$$

$$13\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^5\,\partial_0\partial_0 n+6\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^6\,\partial_0\partial_0 n+\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\tau^7\,\partial_0\partial_0 n\Bigg)+$$

$$104\,A_{32-2}\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\partial_0\partial_1 n+24\,B_{32-2}\,\text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\,\partial_0\partial_1 n-$$

$$184\,A_{32-2}$$

$\text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau\,\partial_0\partial_1 n +$

$56\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau\,\partial_0\partial_1 n -$

$104\, A_{32-2}$

$\text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^2\,\partial_0\partial_1 n -$

$24\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^2\,\partial_0\partial_1 n +$

$264\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^3\,\partial_0\partial_1 n -$

$136\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^3\,\partial_0\partial_1 n +$

$40\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^4\,\partial_0\partial_1 n -$

$40\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^4\,\partial_0\partial_1 n -$

$104\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^5\,\partial_0\partial_1 n +$

$104\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^5\,\partial_0\partial_1 n -$

$56\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^6\,\partial_0\partial_1 n +$

$56\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^6\,\partial_0\partial_1 n +$

$24\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^7\,\partial_0\partial_1 n -$

$24\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^7\,\partial_0\partial_1 n +$

$16\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^8\,\partial_0\partial_1 n -$

$16\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \tau^8\,\partial_0\partial_1 n +$

$208\, A_{32-2}\, \partial_1 n +$

$48\, B_{32-2}\, \partial_1 n -$

$2528\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \partial_1 n +$

$416\, n\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \partial_1 n -$

$288\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\,\tau^2}\right] \partial_1 n +$

$$96\, n\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \partial_1 n -$$
$$160\, A_{32-2}\, \tau\, \partial_1 n +$$
$$160\, B_{32-2}\, \tau\, \partial_1 n +$$
$$4432\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 n -$$
$$320\, n\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 n -$$
$$848\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 n +$$
$$320\, n\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 n -$$
$$368\, A_{32-2}\, \tau^2\, \partial_1 n +$$
$$112\, B_{32-2}\, \tau^2\, \partial_1 n -$$
$$384\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 n -$$
$$736\, n\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 n -$$
$$384\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 n +$$
$$224\, n\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 n +$$
$$160\, A_{32-2}\, \tau^3\, \partial_1 n -$$
$$160\, B_{32-2}\, \tau^3\, \partial_1 n -$$
$$1040\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 n +$$
$$320\, n\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 n +$$
$$1040\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 n -$$
$$320\, n\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 n +$$
$$240\, A_{32-2}\, \tau^4\, \partial_1 n -$$
$$240\, B_{32-2}\, \tau^4\, \partial_1 n -$$
$$1120\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 n +$$
$$480\, n\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 n +$$
$$1120\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 n -$$

$$480\, n\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 n +$$
$$32\, A_{32-2}\, \tau^5\, \partial_1 n -$$
$$32\, B_{32-2}\, \tau^5\, \partial_1 n +$$
$$48\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 n +$$
$$64\, n\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 n -$$
$$48\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 n -$$
$$64\, n\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 n -$$
$$80\, A_{32-2}\, \tau^6\, \partial_1 n +$$
$$80\, B_{32-2}\, \tau^6\, \partial_1 n +$$
$$448\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^6\, \partial_1 n -$$
$$160\, n\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^6\, \partial_1 n -$$
$$448\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^6\, \partial_1 n +$$
$$160\, n\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^6\, \partial_1 n -$$
$$32\, A_{32-2}\, \tau^7\, \partial_1 n +$$
$$32\, B_{32-2}\, \tau^7\, \partial_1 n +$$
$$144\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^7\, \partial_1 n -$$
$$64\, n\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^7\, \partial_1 n -$$
$$144\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^7\, \partial_1 n +$$
$$64\, n\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^7\, \partial_1 n +$$
$$416\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \partial_0 n\, \partial_1 n +$$
$$96\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \partial_0 n\, \partial_1 n -$$
$$736\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau\, \partial_0 n\, \partial_1 n +$$
$$224\, B_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau\, \partial_0 n\, \partial_1 n -$$
$$416\, A_{32-2}\, \mathrm{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^2\, \partial_0 n\, \partial_1 n -$$

$$96\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^2\, \partial_0 n\, \partial_1 n +$$

$$1056\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^3\, \partial_0 n\, \partial_1 n -$$

$$544\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^3\, \partial_0 n\, \partial_1 n +$$

$$160\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^4\, \partial_0 n\, \partial_1 n -$$

$$160\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^4\, \partial_0 n\, \partial_1 n -$$

$$416\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^5\, \partial_0 n\, \partial_1 n +$$

$$416\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^5\, \partial_0 n\, \partial_1 n -$$

$$224\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^6\, \partial_0 n\, \partial_1 n +$$

$$224\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^6\, \partial_0 n\, \partial_1 n +$$

$$96\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^7\, \partial_0 n\, \partial_1 n -$$

$$96\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^7\, \partial_0 n\, \partial_1 n +$$

$$64\, A_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^8\, \partial_0 n\, \partial_1 n -$$

$$64\, B_{32-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^8\, \partial_0 n\, \partial_1 n -$$

$$32\, C_{22-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 (\partial_1 n)^2 +$$

$$16\, D_{22-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 (\partial_1 n)^2 +$$

$$6\, D_{22-2}\, \text{Log}[1-\tau]\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 (\partial_1 n)^2 -$$

$$6\, D_{22-2}\, \text{Log}[1+\tau]\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 (\partial_1 n)^2 -$$

$$36\, D_{22-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau\, (\partial_1 n)^2 +$$

$$96\, C_{22-2}\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^2\, (\partial_1 n)^2 -$$

$$18\, D_{22-2}\, \text{Log}[1-\tau]\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^2\, (\partial_1 n)^2 +$$

$$18\, D_{22-2}\, \text{Log}[1+\tau]\, \text{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]^2 \tau^2\, (\partial_1 n)^2 +$$

$$32\, D_{22\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^3\, (\partial_1 n)^2 -$$

$$96\, C_{22\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^4\, (\partial_1 n)^2 +$$

$$18\, D_{22\text{-}2}\, \text{Log}[1 - \tau]\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^4\, (\partial_1 n)^2 -$$

$$18\, D_{22\text{-}2}\, \text{Log}[1 + \tau]\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^4\, (\partial_1 n)^2 -$$

$$12\, D_{22\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^5\, (\partial_1 n)^2 +$$

$$32\, C_{22\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^6\, (\partial_1 n)^2 -$$

$$6\, D_{22\text{-}2}\, \text{Log}[1 - \tau]\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^6\, (\partial_1 n)^2 +$$

$$6\, D_{22\text{-}2}\, \text{Log}[1 + \tau]\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right]^2 \tau^6\, (\partial_1 n)^2 +$$

$$104\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \partial_1 \partial_0 n +$$

$$24\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \partial_1 \partial_0 n -$$

$$184\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 \partial_0 n +$$

$$56\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau\, \partial_1 \partial_0 n -$$

$$104\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 \partial_0 n -$$

$$24\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^2\, \partial_1 \partial_0 n +$$

$$264\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 \partial_0 n -$$

$$136\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^3\, \partial_1 \partial_0 n +$$

$$40\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 \partial_0 n -$$

$$40\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^4\, \partial_1 \partial_0 n -$$

$$104\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 \partial_0 n +$$

$$104\, B_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^5\, \partial_1 \partial_0 n -$$

$$56\, A_{32\text{-}2}\, \text{Log}\left[\frac{1}{\rho - \rho\, \tau^2}\right] \tau^6\, \partial_1 \partial_0 n +$$

$$56\ B_{32-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_1\partial_0 n\,+$$
$$24\ A_{32-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^7\,\partial_1\partial_0 n\,-$$
$$24\ B_{32-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^7\,\partial_1\partial_0 n\,+$$
$$16\ A_{32-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^8\,\partial_1\partial_0 n\,-$$
$$16\ B_{32-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^8\,\partial_1\partial_0 n\,-$$
$$16\ C_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_1\partial_1 n\,+$$
$$8\ D_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_1\partial_1 n\,+$$
$$3\ D_{22-2}\ \mathrm{Log}[1-\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_1\partial_1 n\,-$$
$$3\ D_{22-2}\ \mathrm{Log}[1+\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\partial_1\partial_1 n\,-$$
$$18\ D_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau\,\partial_1\partial_1 n\,+$$
$$48\ C_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_1\partial_1 n\,-$$
$$9\ D_{22-2}\ \mathrm{Log}[1-\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_1\partial_1 n\,+$$
$$9\ D_{22-2}\ \mathrm{Log}[1+\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^2\,\partial_1\partial_1 n\,+$$
$$16\ D_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^3\,\partial_1\partial_1 n\,-$$
$$48\ C_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^4\,\partial_1\partial_1 n\,+$$
$$9\ D_{22-2}\ \mathrm{Log}[1-\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^4\,\partial_1\partial_1 n\,-$$
$$9\ D_{22-2}\ \mathrm{Log}[1+\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^4\,\partial_1\partial_1 n\,-$$
$$6\ D_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^5\,\partial_1\partial_1 n\,+$$
$$16\ C_{22-2}\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_1\partial_1 n\,-$$
$$3\ D_{22-2}\ \mathrm{Log}[1-\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_1\partial_1 n\,+$$
$$3\ D_{22-2}\ \mathrm{Log}[1+\tau]\ \mathrm{Log}\left[\frac{1}{\rho-\rho\,\tau^2}\right]\tau^6\,\partial_1\partial_1 n\Bigg)\,-$$

$$\frac{1}{128}\rho^8(-1+\tau)^5\left(\frac{1}{\rho-\rho\tau^2}\right)^{3n}$$
$$\Bigg(B_{42-2}\,(1+\tau)^5\Bigg(\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(2-\tau-2\tau^2+\tau^3)\,\partial_0\partial_1 n+$$
$$\Big(4+2\tau-2\tau^2+\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(-23-2\tau+9\tau^2+n\,(8+4\tau-4\tau^2))+4\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]^2$$
$$(2-\tau-2\tau^2+\tau^3)\,\partial_0 n\Big)\,\partial_1 n+\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(2-\tau-2\tau^2+\tau^3)\,\partial_1\partial_0 n\Bigg)+$$
$$A_{42-2}\,(-1+\tau)^3\Bigg(\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(-18-15\tau+12\tau^2+14\tau^3+6\tau^4+\tau^5)\,\partial_0\partial_1 n+$$
$$\Big(-2\,(18+33\tau+21\tau^2+7\tau^3+\tau^4)+\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]$$
$$(567+474\tau+240\tau^2+70\tau^3+9\tau^4-4n\,(18+33\tau+21\tau^2+7\tau^3+\tau^4))+$$
$$4\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]^2\,(-18-15\tau+12\tau^2+14\tau^3+6\tau^4+\tau^5)\,\partial_0 n\Big)\,\partial_1 n+$$
$$\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(-18-15\tau+12\tau^2+14\tau^3+6\tau^4+\tau^5)\,\partial_1\partial_0 n\Bigg)+4\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]$$
$$(-1+\tau)\,(-B_{32-2}\,(1+\tau)^4\,(-3+2\tau)+A_{32-2}\,(-1+\tau)^2\,(13+16\tau+9\tau^2+2\tau^3))$$
$$\Big(2\,\mathrm{Log}\Big[\frac{1}{\rho-\rho\tau^2}\Big]\,(\partial_1 n)^2+\partial_1\partial_1 n\Big)\Bigg)$$

More systematic approach

Extras and induction calculations