Expectations and Frictions: Lessons from a Quantitative Model with Dispersed Information*

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Abstract

What are the macroeconomic implications of informational frictions in a quantitative business cycle model? We develop a general solution method that allows enriching a standard medium-scale DSGE model with dispersed information. We estimate the model, incorporating comprehensive macroeconomic and expectation data, and revisit crucial questions about business cycles. Expectation data identifies strong informational frictions, which dampen general equilibrium effects and change the relative importance of various shocks in driving business cycles. We find that informational frictions complement standard inertial frictions rather than being alternatives. The former is crucial for generating sluggishness in inflation, whereas the latter is important for inertia in real macroeconomic aggregates.

Keywords: Business cycles, DSGE model, informational frictions, higher-order beliefs
JEL Codes: C72, D80, E30, E7

1 Introduction

Expectations play a central role in macroeconomics. Household's and firm's decisions in macroeconomic models are heavily influenced by expectations about future economic developments and economic policy.

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A recent literature has made important efforts to understand how expectations are formed. Coibion and Gorodnichenko (2015) (henceforth, CG) document deviations from full-information rational expectations using empirical measures of information frictions from survey data.[1] They show that forecasts errors and revisions co-move consistently with standard imperfect information theories such as sticky information (Mankiw and Reis; 2002) and noisy information (Woodford; 2002; Sims; 2003).

Despite that, current state-of-the-art DSGE models following the tradition Christiano et al. (2005) and Smets and Wouters (2003, 2007) still rest on the full-information paradigm. These models are essential for understanding macroeconomic dynamics and economic policy, as they successfully fit the empirical properties of key macroeconomic aggregates. However, they typically do not incorporate expectations data to discipline the model, despite the prominent role that expectations play in these models (Milani; 2023).

This paper seeks to fill this gap by estimating a medium-scale DSGE model with heterogeneous and imperfect information using a broad set of macroeconomic aggregates and expectation data. We make two main contributions, one methodological and the other applied.

Solution method. Our methodological contribution is the development of a novel method to solve DSGE models with dispersed and exogenous information. The computational challenge lies in the complexity of solving DSGE models when we relax the assumption of full information: agents must form an infinite regress of expectations, as first pointed out by Townsend (1983). Then, state representations of DSGE models with such informational structure become infinite-dimensional. The method bridges Uhlig's (2001) undermined coefficient method for full-information models with the solution methods from Nimark (2008) and Melosi (2017) for dispersed information models that truncate the hierarchy of beliefs to a finite order.

Our method improves upon existing ones by allowing for the inclusion of endogenous state variables into the system of log-linearized equilibrium conditions, which allows for solving larger-scale models with dispersed information. It also uses a representation of the system of equations that depends only on the average expectation, instead of requiring writing equilibrium equations in terms of higher-order expectations, which simplifies the characterization of equilibrium conditions. Our method is fast enough to perform Bayesian estimation with standard techniques.

Estimated dispersed information model. Our applied contribution consists of estimating a standard medium-scale DSGE model along the lines of Smets and Wouters (2007) augmented

[1] Mankiw et al. (2004) document forecast error persistence and time-varying disagremeent in expectations in the main surveys of expectation in the US. Coibion and Gorodnichenko (2012) explores the forecast error response to shocks to document deviations from full information. Bordalo et al. (2020) extends CG evidence for a broad set of macroeconomic data and individual expectations.

with dispersed information. In the model, both firms and households do not observe perfectly the underlying aggregate shocks hitting the economy. Instead, they form expectations based on exogenous and idiosyncratic signals about them. This form of informational friction is standard in the literature (Woodford; 2002; Nimark; 2008; Melosi; 2017; Angeletos and Huo; 2021, for instance). The model is estimated using U.S. standard macroeconomic data and *average forecast revisions* of several real and nominal variables.

Our estimated macroeconomic model with dispersed information allows us to*revisit* several important questions in the business cycle literature: i) which frictions are more relevant for matching the empirical properties of business cycle *and expectation* dynamics? ii) what are the key driving forces behind output and inflation fluctuations? iii) how do shocks propagate in an economy where agents are not fully informed?

We also explore two open questions of the literature on information frictions: i) Can information frictions replace standard frictions as a explanation of the sluggishness and persistence in macroeconomic variables? ii) Do higher-order expectations quantitatively matter for the shocks' propagation?

To answer these questions, we estimate both the full information (FI) and the dispersed information (DI) models using two different datasets: one including only U.S. macroeconomic data – as common practice in the literature –, and another augmented with forecast revisions data. This helps to understand the role of adding expectation data and information frictions on parameter estimates separately.

Several interesting results arise from our estimation exercises. The DI model has a substantially better fit of expectation data than the FI model, using a marginal likelihood criterion. Moreover, we compute empirical measures of informational frictions using a similar approach to CG in simulated data from the model and compare them to the same estimates using actual data. The model can match reasonably well those untargeted moments for output, consumption, investment, inflation, and interest rate.

When using only macroeconomic data, standard frictions remain remarkably stable when comparing estimates from the DI and FI models. The level of informational frictions varies significantly across shocks, with some being very close to the FI benchmark and others exhibiting substantial information frictions.

By including expectations data in the estimation, we document two key findings. First, data on forecast revisions are key to identifying informational frictions. For instance, preference, investment-specific technological, and price mark-up shocks had negligible information frictions we estimated the model using only macroeconomic data. However, when including expectation data, the estimates change significantly, indicating substantial deviations from full information. In contrast, the monetary policy shock shifted from being the shock with the strongest information friction to the one with the least.

Second, most standard parameter estimates remained largely unchanged when including expectations data, with two notable exceptions: the posterior estimates of the consumption habits parameter increased from 0.87 to 0.97, and the investment adjustment costs parameter almost doubled from 5.47 to 9.73. Moreover, the inclusion of expectation data leads to less persistent shocks in the DI model. The volatility of shocks is much higher in the DI model than in the FI model for both datasets. Both results are compensated by stronger information frictions: under incomplete information, agents react less to shocks since they are imperfectly aware of them, and expectations have their own persistence. In other words, shocks are volatile and slightly less persistent, but agents do not pay much attention to them and slowly learn about them. This varies substantially across shocks.

Business cycle drivers. Performing a forecast error variance decomposition (FEVD) exercise, we find that investment-specific technological shocks and government expenditure shocks are the key drivers for output growth and labor hours. The former has a bigger bite in the short run and decreases in importance over time, whereas the latter does the opposite. Consumption is mainly explained by preference shocks and investment by investment-specific technological shocks. Price mark-up shocks explain most of inflation's fluctuations and have an important and increasing share of explaining nominal rates and real wages dynamics over time. Wage mark-up shocks play a major role for real wage dynamics only, while TFP shocks play a minor role in all observables.

Comparing the FEVD using macroeconomic data only and the complete datasets reveals that adding expectation increases the relative importance of shocks with a direct impact on each variable (e.g., preference shocks for consumption). This is driven by the fact that expectation data requires stronger informational frictions, which generate anchoring as discussed by Angeletos and Huo (2021). Those frictions weaken the general equilibrium forces of the model, increasing the relative importance of partial equilibrium (direct effects) in the model.

Propagation of shocks and the effects of information frictions. We also study the propagation of shocks in the dispersed information model and compare it with the full information model. Moreover, we evaluate the effects of information in shaping the dynamic responses of macroeconomic and expectations variables to shocks. Under DI, agents underestimate the magnitude of shocks in the short run and take time to fully understand their impact.

We focus on the propagation dynamics of three shocks: i) investment-specific technological shocks, the main driver of output fluctuations according to our model; ii) cost-push (price markup) shocks, the main driver of inflation fluctuations; iii) monetary policy shocks, which have been extensively studied in the macroeconomic literature. For investment-specific shocks, the DI and FI models show similar dynamics, with slightly stronger responses in the DI model. Agents take about a year to fully grasp the impact of these shocks. Cost-push shocks exhibit more pronounced differences between DI and FI models, with the DI model showing a more pronounced and hump-shaped response of inflation, larger decreases in real wages, and a stronger nominal rate response. These differences are primarily due to informational frictions rather than different parameter estimates. Monetary policy shocks lead to stronger effects on macroeconomic aggregates and inflation in the DI model, primarily due to changes in parameter estimates rather than informational frictions.

We also decompose the effects of information frictions on impulse responses into first-order uncertainty (gradual learning about shocks) and higher-order uncertainty (uncertainty about other agents' expectations). For investment shocks, imperfect learning (first-order uncertainty) is the key factor behind the dampening effect of information frictions. For cost-push shocks, both first- and higher-order expectations contribute to altering the shock propagation to real variables, while first-order uncertainty explains most of the effect on the impulse response of nominal variables (inflation and nominal interest rate). Interestingly, hump-shape inflation dynamics after the cost-push shock is mainly driven by learning and not by higher-order expectations dynamics.

Information frictions: substitutes or complements for standard frictions? Our application provides a quantitative assessment of the role of information frictions as an alternative source of persistence and inertia in macroeconomic aggregates. Seminal papers by Mankiw and Reis (2002) propose sticky information and Woodford (2002) noisy information as alternatives to price stickiness. Even closer connected, Angeletos and Huo (2021) develop a sharp equivalence between the equilibrium effects of incomplete information and standard frictions that induce inertia such as habits or investment adjustment costs. Our model embeds both types of frictions with reasonably flat priors, leaving the data to pin down their relative importance in a fully-fledged quantitative model.

Surprisingly, when disciplining the model with expectations data and allowing both frictions a chance, parameter estimates of habits and investment adjustment costs *increases*. This does not mean that informational frictions are not important. In fact, their estimated levels are quite high. Instead, standard and information frictions complement each other. To explore this point further, we reestimate the DI model with the complete dataset, shutting down, one at a time, four key standard frictions that generate inertia: habits, investment adjustment costs, and price and wage indexation. We find that habit and investment adjustment costs are essential whereas price and indexation are not very important to explain the data when comparing marginal likelihoods. This suggests a more complementary view of information and standard frictions.

While both types of frictions are important, we document that informational frictions are crucial for generating sluggishness in inflation response to cost-push shocks, its main driver. This corroborates the view that dispersed information is important for inflation despite relevant price rigidity (Angeletos and La'O; 2009).

Related literature. Our paper also integrates the growing but still small quantitative literature estimating macroeconomic models with incomplete information. Melosi (2014) estimates a New Keynesian model in which firms have DI about the money growth and technological shocks. The author shows that the model with DI fits the data better than a model with sticky prices and indexation to past inflation. Melosi (2017) also uses a standard NK model with dispersed information about productivity, demand and monetary shocks. He finds that there is a signaling channel of monetary policy in the US and that the model fits the data better than a New Keynesian model with habits in consumption and indexation.

Our paper also relates to alternative methods for solving general DSGE models with dispersed information. Our setting extends methods from Nimark (2008) and Melosi (2017) that truncate the hierarchy of beliefs to a finite order to avoid the infinite regress of higher-order expectations. There are other alternatives using frequency domain techniques. For instance, Huo and Takayama (2023) use a combination of the Wiener-Hopf prediction formula and the Kalman filter to achieve a tractable finite-state representation for the equilibrium.[2] They apply this methodology to a small-scale dynamic beauty contest model and a HANK-type model with incomplete information. Huo and Takayama (2022) apply the same method to solve an RBC model with TFP and confidence shocks as in Angeletos and La'O (2013). Their model does not include standard nominal and standard frictions that from DSGE literature that we are considering. They show that the model does a good job of matching main business cycle moments and conditional responses identified in empirical VAR. Our application seek to evaluate the role of each type of friction in a fully-fledged DSGE model.

This paper also relates to a literature that evaluates imperfect (and common) information as a source of inertia. Collard et al. (2009) and Collard and Dellas (2010) show that imperfect information plays an important role in accounting for the US business cycle without resorting to price indexation. Levine et al. (2012) also estimate an NK model with habits, indexation, and imperfect information. They show that both habits and imperfect information can generate reasonable endogenous persistence, but the former is preferred in a marginal likelihood comparison. These papers evaluate use small-scale models and macroeconomic data only. Our application uses a medium-scale DSGE model and a broad set of macroeconomic and expectation data.

Outline. The remainder of this paper is organized as follows. Section 2 presents the details of the model and its equilibrium relations. Section 3 explains the new solution method we developed.

[2]For other papers using frequency domain techniques for solving dispersed information models see Kasa (2000); Kasa et al. (2014); Rondina and Walker (2021) and references therein.

Section 4 details the dataset used, the estimation procedure, and its results. Section 5 performs different quantitative exercises to evaluate the ability of the DI model to explain the macroeconomic and expectation data. Section 6 concludes.

2 Model

The model is a standard medium-scale DSGE model along the lines of Christiano et al. (2005) and Smets and Wouters (2007) (henceforth, SW). It features sticky prices and wages, variable capital utilization, fixed cost in intermediate production, and five frictions that generate endogenous persistence in the model: (i) habit formation in consumption, (ii) adjustment cost in investment, (iii, iv) partial indexation of prices and wages to past inflation, and (v) smoothing in the monetary policy rule.

The economic fluctuations are driven by seven orthogonal exogenous shocks: total factor and investment-specific productivity shocks, a preference shock, wage and price mark-up shocks, and government expenditure and monetary policy shocks.

The innovation of our application is the introduction of dispersed information (DI) to this environment. Specifically, households and firms do not observe perfectly the shocks hitting the economy. Instead, they receive noisy idiosyncratic signals about them.

Timing

Time is discrete and each period contains two stages. In the first stage, shocks and signals are realized, intermediate goods firms choose optimal prices, and households choose consumption, investment, installed capital, and its utilization level, and set optimal wages, based on information from their signals. In the second stage, rental rates and wages of differentiated labor are uncovered. Competitive final good firms buy intermediate goods to sell the final good to households. Competitive labor packers use the supply of differentiated labor from households and sell a homogeneous labor bundle to intermediate firms. Intermediate firms rent capital and hire labor to produce the intermediate goods.

This timing protocol is standard in the literature[3] and ensures two features. First, all markets clear. Competitive final good firms ensure that the supply of final good matches consumption and investment demands. Given prices of intermediate goods, differentiated wages, and rental rate chosen at stage 1, firms allocate capital and labor to accommodate the demand from final good firms. Finally, given the wages set at the first stage, labor packers aggregate the differentiated labor services from households and supply homogeneous labor to intermediate firms such that the

[3] See for instance Nimark (2008), Angeletos and La'O (2011).

labor market clears.

Second, intermediate good firms do not use information from the production process to extract information from aggregate variables. This implies that both intermediate firms and households use only information from their signals to form expectations.

Final good firms

The homogeneous final good Y_t is a bundle of intermediate goods, $Y_{i,t}$, where the index $i \in [0,1]$ denotes the continuum of intermediate firms. The production function is given by

$$Y_t = \left(\int_0^1 (Y_{i,t})^{\frac{1}{1+\mu_t^p}} \, di \right)^{1+\mu_t^p}, \tag{1}$$

where μ_t^p is the time-varying price mark-up of intermediate goods following the process $log(1+\mu_t^p) = log(1+\bar{\mu}^p) + x_t^p$, and $\bar{\mu}^p$ is the steady-state value of the price mark-up. The price mark-up shock x_t^p follows

$$x_t^p = \rho_p x_{t-1}^p + \varepsilon_t^p, \quad \varepsilon_t^p \sim \mathcal{N}(0, \sigma_p^2). \tag{2}$$

Final goods firms operate in a perfectly competitive market and sell the final goods to households for a price P_t. They solve a usual profit maximization problem

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{3}$$

subject to (1). From the first-order conditions and the zero-profit condition, we derive the demand for intermediate production

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{1+\mu_t^p}{\mu_t^p}} Y_t, \tag{4}$$

where $P_t = \left(\int_0^1 (P_{i,t})^{-\frac{1}{\mu_t^p}} \, di \right)^{-\mu_t^p}$ is the price level.

Intermediate good firms

Each intermediate producer is a monopolistic competitive firm $i \in [0,1]$ that uses labor and capital to produce their goods. Good i is produce using technology

$$Y_{i,t} = e^{x_t^a} (K_{i,t})^{\alpha} (\gamma^t L_{i,t})^{1-\alpha} - \gamma^t \Phi_p, \tag{5}$$

where γ represents the labor-augmenting deterministic growth rate of productivity and x_t^a is a total factor productivity shock. $K_{i,t}$ and $L_{i,t}$ denote the amount of capital and labor demanded by firm i at period t respectively, and Φ_p is a fixed cost included in the production function.[4] The productivity shock follows

$$x_t^a = \rho_a x_{t-1}^a + \varepsilon_t^a, \quad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2). \tag{6}$$

Intermediate firms are subject to a Calvo (1983)-like pricing friction: in every period, only a fraction $1 - \xi_p$ of firms can adjust prices. Firms not allowed to reset prices apply the following indexation rule

$$P_{i,t} = (\Pi_{t-1})^{\iota_p} \left(\bar{\Pi}\right)^{1-\iota_p} P_{i,t-1}, \tag{7}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is gross inflation and $\bar{\Pi}$ is its steady-state value. Firms that are able to reoptimize prices choose the level $P_{i,t}^*$ that maximizes their expected discounted flow of profits given by

$$E_{it} \sum_{s=0}^{\infty} (\beta\xi_p)^s \Lambda_{t,t+s} \left[\left(X_{t,t+s} P_{i,t}^* - MC_{i,t+s} \right) Y_{i,t+s} \right], \tag{8}$$

subject to $Y_{i,t+s} = \left(\frac{X_{t,t+s} P_{i,t}^*}{P_{t+s}} \right)^{-\frac{1+\mu_{t+s}^p}{\mu_{t+s}^p}} Y_{t+s}$. $MC_{i,t}$ is the marginal cost of firm i at period t. $\Lambda_{t,t+s}$ is the household's stochastic discount factor between periods t and $t+s$ and $X_{t,t+s}$ is the indexation between the same periods. Consistently with the indexation rule (7), $X_{t,t+s}$ is given by

$$X_{t,t+s} = \begin{cases} \bar{\Pi}^{(1-\iota_p)s} \prod_{j=1}^{s} \left(\Pi_{t+j-1}^{\iota_p} \right) & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases} \tag{9}$$

Given the optimal prices and the indexation rule, the price level has the following law of motion

$$P_t = \left[\xi_p \left(\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} P_{t-1} \right)^{-\frac{1}{\mu_t^p}} + (1-\xi_p) \left(\int_0^1 P_{i,t}^* di \right)^{-\frac{1}{\mu_t^p}} \right]^{-\mu_t^p}. \tag{10}$$

Labor packers

There is a continuum of households indexed by $h \in [0, 1]$, each supplying differentiated labor services. Following Erceg et al. (2000), there are labor packers that hire labor from the households and aggregate them according to

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{1}{1+\mu_t^w}} dh \right)^{1+\mu_t^w}, \tag{11}$$

[4] Scaling the fixed cost by γ^t avoids it to decrease over time in terms of output.

where μ_t^w denote the agency's wage mark-up such that $log(1+\mu_t^w) = log(1+\bar{\mu}^w) + x_t^w$, and $\bar{\mu}^w$ is the steady-state value of the wage mark-up. The shock follows the process

$$x_t^w = \rho_w x_{t-1}^w + \varepsilon_t^w, \quad \varepsilon_t^w \sim \mathcal{N}(0, \sigma_w^2). \tag{12}$$

Labor packers pay the wage $W_{h,t}$ for each household h, and sell a homogeneous labor service to intermediate firms at a cost W_t. Agents maximize profits

$$W_t L_t - \int_0^1 W_{h,t} L_{h,t} dh \tag{13}$$

subject to (11). Thus, the labor demand for each household h's labor service is given by

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{1+\mu_t^w}{\mu_t^w}} L_t, \tag{14}$$

where $W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh\right)^{-\mu_t^w}$ is the nominal wage index.

Households

Households $h \in [0,1]$ derive utility from consumption and leisure. In order to maximize their expected utility, they choose consumption ($C_{h,t}$), holdings of government bonds ($B_{h,t}$), installed capital level ($K_{h,t}$) and its utilization rate ($U_{h,t}$). The capital rented to firms, K_t^u, is determined by the installed capital and the utilization rate. The objective function that each household h optimizes is given by

$$U = E_{ht}\left[\sum_{s=0}^{\infty} \beta^s e^{x_{t+s}^c} \left(ln\left(C_{h,t+s} - \varphi C_{h,t+s-1}\right) - \frac{L_{h,t+s}^{1+\chi}}{1+\chi}\right)\right], \tag{15}$$

where $C_{h,t}$ is consumption and $L_{h,t}$ denotes the supply of differentiated labor services of household h at period t. Households' preferences display external habit persistence, captured by the parameter φ, while χ is the inverse of the Frisch elasticity of labor supply. $E_{ht}[\cdot]$ is the expectation operator conditional on households h's information set, and x_t^c is a preference shock that follows

$$x_t^c = \rho_c x_{t-1}^c + \varepsilon_t^c, \quad \varepsilon_t^c \sim \mathcal{N}(0, \sigma_c^2). \tag{16}$$

The capital stock $K_{h,t}$ owned by household h evolves according to

$$K_{h,t} = (1-\delta) K_{h,t-1} + e^{x_t^i}\left(1 - S(I_{h,t}/I_{h,t-1})\right) I_{h,t}, \tag{17}$$

where $S(I_t/I_{t-1})$ is the adjustment investment cost function that denotes the share of investment which does not become new capital. As in Christiano et al. (2005), the cost function $S(\cdot)$ has the following properties: $S(\gamma) = S'(\gamma) = 0$, and $S''(\gamma) = s'' > 0$.

The investment-specific technological shock x_t^i follows

$$x_t^i = \rho_i x_{t-1}^i + \varepsilon_t^i, \quad \varepsilon_t^i \sim \mathcal{N}(0, \sigma_i^2) \tag{18}$$

Households rent to firms an effective amount of capital $K_{h,t}^u$ given by

$$K_{h,t}^u = U_{h,t} K_{h,t-1}, \tag{19}$$

where $U_{h,t}$ is the level of capital utilization. They receive $R^k K_{h,t}^u$ for renting capital but pay a cost $a(U_{h,t})K_{h,t-1}$ in terms of the consumption good. Following Christiano et al. (2005), this function has the properties $a(\bar{U}) = 0$ and $a''(\bar{U}) = a''$, where $\bar{U}$ is the value of the capital utilization rate in the steady-state.

The households' budget constraint is given by

$$\begin{aligned} &P_t C_{h,t} + P_t I_{h,t} + B_{h,t} + P_t a(U_{h,t}) K_{h,t-1} + Q_{t+1,t} A_{h,t} \leq \\ &R_{t-1} B_{h,t-1} + W_{h,t} L_{h,t} + R_t^k U_{h,t} K_{h,t-1} + P_t A_{h,t-1} + T_{h,t}, \end{aligned} \tag{20}$$

in each period. $T_{h,t}$ denote net transfers from the government, $A_{h,t}$ is a vector of one-period state-contingent securities and $Q_{t+1,t}$ is the price of such asset.[5]

Households supply labor in a market with monopolistic competition subject to a wage-setting friction. Following Erceg et al. (2000), in every period only a fraction $1 - \xi_w$ of households are able to optimize their wages. Those households choose their optimal wage by setting a mark-up over the marginal rate of substitution between consumption and labor. Households who are not able to optimize update their wage using a indexation rule given by

$$W_{h,t} = (\Pi_{t-1})^{\iota_w} \left(\bar{\Pi}\right)^{1-\iota_w} \gamma W_{h,t-1}, \tag{21}$$

which is a geometrically weighted average of steady-state wage growth ($\gamma\bar{\Pi}$) and last period wage growth ($\gamma\Pi_{t-1}$).

[5] The assumption of a complete set of state-contingent securities guarantees that all households h make the same consumption and saving choices. This is true despite the fact that they have differentiated wages and form expectations based on idiosyncratic signals. This assumption is for tractability such that standard techniques of log-linearization to solve DSGE models can be applied even with informational frictions in the households' decisions.

Each household minimizes their expected discounted labor disutility

$$E_{ht}\left[\sum_{s=0}^{\infty}(\beta\xi_w)^s\left(-\frac{L_{j,t+s}^{1+\chi}}{1+\chi}\right)\right] \tag{22}$$

subject to the budget constraint (20) at all periods $s \in [0,\infty)$ and to the period $t+s$ nominal wage $W_{h,t+s} = X_{t,t+s}^w W_{h,t}^*$ and

$$X_{t,t+s}^w = \begin{cases} (\gamma\bar{\Pi}^{1-\iota_w})^s \prod_{j=1}^s \left(\Pi_{t+j-1}^{\iota_w}\right) & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases} \tag{23}$$

Given optimal prices and the indexation rule, the aggregate wage level has the following law of motion

$$W_t = \left[\xi_w\left(\gamma\Pi_{t-1}^{\iota_w}\bar{\Pi}^{1-\iota_w}W_{t-1}\right)^{-\frac{1}{\mu_t^w}} + (1-\xi_w)\left(\int_0^1 W_{h,t}^* dh\right)^{-\frac{1}{\mu_t^w}}\right]^{-\mu_t^w}. \tag{24}$$

Government Policies

The central bank sets the nominal interest rate according to a standard Taylor rule with smoothing according to

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\phi_R}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{(1-\phi_R)} e^{x_t^r}, \tag{25}$$

where variables with a bar denote the steady-state values and x_t^r is the monetary policy shock, which follows

$$x_t^r = \rho_r x_{t-1}^r + \varepsilon_t^r, \quad \varepsilon_t^r \sim \mathcal{N}(0,\sigma_r^2). \tag{26}$$

For simplicity, I assume that the monetary authority responds to deviations of output to its steady-state value instead of the natural output.

The government budget constraint is the following

$$P_t G_t + R_{t-1}B_{t-1} = B_t + T_t \tag{27}$$

where T_t and B_t are total lump-sum taxes and bonds, respectively. Government expenditure follows a simple stochastic process given by $G/Y = g_y + x_t^g$, where $g_y \equiv \bar{G}/\bar{Y}$ is the steady-state ratio of government expenditure to output and x_t^g is a government expenditure shock with process

$$x_t^g = \rho_g x_{t-1}^g + \varepsilon_t^g, \quad \varepsilon_t^g \sim \mathcal{N}(0,\sigma_g^2). \tag{28}$$

Resource constraint and market clearing

The aggregate resource constraint

$$C_t + I_t + G_t + a(U_t)K_t = Y_t, \tag{29}$$

is derived by integrating households' budget constraint over h, and combining it with the zero profit condition of final goods firms and labor packers and the government budget constraint.

Market clearing in labor and capital markets holds

$$K_t^u = \int_0^1 K_{i,t} di, \quad L_t = \int_0^1 L_{i,t} di.$$

The bond supply and the transfers

$$B_t = \int_0^1 B_{h,t} dh, \quad T_t = \int_0^1 T_{h,t} dh,$$

are consistent with the government spending rule and the public budget constraint (27).

Information and signal extraction

Intermediate good firms and households do not observe perfectly the structural shocks driving the economy, but instead receive noisy idiosyncratic signals about them. Formally, the signal follows the process

$$s_{j,t}^l = x_t^l + v_{j,t}^l, \quad v_{j,t}^l \sim \mathcal{N}(0, \tau_l^2), \tag{30}$$

where $l \in \{a, c, i, g, p, w, r\}$ denote each type of shock and $j \in [0, 1]$ is a index that pools both intermediate good firms i and households h. This assumption implies that firms and household receive signals with the same properties and are subject to the same degree of informational frictions. Hence, an agent j's information set is described as

$$\mathcal{I}_t^j = \{s_{j,\tau}^a, s_{j,\tau}^c, s_{j,\tau}^i, s_{j,\tau}^g, s_{j,\tau}^p, s_{j,\tau}^w, s_{j,\tau}^r : \tau \leq t\}. \tag{31}$$

Given that households and firms have the same unit mass, their average expectation is the same and denoted by $\bar{E}_t[\cdot] \equiv \int_0^1 E_{j,t}[\cdot] dj$, where $E_{j,t}[\cdot] \equiv E[\cdot | \mathcal{I}_t^j]$ denotes the j's individual expectation. Given the AR(1) structure of the shocks and signal structure (30), j's rational expectation about the shock x_t^l is computed by using the Kalman filter such that

$$E_{j,t}[x_t^l] = E_{j,t-1}[x_t^l] + \bar{k}_l \left[s_{j,t}^l - E_{j,t-1}[s_{j,t}^l] \right] \tag{32}$$

where denotes $\bar{k}_l$ the steady-state Kalman gain of shock l given by

$$\bar{k}_l = \frac{\tilde{p}_l}{\tilde{p}_l + r_l^2} \tag{33}$$

where $r_l \equiv \frac{\tau_l}{\sigma_l}$ denotes the noise-to-signal ratio of signal l and $\bar{p}_l$ can be computed by the Riccati equation

$$\tilde{p}_l^2 - \left[1 - (1 - \rho_l^2) r_l\right] \tilde{p}_l - r_l^2 = 0.$$

Note that $\tilde{p}_l$ is defined as the ratio of the individual expectation steady-state variance and the shock variance, $\tilde{p}_l = \frac{\bar{p}_l}{\sigma_l^2}$.Thus, the Kalman gain is a function of the shock's parameters, (ρ_l, σ_l), and the signal's noise (τ_l), $\bar{k}_l = k(\rho_l, \sigma_l, \tau_l)$.

The Kalman gain is the key parameter that measures the degree of informational friction, *conditional on the shock's parameters*. In other words, conditional on (ρ_l, σ_l) there is a one-to-one relationship between $\bar{k}_l$ and the signal's noise ratio τ_l. We get back on this when discussing priors choices.

Moreover, the average expectation is given by

$$\bar{E}[x_t^l] = (1 - \bar{k}_l)\bar{E}_{t-1}[x_t^l] + \bar{k}_l x_t^l = (1 - \bar{k}_l)\rho_l \bar{E}_{t-1}[x_{t-1}^l] + \bar{k}_l x_t^l. \tag{34}$$

Thus, the current expectation is a weighted average between the past expectation and the actual shock, whose weight is given by the Kalman gain.

If $\bar{k}_l = 1$ $(\tau_l \to \infty)$ the model boils down to the full information benchmark. If $\bar{k}_l = 0$ $(\tau_l \to 0)$, the average expectation is zero (equals to the unconditional expectation).[6]

Equation (34) helps to understand why and when information frictions can generate inertia. Strong informational frictions (low $\bar{k}_l$) has two implications for average expectations: it dampens the response to the actual shock and increases the expectation persistence. The latter also depends on the persistence of the unobserved shock, ρ_l.

In section 4, the model estimation implies that some shocks have a combination of high persistence and high informational frictions, which generates dampening and inertia. However, many shocks combine low persistence with high informational frictions. This cannot generate inertia while preserving a weakened response to shocks. This distinction is key to understanding whether

[6] Zero is the solution for the equation $\bar{E}[x_t^l] = \rho_l \bar{E}_{t-1}[x_{t-1}^l]$.

informational frictions substitutes or complement standard frictions discussed in section 5.4.

2.1 Detrending and log-linearized model

Before showing the system of log-linearized equations that characterizes the model, we discuss the detrending procedure.

All real variables grow along with the productivity trend, so they are detrended as follows: $\hat{Z}_t = \frac{Z_t}{\gamma^t}$, for any real variable Z_t. Nominal variables grow along with the price level P_t, hence they are stationarized using the following procedure: $\hat{Z}_t = \frac{Z_t}{P_t}$.[7]

The optimal prices and wages are detrended by dividing them to the price level of last period: $\hat{P}^*_{i,t} = P^*_{i,t}/P_{t-1}$, while $\hat{W}^*_{h,t} = \hat{W}^*_{h,t}/\gamma^t P_{t-1}$.[8]

Stationary variables are transformed by taking their log-deviation to steady-state value as follows: $z_t = log(Z_t/\bar{Z})$, for any stationary variable Z_t. Thus, lower case variables denote log-deviation from steady-state of the upper case variables.

For brevity, all log-linearized equations are provided in equation (65) in the Appendix A.2. The derivation of the log-linearized equations above can be found in the Online Appendix.

Here we emphasize two key equations to clarify how dispersed information affects the equilibrium conditions.

Consider the Euler equation given by

$$c_t = \frac{\varphi/\gamma}{1+\varphi/\gamma}c_{t-1} + \frac{1}{1+\varphi/\gamma}\bar{E}_t[c_{t+1}] - \frac{(1-\varphi/\gamma)}{1+\varphi/\gamma}\bar{E}_t[r_t - \pi_{t+1} - (x^c_{t+1} - x^c_t)]$$

This is the same Euler equation of Smets and Wouters (2003) with two key differences due to the presence of informational frictions.

First, expectations are heterogeneous, and hence average expectation replaces the full information expectation operator. Second, current endogenous variables and structural shocks are not observed. Thus, their expectations arise in optimal choices. In this case, households must form expectations about the current nominal rate, r_t, and the current preference shock, x^c_t.

Now consider the New Keynesian Phillips (NKPC) curve given by

$$\pi_t = \xi_p \iota_p \pi_{t-1} + \kappa_p \xi_p \bar{E}_t\left[mc_t + x^p_t\right] + \psi_p \bar{E}_t\left[\pi_t\right] + (1-\xi_p)\beta\xi_p \int_0^1 E_{it}[p^*_{i,t+1}]di \qquad (35)$$

where $\psi_p \equiv (1-\xi_p)(1-\iota_p\beta\xi_p)$.

[7]One exception is the Lagrange multiplier of the budget constraint, Λ_t, that must be normalized to $\hat{\Lambda}_t = \Lambda_t\gamma^t P_t$, since it reflects the marginal utility of an additional unit of money, which declines as consumption grows at rate γ and the price level rises.

[8]This simplifies the derivation as $E_{h,t}[\hat{W}^*_{h,t}] = \hat{W}^*_{h,t}$ and $E_{i,t}[\hat{P}^*_{i,t}] = \hat{P}^*_{i,t}$ since observe P_{t-1}. This would not be true if we detrended them using P_t.

An additional difference appears in the NKPC as the average expectation of firms' future *own optimal prices* matters for inflation. Note that $\int_0^1 E_{it}[p^*_{i,t+1}]di \neq \bar{E}_t\left[p^*_{t+1}\right]$, since the law of iterated expectations does not hold for average expectations (Morris and Shin; 2005). Thus, we cannot write the last term in terms of the average expected future *aggregate optimal price*, which is related directly to future inflation.

As pointed out by Nimark (2008), as inflation depends on its own average expectation, higher-order expectations matter for inflation dynamics. By taking the average expectations of this equation and substituting it iteratively, we obtain that

$$\pi_t = \frac{\xi_p \iota_p}{1-\psi_p}\pi_{t-1} + \kappa_p \xi_p \sum_{k=1}^{\infty} \psi_p^{k-1} \bar{E}_t^{(k)}[mc_t + x_t^p] + (1-\xi_p)\beta\xi_p \sum_{k=0}^{\infty} \psi_p^k \bar{E}_t^{(k)}\left[\int_0^1 E_{it}[p^*_{i,t+1}]di\right], \quad (36)$$

where $\bar{E}_s^{(k)}[z_t] \equiv \int_0^1 E_{is}\left[\bar{E}_s^{(k-1)}[z_t]\right]di$, for any variable z_t, all $k \geq 1$ and $s \leq t$, with the convention that $\bar{E}_t^{(0)}[z_t] \equiv z_t$ and $\bar{E}_t^{(1)}[z_t] \equiv \bar{E}_t[z_t]$.

Firms must form not only beliefs about marginal cost, price mark-up shock, and future own price but also higher-order expectations about these variables. $\psi_p \in (0,1)$ is a key parameter that determines how strong the dependence of inflation on higher-order expectations is. One key feature is that inflation dependence on higher-order expectations decreases with the order.

The latter differs from Nimark's (2008) NKPC in two key aspects. First, our model includes partial indexation to past prices, which generates a backward term. Interestingly, ψ_p is decreasing in the indexation parameter, ι_p, i.e., indexation decreases the relevance of higher-order expectations. The intuition is the following. As non-optimizing firms index their prices to past inflation, optimal prices are less responsive to expectations about inflation. Thus, inflation is less dependent on higher-order expectations.

Second, it explicitly considers that $\int_0^1 E_{it}[p^*_{i,t+1}]di \neq \bar{E}_t\left[p^*_{t+1}\right]$, which prevents representing inflation as a function of the future inflation as in the full-information benchmark.[9]

One key feature of the solution method discussed in the next section is that we do not need to represent equilibrium conditions in terms of higher-order expectations as in equation (36) to solve the model. We can find a policy function that depends on higher-order expectations even defining equilibrium conditions in the system of equation in terms of average expectations as in equation (35).

[9]As pointed out in Appendix D of Angeletos and Huo (2021), Nimark (2008) and Melosi (2017) abstract in their derivation that $\int_0^1 E_{it}[p^*_{i,t+1}]di \neq \bar{E}_t\left[p^*_{t+1}\right]$. While this may not be quantitatively relevant, it is important for correctly defining the general system of equilibrium conditions (37) and the guessed policy function (40).

3 Solution method

In this section, we present a novel solution method for DSGE models featuring imperfect and dispersed information.

The log-linearized equilibrium equations (65) can be written as the following general system of linear rational equations

$$\begin{aligned} F_1 \bar{E}_t[Y_{t+1}] + F_2 \int_0^1 E_{it}[Y_{i,t+1}]di + G_1 Y_t + G_2 \bar{E}_t[Y_t] + HY_{t-1} + \\ L\bar{E}_t[x_{t+1}] + M_1 x_t + M_2 \bar{E}_t[x_t] = 0_{m\times 1}, \end{aligned} \tag{37}$$

where $i \in [0,1]$ indexes an agent, $Y_{i,t}$ is a $m \times 1$ vector of individual choices, Y_t is a $m \times 1$ vector of aggregate endogenous variables and x_t is a $n \times 1$ vector of unobservable exogenous shocks.

Structural shocks x_t follow a stationary stochastic process

$$x_t = A_1 x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon) \tag{38}$$

where A_1 is a diagonal matrix storing the persistence of each shock, and Σ_ε is a diagonal covariance matrix. Idiosyncratic signals are given by

$$s_{i,t} = C_x x_t + D v_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0, \Sigma_v), \tag{39}$$

where Σ_v is a diagonal covariance matrix.[10]

Define the k-th order average expectation $E_t^{(k)}[\cdot]$ given information of period t about the vector of unobservable aggregate shocks x_t as $E_s^{(k)}[x_t] \equiv \int_0^1 E_{is}\left[E_s^{(k-1)}[x_t]\right] di$, for all $k \geq 1$ and s, t, with the convention that $E_t^{(0)}[x_t] \equiv x_t$ and $E_t^{(1)}[x_t] \equiv \bar{E}_t[x_t]$.

Following Nimark (2008), it is useful to denote the expectations hierarchy of x_t from order l to s as the vector

$$x_t^{(l:s)} \equiv \begin{bmatrix} E_t^{(l)}[x_t]' & E_t^{(l+1)}[x_t]' & \cdots & E_t^{(s)}[x_t]' \end{bmatrix}',$$

for $s > l \geq 0$.

Following the insight of Nimark (2008), the solution method relies on a truncation of the state space to circumvent the problem of the infinite regression of expectations (see Townsend; 1983).[11] Specifically, we guess that the individual and aggregate equilibrium law of motion and expectation

[10] The model from Section 2 implies a diagonal structure for A_1, Σ_ε and Σ_v. The solution method does not require these assumptions.

[11] Nimark (2017) shows that as long as the impact on equilibrium outcomes of higher-order expectations decreases with the order, there exists a $\bar{k}$ such that the approximation error of the solution is less than any $\epsilon > 0$.

hierarchy are given by

$$Y_{i,t} = \mathbf{R}Y_{i,t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_{i,t}\left[x_t^{(0:\bar{k})}\right], \tag{40}$$

$$Y_t = \mathbf{R}Y_{t-1} + \mathbf{Q}x_t^{(0:\bar{k})} \tag{41}$$

$$x_t^{(0:\bar{k})} = \mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{B}\varepsilon_t, \tag{42}$$

where $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1}, \mathbf{A}, \mathbf{B})$ are finite dimensional matrices to be determined in equilibrium. $\mathbf{Q} = \mathbf{Q_0}e_x + \mathbf{Q_1}T$, e_x is the selection matrix such that $x_t = e_x x_t^{(0:\bar{k})}$ and T is a order transformation matrix such that $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = Tx_t^{(0:\bar{k})}$[12].

The system of equations (37) generalizes the existing methods by allowing: i) average expectation about future own variables (the term post multiplying F_2) and ii) endogenous state variables (the term post multiplying H). The latter allows solving medium-scale DSGE models such as the one in section 2.

Proposition 1 shows the dynamics for expectation hierarchy and the expressions for the fixed-point solution for $(\mathbf{A}, \mathbf{B})$.

Proposition 1. *Suppose that x_t is a stationary process, the expectation hierarchy follow equation (42) and agents use information from signals (39). Assuming common knowledge of rationality, the individual and average expectations about the hierarchy are given by*

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \mathbf{K}C\right)\mathbf{A}E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \mathbf{K}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{K}C\mathbf{B}\varepsilon_t + \mathbf{K}Dv_{it}$$
$$\bar{E}_t\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}\bar{E}_{t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t.$$

where $C = C_x e_x$.

The expectation hierarchy consistent with the expectations above has coefficients $(\mathbf{A}, \mathbf{B})$ that satisfy the matrix equations:

$$\begin{gathered}\left(I_k - T'\bar{\mathbf{K}}C\right)\mathbf{A} = e_x' A_1 e_x + T'\left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}T \\ \left(I_k - T'\bar{\mathbf{K}}C\right)\mathbf{B} = e_x'\end{gathered} \tag{43}$$

such that $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ solve the Riccati equation resulting from equations

$$\begin{gathered}\bar{\mathbf{K}} = \bar{\mathbf{P}}C'\left[C\bar{\mathbf{P}}C' + D\Sigma_v D'\right]^{-1} \\ \bar{\mathbf{P}} = \mathbf{A}\left[\bar{\mathbf{P}} - \bar{\mathbf{K}}C\bar{\mathbf{P}}\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon\mathbf{B}'\end{gathered} \tag{44}$$

[12]See details in Appendix C.

The Ricatti equation resulting from equations (44) solves the steady-state mean square error of the expectation hierarchy. The equations uses the standard Kalman Filter with hierarchy dynamics (42) as state equation and measurement equation as signals (39).

The key difference is that $(\mathbf{A}, \mathbf{B})$ is a result from agents signal extraction. Thus, the hierarchy persistence $(\mathbf{A})$ and response to shocks $(\mathbf{B})$ depend on the Kalman gain matrix $\bar{\mathbf{K}}$. As any signal extraction, the $\bar{\mathbf{K}}$ depends on the unobserved state dynamics $(\mathbf{A}, \mathbf{B})$.

Therefore, the equilibrium solution $(\mathbf{A}, \mathbf{B}, \mathbf{K}, \mathbf{P})$ is the fixed point from equations (43-44).

Since the signals (39) are exogenous, they do not depend on equilibrium conditions (37). This simplifies finding the guessed coefficients for the equilibrium $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1})$ and tremendously diminishes the computational costs.[13]

Before presenting the equilibrium dynamics in Proposition below, we want to emphasize one assumption that simplifies substantially this task. We assume that when forming expectations about Y_t using the guessed solutions (40-41), agents know the past value of endogenous variables, Y_{t-1}.

This assumption contrasts with the expectation formation from Proposition 1 in which expectations depend only on exogenous signals. This avoid learning from (past) endogenous variables, which generates a feedback between the signal extraction and endogenous variable responses to the hierarchy.[14] Ribeiro (2017) considers the general case with endogenous signals.

This assumption is implicit in standard solution methods for imperfect under common information such as Blanchard et al. (2013) and Baxter et al. (2011). This is also a common assumption when studying the New Keynesian Phillips Curve under dispersed information (e.g., Angeletos and La'O; 2009; Angeletos and Huo; 2021).

Proposition 2. *For a given dynamics expectation hierarchy from equation (42), the system of equations (37) has a recursive equilibrium law of motion (41) whose matrix coefficients satisfy:*

1. $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1})$ *satisfy the matrix equations:*

$$F\mathbf{R}^2 + G\mathbf{R} + H = 0_{m\times m} \tag{45}$$

$$[F_1\mathbf{R} + G_1]\,\mathbf{Q_0} + M_1 = 0_{m\times n} \tag{46}$$

$$\begin{aligned}&[F_1\mathbf{R} + G_1]\,\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A} +\\ &[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)]\,e_x = 0_{m\times k}\end{aligned} \tag{47}$$

[13] For instance, Melosi (2017) explains in his replication files that: "Be aware that estimating the DIM may take several weeks or even a few months depending on the computer used for this task." In our application, which has a considerable higher scale model, takes rougly 72 (36) hours for 400k posterior draws using the RB-RWHM (RWMH) algorithm in a standard notebook.

[14] One can think this assumption as a deviation from rationality as agents do not incorporate that their knowledge about Y_{t-1} can be used to learn about $x_{t-1}^{(0,\bar{k})}$, which would also change their beliefs about $x_t^{(0,\bar{k})}$.

where $F \equiv F_1 + F_2$*,* $G \equiv G_1 + G_2$ *and* $k = n(\bar{k}+1)$*.*

2. *(Uhlig; 2001)* $\mathbf{R}$ *has a unique stable solution if all eigenvalues of* $\mathbf{R}$ *are smaller than unity in absolute value.*

3. *Given the solution of* $\mathbf{R}$*, denote the matrices* (V_0, V_1) *such that:*

$$V_0 = F_1\mathbf{R} + G_1 \tag{48}$$

$$V_1 = I_k \otimes (F_1\mathbf{R} + G_1) + T' \otimes (F_2\mathbf{R} + G_2) + \mathbf{A}' \otimes F_1 + T'\mathbf{A}' \otimes F_2 \tag{49}$$

Provided that there exists a inverse for V_0 *and* V_1*, the equilibrium solution for* $(\mathbf{Q_0}, \mathbf{Q_1})$ *is given by*

$$\mathbf{Q_0} = -V_0^{-1} M_1 \tag{50}$$

$$vec(\mathbf{Q_1}) = -V_1^{-1} vec\left(\left[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)\right] e_x\right) \tag{51}$$

where $vec(\cdot)$ *denotes columnwise vectorization.*

Given the individual response to shocks $(\mathbf{Q_0})$ *and to the individual expectation about the hierarchy* $(\mathbf{Q_1})$*, the aggregate response to the hierarchy is given by:* $\mathbf{Q} = \mathbf{Q_0}e_x + \mathbf{Q_1}T$

Proposition 2 connects the solution of imperfect common knowledge models with the standard undetermined coefficients solution for full information DSGE models.

Condition (45) is exactly the same as the usual "brute force" approach of Uhlig (2001). In other words, he equilibrium $\mathbf{R}$ is the same that would happen if agents had full information, as shown in Appendix C.[15]

This has two key implications. First, $\mathbf{R}$ can be computed with standard techniques. Second, informational frictions does not affect how endogenous variables respond to state variables. This does not imply that information frictions are not important their persistence. Endogenous variables persistence also reflects the hierarchy persistence.

Our solution method is a natural extension of Uhlig's (2001) full information and Blanchard et al.'s (2013) imperfect information methods. It is also related to the dispersed information method from Melosi (2017). The key difference is that his method abstracts from state endogenous variables but allows for endogenous signals.

Appendix C further explore the relation with those methods and show the details of the algorithm to find the fixed point solution from Proposition 1.

[15] The key assumption for this result is that agents observe past endogenous variables at period t.

4 Estimation

In this section, we discuss the data, the prior choices, and the posterior results of the Bayesian estimation. We estimate the model featuring dispersed information ('DI model') using two different datasets: one containing macroeconomic data only and other including both macroeconomic and expectation data. For comparison, we also estimate the same model under full-information ('FI model') where we abstract from information frictions in the same datasets.

4.1 Data

We collect U.S. quarterly macroeconomic and expectation data from 4Q:1981 to 4Q:2007. The data on macroeconomic aggregates include the series of real GDP, real consumption, real investment, GDP deflator index, annualized Fed funds rate, hours worked index and nominal wages index, and come from the Federal Reserve Database (FRED).

We also gather the average expectation about one-quarter ahead real GDP, real consumption and real investment, inflation and three-month Treasure bill rate from the US Survey of Professional Forecasters (SPF). A detailed description of the datasets can be found in Appendix B.

When estimating medium scale DSGE models, the inclusion of expectation data in the dataset is scarce. In recent years, few papers have tried to incorporate expectation data to discipline macroeconomic models with complete and incomplete information. For instance, Del Negro and Eusepi (2011) adds one-year ahead inflation expectations to estimate a conventional New Keynesian model. Melosi (2017) estimates a small scale dispersed information model using data on inflation forecasts one-quarter and one-year ahead. Del Negro et al. (2015) uses ten-year inflation expectations from the Blue Chip and SPF.

In this paper, we divert from previous literature in two ways. First, we use expectation data on a broader set of variables. Second, we use data on *current forecast revisions* (the nowcast minus last period forecast). The reasoning for the latter is the following.

By aggregating equation (32), one can see that

$$\overline{Rev}_{t|t-1}[x_t^l] \equiv \bar{E}_t[x_t^l] - \bar{E}_{t-1}[x_t^l] = \bar{k}_l \left[s_t^l - \bar{E}_{t-1}[s_t^l] \right] = \bar{k}_l \overline{Fe}_{t-1}[x_t^l] \tag{52}$$

where $s_t^l = \int_0^1 s_{it}^l di = x_t^l$ is the aggregate signal, which due to the simply signal structure (30) is equal to the shock l. $\overline{Rev}_{t|s}[x_t^l]$ denotes the average expectation revision about x_t^l at period t of a forecast made in period s and $\overline{Fe}_s[x_t^l] = x_t^l - \bar{E}_s[x_t^l]$ denotes the forecast error at period t made from the same forecast.

Note that the current forecast revision of shock l depends on the information frictions and signals only.

By iterating equation (34), one can see that forecast $\bar{E}_t[x_t^l]$ depends on the whole history of previous signals, the information frictions and shock's persistence.

Note that the current forecast revision of shock l depends on the information frictions and signals only. Longer forecast horizons ($\bar{E}_t[x_{t+h}^l]$) and revisions ($\bar{Rev}_{t|t-1}[x_{t+h}^l]$) are likely to be less informative as it depends also on the shocks persistence.[16]

The discussion above is true if we had data on the expectations about shocks. In practice, we have data on endogenous variables, which creates another layer of complexity.

Considering these issues, short-horizon forecast revisions are more appropriate for identifying the degree of informational friction, in our view.

4.2 Measurement equations

The measurement equations that link data with the variables for both models are the following

$$\begin{aligned}
dy_t^{obs} &= \bar{\gamma} + y_t - y_{t-1} \\
dc_t^{obs} &= \bar{\gamma} + c_t - c_{t-1} \\
di_t^{obs} &= \bar{\gamma} + i_t - i_{t-1} \\
dw_t^{obs} &= \bar{\gamma} + w_t - w_{t-1} \\
l_t^{obs} &= l_t \\
\pi_t^{obs} &= \bar{\pi} + 4\pi_t \\
r_t^{obs} &= \bar{r} + 4r_t \\
rev_{dy,t|t-1}^{obs} &= \bar{E}_t[\Delta y_t] - \bar{E}_{t-1}[\Delta y_t] + \varepsilon_{dy,t}^{me} \\
rev_{dc,t|t-1}^{obs} &= \bar{E}_t[\Delta c_t] - \bar{E}_{t-1}[\Delta c_t] + \varepsilon_{dc,t}^{me} \\
rev_{di,t|t-1}^{obs} &= \bar{E}_t[\Delta i_t] - \bar{E}_{t-1}[\Delta i_t] + \varepsilon_{di,t}^{me} \\
rev_{\pi,t|t-1}^{obs} &= 4\left(\bar{E}_t[\pi_t] - \bar{E}_{t-1}[\pi_t]\right) + \varepsilon_{\pi,t}^{me} \\
rev_{r,t|t-1}^{obs} &= 4\left(\bar{E}_t[r_t] - \bar{E}_{t-1}[r_t]\right) + \varepsilon_{r,t}^{me},
\end{aligned} \tag{53}$$

where $\bar{\gamma} \equiv 100(\gamma - 1)$ is the quarterly trend growth rate of productivity. The variables with superscript "*obs*" correspond to the variables in the database. The notation dx_t^{obs} denotes the quarterly growth rate of the variables $x \in \{y, c, i, w\}$ at period t. The data on hours worked (l_t^{obs}) is demeaned, while inflation (π_t^{obs}) and nominal interest rate (r_t^{obs}) are computed in annual terms. $\bar{\pi} \equiv 400(\bar{\Pi} - 1)$ and $\bar{r} \equiv 400(\bar{R} - 1)$ stand for the steady-state annualized inflation and nominal interest rates, respectively. The variables $rev_{\pi,t|t-1}^{obs}$, for $x \in \{dy, dc, di, \pi, r\}$, are the current forecast

[16] The h-step ahead forecast is given by $\bar{E}_t[x_{t+h}^l] = \rho_l^h \bar{E}_t[x_t^l]$ and analogous revision is $\overline{Rev}_{t|t-1}[x_{t+h}^l] = \rho_l^h \overline{Rev}_{t|t-1}[x_t^l]$. They also depend on shocks' persistence, ρ_l and the horizon h.

revision.

Measurement errors ε_x^{me} in the observational equations associated with forecast errors are introduced to avoid stochastic singularity.[17] They also capture the fact that the expectation data from SPF are not completely consistent with the average expectations of the agents populating our model. Measurement errors follow an i.i.d. Gaussian distribution with standard deviations denoted by σ_x^{me}, with $x \in \{dy, dc, di, \pi, r\}$.

4.3 Prior distributions

As standard in the literature, we fix the value of some parameters that are not easily identified. The quarterly depreciation rate δ is set to 2.5%. The ratio of government expenditures to output g_y is calibrated to its historical mean value of 18%. The steady-state wage markup $\bar{\mu}^w$ is fixed at 1.5 as in SW. Finally, the capital share of the production function α is fixed 0.3 to be consistent with the historical capital-output ratio.

The introduction of informational frictions is likely to affect the estimation of real and nominal frictions, so we do not rely completely on estimates from previous papers to choose priors. Instead, most of the priors are relatively loose. Table 1 shows the choices for priors on the left side. The priors are the same for the FI and DI models. By definition, the full information model does not include the informational friction parameters and estimates with macroeconomic data only do not include measurement errors for revision data.

The prior for parameters that have support between 0 and 1, such as the persistence of shocks, partial adjustment of the Taylor rule, indexation of prices and wages to past inflation and habit persistence in consumption, follow a beta distribution ($\mathcal{B}$) with a mean of 0.5 and standard deviation of 0.2. One important exception is the Calvo frictions on prices and wages. For those to parameters we use very tight prices around micro data estimates for prices from Nakamura and Steinsson (2008) and for wages from Grigsby et al. (2021). Those imply average duration of three quarters for prices and one year for wages.

The trend growth, inflation rate, and nominal interest rate steady-state parameters have priors with gamma distribution ($\mathcal{G}$) whose mean equals to their sample average. The standard deviations are 0.10 for the first prior and 0.50 for the others.

The priors on the adjustment cost of investment, capital utilization, and the inverse of Frisch elasticity are taken from Del Negro et al. (2007). The parameters specifying the Taylor rule have priors with gamma distribution whose mean values are $\phi_\pi = 1.5$ and $\phi_y = 0.2$ and the standard deviations are 0.5 and 0.1, respectively.

Priors for the standard deviation of the structural shocks are distributed as an inverse gamma

[17] This is required since the complete dataset includes 12 time series, while the model has only 7 shocks.

($\mathcal{IG}$) with means chosen to match standard deviations of observables in the pre-sample 1957Q1-1981Q1 as in Del Negro and Eusepi (2011). Standard deviations are set to 1.00, which implies fairly loose priors.

Priors for Kalman gains of each shock are set to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The value of 0.5 is similar to estimates from CG using one-year inflation expectations and Bordalo et al. (2020) for one-year data on output, consumption and investment growth rates, inflation, and nominal rates. Note that their estimates of frictions are using macroeconomic aggregates and not actual shocks. We discuss more on this in section 5.1.

We use the Kalman gain ($\bar{k}_l$) directly as a parameter instead of the noise-to-signal ratio ($r_l = \tau_l/\sigma_l$) as Del Negro and Eusepi (2011) or the private signal variances (τ_l) as Melosi (2017). As discussed by Del Negro and Eusepi (2011), forming priors on r_l of a shock implicitly forms a prior for the private signal's variance dependent on the shock's variance, σ_l. Analogously, when placing priors on the Kalman gains of each shock, it implicitly generates a prior for the NRS dependent on the shock's persistence, ρ_l (see equation 33).[18]

Therefore, despite the shock processes (ρ_l, σ_l) having different priors for each shock l, when assuming the same prior for the Kalman gains of all shocks, there is an implicit prior for $\tau_l|\rho_l, \sigma_l$ that allows the same information friction for all shocks a priori.

Finally, priors for the standard deviation of measurement errors on the time series of forecast errors are distributed as inverse gamma with a mean that matches roughly 10% of the total variance of each revision data. The standard deviations are small to ensure very tight priors, which avoids measurement errors to explain more than 25% of the variance in the revision data.

4.4 Posterior distributions

Bayesian techniques are applied to estimate the models, by combining the prior densities described before with the likelihood function computed using the Kalman filter. We use solve the model truncating the average higher-orders at the sixth order, i.e., $\bar{k} = 6$.[19] We use the Random Block Random-Walk Metropolis-Hastings (RB-RWMH) algorithm with five blocks to draw from the posterior distribution.[20] Table 1 reports the posterior estimates for both models (FI and DI models) for both datasets. For each model and dataset, the table shows the mode and the 5%-95%

[18] Del Negro and Eusepi (2011) has also a robustness check that they estimate the Kalman gain directly without taking into account its relationship with other parameters.

[19] Nimark (2017) shows that $\bar{k} = 6$ is a accurate approximation in a simple model. Which order is sufficiently high to provide good approximation to the policy function is model dependent. We find similar results $\bar{k} = 10$ and $\bar{k} = 15$ for complete dataset. Truncating at lower orders speedy-up substantially the estimation.

[20] Estimates from RB-RWMH have substantially better chain convergence diagnostics than standard RWMH as they generate chains with lower autocorrelation. We use 400k draws and a 50% burn-in. The candidate distribution is a multivariate normal with covariance matrix based on the estimate of the Hessian matrix at posterior mode as in An and Schorfheide (2007).

percentiles of the posterior density.

Table 1: Prior and Posterior distributions

Parameters	Prior			Dataset: Macroeconomic data only						Dataset: Macroeconomic and expectations data					
				Posterior: FI model			Posterior: DI model			Posterior: FI model			Posterior: DI model		
	Dist.	Mean	Std. Dev.	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters															
χ	$\mathcal{G}$	2.00	0.75	1.50	1.15	2.49	1.21	1.01	1.83	1.99	1.50	2.80	1.48	1.15	2.02
φ	$\mathcal{B}$	0.50	0.20	0.96	0.83	0.98	0.87	0.55	0.92	0.98	0.95	0.99	0.97	0.95	0.98
a''	$\mathcal{G}$	0.20	0.10	0.40	0.28	0.66	0.37	0.25	0.63	0.42	0.29	0.69	0.41	0.28	0.68
s''	$\mathcal{G}$	4.00	1.50	4.35	2.58	8.30	5.47	3.27	9.21	14.76	11.67	20.00	9.73	7.22	13.72
ϕ_y	$\mathcal{G}$	0.20	0.10	0.06	0.03	0.13	0.06	0.03	0.13	0.06	0.04	0.11	0.07	0.04	0.14
ϕ_π	$\mathcal{G}$	1.50	0.50	3.05	2.39	4.05	2.84	2.24	3.77	2.85	2.35	3.53	3.32	2.71	4.21
ϕ_r	$\mathcal{B}$	0.50	0.20	0.87	0.81	0.90	0.87	0.82	0.91	0.89	0.86	0.91	0.90	0.87	0.92
ι_p	$\mathcal{B}$	0.50	0.20	0.11	0.04	0.28	0.08	0.03	0.23	0.03	0.01	0.09	0.03	0.01	0.10
ι_w	$\mathcal{B}$	0.50	0.20	0.56	0.24	0.85	0.52	0.20	0.83	0.47	0.17	0.80	0.59	0.23	0.86
ξ_p	$\mathcal{B}$	0.67	0.05	0.83	0.80	0.80	0.80	0.75	0.83	0.85	0.83	0.87	0.85	0.82	0.88
ξ_w	$\mathcal{B}$	0.75	0.05	0.75	0.68	0.80	0.70	0.63	0.77	0.79	0.73	0.83	0.75	0.68	0.81
Steady-state parameters															
π_{ss}	$\mathcal{G}$	2.62	0.50	2.29	1.70	2.82	2.22	1.52	2.85	2.76	2.34	3.21	3.06	2.46	3.59
r_{ss}	$\mathcal{G}$	5.15	0.50	6.10	5.40	6.97	6.47	5.69	7.61	6.04	5.46	6.86	6.88	6.03	7.98
γ_{ss}	$\mathcal{G}$	0.50	0.10	0.33	0.28	0.37	0.33	0.27	0.36	0.35	0.32	0.37	0.32	0.29	0.36
Structural shocks parameters															
ρ_c	$\mathcal{B}$	0.50	0.20	0.33	0.18	0.70	0.77	0.19	0.83	0.24	0.16	0.34	0.13	0.07	0.21
ρ_i	$\mathcal{B}$	0.50	0.20	0.62	0.48	0.91	0.46	0.31	0.65	0.25	0.19	0.32	0.24	0.18	0.29
ρ_g	$\mathcal{B}$	0.50	0.20	0.98	0.96	0.99	0.99	0.98	1.00	0.97	0.96	0.98	0.99	0.98	1.00
ρ_p	$\mathcal{B}$	0.50	0.20	0.96	0.92	0.98	0.95	0.92	0.98	0.90	0.87	0.93	0.82	0.77	0.87
ρ_w	$\mathcal{B}$	0.50	0.20	0.22	0.11	0.39	0.22	0.13	0.34	0.24	0.12	0.38	0.32	0.22	0.42
ρ_r	$\mathcal{B}$	0.50	0.20	0.57	0.41	0.70	0.65	0.48	0.82	0.31	0.22	0.41	0.45	0.33	0.57
ρ_a	$\mathcal{B}$	0.50	0.20	0.92	0.86	0.97	0.93	0.86	0.97	0.88	0.84	0.92	0.91	0.86	0.96
σ_c	$\mathcal{IG}$	0.30	1.00	0.17	0.10	0.22	0.11	0.09	0.36	0.20	0.17	0.23	1.70	1.25	2.28
σ_i	$\mathcal{IG}$	1.50	1.00	0.90	0.74	1.13	1.18	0.98	1.92	1.27	1.11	1.44	4.70	3.93	5.57
σ_g	$\mathcal{IG}$	0.30	1.00	0.56	0.49	0.65	0.53	0.45	0.62	0.60	0.53	0.68	0.56	0.48	0.63
σ_p	$\mathcal{IG}$	0.10	1.00	0.05	0.04	0.06	0.06	0.05	0.09	0.04	0.04	0.05	0.17	0.12	0.23
σ_w	$\mathcal{IG}$	0.75	1.00	0.57	0.47	0.67	1.07	0.77	2.71	0.60	0.50	0.71	1.56	0.99	2.48
σ_r	$\mathcal{IG}$	0.10	1.00	0.13	0.11	0.14	0.12	0.11	0.14	0.13	0.11	0.14	0.13	0.11	0.15
σ_a	$\mathcal{IG}$	0.30	1.00	0.40	0.37	0.46	0.39	0.36	0.45	0.41	0.37	0.46	0.41	0.37	0.46
Information friction parameters[a]															
k_c	$\mathcal{B}$	0.50	0.20				0.92	0.73	0.97				0.26	0.20	0.32
k_i	$\mathcal{B}$	0.50	0.20				0.93	0.78	0.98				0.45	0.41	0.50
k_g	$\mathcal{B}$	0.50	0.20				0.58	0.44	0.75				0.57	0.53	0.61
k_p	$\mathcal{B}$	0.50	0.20				0.94	0.81	0.98				0.37	0.29	0.46
k_w	$\mathcal{B}$	0.50	0.20				0.44	0.23	0.72				0.41	0.25	0.57
k_r	$\mathcal{B}$	0.50	0.20				0.22	0.07	0.56				0.72	0.62	0.81
k_a	$\mathcal{B}$	0.50	0.20				0.64	0.32	0.88				0.33	0.08	0.57
Measurement errors standard deviation[b]															
σ_{dy}^{me}	$\mathcal{IG}$	0.10	0.02							0.46	0.41	0.53	0.28	0.25	0.32
σ_{dc}^{me}	$\mathcal{IG}$	0.10	0.02							0.42	0.38	0.47	0.21	0.19	0.23
σ_{di}^{me}	$\mathcal{IG}$	0.50	0.20							2.23	2.01	2.52	1.26	1.12	1.41
σ_{π}^{me}	$\mathcal{IG}$	0.15	0.05							0.74	0.67	0.84	0.54	0.48	0.61
σ_{r}^{me}	$\mathcal{IG}$	0.15	0.05							0.29	0.26	0.33	0.28	0.24	0.33
Marginal Likelihood				-925.46			-942.25			-1544.50			-1333.20		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively.
[a]Applies to the dispersed information model.
[b]Applies to the dataset including macroeconomic and expectations data.

Several interesting results arise from our estimation exercise. First, we compare estimates with macroeconomic data only and the complete dataset including expectation data. The parameters estimates related to endogenous propagation are remarkably stable with two key exceptions. Investment adjustment costs has a three fold increase for the FI model and almost two fold under DI. Consumption habits slightly increases in the DI model from 0.87 to 0.97. The latter estimate is relatively higher than standard values around 0.7-0.8. However, the standard pratice is to impose a very tight prior around 0.7 whereas we use a loose prior centered on 0.5. Moreover, the response to inflation in the Taylor rule also increases reaching a quite strong response of 3.32.

Including data on forecast revisions changes substantially the estimated levels of information frictions. Investment, price mark-up, TFP and preference shocks have sharp decreases in their Kalman gain estimates (higher frictions) while those from monetary shock increases substantially (lower frictions). In the next section, we discuss the drivers of those changes.

Expectation data also has important implications for parameter of exogenous processes for the DI model.[21] It changes substantially estimates for most shocks except government expenditure and TFP shocks. The estimated persistence of some shocks declines moderately for investment-specific technological, price mark-up, and monetary shocks. The decrease is more pronounced in preference shocks (from 0.77 to 0.13). Moreover, standard deviations also increases sharply for investment, price and wage mark-up, and preference shocks.

Overall, the estimates from the complete dataset exhibit stronger habits and adjustment costs, and robust informational frictions across all shocks. Other sources of inertia such as price and wage indexation and smoothing in Taylor rule are relatively stable across datasets and models.

Comparing parameter estimates for the FI and DI models under both datasets reveals a considerable decrease in the variance of measurement errors, a first indication that the DI model provides a better explanation for expectation data, which we confirm further when comparing the marginal likelihoods of the FI and DI models in the bottom of Table 1. For the complete dataset, the difference in favor of the DI model increases considerably to 211 log points.

5 Frictions, shocks and business cycles when accounting for informational frictions

This section is divided into two parts. The first (subsection 5.1) explores whether the model can explain expectations data using empirical measures of informational frictions. In this part, we also explore the relative importance of standard frictions in explaining expectation data. In the second part, we leverage our dispersed information model, disciplined with expectations data, to reassess the relative importance of various shocks in driving business cycles (subsections 5.2 and 5.3).

[21] Interestingly, it does not have much impact on estimates of the FI model because they cannot be compensated by informational frictions as we discuss below.

5.1 Does the model match empirical measures of informational frictions?

A growing body of literature following CG, explores a general feature of models with information frictions that relate forecast errors and revisions to provide simple measures of informational frictions from survey data on expectations.

In this section, we follow a similar empirical strategy. For each variable z, consider the following regression

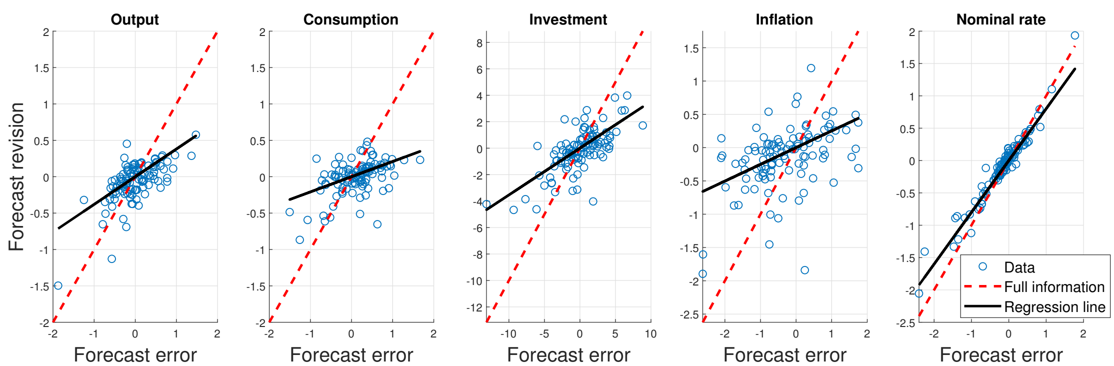
$$\overline{Rev}_{t|t-1}[z_t] = \beta \overline{FE}_{t-1}[z_t] + error_t, \tag{54}$$

where $\overline{Rev}_{t|t-1}[z_t] \equiv \bar{E}_t[z_t] - \bar{E}_{t-1}[z_t]$ is the average forecast revision at time t and $\overline{FE}_{t-1}[z_t] \equiv z_t - \bar{E}_{t-1}[z_t]$ is the average one-step ahead forecast error at time t.

Following the same steps as equation (52), one can see that, if z follows an AR(1) and agents form rational expectations under dispersed information, then β is the Kalman gain, $\bar{k}_z$.

Figure 1 shows the scatter plot with the fitted line in solid black for each variable in the SPF used in the estimation of the model. Hereafter, we refer to those estimates as empirical Kalman gains (eKGs), i.e., Kalman gains directly estimated in the data. If agents had full information,

Figure 1: Empirical Kalman gains from SPF expectation data (1981Q4-2007Q4)



Note: Output, consumption and investment refers to their growth rates. Inflation and interest rates are in levels.

all data points should be in the red dashed line (the 45º line). Thus, Figure 1 suggests a clear departure from the full-information benchmark in the SPF data, in line with previous studies such as CG and BGMS. It is also clear that variables differ in their information friction measure. For instance, the Fed funds rate has forecast revisions that are much more aligned with the forecast errors than any other series.

The AR(1) assumption holds for the dynamics of the exogenous shocks in our model. Thus, the regression (54) strictly applies only if we had data on shocks instead of endogenous variables. The equilibrium dynamics for endogenous variables (41) depend on their lags and the full hierarchy of expectations about all shocks. Therefore, the relationship between forecast errors and forecast revisions from endogenous variables does not have a closed-form representation as in equation (54).

In a dynamic beauty context model with one action and $AR(1)$ fundamental, Angeletos and Huo (2021) show analytically that eKGs are determined by the interaction of informational frictions with other frictions in the model. Therefore, in our general model, estimates from those regressions capture a combination of i) informational frictions of each shock and ii) standard frictions associated with endogenous persistence (e.g., habit formation and price stickiness, among others).[22] Our estimated model can shed light on the relative importance of those frictions.

In the following, we assess whether the data generated by the DI model produces estimates for Kalman gains that align with those found in the SPF data. This exercise verifies the model's ability to match an important untargeted moment of the expectations data.

Let Θ be a vector that collects all parameters of the model and z_t a particular endogenous variable of the model. For a given Θ, we simulate n_{sim} samples of this variable and the respective average expectation $\{z_t, \bar{E}_t[z_t], \bar{E}_{t-1}[z_t]\}$ and measurement errors $\{\varepsilon^{me}_{z,t}\}$ for $t = 1, \cdots, T_{sim}$ and $z \in \{dy, dc, di, \pi, r\}$. Then, we construct n_{sim} time-series for average forecast revisions and forecast errors such that

$$\begin{aligned}\overline{Rev}^{sim}_{z,t|t-1} &= \bar{E}_t[z_t] - \bar{E}_{t-1}[z_t] + \varepsilon^{me}_{z,t} \\ \overline{Fe}^{sim}_{z,t-1} &= z_t - \bar{E}_{t-1}[z_t]\end{aligned}$$

for $t = 1, \cdots, T_{sim}$ and $z \in \{dy, dc, di, \pi, r\}$. Note that we add simulated measurement errors $\varepsilon^{me}_{z,t}$ to the series of forecast revisions so that it is consistent with the measurement equations (53) used in the estimation. Then, we compute the average coefficient and standard error from n_{sim} regressions (54).

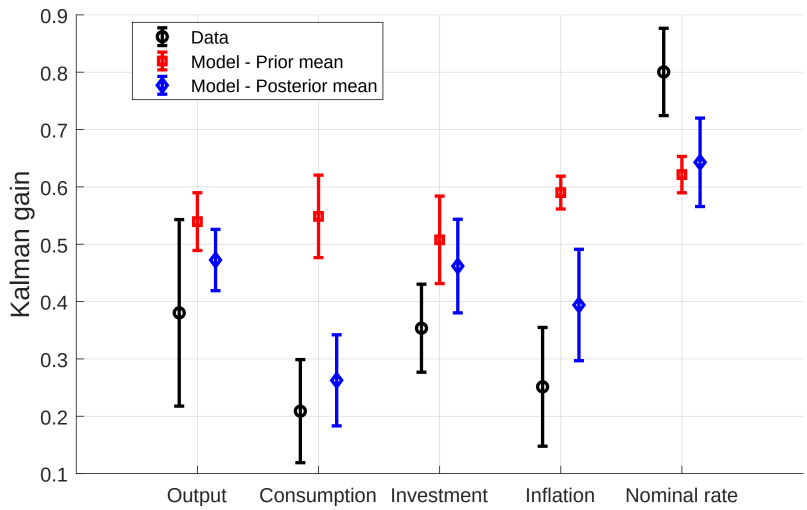
Figure 2 shows the results for $T_{sim} = 105$, $n_{sim} = 100$ and simulations based on the the prior mean ($\Theta = \underline{\Theta}$) and the posterior mean ($\Theta = \bar{\Theta}$).[23] The black error bars (with $\circ$) represent the estimates from the SPF presented in Figure 1.

The eKGs from simulated data using the priors mean (red error bar with $\square$) are similar for all variables and close to the common prior mean for the Kalman gains of exogenous shocks. When comparing the same estimates from the posterior mean (blue error bar with $\diamond$), those are much

[22] CG extends their empirical specification to more general data-generating processes such as $AR(p)$ for inflation or a $VAR(1)$ for some selected macroeconomic models. In our model, the persistence of endogenous variables, represented by the matrix R in our solution (41), is a function of structural parameters.

[23] We use $T_{sim} = 105$ to match the sample size from SPF data. $n_{sim} = 100$ is sufficiently large to avoid sampling variability affecting the Kalman gain estimates meaningfully.

Figure 2: Empirical Kalman gains from simulated data from DI model



Note: Output, consumption and investment refers to their growth rates. Inflation and interest rates are in levels.

more aligned with the direct estimates from the data.

Recall that we use data from forecast revisions at the same horizon in the estimation. Still, the Bayesian estimates do not attempt to match the empirical dependence between forecast revisions and forecast errors.

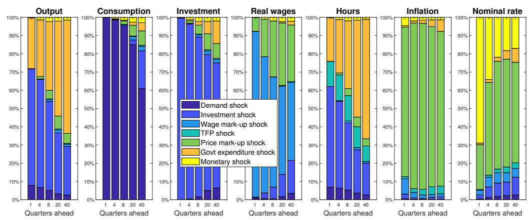
5.2 What are the main drivers of business cycle fluctuations?

We explore the drivers of business cycles in the DI and FI models by performing a forecast error variance decomposition (FEVD) exercise for selected variables at various horizons h. Figure 3 shows the results of this exercise for both models and datasets.

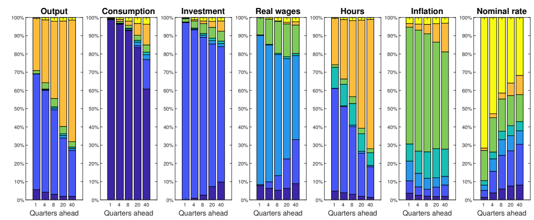
We find that investment-specific technological shocks and government expenditure shocks are the key drivers of fluctuations in output growth and hours worked. Roughly 50-60% of short-run fluctuations in those variables are explained by the investment shock, whereas the government spending shock explains about 25-30%. In the long run, the relevance of these shocks inverts.

This pattern applies to both models and datasets. Our findings are in contrast with those of SW, who suggest that wage mark-up and TFP shocks play a significant role. Our results align with the conclusions of Justiniano et al. (2011), indicating that the difference can be attributed to including inventories in total investment, which SW does not account for. Auclert et al. (2020) also find that investment-specific technological shocks are key to output fluctuations using an estimated

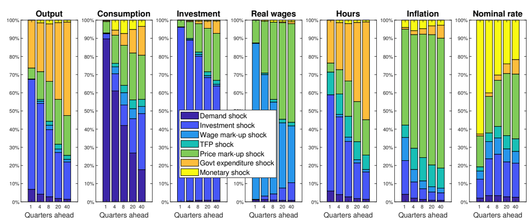
Figure 3: Forecast error variance decomposition for macroeconomic aggregates by model and dataset



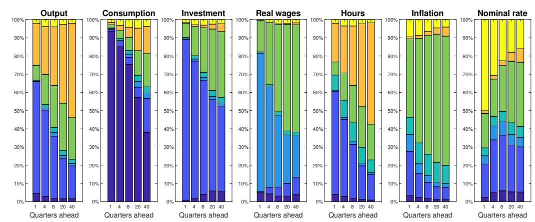
(a) DI model - Forecast revision dataset



(b) FI model - Forecast revision dataset



(c) DI model - Macroeconomic only dataset



(d) FI model - Macroeconomic only dataset

HANK model with sticky information.

In our model, consumption and investment fluctuations are mainly driven by preference shocks and investment-specific technological shocks, respectively. Price mark-up shocks explain most of inflation's fluctuations and have an important role in explaining nominal rates and real wage dynamics, with increasing importance over time. Wage mark-up shocks play a major role in explaining fluctuations in real wages only. TFP shocks play a minor role in explaining fluctuations in all observables.

The FEVD exhibited in Panel (a) reveals that, in the baseline DI model, macroeconomic variables are primarily driven by the shock that directly affects them (e.g., price mark-up shocks explain the bulk of inflation fluctuations). Information frictions can explain the pattern discussed above. The mechanism is anchoring implied by dispersed information: it weakens the general equilibrium forces of the model, increasing the relative importance of partial equilibrium (direct effects) in the model (see Angeletos and Huo; 2021). As we show in the following, this is driven by the underreaction of average expectations and higher-order beliefs to shocks.

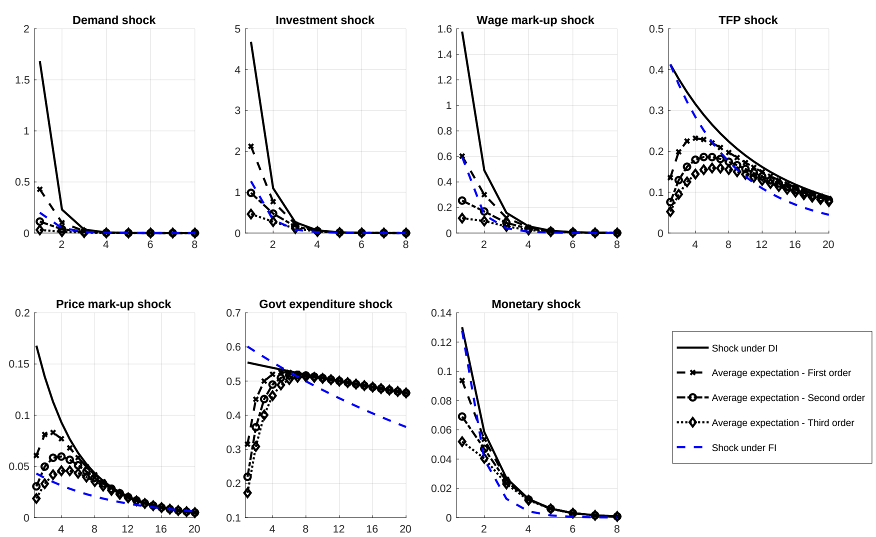
When comparing FEVDs from FI models across both datasets (panels b and d), we observe that the inclusion of data on forecast revisions generates the same pattern (variables being driven by the mostly related shock) to some degree. This pattern is reinforced when introducing informational frictions (compare panels a and c). Interestingly, the DI and FI models have very similar FEVDs when using macroeconomic data only (panels c and d), which highlights the importance of disciplining the model with expectation data.

5.3 Propagation of shocks

In this subsection, we study how shocks propagate in different models by comparing the Bayesian impulse response functions (IRFs) of the DI and FI models estimated using the complete dataset.

Learning and higher-order average expectations about exogenous shocks. Figure 4 shows the IRFs of each shock and the average expectations about them up to the third order. Under full-information rational expectations, all these impulse responses should coincide. The dynamics of shocks differ under full information (dashed blue line) and dispersed information (solid black line) due to distinct estimates for the exogenous processes in each model.

Figure 4: Posterior mean of impulse responses of each shock



Note: IRFs from one standard deviation shock in respective model's estimates.

When using the complete dataset, models suggest that preference, investment-specific technological, and wage mark-up shocks are the least persistent shocks.[24] The key difference is that shocks have a much larger variance under DI, but agents display a high degree of inattention to them. Agents underestimate the magnitude of shocks in the short run and are only fully aware of them after they have dissipated.

The dynamics of monetary policy shocks and the learning process associated with them are very similar for both models. In the DI model, agents take about three quarters to fully learn about them. TFP and government expenditure shocks have high persistence and similar variance

[24]When using macroeconomic data only, FI and DI models differ more regarding the shocks' persistence estimates.

in both models. Moreover, the high estimates of informational frictions generate rich dynamics of higher-order expectations in the DI model. Interestingly, price mark-up shocks differ in terms of variance, but have high persistence for both models. This generates slow-moving higher-order expectations that are similar to the path of the actual shock under the FI model.

We find that shocks when both high persistence and significant departures from full information can accurately generate inertia expectations those shocks. In the following, we show that those shocks are the ones that induce strong inertia in macroeconomic variables. Usual quantitative explorations with small models typically calibrate models consistently with this. In contrast, a combination of low persistence and strong informational frictions (present in half of the shocks) fails to generate sufficient inertia. In that case, informational frictions only dampen the impact of shocks in the short run, without producing the sustained effects characteristic of high inertia.

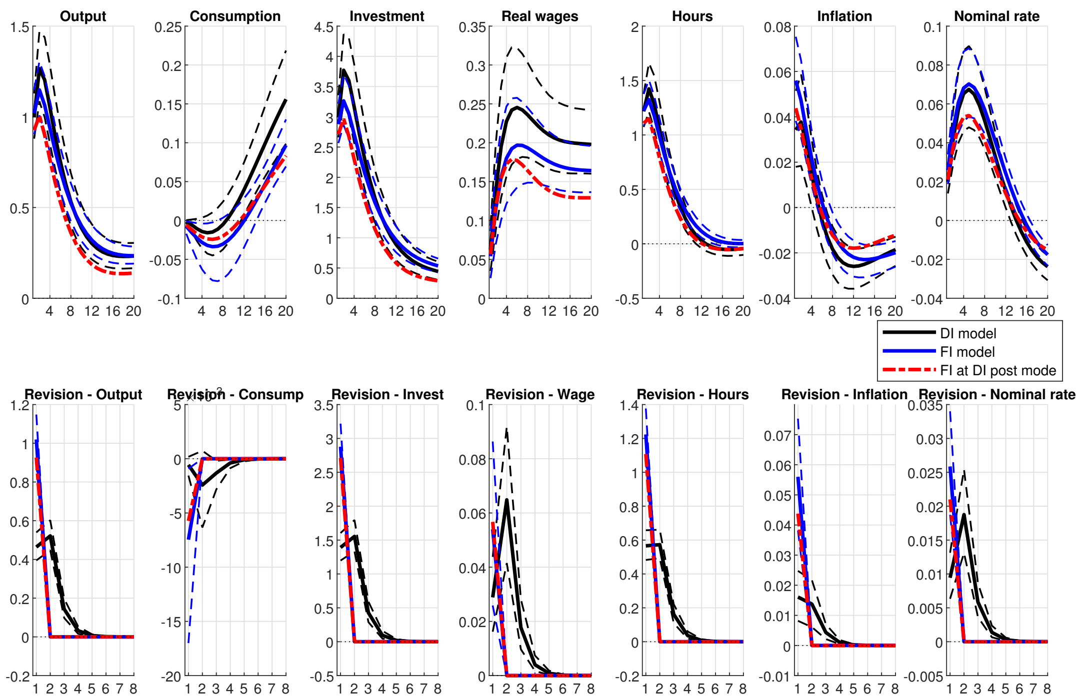
This can be easily seen in equation (34). For each shock l, the persistence depends on both $1 - \bar{k}_l$ and ρ_l and the responsiveness of the average expectation to the shock depends on $\bar{k}_l$ only. The distinction above underscores the importance of correctly specifying both persistence and information structure jointly when estimating macroeconomic models.

Endogenous variables. We now study the propagation of key structural shocks to relevant macroeconomic variables in the DI and FI models. The figures below compare the IRFs from the FI model (blue solid line) and the DI model (black solid line) with 90% Bayesian confidence bands. To highlight the effect of informational frictions on the model's dynamic properties, we also compute the IRFs of a FI model solved at the posterior mean of the DI model (red dot-dashed lines)[25]. This comparison helps to determine whether the dynamics of the DI model differ from those of the FI model due to informational frictions or differences in estimates from the remaining parameters. Indeed, the gap between the black line and the red line is entirely explained by informational friction.

An investment-specific technological shock leads to an increase in output, hours, and investment, with the latter showing the largest percentage increase (Figure 5). Consumption has a weak negative response in the first year and increases in the following years. Inflation increases in the first year, but later decline. The nominal interest rate rise in response to higher inflation and output. The dynamics of the DI and FI models are quite similar with slightly stronger responses in the DI model. The key difference is the response of real wages, which is much more pronounced in the DI model. By comparing red with black lines, one can see that the differences (although small) between the DI and FI models result from the informational frictions and not from the change in common parameter estimates.

[25]The impulse from the FI model with the mean of posterior estimates of the DI model is computed using the standard deviation from the FI model for generating comparable IRFs. Since the model is log-linearized, this normalization does not affect the shape of the IRF.

Figure 5: Impulse responses to an investment-specific technological shock



Note: IRFs from one standard deviation shock in respective model's estimates for DI and FI models. The IRF for FI model using DI's model posterior mode (red dot-dashed line) uses one-standard deviation from the FI's model posterior mode for better comparison. Dashed lines are 90% credible intervals.

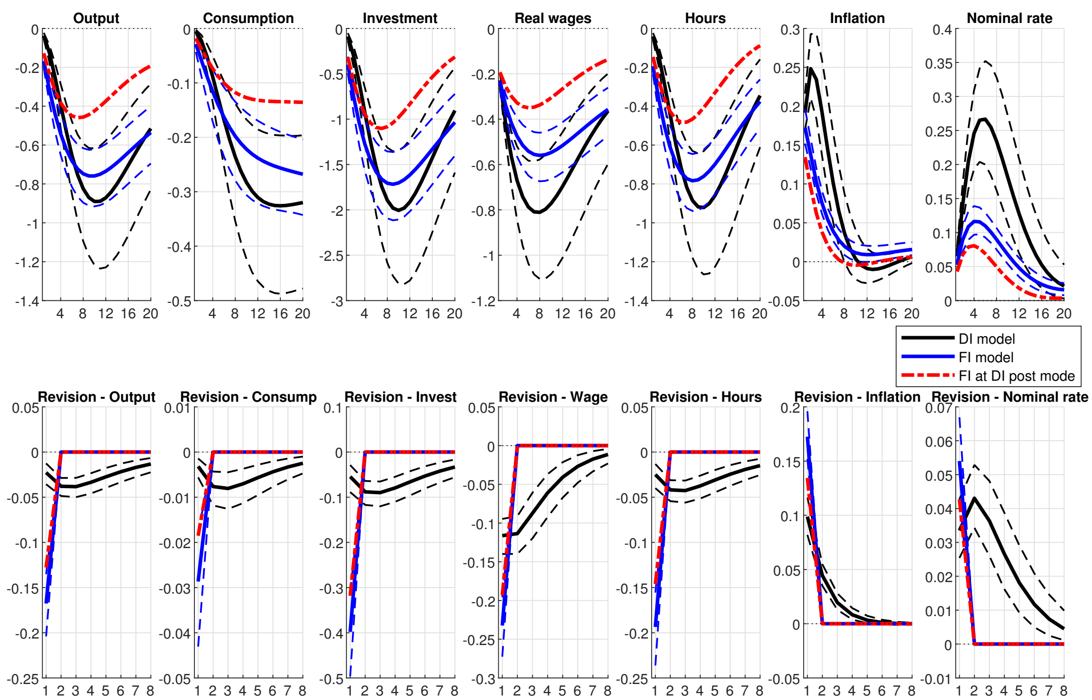
Agents are not fully aware of the boom in output, hours and investment, so they only adjust their forecasts by about half of the actual changes in those variables. It takes them about a year to fully understand the impact of the shock on macroeconomic variables. After about a year, agents fully grasp the impact of the shock on macroeconomic variables. This pattern occurs because of the weak persistence and high informational frictions for this shock implied in the estimated model. In contrast, the FI model generates a spike in the forecast revision on impact followed by an immediate return to zero. This pattern is a feature of any macroeconomic model with full-information rational expectations, and it is one of the key reasons why these models cannot provide a good fit to the expectation data.

There are noticeable differences in the responses of DI and FI dynamics to a cost-push (price markup) shock. These differences are primarily due to informational frictions (Figure 6). The DI model generates a more pronounced and hump-shaped response of inflation, along with a slightly more significant decrease in real aggregates (output, consumption, investment, and hours) and a considerably larger decrease in real wages. The peak effect happens after 10 quarters for output, hours and investment, slightly longer than in the FI model. At the same time, average higher-order expectations converge to the path of the actual shock (Figure 4).

The DI model implies a stronger nominal rate response, with the peak response after 6 quarters roughly two times higher than the FI model. Overall, the DI model implies that a typical cost-push shock has a stronger impact on inflation and real variables, with a stronger response from the central bank than the FI model.

Interestingly, the combination of persistent shocks and strong informational frictions – as discussed in Figure 4 – implies an attenuated and hump-shaped response of forecast revisions for real aggregates. After the shock, agents correctly anticipate the fall in real activity and the increase in inflation which are gradually revised quarter by quarter for more than two years.

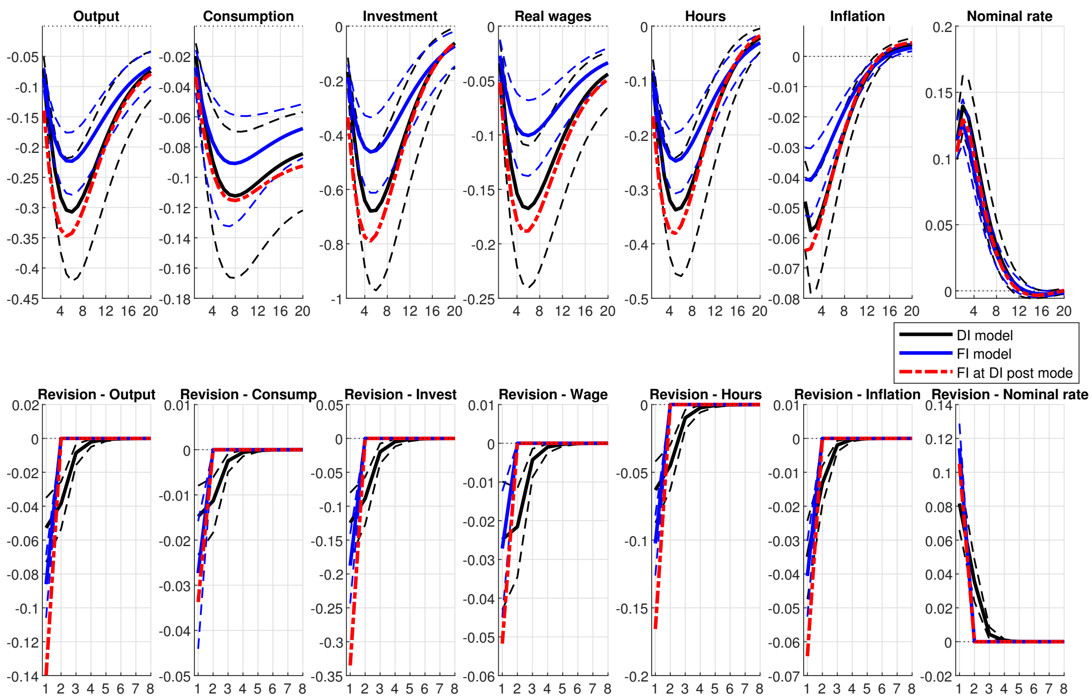
Figure 6: Impulse responses to price markup shock



Note: IRFs from one standard deviation shock in respective model's estimates for DI and FI models. The IRF for FI model using DI's model posterior mode (red dot-dashed line) uses one-standard deviation from the FI's model posterior mode for better comparison. Dashed lines are 90% credible intervals.

Finally, a monetary policy shock leads to a decline in output and investment with peak response after six quarters and eight quarters for consumption. Inflation also decreases with peak response in the quarter ahead (Figure 7). The impulse responses of forecast revisions indicate that agents

Figure 7: Impulse responses to a monetary policy shock



Note: IRFs from one standard deviation shock in respective model's estimates for DI and FI models. The IRF for FI model using DI's model posterior mode (red dot-dashed line) uses one-standard deviation from the FI's model posterior mode for better comparison. Dashed lines are 90% credible intervals.

adjust their expectations about real variables and inflation downward after the shock hits, while revising their expectations about the nominal rate upward.

Comparing the FI and DI models, we find that monetary policy shocks have stronger effects on macroeconomic aggregates and inflation in the DI model. The difference is primarily due to changes in parameter estimates rather than informational frictions, as the red dashed and black lines are very similar. This is consistent with the fact that the monetary shock has the lowest estimated level of informational friction compared to all other shocks.

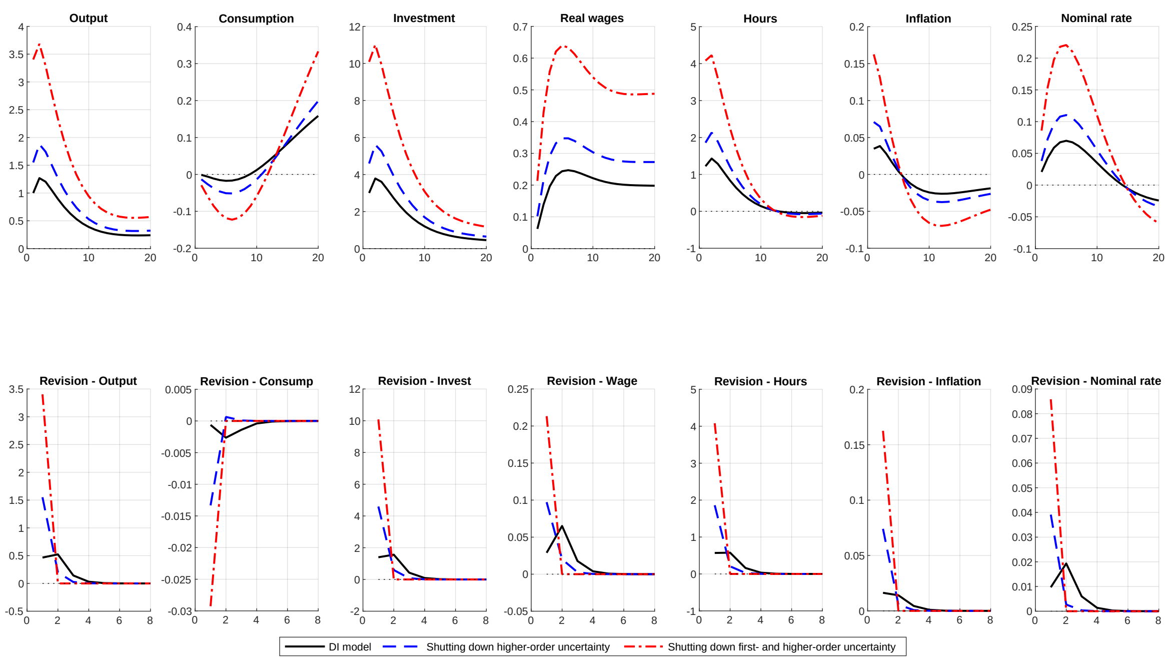
Figures with impulse responses to the remaining shocks are available in Appendix D.

Decomposing the role of information frictions. The implications of incomplete information, in the form of dispersed information in our model, are twofold: i) it generates gradual learning about exogenous shocks (first-order uncertainty), and ii) uncertainty about other agents' expectations (higher-order uncertainty). In this section, we investigate the role of each force in shaping the effects of information frictions on the propagation of shocks. Here, we focus on investment-technological and price mark-up (cost-push) shocks, which informational frictions play a role and are the main drivers of output and inflation, respectively.

We assess the relative importance of those forces by constructing two auxiliary IRFs. The first shuts down higher-order uncertainty by imposing that higher-order beliefs follow the same dynamics as first-order beliefs. This holds in any model with imperfect but common information (dashed blue lines in the following figures). To construct the second, we shut down both first- and higher-order uncertainty, i.e., the full hierarchy of expectations is equal to the true dynamics of the shock (dot-dashed red lines). This corresponds to the full information benchmark, solved with the same parameter values of the DI model.[26]

Figure 8 shows the counterfactual exercise for the investment shock. For almost all variables, higher-order beliefs do not have a great bite, as evidenced by the small difference between the solid black and dashed blue lines. Imperfect learning (first-order uncertainty) is the key factor behind the dampening effect of information frictions, as evidenced by the large differences between the dot-dashed red and solid black lines.

Figure 8: Impulse responses to investment shock



[26] IRFs named as 'FI at DI post mode' in Figures 5-6 depict the IRF in response of shock with the same size of a one-standard-deviation from the FI model for better comparison with the FI model. In Figure 8-9 the impulses from a one-standard deviation from the DI posterior.

Figure 9 shows the same analysis to a cost-push (price markup) shock. We find that for real variables, first- and higher-order expectations contribute roughly the same to the shock propagation, both mitigating the dynamic effects of the shock. The learning contributes in postponing the peak effect on those variables in one quarter (from 7 to 8 quarters) and higher-order expectations in additional two quarters for output, investment and hours (from 8 to 10 quarters).

For inflation, imperfect learning about the shock (first-order uncertainty) explains almost entirely the less inflationary effect of the cost-push shock in the DI model compared to the FI model. Learning is key to generating a hump-shape in the first year following the shock, with a peak in the second quarter.

Interestingly, first-order and higher-order uncertainty drive the nominal rate in different directions. The learning mitigates the nominal rate response in the first two years, while the higher-order expectations push nominal rates higher after the first year. The intuition is that learning about the shock mitigates the inflationary effect of the cost-push shock in the first year. Both first- and higher-order expectations do not play a major role in the inflation dynamics after the first year. In contrast, higher-order expectations have a major role in dampening the shock transmission to output. This drives a slightly tighter policy as the recessionary impact is attenuated. Despite altering largely the output response to the cost-push shock, higher-order uncertainty marginally affects the nominal rate because of the estimated high response to inflation ($\phi_\pi = 3.32$) and small response to output ($\phi_y = 0.07$).

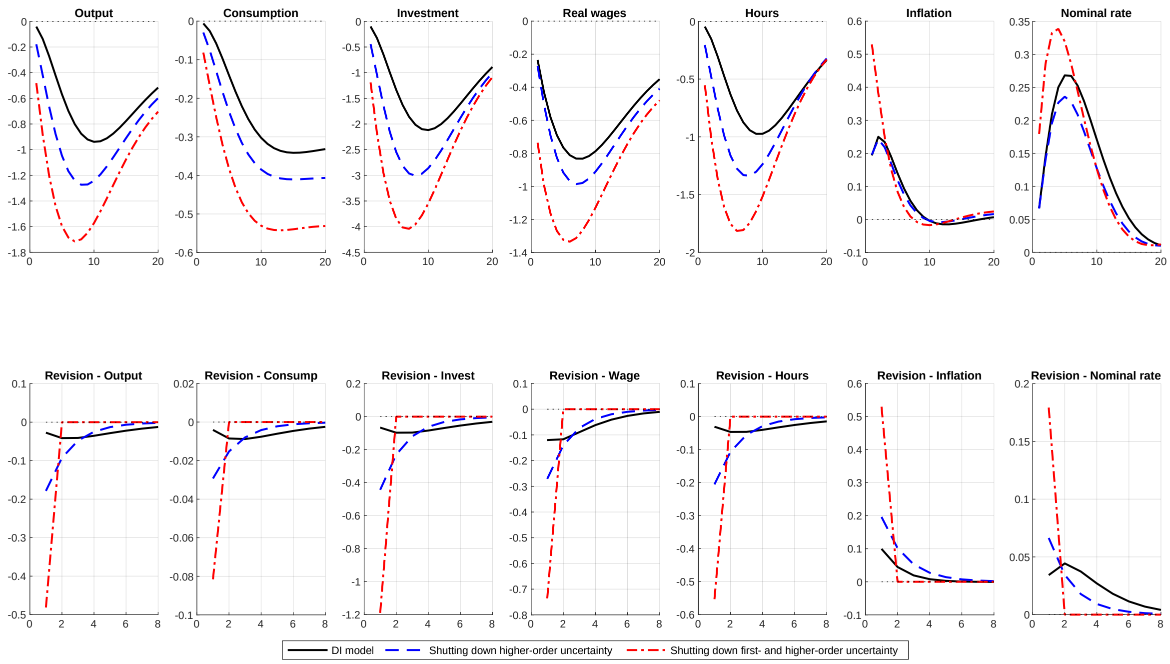
Finally, both first- and higher-order expectations are key for expectations dynamics. The former attenuates the immediate response of expectations to the shock and generates persistence in forecast revisions. The latter weakens further the impact and creates a hump shape with a peak in the third quarter. Higher-order expectations do not generate the same pattern for inflation revision as its peak response is on impact.

5.4 Information frictions: substitutes or complements for standard frictions?

A large body of literature has studied, in more theoretical settings, the potential of information frictions to substitute for "standard" frictions that generate inertia in macroeconomic models, such as habit formation in consumption, adjustment cost in investment, among others. In our quantitative exploration, we find that both standard and information frictions are essential to fit the macroeconomic and expectation data.

When estimating the model with both standard and informational frictions using macroeconomic data only, standard frictions remain at high levels as estimated in mainstream full-information macroeconomic models, while informational frictions are relatively weaker. Moreover,

Figure 9: Impulse responses to price mark-up shock



the comparison of marginal likelihoods suggests that informational frictions do not play such an important role in explaining macroeconomic data when standard frictions are present.

However, our results suggest that informational frictions are not well identified when abstracting from expectational data in the estimation. Interestingly, including revision data doesn't reduce the relevance of standard frictions, even though they could potentially create inertia when combined with informational frictions, as we discussed before. Indeed, estimates of habits and adjustment investment costs increase substantially when including expectation data in the model. Wage indexation increases slightly, and price indexation decreases marginally.

Given this result, we investigate which frictions are most crucial for explaining the data and less replaceable by information frictions. We do this by shutting down each friction one at a time and re-estimating the model. Table 2 shows the marginal likelihood of several nested models that shut down the following forms of sluggish adjustment: i) habit persistence, ii) investment adjustment costs[27], iii) price indexation, iv) wage indexation, and v) all of them together.

When all frictions are removed, the log marginal likelihood is roughly 115 points lower. Eliminating only habits leads to a drop of similar magnitude. While adjustment costs in investment

[27]When shutting down investment adjustment costs, we substitute them with standard capital adjustment costs.

Table 2: Marginal Likelihoods shutting down key frictions

Model	Marginal Likelihood
Baseline	-1333.20
All standard frictions	-1448.70
No habits	-1436.90
No investment adjustment costs	-1377.70
No price indexation	-1327.60
No wage indexation	-1322.90

are important, they only contribute to half of the drop in log marginal likelihood compared to the former. Shutting down price or wage indexation leads to better marginal likelihoods than the baseline model. These results are consistent with Smets and Wouters (2007)'s findings regarding the relevance of habits and investment adjustment costs for fitting the data (now including expectation data) and the minor importance of price and wage indexation.

In summary, our results portray a more complementary view of information and standard frictions, compared to most of the previous literature. The intuition behind this key conclusion is the following. Most shocks have weak persistence and strong informational frictions, which are not sufficient to create the necessary persistence in endogenous variables. Therefore, intrinsic persistence in endogenous variables is needed, which traditionally comes from standard frictions such as habits and investment costs.

One crucial exception is inflation. The forecast error variance decomposition from the DI model shows that mark-up shocks play a major role in explaining inflation fluctuations (section 5.2). Figure 6 shows that the model with information frictions generates a hump-shaped response of inflation to a cost-push shock that the FI model cannot generate. This is due to the high estimates for information frictions and persistence for this particular shock, along with a low estimated price indexation. Angeletos and La'O (2009) shows in a Calvo model with incomplete information that higher-order expectations play an important role in inflation dynamics even in the presence of nominal rigidity and learning about shocks. Our results corroborate this view[28].

6 Conclusion

We develop a general solution method that allows enriching a standard medium-scale DSGE model with dispersed information and estimate using Bayesian techniques with comprehensive macroeconomic and expectation data. We draw important conclusions regarding the role information

[28]The key difference is that they study a nominal demand shock while we find price mark-up shocks as the main driver of inflation.

frictions play in business cycles. The degree of informational friction varies significantly across shocks and indicates important departures from perfect information. Relatedly, simulated data from the model can match standard empirical measures of informational frictions when using data on forecast revisions in the estimation.

Moreover, we contribute to the business cycle literature by showing that the inclusion of informational frictions and expectation data changes the relative importance of shocks, as the general equilibrium effects are dampened due to a weaker response to expectations about future conditions. In general, shocks that directly impact some particular variables are more relevant to explaining their fluctuations in the DI model. For instance, price mark-up shocks play a stronger role in explaining inflation dynamics in the DI model compared to the FI model.

We also contribute to the literature by re-assessing a long-standing view that information frictions could be an alternative source of sluggishness in macroeconomic data. Our results suggest a more complementary view of the standard and informational frictions. When disciplining our model with expectation data, standard frictions such as habits and investment costs increase substantially despite the pervasive informational frictions. Hence, both types of frictions are important to explain the data.

In our estimated model, shocks with stronger informational frictions also exhibit weak persistence. This combination helps to lessen the impact of shocks and expectations on endogenous variables, but it does not produce the necessary high inertia, as observed in the data. Strong standard frictions instead provide this inertia. One important exception is the price-mark-up shock, for which information frictions introduce additional sluggishness in inflation and nominal rates that cannot be produced by standard frictions alone.

A natural extension for our work is embedding a richer informational structure, including both public endogenous and exogenous signals, for instance. Additionally, our model provides a natural setting to study the role of additional shocks related to agents' information sets such as noise shocks (Lorenzoni; 2010; Blanchard et al.; 2013) or confidence shocks (Angeletos and La'O; 2013; Angeletos et al.; 2018). In conclusion, we see an open venue for exploring the quantitative potential of DSGE models with informational frictions.

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Appendix

A Log-linear model and full derivation

A.1 Optimality conditions

Households

From the households' utility maximization problem, we get the following first-order conditions for consumption and bonds, respectively

$$E_{ht}[\Lambda_{h,t}P_t] = \frac{E_{ht}\left[e^{\eta_t^c}\right]}{C_{h,t} - \varphi C_{h,t-1}} \tag{55}$$

$$E_{ht}[\Lambda_{h,t}] = \beta E_{ht}\left[\Lambda_{h,t+1}R_t\right] \tag{56}$$

where $\Lambda_{h,t}$ is the Lagrange multiplier of households' budget constraint (20). The optimal conditions for capital and investment are given by, respectively:

$$\Phi_{h,t} = \beta E_{ht}\left[\Lambda_{h,t+1}\left(R_{t+1}^k U_{h,t+1} - P_{t+1}a(U_{h,t+1})\right)P_{t+1}\right] + (1-\delta)\beta E_{ht}\left[\Phi_{h,t+1}\right] \tag{57}$$

$$\begin{aligned} E_{ht}[\Lambda_{h,t}P_t] = {} & \Phi_{h,t}E_{ht}\left[e^{\eta_t^i}\right]\left(1 - S\left(\frac{I_{h,t}}{I_{h,t-1}}\right) - S'\left(\frac{I_{h,t}}{I_{h,t-1}}\right)\frac{I_{h,t}}{I_{h,t-1}}\right) \\ & +\beta E_{ht}\left[\Phi_{h,t+1}e^{\eta_{t+1}^i}S'\left(\frac{I_{h,t+1}}{I_{h,t}}\right)\left(\frac{I_{h,t+1}}{I_{h,t}}\right)^2\right] \end{aligned} \tag{58}$$

where $\Phi_{h,t}$ is the Lagrange multiplier associated with the law of motion of capital. The optimal value of the capital utilization rate solves

$$E_{ht}[R_t^k] = a'(U_{h,t})E_{ht}[P_t]. \tag{59}$$

Recall that households can buy state-contingent assets in terms of consumption (but not for leisure). Therefore, it must hold that for all households $h \in [0,1]$ that $C_{h,t} = C_t$, $I_{h,t} = I_t$, $U_{h,t} = U_t$, $\Lambda_{h,t} = \Lambda_t$ and $\Phi_{h,t} = \Phi_t$. However, households still have heterogeneous expectations, $E_{ht}[\cdot]$, wages $w_{h,t}$ and labor supply $L_{h,t}$.

The optimal condition of the wage setting problem is

$$E_{ht}\left[\sum_{s=0}^{\infty}(\beta\xi_w)^s\left(\Lambda_{t,t+s}\frac{X_{t,t+s}^w}{\mu_{t+s}^w} - \frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}\frac{L_{h,t+s}^{\chi}}{W_{h,t}^*}\right)L_{h,t+s}\right] = 0 \tag{60}$$

where $\Lambda_{t,t+s} \equiv \frac{\Lambda_{t+s}}{\Lambda_t}$ is the stochastic discount factor from period t to period $t+s$. The marginal rate of substitution of labor and consumption is defined as

$$MRS_{h,t} \equiv -\frac{U_{L_{h,t}}}{U_{C_{h,t}}} = L_{h,t}^{\chi} \left(C_{h,t} - \varphi C_{h,t-1} \right). \tag{61}$$

Intermediate good firms

At stage 1, firms choose their optimal price based on the information from signals. The optimal price $P_{i,t}^*$ that maximize intermediate firm i's profit solve the following first-order condition

$$E_{it} \left[\sum_{s=0}^{\infty} (\beta \xi_p)^s \Lambda_{t,t+s} \left(\frac{X_{t,t+s}}{\mu_{t+s}^p} - \frac{1+\mu_{t+s}^p}{\mu_{t+s}^p} \frac{MC_{i,t+s}}{P_{i,t}^*} \right) Y_{i,t+s} \right] = 0. \tag{62}$$

At stage 2, they hire labor at the nominal wage W_t and rent capital at the rental rate R_t^k. Cost minimization subject to production function (5) implies that the capital-labor ratio is given by[29]

$$\frac{K_{i,t}}{L_{i,t}} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}. \tag{63}$$

We obtain that the marginal cost is given by

$$MC_{i,t} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(R_t^k)^\alpha \left(W_t \right)^{1-\alpha}}{a_t}. \tag{64}$$

Since the production function displays constant return of scale, capital-labor ratios and marginal costs are the same across firms in equilibrium.

[29]Since factor prices are revealed at stage 2, firms does not need to form expectations for W_t and R_t^k.

A.2 Log-linearized model

The log-linearization of optimal conditions leads to the following system of equations

$$
\begin{aligned}
c_t =& \frac{\varphi/\gamma}{1+\varphi/\gamma} c_{t-1} + \frac{1}{1+\varphi/\gamma}\bar{E}_t[c_{t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma}\bar{E}_t[r_t - \pi_{t+1} - (x^c_{t+1} - x^c_t)] && (65.1)\\
i_t =& \left(\frac{1}{1+\beta}\right) i_{t-1} + \left(\frac{\beta}{1+\beta}\right)\bar{E}_t[i_{t+1}] + \left(\frac{1}{s''(1+\beta)\gamma^2}\right)\left(q_t + \bar{E}_t\left[x^i_t\right]\right) && (65.2)\\
q_t =& \left(\frac{\beta(1-\delta)}{\gamma}\right)\bar{E}_t[q_{t+1}] + \left(1 - \frac{\beta(1-\delta)}{\gamma}\right)\bar{E}_t\left[r^k_{t+1}\right] - \bar{E}_t[r_t - \pi_{t+1}] && (65.3)\\
k_t =& \left(\frac{1-\delta}{\gamma}\right) k_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right)\left(i_t + x^i_t\right) && (65.4)\\
u_t =& \left(\bar{R}^k/a''\right)\bar{E}_t[r^k_t] && (65.5)\\
k^u_t =& k_{t-1} + u_t && (65.6)\\
mrs_t =& \left(\frac{1}{1-\varphi/\gamma}\right)(c_t - \varphi/\gamma c_{t-1}) + \chi l_t && (65.7)\\
y_t =& \left(1 + \frac{\Phi_p}{\bar{Y}}\right)(\alpha k^u_t + (1-\alpha) l_t + x^a_t) && (65.8)\\
k^u_t - l_t =& w_t - r^k_t && (65.9)\\
mc_t =& (1-\alpha) w_t + \alpha r^k_t - x^a_t && (65.10)\\
y_t =& \frac{1}{1-g_y}\left(\frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{I}}{\bar{Y}} i_t + \frac{\bar{R}^k \bar{K}}{\bar{Y}} u_t + x^g_t\right) && (65.11)\\
r_t =& \phi_r r_{t-1} + (1-\phi_r)(\phi_\pi \pi_t + \phi_y y_t) + x^r_t && (65.12)\\
\pi_t =& \xi_p \iota_p \pi_{t-1} + (1-\xi_p) p^*_t && (65.13)\\
p^*_t =& (1-\beta\xi_p)\bar{E}_t\left[mc_t + x^p_t\right] + (1-\iota_p\beta\xi_p)\bar{E}_t[\pi_t] + \beta\xi_p \int_0^1 E_{it}[p^*_{i,t+1}]di && (65.14)\\
w_t + \pi_t =& \xi_w(\iota_w \pi_{t-1} + w_{t-1}) + (1-\xi_w) w^*_t && (65.15)\\
w^*_t =& \frac{(1-\beta\xi_w)}{1+\chi\theta_w}\bar{E}_t\left[mrs_t + w_t + x^w_t\right] + (1-\iota_w\beta\xi_w)\bar{E}_t[\pi_t] + \beta\xi_p \int_0^1 E_{ht}\left[w^*_{h,t+1}\right] dh && (65.16)
\end{aligned}
$$

where $\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$, $\kappa_w = \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(1+\chi\theta_w)}$ and $\theta_w = \frac{1+\bar{\mu}^w}{\bar{\mu}^w}$. $\bar{E}_t[\cdot] \equiv \int_0^1 E_{it}[\cdot]di$ denotes average expectation, where $E_{it}[\cdot] \equiv E[\cdot \mid \mathcal{I}_{it}]$ is agent i's expectation based on her information set at period t, $\mathcal{I}_{it}$.

All equations are standard in macroeconomic models. The first equation is the Euler equation for consumption, which is derived by combining the log-linearized versions of equations (55) and (56). The second equation is the equilibrium condition for investment from (58) and the third determines the marginal real value of a unit of capital q_t that come from condition (57). The forth is log-linearized version of the usual law of motion of capital (17). The fifth is the log-linearized version of the optimal condition for capital utilization (59) and the sixth comes from the definition of utilized capital (19). The seventh and eighth are log-linearized versions of the aggregation of the marginal rate of substitution between consumption and labor from equation (61) and the production function (5), respectively. The ninth and tenth come from aggregating firms' cost minimization condition (63) and marginal cost (63), respectively. The eleventh derives from the aggregate resource constraint (29), while twelfth is the log-linearized version of the Taylor rule (25). The thirteenth is log-linearized inflation dynamics implied by the law of motion of the price level (10) and the fourteen is the optimal price dynamics from the log-linearization of equation (62). Combining the those two equations leads to the New Keynesian Phillips Curve (35) in the main text. The last two equations are the real wage dynamics implied by the law of motion of the price level (24) and the optimal wage dynamics from the log-linearization of equation (60).

The derivation of the model can be found in the Online Appendix.

B Data

Data on macroeconomic aggregates are collected from the Federal Reserve Database (FRED) and expectation data come from the Survey of Professional Forecasters (SPF). The time series collected to construct the database are displayed in Table 3. It starts in 4Q81, the first quarters with data for all time series on expectations that I use in this work, and ends in 4Q07, before the Great Recession.

Table 3: Database

Series	**Description**	**Measure**	**Source**
GDPC1	Real Gross Domestic Product	Billions of Chained 2012 Dollars	BEA
PCECC96	Real Personal Consumption Expenditures	Billions of Chained 2012 Dollars	BEA
GPDIC1	Real Gross Private Domestic Investment	Billions of Chained 2012 Dollars	BEA
GDPDEF	Gross Domestic Product: Implicit Price Deflator	Index 2012=100	BEA
FF	Effective Federal Funds Rate	Percentage points	FED Board
PRS85006063	Nonfarm Business Sector: Compensation	Index 2012=100	BEA
HOANBS	Nonfarm Business Sector: Hours of All Persons	Index 2012=100	BLS
CNP16OV	Civilian Noninstitutional Population	Thousands of People	BLS
RGDP	Forecast for the real GDP	Chained Dollars (base year varies)	SPF
RCONSUM	Forecast for the real PCE	Chained Dollars (base year varies)	SPF
RNRESINV	Forecast for the nonresidential fixed investment	Chained Dollars (base year varies)	SPF
PGDP	Forecast for the GDP price index	Index (base year varies)	SPF
TBILL	Forecast for the annual three-month Treasury bill rate	Percentage points	SPF

Note: BEA, BLS, Fed Board and SPF stand for, respectively: U.S. Bureau of Economic Analysis, U.S. Bureau of Labor Statistics, Board of Governors of the Federal Reserve and Survey of Professional Forecasters.

Real output (GDPC1), consumption (PCE) and investment (GDPIC1) are computed as per capita aggregates by dividing them to the Hodrick-Prescott filtered population index (CNP16OV). Real wage is constructed by dividing the compensation in the nonfarm business sector (PRS85006063) by the GDP deflator (GDPDEF). Per capita total hours is computed by dividing the hours of all persons out nonfarm business sector (HOANBS) from the smoothed population index. Inflation is defined as the annualized quarterly growth rate of GDP deflator (GDPDEF). The nominal interest rate is computed by the log of the gross federal funds rate (FF) to be consistent with the log-linearization procedure.

$$
\begin{aligned}
Y_t^{obs} &= GDPC1_t/HP(CNP16OV_t) \\
C_t^{obs} &= PCE_t/HP(CNP16OV_t) \\
I_t^{obs} &= GPDIC1_t/HP(CNP16OV_t) \\
W_t^{obs} &= PRS85006063_t/GDPDEF_t \\
dy_t^{obs} &= 100\log(Y_t^{obs}/Y_{t-1}^{obs}) \\
dc_t^{obs} &= 100\log(C_t^{obs}/C_{t-1}^{obs}) \\
di_t^{obs} &= 100\log(I_t^{obs}/I_{t-1}^{obs}) \\
dw_t^{obs} &= 100\log(W_t^{obs}/W_{t-1}^{obs}) \\
L_t^{obs} &= HOANBS_t/HP(CNP16OV_t) \\
\Pi_t^{obs} &= 400\log(GDPDEF_t/GDPDEF_{t-1}) \\
R_t^{obs} &= 100\log(1 + FF_t/100)
\end{aligned}
$$

Expectation data are constructed as follows

$$
\begin{aligned}
\bar{E}_t[Y^{obs}_{t+1}] &= RGDP3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[Y^{obs}_{t}] &= RGDP2_t/HP(CNP16OV_{t}) \\
\bar{E}_t[C^{obs}_{t+1}] &= RCONSUM3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[C^{obs}_{t}] &= RCONSUM2_t/HP(CNP16OV_{t}) \\
\bar{E}_t[I^{obs}_{t+1}] &= RINVEST3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t[I^{obs}_{t}] &= RINVEST32_t/HP(CNP16OV_{t}) \\
\bar{E}_t[dy^{obs}_{t}] &= 100\log\left(\bar{E}_t[Y^{obs}_{t+1}]/\bar{E}_t[Y^{obs}_{t}]\right) \\
\bar{E}_t[dy^{obs}_{t}] &= 100\log\left(\bar{E}_t[Y^{obs}_{t}]/\bar{E}_t[Y^{obs}_{t-1}]\right) \\
\bar{E}_t[dc^{obs}_{t}] &= 100\log\left(\bar{E}_t[C^{obs}_{t}]/\bar{E}_t[C^{obs}_{t-1}]\right) \\
\bar{E}_t[di^{obs}_{t}] &= 100\log\left(\bar{E}_t[I^{obs}_{t}]/\bar{E}_t[I^{obs}_{t-1}]\right) \\
\bar{E}_t[\pi^{obs}_{t}] &= 400\log(PGDP3t/PGDP2t) \\
\bar{E}_t[R^{obs}_{t}] &= \log(1+TBILL2_t/100) \\
\bar{F}[dy_t] &= dy^{obs}_t - \bar{E}_{t-1}[dy^{obs}_t] \\
\bar{F}[dc_t] &= dc^{obs}_t - \bar{E}_{t-1}[dc^{obs}_t] \\
\bar{F}[di_t] &= di^{obs}_t - \bar{E}_{t-1}[di^{obs}_t] \\
\bar{F}[\pi_t] &= \pi^{obs}_t - \bar{E}_{t-1}[\pi^{obs}_t] \\
\bar{F}[R_t] &= R^{obs}_t - \bar{E}_{t-1}[R^{obs}_t]
\end{aligned}
$$

C Solution method and Algorithm

We first start with two Lemmas. The first writes the first-order expectation of the hierarchy as a linear transformation of the hierarchy.

Lemma 1. *The first-order expectation of the hierarchy of expectations satisfy*

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = Tx_t^{(0:\bar{k})} \tag{66}$$

where $T = \begin{bmatrix} 0_{n\bar{k}\times n} & I_{n\bar{k}} \\ 0_{n\times n} & 0_{n\times n\bar{k}} \end{bmatrix}$ *is the order transformation matrix.*

Proof. By the definition of $x_t^{(0:\bar{k})}$ one can see that $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = x_t^{(1:\bar{k}+1)}$. By definition for $\bar{k}$, any order s such that $s > \bar{k}$ does not affect the equilibrium. Then, without loss of generality, one can set $E_t^{(s)}[x_t] = 0$ if $s > \bar{k}$. Therefore, one can rewrite $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$ as

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)}[x_t] \end{bmatrix} = \begin{bmatrix} x_t^{(1:\bar{k})} \\ 0_{n\times 1} \end{bmatrix} = \begin{bmatrix} 0_{n\bar{k}\times n} & I_{n\bar{k}} \\ 0_{n\times n} & 0_{n\times n\bar{k}} \end{bmatrix} \begin{bmatrix} x_t \\ x_t^{(1:\bar{k})} \end{bmatrix} = Tx_t^{(0:\bar{k})}, \tag{67}$$

where the first equality uses the definition of $x_t^{(1:\bar{k}+1)}$ and the second equality uses that $E_t^{(\bar{k}+1)}[x_t] = 0$. In the last equality, T is defined accordingly.

□

This is Lemma builds on Proposition 1 from Online Appendix of Melosi (2017). It explores the truncation of the hierarchy.

The second Lemma does the opposite: writes the hierarchy as a linear transformation of the first-order expectation of the hierarchy.

Lemma 2. $x_t^{(0:\bar{k})}$ *can be rewritten as a linear function of* x_t *and its average expectation such that*

$$x_t^{(0:\bar{k})} \equiv e_x' x_t + T' E_t^{(1)}\left[x_t^{(0:\bar{k})}\right],$$

Proof. By definition, $x_t^{(0:\bar{k})}$ can be decomposed as $x_t^{(0:\bar{k})} = \left[x_t' \quad (x_t^{(\bar{1}:\bar{k})})'\right]'$. Using this decomposition, the average expectation decomposed as $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \left[(x_t^{(\bar{1}:\bar{k})})' \quad E_t^{(\bar{k}+1)}[x_t]'\right]'$. Therefore, one can use this two definitions such that

$$x_t^{(0:\bar{k})} \equiv \begin{bmatrix} x_t \\ x_t^{(\bar{1}:\bar{k})} \end{bmatrix} = \begin{bmatrix} I_n \\ 0_{n\bar{k}\times n} \end{bmatrix} x_t + \begin{bmatrix} 0_{n\times n\bar{k}} & 0_{n\times n} \\ I_{n\bar{k}} & 0_{n\bar{k}\times n} \end{bmatrix} \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)}[x_t] \end{bmatrix} = e_x' x_t + T' E_t^{(1)}\left[x_t^{(0:\bar{k})}\right],$$

where last equality uses the definitions of T in Lemma 1 and e_x. □

Lemmas are taken from Ribeiro (2018, chap. 3).

C.1 Proof of Proposition 1

Individual and average expectations about the hierarchy. The Kalman filter delivers the individual conditional expectation, $E_{it}[\cdot] \equiv E[\cdot|\mathcal{I}_t^i]$, where $\mathcal{I}_t^i = \{s_{i,\tau}, \tau \leq t\}$ is the information set of individual i in period t.

The state equation is the hierarchy of expectations that is given by the state equation (42), restated for convenience:

$$x_t^{(0:\bar{k})} = \mathbf{A} x_{t-1}^{(0:\bar{k})} + \mathbf{B}\varepsilon_t, \tag{68}$$

and the agent i with the observational equation (39), also restated:

$$s_{i,t} = C_x x_t + Dv_{i,t}.$$

Note that $s_{i,t}$ is a signal about the shocks and not the whole hierarchy of expectations.

Let the selection matrix $e_x \equiv \left[\begin{array}{cc} I_n & 0_{n\times n\bar{k}} \end{array}\right]$ such that $x_t = e_x x_t^{(0:\bar{k})}$. Then, we can rewrite the signal in terms of the hierarchy as

$$s_{i,t} = Cx_t^{(0:\bar{k})} + Dv_{it}. \tag{69}$$

where $C = C_x e_x$.

Each agent i uses the Kalman filter and find the update equation given by

$$E_{i,t}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[s_{i,t} - E_{i,t-1}\left[s_{i,t}\right]\right], \tag{70}$$

where $\mathbf{K_t}$ is the Kalman gain given by

$$\mathbf{K_t} = \mathbf{P_{t/t-1}}C'\left[C\mathbf{P_{t/t-1}}C' + D\Sigma_v D'\right]^{-1}. \tag{71}$$

As usual, the mean squared error (MSE) of the one-period ahead prediction error is given by

$$\mathbf{P_{t+1/t}} = \mathbf{A}\left[\mathbf{P_{t/t-1}} - \mathbf{K_t}\left[C\mathbf{P_{t/t-1}}\right]\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon\mathbf{B}'. \tag{72}$$

For details of this deviation, see for instance Hamilton (1995, chap. 13).

Using the observational equation, (69), taking expectations and inserting in (70) one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[Cx_t^{(0:\bar{k})} + Dv_{it} - CE_{i,t-1}\left[x_t^{(0:\bar{k})}\right]\right] \tag{73}$$

Therefore, one can rewrite the equation above as

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \mathbf{K_t}C\right)E_{it-1}\left[x_t^{(0:\bar{k})}\right] + \mathbf{K_t}\left[Cx_t^{(0:\bar{k})} + Dv_{it}\right] \tag{74}$$

where $k = n(\bar{k}+1)$. Using the fact that $E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] = \mathbf{A}E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right]$ and substituting equation (68), one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \mathbf{K_t}C\right)\mathbf{A}E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \mathbf{K_t}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{K_t}C\mathbf{B}\varepsilon_t + \mathbf{K_t}Dv_{it} \tag{75}$$

We follow the literature by focusing in the stationary equilibrium. Therefore, the expectation of each individual i in the stationary equilibrium is the one which the MSE is in steady-state, i.e., agents update their forecast based on the steady-state Kalman gain. In other words, the dynamics of expectations depends only in the properties of the process they are forecasting and signals, but do not depend in the period t.

Combining equations (71-72), one can find the Riccatti equation

$$\mathbf{P_{t+1/t}} = \mathbf{A}\left[\mathbf{P_{t/t-1}} - \mathbf{P_{t/t-1}}C'\left[C\mathbf{P_{t/t-1}}C' + D\Sigma_v D'\right]^{-1} C\mathbf{P_{t/t-1}}\right]\mathbf{A}' + \mathbf{B}\Sigma_\varepsilon \mathbf{B}' \tag{76}$$

Therefore, one need to iterate this equation to find the steady-state MSE, $\bar{\mathbf{P}}$, and compute its counterpart Kalman gain, $\bar{\mathbf{K}}$. Nimark (2017) shows that if is x_t stationary process, then the expectations hierarchy about this process, $x_t^{(0:\bar{k})}$, is also stationary. This and the fact that Σ_ε is positive definite, then there exists a steady-state solution such that $\bar{\mathbf{P}} = \mathbf{P_{t+1|t}} = \mathbf{P_{t|t-1}}$ which implies the steady-state Kalman gain $\bar{\mathbf{K}} = \mathbf{K_t} = \mathbf{K_{t-1}}$ (see Hamilton; 1995, chap. 13).

Expressions (44) in the Proposition 1 are equations (71-72) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\bar{\mathbf{K}}_\mathbf{t}$.

Moreover, the average expectation is easily computed by

$$\bar{E}_t\left[x_t^{(0:\bar{k})}\right] \equiv \int_0^1 E_{it}\left[x_t^{(0:\bar{k})}\right] di = \left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}\bar{E}_{t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t \tag{77}$$

Analogously, the individual and average expectation of Proposition 1 are equations (75) and (77) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\bar{\mathbf{K}}_\mathbf{t}$.

Verify guess for $x_t^{(0:\bar{k})}$. In the first part of the proof, we found the average expectation (77) for the guessed the dynamics of the hierarchy of expectations (68).

Now we verify the guessed hierarchy and find the coefficients $(\mathbf{A}, \mathbf{B})$ consistent with the average expectation.

Substituting the average expectation from equation (77) into the expression from Lemma 2 one can find that:

$$x_t^{(0:\bar{k})} = e_x' x_t + T'\left[\left(I_k - \bar{\mathbf{K}}C\right)\mathbf{A}E_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t\right]$$

Then, using the shocks definition (38) and the fact that $x_t = e_x x_t^{(0:\bar{k})}$ one can rewrite equation above as

$$x_t^{(0:\bar{k})} = e_x'\left(A_1 e_x x_{t-1}^{(0:\bar{k})} + \varepsilon_t\right) + T'\left(I_k - \bar{\mathbf{K}}C\right)AE_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + T'\bar{\mathbf{K}}CAx_{t-1}^{(0:\bar{k})} + T'\bar{\mathbf{K}}C\mathbf{B}\varepsilon_t$$

Using Lemma 1 at period $t-1$ into equation above and rearranging:

$$x_t^{(0:\bar{k})} = \left[e_x' A_1 e_x + T' \left(I_k - \bar{\mathbf{K}} C\right) AT + T' \bar{\mathbf{K}} CA\right] x_{t-1}^{(0:\bar{k})} + \left[T' \bar{\mathbf{K}} C \mathbf{B} + e_x'\right] \varepsilon_t. \tag{78}$$

This expression shows that the expectations hierarchy is a function of its lag and structural shocks, as guessed in equation (68). Therefore, the expression above verifies that $x_t^{(0:\bar{k})}$ follows the guessed form and the square brackets terms provide identities for A and $\mathbf{B}$ in equations (43)

□

C.2 Proof of Proposition 2

This proof builds on techniques developed by Ribeiro (2018, chap. 3). The key difference is that we guess a law of motion for individual endogenous variables instead of guessing the aggregate. This allows recovering expressions for solving for $(\mathbf{Q_0}, \mathbf{Q_1})$ and then get the solution for $\mathbf{Q}$. We also consider the exogenous information case only.

The guessed law of motion for individual endogenous variables is given (40), restated for convenience:

$$Y_{i,t} = \mathbf{R} Y_{i,t-1} + \mathbf{Q_0} x_t + \mathbf{Q_1} E_{i,t}\left[x_t^{(0:\bar{k})}\right]. \tag{79}$$

Aggregating the individual law of motion (79):

$$Y_t = \mathbf{R} Y_{t-1} + \mathbf{Q_0} x_t + \mathbf{Q_1} E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] \tag{80}$$

$$= \mathbf{R} Y_{t-1} + \mathbf{Q_0} x_t + \mathbf{Q_1} T x_t^{(0:\bar{k})} \tag{81}$$

where the last equality uses Lemma 1.

By computing the individual expectation of the endogenous aggregate variables, one can find that

$$\begin{aligned} E_{it}\left[Y_t\right] &= \mathbf{R} Y_{t-1} + \mathbf{Q_0} E_{it}\left[x_t\right] + \mathbf{Q_1} T E_{it}\left[x_t^{(0:\bar{k})}\right] \\ &= \mathbf{R} Y_{t-1} + (\mathbf{Q_0} e_x + \mathbf{Q_1} T) E_{it}\left[x_t^{(0:\bar{k})}\right] \\ E_t^{(1)}\left[Y_t\right] &= \mathbf{R} Y_{t-1} + (\mathbf{Q_0} e_x + \mathbf{Q_1} T) E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] \end{aligned} \tag{82}$$

where the first equality uses that Y_{t-1} is known, second equality uses the definition of the selection matrix, e_x, and the third equality aggregates.

Similarly, the individual expectation for endogenous variables in $t+1$:

$$\begin{aligned}
E_{it}\left[Y_{t+1}\right] &= \mathbf{R}E_{it}\left[Y_t\right] + \mathbf{Q_0}E_{it}\left[x_{t+1}\right] + \mathbf{Q_1}TE_{it}\left[x_{t+1}^{(0:\bar{k})}\right] \\
&= \mathbf{R}E_{it}\left[Y_t\right] + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})E_{it}\left[x_t^{(0:\bar{k})}\right] \\
&= \mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_{it}\left[x_t^{(0:\bar{k})}\right] \\
E_t^{(1)}\left[Y_{t+1}\right] &= \mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]
\end{aligned} \tag{83}$$

where the second equality uses equations (38) and (42), and the definition of e_x. Third equation uses equation (82) and the fourth aggregates.

Finally, taking the individual expectation of the law of motion (40) in period $t+1$ implies that

$$\begin{aligned}
E_{it}[Y_{i,t+1}] &= \mathbf{R}Y_{i,t} + \mathbf{Q_0}E_{it}[x_{t+1}] + \mathbf{Q_1}E_{it}\left[E_{i,t+1}\left[x_{t+1}^{(0:\bar{k})}\right]\right] \\
&= \mathbf{R}Y_{i,t} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_{it}\left[x_t^{(0:\bar{k})}\right]
\end{aligned}$$

where the second equality uses the law of iterated expectations (which holds for i's expectation but not for the average expectation), equations (38) and (42) as before.

Aggregating the expectation above leads to

$$\begin{aligned}
\int_0^1 E_{it}[Y_{i,t+1}]di &= \mathbf{R}Y_t + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_t^{(1)}[x_t^{(0:\bar{k})}] \\
&= \mathbf{R}\left[\mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})E_t^{(1)}[x_t^{(0:\bar{k})}] \\
&= \mathbf{R}^2Y_{t-1} + \mathbf{R}\mathbf{Q_0}x_t + \left[\mathbf{R}\mathbf{Q_1} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})\right]E_t^{(1)}[x_t^{(0:\bar{k})}]
\end{aligned} \tag{84}$$

where the second equality uses equation (81). Note that $\int_0^1 E_{it}[Y_{i,t+1}]di \neq E_t^{(1)}\left[Y_{t+1}\right]$.

Substituting the guessed solution (81) and the expectations (82-84) into the system of equations (37) one can find:

$$\begin{aligned}
&F_1\left[\mathbf{R}^2Y_{t-1} + \mathbf{R}\mathbf{Q_0}x_t + \left[\mathbf{R}\mathbf{Q_1} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})\right]E_t^{(1)}[x_t^{(0:\bar{k})}]\right] + \\
&F_2\left[\mathbf{R}^2Y_{t-1} + \left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right]E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + \\
&G_1\left[\mathbf{R}Y_{t-1} + \mathbf{Q_0}x_t + \mathbf{Q_1}E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + G_2\left[\mathbf{R}Y_{t-1} + (\mathbf{Q_0}e_x + \mathbf{Q_1}T)E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]\right] + \\
&HY_{t-1} + \left[(L_1A_1 + L_2)\right]e_xE_t^{(1)}\left[x_t^{(0:\bar{k})}\right] + M_1x_t = 0_{m\times 1}
\end{aligned}$$

which can be rearranged to

$$
\begin{aligned}
&\left[F\mathbf{R}^2 + G\mathbf{R} + H\right] Y_{t-1} + \left[\left(F_1\mathbf{R} + G_1\right)\mathbf{Q_0} + M_1\right] x_t \\
&\left[F_1\left[\mathbf{R}\mathbf{Q_1} + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}\mathbf{A})\right] + F_2\left[\mathbf{R}(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (\mathbf{Q_0}A_1e_x + \mathbf{Q_1}T\mathbf{A})\right] + \right. \\
&\left. G_1\mathbf{Q_1} + G_2(\mathbf{Q_0}e_x + \mathbf{Q_1}T) + (L_1A_1 + L_2 + M)e_x\right] E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = 0_{m\times 1}
\end{aligned}
$$

which can be simplified to

$$
\begin{aligned}
&\left[F\mathbf{R}^2 + G\mathbf{R} + H\right] Y_{t-1} + \left[\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_0} + M_1\right] x_t + \\
&\left[\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A}\right. \\
&\left.\left[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + (F_1 + F_2)\mathbf{Q_0}A_1 + (L_1A_1 + L_2)\right] e_x\right] E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = 0_{m\times 1}
\end{aligned}
$$

This condition must hold for all realizations of Y_{t-1}, x_t and $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$. Therefore, all coefficients between square brackets must be zero, which leads to

$$
\begin{aligned}
&F\mathbf{R}^2 + G\mathbf{R} + H = 0_{m\times m} \\
&\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_0} + M_1 = 0_{m\times n} \\
&\left[F_1\mathbf{R} + G_1\right]\mathbf{Q_1} + F_1\mathbf{Q_1}\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q_1}T + F_2\mathbf{Q_1}T\mathbf{A} \\
&\left[(F_2\mathbf{R} + G_2)\mathbf{Q_0} + F\mathbf{Q_0}A_1 + (LA_1 + M_2)\right] e_x = 0_{m\times k}
\end{aligned}
$$

which leads to the same equations from Proposition 2. $\mathbf{R}$ can be solved using Uhlig (2001) method. For a given solution of $\mathbf{R}$, $\mathbf{Q_0}$ and $\mathbf{Q_1}$ can be solved by straightforward vectorization as discussed in the Proposition.

□

C.3 Algorithm

Algorithm. Set the initial values $(\mathbf{A}^{(0)}, \mathbf{B}^{(0)})$, a small tolerance $\epsilon > 0$ and set $i = 1$. Then, follow the steps:

1. Given $\mathbf{A} = \mathbf{A}^{(i-1)}$ and $\mathbf{B}^{(i-1)}$, compute $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ using equations (44) using standard solver for Ricatti equations. Set $\bar{\mathbf{K}}^{(i)} = \bar{\mathbf{K}}$ and $\bar{\mathbf{P}}^{(i)} = \bar{\mathbf{P}}$.

2. Given $\mathbf{A}^{(i-1)}$ and $\bar{\mathbf{K}}^{(i)}$, compute the right hand side of equations (43) and solve for $\mathbf{B}$ and $\mathbf{A}$ the equations by matrix inversion. Set $\mathbf{B}^{(i)} = \mathbf{B}$, $\mathbf{A}^{(i)} = \mathbf{A}$.

3. If $\max\left\{||\mathbf{B}^{(i)} - \mathbf{B}^{(i-1)}||, ||\mathbf{A}^{(i)} - \mathbf{A}^{(i-1)}||, ||\mathbf{P}^{(i)} - \mathbf{P}^{(i-1)}||\right\} < \epsilon$, stop iterating. Otherwise, set $i = i + 1$ and go back to step 1.

Given the solution for $\mathbf{A}$, one can use standard techniques for solving for $(\mathbf{R}, \mathbf{Q_0}, \mathbf{Q_1}, \mathbf{Q})$ using Proposition 2.

D Impulse response functions

In this section, the remaining impulse responses for the FI and ICK models estimated with the dataset including expectation data are displayed.

Figure 10: Impulse responses to preference shock

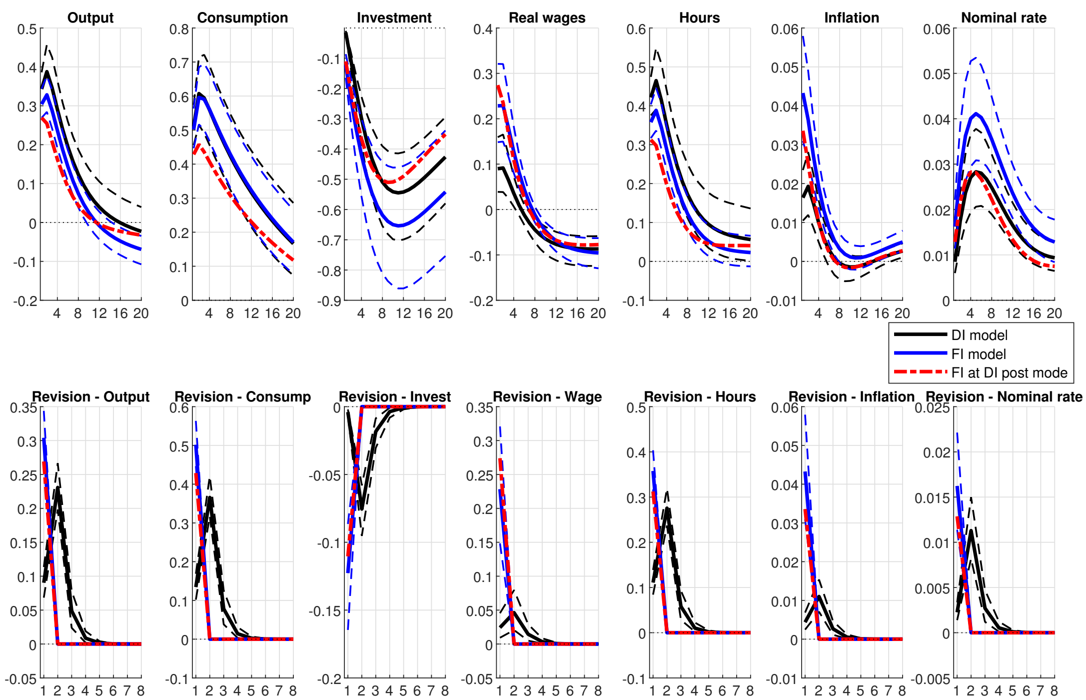

Figure 11: Impulse responses to TFP shock

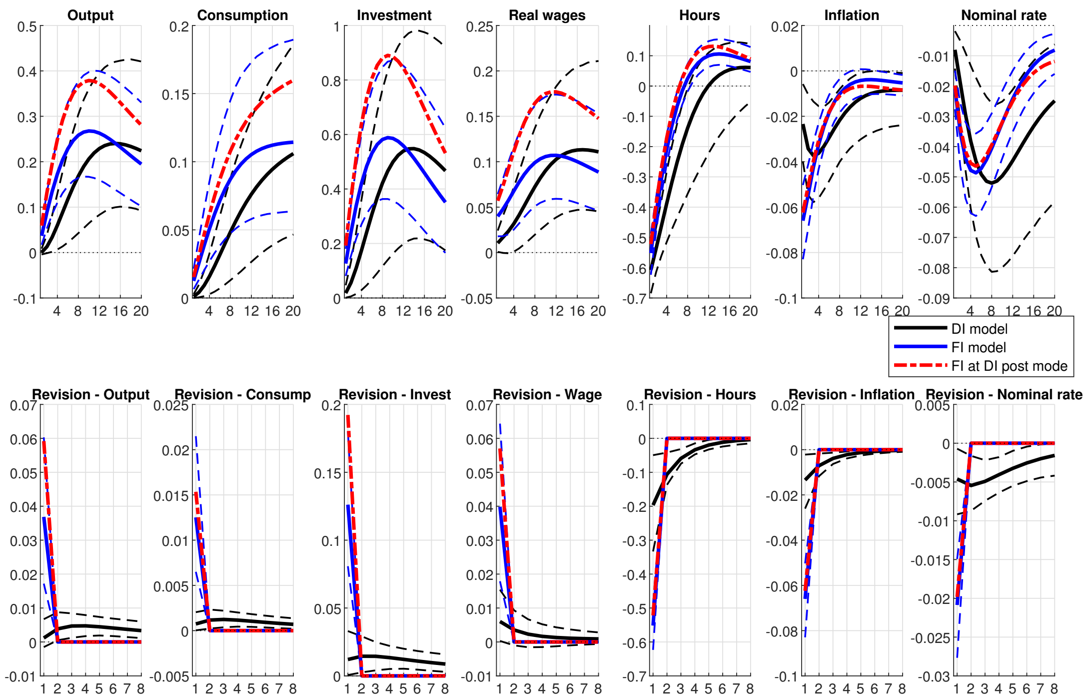

Figure 12: Impulse responses to wage mark-up shock

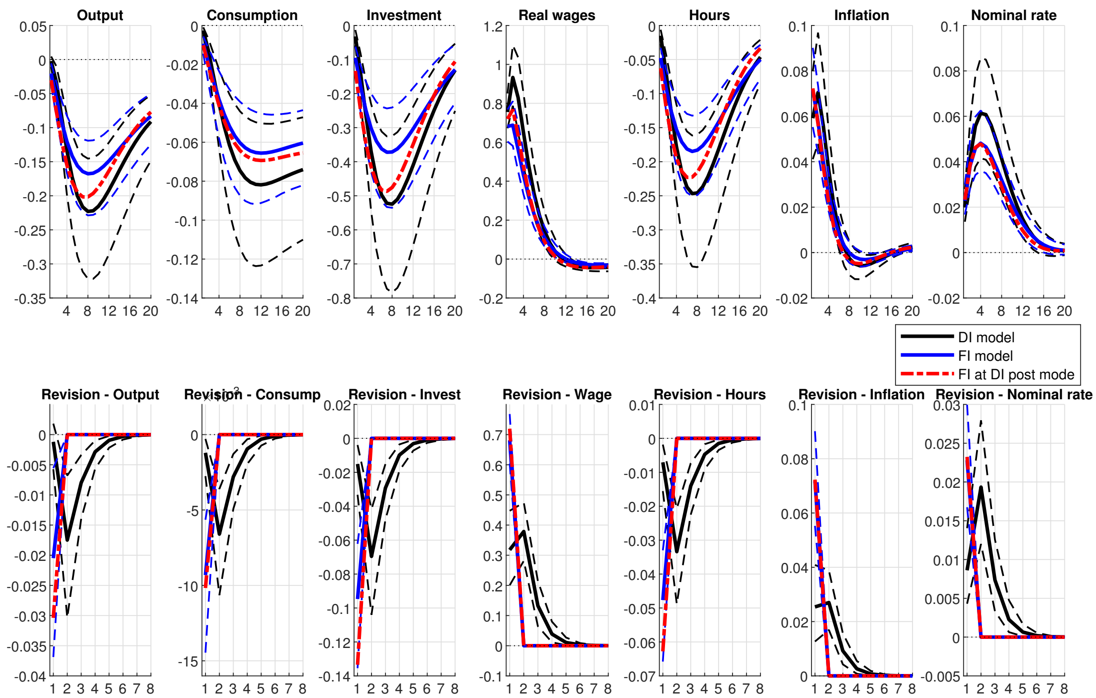

Figure 13: Impulse responses to government expenditure shock