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HW #4

1. $C = AB = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 6 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 6+9 & 5+6 \\ 24+6 & 20+4 \end{pmatrix} = \begin{pmatrix} 15 & 11 \\ 30 & 24 \end{pmatrix}$

$D = BA = \begin{pmatrix} 6 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 6+20 & 18+10 \\ 3+8 & 9+4 \end{pmatrix} = \begin{pmatrix} 26 & 28 \\ 11 & 13 \end{pmatrix}$

2. $C = AB = \begin{pmatrix} 2 & 2 \\ 4 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 6 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 12+6 & 10+4 \\ 24+9 & 20+6 \\ 18+9 & 15+6 \end{pmatrix} = \begin{pmatrix} 18 & 14 \\ 33 & 26 \\ 27 & 21 \end{pmatrix}$

• BA does NOT exist

3. $\int V\,dt = \sum_{j \le i} V_j$

$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix} \quad V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_3 \end{bmatrix} \qquad S = LV \longrightarrow \begin{bmatrix} V_1 \\ V_1 + V_2 \\ V_1 + V_2 + V_3 \\ \vdots \\ V_1 + V_2 + \cdots + V_i \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_i \end{bmatrix} \quad \therefore S_i = \sum_{j \le i} V_j$

4. • orthogonality:

$\hat{b}_1 \cdot \hat{b}_2 = \begin{vmatrix} 1 \\ 2 \\ -1 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \\ 2 \end{vmatrix} = 0 + 2 - 2 = 0 \leftarrow$ orthogonal ✓

$b_1 \cdot b_3 = \begin{vmatrix} 1 \\ 2 \\ -1 \end{vmatrix} \cdot \begin{vmatrix} 5 \\ -2 \\ 1 \end{vmatrix} = 5 - 4 - 1 = 0 \leftarrow$ orthogonal ✓

$b_2 \cdot b_3 = \begin{vmatrix} 0 \\ 1 \\ 2 \end{vmatrix} \cdot \begin{vmatrix} 5 \\ -2 \\ 1 \end{vmatrix} = 0 - 2 + 2 = 0 \leftarrow$ orthogonal ✓

Length 1?:

$\|\hat{b}_1\| = \sqrt{(1/\sqrt{6})^2 + (2/\sqrt{6})^2 + (-1/\sqrt{6})^2} = 1$ ✓

$\|\hat{b}_2\| = \sqrt{(0)^2 + (1/\sqrt{5})^2 + (2/\sqrt{5})^2} = 1$ ✓

$\|\hat{b}_3\| = \sqrt{(5/\sqrt{30})^2 + (-2/\sqrt{30})^2 + (1/\sqrt{30})^2} = 1$ ✓

5.

$$B = \begin{vmatrix} \frac{1}{\sqrt6} & \frac{2}{\sqrt6} & \frac{-1}{\sqrt6} \\ 0 & 1/\sqrt5 & 2/\sqrt5 \\ 5/\sqrt{30} & -2/\sqrt{30} & 1/\sqrt{30} \end{vmatrix} \qquad B^T = \begin{vmatrix} \frac{1}{\sqrt6} & 0 & 5/\sqrt{30} \\ \frac{2}{\sqrt6} & 1/\sqrt5 & -2/\sqrt{30} \\ \frac{-1}{\sqrt6} & 2/\sqrt5 & 1/\sqrt{30} \end{vmatrix}$$

$$BB^T = \begin{vmatrix} \cancelto{1}{\frac16 + \frac46 + \frac16} & \cancelto{0}{0 + \frac{2}{\sqrt5\cdot\sqrt6} - \frac{2}{\sqrt5\cdot\sqrt6}} & \cancelto{0}{\frac{5}{\sqrt{30}\cdot\sqrt6} - \frac{4}{\sqrt{30}\cdot\sqrt6} - \frac{1}{\sqrt{30}\sqrt6}} \\ \cancelto{0}{0 + \frac{2}{\sqrt6\cdot\sqrt5} - \frac{2}{\sqrt6\cdot\sqrt5}} & \cancelto{1}{0 + \frac15 + \frac45} & \cancelto{0}{0 - \frac{2}{\sqrt5\sqrt{30}} + \frac{2}{\sqrt5\sqrt{30}}} \\ \cancelto{0}{\frac{5}{\sqrt{30}\sqrt6} - \frac{4}{\sqrt{30}\sqrt6} - \frac{1}{\sqrt{30}\sqrt6}} & \cancelto{0}{0 - \frac{2}{\sqrt{30}\sqrt5} + \frac{2}{\sqrt{30}\sqrt5}} & \cancelto{1}{\frac{25}{30} + \frac{4}{30} + \frac{1}{30}} \end{vmatrix}$$

$$BB^T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

6.

$$A^{-1} = \frac12 \begin{vmatrix} 2 & -1 \\ -4 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1/2 \\ -2 & 3/2 \end{vmatrix}$$

$$B^{-1} = \frac{1}{-2} \begin{vmatrix} 0 & -2 \\ -1 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1/2 & -5/2 \end{vmatrix}$$

$$B^{-1}A^{-1} = \begin{vmatrix} 0-2 & 0+3/2 \\ 1/2+5 & -\frac14 - \frac{15}{4} \end{vmatrix} = \begin{vmatrix} -2 & 3/2 \\ 11/2 & -4 \end{vmatrix}$$

$$AB = \begin{vmatrix} 16 & 6 \\ 22 & 8 \end{vmatrix} \qquad (AB)^{-1} = \frac{-1}{4} \begin{vmatrix} 8 & -6 \\ -22 & 16 \end{vmatrix} = \begin{vmatrix} -2 & 3/2 \\ 11/2 & -4 \end{vmatrix} = B^{-1}A^{-1}$$

7. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad B = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

· $AB = I$

$$\hookrightarrow AB = \begin{vmatrix} \frac{a\cdot d}{ad-bc} - \frac{bc}{ad-bc} & \frac{ab-ab}{ad-bc} \\ \frac{cd-cd}{ad-bc} & \frac{ad-cb}{ad-bc} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$$

· $BA = I$

$$\hookrightarrow BA = \begin{vmatrix} \frac{ad-bc}{ad-bc} & \frac{bd-bd}{ad-bc} \\ \frac{ac-ac}{ad-bc} & \frac{ad-bc}{ad-bc} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$$

· when $ad = bc$, the matrix does <u>NOT</u> have any possible inverse

8.

I) $A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$

$\vec{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$Av = \begin{vmatrix} 4 & -2 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 2 \\ 1 \end{vmatrix} = \begin{vmatrix} 6 \\ 3 \end{vmatrix}$

$\lambda v = \lambda \begin{vmatrix} 2 \\ 1 \end{vmatrix} \longleftarrow \lambda = 3$

$\vec{x}$ = eigenvector

& $\lambda = 3$

II) $|A - \lambda I| = 0$

$\begin{vmatrix} 4 & -2 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = \begin{vmatrix} 4-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix}$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 3)(\lambda - 2) = 0$

$\left.\begin{matrix} \lambda = 3 \\ \lambda = 2 \end{matrix}\right\}$ eigenvalue

& eigenvector: $Av = \lambda v$

$(A - \lambda)v = 0$

$\begin{vmatrix} 4-2 & -2 \\ 1 & 1-2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = 0$

$\begin{vmatrix} 2 & -2 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = 0$

$2x_1 - 2x_2 = 0 \longrightarrow x_1 = x_2 \therefore$

eigenvector $= x = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

9. $A = \begin{vmatrix} 3 & 2 \\ 3 & -2 \end{vmatrix}$

$Tr = 3 + (-2) = 1$

$\det = (3)(-2) - (2)(3) = -6 - 6 = -12$

$\lambda = -3, 4$

eigenvalues $= \dfrac{1 \pm \sqrt{1^2 - 4(-12)}}{2} = \dfrac{1 \pm \sqrt{49}}{2} = \dfrac{1 \pm 7}{2} \longrightarrow = 4, -3$