

# Graph Learning

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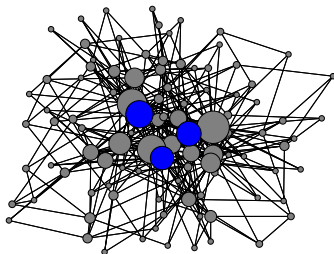
2024 – 2025



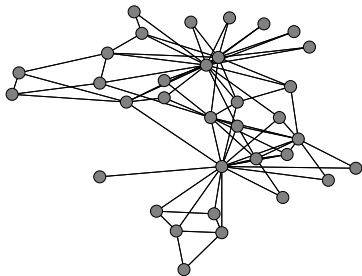
# Graph data

Graphs describe **links** between objects:

- ▶ **Social networks** → contacts
- ▶ **Web** → hyperlinks
- ▶ **Knowledge bases** → facts
- ▶ **Documents** → references
- ▶ **Commerce** → transactions
- ▶ **Biology** → interactions

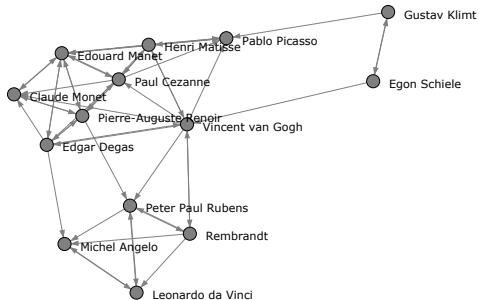


# Graphs



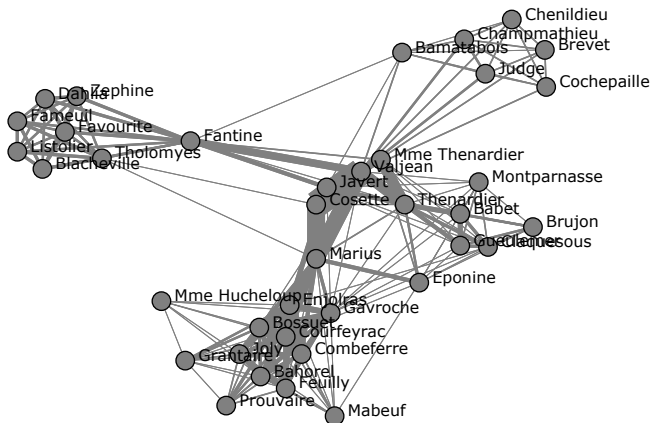
Karate club graph

# Directed graphs



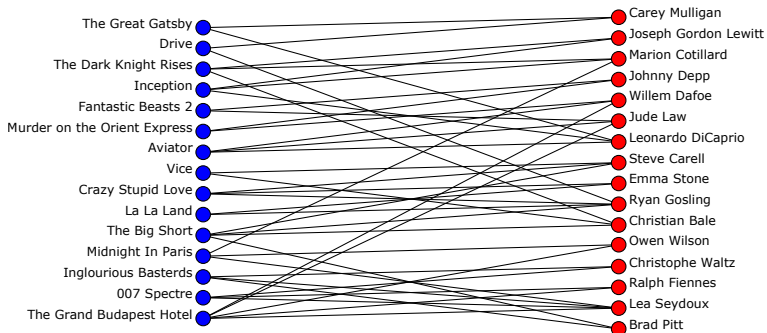
Links between some Wikipedia articles

# Weighted graphs



Co-occurrence of some characters of the novel *Les Misérables*

# Bipartite graphs



Actors starring in movies

# Tabular data

Source: Adult income dataset (Kaggle)

	gender	age	workclass	education	family	occupation
0	Male	40	State-gov	Bachelors	Not-in-family	Adm-clerical
1	Male	50	Self-emp-not-inc	Bachelors	Husband	Exec-managerial
2	Male	40	Private	HS-grad	Not-in-family	Handlers-cleaners
3	Male	50	Private	11th	Husband	Handlers-cleaners
4	Female	40	Private	Masters	Wife	Exec-managerial
...	...	...	...	...	...	...
31973	Male	20	Private	Bachelors	Not-in-family	Prof-specialty
31974	Male	60	Self-emp-inc	HS-grad	Husband	Transport-moving
31975	Male	60	Private	Assoc-voc	Husband	Craft-repair
31976	Female	50	Private	HS-grad	Wife	Adm-clerical
31977	Female	30	Private	Some-college	Other-relative	Machine-op-inspct

31978 rows × 6 columns

# Tabular data as bipartite graph

	gender_Female	gender_Male	age_20	age_30	age_40	age_50	age_60	age_70	age_80	age_90	...
0	0	1	0	0	1	0	0	0	0	0	...
1	0	1	0	0	0	1	0	0	0	0	...
2	0	1	0	0	1	0	0	0	0	0	...
3	0	1	0	0	0	1	0	0	0	0	...
4	1	0	0	0	1	0	0	0	0	0	...
...	...	...	...	...	...	...	...	...	...	...	...
31973	0	1	1	0	0	0	0	0	0	0	...
31974	0	1	0	0	0	0	1	0	0	0	...
31975	0	1	0	0	0	0	1	0	0	0	...
31976	1	0	0	0	0	1	0	0	0	0	...
31977	1	0	0	1	0	0	0	0	0	0	...

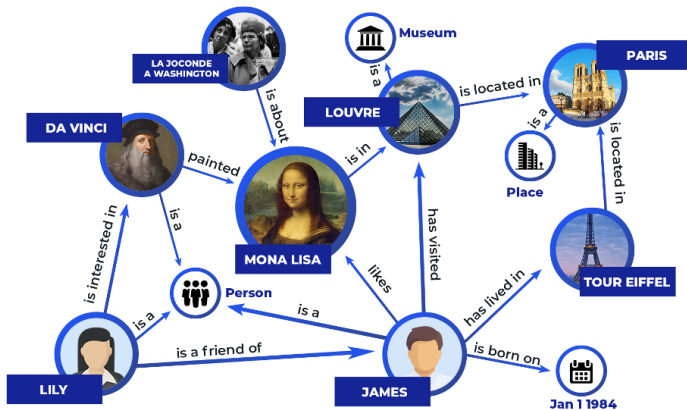
31978 rows x 56 columns

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```
> pd.get_dummies(dataframe)    # one-hot encoding
```



# Attributed graphs



## Large graphs are sparse

Dataset	#nodes	#edges	Density
Flights	2,939	30,500	$\approx 10^{-3}$
Amazon products	335k	925k	$\approx 10^{-5}$
Actors	382k	33M	$\approx 10^{-4}$
Wikipedia	12M	378M	$\approx 10^{-6}$
Twitter	42M	1.5G	$\approx 10^{-6}$
Friendster	68M	2.5G	$\approx 10^{-7}$

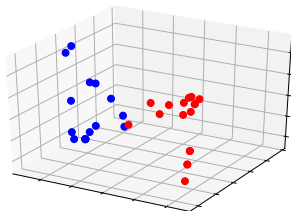
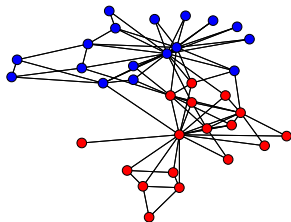
# Machine learning on graphs

## **Supervised** learning

- ▶ Classification
- ▶ Regression

## **Unsupervised** learning

- ▶ Ranking
- ▶ Clustering
- ▶ Embedding
- ▶ Link prediction
- ▶ Anomaly detection



# Outline of the course

1. Sparse matrices  
Graph structure
2. PageRank
3. Clustering
4. Hierarchical clustering
5. Heat diffusion
6. Spectral embedding
7. Graph neural networks

Each course = **lecture** + **quiz** + **lab**

The **quiz** must be done in class

The **lab** is due in 3 working days

# Information & Grading

- ▶ **Moodle**

<https://moodle.r2.enst.fr/>

For general information, slides, notebooks, etc.

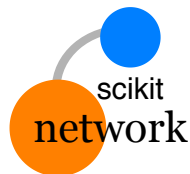
- ▶ **Grading**

Exam (on Moodle, 50%)

Quizzes\* (25%)

Labs\* (25%)

\* Starting next week



A Python library for graph analysis

- ▶ **easy** to install `pip install scikit-network`
- ▶ **easy** to use `algorithm.fit(data)`
- ▶ **fast** and **memory-efficient**

Relies on **NumPy** and **SciPy** only

BSD license

# Benchmark



NetworkX  
Network Analysis in Python




igraph




graph-tool

Test on the **Orkut graph** (3M nodes, 117M edges)

## Memory

NetworkX	iGraph	graph-tool	scikit-network
	18G	10G	<b>1G</b>

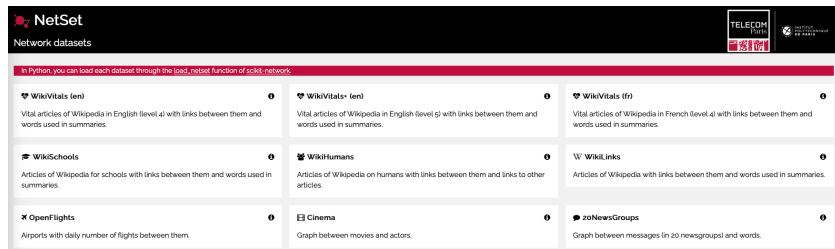
## Running time

	iGraph	graph-tool	scikit-network
PageRank	3 min 56 s	<b>45 s</b>	48 s
Louvain	33 min		<b>2 min</b>

# The NetSet collection

A collection of network datasets maintained by Télécom Paris

<https://netset.telecom-paris.fr>



The screenshot shows the NetSet website interface. At the top, there is a black header with the 'NetSet' logo on the left and the 'TELECOM Paris' logo on the right. Below the header, a pink banner contains the text: 'In Python, you can load each dataset through the load\_netset function of scikit-network.' The main content area is a grid of dataset cards. Each card includes an icon, the dataset name, a brief description, and a small circular icon with a lowercase 'i'.

Dataset Name	Description
WikiVitals (en)	Vital articles of Wikipedia in English (level 4) with links between them and words used in summaries.
WikiVitals+ (en)	Vital articles of Wikipedia in English (level 5) with links between them and words used in summaries.
WikiVitals (fr)	Vital articles of Wikipedia in French (level 4) with links between them and words used in summaries.
WikiSchools	Articles of Wikipedia for schools with links between them and words used in summaries.
WikiHumans	Articles of Wikipedia on humans with links between them and links to other articles.
WikiLinks	Articles of Wikipedia with links between them and words used in summaries.
OpenFlights	Airports with daily number of flights between them.
Cinema	Graph between movies and actors.
20NewsGroups	Graph between messages (in 20 newsgroups) and words.

Easy to import with scikit-network!