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Revenue monotonicity in deterministic, dominant-strategy combinatorial auctions

Baharak Rastegari *, Anne Condon, Kevin Leyton-Brown

Department of Computer Science, University of British Columbia, 2366 Main Mall, Vancouver, B.C., Canada, V6T 1Z4

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ABSTRACT

In combinatorial auctions using VCG, a seller can sometimes increase revenue by *dropping* bidders. In this paper we investigate the extent to which this counterintuitive phenomenon can also occur under other deterministic, dominant-strategy combinatorial auction mechanisms. Our main result is that such failures of "revenue monotonicity" can occur under *any* such mechanism that is *weakly maximal*—meaning roughly that it chooses allocations that cannot be augmented to cause a losing bidder to win without hurting winning bidders—and that allows bidders to express arbitrary known single-minded preferences. We also give a set of other impossibility results as corollaries, concerning revenue when the set of goods changes, false-name-proofness, and the core.[1]

1. Introduction

In combinatorial auctions (CAs), multiple goods are sold simultaneously and bidders are allowed to place bids on bundles, rather than just on individual goods. These auctions are interesting in settings where bidders have non-additive—and particularly, superadditive—values for goods. (For an introduction, see Cramton et al. [12].) As with other applications of mechanism design, the design of combinatorial auctions has tended to focus on the theoretical properties that a given design can guarantee.

It is often desired for an auction mechanism to offer bidders the dominant strategy of truthfully revealing their private information to the mechanism. (By the revelation principle, the assumption that bidders declare *truthfully* is without loss of generality; however, not all mechanisms offer dominant strategies.) Another useful property is revenue monotonicity, the guarantee that the seller's revenue weakly increases as the number of bidders grows. Revenue monotonicity is important because, without it the auctioneer has an incentive to disqualify bidders to increase revenue. Similarly, in such auctions a bidder might find it profitable to place pseudonymous bids in order to reduce the seller's revenue.

We begin with a discussion of related work in the literature concerning dominant strategy truthfulness and revenue monotonicity.

1.1. Dominant strategy implementation

Considerable research has characterized the space of social choice functions that can be implemented in dominant strategies. A classic result of Roberts [26] showed that for bidders with unrestricted quasilinear valuations, affine maximizers are

* Corresponding author.
E-mail addresses: baharak@cs.ubc.ca (B. Rastegari), condon@cs.ubc.ca (A. Condon), kevinlb@cs.ubc.ca (K. Leyton-Brown).

[1] We previously published a six-page preliminary version of our main result at a conference (Rastegari et al., 2007) [24]. Among other differences, this work considered a very limited version of our weak maximality condition that can be understood as requiring Pareto efficiency with respect to bidder valuations (i.e., ignoring payments).

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the only dominant-strategy-implementable social choice functions. Subsequent work has focused mainly on restricted classes of preferences [27,17,9,29,11].

The VCG mechanism [31,10,16] has gained substantial attention in mechanism design literature because of its strong theoretical properties. In particular, it offers dominant strategies and achieves efficiency. Indeed, no substantially different (technically, no non-Groves) mechanism can guarantee these properties for agents with general quasilinear valuations [15]. VCG is computationally intractable,[2] thus, there have been many attempts to design feasible dominant strategy truthful mechanisms, even if for restricted classes of valuations. Archer and Tardos [2], Andelman and Mansour [1] and Mu'alem and Nisan [19] studied the design of dominant strategy truthful mechanisms for combinatorial settings with single-parameter agents: agents whose private information can be encoded in a single positive real number. Babaioff et al. [6,7] studied CA design in single-value domains under the further assumption that each agent has the same value for all desired outcomes. Yokoo et al. [34,35,33] studied the design of dominant strategy truthful mechanisms for settings in which bidders may submit multiple bids using pseudonyms.

1.2. Revenue monotonicity

A main concern for auctioneers is an auction's revenue. In this paper we focus on a particular revenue-related property: that a seller's revenue from an auction is guaranteed weakly to increase as the number of bidders grows. Ausubel and Milgrom [4] dubbed this property *bidder monotonicity*. In order to emphasize that we are concerned with monotonicity of revenue—as compared to some other auction property—we prefer the term *bidder revenue monotonicity*. This can be contrasted with e.g., *good revenue monotonicity*, the property that a seller's revenue from an auction is guaranteed to weakly increase as the number of goods at the auction grows. We are primarily interested in the former property; thus, as a shorthand we abbreviate bidder revenue monotonicity simply as *revenue monotonicity*.

It is easy to see that even VCG is not (bidder) revenue monotonic. Following an example due to Ausubel and Milgrom [5], consider an auction with three bidders and two goods for sale. Suppose that bidder 2 values the bundle of both goods at \$2 billion whereas bidder 1 and bidder 3 value the first and the second goods at \$2 billion respectively. The VCG mechanism awards the goods to bidders 1 and 3 for the price of zero, yielding the seller zero revenue. However, in the absence of either bidder 1 or bidder 3, the auction would generate \$2 billion in revenue.

Different approaches have been proposed for understanding the extent of revenue non-monotonicity problems. One approach has considered VCG's performance under restricted valuation classes. Say that *the combined valuation of bidders satisfies bidder submodularity* if and only if for any bidder i and any two sets of bidders S and S' with $S \subseteq S'$, it is the case that $V^*_{S\cup\{i\}} - V^*_S \geqslant V^*_{S'\cup\{i\}} - V^*_{S'}$, where V^*_S is the maximum social welfare achievable under S. Ausubel and Milgrom [4] showed that if the combined valuation of bidders satisfies bidder submodularity then VCG is guaranteed to be revenue monotonic. Bidder submodularity is implied by the *goods are substitutes* condition (see, e.g., Ausubel and Milgrom [4] for a definition). However, in many application domains for which combinatorial auctions have been proposed, goods are not substitutes and bidders' valuations exhibit complementarity. We therefore wish to investigate revenue monotonicity in domains where arbitrary complementarity may exist. The simplest such domain is that of known single-minded bidders. Roughly speaking, a known single-minded bidder desires only a specific, known bundle, valuing that bundle and all its supersets at the same amount and any other bundle at zero. Note that VCG is not revenue monotonic in this domain, as demonstrated in the above example.

Day and Milgrom [13] showed that, under the assumption that bidders follow a so-called "best-response truncation strategy," auctions that always select an outcome that is in the core with respect to declared valuations (so-called core-selecting auctions) are revenue monotonic when they select a core outcome that minimizes the seller's revenue. (A precursor to this result also appeared in Ausubel and Milgrom [4].) Thus, the ascending proxy auction proposed by Ausubel and Milgrom [4] and the clock-proxy auction proposed by Ausubel et al. [3] are both revenue monotonic when agents play such a best-response strategy, but do not offer dominant strategies. Other mechanisms that have been proposed for use in practice similarly lack dominant strategies (see, e.g., Bernheim and Whinston [8], Rassenti et al. [23] and Porter et al. [22]); we are not aware of any result in the literature that shows whether or not they are revenue monotonic.

While revenue monotonicity *is* a feature of some auction mechanisms that have been deployed in practice, dominant strategies *are* (perhaps surprisingly) uncommon. This fact underscores the practical importance of revenue properties like revenue monotonicity, while pointing out that auctioneers are willing to sacrifice the strategic simplicity of dominant strategies.

1.3. Overview of our work

In our work, we ask whether there exists a combinatorial auction mechanism that allows bidders to express arbitrary known single-minded preferences and that is both dominant strategy truthful and revenue monotonic.

If dominant strategy truthfulness and revenue monotonicity are the only conditions we require, it is easy to answer the above question in the affirmative. Specifically, we can offer all goods as one indivisible bundle using a second-price

[2] Indeed, VCG has a host of other drawbacks too; see, e.g., Rothkopf [28].

sealed-bid auction. However, this mechanism is unappealing, because it is combinatorial only in a degenerate sense. If we want to require the mechanism to allocate the goods more sensibly than through a static pre-bundling, we must ask for some further property. Very restrictively, we could require efficiency. In this work, we exchange efficiency for the much more inclusive notion of *weak maximality*. While efficiency requires the mechanism to choose an allocation that maximizes social welfare, weak maximality requires that the mechanism choose an allocation that cannot be augmented to make some bidder better off, while making none worse off.

Our main result roughly states that, when bidders are allowed to express arbitrary known single-minded preferences, no deterministic, dominant-strategy combinatorial auction mechanism is revenue monotonic (we further assume "weak maximality", "consumer sovereignty" and "participation"; see Section 2). As noted above, none of the auctions in practical use offer dominant strategies; in contrast, at least some are revenue monotonic (given behavioral assumptions). Our impossibility result helps to explain this phenomenon: if revenue monotonicity is an important property in practice, deployed deterministic mechanisms will be unable to offer dominant strategies.[3]

In Section 2 we define terminology for discussing combinatorial auction mechanisms and their properties. We present our main impossibility result in Section 3. As corollaries, in Section 4 we prove similar impossibility results concerning the existence of mechanisms that yield weakly increasing revenue as the set of goods (rather than bidders) increases, that are false-name-proof (i.e., that offer truthful dominant strategies when agents are able to submit multiple bids under different identities), and that choose outcomes guaranteed to belong to the core.

2. Preliminaries

In this section we define terminology for discussing combinatorial auction mechanisms. To keep our presentation concise and convey intuition, we provide informal and intuitive definitions and defer many technicalities to the appendix. For the same purpose, we restrict our attention to the most restricted class of bidders that our result applies to, *known single-minded bidders*. Note that an impossibility result, such as ours, becomes stronger under a more restricted setting.

2.1. Bidders

We use N and G to denote the set of bidders present in an auction and the set of goods available for sale, respectively. Auction mechanisms generally work for any given set of bidders and goods. In this work we want to reason about changing the setting to include or exclude bidders or goods. Therefore, we define a universal set of possible bidders $\mathbb{N}$ to be $\{1, 2, \ldots, n\}$, and a universal set of goods $\mathbb{G}$. In any auction, $N \subseteq \mathbb{N}$ and $G \subseteq \mathbb{G}$.

A *valuation function* describes the values that a bidder holds for subsets of the set of goods in G. Let v_i denote bidder i's valuation function.

A *valuation profile* is an n-tuple $v = (v_1, \ldots, v_n)$, where, for every participating bidder i, v_i is a valuation function. We adopt the convention that valuation profiles always have one entry for every potential bidder, regardless of the number of bidders who participate in the auction. We use the symbol $\varnothing$ in such tuples as a placeholder for each non-participating bidder (i.e., each bidder $i \notin N$). Let v_{-i} denote $(v_1, \ldots, v_{i-1}, \varnothing, v_{i+1}, \ldots, v_n)$. Note that if $i \notin N$, then $v = v_{-i}$.

If asked to reveal her valuation, a bidder may not tell the truth. Denote the declared valuation function of a (participating) bidder i as $\widehat{v}_i$. Let $\widehat{v}$ be the declared valuation profile. Use the same notation to describe declared valuation profiles as valuation profiles (e.g., all declared valuation profiles are n-tuples), and furthermore write $(v_i, \widehat{v}_{-i})$ to denote $(\widehat{v}_1, \ldots, \widehat{v}_{i-1}, v_i, \widehat{v}_{i+1}, \ldots, \widehat{v}_n)$.

The class of *unknown single-minded bidders*, or simply *single-minded bidders*, was first introduced by Lehmann et al. [18]. Informally, a participating bidder i is single-minded if there exists a particular bundle b_i such that bidder i has a nonzero valuation only for bundles that contain b_i, and values all these bundles equally.

Definition 1 *(Single-minded bidder).* Bidder i is *single-minded* if her valuation function is defined as

$$v_i(b'_i) = \begin{cases} \mathit{v}_i & b'_i \supseteq b_i, \\ 0 & \text{otherwise}, \end{cases}$$

where $\mathit{v}_i > 0$ and $b_i \subseteq \mathbb{G}$.

A single-minded bidder i's valuation can be characterized by two parameters: $\langle b_i, \mathit{v}_i\rangle$. Therefore, we use $\langle b_i, \mathit{v}_i\rangle$ and v_i interchangeably when a bidder is single-minded. We let $\widehat{\langle b_i, \widehat{\mathit{v}}_i\rangle}$ denote the *declared valuation* of single-minded bidder i.

Known single-minded bidders, an even more restricted bidder model, was first introduced by Mu'alem and Nisan [19]. A bidder i is known single-minded if she is single-minded and her bundle of interest b_i is known to the mechanism designer. The valuation of a known single-minded bidder i can be characterized by the single parameter v_i, representing i's valuation for any weak superset of bundle b_i. Thus in this case we use v to denote a valuation profile for a group of single-minded bidders, $\widehat{\mathit{v}}_i$ to denote the declared valuation of a participating bidder i, and $\widehat{\mathit{v}}$ to denote a tuple consisting of declared valuations for participating bidders and $\varnothing$ symbols for non-participating bidders.

[3] In contrast, we show in [25] that there exist *randomized*, dominant-strategy mechanisms that are revenue monotonic.

2.2. Combinatorial auction mechanisms

A combinatorial auction mechanism is capable of selling multiple goods simultaneously and allows bidders to place bids on bundles. In a known single-minded setting, the mechanism already knows the bundle each bidder is interested in. Thus, the bidders only have to express their value for their bundles of interest.

A *deterministic direct Combinatorial Auction* (*CA*) *mechanism* (CA mechanism) $M^{(ksm)}$ defined for known single-minded bidders over $\mathbb{N}$ and $\mathbb{G}$ produces an allocation of goods $a = (a_1(\widehat{v}), \ldots, a_n(\widehat{v}))$ and a payment vector $p = (p_1(\widehat{v}), \ldots, p_n(\widehat{v}))$ for all $G \subseteq \mathbb{G}$ and $N \subseteq \mathbb{N}$, given bidders' declared valuation profile $\widehat{v}$. Note that for allocations to be feasible, the following must hold: (i) $\bigcup_i a_i(\widehat{v}) \subseteq G$, and (ii) $a_i(\widehat{v}) \cap a_j(\widehat{v}) = \emptyset$ if $i \neq j$. Furthermore, a CA mechanism does not allocate goods to, and does not require a payment from, a bidder who is not participating in the auction. That is, $a_i(\widehat{v}) = \emptyset$ and, $p_i(\widehat{v}) = 0$ if $i \notin N$.

We refer to (a, p) as the outcome of a CA mechanism. We refer to a_i and p_i as bidder i's allocation and payment functions respectively. Note that the allocation and payment functions may depend not only on $\widehat{v}$, but also on b. We denote by $\mathbb{A}_{N,G}$ the set of all possible ways of allocating goods. The seller may fail to allocate some or all of the goods. For any given allocation $\mathrm{a} \in \mathbb{A}_{N,G}$, we denote by a_i the set of goods that are allocated to bidder i under a. Whenever $\widehat{v}$ can be understood from the context, we refer to $a_i(\widehat{v})$ and $p_i(\widehat{v})$ by a_i and p_i, respectively. If $\widehat{v}_i(a_i) > 0$, i.e. if $a_i \supseteq b_i$, we say that bidder i "wins". A losing bidder is a bidder who does not win. We assume that bidders have quasilinear utility functions; that is, bidder i's utility for bundle a_i is $v_i(\mathrm{a}_i) - p_i$.

In the next section, we give intuitive and informal definitions for various properties that we would like to require of CA mechanisms. We provide references to formal definitions in Appendix A throughout the section.

2.2.1. Dominant strategy truthfulness

In mechanism design, it is especially desirable for a mechanism to give rise to dominant strategies, as then there is no need for bidders to reason about each others' behavior in order to maximize their utilities. A CA mechanism M is *truthful* if bidders declare their true valuations to the mechanism in equilibrium. M is *dominant strategy* (*DS*) *truthful* if it is a dominant strategy for every bidder to reveal her true preferences (see Definition 19 in Appendix A). Observe that the revelation principle tells us that any social choice function that can be implemented in dominant strategies can also be implemented truthfully in dominant strategies. This means that our conflation of dominant strategies with truth-telling is without loss of generality.

2.2.2. Participation

It is natural to require that no bidder be made to pay unless she wins. A CA mechanism M satisfies *participation* if and only if any bidder i for whom $v_i(a_i) = 0$ makes a zero payment to the mechanism (see Definition 20 in Appendix A). Unlike the property of individual rationality (IR), which requires roughly that no bidder has to make a payment more than her value for the bundle she gets, participation does not constrain the payments of bidders who win. Participation is therefore a weaker condition than IR.

2.2.3. Efficiency

As discussed earlier, one of the most commonly-desired properties for an auction mechanism is efficiency. A CA mechanism is *efficient* if its chosen allocation in equilibrium maximizes the social welfare (see Definition 21 in Appendix A).

2.2.4. Revenue monotonicity

The revenue of an auction mechanism is the sum over all the payments made by the bidders to the auctioneer. An auction mechanism is *revenue monotonic* if and only if the auctioneer never collects more money when a bidder is dropped (see Definition 22 in Appendix A). Our goal in this paper is to investigate whether broad families of dominant-strategy truthful CA mechanisms satisfy revenue monotonicity.

2.2.5. Weak maximality

Consider the *set protocol*, a simple mechanism that offers all goods as one indivisible bundle and uses the second price sealed-bid auction to determine the winner and the payment. It is trivial to show that the set protocol is dominant-strategy truthful and satisfies participation. This mechanism is also revenue monotonic since dropping a bidder cannot cause the second-price bid to increase. However, the set protocol is a combinatorial auction only in a degenerate sense: it pre-bundles all goods and treats them as a single indivisible good. If we are not happy with this solution, we need to require a property that rules out mechanisms like the set protocol.

One popular option is efficiency (see Definition 21). As mentioned above, the only dominant-strategy truthful and efficient CA mechanisms are Groves mechanisms [15]. We have already seen that VCG fails revenue monotonicity; therefore, efficiency may be too strong a condition to require.

We propose instead requiring *weak maximality*. Intuitively, weak maximality requires that the mechanism does not withhold any good, or give it away to a bidder who does not value it, when it is sufficiently valued by a losing bidder. Weak maximality is a less standard property than those we have discussed so far. Thus, we provide a more formal definition of it here. For the fully formal treatment, see Definition 23 in Appendix A.

Consider a truthful CA mechanism $M^{(ksm)}$ defined for known single-minded bidders, with set of bidders N, set of goods G and set of (known) bundles $b = (b_1, \dots, b_n)$. M is *weakly maximal with respect to bidder i* if and only if there exists a set of nonnegative finite constants $\alpha_{N,G,b,i,g}$,[4] $\forall g \in G$, such that $M^{(ksm)}$ always chooses an allocation a where either:

1. $v_i(\mathrm{a}_i) > 0$; or
2. for all allocations a′ with $v_i(\mathrm{a}'_i) > \alpha_{N,G,b,i,\mathrm{a}'_i}$, $|\mathrm{a}'_i| = 1$, and $\mathrm{a}'_j = \mathrm{a}_j \setminus \mathrm{a}'_i$ for all $j \neq i$, for some j, $v_j(\mathrm{a}'_j) < v_j(\mathrm{a}_j)$.

Equivalently, we could write condition (1) as $b_i \subseteq \mathrm{a}_i$, and condition (2) as $b_j \subseteq \mathrm{a}_j$ and $b_j \not\subseteq \mathrm{a}'_j$.

When a mechanism is weakly maximal with respect to bidder i, if i's valuation for a single good g is at least $\alpha_{N,G,b,i,g}$ and if g is not allocated to any other bidder who gets additional value for it, then i is awarded g. The quantities $\{\alpha_{N,G,b,i,g} \mid g \in G\}$ can be thought of as bidder- and item-specific reserve prices. Observe that the set protocol does not satisfy weak maximality with respect to any bidder, because the winning bidder may be given a good g that she does not value, even when there exists another bidder i who values g and bids more than an arbitrary constant amount.

Many interesting mechanisms are weakly maximal. (The definitions of the concepts that follow, along with the formal statement of the claims and the proofs are given in Appendix B.) Efficient mechanisms, a broad class of affine-maximizing mechanisms, and all mechanisms that are strongly Pareto efficient with respect to bidders' valuations are weakly maximal. Indeed, we can show that all these mechanisms satisfy an even stronger constraint, *maximality*. Note that requiring a weaker constraint makes our impossibility result stronger.

Our weak maximality property is related to the *reasonableness* condition of Nisan and Ronen [21], which says that whenever an item is desired by a single agent only, that agent must receive the item. If $\alpha_{N,G,b,i,g}$'s are all set to zero, then weak maximality implies reasonableness. However, reasonableness does not imply weak maximality. Consider the case where there are exactly two bidders who desire an item. Reasonableness still holds even if that item is never allocated to either of the agents, regardless of their declarations; however, such an allocation rule would violate weak maximality.

2.2.6. Consumer sovereignty

A CA mechanism M satisfies *consumer sovereignty* if any bidder can win any bundle she desires by bidding at or above a finite amount that may depend on the declarations of the other bidders (see Definition 24 in Appendix A). Practical auctions usually satisfy consumer sovereignty; see Feigenbaum et al. [14], upon which we base our definition.

It is useful at this point to contrast consumer sovereignty with weak maximality. Consumer sovereignty implies that e.g. a bidder i surely wins the good she desires, g, by bidding at or above some finite amount. In contrast, weak maximality does not imply that i necessarily wins g if she values it at or above the bidder-specific reserve price.

2.3. Criticality for known single-minded bidders

Consider a mechanism defined for known single-minded bidders. We say that the mechanism *offers critical values to bidder i* if two properties hold. First, bidder i wins whenever she bids more than some critical value that depends only on the other bidders' declarations, and loses whenever she bids less. Second, bidder i's payment is equal to this critical value if she wins, and is zero otherwise. A mechanism defined for known single-minded bidders satisfies criticality if it offers critical values to all bidders.

Definition 2 *(Criticality).* A CA mechanism $M^{(ksm)}$ defined for known single-minded bidders satisfies *criticality* if and only if $\forall b = (b_1, b_2, \dots, b_n) \in (2^G)^n$ and $\forall \widehat{v}_{-i}$, there exists a *critical value* $cv_{b,i}(\widehat{v}_{-i}) \in \mathbb{R} \cup \{\infty\}$ where:

- if $\widehat{v}_i > cv_{b,i}(\widehat{v}_{-i})$, i wins and pays $cv_{b,i}(\widehat{v}_{-i})$;
- if $\widehat{v}_i < cv_{b,i}(\widehat{v}_{-i})$, i loses and pays 0.

Whenever b is understood from the context, we drop it from the subscript, writing $cv_i(\widehat{v}_{-i})$. The following result can be straightforwardly obtained from necessary and sufficient conditions for dominant-strategy truthfulness (see, e.g., Mu'alem and Nisan [19] and Nisan [20]).

Theorem 3. *(Following Lehmann [18] and Mu'alem and Nisan [19].) Any CA mechanism defined for known single-minded bidders that satisfies dominant-strategy truthfulness and participation also satisfies criticality.*

The following corollary, which immediately follows from Theorem 3 and consumer sovereignty, is used in the proof of our main theorem.

[4] The reader may be surprised that these constants depend on N, G and b as well as i and g. A full explanation requires the formal details developed in Appendix A. Intuitively, the constants are allowed to vary for different sets of participating bidders $N \in \mathbb{N}$, available goods $G \in \mathbb{G}$, and known bundles $b \in (2^G)^n$.

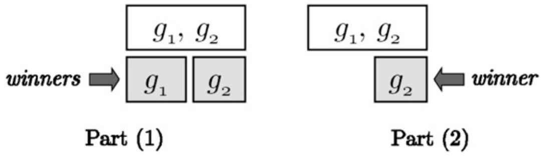

Fig. 1. A high-level illustration of Theorem 5: Given $\langle\{g_1\}, v_1\rangle$, $\langle\{g_1, g_2\}, v_2\rangle$ and $\langle\{g_3\}, v_3\rangle$—$v_i$'s as constructed in the proof of the theorem—(1) Bidders 1 and 3 win bundle $\{g_1\}$ and bundle $\{g_2\}$ respectively and each pay more than a predefined constant amount, (2) bidder 3 wins bundle $\{g_2\}$ and pays more than the sum of the payments in part (1).

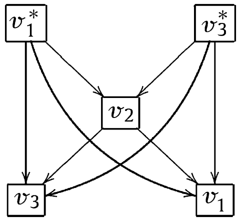

Fig. 2. Illustration of dependencies between the constructed values in the proof of Theorem 5: For example, the edge from v_1^* to v_3 shows that v_3 depends on v_1^*; the lack of edges to v_1^* shows that v_1^* can be set freely.

Corollary 4. *Any CA mechanism defined for known single-minded bidders that satisfies dominant-strategy truthfulness, participation, and consumer sovereignty offers finite critical values to all bidders.*

3. Impossibility of revenue monotonicity

In this section we turn to our main claim, that no CA mechanism can be revenue monotonic if it satisfies our desired properties of dominant-strategy truthfulness, participation, consumer sovereignty and weak maximality with respect to at least two bidders. We begin by giving an example of how an existing inefficient mechanism fails revenue monotonicity, and then prove the general result.

3.1. An example with an inefficient mechanism

In the introduction we gave a well-known example showing that VCG does not satisfy revenue monotonicity. Now we show—we believe, for the first time—that another widely-studied mechanism also fails revenue monotonicity, even though it does not have an efficient allocation rule. The existence of such an example is not surprising given our impossibility theorem; however, it offers intuition for what follows.

Lehmann et al. [18] introduced an inefficient, dominant-strategy truthful, direct CA mechanism for single-minded bidders. Naming it after its authors, we call the mechanism LOS. Like VCG, LOS satisfies participation and consumer sovereignty. LOS is also strongly Pareto efficient with respect to bidders' valuations (see Definition 28 in Appendix A), and so satisfies weak maximality with respect to all bidders.

Let $ppg_i = v_i/|b_i|$, bidder i's declared price per good. LOS ranks bids in a list L in decreasing order of ppg, and then greedily allocates bids starting from the top of L. Thus, each bidder i's bid is granted if b_i does not conflict with any previously allocated bids. If i's bid is allocated she is made to pay $|b_i| * v_{inext}/|b_{inext}|$, where *inext* is the first bidder following i in L whose bid was denied but would have been allocated if i's bid were not present. Bidder i pays zero if she does not win or if there is no bidder *inext*.

Consider three bidders $\{1, 2, 3\}$ and two goods $\{g_1, g_2\}$. Let the true valuations of bidder 1, 2 and 3 be $\langle\{g_1\}, v_1\rangle$, $\langle\{g_1, g_2\}, v_2\rangle$ and $\langle\{g_2\}, v_3\rangle$, respectively. Now consider the following conditions on the bidders' valuations:

(1) $v_1 > v_3 > v_2/2$;
(2) $v_2 > 0$.

It is possible to assign values to the v_i's in a way that satisfies both conditions: e.g., $v_1 = 5$, $v_2 = 4$ and $v_3 = 3$.

We will demonstrate that the auctioneer's revenue under LOS can be increased by dropping a bidder whenever the bidders and their valuations are as described above. From Condition 1, $ppg_1 > ppg_3 > ppg_2$ and therefore bidders 1 and 3 win. Each pays zero, so the total revenue is zero. To see this, note that the next bidder in the list after bidder 1 whose bid conflicts with b_1 is bidder 2. However, bidder 2 would not win even if bidder 1 were not present, since b_2 also conflicts with b_3. Therefore bidder 1 pays zero. The same is true for bidder 3, and thus she also pays zero. If bidder 1 is dropped, bidder 3 wins and must pay $ppg_2 = v_2/2$. Since $v_2 > 0$ (Condition 2), this payment is more than zero and so revenue monotonicity fails.

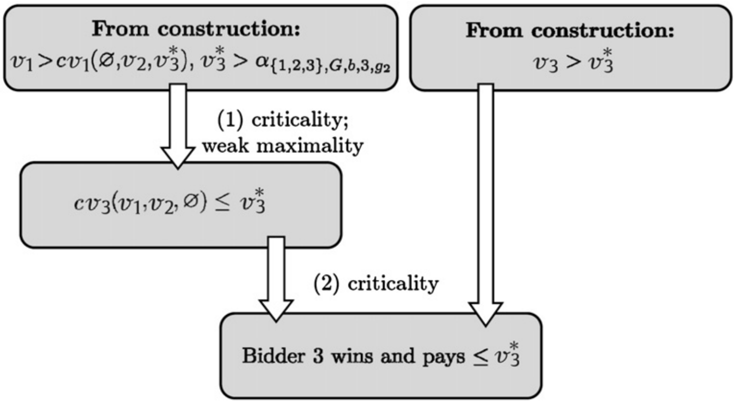

Fig. 3. Sketch of the proof of Theorem 5: Part 1.

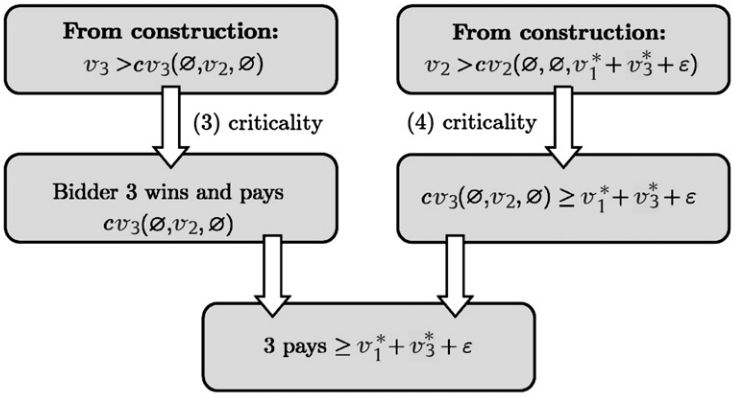

Fig. 4. Sketch of the proof of Theorem 5: Part 2.

3.2. Impossibility theorem

We are now ready to prove our main result.

Theorem 5. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders and satisfies participation,[5] consumer sovereignty, and weak maximality with respect to at least two bidders. Then $M^{(ksm)}$ is not revenue monotonic.*

Proof. Without loss of generality (by the revelation principle) assume that $M^{(ksm)}$ is dominant-strategy truthful. We will further assume that bidders follow their dominant strategies and bid truthfully. Since $|\mathbb{N}| \geqslant 3$, there are at least three bidders; let us name the first three 1, 2, and 3. (For notational simplicity in what follows, we will write the proof as though $|\mathbb{N}| = 3$. If in fact $|\mathbb{N}| > 3$ our argument would not change, except that all valuation profiles would include extra $\varnothing$ entries.) Assume without loss of generality that $M^{(ksm)}$ is weakly maximal with respect to bidders 1 and 3. Since $|\mathbb{G}| \geqslant 2$, there are at least two goods; let us name the first two g_1 and g_2. Let the bundles valued by bidders 1, 2, and 3 be $b_1 = \{g_1\}$, $b_2 = \{g_1, g_2\}$ and $b_3 = \{g_2\}$ respectively.

The five properties in which we are interested (DS truthfulness, participation, revenue monotonicity, weak maximality, and consumer sovereignty) are all universally quantified over $N \subseteq \mathbb{N}$ and $G \subseteq \mathbb{G}$ (see Definitions 19, 20, 22, 23, and 24 in Appendix A). Thus, to prove the theorem, it is enough to demonstrate that no mechanism satisfies all of these properties for some given N and G. Specifically, we consider $N = \{1, 2, 3\}$ and $G = \{g_1, g_2\}$.

[5] In the case of single-parameter domains—which includes known single-minded bidders—it is possible to satisfy participation for free. That is, the space of social choice functions that are implementable in dominant strategies is the same with or without adding a participation constraint. This is mainly because each bidder has to pay either of the two specific amounts: one if she wins and one if she loses. If we "normalize" the payment function and unconditionally pay each bidder the losing amount—which could be negative—then we achieve a dominant-strategy mechanism that satisfies participation. However, there are revenue implications to these unconditional payments that vary as the number of bidders in the auction varies. Therefore, we nevertheless state the participation condition explicitly.

We now show how to construct valuations for the three bidders. First pick an arbitrary positive constant k, and then define $v_1^* = \alpha_{\{1,2,3\},G,b,1,g_1} + k$ and $v_3^* = \alpha_{\{1,2,3\},G,b,3,g_2} + k$. Next pick an arbitrary positive constant ε, and then pick an arbitrary value for v_2 that satisfies

$$v_2 > cv_2\big(\emptyset, \emptyset, v_1^* + v_3^* + \varepsilon\big).$$

Finally, pick values for v_1 and v_3 that satisfy

$$v_1 > \max\big\{cv_1\big(\emptyset, v_2, v_3^*\big), cv_1(\emptyset, v_2, \emptyset), v_1^*\big\}, \quad \text{and}$$
$$v_3 > \max\big\{cv_3\big(v_1^*, v_2, \emptyset\big), cv_3(\emptyset, v_2, \emptyset), v_3^*\big\}.$$

By Corollary 4 the above critical values are all finite. Dependencies between v_1^*, v_3^*, v_2, v_1, and v_3 are shown in Fig. 2, illustrating the fact that it is possible to pick values for these variables that satisfy all our constraints by following the ordering given.

The rest of the proof is divided into two parts. In Part 1 we consider $N = \{1, 2, 3\}$ and construct an expression for the auction's revenue. In Part 2 we consider $N = \{2, 3\}$ and show that more revenue is obtained than in Part 1. Sketches of the arguments in each of these parts are given in Figs. 3 and 4 respectively. Fig. 1 gives a high-level illustration of the proof.

Part 1: Since $v_1 > cv_1(\emptyset, v_2, v_3^*)$ (by construction), if bidder 3 were to bid v_3^* then bidder 1 would win (by criticality). By construction, bidder 3 is the only bidder whose bundle does not overlap with b_1, and $v_3^* > \alpha_{\{1,2,3\},G,b,3,g_2}$; thus, by weak maximality bidder 3 would also win and, by criticality,

$$cv_3(v_1, v_2, \emptyset) \leqslant v_3^* \quad \big(\text{see (1) in Fig. 3}\big). \tag{1}$$

Symmetrically, from $v_3 > cv_3(v_1^*, v_2, \emptyset)$ we can also conclude that

$$cv_1(\emptyset, v_2, v_3) \leqslant v_1^*. \tag{2}$$

By construction, $v_1 > v_1^*$ and $v_3 > v_3^*$. Then, using inequalities (1) and (2) and by criticality, bidders 1 and 3 win (see (2) in Fig. 3). By participation, since bidder 2 loses she must pay zero. Therefore the revenue of the auction, by criticality, is $R = cv_1(\emptyset, v_2, v_3) + cv_3(v_1, v_2, \emptyset) \leqslant v_1^* + v_3^*$.

Part 2: If bidder 1 is not present, then only bidders 2 and 3 compete. Since $v_3 > cv_3(\emptyset, v_2, \emptyset)$, by criticality, bidder 3 wins and pays $cv_3(\emptyset, v_2, \emptyset)$ (see (3) in Fig. 4). Since b_2 and b_3 overlap, bidder 2 loses and by participation pays zero. The revenue of the auction is therefore $R_{-1} = cv_3(\emptyset, v_2, \emptyset)$. By construction, $v_2 > cv_2(\emptyset, \emptyset, v_1^* + v_3^* + \varepsilon)$. Thus if bidder 3 were to bid $v_1^* + v_3^* + \varepsilon$ then bidder 2 would win (by criticality) and so bidder 3 would lose. This tells us (again by criticality) that $cv_3(\emptyset, v_2, \emptyset) \geqslant v_1^* + v_3^* + \varepsilon$ (see (4) in Fig. 4). Therefore, $R_{-1} = cv_3(\emptyset, v_2, \emptyset) \geqslant v_1^* + v_3^* + \varepsilon > v_1^* + v_3^* \geqslant R$. Thus, $M^{(ksm)}$ is not revenue monotonic. $\square$

One might have imagined that weak maximality would increase an auction mechanism's revenue by avoiding "leaving money on the table," augmenting allocations to award available goods to the bidders who value them. Instead, we have shown above that any dominant-strategy truthful combinatorial auction mechanism that satisfies weak maximality with respect to at least two bidders—along with some more standard conditions—can sometimes collect no more than a predefined constant amount of revenue despite competition between bidders. That is, given the constructed valuations, the payments of bidders 1 and 3 are bounded from above by an amount that does not depend on bidder 2's losing bid; bidder 2 effectively fails to offer competition to bidders 1 and 3, who also offer each other no competition as they bid on separate bundles. On the other hand, when bidder 1 is dropped then bidders 2 and 3 do compete. Although bidder 3 still wins, she pays more than before and more than the sum of the payments in the three-bidder case. The mechanism can thus achieve arbitrarily higher revenue in the two-bidder case than in the three-bidder case, since ε and k can be set to be arbitrarily large and arbitrarily small, respectively.

We have mentioned that an impossibility result becomes stronger as the set of mechanisms it describes becomes more restricted, hence our focus on known single-minded bidders. However, our claim is only really true if we can state our impossibility theorem for mechanisms that are not restricted to known single-minded bidders. It may seem intuitive that such a result would be very close to, and somewhat straightforward to derive from, what we have already shown. While this is indeed the case, the general statement of our result requires considerable formal setup beyond what we have provided so far. We formally state our result for the general case and prove it in Appendix C (Theorem 29). The extension of all theorems and corollaries in Section 4 to the same general case are also given in the same appendix.

4. Related impossibility results

Our main impossibility result straightforwardly implies several other impossibility results. Here we demonstrate that arguments of the same form suffice to show that it is impossible to achieve false-name-proofness and monotonicity in the set of goods rather than the set of bidders, and to guarantee that an auction's outcome belongs to the core.

4.1. False-name-proofness

False-name (pseudonymous) bidding has been studied extensively (e.g., Yokoo [32] and Yokoo et al. [34,35]). This work is concerned with auctions in which a bidder may submit multiple bids using pseudonyms. An auction mechanism is said to be false-name-proof if truth-telling without using false-name bids is a dominant strategy for each bidder. Yokoo et al. [34] proved that there does not exist any combinatorial auction mechanism that is false-name-proof and efficient. Observe that this is a somewhat narrow result, because—as discussed earlier—only Groves mechanisms are both dominant-strategy truthful and efficient [15].

There is a connection between false-name-proofness and revenue monotonicity. From the seller's perspective, false-name bidding is the same as having more bidders in the auction. If an auction is not revenue monotonic, more bidders can mean less revenue. Our results are therefore relevant to research on false-name bidding (Todo et al. [30] indeed recently investigated this connection). For technical reasons, we have to make minor changes to our formal model to capture false-name bidding (e.g., we have assumed that mechanisms know bidders' identities.) We can then prove the following corollary which generalizes the result of Yokoo et al. [34] by replacing their requirement of efficiency with the much weaker criterion of weak maximality. Recall that all efficient mechanisms are weakly maximal with respect to all bidders, but there exist other mechanisms that are inefficient and still weakly maximal.

Corollary 6. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders, and that satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then $M^{(ksm)}$ is not false-name-proof.*

Proof. Given the valuations constructed in the proof of Theorem 5, bidder 3 gains by pseudonymously bidding also as bidder 1, and so truth telling is not a dominant strategy for bidder 3. $\square$

4.2. Monotonicity in the set of goods

Here we show that we can also obtain the same impossibility results as in Theorem 5 when we define revenue monotonicity over the set of goods instead of over the set of bidders. This result may be more intuitive than our first result, as it relies on the fact that adding goods to an auction can reduce the level of competition between the bidders.

An auction mechanism is *good revenue monotonic* if and only if the auctioneer never collects more money by dropping a good (see Definition 31 in Appendix C).

Corollary 7. *Let $|\mathbb{G}| \geqslant 3$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then $M^{(ksm)}$ is not good revenue monotonic.*

Proof. The claim follows from the proof of Theorem 5 with some minor modifications: (i) add an extra good g_3 to bidder 1's bundle b_1; (ii) instead of dropping bidder 1 in Part 2, drop g_3—then bidder 1's valuation for all available bundles is 0. $\square$

4.3. Outcomes in the core

Coalitional game theory focuses on groups of players and the utility they can achieve together. It is relatively standard (see, e.g., Ausubel and Milgrom [4]; Day and Milgrom [13]) to describe efficient auction mechanisms as coalitional games. This theory can be useful for discussing what happens to an auction's revenue when bidders are added or removed.

4.3.1. Modeling efficient mechanisms as coalitional games

A *transferable utility (TU)* coalitional game is defined by a set of *players* N_p and a *characteristic function* w that maps each coalition of players S to the coalition's value, $w(S)$. The grand coalition is the coalition of all players. An imputation is a payoff profile in which each player receives a nonnegative payoff and the sum of the payoffs does not exceed the grand coalition's value.[6] An efficient combinatorial auction naturally defines a TU coalitional game. Define N_p as the set of participating bidders, N, plus the seller whom we denote by 0. An efficient auction game is then defined as follows.

Definition 8 *(Efficient auction game).* For any coalition $S \subseteq N_p$, define the coalition's value as

$$w(S) = \begin{cases} \max_{\mathrm{a} \in \mathbb{A}_{S,G}} \sum_{i \in S \setminus \{0\}} v_i(\mathrm{a}_i) & 0 \in S, \\ 0 & 0 \notin S. \end{cases}$$

[6] In the literature, it is usually also required that imputations be efficient. Here, we slightly modified the common definition so that we can later extend it to inefficient mechanisms.

Intuitively, in an efficient auction game, the value of a coalition consisting of any set of players S including the seller is the maximum social welfare achievable under S. When the seller does not belong to a coalition, the coalition's value is zero.

In an auction, the mechanism picks a specific imputation by imposing the chosen allocation and payments. We call this *auction's imputation*. In an auction game, define the payoff of the seller under the auction's imputation as the auction's revenue, $\pi_0 = R = \sum_{i \in N} p_i$. Define bidder i's payoff as her utility from the auction, $\pi_i = u_i = v_i - p_i$. Observe that in an efficient auction game $\sum_{i \in N_p} \pi_i = w(N_p)$.

Definition 9 *(Core in TU coalitional game).* An imputation π is in the *core* of a TU coalitional game if and only if no subset of players can achieve higher payoff:

$$\forall S \subseteq N_p, \quad \sum_{j \in S} \pi_j \geqslant w(S).$$

If an auction's imputation is in the core, no coalition has an incentive to deviate from it. (Observe that all coalitions that do not involve the seller have zero value; thus, a deviating coalition would always involve the seller and a subset of the bidders.) Note that we allow for the possibility that the grand coalition (in addition to smaller coalitions) would make such a deviation. We say that the outcome of an auction mechanism is in the core if the auction's imputation is in the core. Recall that any efficient mechanism is maximal with respect to all bidders. Our impossibility result then implies the following corollary.

Corollary 10. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders, and that satisfies participation, consumer sovereignty, and efficiency. Then, there exists a valuation profile for which the auction's imputation does not belong to the core.*

This result follows as a special case of Corollary 13, so we omit the proof.

4.3.2. Modeling inefficient mechanisms as coalitional games

The literature on modeling auctions as coalitional games focuses on efficient mechanisms. This makes sense under the assumption that any deviating coalition can achieve a social welfare maximizing outcome. Recall that in an auction game, the payoff of the seller is the auction's revenue and the payoff of each bidder is her utility. If one attempts to describe an inefficient auction mechanism as a TU game following Definition 8, the outcome of the auction is not guaranteed to be in the core. This is because the sum of the payoffs may not add up to the grand coalition's value. In other words, if the auction mechanism chooses an inefficient outcome then the grand coalition has an incentive to deviate to an efficient outcome. However, if a seller elects to use an inefficient mechanism, it is inconsistent to then imagine all bidders and the seller jointly deviating to an efficient allocation. The use of an inefficient mechanism can nevertheless make sense, e.g., because regulatory or computational constraints may limit the set of outcomes that can be achieved. Therefore, here we aim to model inefficient mechanisms as coalitional games. Specifically, we discuss three alternate coalitional game models of the auction game, none of which obviously dominates the others.

In the first alternative, which makes minimal changes to Definition 8, we assume that players can reach the efficient allocation for all but the grand coalition. (That is, we assume that the coalition's value for all coalitions except the grand coalition is as stated in Definition 8.)

In what follows, let v_S denote the valuation profile of the bidders in S. That is, v_S is the restriction of v to the bidders in S and is derived from v by replacing the valuation of any bidder $i \notin S$ by $\varnothing$.

Definition 11 *(Inefficient auction game (first alternative)).* For any coalition $S \subseteq N_p$, define the coalition's value as

$$w(S) = \begin{cases} \sum_{i \in S \setminus \{0\}} v_i(a_i(v_S)) & S = N_p, \\ \max_{\mathrm{a} \in \mathbb{A}_{S,G}} \sum_{i \in S \setminus \{0\}} v_i(\mathrm{a}_i) & 0 \in S \text{ and } S \neq N_p, \\ 0 & 0 \notin S. \end{cases}$$

In the second alternative, we assume that players have to obey the mechanism's allocation choice under all coalitions, rather than only under the grand coalition.

Definition 12 *(Inefficient auction game (second alternative)).* For any coalition $S \subseteq N_p$, define the coalition's value as

$$w(S) = \begin{cases} \sum_{i \in S \setminus \{0\}} v_i(a_i(v_S)) & 0 \in S, \\ 0 & 0 \notin S. \end{cases}$$

The second alternative may seem more plausible than the first one. We do not need to choose between them, however, as both lead to the following impossibility result.

Corollary 13. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Define the auction game as in Definition* 11 *or Definition* 12. *Then, there exists a valuation profile for which the auction's imputation does not belong to the core.*

Proof. The proof can be derived from the proof of Theorem 5 by slight modifications. First, construct valuations as in the proof of Theorem 5, but now choose v_2 to satisfy the constraint $v_2 > \max(cv_2(\varnothing, \varnothing, v_1^* + v_3^* + \varepsilon), cv_2(\varnothing, \varnothing, \varnothing), v_1^* + v_3^*)$. Then, notice that in the auction game defined as in either of Definitions 11 or 12, the auctioneer must award bidder 2 her desired bundle when she is the only present bidder, as by construction $v_2 > cv_2(\varnothing, \varnothing, \varnothing)$. Therefore, $w(\{2, 0\}) = v_2$. Furthermore, the coalition of the seller and bidder 2 has an incentive to deviate from the grand coalition since $w(\{2, 0\}) = v_2 > v_1^* + v_3^* \geqslant u_2 + R$. In other words, the seller can sell the bundle to bidder 2 for the price of p_2', $v_1^* + v_3^* < p_2' < v_2$, making both himself and bidder 2 better off. □

In the coalitional game formulations that we have considered so far, the mechanism only dictates its choice of allocation to some or all of the coalitions—specifically, to the grand coalition in Definition 11 and to all coalitions in Definition 12. We may want to assume that the mechanism imposes not only its choice of allocation, but also its choice of payments, thus, disallowing side payments between bidders. This motivates our third coalitional game model, which describes an inefficient auction game as a coalitional game with *nontransferable utility* (*NTU*).

Formally, an NTU coalitional game is defined by a set of players N_p and a characteristic function w that maps each coalition of players S to a set of real-valued vectors describing different sets of payoffs achievable by the members.

Definition 14 *(Core in an NTU coalitional game).* A payoff vector $\pi \in w(N_p)$ is in the *core* of an NTU coalitional game if and only if $\forall S \subseteq N_p$, $\neg\exists x \in w(S)$ such that $\forall i \in S$, $\pi_i \leqslant x_i$ and $\exists j \in S, \pi_j < x_j$.

Definition 15 *(Inefficient auction game (third alternative)).* Let the characteristic function w map each coalition $S \subseteq N_p$ to a single real-valued vector in which each player's payoff is exactly her utility under the mechanism's chosen allocation and taking into account her payment to the mechanism, when the set of participating bidders is $S \setminus \{0\}$.

For known single-minded bidders, all mechanisms that involve only a single bidder i, that satisfy participation, and that offer dominant strategies can be understood as offering i her desired bundle at a fixed price, $cv_i(\varnothing, \ldots, \varnothing)$. The following result can be understood as showing that any mechanism satisfying our conditions either already sets $cv_i(\varnothing, \ldots, \varnothing)$ in such a way that both the seller and i can gain when all other bidders are excluded from the mechanism, or can be modified to do so. Intuitively, our counterexample cannot be used to show that a given (unmodified) mechanism *always* suffers from this problem because, while i is always better off when the other bidders are dropped, the seller could be worse off if $cv_i(\varnothing, \ldots, \varnothing)$ is set too low.

Corollary 16. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let $M^{(ksm)}$ be a CA mechanism defined for known single-minded bidders that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then there exists a CA mechanism M' that allows bidders to express the same preferences as in $M^{(ksm)}$ and that*

1. *has the same allocation and payment functions as $M^{(ksm)}$, except that it may have a different $cv_i(\varnothing, \ldots, \varnothing)$ for some* (*single*) *$i \in \mathbb{N}$*;
2. *satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders*; *and*
3. *chooses an outcome that is not guaranteed to belong to the core.*

Proof. The result follows from the proof of Theorem 5 by slight modification. Consider the three-bidder two-good setting in the proof of Theorem 5. To emphasize that $M^{(ksm)}$ may already choose outcomes that do not belong to the core, our proof considers two cases.

Case 1. $cv_2(\varnothing, \varnothing, \varnothing) > \alpha_{\{1,2,3\},G,b,1,g_1} + \alpha_{\{1,2,3\},G,b,3,g_3}$.

Pick an arbitrary positive $k < \frac{1}{2}(cv_2(\varnothing, \varnothing, \varnothing) - \alpha_{\{1,2,3\},G,b,1,g_1} - \alpha_{\{1,2,3\},G,b,3,g_3})$. Construct valuations as in the proof of Theorem 5, given the chosen k, but now choose v_2 to satisfy the constraint $v_2 > \max(cv_2(\varnothing, \varnothing, v_1^* + v_3^* + \varepsilon), cv_2(\varnothing, \varnothing, \varnothing), v_1^* + v_3^*)$. By Corollary 4, $M^{(ksm)}$'s revenue when only bidder 2 participates is $R_2 = cv_2(\varnothing, \varnothing, \varnothing) > v_1^* + v_3^* \geqslant R$. The utility of bidder 2 in this case—i.e., when only bidder 2 participates—is $u_2 = v_2 - cv_2(\varnothing, \varnothing, \varnothing) > 0$, which is strictly greater than bidder 2's utility when all three bidders participate. Thus, the outcome chosen by $M^{(ksm)}$ does not belong to the core; let $M' = M^{(ksm)}$.

Case 2. $cv_2(\varnothing, \varnothing, \varnothing) \leqslant \alpha_{\{1,2,3\},G,b,1,g_1} + \alpha_{\{1,2,3\},G,b,3,g_3}$.

Construct M' to be the same as $M^{(ksm)}$, except choose $cv_2(\varnothing,\varnothing,\varnothing) > \alpha_{\{1,2,3\},G,b,1,g_1} + \alpha_{\{1,2,3\},G,b,3,g_3}$. Observe that this change preserves dominant strategies (this property is unaffected by the specific value taken by $cv_2(\varnothing,\varnothing,\varnothing)$), participation (bidder 2 pays nothing if she loses), consumer sovereignty ($cv_2(\varnothing,\varnothing,\varnothing)$ is finite), and weak maximality with respect to bidders 1 and 3 (nothing changes for these bidders). Then, the proof follows from the argument in Case 1. □

Earlier, when we modeled inefficient auctions as TU games, we assumed that bidder 2 and the seller could divide gains between them, meaning that the pair were always better off forming a coalition. Under the NTU model, that division must be described explicitly through the auction's payment rule. The proof of Corollary 16 shows that such a division can always be accomplished by an appropriate choice of $cv_i(\varnothing,\ldots,\varnothing)$.

5. Conclusions and future work

In this work, we investigated whether there exists any deterministic, dominant-strategy truthful CA mechanism that satisfies participation, consumer sovereignty and weak maximality with respect to at least two bidders and that is revenue monotonic. We showed that no such mechanism exists, whenever bidders are allowed to express arbitrary known single-minded preferences; as corollaries, we were able to show similar results concerning mechanisms that yield weakly decreasing revenue when goods are dropped and false-name-proof mechanisms. Also, we investigated the relationship between a mechanism being revenue monotonic and the mechanism yielding an outcome that belongs to the core. More specifically, we showed that for any mechanism that satisfies our desired properties, the outcome of the mechanism is not guaranteed to belong to the core.

In future work, we are interested in investigating the probability that such revenue monotonicity failures occur in practical auctions. It is also interesting to ask what dominant-strategy truthful CA mechanism has all the properties we demanded before and has the minimum probability of violating revenue monotonicity. Finally, we hope to gain a better understanding of the existence or nonexistence of revenue monotonic mechanisms that satisfy only a subset of our desired properties.

Appendix A. Formal definitions

In this section we formally define the concepts introduced in Section 2. Let valuation function $v_{\mathbb{G},i}$ for bidder $i \in \mathbb{N}$ map $2^{\mathbb{G}}$ to the nonnegative reals. For every $G \subset \mathbb{G}$ let valuation function $v_{G,i}$ be the restriction of $v_{\mathbb{G},i}$ to 2^G. Whenever G is understood, we drop it from the subscript.

Let $\mathbb{V}$ denote the universal set of all possible valuation profiles. Let $V_{N,G}$ denote a set of valuation profiles for the set of participating bidders N and the set of goods for sale G. Let $\mathbb{V}_{N,G}$ denote the set of all valuation profiles given a set of participating bidders N and a set of goods for sale G; that is, the set of all valuation profiles v_G for which $v_i = \varnothing$ if and only if $i \notin N$.

In a particular auction, bidders' valuation functions may be drawn from some restricted set. Let $V_{N,G} \subseteq \mathbb{V}_{N,G}$ denote a subspace of the universal set of valuation profiles for the set of participating bidders N and the set of goods for sale G. Let $V_{N,G,i}$ denote the set $\{v_i \mid (v_1,\ldots,v_i,\ldots,v_n) \in V_{N,G}\}$. Let $\mathcal{V}_{\mathbb{N},\mathbb{G}}$ denote the universal set of valuation profile subspaces, that is $\mathcal{V}_{\mathbb{N},\mathbb{G}} = \{V_{N,G} \mid N \subseteq \mathbb{N}, G \subseteq \mathbb{G}, V_{N,G} \subseteq \mathbb{V}_{N,G}\}$. Let $\mathcal{V}$ denote a set of valuation profile subspaces with at least one member corresponding any $N \subseteq \mathbb{N}$ and $G \subseteq \mathbb{G}$. That is, $\mathcal{V} \subseteq \mathcal{V}_{\mathbb{N},\mathbb{G}}$ and $\exists V_{N,G} \in \mathcal{V}, \forall N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$. Note that there could be more than one subspace corresponding to a fixed N and a fixed G in $\mathcal{V}$.

Consider the case of known single-minded bidders. Let $N \subseteq \mathbb{N}$ and $G \subseteq \mathbb{G}$ be fixed. Let $b = (b_1, b_2, \ldots, b_n) \in (2^G)^n$. If i is a participating bidder, let $V^{(b)}_{N,G,i}$ be the set of all possible single-minded valuation functions, taken over all possible choices of v_i, and otherwise let $V^{(b)}_{N,G,i} = \varnothing$. Let $V^{(b)}_{N,G} = V^{(b)}_{N,G,1} \times \cdots \times V^{(b)}_{N,G,n}$. Then, $V^{(b)}_{N,G}$ is simply the space of valuation profiles in which participating bidders are all single-minded and each participating bidder i values bundle b_i. Let $\mathcal{V}^{(ksm)}$ denote the set of valuation profile subspaces for known single-minded bidders, $\mathcal{V}^{(ksm)} = \{V^{(b)}_{N,G} \mid N \subseteq \mathbb{N}, G \subseteq \mathbb{G}, b \in (2^G)^n\}$.

A.1. CA mechanisms

We first formally define a combinatorial auction mechanism. Observe that our definition requires a mechanism to define allocations and payments for all possible sets of bidders, all possible sets of goods, and all corresponding valuation profiles belonging to a given, possibly restricted set. Also, note the implicit assumption that the auction setting—i.e., N, G and $V_{N,G}$—is common knowledge among all bidders and the auctioneer.

Definition 17 *(CA mechanism).* Let the set of valuation profile subspaces $\mathcal{V}$ be given. A *deterministic direct Combinatorial Auction* (*CA*) *mechanism M* (CA mechanism) maps each $V_{N,G} \in \mathcal{V}$, $N \subseteq \mathbb{N}$ and $G \subseteq \mathbb{G}$, to a pair (a, p) where

- a, the allocation scheme, maps each $\widehat{v} \in V_{N,G}$ to an allocation tuple $a = (a_1(\widehat{v}),\ldots,a_n(\widehat{v}))$ of goods, where $\bigcup_i a_i(\widehat{v}) \subseteq G$, $a_i(\widehat{v}) \cap a_j(\widehat{v}) = \emptyset$ if $i \neq j$, and $a_i(\widehat{v}) = \emptyset$ if $\widehat{v}_i = \varnothing$.
- p, the payment scheme, maps each $\widehat{v} \in V_{N,G}$ to a payment tuple $p = (p_1(\widehat{v}),\ldots, p_n(\widehat{v}))$, where $p_i(\widehat{v})$ is the payment from bidder i to the auctioneer such that $p_i(\widehat{v}) = 0$ if $\widehat{v}_i = \varnothing$.

We say that CA mechanism M is defined for $\mathcal{V}$. A CA mechanism is *defined for known single-minded bidders* if its set of valuation profile subspaces is $\mathcal{V}^{(ksm)}$. From the definition it follows that the allocation and payment functions depend on the set $V_{N,G}^{(b)} \in \mathcal{V}^{(ksm)}$ from which bidders' valuation profiles are drawn. Informally, b is *known*, since the allocation and payments depend on b. Observe that our definition requires that the mechanism be defined for *all* possible known single-minded valuations, not just for the set of bundles that a given set of bidders might value.

A set of valuation subspaces $\mathcal{V}$ *subsumes* another set of valuation subspaces $\mathcal{V}'$ if and only if for all $V'_{N,G} \in \mathcal{V}'$, there exists $V_{N,G} \in \mathcal{V}$ such that $V'_{N,G} \subseteq V_{N,G}$. For example, the set of valuation subspaces for single-minded bidders subsumes the set of valuation subspaces for known single-minded bidders.

We can say that the class of mechanisms defined for $\mathcal{V}$ is a subset of the class of mechanisms defined for known single-minded bidders when $\mathcal{V}$ subsumes known single-minded valuations. The preceding claim is in fact true in a general sense, that is even if we replace known single-minded valuations with any $\mathcal{V}'$. The following lemma states it formally. On some level this result is obvious; however, we were not able to find any formal discussion of it in the literature and so present it here for completeness.

Lemma 18. *If $\mathcal{V}$ subsumes $\mathcal{V}'$, then every social choice function that can be implemented by a mechanism defined for $\mathcal{V}$ can also be implemented by a mechanism defined for $\mathcal{V}'$.*

Proof. Without loss of generality (by the revelation principle), we can restrict ourselves to truthful mechanisms. The allocation function in a truthful mechanism is precisely the social choice function. Let $M^{(\mathcal{V})}$ be a truthful mechanism defined for $\mathcal{V}$. Modify $M^{(\mathcal{V})}$ such that given declared valuation profile $\widehat{v} \in V'_{N,G}$, $V'_{N,G} \in \mathcal{V}'$, runs the same allocation and payment functions as $M^{(\mathcal{V})}$ would run on $\widehat{v} \in V_{N,G}$, $V_{N,G} \in \mathcal{V}$, where $V'_{N,G} \subseteq V_{N,G}$. As $\mathcal{V}$ subsumes $\mathcal{V}'$, such a $V_{N,G}$ exists. Let $M^{(\mathcal{V}')}$ be this new mechanism that is defined for $\mathcal{V}'$. $M^{(\mathcal{V}')}$ is clearly truthful, since is $M^{(\mathcal{V})}$. □

Informally speaking, for each mechanism M defined for $\mathcal{V}$, there is a corresponding mechanism M' defined for $\mathcal{V}'$. We will use the above claim in Section 3 to state our result for general CA mechanisms.

A.2. Properties of CA mechanisms

Here, we formally define properties that we may require of a CA mechanism.

Definition 19 *(Dominant-strategy truthfulness).* A CA mechanism M is *dominant strategy truthful* (or *DS truthful*) if and only if for all fixed sets of participating bidders, it is a best response for each participating bidder to declare her true valuation regardless of the declarations of the other participating bidders. That is, for all $N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, $\widehat{v} \in V_{N,G}$, $v_i \in V_{N,G,i}$, and for every bidder i we have that

$$v_i\big(a_i(v_i, \widehat{v}_{-i})\big) - p_i(v_i, \widehat{v}_{-i}) \geqslant v_i\big(a_i(\widehat{v})\big) - p_i(\widehat{v}).$$

Definition 20 *(Participation).* A truthful CA mechanism M satisfies *participation* if and only if for all $N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, and $v \in V_{N,G}$, $p_i(v) = 0$ for all bidder i for whom $v_i(a_i) = 0$ (i.e., who does not win).

Definition 21 *(Efficiency).* A CA mechanism M is efficient if its chosen allocation in equilibrium, a^*, maximizes the social welfare; that is, for all $N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, and $v \in V_{N,G}$,

$$\mathrm{a}^* \in \arg\max_{\mathrm{a} \in \mathbb{A}_{N,G}} \sum_{i \in N} v_i(\mathrm{a}_i).$$

Definition 22 *(Revenue monotonicity).* A truthful CA mechanism M is bidder revenue monotonic (or *revenue monotonic*) if and only if for all $N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, $v \in V_{N,G}$ and for all bidders j,

$$\sum_{i \in \mathbb{N}} p_i(v) \geqslant \sum_{i \in \mathbb{N} \setminus \{j\}} p_i(v_{-j}).$$

Definition 23 *(Weak maximality).* A truthful CA mechanism M is *weakly maximal with respect to bidder i* if and only if for all $N \subseteq \mathbb{N}$ where $i \in N$, for all $G \subseteq \mathbb{G}$, and for all $V_{N,G} \in \mathcal{V}$, there exists a set of nonnegative finite constants $\{\alpha_{N,G,V_{N,G},i,g} \mid g \in G\}$ such that the following holds. For all $v \in V_{N,G}$, M always chooses an allocation a where either:

1. $v_i(\mathrm{a}_i) > 0$; or
2. for all allocations a' with $v_i(\mathrm{a}'_i) > \alpha_{N,G,V_{N,G},i,\mathrm{a}'_i}$, $|\mathrm{a}'_i| = 1$, and $\mathrm{a}'_j = \mathrm{a}_j \setminus \mathrm{a}'_i$ for all $j \neq i$, it must be the case that for some j, $v_j(\mathrm{a}'_j) < v_j(\mathrm{a}_j)$.

Note that α is subscripted by N, G and $V_{N,G}$. This is because a CA mechanism may set different bidder-specific reserve prices for different settings, i.e. for different sets of participating bidders N, available goods G, and different valuation profiles subspaces $V_{N,G}$. Also note that in the informal definition of weak maximality that we provided in Section 2, $V_{N,G}$ in the subscript of α was replaced by b, bidders' bundles of interest. This change is innocuous as, in the case of known single-minded bidders, all the knowledge that a mechanism extracts from $V_{N,G}$ can be learned from b.

In what follows let $(V_{N,G})_{-i}$ denote the set $\{v_{-i} \mid v \in V_{N,G}\}$.

Definition 24 *(Consumer sovereignty).* A CA mechanism M satisfies consumer sove/-reignty if and only if for all $N \subseteq \mathbb{N}$, $G \subseteq \mathbb{G}$, and $V_{N,G} \in \mathcal{V}$, $\forall i \in N$ and $\forall \widehat{v}_{-i} \in (V_{N,G})_{-i}$, for all $s \subseteq G$ that i may value above zero according to $V_{N,G}$, there exists some finite amount $k_i^s(\widehat{v}_{-i}) \in \mathbb{R}$, $k_i^s(\widehat{v}_{-i}) > 0$, such that i can win s by reporting that she values s at amount at least $k_i^s(\widehat{v}_{-i})$.

Appendix B. Maximality

We use a weakened version of maximality in our work as it is sufficient for our purpose and, being a weaker constraint, makes our impossibility result stronger. Maximality, on the other hand, is useful when one is trying to prove a possibility result (see, e.g., Rastegari et al. [25]).

Definition 25 *(Maximality).* A truthful CA mechanism M is *maximal with respect to bidder i* if and only if for all $N \subseteq \mathbb{N}$ where $i \in N$, for all $G \subseteq \mathbb{G}$, and for all $V_{N,G} \in \mathcal{V}$, there exists a set of nonnegative finite constants $\{\alpha_{N,G,V_{N,G},i,s} \mid s \subseteq G\}$ such that the following holds. For all $v \in V_{N,G}$, M always chooses an allocation a where either:

1. $v_i(\mathrm{a}_i) > 0$; or
2. for all allocations a' with $v_i(\mathrm{a}'_i) > \alpha_{N,G,V_{N,G},i,\mathrm{a}'_i}$, and $\mathrm{a}'_j = \mathrm{a}_j \setminus \mathrm{a}'_i$ for all $j \neq i$, it must be the case that for some j, $v_j(\mathrm{a}'_j) < v_j(\mathrm{a}_j)$.

Observe that the definition of weak maximality is simply derived from the definition of maximality by restricting a'_i to be of size 1. Intuitively, maximality ensures that the mechanism does not withhold any subset of goods, or give the goods away to the bidders who do not value them, when they are sufficiently valued by a losing bidder. The quantities $\{\alpha_{N,G,V_{N,G},i,s} \mid s \subseteq G\}$ can be thought of as bidder- and bundle-specific reserve prices.[7]

Many interesting mechanisms are maximal. First, it is straightforward to show that efficiency implies maximality. Second, we show here that a broad class of affine maximizing mechanisms are maximal.

Affine maximizers generalize the idea behind the VCG mechanism's allocation rule (which aims to maximize the social welfare) by allowing the mechanism to restrict the set of possible allocations, to assign different nonnegative weights ω_i to different players, and to assign different additive weights γ_a to different allocations.

Definition 26 *(Affine maximizer).* A CA mechanism is an affine maximizer if for some $\mathbb{A}'_{N,G} \subseteq \mathbb{A}_{N,G}$, nonnegative $\{\omega_i\}_{i\in\mathbb{N}}$ and $\{\gamma_\mathrm{a}\}_{\mathrm{a}\in\mathbb{A}'_{N,G}}$, for all $N \subseteq \mathbb{N}$, $G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, $v \in V_{N,G}$, its chosen allocation in equilibrium, a^* satisfies the following:

$$\mathrm{a}^* \in \arg\max_{\mathrm{a}\in\mathbb{A}'_{N,G}} \left(\sum_i \omega_i v_i(\mathrm{a}_i) + \gamma_\mathrm{a}\right).$$

We call $\{\omega_i\}_{i\in\mathbb{N}}$ and $\{\gamma_\mathrm{a}\}_{\mathrm{a}\in\mathbb{A}'_{N,G}}$ the allocation parameters of affine maximizer M.

Theorem 27. *Let M be an affine maximizing truthful CA mechanism with finite allocation parameters $\{\omega_i\}_{i\in\mathbb{N}}$ and $\{\gamma_\mathrm{a}\}_{\mathrm{a}\in\mathbb{A}_{N,G}}$. Suppose that for some $i \in \mathbb{N}$, $\omega_i > 0$. Then M is maximal with respect to bidder i.*

Proof. Let $\alpha_{N,G,V_{N,G},i,s} = \frac{\max_\mathrm{a}\{\gamma_\mathrm{a}\}}{\omega_i}$, $\forall N \subseteq \mathbb{N}$ where $i \in N$, $\forall G \subseteq \mathbb{G}$ and $\forall s \subseteq G$. We prove that M is maximal with respect to bidder i. Assume for contradiction that M is not maximal with respect to i. Then, for some v, M's allocation scheme maps v to an allocation a that satisfies the following properties: (i) $v_i(\mathrm{a}_i) = 0$, and (ii) $\exists s \subseteq G$, $\exists \mathrm{a}' \in \mathbb{A}_{N,G}$: $\mathrm{a}'_i = s$ and $\forall j \neq i$, $\mathrm{a}'_j = \mathrm{a}_j \setminus s$ such that $v_i(\mathrm{a}'_i) > \alpha_{N,G,V_{N,G},i,s}$ and $v_j(\mathrm{a}'_j) \geqslant v_j(\mathrm{a}_j)$.

From construction and (i) and (ii) we have that

$$\sum_{k\in N} \omega_k v_k(\mathrm{a}'_k) + \gamma_{\mathrm{a}'} > \sum_{k\in N\setminus\{i\}} \omega_k v_k(\mathrm{a}'_k) + \max_\mathrm{a}\{\gamma_\mathrm{a}\} + \gamma_{\mathrm{a}'} \geqslant \sum_{k\in N} \omega_k v_k(\mathrm{a}_k) + \gamma_\mathrm{a}.$$

Since M is affine maximizing it would not choose allocation a, giving us our contradiction. □

[7] Note that in the definition of maximality there exist distinct α variables for each different subset of goods $s \subseteq G$; in contrast, in the definition of weak maximality, there exist distinct α's only for each single good $g \in G$. Thus, we can understand the α's as representing bundle-specific reserve prices in maximal mechanisms, and item-specific reserve prices in weakly maximal mechanisms.

Finally, all mechanisms that are *strongly Pareto efficient with respect to bidders' valuations* are also maximal.

Definition 28 *(Strong Pareto efficiency with respect to bidders' valuations).* A mechanism is *strongly Pareto efficient with respect to bidders' valuations* if it chooses allocations that would be strongly Pareto efficient if monetary transfers were disallowed.

Note that these allocations are a superset of those that are strongly Pareto efficient when transfers are permitted. Thus, a broader class of mechanisms achieves strong Pareto efficiency with respect to bidders' valuations than achieve strong Pareto efficiency.

In case of single-minded bidders, strong Pareto efficiency with respect to bidder's valuations is equivalent to maximality with respect to all bidders when all bidder- and bundle-specific reserve prices, α's, are set to zero.

In general, however, maximality is a weaker condition than strong Pareto efficiency. To see this, consider an example with a single bidder, 1, and two goods $\{g_1, g_2\}$. Assume that bidder 1 values each good individually, and also (e.g., by free disposal) values a bundle of both goods. A mechanism that awards either of the goods, but not both, to bidder 1 is maximal but does not satisfy strong Pareto efficiency. Alternatively, if bidder 1 were single-minded and interested in $\{g_1, g_2\}$, then *any* mechanism would be weakly maximal with respect to bidder 1, whereas a mechanism that is maximal with respect to bidder 1 has to award both goods to her as long as she values them above the bidder- and bundle-specific reserve price.

Appendix C. Impossibility results for general CA mechanisms

Theorem 29. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces $\mathcal{V}$ subsumes known single-minded bidders, that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then M is not revenue monotonic.*

Proof. The proof directly follows from Lemma 18 and Theorem 5. Following Lemma 18, if there is a mechanism M defined for $\mathcal{V}$—i.e., M's set of valuation subspaces is $\mathcal{V}$—that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, weak maximality with respect to at least two bidders and revenue monotonicity, then there exists a mechanism $M^{(ksm)}$ defined for known single-minded bidders that has all the above properties. By Theorem 5, such a mechanism $M^{(ksm)}$ does not exist, and thus nor does such a mechanism M. $\square$

The following corollaries are the extension of those in Section 4 to the general case. We omit the proofs as they are very similar to the proofs given in Section 4 except that they have to also appeal to Lemma 18, making an argument similar to the one in the proof of Theorem 29.

Corollary 30. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces subsumes known single-minded bidders, that offers dominant strategies to the bidders, and that satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then M is not false-name-proof.*

Let $p_i^G(v)$ denote bidder i's payment to a DS truthful CA mechanism when all bidders' valuations are v, and where the set of goods at auction is G. Then we can give the following definition.

Definition 31 *(Good revenue monotonicity).* A truthful CA mechanism M is *good revenue monotonic* if and only if for all $N \subseteq \mathbb{N}, G \subseteq \mathbb{G}$, $V_{N,G} \in \mathcal{V}$, $v \in V_{N,G}$ and for all goods $g \in G$,

$$\sum_{i \in \mathbb{N}} p_i^G(v) \geqslant \sum_{i \in \mathbb{N}} p_i^{G \setminus \{g\}}(v).$$

Corollary 32. *Let $|\mathbb{G}| \geqslant 3$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces subsumes known single-minded bidders, that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then M is not good revenue monotonic.*

Corollary 33. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces subsumes known single-minded bidders, that offers dominant strategies to the bidders, and that satisfies participation, consumer sovereignty, and efficiency. Then, there exists a valuation profile for which the auction's imputation does not belong to the core.*

Corollary 34. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces $\mathcal{V}$ subsumes known single-minded bidders, that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Define the auction game as in Definition 11 or Definition 12. Then, there exists a valuation profile for which the auction's imputation does not belong to the core.*

Corollary 35. *Let $|\mathbb{G}| \geqslant 2$ and $|\mathbb{N}| \geqslant 3$. Let M be a CA mechanism whose set of valuation subspaces $\mathcal{V}$ subsumes known single-minded bidders, that offers dominant strategies to the bidders and satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders. Then there exists a CA mechanism M' that allows bidders to express the same preferences as in M and that*

1. *has the same allocation and payment functions as M, except that it may have a different $cv_i(\varnothing, \ldots, \varnothing)$ for some (single) $i \in \mathbb{N}$;*
2. *satisfies participation, consumer sovereignty, and weak maximality with respect to at least two bidders; and*
3. *chooses an outcome that is not guaranteed to belong to the core.*

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