Augmenting tractable fragments of abstract argumentation ☆,☆☆

Wolfgang Dvořák, Sebastian Ordyniak, Stefan Szeider *

Institute of Information Systems, Vienna University of Technology, Austria

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We present a new approach to the efficient solution of important computational problems that arise in the context of abstract argumentation. Our approach makes known algorithms defined for restricted fragments generally applicable, at a computational cost that scales with the distance from the fragment. Thus, in a certain sense, we gradually augment tractable fragments. Surprisingly, it turns out that some tractable fragments admit such an augmentation and that others do not.

More specifically, we show that the problems of Credulous and Skeptical Acceptance are fixed-parameter tractable when parameterized by the distance from the fragment of acyclic argumentation frameworks—for most semantics. Other tractable fragments such as the fragments of symmetrical and bipartite frameworks seem to prohibit an augmentation: the acceptance problems are already intractable for frameworks at distance 1 from the fragments.

For our study we use a broad setting and consider several different semantics. For the algorithmic results we utilize recent advances in fixed-parameter tractability.

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1. Introduction

The study of arguments as abstract entities and their interaction in form of *attacks* as introduced by Dung [13] has become one of the most active research branches within Artificial Intelligence, Logic and Reasoning [3,4,35]. Argumentation handles possible conflicts between arguments in form of attacks. Arguments may either originate from a dialogue between several agents or from the pieces of information available to a single agent, this information may even contain contradictions. A main issue for any argumentation system is the selection of acceptable sets of arguments, called extensions, where an acceptable set of arguments must be in some sense coherent and be able to defend itself against all attacking arguments. Abstract argumentation provides suitable concepts and formalisms to study, represent, and process various reasoning problems most prominently in defeasible reasoning (see, e.g., [5,34]) and agent interaction (see, e.g., [33]).

Unfortunately, important computational problems such as determining whether an argument belongs to some extension (Credulous Acceptance) or to all extensions (Skeptical Acceptance), are intractable (see, e.g., [11,16]). In order to solve these problems on medium or large-sized real world instances, it is significant to identify efficient algorithms. However, a few tractable fragments are known where the acceptance problems can be efficiently solved: the fragments of acyclic [13], symmetric [10], bipartite [14], and—for most semantics—noeven [16] argumentation frameworks.

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* Corresponding author.
E-mail address: stefan@szeider.net (S. Szeider).

Table 1
Complexity of acceptance problems, parameterized by the distance from a fragment.

Fragment	adm	com	prf	sem	stb	stg
	CA	CA	CA/SA	CA/SA	CA/SA	CA/SA
Acyc	FPT	FPT	FPT	FPT	FPT	hard
Noeven	XP	XP	XP	XP	XP	hard
Bip	hard	hard	hard	hard	hard	hard
Sym	hard	hard	hard	hard	hard	hard

It seems unlikely that an argumentation framework originating from a real-world application belongs to one of the known tractable fragments, but it might be "close" to a tractable fragment.

In this paper we study the natural and significant question of whether we can solve the relevant problems efficiently for argumentation frameworks that are of small distance to a tractable fragment. As the distance we take the smallest number of arguments that must be deleted to put the framework into the tractable fragment under consideration. One would certainly have to pay some extra computational cost that increases with the distance from the tractable fragment, but ideally this extra cost should scale gradually with the distance. To get a broad picture of the complexity landscape we take several popular semantics into consideration, namely the semantics introduced by Dung [13], i.e., admissible, complete, preferred, and stable semantics, and further semi-stable [6,7,40], and stage [40] semantics (see [2] for a survey). Our approach is inspired by the notion of "backdoors" which originates from the area of propositional satisfiability (see, e.g., [28,39,42]), and has been successfully used in other problem areas, including quantified Boolean formulas and nonmonotonic reasoning [24,38].

Results. On the positive side we show that for all the considered semantics, except for stage semantics, the fragments of *acyclic* and *noeven* argumentation frameworks admit an augmentation. In particular, we show that we can solve Credulous and Skeptical Acceptance in polynomial time for argumentation frameworks that are of bounded distance from either of the two fragments. We further show that with respect to the acyclic fragment, the order of the polynomial time bound is independent of the distance, which means that both acceptance problems are *fixed-parameter tractable* (see [12]) when parameterized by the distance from the acyclic fragment. To obtain these results we introduce the new notion of partial labelings of argumentation frameworks and apply recent results from fixed-parameter theory. We use partial labelings to capture and propagate the acceptance state of certain key arguments (forming a backdoor) of the argumentation framework.

On the negative side, we show that the fragments of *bipartite* and *symmetric* argumentation frameworks do not admit an augmentation. In particular, we show that the problems Credulous and Skeptical Acceptance are already *intractable* for argumentation frameworks at distance 1 from either of the two fragments. We also show that the acyclic and noeven fragments do not admit an augmentation with respect to the stage semantics, in contrast to the other five considered semantics. In particular, we show that the acceptability problems for the stage semantics are already intractable for noeven argumentation frameworks, and for argumentation frameworks at distance 1 from the acyclic fragment.

To put our tractability results into context, we compare the parameters "distance to the acyclic fragment" and "distance to the noeven fragment" with other parameters that, if bounded, make acceptance problems tractable. We show that our distance-based parameters are *incomparable* with the previously considered parameters treewidth and clique-width [14,23]. Hence our augmentation approach provides an efficient solution for instances that are hard for other known methods.

Table 1 summarizes our results for the different semantics and fragments. The table can be read as follows: A column marked CA concerns Credulous Acceptance, and a column marked CA/SA concerns both Credulous and Skeptical Acceptance, each with respect to a particular semantics as indicated. For the admissible and complete semantics we omit Skeptical Acceptance because the corresponding problems are already tractable for arbitrary frameworks [10,13]. An entry "XP" means that acceptance can be decided in polynomial time for argumentation frameworks whose distance to the fragment is bounded by a constant (the order of the polynomial may depend on the distance); an entry "FPT" means that the acceptance problem is fixed-parameter tractable, parameterized by the distance from the fragment; an entry "hard" means that Credulous or Skeptical Acceptance are at least NP-hard or coNP-hard, respectively, even for instances of distance 1 from the fragment.

The reminder of the paper is organized as follows. In Section 2 we provide basic definitions and preliminaries. In Section 3 we introduce the important concept of partial labelings that will be an important tool to obtain our tractability results. In Section 4 we give efficient algorithms for the augmentation of the acyclic and noeven fragments. In Section 5 we establish intractability for the bipartite and symmetric fragments. In Section 6 we strengthen the tractability results from Section 4 with the help of strong backdoors. In Section 7 we compare our newly found parameters with already known structural parameters such as treewidth and clique-width. We close in Section 8 with concluding remarks. Some proofs of technical lemmas and theorems are given in Appendix A.

2. Preliminaries

An *abstract argumentation system* or *argumentation framework* (*AF*, for short) is a pair (X, A) where X is a (possibly infinite) set of elements called *arguments* and $A \subseteq X \times X$ is a binary relation called *attack relation*. In this paper we will
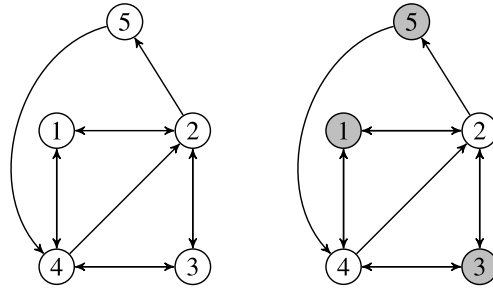
Fig. 1. Left: the AF F from Example 1. Right: indicated in gray the only non-empty complete extension of F.

restrict ourselves to finite AFs, i.e., to AFs for which X is a finite set. If $(x, y) \in A$ we say that x *attacks* y and that x is an *attacker* of y.

An AF $F = (X, A)$ can be considered as a directed graph, and therefore it is convenient to borrow notions and notation from graph theory. For a set of arguments $Y \subseteq X$ we denote by $F[Y]$ the AF $(Y, \{(x, y) \in A \mid x, y \in Y\})$ and by $F - Y$ the AF $F[X \setminus Y]$.

Example 1. An AF with arguments $1, \ldots, 5$ and attacks $(1, 2), (1, 4), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3), (5, 4)$ is displayed in Fig. 1.

Let $F = (X, A)$ be an AF, $S \subseteq X$ and $x \in X$. We say that x is *defended* (in F) by S if for each $x' \in X$ such that $(x', x) \in A$ there is an $x'' \in S$ such that $(x'', x') \in A$. We define the range of a set S of arguments, denoted as S_F^+, as the set of arguments $x \in X$ such that either $x \in S$ or there is an $x' \in S$ with $(x', x) \in A$, and we omit the subscript if F is clear from the context. Note that in our setting the set S is contained in S_F^+. We say S is *conflict-free* if there are no arguments $x, x' \in S$ with $(x, x') \in A$.

Next we define commonly used semantics of AFs, see the survey of Baroni and Giacomin [2]. We consider a semantics σ as a mapping that assigns to each AF $F = (X, A)$ a family $\sigma(F) \subseteq 2^X$ of sets of arguments, called *extensions*. We denote by adm, com, prf, sem, stb, and stg the *admissible*, *complete*, *preferred*, *semi-stable*, *stable*, and *stage* semantics, respectively. These six semantics are characterized by the following conditions which hold for each AF $F = (X, A)$ and each conflict-free set $S \subseteq X$.

- $S \in \text{adm}(F)$ if and only if each $s \in S$ is defended by S.
- $S \in \text{com}(F)$ if and only if $S \in \text{adm}(F)$ and every argument that is defended by S is contained in S.
- $S \in \text{prf}(F)$ if and only if $S \in \text{adm}(F)$ and there is no $T \in \text{adm}(F)$ with $S \subsetneq T$.
- $S \in \text{sem}(F)$ if and only if $S \in \text{adm}(F)$ and there is no $T \in \text{adm}(F)$ with $S^+ \subsetneq T^+$.
- $S \in \text{stb}(F)$ if and only if $S^+ = X$.
- $S \in \text{stg}(F)$ if and only if there is no conflict-free set $T \subseteq X$ with $S^+ \subsetneq T^+$.

Moreover we say that an AF F is *coherent* if the preferred, stable, semi-stable, and stage extensions of F coincide.

Let $F = (X, A)$ be an AF, $x \in X$, and $\sigma \in \{\text{adm, com, prf, sem, stb, stg}\}$. The argument x is *credulously accepted* in F with respect to σ if x is contained in some extension $S \in \sigma(F)$, and x is *skeptically accepted* in F with respect to σ if x is contained in all extensions $S \in \sigma(F)$.

Each semantics σ gives rise to the following two fundamental computational problems: σ-CREDULOUS ACCEPTANCE and σ-SKEPTICAL ACCEPTANCE, in symbols CA_σ and SA_σ, respectively. Both problems take as input an AF $F = (X, A)$ together with an argument $x \in X$. Problem CA_σ asks whether x is credulously accepted in F, and problem SA_σ asks whether x is skeptically accepted in F. Table 2 summarizes the complexities of these problems for the considered semantics (see [10,11, 13,16–19]). A brief description of the complexity classes Σ_2^P and Π_2^P and their relationship to NP and coNP can be found in Appendix A.

Example 2. Consider the AF F from Example 1 and the complete semantics (com). F has two complete extensions $\emptyset$ and $\{1, 3, 5\}$, see Fig. 1. Consequently, the arguments 1, 3, and 5 are credulously accepted in F and the arguments 2 and 4 are not. Furthermore, because of the complete extension $\emptyset$, no argument of F is skeptically accepted.

In the following we list classes of AFs for which CA and SA are known to be solvable in polynomial time for the admissible, complete, preferred, and stable semantics [2,10,13,14].

- ACYC is the class of *acyclic* argumentation frameworks, i.e., of AFs that do not contain directed cycles.
- NOEVEN is the class of *noeven* argumentation frameworks, i.e., of AFs that do not contain directed cycles of even length.

Table 2
Complexity of Credulous and Skeptical Acceptance for various semantics σ.

σ	CA$_\sigma$	SA$_\sigma$
adm	NP-complete	trivial
com	NP-complete	P-complete
prf	NP-complete	Π_2^P-complete
sem	Σ_2^P-complete	Π_2^P-complete
stb	NP-complete	coNP-complete
stg	Σ_2^P-complete	Π_2^P-complete

- SYM is the class of *symmetric* argumentation frameworks, i.e., of AFs whose attack relation is symmetric and irreflexive.
- BIP is the class of *bipartite* argumentation frameworks, i.e., of AFs whose sets of arguments can be partitioned into two conflict-free sets.

Observe that the original tractability results for the above classes are not stated for the semi-stable, or stage semantics. Indeed, as we will show in Section 5, they do not hold for stage semantics and the class of noeven argumentation frameworks. However, in all other cases it is easy to show the above tractability results for the semi-stable, and stage semantics, as follows: The fragments ACYC and BIP are free of odd-length directed cycles and thus propose at least one stable extension [13]. The same holds for the SYM fragment of AFs, i.e., every symmetric AF has at least one stable extension [10]. It follows that the stable, stage, and semi-stable semantics coincide on these fragments. Consequently, tractability of the stable semantics for the classes ACYC, SYM, and BIP passes over to the semi-stable and stage semantics. When considering the NOEVEN fragment, we know that every such AF has a unique preferred extension [14] which then is also the unique semi-stable extension. This follows from using the well-known fact that every AF has at least one semi-stable extension, and every semi-stable extension is also a preferred one. Again tractability passes over to semi-stable semantics.

Lemma 1. *The classes* ACYC, NOEVEN, SYM, *and* BIP *can be recognized in polynomial time* (*i.e., given an AF F, we can decide in polynomial time whether F belongs to any of the four classes*).

Proof. The statement of the lemma is easily seen for the classes ACYC, BIP and SYM. For the class NOEVEN it follows by a result of Robertson et al. [37]. □

Since the recognition and the acceptance problems are polynomial for these classes, we consider them as "tractable fragments of abstract argumentation".

Parameterized complexity. For our investigation we need to take two measurements into account: the input size n of the given AF F and the distance k of F from a tractable fragment. The theory of *parameterized complexity*, introduced and pioneered by Downey and Fellows [12], provides the adequate concepts and tools for such an investigation. We outline the basic notions of parameterized complexity that are relevant for this paper, for an in-depth treatment we refer to other sources [25,31].

An instance of a parameterized problem is a pair (I, k) where I is the *main part* and k is the *parameter*; the latter is usually a non-negative integer. A parameterized problem is *fixed-parameter tractable* (FPT) if there exists a computable function f such that instances (I, k) of size n can be solved in time $f(k) \cdot n^{O(1)}$. Fixed-parameter tractable problems are also called *uniform polynomial-time tractable* because if k is considered constant, then instances with parameter k can be solved in polynomial time where the order of the polynomial is independent of k, in contrast to *non-uniform polynomial-time* running times such as $n^{O(k)}$. Thus we have three complexity categories for parameterized problems: (1) problems that are fixed-parameter tractable (uniform polynomial-time tractable), (2) problems that are non-uniform polynomial-time tractable, and (3) problems that are NP-hard or coNP-hard if the parameter is fixed to some constant (such as k-SAT which is NP-hard for $k = 3$).

Backdoors. For our approach to abstract argumentation we borrow the concept of backdoors from the areas of propositional satisfiability and constraint satisfaction (see, e.g., [27,28,39,42]). A SAT backdoor is a small set of key variables that represent a "clever reasoning shortcut" through the search space. By deciding the truth values of the atoms in the backdoor, one can reduce a SAT instance to several instances belonging to a target class. Backdoors have also been used for quantified Boolean formulas [38] and answer-set programming [24].

Let us adopt this notion for abstract argumentation. Let $\mathcal{C}$ be a class of AFs, $F = (X, A)$ an AF, and $Y \subseteq X$. We call Y a $\mathcal{C}$-*backdoor of* F if $F - Y \in \mathcal{C}$. We write dist$_\mathcal{C}(F)$ for the cardinality (size) of a smallest $\mathcal{C}$-backdoor of F, i.e., dist$_\mathcal{C}(F)$ represents the *distance* of F from the class $\mathcal{C}$.
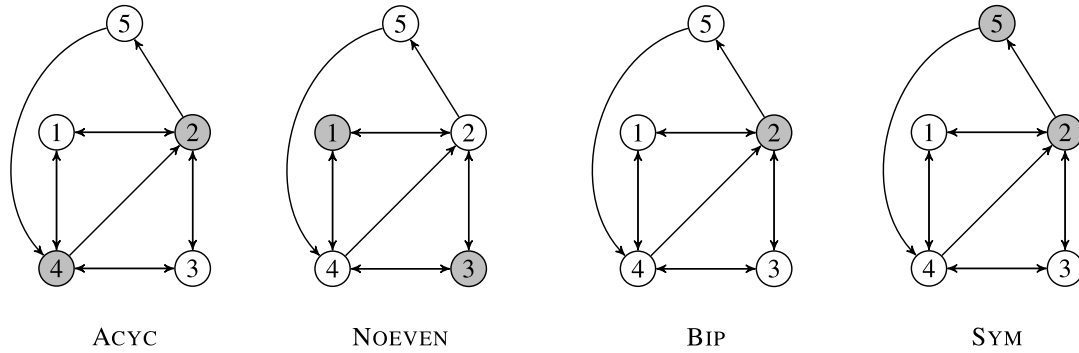
Fig. 2. Backdoors of the AF F from Example 1, with respect to the indicated classes.

Example 3. Fig. 2 illustrates backdoors of the AF F from Example 1 for the classes Acyc, Noeven, Bip, and Sym. The indicated backdoors are the smallest possible, hence the considered AF is of distance 2 from the classes Acyc, Noeven, and Sym, and of distance 1 from the class Bip.

In the following we consider CA and SA parameterized by the distance to a tractable fragment $\mathcal{C}$.

3. Partial labelings

In this section we will introduce the novel concept of partial labelings,[1] which we will be key for establishing our tractability results. Partial labelings generalize total labelings which are defined on the entire set X of arguments (see, e.g., [8,30,41]). We use partial labelings to capture and propagate the acceptance state of the arguments that form the backdoor. In contrast to just distinguishing whether an argument in the backdoor belongs to some extension or not, a partial labeling allows us to assign 3 distinct statuses to every argument, i.e., an argument can be either in the extension, attacked by the extension or neither in the extension nor attacked by the extension (undecided). It is this property of labelings that make them particularly suited for our algorithm which is based on the propagation of partial labelings.

Let $F = (X, A)$ be an AF. A *partial labeling* of F, or *labeling* for short, is a function $\lambda : Y \to \{\text{IN}, \text{OUT}, \text{UND}\}$ defined on a subset Y of X. We denote by $\text{IN}(\lambda)$, $\text{OUT}(\lambda)$ and $\text{UND}(\lambda)$ the sets of arguments $x \in X$ with $\lambda(x) = \text{IN}$, $\lambda(x) = \text{OUT}$ and $\lambda(x) = \text{UND}$ respectively. Furthermore, we set $\text{DEF}(\lambda) = Y$ and $\text{UD}(\lambda) = X \setminus \text{DEF}(\lambda)$ and denote by $\lambda_\emptyset$ the *empty labeling*, i.e., the labeling with $\text{DEF}(\lambda_\emptyset) = \emptyset$. For a set $S \subseteq X$ we define $\text{lab}(F, S)$ to be the *labeling of F with respect to S* by setting $\text{IN}(\text{lab}(F, S)) = S$, $\text{OUT}(\text{lab}(F, S)) = S^+ \setminus S$ and $\text{UND}(\text{lab}(F, S)) = X \setminus S^+$. We say a set $S \subseteq X$ is *compatible* with a labeling λ if $\lambda(x) = \text{lab}(F, S)(x)$ for every $x \in \text{DEF}(\lambda)$.

Let $F = (X, A)$ be an AF and λ a partial labeling of F. The *propagation* of λ with respect to F, denoted λ^*, is the labeling that is obtained from λ by initially setting $\lambda^*(x) = \lambda(x)$, for every $x \in \text{DEF}(\lambda)$, and subsequently applying one of the following three rules to unlabeled arguments $x \in X$ as long as possible.

Rule 1. x is labeled OUT if x has at least one attacker that is labeled IN.
Rule 2. x is labeled IN if all attackers of x are labeled OUT.
Rule 3. x is labeled UND if all attackers of x are either labeled OUT or UND and at least one attacker of x is labeled UND.

It is easy to see that λ^* is well-defined and unique.

For an AF F, we define the *reduced* AF F^* as the AF $F - \text{DEF}(\lambda_\emptyset^*)$. In other words, F^* is obtained from F after deleting all arguments from F that, starting from the empty labeling, are labeled according to Rules 1–3. We observe that because we start from the empty labeling, Rule 3 will not be invoked. We further note that $\text{DEF}(\lambda_\emptyset^*)$ is the range of the grounded extension and F^* is what Baroni et al. call the "cut of F" in their work on resolution-based semantics [1].

The following lemmas illustrate the connection between partial labelings and complete extensions.

Lemma 2. *Let $F = (X, A)$ be an AF, λ a partial labeling of F, and S a complete extension that is compatible with λ. Then the propagation λ^* of λ is compatible with S.*

Proof. We show the claim by induction on the number of arguments that have been labeled according to Rules 1–3. Because S is compatible with λ it holds that $\lambda^*(x) = \lambda(x) = \text{lab}(F, S)(x)$ for every $x \in \text{DEF}(\lambda)$ and hence the proposition holds before the first argument has been labeled according to one of the rules. Now, suppose that λ' is the labeling that is obtained from

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[1] The term "partial labeling" was previously used in [29] to denote a specific class of four-valued labelings. The main difference to our setting is that the definition in [29] is tied to a specific semantics while our approach does not propose any constraints on the labels.

λ after labeling the first i arguments according to one of the rules and that x is the $(i + 1)$-th argument that is labeled according to the rules. We distinguish three cases.

First we assume that x is labeled according to Rule 1. In this case $\lambda^*(x) = \text{OUT}$ and we need to show that $x \in S^+ \setminus S$. It follows from the definition of Rule 1 that x has at least one attacker n with $\lambda'(n) = \text{IN}$. Using the induction hypothesis it follows that $\text{lab}(F, S)(n) = \text{IN}$ and hence $n \in S$. Because S is conflict-free it follows that $x \notin S$ but since x is attacked by n it follows that $x \in S^+ \setminus S$.

Second we assume that x is labeled according to Rule 2. In this case $\lambda^*(x) = \text{IN}$ and we need to show that $x \in S$. Let $n_1, \ldots, n_r$ be all the attackers of x in F. It follows from the definition of Rule 2 that $\lambda'(n_j) = \text{OUT}$ for every $1 \leqslant j \leqslant r$. Using the induction hypothesis it follows that $\text{lab}(F, S)(n_j) = \text{OUT}$ and hence $n_j \in S^+ \setminus S$ for every $1 \leqslant j \leqslant r$. It follows that no argument attacked by x can be contained in S otherwise this argument would be attacked by x but x is not attacked by any argument in S. Hence $S \cup \{x\}$ is also admissible. Because x is defended by S it follows that $x \in S$.

Finally, we assume that x is labeled according to Rule 3. In this case $\lambda^*(x) = \text{UND}$ and we need to show that $x \notin S^+$. Using the definition of Rule 3 it follows that the set of all attackers of x can be partitioned into two sets U and O such that $\lambda'(u) = \text{UND}$ for every $u \in U$ and $\lambda'(o) = \text{OUT}$ for every $o \in O$ and $U \neq \emptyset$. Using the induction hypothesis it follows that $\lambda'(n) = \text{lab}(F, S)(n)$ for every $n \in U \cup O$. Hence, no attacker of x belongs to S and so x cannot be contained in $S^+ \setminus S$. Furthermore, because S is admissible and x has an attacker that is not contained in S^+ it follows that x cannot be contained in S. Hence, x is not contained in S^+. $\quad\square$

For an AF F, a set B of arguments of F, and a partial labeling λ of F we set:

$$\text{com}^*(F, \lambda) = \left\{ \text{IN}(\lambda^*) \cup S \mid S \in \text{adm}(F - \text{DEF}(\lambda^*)) \right\},$$

$$\text{com}^*(F, B) = \bigcup_{\lambda : B \to \{\text{IN}, \text{OUT}, \text{UND}\}} \text{com}^*(F, \lambda).$$

The set $\text{com}^*(F, B)$ can be seen as a set of candidates for complete extensions of the AF F. In particular, as shown by the following lemma, for every set B of arguments the set $\text{com}^*(F, B)$ contains all the complete extensions of the AF F.

Lemma 3. *Let $F = (X, A)$ be an AF and $B \subseteq X$. Then $\text{com}(F) \subseteq \text{com}^*(F, B)$.*

Proof. Let $F = (X, A)$ be the given AF, $B \subseteq X$ and $S \in \text{com}(F)$. We show that $S \in \text{com}^*(F, \lambda) = \{\text{IN}(\lambda^*) \cup S \mid S \in \text{adm}(F - \text{DEF}(\lambda^*))\}$ for the unique partial labeling λ defined on B that is compatible with S. We set $S_1 = S \cap \text{DEF}(\lambda^*)$, $S_2 = S \setminus S_1$, and $F_2 = F - \text{DEF}(\lambda^*)$.

It follows from Lemma 2 that $S_1 = \text{IN}(\lambda^*)$. It remains to show that S_2 is admissible in F_2. Clearly, S_2 is conflict-free. To see that S_2 is admissible suppose to the contrary that there is an argument $x \in S_2$ that is not defended by S_2 in F_2, i.e., x has an attacker y in F_2 that is not attacked by an argument in S_2. Because S is a complete extension of F the argument x is defended by S in F. Hence, there is a $z \in S_1 = \text{IN}(\lambda^*)$ that attacks y. But then, using Rule 1, $\lambda^*(y) = \text{OUT}$, and hence y cannot be an argument of F_2. Hence S_2 is admissible in F_2. $\quad\square$

4. Tractability results

Regarding the fragments of acyclic and noeven argumentation frameworks we obtain the following two results which show that these two fragments admit an augmentation.

Theorem 1. *The problems CA_σ and SA_σ are fixed-parameter tractable for parameter $\text{dist}_{\text{ACYC}}$ and any semantics $\sigma \in \{\text{adm, com, prf, sem, stb}\}$.*

Theorem 2. *The problems CA_σ and SA_σ are solvable in non-uniform polynomial-time for parameter $\text{dist}_{\text{NOEVEN}}$ and any semantics $\sigma \in \{\text{adm, com, prf, sem, stb}\}$.*

The remainder of this section is devoted to establishing Theorems 1 and 2.

For a class $\mathcal{C}$ of AFs and a semantic σ, the solution of the acceptance problems involves two tasks:

1. *Backdoor Detection*: to find a $\mathcal{C}$-backdoor B of F of size at most k, and
2. *Backdoor Evaluation*: to use the $\mathcal{C}$-backdoor B of F for deciding whether x is credulously/skeptically accepted in F with respect to the semantics σ.

For backdoor detection we utilize recent results from fixed-parameter theory. For backdoor evaluation we use our new concept of partial labelings.

Table 3
Calculation of all complete extensions for the AF F of Example 1 using the Acyc-backdoor $\{2, 4\}$.

λ		λ^*			$\text{IN}(\lambda^*)$	$\text{IN}(\lambda^*) \in \text{com}(F)$?
2	4	1	3	5		
IN	IN	OUT	OUT	OUT	$\{2, 4\}$	no
IN	OUT	OUT	OUT	OUT	$\{2\}$	no
IN	UND	OUT	OUT	OUT	$\{2\}$	no
OUT	IN	OUT	OUT	IN	$\{4, 5\}$	no
OUT	OUT	IN	IN	IN	$\{1, 3, 5\}$	yes
OUT	UND	UND	UND	IN	$\{5\}$	no
UND	IN	OUT	OUT	UND	$\{4\}$	no
UND	OUT	UND	UND	UND	$\emptyset$	yes
UND	UND	UND	UND	UND	$\emptyset$	yes

4.1. Backdoor evaluation

We say a class $\mathcal{C}$ of AFs is *fully tractable* if (i) for every $F \in \mathcal{C}$ the set $\text{adm}(F^*)$ can be computed in polynomial time, and (ii) $\mathcal{C}$ is closed under the deletion of arguments, i.e., for each $F = (X, A) \in \mathcal{C}$ and each $Y \subseteq X$, also $F - Y \in \mathcal{C}$.

Theorem 3. *Let $\mathcal{C}$ be a fully tractable class of AFs, $F = (X, A)$ an AF and B a $\mathcal{C}$-backdoor of F with $|B| \leqslant k$. Then the computation of the sets $\text{com}(F)$, $\text{prf}(F)$, $\text{sem}(F)$ and $\text{stb}(F)$ can be carried out in time $3^k |X|^{O(1)}$ and is therefore fixed-parameter tractable for parameter k.*

Proof. We first show that the computation of $\text{com}(F)$ is fixed-parameter tractable for parameter k. Let λ be one of the at most 3^k partial labelings of F defined on B. We show that we can compute $\text{com}^*(F, \lambda) = \{\text{IN}(\lambda^*) \cup S \mid S \in \text{adm}(F - \text{DEF}(\lambda^*))\}$ in polynomial time, i.e., in time $|X|^{O(1)}$. Clearly, we can compute the propagation λ^* of λ in polynomial time. Furthermore, because $F - B \in \mathcal{C}$ (B is a $\mathcal{C}$-backdoor) also $F - \text{DEF}(\lambda^*) \in \mathcal{C}$; this follows since $B \subseteq \text{DEF}(\lambda^*)$ and $\mathcal{C}$ is closed under argument deletion, as $\mathcal{C}$ is assumed to be fully tractable. Moreover, since $\mathcal{C}$ is assumed to be fully tractable and $F - \text{DEF}(\lambda^*) \in \mathcal{C}$, we can compute $\text{adm}((F - \text{DEF}(\lambda^*))^*) = \text{adm}((F - \text{DEF}(\lambda^*)) - \text{DEF}(\lambda^*)) = \text{adm}(F - \text{DEF}(\lambda^*))$ in polynomial time. Consequently, we can compute the set $\text{com}^*(F, \lambda)$ in polynomial time. Since there are at most 3^k partial labelings of F defined on B, it follows that we can compute the entire set $\text{com}^*(F, B)$ in time $3^k |X|^{O(1)}$.

By Lemma 3 we have $\text{com}(F) \subseteq \text{com}^*(F, B)$. Thus we can obtain $\text{com}(F)$ from $\text{com}^*(F, B)$ by simply testing for each $S \in \text{com}^*(F, B)$ whether S is a complete extension of F. It is a well-known fact that each such a test can be carried out in polynomial time (see e.g. [20]). Hence, we conclude that indeed $\text{com}(F)$ can be computed in time $3^k |X|^{O(1)}$.

For the remaining sets $\text{prf}(F)$, $\text{sem}(F)$ and $\text{stb}(F)$ we note that each of them is a subset of $\text{com}(F)$. Furthermore, the extensions in $\text{prf}(F)$ are exactly the extensions in $\text{com}(F)$ which are maximal with respect to set inclusion. Similarly, the extensions in $\text{sem}(F)$ are exactly the extensions $S \in \text{com}(F)$ where the set S^+ is maximal with respect to set inclusion, and $\text{stb}(F)$ are exactly the extensions $S \in \text{com}(F)$ where $S^+ = X$. Clearly, these observations can be turned into an algorithm that computes from $\text{com}(F)$ the sets $\text{prf}(F)$, $\text{sem}(F)$, and $\text{stb}(F)$ in polynomial time. $\quad\square$

Lemma 4. *The classes Acyc and Noeven are fully tractable.*

Proof. It is easy to see that both classes are closed under the deletion of arguments. It remains to show that for every $F \in \text{Acyc} \cup \text{Noeven} = \text{Noeven}$, the set $\text{adm}(F^*)$ can be computed in polynomial time. Dunne and Bench-Capon [15] have shown that if $F \in \text{Noeven}$ and every argument of F is contained in at least one directed cycle, then $\text{adm}(F) = \{\emptyset\}$. It follows that if every argument in F^* lies on a directed cycle then for every AF $F \in \text{Noeven}$, then $\text{adm}(F^*) = \{\emptyset\}$ and hence $\text{adm}(F^*)$ can be computed in polynomial (constant) time. To see this it suffices to show that every argument x of F^* has at least one attacker in F^*. Suppose the contrary, i.e., there is an argument $x \in X \setminus \text{DEF}(\lambda^*_\emptyset)$ with no attacker in F^*. It follows that every attacker of x must be labeled and hence $x \in \text{DEF}(\lambda^*_\emptyset)$, a contradiction. $\quad\square$

Combining Theorem 3 with Lemma 4 we conclude that if $\mathcal{C} \in \{\text{Acyc}, \text{Noeven}\}$ then the backdoor evaluation problem is fixed-parameter tractable parameterized by the size of the backdoor for the semantics $\sigma \in \{\text{com}, \text{prf}, \text{sem}, \text{stb}\}$. For the remaining case of admissible semantics, we recall from Table 2 that SA_{adm} is trivial. Furthermore, we observe that by Dung's fundamental lemma [13] every admissible extension is contained in some complete extension, and by definition every complete extension is also admissible. We conclude that an argument is credulously accepted with respect to the admissible semantics if and only if the argument is credulously accepted with respect to the complete semantics. Hence, we have shown that backdoor evaluation is also fixed-parameter tractable with respect to the admissible semantics.

Corollary 1. *For any class $\mathcal{C} \in \{\text{Acyc}, \text{Noeven}\}$ and any semantics $\sigma \in \{\text{adm}, \text{com}, \text{prf}, \text{sem}, \text{stb}\}$, the problems σ-Credulous Acceptance and σ-Skeptical Acceptance are fixed-parameter tractable, parameterized by the size of a given $\mathcal{C}$-backdoor set.*

Example 4. Consider again the AF F from Example 1. We have observed above that F has an Acyc-backdoor B consisting of the arguments 2 and 4. We now show how to use the backdoor B to compute all complete extensions of F using the procedure given in the proof of Theorem 3. Table 3 shows the propagation for all partial labelings of F defined on B together with the set $\text{IN}(\lambda^*)$ and for every λ it is indicated whether the set $\text{IN}(\lambda^*)$ is a complete extension of F. Because $F - B$ is acyclic it follows that $\text{adm}(F - \text{DEF}(\lambda^*)) = \{\emptyset\}$ (see the proof of Lemma 4) and hence $\text{com}^*(F, \lambda) = \{\text{IN}(\lambda^*)\}$. It is now easy to compute $\text{com}^*(F, B)$ as the union of all the sets $\text{IN}(\lambda^*)$ given in Table 3. Furthermore, using the rightmost column of Table 3 we conclude that $\text{com}(F) = \{\emptyset, \{1, 3, 5\}\}$, which agrees with our original observation in Example 2.

4.2. Backdoor detection

The following lemma gives an easy upper bound for the complexity of detecting a $\mathcal{C}$-backdoor of size at most k for any class $\mathcal{C}$ of AFs that can be recognized in polynomial time.

Proposition 1. *Let $\mathcal{C}$ be a class of AFs that can be recognized in polynomial time and $F = (X, A)$ an AF with $\text{dist}_{\mathcal{C}}(F) \leqslant k$. Then a $\mathcal{C}$-backdoor of F of size at most k can be found in time $|X|^{O(k)}$ and hence in non-uniform polynomial-time for parameter k.*

Proof. To find a $\mathcal{C}$-backdoor of F of size at most k we simply check for every subset $B \subseteq X$ of size $\leqslant k$ whether $F - B \in \mathcal{C}$. There are $O\left(\sum_{i=0}^{k} \binom{|X|}{i}\right) = O(|X|^k)$ such sets and each check can be carried out in polynomial time. $\square$

Together with Lemma 1 we obtain the following consequence of Proposition 1.

Corollary 2. *Let $\mathcal{C} \in \{\text{Acyc}, \text{Noeven}, \text{Sym}, \text{Bip}\}$ and $F = (X, A)$ an AF with $\text{dist}_{\mathcal{C}}(F) \leqslant k$. Then a $\mathcal{C}$-backdoor of F of size at most k can be found in time $|X|^{O(k)}$ and hence in non-uniform polynomial-time for parameter k.*

It is a natural question to ask whether the above result can be improved to uniform-polynomial time. We provide an affirmative answer for three of the four classes under consideration.

Lemma 5. *Let $\mathcal{C} \in \{\text{Acyc}, \text{Sym}, \text{Bip}\}$ and $F = (X, A)$ an AF with $\text{dist}_{\mathcal{C}}(F) \leqslant k$. Then a $\mathcal{C}$-backdoor of F of size at most k can be found in time $f(k)|X|^{O(1)}$ for some function f; hence the detection of $\mathcal{C}$-backdoors is fixed-parameter tractable and for parameter k.*

Proof. The detection of Acyc-backdoors is equivalent to the so-called *directed feedback vertex set* problem, which, given a directed graph, asks for a set of vertices whose deletion makes the remaining graph acyclic (when we delete a vertex from a graph, we also delete all edges incident to the vertex). The directed feedback vertex set problem, parameterized by the number of deleted vertices, has recently been shown to be fixed-parameter tractable by Chen et al. [9].

Similarly, the detection of Bip-backdoors is equivalent to the so-called *odd cycle traversal* problem which, given an undirected graph, asks for a set of vertices whose deletion makes the remaining graph bipartite. The odd cycle traversal problem, parameterized by the number of deleted vertices, is fixed-parameter tractable due to a result of Reed et al. [36].

Finally, the detection of a Sym-backdoor can be easily translated to the *vertex cover problem* which, given an undirected graph, asks for a set of vertices whose deletion leaves the remaining graph without edges. Given an AF $F = (X, A)$, we construct an undirected graph $G = (X, E)$ where E is the set of edges $\{x, y\}$ such that $(x, y) \in A$ and $(y, x) \notin A$. Then a set $B \subseteq X$ is a Sym-backdoor of F if and only if B is a vertex cover of G. Using this translation, the fixed-parameter tractability of Sym-backdoor detection follows from the fixed-parameter tractability of the vertex cover problem [12]. $\square$

We must leave it open whether the detection of Noeven-backdoors of size at most k is fixed-parameter tractable for parameter k. Since already the polynomial-time recognition of Noeven is highly nontrivial [37], a solution for the backdoor problem seems very challenging. However, it is easy to see that $\mathcal{C}$-backdoor detection, considered as a non-parameterized problem, where k is just a part of the input, is NP-complete for $\mathcal{C} \in \{\text{Acyc}, \text{Noeven}, \text{Sym}, \text{Bip}\}$. Hence it is unlikely that Lemma 5 can be improved to a polynomial-time result (as this would imply that $P = NP$).

Combining Lemmas 5 and 1 with Corollary 1 establishes our main results Theorems 1 and 2 of this section.

5. Hardness results

In this section we show that the fragments Bip and Sym are not amenable for the backdoor approach. In particular, we show that the acceptance problems for all the considered semantics are (co)NP-hard for AFs that are of distance 1 to any of these two fragments. Moreover we show that none of the fragments considered in this paper admits an augmentation for the stage semantics. In particular, we show that the acceptance problems for the stage semantics are (co)NP-hard for AFs of distance 1 from Acyc, and also (co)NP-hard for AFs of distance 0 from Noeven.

Theorem 4. *Let $\sigma \in \{\text{adm}, \text{com}, \text{prf}, \text{stb}, \text{stg}\}$. Then the problem CA_σ is NP-hard for AFs F with $\text{dist}_{\text{BIP}}(F) = \text{dist}_{\text{SYM}}(F) = 1$.*
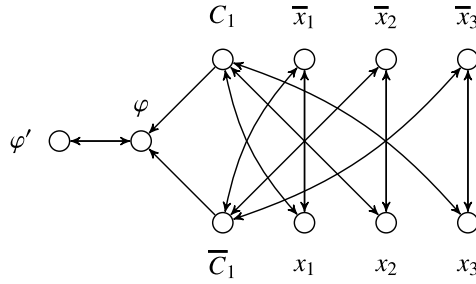
Fig. 3. Illustration for the reduction used in the proof of Theorems 4 and 5, showing the AF F, obtained from the monotone 3-CNF formula $\varphi = C_1 \wedge \overline{C}_1$ with $C_1 = x_1 \vee x_2 \vee x_3$ and $\overline{C}_1 = \neg x_1 \vee \neg x_2 \vee \neg x_3$. The set $\{\varphi\}$ is a BIP- and an SYM-backdoor of F.

Proof. We devise a polynomial reduction from a restricted version of propositional satisfiability (Monotone 3-SAT), where every clause contains either only positive or only negative literals. This version of propositional satisfiability is still NP-complete [26]. Let φ be a CNF formula with the variables $x_1, \ldots, x_n$ and the clauses $C_1, \ldots, C_k$ and $\overline{C}_1, \ldots, \overline{C}_l$, where for every $1 \leqslant j \leqslant k$ the clause C_j contains only positive literals and for every $1 \leqslant j \leqslant l$ the clause $\overline{C}_j$ contains only negative literals.

We construct an instance $I = (F, x)$ of CA_σ such that $\mathrm{dist}_{BIP}(F) = \mathrm{dist}_{SYM}(F) = 1$ and the formula φ is satisfiable if and only if the argument x is credulously accepted in F with respect to σ. F contains the following arguments:

- Two arguments x_i and $\bar{x}_i$ for every $1 \leqslant i \leqslant n$.
- One argument C_j for every $1 \leqslant j \leqslant k$.
- One argument $\overline{C}_j$ for every $1 \leqslant j \leqslant l$.
- The arguments φ, and φ'.

F contains the following attacks:

- For every $1 \leqslant i \leqslant n$ the attacks $(x_i, \bar{x}_i)$, and $(\bar{x}_i, x_i)$.
- For every $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant k$ the attacks (x_i, C_j), and (C_j, x_i) if $x_i \in C_j$.
- For every $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant l$ the attacks $(\bar{x}_i, \overline{C}_j)$, and $(\overline{C}_j, \bar{x}_i)$ if $\bar{x}_i \in \overline{C}_j$.
- For every $1 \leqslant j \leqslant k$ the attack (C_j, φ).
- For every $1 \leqslant j \leqslant l$ the attack $(\overline{C}_j, \varphi)$.
- The attacks (φ, φ'), and (φ', φ).

Furthermore, we set $X = \{x_i \mid 1 \leqslant i \leqslant n\}$, $C = \{C_j \mid 1 \leqslant j \leqslant k\} \cup \{\overline{C}_j \mid 1 \leqslant j \leqslant l\}$, and for a set M of arguments of F we set $\overline{M} = \{\overline{m} \mid m \in M\}$. Finally, we identify the argument x with φ.

An illustration of this construction is given in Fig. 3.

By construction, $F - \{\varphi\}$ is symmetric. $F - \{\varphi\}$ is bipartite with the bipartition $\{\{C_j \mid 1 \leqslant j \leqslant k\} \cup \overline{X}$ and $\{\overline{C}_j \mid 1 \leqslant j \leqslant l\} \cup X\}$. Consequently, $\mathrm{dist}_{BIP}(F) = \mathrm{dist}_{SYM}(F) = 1$.

Because F does not contain an odd-length cycle it follows that F is coherent [13]. Consequently, the preferred, stable, semi-stable, and stage extensions of F coincide. Moreover it is well known that the problems of credulous acceptance for admissible, complete, and preferred semantics are just different formulations of the same problem (see, e.g., [18]). Hence it suffices to show correctness for the stable semantics.

Suppose that the formula φ is satisfiable and let M be a model of φ witnessing this (by model we mean a set M of literals that contains no complementary pairs such that each clause of φ contains a literal of M). It is easy to see that the set $S = \{\varphi\} \cup M \cup \overline{X \setminus M}$ is a stable extension of F that contains the argument φ.

To see the converse suppose there is a stable extension S of F that contains the argument φ. Because S is admissible and the argument φ is attacked from every argument in C it follows that $S \cap C = \emptyset$ and that every argument in C is attacked by some argument in S. Because for every $1 \leqslant i \leqslant n$ at most one of the arguments x_i and $\bar{x}_i$ can be contained in S it follows that $S \cap X$ is a model of φ. □

Theorem 5. *Let* $\sigma \in \{\mathrm{prf}, \mathrm{sem}, \mathrm{stb}, \mathrm{stg}\}$. *Then the problem* SA_σ *is coNP-hard for AFs* F *with* $\mathrm{dist}_{BIP}(F) = \mathrm{dist}_{SYM}(F) = 1$.

Proof. The proof uses the same construction as the proof of Theorem 4. Let the formula φ and the AF $F = (X, A)$ be defined as in the proof of Theorem 4. We show that the argument φ' of F is skeptically accepted in F with respect to σ if and only if the formula φ is not satisfiable.

Again because F is coherent it suffices to show the theorem for the stable semantics. It follows from the proof of Theorem 4 that the formula φ is satisfiable if and only if the argument φ is credulously accepted in F. Furthermore,
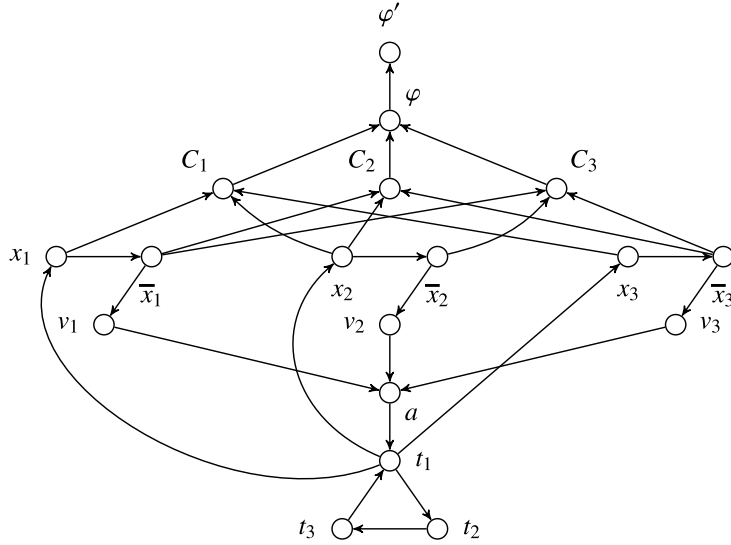
Fig. 4. Illustration for the reduction used in the proof of Theorems 6 and 7, showing the AF F, obtained from the 3-CNF formula $\varphi = C_1 \wedge C_2 \wedge C_3$ with $C_1 = x_1 \vee x_2 \vee x_3$, $C_2 = \neg x_1 \vee x_2 \vee \neg x_3$ and $C_3 = \neg x_1 \vee \neg x_2 \vee \neg x_3$. The set $\{t_1\}$ is an Acyc-backdoor of F.

because the argument φ' participates only in the attacks (φ, φ') and (φ', φ), it follows that φ' is skeptically accepted in F if and only if the argument φ is not credulously accepted in F. Consequently, the argument φ' is skeptically accepted in F if and only if the formula φ is not satisfiable. This establishes the theorem. $\quad\square$

Theorem 6. *The problem* CA_{stg} *is* NP-*hard for noeven AFs F with* $dist_{ACYC}(F) = 1$.

Proof. We devise a polynomial reduction from propositional satisfiability which is well-known to be NP-complete [26]. Let φ be a CNF formula with the variables $x_1, \ldots, x_n$ and the clauses $C_1, \ldots, C_m$.

We construct an instance $I = (F, x)$ of CA_{stg} with $F = (X, A)$, such that F does not contain an even cycle, $dist_{ACYC}(F) = 1$ and the formula φ is satisfiable if and only if the argument x is credulously accepted in the AF F with respect to the stage semantics. F contains the following arguments:

- Three arguments x_i, $\bar{x}_i$, and v_i for every $1 \leqslant i \leqslant n$.
- One argument C_j for every $1 \leqslant j \leqslant m$.
- The arguments t_1, t_2, t_3, and a.
- Two arguments φ and φ'.

F contains the following attacks:

- For every $1 \leqslant i \leqslant n$ the attacks $(x_i, \bar{x}_i)$, $(\bar{x}_i, v_i)$, (v_i, a), and (t_1, x_i).
- The attacks (a, t_1), (t_1, t_2), (t_2, t_3), (t_3, t_1) and (φ, φ').
- For every $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$ the attack (x_i, C_j) if $x_i \in C_j$.
- For every $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$ the attack $(\bar{x}_i, C_j)$ if $\bar{x}_i \in C_j$.
- For every $1 \leqslant j \leqslant m$ the attack (C_j, φ).

Furthermore, we set $X = \{x_i \mid 1 \leqslant i \leqslant n\}$, $C = \{C_j \mid 1 \leqslant j \leqslant m\}$, and for a set M of arguments of F we set $\overline{M} = \{\overline{m} \mid m \in M\}$. Finally, we identify the argument x with φ. See Fig. 4.

It is easy to see that F does not contain an even directed cycle and that $F - \{t_1\}$ is acyclic. Consequently, F is a noeven AF, and $dist_{ACYC}(F) = 1$. Before we proceed to prove the theorem we need the following claims.

Claim 1. *If* $S \in stg(F)$, *then* $C \subseteq S^+$.

Suppose the claim does not hold, i.e., there is an $S \in stg(F)$ such that $C_j \notin S^+$ for some $1 \leqslant j \leqslant m$. If $\varphi \notin S$ then the set $T = S \cup \{C_j\}$ is a conflict-free set, and $S^+ \subsetneq S^+ \cup \{C_j\} \subseteq T^+$, which contradicts the maximality of S^+. So assume that $\varphi \in S$. Then $T = (S \setminus \{\varphi\}) \cup \{C_j, \varphi'\}$ is a conflict-free set, and $S^+ \subsetneq S^+ \cup \{C_j\} \subseteq T^+$ again a contradiction to the maximality of S^+.
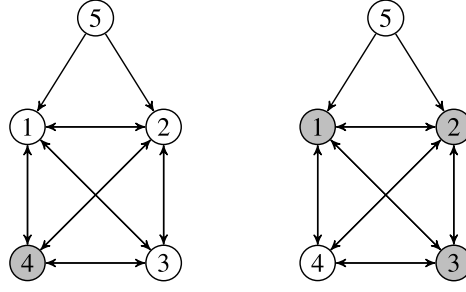
Fig. 5. Left: A smallest strong Acyc-backdoor of the AF *F* from Example 5 indicated in gray. Right: A smallest deletion Acyc-backdoor of *F* indicated in gray.

Claim 2. *If* $S \in \text{stg}(F)$ *with* $\{t_1, t_2, t_3\} \subseteq S^+$, *then for every* $1 \leqslant i \leqslant n$ *at most one of the arguments* x_i, *and* v_i *is contained in* S^+.

It is easy to see that every $S \in \text{stg}(F)$ such that S^+ contains the set $\{t_1, t_2, t_3\}$ must contain the argument *a*. It follows that the set *S* can neither contain the argument t_1 nor an argument v_i for any $1 \leqslant i \leqslant n$. Because *F* contains an attack from x_i to $\bar{x}_i$, the set *S* can contain at most one x_i and $\bar{x}_i$. If *S* contains x_i, then $v_i \notin S^+$, and otherwise $x_i \notin S^+$. This shows the claim.

Suppose that there is a stage extension *S* of *F* that contains the argument φ. Because *S* is conflict-free it follows that *S* cannot contain an argument from *C*. Consequently, using Claim 1 it follows that every argument in *C* must be attacked by some argument in $S \cap (X \cup \bar{X})$. Because *S* is conflict-free it follows that $S \cap X$ is a model of φ.

To see the converse, suppose φ is satisfiable and let *M* be a model of φ witnessing this. It suffices to show that $S = \{\varphi, a, t_2\} \cup M \cup \overline{X \setminus M}$ is a stage extension of *F*. It is easy to see that *S* is conflict-free, and $S^+ = X \setminus ((X \setminus M) \cup \{v_i \mid x_i \in M\})$. Consequently, the maximality of S^+ follows from Claim 2. □

Theorem 7. *The problem* SA_{stg} *is coNP-hard for noeven AFs F with* $\text{dist}_{\text{ACYC}}(F) = 1$.

Proof. The proof uses the same construction as the proof of Theorem 6. Hence let the formula φ and the AF $F = (X, A)$ be defined as in Theorem 6. We show that the argument φ' of *F* is skeptically accepted in *F* with respect to the stage semantics if and only if the formula φ is not satisfiable.

It is easy to see that each stage extension either contains φ or φ' but not both of them. Thus we have that the argument φ' is skeptically accepted if and only if the argument φ is not credulously accepted in *F* with respect to the stage semantics. Because of the proof of Theorem 6 this holds if and only if φ is not satisfiable. □

In view of the complexities of the acceptance problems in general, as summarized in Table 2, we conclude from Theorem 4 that for $\sigma \in \{\text{adm}, \text{com}, \text{prf}, \text{stb}\}$ the problem CA_σ is NP-complete for AFs *F* with $\text{dist}_{\text{BIP}}(F) = \text{dist}_{\text{SYM}}(F) = 1$, and from Theorem 5 that for $\sigma = \text{stb}$ the problem SA_σ is coNP-complete for AFs *F* with $\text{dist}_{\text{BIP}}(F) = \text{dist}_{\text{SYM}}(F) = 1$. Hence these problems reach their full hardness already at a constant distance from the considered fragments. It is natural to ask whether the same is true for acceptance problems with respect to other semantics which are in general harder than NP or coNP (recall Table 2). Note that the original reductions provided in [16,19] produce argumentation frameworks of arbitrary high distance to each of the tractable fragments under our consideration, hence the known reductions do not answer the above question. However, we can show that all the hardness results in this section can be extended to completeness results as specified in Table 2, considering AFs of constant distance from the fragment under consideration. In several cases distance 1 suffices to obtain Σ_2^P-completeness or Π_2^P-completeness, but we do not know whether distance 1 suffices in all cases. The completeness proofs are rather long and somewhat tedious, therefore we restrict ourselves to providing one Π_2^P-completeness proof as an example in Appendix A; the remaining completeness proofs can be found in a technical report [21].

6. An extension of the tractable classes

In this section we introduce a more general and powerful form of backdoors, the *strong backdoors*, and show that these can be used to make our approach applicable to an even larger class of AFs. Our notion of strong backdoor sets for abstract argumentation is inspired by strong backdoor sets for satisfiability [42]. Let $\mathcal{C}$ be a class of AFs, $F = (X, A)$ an AF, and $Y \subseteq X$. We call *Y* a *strong* $\mathcal{C}$-*backdoor of F* if $F - \text{DEF}(\lambda^*) \in \mathcal{C}$ for every partial labeling λ with $\text{DEF}(\lambda) = Y$. We write s-$\text{dist}_{\mathcal{C}}(F)$ for the size of a smallest strong $\mathcal{C}$-backdoor of *F*. Furthermore, to distinguish strong backdoors from the backdoors introduced in Section 2 we refer to the latter as *deletion backdoors* for the remainder of this section.

Example 5. Consider the AF *F* with arguments $1, \ldots, 5$ and attacks $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 1)$, $(2, 3)$, $(2, 4)$, $(3, 1)$, $(3, 2)$, $(3, 4)$, $(4, 1)$, $(4, 2)$, $(4, 3)$, $(5, 1)$, $(5, 2)$, which is displayed in Fig. 5. The set $\{4\}$ is a smallest strong Acyc-backdoor of *F*

because for every partial labeling λ of the argument 4 the AF $F - \text{DEF}(\lambda^*)$ is empty, and hence acyclic. Note also that the empty set is not a strong Acyc-backdoor of F because $F - \text{DEF}(\lambda^*)$ contains the directed cycle on the arguments 3 and 4. Fig. 5 also shows a smallest deletion Acyc-backdoor of F containing three arguments. □

It is easy to see that for every class $\mathcal{C}$ of AFs that is closed under the deletion of arguments, every deletion $\mathcal{C}$-backdoor is also a strong $\mathcal{C}$-backdoor. However, as can be observed from Example 5, the converse does not hold. Indeed, as shown by the following proposition, strong backdoors can be arbitrarily smaller and hence arbitrarily more powerful than deletion backdoors.

Proposition 2. *Let $\mathcal{C} \in \{\text{Acyc}, \text{NoEven}\}$. There are AFs where* $\text{dist}_\mathcal{C}$ *is arbitrarily high but* s-$\text{dist}_\mathcal{C}$ *is zero.*

Proof. Consider the AF $F = (X, A)$, where $X = \{c_1, \ldots, c_n, a\}$ and $A = \{(a, c_i) \mid 1 \leqslant i \leqslant n\} \cup \{(c_i, c_j) \mid i \neq j\}$, for an arbitrary positive integer n. Then it is easy to see that $\text{dist}_{\text{Acyc}}(F) = \text{dist}_{\text{NoEven}}(F) = n - 1$ but $F^* = F - \text{DEF}(\lambda_\emptyset^*) = (\emptyset, \emptyset)$ and consequently s-$\text{dist}_{\text{Acyc}}(F) = $ s-$\text{dist}_{\text{NoEven}}(F) = 0$. □

We are however able to show that for an AF F both backdoors coincide on the reduced AF $F^* = F - \text{DEF}(\lambda_\emptyset)$. Consequently, strong backdoor sets can be interpreted as deletion backdoor sets with some additional preprocessing.

Lemma 6. *Let $\mathcal{C} \in \{\text{Acyc}, \text{NoEven}\}$, $F = (X, A)$ be an AF, and $B \subseteq X$ a strong $\mathcal{C}$-backdoor of F^*. Then B is a deletion $\mathcal{C}$-backdoor of F^*.*

Proof. Suppose for a contradiction that the set B is not a deletion $\mathcal{C}$-backdoor of F^*. Because $\mathcal{C} \in \{\text{Acyc}, \text{NoEven}\}$ there must be some (even) directed cycle C in $F^* - B$. Let λ be the partial labeling defined on B with $\lambda(b) = \text{UND}$ for every $b \in B$. We show that C must already be contained in $F - \text{DEF}(\lambda^*)$, contradicting the assumption that B is a strong $\mathcal{C}$-backdoor of F.

Let $O = (x_1, \ldots, x_n)$ be an arbitrary but fixed ordering of the arguments in $\text{DEF}(\lambda^*) \setminus \text{DEF}(\lambda)$ such that the application of the rules P1–P3 to the arguments $x_1, \ldots, x_n$ in the given order leads from λ to λ^*.

Before we proceed we need to show the following claim.

Claim 3. *For every $x \in (\text{DEF}(\lambda^*) \setminus \text{DEF}(\lambda_\emptyset^*))$ it holds that $\lambda^*(x) = \text{UND}$.*

Let x_i be the first argument in the sequence O with $\lambda^*(x_i) \neq \text{UND}$. If $\lambda^*(x) = \text{OUT}$ then, according to rule P1 there is an attacker y of x with $\lambda^*(y) = \text{IN}$. It follows that $y \in \text{DEF}(\lambda_\emptyset^*)$ and hence $x \in \text{DEF}(\lambda_\emptyset^*)$ a contradiction. If $\lambda^*(x) = \text{IN}$ then, according to rule P2 it holds that $\lambda^*(y) = \text{OUT}$ for every attacker y of x. Again, this implies $y \in \text{DEF}(\lambda_\emptyset^*)$ for every attacker y of x and hence $x \in \text{DEF}(\lambda_\emptyset^*)$. This shows the claim.

We now proceed to show that the cycle C is contained in $F - \text{DEF}(\lambda^*)$. Suppose C is not contained in $F - \text{DEF}(\lambda^*)$ and let c be the first labeled argument of C in the ordering O. It follows that c has at least one attacker, i.e., the attacker on the cycle C that has not been labeled yet. Hence, c must have been labeled OUT according to rule P2 which contradicts Claim 3. □

Using the above lemma we are able to apply the tractability results from Section 4 to the case of strong backdoors.

Theorem 8. *The tractability results from Section 4, i.e., Theorems 1 and 2, do also apply to strong backdoors.*

Proof. Lemma 6 shows that we can compute a strong $\mathcal{C}$-backdoor of an AF F of size at most k by first computing F^*, and then computing a deletion $\mathcal{C}$-backdoor of F^* of size at most k. Using the argumentation from Section 4.1 it follows that we can compute the set of all complete extensions of the AF F^*. Furthermore, because $\lambda_\emptyset^*$ is the range of the grounded extension and the grounded extension of F is contained in every complete extension of F we obtain the set of all complete extensions of F by adding the grounded extension obtained from $\lambda_\emptyset^*$ to every complete extension of F^*. The theorem now follows because the acceptance problems for F can be efficiently solved given the set of all complete extensions of F (see Section 4.1) and the partial labeling $\lambda_\emptyset^*$ can be efficiently computed. □

The following theorem shows that also the intractability results from Section 5 remain true for strong backdoors.

Theorem 9. *The hardness results from Section 5, i.e., Theorems 4–10, do also apply to strong backdoors.*

Proof. Because $F = F^*$ for every AF constructed in the proofs of Theorems 4, 5, 6, 7 and 10 it follows from Lemma 6 that strong and deletion backdoors coincide on these AFs. Consequently, the theorems remain true for strong backdoors. □

7. Comparison with other parameters

In this section we compare our new structural parameters $\text{dist}_{\text{ACYC}}$, $\text{dist}_{\text{NOEVEN}}$, s-$\text{dist}_{\text{ACYC}}$, and s-$\text{dist}_{\text{NOEVEN}}$ to the parameters treewidth and clique-width that have been introduced to the field of abstract argumentation by Dunne [14,22] and Dvořák et al. [23], respectively. Both problems Credulous and Skeptical Acceptance with respect to the preferred semantics can be solved in linear time for AFs of bounded tree-width, and bounded clique-width. We do not consider any notions of "directed treewidth" because it has recently been shown that none of these notions provide tractable fragments for abstract argumentation [19,22].[2]

Treewidth of AFs. The *treewidth* of an AF $F = (X, A)$ is defined via the following notion of decomposition: a *tree decomposition* of F is a pair (T, χ) where T is a tree and χ is a labeling function with $\chi(t) \subseteq X$ for every tree node t such that the following conditions hold:

1. Every argument of F occurs in $\chi(t)$ for some tree node t.
2. For every attack (u, v) of F there is a tree node t such that $u, v \in \chi(t)$.
3. For every argument x of F, let T_x be the subgraph of T induced by all nodes t such that $x \in \chi(t)$. Then T_x is a (connected) subtree of T ("Connectedness Condition").

The *width* of a tree decomposition (T, χ) is the size of a largest set $\chi(t)$ minus 1 among all nodes t of T. A tree decomposition of smallest width is *optimal*. The *treewidth* of an AF F is the width of an optimal tree decomposition of F.

Clique-width of AFs. Let k be a positive integer. A *k-AF* is an AF whose arguments are labeled by integers from $\{1, \ldots, k\} =:$ $[k]$. The labeling of an AF $F = (X, A)$ is formally denoted by a function $L : X \to [k]$. We consider an arbitrary AF as a *k-AF* with all arguments labeled by 1. We call the *k-AF* consisting of exactly one argument x (say, labeled by $i \in [k]$) an *initial k-AF* and denote it by $i(x)$. AFs can be constructed from initial *k-AFs* by means of repeated application of the following three operations.

- Disjoint union (denoted by $\oplus$);
- Relabeling: changing all labels i to j (denoted by $\rho_{i,j}$);
- Attack insertion: adding attacks from all arguments labeled by i to all arguments labeled by j (denoted by $\eta_{i,j}$); already existing attacks are not doubled.

A construction of a *k-AF* $F = (X, A)$ using the above operations can be represented by an algebraic term composed of $i(x)$, $\oplus$, $\rho_{i,j}$, and $\eta_{i,j}$, for $i, j \in [k]$, and $x \in X$. Such a term is then called a *cwd-expression* defining F. A *k-expression* is a cwd-expression in which at most k different labels occur. The *clique-width* of an AF F is the smallest integer k such that F can be defined by a *k-expression*.

The following two propositions show that treewidth and clique-width are both incomparable to our distance parameters.

Proposition 3. *There are acyclic and noeven AFs that have arbitrarily high treewidth and clique-width.*

Proof. Consider any symmetric AF F of high treewidth or clique-width together with an arbitrary but fixed ordering $<$ of the arguments of F. By deleting all attacks from an argument x to an argument y with $y < x$ we obtain an acyclic AF F' that has the same underlying undirected graph as F. Because the treewidth of an AF is equal to the treewidth of its underlying undirected graph it immediately follows that the treewidth of F' is equal to the treewidth of F. Similarly, it is easy to see that the clique-width of any AF is at least as high as the clique-width of the AF obtained by adding all arcs (x, y) such that (y, x) is an attack in the original AF. Hence, the clique-width of F' is at least as high as the clique-width F. It follows that F' has arbitrary high treewidth and clique-width, but $\text{dist}_{\text{NOEVEN}}(F) = \text{dist}_{\text{ACYC}}(F) = \text{s-dist}_{\text{NOEVEN}}(F) = \text{s-dist}_{\text{ACYC}}(F) = 0$. $\square$

Proposition 4. *There are AFs with bounded treewidth and clique-width where* $\text{dist}_{\text{NOEVEN}}$, $\text{dist}_{\text{ACYC}}$, s-$\text{dist}_{\text{NOEVEN}}$, *and* s-$\text{dist}_{\text{ACYC}}$ *are arbitrarily high.*

Proof. Consider an AF F that consists of n disjoint directed cycles of even length. It is easy to see that the treewidth and the clique-width of F are bounded by a constant but $\text{dist}_{\text{NOEVEN}}(F) = \text{dist}_{\text{ACYC}}(F)$. Furthermore, using Lemma 6 and the fact that $F = F^*$ we also have $\text{s-dist}_{\text{NOEVEN}}(F) = \text{s-dist}_{\text{ACYC}}(F) = n$. $\square$

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[2] Note that although stable semantics are not explicitly mentioned there, the hardness results immediately apply to stable semantics.
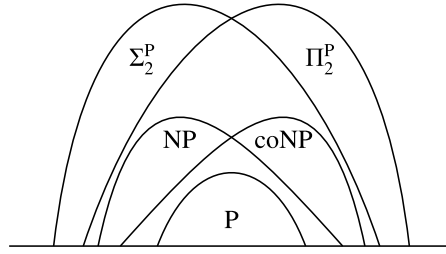
Fig. 6. The relationship between the complexity classes P, NP, coNP, Π_2^P, and Σ_2^P, assuming they are all distinct.

8. Conclusion

We have introduced a novel approach to the efficient solution of acceptance problems for abstract argumentation frameworks by "augmenting" tractable fragments. This way the efficient solving techniques known for a restricted fragment, like the fragment of acyclic argumentation frameworks, become generally applicable to a wider range of argumentation frameworks and thus relevant for real-world instances. Our approach is orthogonal to decomposition-based approaches and thus we can solve instances efficiently that are hard for known methods.

The augmentation approach entails two tasks, the detection of small backdoors and the evaluation of small backdoors. For the first task we could utilize recent results from fixed-parameter algorithm design, thus making results from a different research field applicable to abstract argumentation. For the second task we have introduced the concept of partial labeling, which seems to us a useful tool that may be of independent interest. In view of the possibility of an augmentation, our results add significance to known tractable fragments and motivate the identification of new tractable fragments. For future research it would be interesting to consider other distance measures and new tractable fragments.

Acknowledgements

Appendix A. Completeness for the second level of the polynomial hierarchy

Before we give the completeness proof, we briefly define the relevant complexity classes in terms of oracle machines and quantified Boolean formulas. For further information and background on the complexity classes we refer to other sources [32].

By an NP-oracle machine we mean a Turing machine that, in each computation step, can access an oracle that decides problems within the class NP. The complexity class $\Sigma_2^P = NP^{NP}$ consists of the problems that can be decided in polynomial time by a non-deterministic NP-oracle machine, the complexity class $\Pi_2^P = coNP^{NP}$ is defined as the complementary class of Σ_2^P, i.e., $\Pi_2^P = co\Sigma_2^P$. The complexity classes NP and coNP form the first level of the *Polynomial Hierarchy* (PH), the classes Σ_2^P and Π_2^P form the second. Classes of the $(i+1)$-th level of the PH are defined in terms of oracle machines that can access oracles for deciding problems of the i-th level. Fig. 6 gives an overview of the relations between the complexity classes of the first and second level of the PH.

The validity of quantified Boolean formulas (QBF) in a particular form provides complete problems for the various classes of the PH. A $QBF_2^\forall$ formula is a QBF formula of the form $\forall Y \exists X \varphi$ where X and Y are strings of propositional atoms and φ is a propositional formula over the atoms $X \cup Y$ (we may assume that φ is in 3-CNF). We say that a QBF $\Phi = \forall Y \exists X \varphi$ is *valid* if for each $M_Y \subseteq Y$ there exists an $M_X \subseteq X$ such that $M = M_Y \cup M_X$ is a model of φ (recall from the proof of Theorem 4 that a model is a set M of literals that contains no complementary pairs such that each clause of φ contains a literal of M). The problem $QSAT_2^\forall$, which asks to decide whether a given $QBF_2^\forall$ formula Φ is valid, is a Π_2^P-complete problem.

Theorem 10. *The problem* SA_{prf} *is* Π_2^P*-complete for AFs F with* $dist_{BIP}(F) = 1$.

Proof. It suffices to show Π_2^P-hardness for since the membership in the class Π_2^P follows from the general case. We devise a polynomial reduction from the restricted version of $QSAT_2^\forall$ where every clause contains either only positive or only negative literals. Again this restricted version is Π_2^P-complete [26].

Let Φ be a QBF of the form $\forall Y \exists X \varphi(X, Y)$ where φ is a CNF formula with variables $x_1, \ldots, x_n, y_1, \ldots, y_m$ and clauses $C_1, \ldots, C_k$ and $\bar{C}_1, \ldots, \bar{C}_l$ where for every $1 \leqslant j \leqslant k$ the clause C_j contains only positive literals and for every $1 \leqslant j \leqslant l$ the clause $\bar{C}_j$ contains only negative literals. Moreover each clause contains either a literal x_i or a literal $\bar{x}_i$.

We construct an instance $I = (F, x)$ of SA_{prf} such that $dist_{BIP}(F) = 1$ and the formula Φ is valid if and only if the argument x is skeptically accepted in the AF F with respect to the preferred semantics. F contains the following arguments:
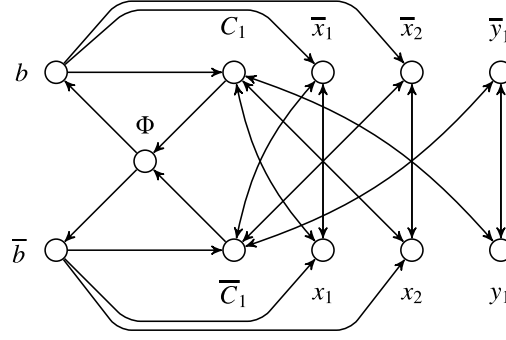
Fig. 7. Illustration for the reduction in the proof of Theorem 10 showing the AF F, obtained from the monotone QSAT$_\forall^2$ formula $\Phi = \forall y_1 \exists x_1 \exists x_2 C_1 \wedge \bar{C}_1$ with $C_1 = x_1 \vee x_2 \vee y_1$ and $\bar{C}_1 = \neg x_1 \vee \neg x_2 \vee \neg y_1$. The set $\{\varphi\}$ is a B$_{\text{IP}}$-backdoor of F.

- Two arguments x_i and $\bar{x}_i$ for every $1 \leqslant i \leqslant n$.
- Two arguments y_i and $\bar{y}_i$ for every $1 \leqslant i \leqslant m$.
- One argument C_j for every $1 \leqslant j \leqslant k$.
- One argument $\bar{C}_j$ for every $1 \leqslant j \leqslant l$.
- The arguments Φ, $\bar{b}$, and b.

F contains the following attacks:

- The attacks $(\Phi, \bar{b})$, and (Φ, b).
- For every $1 \leqslant j \leqslant k$ the attacks (C_j, Φ), and (b, C_j).
- For every $1 \leqslant j \leqslant l$ the attacks $(\bar{C}_j, \Phi)$, and $(\bar{b}, \bar{C}_j)$.
- For every $1 \leqslant i \leqslant n$:
  – The attacks $(x_i, \bar{x}_i)$, and $(\bar{x}_i, x_i)$.
  – The attacks $(b, \bar{x}_i)$, and $(\bar{b}, x_i)$.
  – For every $1 \leqslant j \leqslant k$, the attacks (x_i, C_j), and (C_j, x_i) if $x_i \in C_j$.
  – For every $1 \leqslant j \leqslant l$, the attacks $(\bar{x}_i, \bar{C}_j)$, and $(\bar{C}_j, \bar{x}_i)$ if $\bar{x}_i \in \bar{C}_j$.
- For every $1 \leqslant i \leqslant m$:
  – The attacks $(y_i, \bar{y}_i)$, and $(\bar{y}_i, y_i)$.
  – For every $1 \leqslant j \leqslant k$, the attacks (y_i, C_j), and (C_j, y_i) if $y_i \in C_j$.
  – For every $1 \leqslant j \leqslant l$, the attacks $(\bar{y}_i, \bar{C}_j)$, and $(\bar{C}_j, \bar{y}_i)$ if $\bar{y}_i \in \bar{C}_j$.

Furthermore, we set $X = \{x_i \mid 1 \leqslant i \leqslant n\}$, $Y = \{y_i \mid 1 \leqslant i \leqslant m\}$, $C = \{C_j \mid 1 \leqslant j \leqslant k\}$, and for a set M of arguments of F we set $\overline{M} = \{\bar{m} \mid m \in M\}$. Finally, we identify the argument x with Φ. For an illustration of this reduction see Fig. 7.

Before proving the correctness of the reduction we show some general observations:

Claim 4. *For the QBF Φ and the AF F constructed above the following hold*:

1. *The arguments $\bar{b}, b, c \in C \cup \bar{C}$ are not contained in any admissible set of F.*
2. *For each set $S \subseteq Y$ the set $S \cup \overline{Y \setminus S}$ is admissible in F and each set $E \subsetneq Y \cup \bar{Y}$ not of this form is not a preferred extension.*
3. *For any preferred extension E if $\Phi \notin E$ then $E = S \cup \overline{Y \setminus S}$ for some $S \subseteq Y$.*
4. *If $\Phi \in E$ then $E \cap (X \cup Y)$ is a model of φ.*
5. *If M is a model of φ then $M \cup \overline{(X \cup Y) \setminus M} \cup \{\Phi\}$ is a preferred extension of F.*

We prove each point separately:

1. In order to obtain a contradiction let us assume that there exists an admissible set E containing an argument $c \in C \cup \bar{C}$. We have that c is either attacked by $\bar{b}$ or b, and as E is an admissible set it must contain an argument that defends c. However, Φ is the only argument that attacks $\bar{b}$ or b, thus $\Phi \in E$. But as c attacks Φ this contradicts the conflict-freeness of E. Hence we conclude that no admissible set of F contains an argument from $C \cup \bar{C}$.
   Next let us assume that there exists an admissible set E containing $\bar{b}$ or b. Then E defends $\bar{b}$ or b, respectively, and thus contains an argument attacking Φ. As the only arguments attacking Φ are those in the set $C \cup \bar{C}$ this contradicts the above observation.
2. As all attacks concerning arguments in $Y \cup \bar{Y}$ are mutual attacks we conclude that a subset of $Y \cup \bar{Y}$ is admissible if and only if it is conflict-free. One can see that the maximal conflict free subsets of $Y \cup \bar{Y}$ are the sets $S \cup \overline{Y \setminus S}$ with

$S \subseteq Y$. Hence we can conclude that (i) these sets are admissible, (ii) each admissible subset of $Y \cup \overline{Y}$ is either of the form $S \cup \overline{Y \setminus S}$ with $S \subseteq Y$ or the subset of such a set, and (iii) no subset of such a set can be maximal admissible, i.e., preferred.

3. As mentioned in (1) the arguments $\bar{b}, b, c \in C \cup \overline{C}$ are not contained in any admissible set. Further, since $\Phi \notin E$ we obtain that the arguments $x \in X \cup \overline{X}$ are not defended by E and thus not contained in E. The only arguments that are left belong to $Y \cup \overline{Y}$ and hence by (2) $E = S \cup \overline{Y \setminus S}$.

4. As $\Phi \in E$ it follows that each argument in $C \cup \overline{C}$ is attacked by E. Further as $\bar{b}, b \notin E$ each argument in $C \cup \overline{C}$ is attacked by an argument in $E \cap (X \cup Y \cup \overline{X} \cup \overline{Y})$. Thus by construction, $E \cap (X \cup Y)$ is a model of φ.

5. Clearly $M \cup \overline{(X \cup Y) \setminus M} \cup \{\Phi\}$ is conflict-free. We have mutual attacks between arguments $x \in X \cup Y$ and the corresponding arguments $\bar{x} \in \overline{X} \cup \overline{Y}$. Hence all arguments in $X \cup Y \cup \overline{X} \cup \overline{Y}$ are either in the set $M \cup \overline{(X \cup Y) \setminus M}$ or attacked by some argument from this set. Further, as M is a model of Φ, it follows by construction that the arguments $M \cup \overline{(X \cup Y) \setminus M}$ attack all the arguments in $C \cup \overline{C}$ and thus defend Φ. Finally the argument Φ attacks both b and $\bar{b}$. That is, each argument of F either is in the set $M \cup \overline{(X \cup Y) \setminus M} \cup \{\Phi\}$ or attacked by an argument from this set. Hence the set $M \cup \overline{(X \cup Y) \setminus M} \cup \{\Phi\}$ is a stable extension of F, and thus also a preferred extension of F.

We are going to show that the formula Φ is valid only if the argument Φ is skeptically accepted in F with respect to prf. To this end we consider a valid formula $\Phi = \forall Y \exists X \varphi(X, Y)$. In order to obtain a contradiction let us assume that there exists a preferred extension E such that $\Phi \notin E$. Then we have that $E = S \cup \overline{Y \setminus S}$ for some $S \subseteq Y$. Using that the formula Φ is valid, we conclude that there exists a model M of φ such that $S \subsetneq M$. But then $E' = M \cup \overline{(X \cup Y) \setminus M} \cup \{\Phi\}$ is a preferred extension of F and $E \subsetneq E'$, a contradiction.

It remains to show that the formula Φ is valid if the argument Φ is skeptically accepted in F with respect to prf. Towards a contradiction, let us assume that Φ is not valid, i.e., there exists an $S \subsetneq Y$ which is not contained in any model of φ and that Φ is skeptically accepted in F. Now let us consider an arbitrary preferred extension E such that $S \cup \overline{Y \setminus S} \subseteq E$. Such an E must exist as $S \cup \overline{Y \setminus S}$ is an admissible set. By assumption $\Phi \in E$. It follows that $E \cap (X \cup Y)$ is a model of φ containing S and we obtain the desired contradiction. □

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