



On the adoption of abductive reasoning for time series interpretation

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ABSTRACT

Time series interpretation aims to provide an explanation of what is observed in terms of its underlying processes. The present work is based on the assumption that the common classification-based approaches to time series interpretation suffer from a set of inherent weaknesses, whose ultimate cause lies in the monotonic nature of the deductive reasoning paradigm. In this document we propose a new approach to this problem, based on the initial hypothesis that abductive reasoning properly accounts for the human ability to identify and characterize the patterns appearing in a time series. The result of this interpretation is a set of conjectures in the form of observations, organized into an abstraction hierarchy and explaining what has been observed. A knowledge-based framework and a set of algorithms for the interpretation task are provided, implementing a hypothesize-and-test cycle guided by an attentional mechanism. As a representative application domain, interpretation of the electrocardiogram allows us to highlight the strengths of the proposed approach in comparison with traditional classification-based approaches.

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1. Introduction

The interpretation and understanding of the behavior of a complex system involves the deployment of a cognitive apparatus aimed at guessing the processes and mechanisms underlying what is observed. The human ability to recognize patterns plays a paramount role as an instrument for highlighting evidence which should require an explanation, by matching information from observations with information retrieved from memory. Classification naturally arises as a pattern recognition task, defined as the assignment of observations to categories.

Let us first state precisely at this point what is the problem under consideration: we wish to interpret the behavior of a complex system by measuring a physical quantity along time. This quantity is represented as a time series.

The Artificial Intelligence community has devoted a great deal of effort on different paradigms, strategies, methodologies and techniques for time series classification. Nonetheless, in spite of the wide range of proposals for building classifiers, either by eliciting domain knowledge or by induction from a set of observations, the resulting classifiers behaves as deductive system. The present work is premised on the assumption that some of the important weaknesses of this approach lie in its deductive nature, and that an abductive approach can address these shortcomings, which are described below.

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Let us remember that a deduction contains in its conclusions information that is already implicitly contained in the premises, and thus it is truth-preserving. In this sense, a classifier ultimately assigns a label or a set of labels to observations. This label can designate a process or a mechanism of the system being observed, but it is nothing more than a term that summarizes the premises implied by the observations. Conversely, abduction, or inference to the best explanation, is a form of inference that goes from data to a hypothesis that best explains or accounts for the data [21]. Abductive conclusions contain new information not contained in the premises, and are capable of predicting new evidence, although they are fallible. Abductions are thus truth-widening, and they can make the leap from the language of observations to the language of the underlying processes and mechanisms, responding to the aforementioned problem in a natural way [24]. For example, consider a simple rule stating that if a patient experiences a sudden tachycardia and a decrease in blood pressure, then we can conclude that he or she is suffering from shock due to a loss of blood volume. From a deductive perspective, *loss of blood volume* is just a name provided by the rule for the satisfaction of the two premises. However, from an abductive perspective, *loss of blood volume* is an explanatory hypothesis, a conjecture, that expands the truth contained in the premises, enabling the observer to predict additional consequences such as, for example, pallid skin, faintness, dizziness or thirst.

Of course, the result of a classifier can be considered as a conjecture, but always from an external agent, since a classifier is monotonic as a logical system and its conclusions cannot be refuted from within. Classifier ensembles aim to overcome the errors of individual classifiers by combining different classification instances to obtain a better result; thus, a classifier can be amended by others in the final result of the ensemble. However, even an ensemble represents a bottom-up mapping, and classification invariably fails above a certain level of distortion within the data. The interpretation and understanding of a complex system usually unfolds along a set of abstraction layers, where at each layer the temporal granularity of the representation is reduced from below. A classification strategy provides an interpretation as the result of connecting a set of classifiers along the abstraction structure, and the monotonicity of deduction entails a propagation of errors from the first abstraction layers upwards, narrowing the capability of making a proper interpretation as new abstraction layers are successively added. Following an abductive process instead, an observation is conjectured at each abstraction layer as the best explanatory hypothesis for the data from the layer or layers below, within the context of information from above, and the non-monotonicity of abduction supports the retraction of any observation at any abstraction layer in the search for the best global explanation. Thus, bottom-up and top-down processing complement one another and provide a joint result. As a consequence, abduction can guess the underlying processes from corrupted data or even in the temporary absence of data.

On the other hand, a classifier is based on the assumption that the underlying processes or mechanisms are mutually exclusive. Superpositions of two or more processes are excluded; they must be represented by a new process, corresponding to a new category which is different and usually unrelated to previous ones. Therefore, an artificial casuistry-based heuristics is adopted, increasing the complexity of the interpretation and reducing its adaptability to the variability of observations. In contrast, abduction can reach a conclusion from the availability of partial evidence, refining the result by the incremental addition of new information. This makes it possible to discern different processes just from certain distinguishable features, and at the end to infer a set of explanations as far as the available evidence does not allow us to identify the best one, and they are not incompatible with each other.

In a classifier, the truth of the conclusion follows from the truth of all the premises, and missing data usually demand an imputation strategy that results in a conjecture: a sort of abducting to go on deducing. In contrast, an abductive interpretation is posed as a hypothesize-and-test cycle, in which missing data are naturally managed, since a hypothesis can be evoked by every single piece of evidence in isolation and these can be incrementally added to reasoning. This fundamental property of abduction is well suited to the time-varying requirements of the interpretation of time series, where future data can compel changes to previous conclusions, and the interpretation task may be requested to provide the current result as the best explanation at any given time.

Abduction has primarily been proposed for diagnostic tasks [10,33], but also for question answering [15], language understanding [22], story comprehension [6], image understanding [36] or plan recognition [28], amongst others. Some studies have proposed that perception might rely on some form of abduction. Even though abductive reasoning has been proven to be NP-complete or worse, a compiled form of abduction based on a set of pre-stored hypotheses could narrow the generation of hypotheses [24]. The present work takes this assumption as a starting point and proposes a model-based abductive framework for time series interpretation supported on a set of temporal abstraction patterns. An abstraction pattern represents a set of constraints that must be satisfied by some evidence in order to be interpreted as the hypothetical observation of a certain process, together with an observation procedure providing a set of measurements for the features of the conjectured observation. A set of algorithms is devised in order to achieve the best explanation through a process of successive abstraction from raw data, by means of a hypothesize-and-test strategy.

Some previous proposals have adopted a non-monotonic schema for time series interpretation. TrendX system detects significant trends in time series by matching data to predefined trend patterns [19,20]. One of these patterns plays the role of the expected or normal pattern, and the other patterns are fault patterns. A matching score of each pattern is based on the error between the pattern and the data. Multiple trend patterns can be maintained as competing hypotheses according to their matching score; as additional data arrive some of the patterns can be discarded and new patterns can be triggered. This proposal has been applied to diagnose pediatric growth trends. A similar proposal can be found in [27], taking a step further by providing complex temporal abstractions, the result of finding out specific temporal relationships between a set of significant trends. This proposal has been applied to the infectious surveillance of heart transplanted patients. Another example is the Résumé system, a knowledge-based temporal abstraction framework [42,39]. Its goal is to provide

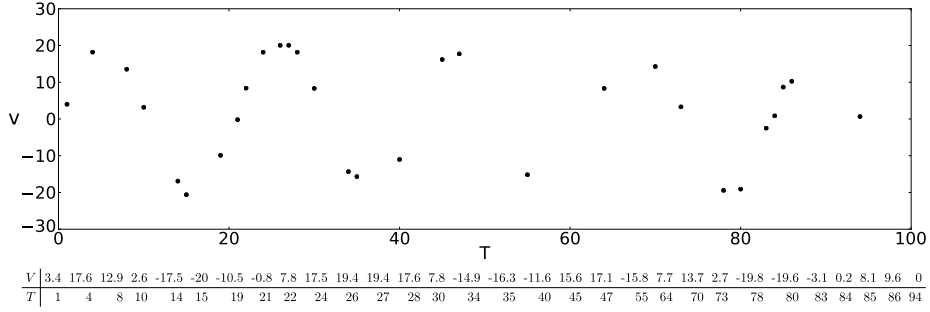


Fig. 1. Initial temporal observations.

a set of interval-based temporal abstractions from time-stamped input data, distinguishing four output abstraction types: state, gradient, rate and pattern. It uses a truth maintenance system to retract inferred intervals that are no longer true, and propagate new abstractions. Furthermore, this framework includes a non-monotonic interpolation mechanism for trend detection [41]. This approach has been applied to several clinical domains (protocol-based care, monitoring of children's growth and therapy of diabetes) and to an engineering domain (monitoring of traffic control).

The present work includes several examples and results from the domain of electrocardiography. The electrocardiogram (ECG) is the recording at the body's surface of the electrical activity of the heart as it changes with time, and is the primary method for the study and diagnosis of cardiac disease, since the processes involved in cardiac physiology manifest in characteristic temporal patterns on the ECG trace. In other words, a correct reading of the ECG has the potential to provide valuable insight into cardiac phenomena. Learning to interpret the ECG involves the acquisition of perceptual skills from an extensive bibliography with interpretation criteria and worked examples. In particular, pattern recognition is especially important in order to build a bottom-up representation of cardiac phenomena in multiple abstraction levels. This has encouraged extensive research on classification techniques for interpreting the ECG; however, in spite of all these efforts, this is still considered an open problem. We shall try to demonstrate that the problem lies in the nature of deduction itself.

The rest of this paper is structured as follows: Section 2 introduces the main concepts and terminology used in the paper in an informal and intuitive way. Following this, in Sections 3, 4 and 5 we formally describe all the components of the interpretation framework, including the knowledge representation model and the algorithms used to obtain effective interpretations within an affordable time. Section 6 illustrates the capabilities of the framework in overcoming some of the most important shortcomings of deductive classifiers. Section 7 presents the main experimental results derived from this work. Finally, in section 8 we discuss the properties of the model compared with other related approaches and draw several conclusions.

2. Interpretation as a process-guessing task

We propose a knowledge-based interpretation framework upon the principles of abductive reasoning, on the basis of a strategy of hypothesis formation and testing. Taking as a starting point a time series of physical measurements, a set of observations are guessed as conjectures of the underlying processes, through successive levels of abstraction. Each new observation will be generated from previous levels as the underlying processes aggregate, superimpose or concatenate to form more complex processes with greater duration and scope, and are organized into an abstraction hierarchy.

The knowledge of the domain is described as a set of abstraction patterns as follows:

$$[h_{\psi}(\mathbf{A}_h, T_h^b, T_h^e) = \Theta(\mathbf{A}_1, T_1, \dots, \mathbf{A}_n, T_n)] \text{ abstracts } m_1(\mathbf{A}_1, T_1), \dots, m_n(\mathbf{A}_n, T_n) \\ \{C(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, T_1, \dots, \mathbf{A}_n, T_n)\}$$

where $h_{\psi}(\mathbf{A}_h, T_h^b, T_h^e)$ is an observable of the domain playing the role of a hypothesis on the observation of an underlying process ψ . \mathbf{A}_h represents a set of attributes, and T_h^b and T_h^e are two temporal instants representing the beginning and the end of the hypothesis. $m_1(\mathbf{A}_1, T_1), \dots, m_n(\mathbf{A}_n, T_n)$ is a set of observables of the domain which plays the role of the evidence suggesting the observation of h_{ψ} . Each piece of evidence has its own set of attributes \mathbf{A}_i and temporal support T_i , represented here as a single instant for the sake of simplicity, but it could also be an interval (T_i^b, T_i^e) . C is a set of constraints among the variables involved in the abstraction pattern, which are interpreted as necessary conditions in order for the evidence $m_1(\mathbf{A}_1, T_1), \dots, m_n(\mathbf{A}_n, T_n)$ to be abstracted into $h_{\psi}(\mathbf{A}_h, T_h^b, T_h^e)$. Finally, $\Theta(\mathbf{A}_1, T_1, \dots, \mathbf{A}_n, T_n)$ is an observation procedure that gives as a result an observation of $h_{\psi}(\mathbf{A}_h, T_h^b, T_h^e)$ from a set of observations for $m_1(\mathbf{A}_1, T_1), \dots, m_n(\mathbf{A}_n, T_n)$.

To illustrate this concept, consider the sequence of observations in Fig. 1. Each of these observations is an instance of an observable we call *point* (p), represented as $p(\mathbf{A} = \{V\}, T)$, where T determines the temporal location of the observation and V is a value attribute.

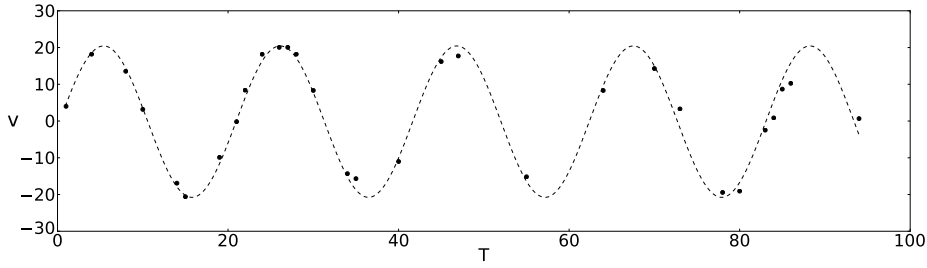


Fig. 2. Abstracted sinusoidal process.

If we analyze these observations visually, we may hypothesize the presence of an underlying sinusoidal process. Let us define an observable *sinus* for such a sinusoidal process, with two attributes: the amplitude of the process (α) and its frequency (ω). The knowledge necessary to conjecture this hypothesis is collected in the following abstraction pattern:

$$[h_{\text{sinus}}(\{\alpha, \omega\}, T_h^b, T_h^e) = \Theta(V_1, T_1, \dots, V_n, T_n)] \text{ abstracts } p(V_1, T_1), \dots, p(V_n, T_n) \\ \{C(\alpha, \omega, T_h^b, T_h^e, V_1, T_1, \dots, V_n, T_n)\}$$

We can estimate the attribute values ($\alpha, \omega, T_h^b, T_h^e$) of this process by a simple observation procedure Θ that calculates $\alpha = \max(|V_i|)$, for $1 \leq i \leq n$, i.e., the amplitude α is obtained as the maximum absolute value of the observations; $\omega = \pi / \text{mean}(T_j^{\text{peak}} - T_{j-1}^{\text{peak}})$, where T_j^{peak} are point observations representing a peak, satisfying $(V_j^{\text{peak}} = V_k, T_j^{\text{peak}} = T_k) \wedge \text{sign}(V_k - V_{k-1}) \neq \text{sign}(V_{k+1} - V_k)$, so that the frequency ω is obtained as the inverse of the mean temporal separation between consecutive peaks in the sequence of observations; and $T_h^b = T_1, T_h^e = T_n$, i.e., the temporal support of the hypothesis is the time interval between the first and the last evidence points.

We can impose the following constraint $C(\alpha, \omega, T_h^b, T_h^e, V_1, T_1, \dots, V_n, T_n)$ for every pair (V_i, T_i) in the sequence:

$$|\alpha \cdot \sin(\omega \cdot T_i) - V_i| \leq \epsilon.$$

This constraint provides a model of a sinusoidal process and a measure of how well it fits a set of observations by means of a maximum error ϵ . Fig. 2 shows the continuous representation of the abstracted process, whose resulting observation is $h_{\text{sinus}}(\alpha = 20, \omega = 0.3, T_h^b = 1, T_h^e = 94)$. A value of $\alpha/3$ has been chosen for ϵ .

Of course, various observation procedures can be devised in order to estimate the same or different characteristics of the process being guessed. These procedures can provide one or several valid estimations in terms of their consistency with the abovementioned necessary constraints. In addition, different processes can be guessed from the same set of observations, all of them being valid in terms of their consistency. Hence, further criteria may be needed in order to rank the set of interpretations.

This simple example summarizes the common approach to the interpretation of experimental results in science and technology, when the knowledge is available as a model or a set of models. The challenge is to assume that this knowledge is not available in an analytical but in a declarative form, as a pattern or a set of patterns, and that the interpretation task is expected to mimic certain mechanisms of human perception.

3. Definitions

In this section we formally define the main pieces of our interpretation framework: *observables* and *observations* for representing the behavior of the system under study, and *abstraction patterns* for representing the knowledge about this system.

3.1. Representation entities

An *observation* is the result of measuring something with the quality of being *observable*. We call $\mathcal{Q} = \{q_0, q_1, \dots, q_n\}$ the set of observables of a particular domain.

Definition 1. We define an **observable** as a tuple $q = \langle \psi, \mathbf{A}, T^b, T^e \rangle$, where ψ is a name representing the underlying process being observable, $\mathbf{A} = \{A_1, \dots, A_{n_q}\}$ is a set of attributes to be valued, and T^b and T^e are two temporal variables representing the beginning and the end of the observable.

We call $V_q(A_i)$ the domain of possible values for the attribute A_i . We assume a representation of the time domain τ isomorphic to the set of real numbers \mathbb{R} . In the case of an instantaneous observable, this is represented as $q = \langle \psi, \mathbf{A}, T \rangle$. Some observables can be dually represented from the temporal perspective, as either an observable supported by a temporal

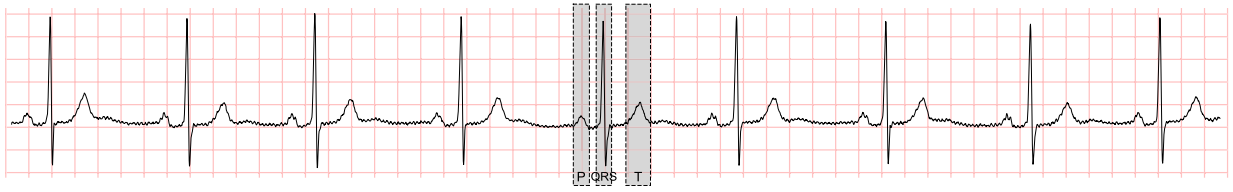


Fig. 3. Example of the ECG basic waveforms. [Source: MIT-BIH arrhythmia DB [18], recording: 123, between 12:11.900 and 12:22.400].

interval or as an observable supported by a temporal instant, according to the task to be carried out. A paradigmatic example is found in representing the heart beat, since it can be represented as a domain entity with a temporal extension comprising its constituent waves, and it can also be represented as an instantaneous entity for measuring heart rate.

Example 3.1. In the ECG signal, several distinctive waveforms can be identified, corresponding to the electrical activation-recovery cycle of the different heart chambers. The so-called P wave represents the activation of the atria, and is the first wave of the cardiac cycle. The next group of waves recorded is the QRS complex, representing the simultaneous activation of the right and left ventricles. Finally, the wave that represents the ventricular recovery is called the T wave. Together, these waveforms devise the characteristic pattern of the heart cycle, which is repeated in a normal situation with every beat [46]. An example of a common ECG strip is shown in Fig. 3.

According to this description, the observable $q_{pw} = \langle \text{atrial_activation}, \{\text{amplitude}\}, T^b, T^e \rangle$ represents a P wave resulting from an atrial activation process with an unknown amplitude, localized in a still unknown temporal interval.

Definition 2. We define an **observation** as a tuple $o = \langle q, \mathbf{v}, t^b, t^e \rangle$, an instance of the observable q resulting from assigning a specific value to each attribute and to the temporal variables, where $\mathbf{v} = (v_1, \dots, v_{n_q})$ is the set of attribute values such that $\mathbf{v} \in V_q(A_1) \times \dots \times V_q(A_{n_q})$ and $t^b, t^e \in \tau$ are two precise instants limiting the beginning and the end of the observation.

We also use the notation $(A_1 = v_1, \dots, A_{n_q} = v_{n_q})$ to represent the assignment of values to the attributes of the observable and $T^b = t^b$ and $T^e = t^e$ for representing the assignment of temporal limits to the observation.

Example 3.2. The tuple $o = \langle q_{pw}, 0.17\text{mV}, 12:16.977, 12:17.094 \rangle$ represents the particular occurrence of the P wave observable highlighted in Fig. 3.

Some notions involving observables and observations are defined below that will be useful in describing certain properties and constraints of the domain concepts, as well as in temporally arranging the interpretation process.

Definition 3. Given a set of observables \mathcal{Q} , a **generalization relation** can be defined between two different observables $q = \langle \psi, \mathbf{A}, T^b, T^e \rangle$ and $q' = \langle \psi', \mathbf{A}', T'^b, T'^e \rangle$, denoted by q' is a q , meaning that q generalizes q' if and only if $\mathbf{A} \subseteq \mathbf{A}'$ and $V_{q'}(A_i) \subseteq V_q(A_i) \forall A_i \in \mathbf{A}$.

The generalization relation is reflexive, antisymmetric and transitive. The inverse of a generalization relation is a specification relation. From a logical perspective, a generalization relation can be read as an implication $q' \rightarrow q$, meaning that q' is more specific than q . It holds that every observation $o = \langle q', \mathbf{v}, t^b, t^e \rangle$ of the observable q' is also an observation of q .

Example 3.3. A common example of a generalization relation can be defined from a domain partition of an attribute. For example, the observable $q_1 = \langle \text{Sinus_Rhythm}, \{\text{RR} \in [200 \text{ ms}, 4000 \text{ ms}]\}, T^b, T^e \rangle$ is a generalization of the observables $q_2 = \langle \text{Sinus_Tachycardia}, \{\text{RR} \in [200 \text{ ms}, 600 \text{ ms}]\}, T^b, T^e \rangle$, $q_3 = \langle \text{Normal_Rhythm}, \{\text{RR} \in [600 \text{ ms}, 1000 \text{ ms}]\}, T^b, T^e \rangle$ and $q_4 = \langle \text{Sinus_Bradycardia}, \{\text{RR} \in [1000 \text{ ms}, 4000 \text{ ms}]\}, T^b, T^e \rangle$. The RR attribute represents the measure of the mean time distance between consecutive beats, while q_2, q_3 and q_4 represent the normal cardiac rhythm denominations according to the heart rate [46].

Definition 4. Given a set of observables \mathcal{Q} , an **exclusion relation** can be defined between two different observables $q = \langle \psi, \mathbf{A}, T^b, T^e \rangle$ and $q' = \langle \psi', \mathbf{A}', T'^b, T'^e \rangle$, denoted by q excludes q' , meaning that they are mutually exclusive if and only if their respective processes ψ and ψ' cannot concurrently occur.

The exclusion relation is defined by extension from the knowledge of the domain, and its rationale lies in the nature of the underlying processes and mechanisms. Inasmuch as the occurrence of a process can only be hypothesized as long as it is observable, the exclusion relation behaves as a restriction on observations. Thus, given two observables q and q' , q excludes q' entails that they cannot be observed over two overlapping intervals, i.e., every two observations $o =$

$\langle q, \mathbf{v}, t^b, t^e \rangle$ and $o' = \langle q', \mathbf{v}', t'^b, t'^e \rangle$ satisfy either $t^e < t'^b$ or $t'^e < t^b$. The opposite is not generally true. The exclusion relation is symmetric and transitive. As an example, in the domain of electrocardiography, the knowledge about the physiology of the heart precludes the observation of a *P wave* during an episode of *Atrial fibrillation* [46], so these two observables are mutually exclusive.

We call \mathcal{O} the set of observations available for the observables in \mathcal{Q} . In order to index this set of observations, they will be represented as a sequence by defining an order relation between them. This ordering aims to prioritize the interpretation of the observations as they appear.

Definition 5. Let $<$ be an **order relation** between two observations $o_i = \langle q_i, \mathbf{v}_i, t_i^b, t_i^e \rangle$ and $o_j = \langle q_j, \mathbf{v}_j, t_j^b, t_j^e \rangle$ such that $(o_i < o_j) \Leftrightarrow (t_i^b < t_j^b) \vee ((t_i^b = t_j^b) \wedge (t_i^e < t_j^e)) \vee ((t_i^b = t_j^b) \wedge (t_i^e = t_j^e) \wedge (q_i < q_j))$, assuming a lexicographical order between observable names.

A sequence of observations is an ordered set of observations $\mathcal{O} = (o_1, \dots, o_i, \dots)$ where for all $i < j$ then $o_i < o_j$. Every subset of a sequence of observations is also a sequence. The q -sequence of observations from \mathcal{O} , denoted as $O(q)$, is the subset of the observations for the observable q . The exclusion relation forces that any two observations $o_i = \langle q, \mathbf{v}_i, t_i^b, t_i^e \rangle$ and $o_j = \langle q, \mathbf{v}_j, t_j^b, t_j^e \rangle$ in $O(q)$ satisfy $o_i < o_j \Rightarrow t_i^e < t_j^b$ for the current application domain. By $\text{succ}(o_i)$ we denote the successor of the observation o_i in the sequence \mathcal{O} , according to the order relation $<$. By $q\text{-succ}(o_i)$ we denote the successor of the observation $o_i \in O(q)$ in its q -sequence $O(q)$. Conversely to this notation, we denote by $q(o_i)$ the observable corresponding to the o_i observation.

3.2. Abstraction patterns

We model an abstraction process as an abduction process, based on the conjectural relation $m \leftarrow h$ [21], which can be read as ‘the observation of the finding m allows us to conjecture the observation of h as a possible explanatory hypothesis’. For example, a very prominent peak in the ECG signal allows us to conjecture the observation of a heartbeat. A key aspect of the present proposal is that both the hypothesis and the finding are observables, and therefore formally identical, i.e., there exists $q_i, q_j \in \mathcal{Q}$, with $q_i \neq q_j$, such that $h \equiv q_i = \langle \psi_i, \mathbf{A}_i, T_i^b, T_i^e \rangle$ and $m \equiv q_j = \langle \psi_j, \mathbf{A}_j, T_j^b, T_j^e \rangle$. In general, an abstraction process can involve a number of different findings, even multiple findings of the same observable, and a set of constraints among them; thus, for example, a regular sequence of normal heartbeats allows us to conjecture the observation of a sinus rhythm. Additionally, an observation procedure is required in order to produce an observation of the hypothesis from the observation of those findings involved in the abstraction process.

We devise an abstraction process as a knowledge-based reasoning process, supported by the notion of abstraction pattern, which brings together those elements required to perform an abstraction. Formally:

Definition 6. An **abstraction pattern** $P = \langle h, M_P, C_P, \Theta_P \rangle$ consists of a hypothesis h , a set of findings $M_P = \{m_1, \dots, m_n\}$, a set of constraints $C_P = \{C_1, \dots, C_t\}$ among the findings and the hypothesis, and an observation procedure $\Theta_P(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_n, T_n^b, T_n^e) \in O(h)$.

Every constraint $C_i \in C_P$ is a relation defined on a subset of the set of variables taking part in the set of findings and the hypothesis $\{\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_n, T_n^b, T_n^e\}$. Thus, a constraint is a subset of the Cartesian product of the respective domains, and represents the simultaneously valid assignments to the variables involved. We will denote each constraint by making reference to the set of variables being constrained, as in $C_P(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_n, T_n^b, T_n^e)$ for the whole abstraction pattern.

An abstraction pattern establishes, through the set C_P , the conditions for conjecturing the observation of h from a set of findings M_P , and through the observation procedure Θ_P , the calculations for producing a new observation $o_h \in O(h)$ from the observation of these findings. We call $M_P^q = \{m_1^q, m_2^q, \dots, m_s^q\}$ the set of findings of the observable q in P , being $M_P = \bigcup_{q \in \mathcal{Q}} M_P^q$. Thus, a set of findings allows the elements of a multiset of observables to be distinguished. The interpretation procedure will choose, as we will see later, from the available observations for every observable q satisfying the constraints C_P , which are to be assigned to the findings in M_P^q in order to calculate o_h .

The set of findings M_P is divided into two disjoint sets A_P and E_P , where A_P is the set of findings that is said to be abstracted in o_h , and E_P is the set of findings that constitute the observation environment of o_h , that is, the set of findings needed to properly conjecture o_h , but which are not synthesized in o_h .

A temporal covering assumption can be made as a *default assumption* [36] on a hypothesis $h = \langle \psi_h, \mathbf{A}_h, T_h^b, T_h^e \rangle$ with respect to those findings $m = \langle \psi_m, \mathbf{A}_m, T_m^b, T_m^e \rangle$ appearing in an abstraction pattern:

Default Assumption 1. (Temporal covering) Given an abstraction pattern P , it holds that $T_h^b \leq T_m^b$ and $T_m^e \leq T_h^e$, for all $m \in A_P \subseteq M_P$.

The temporal covering assumption allows us to define the exclusiveness of an interpretation as the impossibility of including competing abstractions in the same interpretation.

Example 3.4. According to [11], in the electrocardiography domain a “wave” is a discernible deviation from a horizontal reference line called baseline, where at least two opposite slopes can be identified. The term discernible means that both the amplitude and the duration of the deviation must exceed some minimum values, agreed as 20 μV and 6 ms respectively. A wave can be completely described by a set of attributes: its amplitude (A), voltage polarity ($VP \in \{+, -\}$) and its main turning point T^p , resulting in the following observable:

$$q_{\text{wave}} = \langle \text{electrical_activity}, \{A, VP, T^p\}, T^b, T^e \rangle$$

Let us consider the following abstraction pattern:

$$P_{\text{wave}} = \langle \text{wave}, M_P = \{m_0^{\text{ECG}}, \dots, m_n^{\text{ECG}}\}, C_{P_{\text{wave}}}, \text{wave_observation}() \rangle$$

where m_i^{ECG} is a finding representing an ECG sample, with a single attribute V_i representing the sample value, and a temporal variable T_i representing its time point. We set the onset and end of a wave to the time of the second m_1^{ECG} and second-to-last m_{n-1}^{ECG} samples, considering m_0^{ECG} and m_n^{ECG} as environmental observations which are used to check the presence of a slope change just before and after the wave; thus $E_{P_{\text{wave}}} = \{m_0^{\text{ECG}}, m_n^{\text{ECG}}\}$, and $A_{P_{\text{wave}}} = \{m_1^{\text{ECG}}, \dots, m_{n-1}^{\text{ECG}}\}$.

A set of temporal constraints are established between the temporal variables: $c_1 = \{T^e - T^b \geq 6 \text{ ms}\}$, $c_2 = \{T^b = T_1\}$, $c_3 = \{T^e = T_{n-1}\}$ and $c_4 = \{T^b < T^p < T^e\}$. Another set of constraints limit the amplitude and slope changes of the samples included in a wave: $c_5 = \{\text{sign}(V_1 - V_0) \neq \text{sign}(V_2 - V_1)\}$, $c_6 = \{\text{sign}(V_n - V_{n-1}) \neq \text{sign}(V_{n-1} - V_{n-2})\}$, $c_7 = \{\text{sign}(V_{tp} - V_{tp-1}) = -\text{sign}(V_{tp+1} - V_{tp})\}$ and $c_8 = \{\min\{|V_{tp} - V_1|, |V_{tp} - V_{n-1}|\} \geq 20 \mu\text{V}\}$. These two sets form the complete set of constraints of the pattern $C_{P_{\text{wave}}} = \{c_1, \dots, c_8\}$.

Once a set of ECG samples has satisfied these constraints, they support the observation of a wave: $o_{\text{wave}} = \langle q_{\text{wave}}, (a, vp, t^p), t^b, t^e \rangle$. The values of t^b and t^e are completely determined by the constraints c_2 and c_3 , while the observation procedure `wave_observation()` provides a value for the attributes as follows: $vp = \text{sign}(V_{tp} - V_1)$, $a = \max\{|V_{tp} - V_1|, |V_{tp} - V_{n-1}|\}$, and $t^p = t^b + tp$, where $tp = \arg \min_k \{V_k | 1 \leq k \leq n-1\}$, if $V_1 < V_0$, or $tp = \arg \max_k \{V_k | 1 \leq k \leq n-1\}$, if $V_1 > V_0$.

3.3. Abstraction grammars

According to the definition, an abstraction pattern is defined over a fixed set of evidence findings M_P . In general, however, an abstraction involves an undetermined number of pieces of evidence – in the case of an ECG wave, the number of samples. Hence, we provide a procedure for dynamically generating abstraction patterns, based on the theory of formal languages. The set \mathcal{Q} of observables can be considered as an alphabet. Given an alphabet \mathcal{Q} , the special symbols \emptyset (empty set), and λ (empty string), and the operators $|$ (union), \cdot (concatenation), and $*$ (Kleene closure), a formal grammar G denotes a pattern of symbols of the alphabet, describing a language $L(G) \subseteq \mathcal{Q}^*$ as a subset of the set of possible strings of symbols of the alphabet.

Let G^{ap} be the class of formal grammars of abstraction patterns. An *abstraction grammar* $G \in G^{\text{ap}}$ is syntactically defined as a tuple (V_N, V_T, H, R) . For the production rules in R the expressiveness of right-linear grammars is adopted [23]:

$$H \rightarrow qD$$

$$D \rightarrow qF | q | \lambda$$

H is the initial symbol of the grammar, and this plays the role of the hypothesis guessed by the patterns generated by G . V_N is the set of non-terminal symbols of the grammar, satisfying $H \in V_N$, although H cannot be found on the right-hand side of any production rule, since a hypothesis cannot be abstracted by itself. V_T is the set of terminal symbols of the grammar, representing the set of observables $\mathcal{Q}_G \subseteq \mathcal{Q}$ that can be abstracted by the hypothesis.

Given a grammar $G \in G^{\text{ap}}$, we devise a constructive method for generating a set of abstraction patterns $P_G = \{P_1, \dots, P_i, \dots\}$. Since a formal grammar is simply a syntactic specification of a set of strings, every grammar $G \in G^{\text{ap}}$ is semantically extended to an attribute grammar [1], embedded with a set of actions to be performed in order to incrementally build an abstraction pattern by the application of production rules. An abstraction grammar is represented as $G = ((V_N, V_T, H, R), B, BR)$, where $B(\alpha)$ associates each grammar symbol $\alpha \in V_N \cup V_T$ with a set of attributes, and $BR(r)$ associates each rule $r \in R$ with a set of attribute computation rules. An abstraction grammar associates the following attributes: i) $P(\text{attern})$, with each non-terminal symbol of the grammar; this will be assigned an abstraction pattern; ii) $A(\text{bstracted})$, with each terminal symbol corresponding to an observable $q \in \mathcal{Q}_G$; this allows us to assign each finding either to the set A_P or E_P , depending on its value of true or false; iii) $C(\text{onstraint})$, with each terminal symbol corresponding to an observable; this will be assigned a set of constraints. There are approaches in the bibliography dealing with different descriptions of Constraint Satisfaction Problems and their semantic expression in different formalisms [2,5,12]. By explicitly specifying a constraint as a relation a clear description is provided on its underlying meaning, but this can lead to cumbersome knowledge representation processes. Multiple mathematical conventions can concisely and conveniently describe a constraint as a Boolean-valued function over the variables of a set of observables. However, we will focus on the result of applying a set of constraints among the variables involved.

In the following, the set of attribute computation rules associated with the grammar productions is specified to provide a formal method for building abstraction patterns $P \in P_{G_h}$ from a grammar $G_h \in G^{ap}$. P_{G_h} gathers the set of abstraction patterns that share the same observable h as a hypothesis; thus, these represent the different ways to conjecture h . Using this method, the application of every production incrementally adds a new observable as a finding and a set of constraints between this finding and previous entities, as follows:

1. The initial production $H \rightarrow qD$ entails:

$$\begin{aligned} P_H &:= \langle h, M_H = \emptyset, C_H = \emptyset, \Theta_H = \emptyset \rangle \\ C_q &:= C(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, T_1^b, T_1^e) \\ A_q &\in \{true, false\} \\ P_D &:= \langle h, M_D = M_H \cup \{m_1^q\}, C_D = C_H \cup C_q, \Theta_D(\mathbf{A}_1, T_1^b, T_1^e) \rangle \end{aligned}$$

2. All productions in the form $D \rightarrow qF$ entail:

$$\begin{aligned} P_D &:= \langle h, M_D, C_D, \Theta_D(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_k, T_k^b, T_k^e) \rangle \\ C_q &:= C(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, \dots, \mathbf{A}_{k+1}, T_{k+1}^b, T_{k+1}^e) \\ A_q &\in \{true, false\} \\ P_F &:= \langle h, M_F = M_D \cup \{m_{k+1}^q\}, C_F = C_D \cup C_q, \Theta_F(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_{k+1}, T_{k+1}^b, T_{k+1}^e) \rangle \end{aligned}$$

3. Productions in the form $D \rightarrow q$ conclude the generation of a pattern $P \in P_{G_h}$:

$$\begin{aligned} P_D &:= \langle h, M_D, C_D, \Theta_D(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_k, T_k^b, T_k^e) \rangle \\ C_q &:= C(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, \dots, \mathbf{A}_{k+1}, T_{k+1}^b, T_{k+1}^e) \\ A_q &\in \{true, false\} \\ P &:= \langle h, M_P = M_D \cup \{m_{k+1}^q\}, C_P = C_D \cup C_q, \Theta_P(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_{k+1}, T_{k+1}^b, T_{k+1}^e) \rangle \end{aligned}$$

4. Productions in the form $D \rightarrow \lambda$ also conclude the generation of a pattern:

$$\begin{aligned} P_D &:= \langle h, M_D, C_D, \Theta_D(\mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_k, T_k^b, T_k^e) \rangle \\ P &:= P_D \end{aligned}$$

This constructive method enables the incremental addition of new constraints as new findings are included in the representation of the abstraction pattern, providing a dynamic mechanism for knowledge assembly by language generation. The final constraints in C_P are obtained from the conjunction of the constraints added at each step. Moreover, it is possible to design an adaptive observation procedure as new evidence becomes available, since the observation procedure may be different at each step.

In the case that no temporal constraints are attributed to a production, a ‘hereafter’ temporal relationship will be assumed by default to exist between the new finding and the set of previous findings. For instance, a production of the form $D \rightarrow qF$ entails that $C_F = C_P \cup \{T_i^b \leq T_{k+1}^b \mid m_i \in M_P\}$.

Hence, in the absence of any temporal constraint, an increasing temporal order among consecutive findings in every abstraction pattern is assumed. Moreover, every temporal constraint must be consistent with this temporal order.

According to the limitation imposed on observations of the same observable which prevents two different observations from occurring at the same time, an additional constraint is added on any two findings of the same observable, and thus $\forall m_i^q, m_j^q \in M_P, (T_i^e < T_j^b \vee T_j^e < T_i^b)$.

Several examples of abstraction pattern grammars modeling common knowledge in electrocardiography are given below, in order to illustrate the expressiveness of the G^{ap} grammars.

Example 3.5. The grammar $G_N = (V_N, V_T, H, R)$ is designed to generate an abstraction pattern for a *normal cardiac cycle*, represented by the observable q_N , including the descriptions of common durations and intervals [46]. In this grammar, $V_N = \{H, D, E\}$, $V_T = \{q_{PW}, q_{QRS}, q_{TW}\}$, and R is given by:

$$\begin{aligned} H &\rightarrow q_{PW}D & (P_H := \langle q_N, M_H = \emptyset, C_H = \emptyset, \Theta_H = \emptyset \rangle, \\ & & C_{PW} := \{T_N^b = T_{PW}^b; 50 \text{ ms} \leq T_{PW}^e - T_{PW}^b \leq 120 \text{ ms}\}, \\ & & A_{PW} := true, \end{aligned}$$

$$\begin{aligned}
& P_D := \langle q_N, M_D = \{m^{Pw}\}, C_D = C_{Pw}, \Theta_D = \emptyset \rangle \\
& \} \\
D \rightarrow q_{QRS} E & \{ P_D := \langle q_N, M_D = \{m^{Pw}\}, C_D = C_{Pw}, \Theta_D = \emptyset \rangle, \\
& C_{QRS} := \{50 \text{ ms} \leq T_{QRS}^e - T_{QRS}^b \leq 150 \text{ ms}; 100 \text{ ms} \leq T_{QRS}^b - T_{Pw}^b \leq 210 \text{ ms}\}, \\
& A_{QRS} := \text{true}, \\
& P_E := \langle q_N, M_E = M_D \cup \{m^{QRS}\}, C_E = C_D \cup C_{QRS}, \Theta_E = \emptyset \rangle \\
& \} \\
E \rightarrow q_{Tw} & \{ P_E := \langle q_N, M_E = \{m^{Pw}, m^{QRS}\}, C_E, \Theta_E = \emptyset \rangle, \\
& C_{Tw} := \{80 \text{ ms} \leq T_{Tw}^b - T_{QRS}^e \leq 120 \text{ ms}; T_{Tw}^e - T_{QRS}^b \leq 520 \text{ ms}; T_N^e = T_{Tw}^e\}, \\
& A_{Tw} := \text{true}, \\
& P := \langle q_N, M_P = M_E \cup \{m^{Tw}\}, C_P = C_E \cup C_{Tw}, \Theta_P = \emptyset \rangle \\
& \}
\end{aligned}$$

This grammar generates a single abstraction pattern, which allows us to interpret the sequence of a P wave, a QRS complex, and a T wave as the coordinated contraction and relaxation of the heart muscle, from the atria to the ventricles. Some additional temporal constraints are required and specified in the semantic description of the production rules. In this case, an observation procedure Θ is not necessary since the attributes of the hypothesis are completely determined by the constraints in the grammar, and do not require additional calculus.

The next example shows the ability of an abstraction grammar to generate abstraction patterns dynamically with an undefined number of findings.

Example 3.6. A *bigeminy* is a heart arrhythmia in which there is a continuous alternation of long and short heart beats. Most often this is due to ectopic heart beats occurring so frequently that there is one after each normal beat, typically premature ventricular contractions (PVCs) [46]. For example, a normal beat is followed shortly by a PVC, which is then followed by a pause. The normal beat then returns, only to be followed by another PVC. The grammar $G_{VB} = (V_N, V_T, H, R)$ generates a set of abstraction patterns for *ventricular bigeminy*, where $V_N = \{H, D, E, F\}$, $V_T = \{q_N, q_V\}$, and R is given by:

$$\begin{aligned}
H \rightarrow q_N D & \{ P_H := \langle q_{VB}, M_H = \emptyset, C_H = \emptyset, \Theta_H = \emptyset \rangle, \\
& C_N := \{T_{VB}^b = T_1\}, \\
& A_N := \text{true}, \\
& P_D := \langle q_{VB}, M_D = \{m_1^N\}, C_D = C_N, \Theta_D = \emptyset \rangle \\
& \} \\
D \rightarrow q_V E & \{ P_D := \langle q_{VB}, M_D = \{m_1^N\}, C_D = C_N, \Theta_D = \emptyset \rangle, \\
& C_V := \{200 \text{ ms} \leq T_2 - T_1 \leq 800 \text{ ms}\}, \\
& A_V := \text{true}, \\
& P_E := \langle q_{VB}, M_E = M_D \cup \{m_2^V\}, C_E = C_D \cup C_V, \Theta_E = \emptyset \rangle \\
& \} \\
E \rightarrow q_N F & \{ P_E := \langle q_{VB}, M_E = \{m_1^N, \dots, m_{k-1}^V\}, C_E, \Theta_E = \emptyset \rangle, \\
& C_N := \{1.5 \cdot 200 \text{ ms} \leq T_k - T_{k-1} \leq 4 \cdot 800 \text{ ms}\}, \\
& A_N := \text{true}, \\
& P_F := \langle q_{VB}, M_F = M_E \cup \{m_k^N\}, C_F = C_E \cup C_N, \Theta_F = \emptyset \rangle \\
& \} \\
F \rightarrow q_V E & \{ P_F := \langle q_{VB}, M_F = \{m_1^N, m_2^V, \dots, m_k^N\}, C_F, \Theta_F = \emptyset \rangle, \\
& C_V := \{200 \text{ ms} \leq T_{k+1} - T_k \leq 800 \text{ ms}\}, \\
& A_V := \text{true}, \\
& P_E := \langle q_{VB}, M_E = M_F \cup \{m_{k+1}^V\}, C_E = C_F \cup C_V, \Theta_E = \emptyset \rangle \\
& \}
\end{aligned}$$

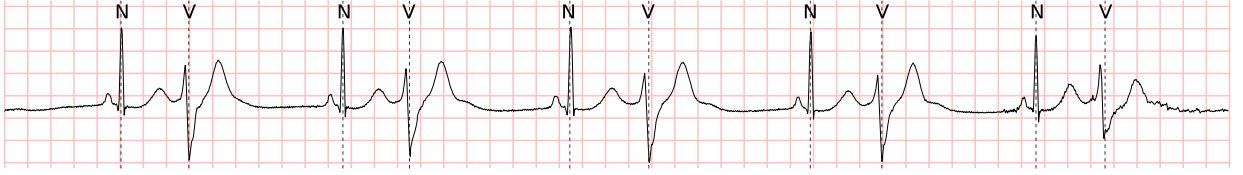


Fig. 4. Example of ventricular bigeminy. [Source: MIT-BIH arrhythmia DB, recording: 106, between 25:06.350 and 25:16.850].

$$\begin{aligned}
 F \rightarrow q_V \quad & \{P_F := \langle q_{VB}, M_F = \{m_1^N, m_2^V, \dots, m_{n-1}^N\}, C_F, \Theta_F = \emptyset \rangle, \\
 & C_V := \{200 \text{ ms} \leq T_n - T_{n-1} \leq 800 \text{ ms}; T_{VB}^e = T_n\}, \\
 & A_V := \text{true}, \\
 & P := \langle q_{VB}, M_P = M_F \cup \{m_n^V\}, C_P = C_F \cup C_V, \Theta_P = \emptyset \rangle \\
 & \}
 \end{aligned}$$

For simplicity, we have referenced each N and V heart beat with a single temporal variable. Thus T_i represents the time point of the i th heart beat, and is a normal beat if i is odd, and a PVC if i is even. With the execution of these production rules, an unbounded sequence of alternating normal and premature ventricular QRS complexes is generated, described above as ventricular bigeminy. Note that in terms of the $\{N, V\}$ symbols the G_{VB} grammar is syntactically equivalent to the regular expression $NV(NV)^+$.

In this example, as in 3.5, an observation procedure Θ_P is not necessary, since the constraints in the grammar completely determine the temporal endpoints of the hypothesis and there are no more attributes to be valued. Fig. 4 shows an example of a ventricular bigeminy pattern.

4. An interpretation framework

In this section, we define and characterize an interpretation problem. Informally, an interpretation problem arises from the availability of a set of initial observations from a given system, and of domain knowledge formalized as a set $\mathcal{G} = \{G_{q_1}, \dots, G_{q_n}\}$ of G^{ap} grammars. Every abstraction grammar $G_h \in \mathcal{G}$ generates a set of abstraction patterns that share the same hypothesis h . The whole set of abstraction patterns that can be generated by \mathcal{G} is denoted as \mathcal{P} .

Definition 7. Let \mathcal{Q} be a set of observables and \mathcal{G} a set of abstraction grammars. We say \mathcal{G} induces an **abstraction relation** in $\mathcal{Q} \times \mathcal{Q}$, denoted by $q_i \prec q_j$ if and only if there exists an abstraction pattern P generated by some $G_h \in \mathcal{G}$ such that:

1. $q_j = h$
2. $M_P^{q_i} \cap A_P \neq \emptyset$
3. $q_i \prec^+ q_j$, where \prec^+ is the transitive closure of \prec

The relation $q_i \prec q_j$ is a sort of *conjectural consequence relation* [16] that allows us to conjecture the presence of q_j from the observation of q_i . The transitive closure of the abstraction relation is a strict partial order relation between the domain observables, such that $q_i \prec q_j \Leftrightarrow q_i \prec^+ q_j$; that is, if and only if $\exists q_{k_0}, \dots, q_{k_n} \in \mathcal{Q}$ such that $q_{k_0} = q_i$, $q_{k_n} = q_j$ and for all m , with $0 \leq m < n$, it holds that $q_{k_m} \prec q_{k_{m+1}}$. We denote by $q_i = q_{k_0} \prec q_{k_1} \prec \dots \prec q_{k_n} = q_j$ an *abstraction sequence* in n steps that allows the conjecture of q_j from q_i . This order relation defines an abstraction hierarchy among the observables in \mathcal{Q} . From the definition of a strict partial order, there must be at the base of this hierarchy at least one observable we call q_0 , corresponding in the domain of electrocardiography to the digital signal.

Example 4.1. Let $\mathcal{Q} = \{q_{PW}, q_{QRS}, q_{TW}, q_N, q_V, q_{VB}\}$ and $\mathcal{G} = \{G_N, G_{VB}\}$, containing the knowledge represented in Examples 3.5 and 3.6. The derived abstraction relation states that $q_{PW}, q_{QRS}, q_{TW} \prec q_N$, and $q_N, q_V \prec q_{VB}$. Intuitively, we can see that this relation splits the observables into three abstraction levels: the wave level, describing the activation/recovery of the different heart chambers; the heartbeat level, describing each cardiac cycle by its origin in the muscle tissue; and the rhythm level, describing the dynamic behavior of the heart over multiple cardiac cycles. These levels match those commonly used by experts in electrocardiogram analysis [46].

It is worth noting that the abstraction relation is only established between observables in the A_P set. This provides flexibility in defining the evidence forming the context of a pattern, as this may belong to different abstraction levels.

Definition 8. We define an **abstraction model** as a tuple $\mathcal{M} = \langle \mathcal{Q}, \prec, \mathcal{G} \rangle$, where \mathcal{Q} is the set of domain observables, \prec is an abstraction relation between such observables, and \mathcal{G} is the available knowledge as a set of abstraction grammars.

The successive application of the available abstraction grammars results in a series of observations organized in a hierarchy of abstraction, according to the order relation between observables as described above. We are able to define an interpretation problem as follows.

Definition 9. We define an **interpretation problem** as a pair $IP = \langle \mathcal{O}, \mathcal{M} \rangle$, where $\mathcal{O} = (o_1, o_2, \dots, o_i, \dots)$ is a sequence of observations requiring interpretation and \mathcal{M} is an abstraction model of the domain.

It is worth mentioning that this definition of an abductive interpretation problem differs from the common definition of an abductive diagnosis problem, where the difference between normal and faulty behaviors is explicit, leading to the role of faulty manifestations that guide the abductive process of diagnosis. In contrast, in the present framework all the observations have the same status, and the objective of the interpretation process is to provide an interpretation of what is observed at the highest possible abstraction level in terms of the underlying processes. As we will see later, some observables may stand out amongst others regarding the efficiency of the interpretation process, as salient features that can draw some sort of perceptual attention.

As discussed above, any observable $q \in Q_P$ can appear multiple times as different pieces of evidence for an abstraction pattern P , in the form of findings collected in the set M_P . As a consequence, P can predict multiple observations of the set \mathcal{O} for a given observable $q \in Q_P$, each of these corresponding to one of the findings of the set M_P through a matching relation. This matching relation is a matter of choice for the agent in charge of the interpretation task, by selecting from the evidence the observation corresponding to each finding in a given pattern.

Definition 10. Given an interpretation problem IP , a **matching relation** for a pattern $P \in \mathcal{P}$ is an injective relation in $M_P \times \mathcal{O}$, defined by $m^q \leftrightarrow o$ if and only if $o = \langle q, \mathbf{v}, t^b, t^e \rangle \in \mathcal{O}(q) \subseteq \mathcal{O}$ and $m^q = \langle \psi, \mathbf{A}, T^b, T^e \rangle \in M_P$, such that $(A_1 = v_1, \dots, A_{n_q} = v_{n_q}), T^b = t^b$ and $T^e = t^e$.

A matching relation makes an assignment of a set of observations to a set of findings of a certain pattern, leading us to understand the interpretation problem as a search within the available evidence for a valid assignment for the constraints represented in an abstraction pattern.

From the notion of matching relation we can design a mechanism for abductively interpreting a subset of observations in \mathcal{O} through the use of abstraction patterns. Thus, a matching relation for a given pattern allows us to hypothesize new observations from previous ones, and to iteratively incorporate new evidence into the interpretation by means of a hypothesize-and-test cycle. The notion of abstraction hypothesis defines those conditions that a subset of observations must satisfy in order to be abstracted by a new observation, and makes it possible to incrementally build an interpretation from the incorporation of new evidence.

Definition 11. Given an interpretation problem IP , we define an **abstraction hypothesis** as a tuple $h = \langle o_h, P, \leftrightarrow \rangle$, where $P = \langle h, M_P, C_P, \Theta_P \rangle \in \mathcal{P}$, $\leftrightarrow \subseteq M_P \times \mathcal{O}$, and we denote $O_h = \text{codomain}(\leftrightarrow)$, satisfying:

1. $o_h \in \mathcal{O}(h)$.
2. $o_h = \Theta_P(O_h)$.
3. $C_P(\mathbf{A}_h, T_h^b, T_h^e, \mathbf{A}_1, T_1^b, T_1^e, \dots, \mathbf{A}_n, T_n^b, T_n^e) |_{o_h, o_1, \dots, o_n \in O_h}$ is satisfied.

These conditions entail: (1) an abstraction hypothesis guesses an observation of the observable hypothesized by the pattern; (2) a new observation is obtained from the application of the observation procedure to those observations being assigned to the set of findings M_P by the matching relation; and (3) the observations taking part in an abstraction hypothesis must satisfy those constraints of the pattern whose variables are assigned a value by the observations.

Even though the matching relation is a matter of choice, and therefore a conjecture in itself, some additional constraints may be considered as default assumptions. An important default assumption in the abstraction of a periodic process states that consecutive observations are related by taking part in the same hypothesis, defining the basic period of the process. This assumption functions as a sort of operative hypothesis of the abstraction task:

Default Assumption 2. (Basic periodicity) *Periodic findings in an abstraction pattern must be assigned consecutive observations by any matching relation:*

$$\forall m_i^q, m_{i+1}^q \in M_P^q, m_i^q \leftrightarrow o_j \wedge \text{q-succ}(o_j) \in O_h \Rightarrow m_{i+1}^q \leftrightarrow \text{q-succ}(o_j)$$

This default assumption allows us to avoid certain combinations of abstraction hypotheses that, although formally correct, are meaningless from an interpretation point of view. For example, without the assumption of basic periodicity, a normal rhythm fragment might be abstracted by two alternating bradycardia hypotheses, as shown in Fig. 5.

The set of observations that may be abstracted in an interpretation problem IP is $\mathcal{O}(\text{domain}(\leftarrow))$, that is, observations corresponding to observables involved in the set of findings to be abstracted by some abstraction pattern. An abstraction

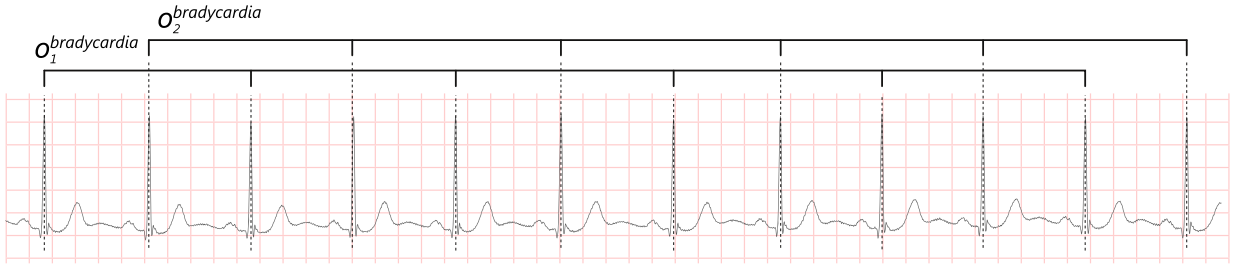


Fig. 5. Motivation for the assumption of basic periodicity. [Source: MIT-BIH arrhythmia DB, recording: 103, between 00:40.700 and 00:51.200].

hypothesis defines in the set of observations \mathcal{O} a counterpart of the subsets A_P and E_P of the set of findings M_P of a pattern P , resulting from the selection of a set of observations $O_h \subseteq \mathcal{O}$ by means of a matching relation, satisfying those requirements shown in the Definition 11.

Definition 12. Given an interpretation problem IP and an abstraction hypothesis $\hat{h} = \langle o_h, P, \leftarrow \rangle$, we define the following sets of observations:

- $\text{abstracted_by}(o_h) = \{o \in O_h \mid m_i^q \leftarrow o \wedge m_i^q \in A_P\}.$
- $\text{environment_of}(o_h) = \{o \in O_h \mid m_i^q \leftarrow o \wedge m_i^q \in E_P\}.$
- $\text{evidence_of}(o_h) = \text{abstracted_by}(o_h) \cup \text{environment_of}(o_h).$

We denote by $\text{abstracted_by}(o_h)$ the set of observations abstracted by o_h and which are somehow its constituents, while $\text{environment_of}(o_h)$ denotes the evidential context of o_h . We denote by $\text{evidence_of}(o_h)$ the set of all observations supporting a specific hypothesis. Since the matching relation is injective, it follows that $\text{abstracted_by}(o_h) \cap \text{environment_of}(o_h) = \emptyset$.

The definition of these sets can be generalized to include as arguments a set of observations $O = \{o_{h_1}, \dots, o_{h_m}\}$ from a set of abstraction hypotheses $\hat{h}_1, \dots, \hat{h}_m$:

- $\text{abstracted_by}(O) = \bigcup_{o_h \in O} \text{abstracted_by}(o_h)$
- $\text{environment_of}(O) = \bigcup_{o_h \in O} \text{environment_of}(o_h).$
- $\text{evidence_of}(O) = \bigcup_{o_h \in O} \text{evidence_of}(o_h).$

As a result of an abstraction hypothesis, a new observation o_h is generated which can be included in the set of domain observations, so that $\mathcal{O} = \mathcal{O} \cup \{o_h\}$. In this way, an interpretation can be incrementally built from the observations, by means of the aggregation of abstraction hypotheses.

Definition 13. Given an interpretation problem IP , an **interpretation** is defined as a set of abstraction hypotheses $I = \{\hat{h}_1, \dots, \hat{h}_m\}$.

An interpretation can be rewritten as $I = \langle O_I, P_I, \leftarrow_I \rangle$, where $O_I = \{o_{h_1}, \dots, o_{h_m}\}$ is the set of observations guessed by performing multiple abstraction hypotheses; $P_I = \{P_1, \dots, P_m\}$ is the set of abstraction patterns used in the interpretation; and $\leftarrow_I = \leftarrow_{h_1} \cup \dots \cup \leftarrow_{h_m} \subseteq (M_1 \cup \dots \cup M_m) \times \mathcal{O}$ is the global matching relation. It should be noted that the global matching relation \leftarrow_I is not necessarily injective, since some observations may simultaneously belong to both the $\text{abstracted_by}()$ and $\text{environment_of}()$ sets of different observations.

From a given interpretation problem IP , multiple interpretations can be abductively proposed through different sets of abstraction hypotheses. Indeed, the definition of interpretation is actually weak, since even an empty set $I = \emptyset$ is formally a valid interpretation. Thus, we need additional criteria in order to select the solution to the interpretation problem as the best choice among different possibilities [33].

Definition 14. Given an interpretation problem IP , an interpretation I is a **cover** of IP if the set of observations to be interpreted $O(\text{domain}(\leftarrow)) \subseteq \mathcal{O}$ is included in the set of observations abstracted by I , that is, $O(\text{domain}(\leftarrow)) \subseteq \text{abstracted_by}(O_I).$

Definition 15. Given an interpretation problem IP , two different abstraction hypotheses \hat{h} and \hat{h}' of the mutually exclusive observables q_h and $q_{h'}$ are **alternative hypotheses** if and only if $\text{abstracted_by}(o_h) \cap \text{abstracted_by}(o_{h'}) \neq \emptyset$.

Example 4.2. A *ventricular trigeminy* is an infrequent arrhythmia very similar to *ventricular bigeminy*, except that the ectopic heart beats occur after every pair of normal beats instead of after each one. The grammar for hypothesizing a ventricular

trigeminy q_{VT} would therefore be very similar to that described in Example 3.6, with the difference that each q_V finding would appear after every pair of q_N findings. These two processes are mutually exclusive, insofar as the heart can develop just one of these activation patterns at a given time. For this reason, in the event of an observation of q_V , this may be abstracted by either a q_{VB} or a q_{VT} hypothesis, but never by both simultaneously.

Definition 16. Given an interpretation problem IP , a cover I for IP is **exclusive** if and only if it contains no alternative hypotheses.

Thus, two or more different hypotheses of mutually exclusive observables abstracted from the same observation will be incompatible in the same interpretation, since inferring both a statement and its negation is logically prevented, and therefore only one of them can be selected.

On the other hand, a parsimony criterion is required, in order to disambiguate the possible interpretations to select as the most plausible those of which the complexity is minimum [33]. We translate this minimum complexity in terms of minimal cardinality.

Definition 17. Given an interpretation problem IP , a cover I for IP is **minimal**, if and only if its cardinality is the smallest among all covers for IP .

Minimality introduces a parsimony criterion on hypothesis generation, promoting temporally maximal hypotheses, that is, those hypotheses of a larger scope rather than multiple equivalent hypotheses of smaller scope. For example, consider an abstraction pattern that allows the conjecture of a regular cardiac rhythm from the presence of three or more consecutive heart beats. Without a parsimony criterion, a sequence of nine consecutive beats could be abstracted by up to three consecutive rhythm observations, even when a single rhythm observation would be sufficient and better.

Definition 18. The **solution** of an interpretation problem IP is the set of all minimal and exclusive covers of IP .

This definition of solution is very conservative and has limited practical value, since the usual objective is to obtain a small set of interpretations explaining what has been observed (and ideally only a single one). However, it allows us to characterize the problem in terms of complexity. Abduction has been formulated under different frameworks according to the task to be addressed, but has always been found an intractable problem in the general case [24]. The next theorem proves that an interpretation problem is also an intractable problem.

Theorem 1. Finding the solution to an interpretation problem is NP-hard.

Proof. We will provide a polynomial-time reduction of the well-known set covering problem to an interpretation problem. Given a set of elements $U = \{u_1, \dots, u_m\}$ and a set S of subsets of U , a cover is a set $C \subseteq S$ of subsets of S whose union is U . In terms of complexity analysis, two different problems of interest are identified:

- A set covering decision problem, stating that given a pair (U, S) and an integer k the question is whether there is a set covering of size k or less. This decision version of set covering is NP-complete.
- A set covering optimization problem, stating that given a pair (U, S) the task is to find a set covering that uses the fewest sets. This optimization version of set covering is NP-hard.

We will therefore reduce the set covering problem to an interpretation problem by means of a polynomial-time function φ . Thus, we shall prove that $\varphi(U, S)$ is an interpretation problem, and there is a set covering of $\varphi(U, S)$ of size k or less if and only if there is a set covering of U in S of size k or less.

Given a pair (U, S) , let $\varphi(U, S) = \langle \mathcal{O}, \mathcal{M} \rangle$ where:

1. $\mathcal{O} = U = \{u_1, \dots, u_m\}$, such that $u_i = \langle q, \text{true}, i \rangle$ and $q = \langle \psi, \text{present}, T \rangle$.
2. $\mathcal{M} = \langle Q, \prec, \mathcal{P} \rangle$, such that $\text{domain}(\prec) = q$.
3. $\forall s = \{u_{i_1}, \dots, u_{i_n}\} \in S$, $\exists P \in \mathcal{P}$, being $P = \langle q_P, M_P, C_P, \Theta_P \rangle$, where:
 - $q \prec q_P$ and $P \neq P' \Rightarrow q_P \neq q_{P'}$.
 - $M_P = A_P = M_P^q = \{m_1^q = \langle \psi, \text{present}_1, T_1 \rangle, \dots, m_n^q\}$.
 - $C_P = \{\bigwedge_{k=1}^n T_k = k; T_h^b = \min\{T_k\}; T_h^e = \max\{T_k\}\}$.
 - $\text{present}_P = \Theta_P(m_1^q, \dots, m_n^q) = \bigwedge_{k=1}^n \text{present}_k$.

Thus, $\varphi(U, S)$ is an interpretation problem according to this definition. On the other hand, $\varphi(U, S)$ can be built in polynomial time. In addition, for all $s \in S$ there exists an abstraction hypothesis $\hat{h} = \langle o_h, P, \leftarrow \rangle$ such that:

1. $o_h = \langle h, \text{true}, \min_{u_i \in s} \{i\}, \max_{u_i \in s} \{i\} \rangle$.

2. $u_i \in s \Rightarrow u_i \in \text{codomain}(\leftarrow)$.
3. \leftarrow provides a valid assignment, since the set of observations satisfying $\Theta_P = \text{true}$ also satisfies the constraints in C_P .

Since each abstraction hypothesis involves a different abstraction pattern there are no alternative hypotheses in any interpretation of $\varphi(U, S)$.

Suppose there is a set covering $C \subseteq S$ of U of size k or less. For all $u \in U$ there exists $c_i \in C - \{\emptyset\}$ such that $u \in c_i$ and, by the above construction, there exists $h_i \in I$ such that $\text{abstracted_by}(o_{h_i}) = \{u \in \text{codomain}(\leftarrow_{h_i})\} = \{u \in c_i\} = c_i$, and therefore, $O(\text{domain}(\leftarrow)) \subseteq \bigcup_{h_i \in I} \text{abstracted_by}(o_{h_i}) = \bigcup_i c_i = C$. That is, the set of abstraction hypotheses I is an exclusive cover of the interpretation problem $\varphi(U, S)$ of size k or less.

Following the same reasoning as for the set covering optimization problem, finding a minimal and a exclusive cover of an interpretation problem $\varphi(U, S)$ is NP-hard, since we can use the solution of this problem to check whether there is an exclusive cover of the interpretation problem of size k or less, and this has been proven above to be NP-complete. \square

5. Solving an interpretation problem: A heuristic search approach

The solution set for an interpretation problem IP consists of all exclusive covers of IP having the minimum possible number of abstraction hypotheses. Obtaining this solution set can be stated as a search on the set of interpretations of IP . The major source of complexity of searching for a solution is the local selection, from the available evidence in \mathcal{O} , of the most appropriate matching relation for a number of abstraction hypotheses that can globally shape a minimal and exclusive cover of IP .

Nevertheless, the whole concept of solution must be revised in practical terms, due to the intractability of the task and the incompleteness of the abstraction model, that is, of the available knowledge. Indeed, we assume that any realistic abstraction model can hardly provide a cover for every possible interpretation problem. Hence the objective should shift from searching for a solution to searching for an approximate solution.

Certain principles applicable to the interpretation problem can be exploited in order to approach a solution in an iterative way, bounding the combinatorial complexity of the search. These principles can be stated as a set of heuristics that make it possible to evaluate and discriminate some interpretations against others from the same base evidence:

- A *coverage principle*, which states the preference for interpretations explaining more initial observations.
- A *simplicity principle*, which states the preference for interpretations with fewer abstraction hypotheses.
- An *abstraction principle*, which states the preference for interpretations involving higher abstraction levels.
- A *predictability principle*, which states the preference for interpretations that properly predict future evidence.

The coverage and simplicity principles are used to define a cost measure for the heuristic search process [14], while the abstraction and predictability principles are used to guide the reasoning process, in an attempt to emulate the same shortcuts used by humans.

Given an interpretation problem IP , a heuristic vector for a certain interpretation I can be defined to guide the search, as $\epsilon(I) = (1 - \zeta(I), \kappa(I))$, where $\zeta(I) = |\text{abstracted_by}(O_I)| / |O(\text{domain}(\leftarrow))|$ is the *covering ratio* of I , and $\kappa(I) = |O_I|$ is the *complexity* of I . The main goal of the search strategy is to approach a solution with a maximum covering ratio and a minimum complexity, which is equivalent to the minimization of the heuristic vector. The covering ratio will be considered the primary heuristic, and complexity will be considered for ranking interpretations with the same covering ratio. The $\epsilon(I)$ heuristic is intuitive and very easy to calculate, but as a counterpart it is a non-admissible heuristic, since it is not monotone and may underestimate or overestimate the true goal covering. Therefore optimality cannot be guaranteed and we require an algorithm efficient with this type of heuristic. We propose the CONSTRUE() algorithm, whose pseudocode is shown in Algorithm 1. This algorithm is a minor variation of the K-Best First Search algorithm [14], with partial expansion to reduce the number of explored nodes.

The CONSTRUE() algorithm takes as its input an interpretation problem IP , and returns the first interpretation found with full coverage, or the interpretation with the maximum covering ratio and minimum complexity if no covers are found, using the abstraction and predictability principles in the searching process. To do this, it manages two ordered lists of interpretations, named *open* and *closed*. Each interpretation is annotated with the computed values of the heuristic vector. The *open* list contains those partial interpretations that can further evolve by (1) appending new hypotheses or (2) extending previously conjectured hypotheses to subsume or predict new evidence. This *open* list is initialized with the trivial interpretation $I_0 = \emptyset$. The *closed* list contains those interpretations that cannot explain more evidence.

At each iteration, the algorithm selects the K most promising interpretations according to the heuristic vector (line 8), and partially expands each one of them to obtain the next descendant node I' . If this node is a solution, then the process ends by returning it (line 13), otherwise it is added to the *open* list. The partial expansion ensures that the *open* list grows at each iteration by at most K new nodes, in order to save memory. When a node cannot expand further, it is added to the *closed* list (line 12), from which the solution is taken if no full coverages are found (line 20).

The selection of a value for the K parameter depends on the problem at hand. We select its value as $K = \max(|\{q_j \in \mathcal{Q} \mid q_i \prec q_j, q_i \in \mathcal{Q}\}|)$, that is, as the maximum number of observables that can be abstracted from any observable q_i . The

Algorithm 1 CONSTRUE search algorithm.

```

1: function CONSTRUE( $IP$ )
2:   var  $I_0 = \emptyset$ 
3:   var  $K = \max(|\{q_j \in \mathcal{Q} \mid q_i \vdash q_j, q_i \in \mathcal{Q}\}|)$ 
4:   SET_FOCUS( $I_0, o_1$ )
5:   var  $open = \text{SORTED}(\{(\epsilon(I_0), I_0)\})$ 
6:   var  $closed = \text{SORTED}(\{\})$ 
7:   while  $open \neq \emptyset$  do
8:     for all  $I \in open[0 \dots K]$  do
9:        $I' = \text{NEXT}(\text{GET\_DESCENDANTS}(I))$ 
10:      if  $I'$  is null then
11:         $open = open - \{(\epsilon(I), I)\}$ 
12:         $closed = closed \cup \{(\epsilon(I), I)\}$ 
13:      else if  $\zeta(I') = 1.0$  then
14:        return  $I'$ 
15:      else
16:         $open = open \cup \{(\epsilon(I'), I')\}$ 
17:      end if
18:    end for
19:  end while
20:  return  $\min(closed)$ 
21: end function

```

intuition behind this choice is that at any point in the interpretation process, and with the same heuristic values, the same chance is given to any plausible abstraction hypothesis in order to explain a certain observation.

In order to expand the current set of interpretations, the GET_DESCEND-ANTS() function relies on different reasoning modes, that is, different forms of abduction and deduction, which are brought into play under the guidance of an attentional mechanism. Since searching for a solution finally involves the election of a matching relation, both observations and findings should be included in the scope of this mechanism. Hence, a focus of attention can be defined to answer the following question: which is the next observation or finding to be processed? The answer to this question takes the form of a hypothesize-and-test cycle: if the attention focuses on an observation, then an abstraction hypothesis explaining this observation should be generated (hypothesize); however, if the attention focuses on a finding predicted by some hypothesis, an observation should be sought to match such finding (test). Thus, the interpretation problem is solved by a reasoning strategy that progresses incrementally over time, coping with new evidence through the dynamic generation of abstraction patterns from a finite number of abstraction grammars, and bounding the theoretical complexity by a parsimony criterion.

To illustrate and motivate the reasoning modes implemented in building interpretations and supporting the execution of the CONSTRUE() algorithm, we use a simple, but complete, interpretation problem.

Example 5.1. Let $\mathcal{Q} = \{q_{wave}, q_{PW}, q_{QRS}, q_{TW}, q_N\}$, $\mathcal{G} = \{G_w, G_N, G_{TW}\}$, where G_w models the Example 3.4, G_N is described in Example 3.5, and $G_{TW} = (\{H, D\}, \{q_{QRS}, q_{wave}\}, H, R)$ describes the knowledge to conjecture a T wave with the following rules:

$$\begin{aligned}
 H \rightarrow q_{QRS} D \quad & \{P_H := \langle q_{TW}, M_H = \emptyset, C_H = \emptyset, \Theta_H = \emptyset \rangle, \\
 & C_{QRS} := \{80 \text{ ms} \leq T_{TW}^b - T_{QRS}^e \leq 120 \text{ ms}; T_{TW}^e - T_{QRS}^b \leq 520 \text{ ms}\}, \\
 & A_{QRS} := \text{false}, \\
 & P_D := \langle q_{TW}, M_D = \{m^{QRS}\}, C_D = C_{QRS}, \Theta_D = \emptyset \rangle \\
 & \} \\
 D \rightarrow q_{wave} \quad & \{P_D := \langle q_{TW}, M_D = \{m^{QRS}\}, C_D = C_{QRS}, \Theta_D = \emptyset \rangle, \\
 & C_{wave} := \{T_{TW}^b = T_{wave}^b; T_{TW}^e = T_{wave}^e; \max(\text{diff}(\text{sig}[m^{wave}]) \leq 0.7 \cdot \max(\text{diff}(\text{sig}[m^{QRS}]))\}, \\
 & A_{wave} := \text{true}, \\
 & P := \langle q_{TW}, M_P = M_D \cup \{m^{wave}\}, C_P = C_D \cup C_{wave}, \Theta_P = \text{Tw_delin}(T_{QRS}^b, T_{QRS}^e, T_{wave}^b, T_{wave}^e) \rangle \\
 & \}
 \end{aligned}$$

This grammar hypothesizes the observation of a T wave from a wave appearing shortly after the observation of a QRS complex, requiring a significant decrease in the maximum slope of the signal (in the constraint definition C_{wave} , the expression “ $\max(\text{diff}(\text{sig}[m]))$ ” stands for the maximum absolute value of the derivative of the ECG signal between T_m^b and T_m^e). The observation procedure of the generated pattern is denoted as Tw_delin(), and may be any of the methods described in the literature for the delineation of T waves, such as in [26].

In addition to the P_{wave} pattern generated by G_w and detailed in Example 3.4, G_N and G_{TW} generate the following abstraction patterns:

$$P_N = \langle q_N, A_{P_N} = \{m^{Pw}, m^{QRS}, m^{Tw}\} \cup E_{P_N} = \emptyset, C_{P_N}, \Theta_{P_N} = \emptyset \rangle$$

$$P_{Tw} = \langle q_{Tw}, A_{P_{Tw}} = \{m^{wave}\} \cup E_{P_{Tw}} = \{m^{QRS}\}, C_{QRS} \cup C_{wave}, Tw_delin() \rangle$$

Finally, let $\mathcal{O} = \{o_1^{wave} = \langle q_{wave}, \emptyset, 0.300, 0.403 \rangle, o_2^{wave} = \langle q_{wave}, \emptyset, 0.463, 0.549 \rangle, o^{Pw} = \langle q_{Pw}, \emptyset, 0.300, 0.403 \rangle, o^{QRS} = \langle q_{QRS}, \emptyset, 0.463, 0.549 \rangle\}$ be a set of initial observations including a P wave and a QRS complex abstracting two wave observations located at specific time points.

Given this interpretation problem, Fig. 6 shows the starting point for the interpretation, where the root of the interpretation process is the trivial interpretation I_0 , and the attention is focused on the first observation. The sequence of reasoning steps towards the resolution of this interpretation problem will be explained in the following subsections.

5.1. Focus of attention

The focus of attention is modeled as a stack; thus, once the focus is set on a particular observation (or finding), any observation that was previously under focus will not return to be focused on until the reasoning process on the current observation is finished. Algorithm 2 shows how the different reasoning modes are invoked based on the content of the focus of attention, resulting in a hypothesize-and-test cycle.

Algorithm 2 Method for obtaining the descendants of an interpretation using different reasoning modes based on the content of the focus of attention.

```

1: function GET_DESCENDANTS(I)
2:   var focus = GET_FOCUS(I).TOP()
3:   var desc = ∅
4:   if IS_OBSERVATION(focus) then
5:     if focus = oh | h ∈ I then
6:       desc = DEDUCE(I, focus)
7:     end if
8:     desc = desc ∪ ABDUCE(I, focus) ∪ ADVANCE(I, focus)
9:   else if IS_FINDING(focus) then
10:    desc = SUBSUME(I, focus) ∪ PREDICT(I, focus)
11:   end if
12:   return desc
13: end function

```

Lines 4–8 generate the descendants of an interpretation I when there is an observation at the top of the stack. These descendants are the result of two possible reasoning modes: the deduction of new findings, performed by the DEDUCE() function, provided that the observation being focused on is an abstraction hypothesis; and the abduction of a new hypothesis explaining the observation being focused on, performed by the ABDUCE() function. A last descendant is obtained using the ADVANCE() function, which simply restores the previous focus of attention by means of a POP() operation. If the focus is then empty, ADVANCE() inserts the next observation to explain, which may be selected by temporal order in the general case, or by some domain-dependent saliency criterion to prioritize certain observations over others. By removing the observation at the top of the focus of attention, the ADVANCE() function sets aside that observation as unintelligible in the current interpretation, according to the available knowledge.

If the top of the stack contains a finding, then Algorithm 2 obtains the descendants of the interpretation from the SUBSUME() and PREDICT() functions (line 10). The first of these functions looks for an existing observation satisfying the constraints on the finding focused on, while the second makes predictions about observables that have not yet been observed. All of these reasoning modes are described separately and detailed below; we will illustrate how the CONSTRUCT() algorithm combines these in order to solve the interpretation problem in Example 5.1.

5.2. Building an interpretation: Abduction

Algorithm 3 enables the abductive generation of new abstraction hypotheses. It is applied when the attention is focused on an observation that can be abstracted by some abstraction pattern, producing a new observation at a higher level of abstraction.

The result of ABDUCE() is a set of interpretations I' , each one adding a new abstraction hypothesis with respect to the parent interpretation I . To generate these hypotheses, we iterate through those grammars that can make a conjecture from the observation o_i under focus (line 3). Then, for each grammar, each production including the corresponding observable $q(o_i)$ (line 4) initializes an abstraction pattern with a single finding of this observable (line 5), and a new hypothesis is conjectured with a matching relation involving both the observation under focus and the finding (line 6). A list structure L_h and two additional variables B_h and E_h are initialized to trace the sequence of productions used to generate the findings in the abstraction pattern; these will play an important role in subsequent reasoning steps (line 7). Finally the new hypothesis opens a new interpretation (lines 8–9) focused on this hypothesis (line 11).

In this way, the ABDUCE() function implements, from a single piece of evidence, the hypothesize step of the hypothesize-and-test cycle. Below we explain the reasoning modes involved in the test step of the cycle.

Algorithm 3 Moving forward an interpretation through abduction.

```

1: function ABDUCE( $I, o_i$ )
2:   var desc =  $\emptyset$ 
3:   for all  $G_h = \langle V_N, V_T, H, R \rangle \in \mathcal{G} \mid q(o_i) \vdash h$  do
4:     for all  $(U \rightarrow qV) \in R \mid q(o_i) \text{ i.s.a } q \wedge A_q = \text{true}$  do
5:        $P_V = \langle h, M_V = \{m^q\}, C_V, \Theta_V \rangle$ 
6:        $\bar{h} = \langle o_h, P_V, \leftarrow_h = \{m^q \leftarrow o_i\} \rangle$ 
7:        $L_h = [(U \rightarrow qV)]; B_h = U; E_h = V$ 
8:        $I' = \langle O_I \cup \{o_h\}, P_I \cup \{P_V\}, \leftarrow_I \cup \leftarrow_h \rangle$ 
9:        $\mathcal{O} = \mathcal{O} \cup \{o_h\}$ 
10:      GET_FOCUS( $I'$ ).POP()
11:      GET_FOCUS( $I'$ ).PUSH( $o_h$ )
12:      desc = desc  $\cup \{I'\}$ 
13:     end for
14:   end for
15:   return desc
16: end function

```

Algorithm 4 Moving forward an interpretation through the deduction of new findings.

```

1: function DEDUCE( $I, o_h$ )
2:   var desc =  $\emptyset$ 
3:   if  $B_h \neq H$  then
4:     for all  $(X \rightarrow qB_h) \in R$  do
5:        $P_{B_h} = \langle h, M_{B_h} = \{m^q\}, C_{B_h}, \Theta_{B_h} \rangle$ 
6:       for all  $(U \rightarrow q'V) \in L_h$  do
7:          $P_V = \langle h, M_U \cup \{m^q\}, C_U \cup C_V, \Theta_V \rangle$ 
8:       end for
9:        $\bar{h} = \langle o_h, P_{E_h}, \leftarrow_h \rangle$ 
10:       $I' = \langle O_I, P_I \cup \{P_{E_h}\}, \leftarrow_I \rangle$ 
11:      INSERT( $L_h, (X \rightarrow qB_h), \text{begin}$ );  $B_h = X$ 
12:      GET_FOCUS( $I'$ ).PUSH( $m^q$ )
13:      desc = desc  $\cup \{I'\}$ 
14:     end for
15:   else
16:     for all  $(E_h \rightarrow qX) \in R$  do
17:        $P_X = \langle h, M_{E_h} \cup \{m^q\}, C_{E_h} \cup C_X, \Theta_X \rangle$ 
18:        $\bar{h} = \langle o_h, P_X, \leftarrow_h \rangle$ 
19:        $I' = \langle O_I, P_I \setminus \{P_{E_h}\} \cup \{P_X\}, \leftarrow_I \rangle$ 
20:       INSERT( $L_h, (E_h \rightarrow qX), \text{end}$ );  $E_h = X$ 
21:       GET_FOCUS( $I'$ ).PUSH( $m^q$ )
22:       desc = desc  $\cup \{I'\}$ 
23:     end for
24:   end if
25:   return desc
26: end function

```

Example 5.2. Let us consider the interpretation problem set out in Example 5.1 and the interpretation I_0 shown in Fig. 6. According to Algorithm 2, the ABDUCE() function is used to move forward the interpretation, since the focus of attention points to an observation o^{pw} . The abstraction pattern that supports this operation is P_N , and a matching relation is established with the m^{pw} finding. As a result, the following hypothesis is generated:

$$\bar{h}_1 = \langle o^N, P_N, \{m^{pw} \leftarrow o^{pw}\} \rangle$$

Fig. 6 shows the result of this reasoning process, in a new interpretation called I_1 . Note that the focus of attention has been moved to the newly created hypothesis (lines 10–11 of the ABDUCE() function).

5.3. Building an interpretation: Deduction

This reasoning mode is applied when the attention is focused on an observation o_h previously conjectured as part of an abstraction hypothesis \bar{h} (see Algorithm 4). The DEDUCE() function takes the evidence that has led to conjecture o_h and tries to extend it with new findings which can be expected, i.e., deduced, from the abstraction grammar G_h used to guess the observation. The key point is that this deduction process follows an iterative procedure, as the corresponding abstraction pattern is dynamically generated from the grammar. Hence the DEDUCE() function aims to extend a partial matching relation by providing the next finding to be tested, as part of the test step of the hypothesize-and-test cycle.

Since the first finding leading to conjecture o_h does not necessarily appear at the beginning of the grammar description, the corresponding abstraction pattern will not, in general, be generated incrementally from the first production of the grammar. Taking as a starting point the production used to conjecture o_h (line 4 in Algorithm 3), the goal is to add a new finding by applying a new production at both sides, towards the beginning and the end of the grammar, using the

information in the L_h list. The B_h variable represents the non-terminal at the left-hand side of the first production in L_h , while E_h represents the non-terminal at the right-hand side of the last production in L_h . Hence, this list has the form $L_h = [(B_h \rightarrow q'V'), (V' \rightarrow q''V''), \dots, (V'^{n-1} \rightarrow q'^n E_h)]$. In case L_h is empty, both variables B_h and E_h represent the H non-terminal. With this information the sequence of findings supporting the hypothesis h can be updated in two opposite directions:

- Towards the beginning of the grammar (lines 3–14): we explore the set of observables that may occur before the first finding according to the productions of the grammar (line 4), and a new finding is deduced for each of these in different descendant interpretations. A new pattern P_{B_h} associated with the B_h non-terminal is initialized with the new finding (line 5), and by moving along the sequence of productions generating the previous set of findings (lines 6–8) the pattern associated to the rightmost non-terminal P_{E_h} is updated with a new set of findings containing m^q . Consequently, the hypothesis and the interpretation are also updated (lines 9 and 10), and the applied production is inserted at the beginning of L_h (line 11). Finally the newly deduced finding is focused on (line 12).
- Towards the end of the grammar (lines 15–23): for each one of the observables that may occur after the last finding, a new finding m^q is deduced, expanding the abstraction pattern associated with the new rightmost non-terminal X . After updating the hypothesis h , the previous pattern P_{E_h} in the resulting interpretation I' is replaced by the new one, P_X , and the applied production is inserted at the end of L_h . Finally, the new finding is focused on (line 21).

Example 5.3. Let us consider the interpretation problem set out in Example 5.1 and the interpretation I_1 shown in Fig. 6. Remember that the grammar used to generate the hypothesis in the focus of attention, G_N , has the following form:

$$\begin{aligned} H &\rightarrow q_{Pw}D \\ D &\rightarrow q_{QRS}E \\ E &\rightarrow q_{Tw} \end{aligned}$$

In this situation, it is possible to deduce new findings from the o^N hypothesis. Following Algorithm 3 we can check that $B_h = H$ and $E_h = D$, since the only finding in the matching relation is m^{Pw} . Deduction then has to be performed after this last finding, using the production $D \rightarrow q_{QRS}E$. After constraint checking, the resulting finding is as follows:

$$m_{n+1}^q = m^{QRS} = \langle q_{QRS}, \emptyset, T_{QRS}^b \in [0.400, 0.520], T_{QRS}^e \in [0.450, 0.660] \rangle$$

Fig. 6 illustrates the outcome of this reasoning process and the uncertainty in the temporal limits of the predicted finding, which is now focused on in the interpretation I_2 .

5.4. Building an interpretation: Subsumption

Subsumption is performed when the attention is focused on a finding previously deduced from some abstraction grammar (see Algorithm 5). This reasoning mode avoids the generation of a new hypothesis for every piece of available evidence if it can be explained by a previous hypothesis. The SUBSUME() function explores the set of observations \mathcal{O} and selects those consistent with the constraints on the finding in the focus of attention (line 3), expanding the matching relation of the corresponding hypothesis in different descendant interpretations (line 4). The focus of attention is then restored to its previous state (line 5), allowing the deduction of new findings from the same hypothesis. The SUBSUME() function clearly enforces the simplicity principle.

Algorithm 5 Moving forward an interpretation through subsumption.

```

1: function SUBSUME( $I, m_i$ )
2:   var desc =  $\emptyset$ 
3:   for all  $o_j \in \mathcal{O} \mid m_i \leftarrow o_j$  do
4:      $I' = \langle \mathcal{O}_I, P_I, \leftarrow_I \cup \{m_i \leftarrow o_j\} \rangle$ 
5:     GET_FOCUS( $I'$ ).POP( $m_i$ )
6:     desc = desc  $\cup \{I'\}$ 
7:   end for
8:   return desc
9: end function

```

Example 5.4. Let us consider the interpretation I_2 shown in Fig. 6. If we apply the subsumption procedure, it is possible to set a matching relation between o^{QRS} and m^{QRS} , since this observation satisfies all the constraints on the finding. The result is shown in the interpretation I_3 . Note that the uncertainty in the end time of the o^N hypothesis is now reduced after the matching, having $T_N^e \in [0.631, 1.030]$. Following this, the attention focuses once again on this hypothesis, and a new deduction operation may be performed.

5.5. Building an interpretation: Prediction

This reasoning mode is also performed when the attention is focused on a finding deduced from some abstraction grammar (see Algorithm 6). In this case, if a finding previously deduced has not yet been observed, it will be predicted.

Algorithm 6 Moving forward an interpretation through the prediction of non-available evidence.

```

1: function PREDICT( $I, m_i$ )
2:   var desc =  $\emptyset$ 
3:   for all  $G_h = \langle V_N, V_T, H, R \rangle \in \mathcal{G} \mid h \text{ is\_a } q(m_i)$  do
4:      $P_H = \langle h, M_H = \emptyset, C_H = \emptyset, \Theta_H = \emptyset \rangle$ 
5:      $h = \langle o_h, P_H, \leftarrow_h = \emptyset \rangle$ 
6:      $L_h = \emptyset; B_h = E_h = H$ 
7:      $I' = \langle O_I \cup \{o_h\}, P_I \cup \{P_H\}, \leftarrow_I \cup \{m_i \leftarrow o_h\} \rangle$ 
8:      $\mathcal{O} = \mathcal{O} \cup \{o_h\}$ 
9:     GET_FOCUS( $I'$ ).POP( $m_i$ )
10:    GET_FOCUS( $I'$ ).PUSH( $o_h$ )
11:    desc = desc  $\cup \{I'\}$ 
12:   end for
13:   return desc
14: end function

```

The goal of the PREDICT() function is to conjecture a new observation to match the focused finding. For this, the abstraction model is explored and those grammars whose hypothesized observable is more specific than the predicted observable are selected (line 3). Then, a new pattern is initialized with no evidence supporting it, and a new abstraction hypothesis with an empty matching relation is generated (lines 4–5). Finally, the attention focuses on the observation being guessed (lines 9–10) to enable the DEDUCE() function to start a new test step at a lower abstraction level. Since L_h is initialized as an empty list (line 6), B_h and E_h point to the initial symbol of the grammar, and the corresponding abstraction pattern will be generated only towards the end of the grammar.

Example 5.5. Starting from the I_3 interpretation shown in Fig. 6, the next step we can take to move forward the interpretation is a new deduction on the o^N hypothesis, generating a new finding m^{Tw} and leading to the I_4 interpretation. Since there is no available observation of the T wave, a matching with this new finding m^{Tw} cannot be made by the SUBSUME() function, thus, the only option for moving forward this interpretation is through prediction. Following the PREDICT() function, the G_{Tw} grammar can be selected, and a new observation o^{Tw} can be conjectured, generating the I_5 interpretation.

From I_5 we can continue the deduction on the o^{Tw} hypothesis. If we apply the DEDUCE() function we obtain the $m^{QRS'}$ finding from the environment, shown in Fig. 6 as I_6 . To move on, we can apply the SUBSUME() function, establishing the matching relation $\{m^{QRS'} \leftarrow o^{QRS}\}$. This leads to the I_7 interpretation, in which the uncertainty on the o^{Tw} observation is reduced; however, the evidence for the P_{Tw} pattern is not yet complete. A new DEDUCE() step is necessary, which deduces the m^{wave} necessary finding in the I_8 interpretation. This finding is also absent, so another PREDICT() step is required. In this last step, the P_{wave} pattern can be applied to observe the deviation in the raw ECG signal, generating the o_3^{wave} observation and completing the necessary evidence for the o^{Tw} observation and thus also for o^N . Constraint solving assigns the value of t_{Tw}^b , t_{Tw}^e and t_N^e , so the result is a cover of the initial interpretation problem in which all the hypotheses have a necessary and sufficient set of evidence. This solution is depicted in I_9 .

It is worth noting that in this example the global matching relation \leftarrow_I is not injective, since $m^{QRS} \leftarrow o^{QRS}$ and $m^{QRS'} \leftarrow o^{QRS}$. Also note that each interpretation only generates one descendant; in a more complex scenario, however, the possibilities are numerous, and the responsibility of finding the proper sequence of reasoning steps lies with the CONSTRUCT() algorithm.

5.6. Improving the efficiency of interpretation through saliency

Starting a hypothesize-and-test cycle for every single sample is not feasible for most of the time series interpretation problems. Still, many problems may benefit from certain saliency features that can guide the attention focus to some limited temporal fragments that can be easily interpretable. Thus, the interpretation of the whole time series can pivot on a reduced number of initial observations, thereby speeding up the interpretation process.

A saliency-based attentional strategy can be devised from the definition of abstraction patterns using a subset of their constraints as a coarse filter to identify a set of plausible observations. For example, in the ECG interpretation problem the most common strategy is to begin the analysis by considering a reduced set of time points showing a significant slope in the signal, consistent with the presence of QRS complexes [47]. This small set of evidence allows us to focus the interpretation on the promising signal segments, in the same way that a cardiologist focuses on the prominent peaks to start the analysis [46]. It should be noted that this strategy is primarily concerned with the behavior of the focus of attention, and that it does not discard the remaining, non-salient observations, as these are included later in the interpretation by means of the subsumption and prediction reasoning modes.

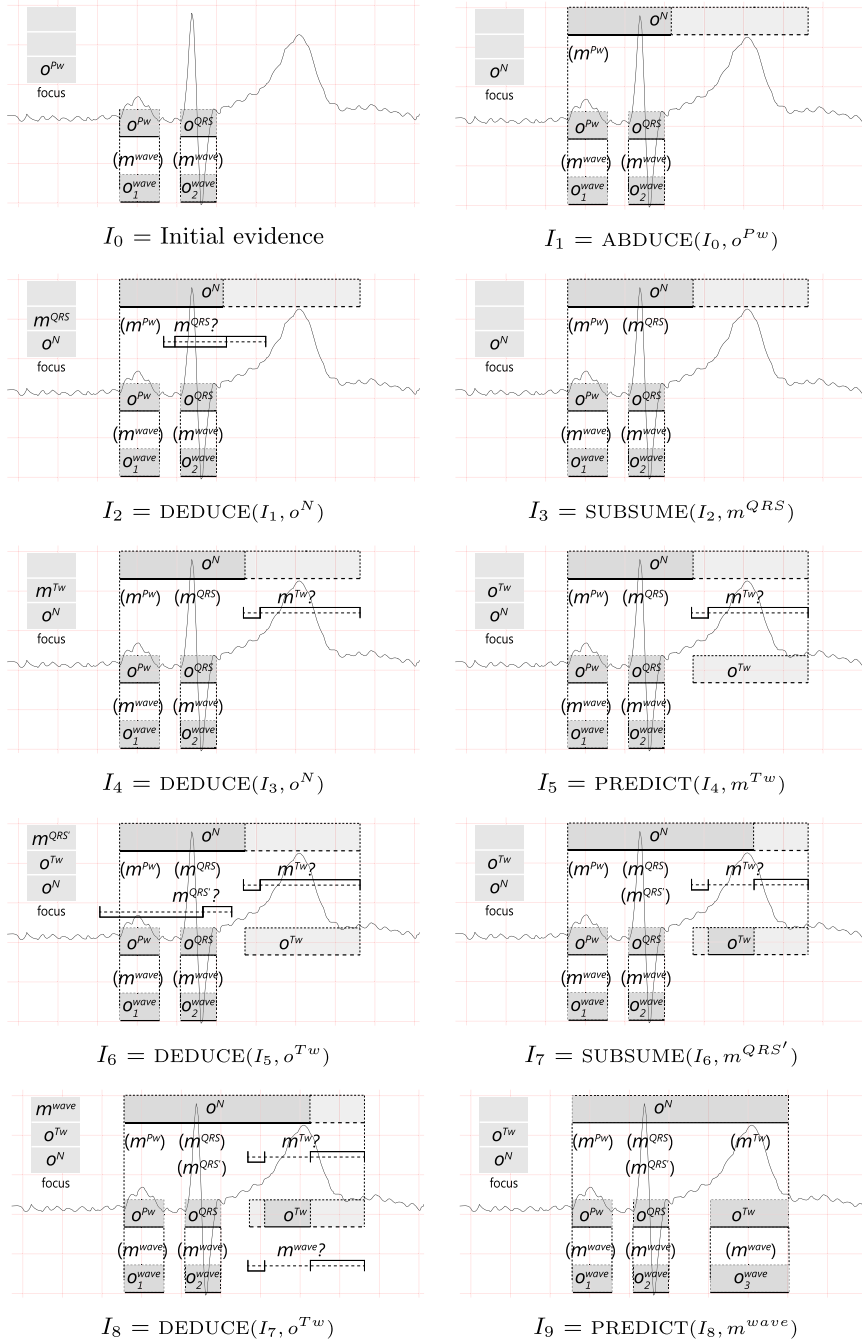


Fig. 6. Sequence of reasoning steps for solving a simple interpretation problem.

6. Advantages of the framework

In this section we provide several practical examples which illustrate some of the strengths of the proposed interpretation framework and its ability to overcome typical weaknesses of the strategies based solely on a classification approach.

6.1. Avoiding a casuistry-based interpretation

In the time domain, classification-based recognition of multiple processes occurring concurrently usually leads to a casuistry-based proliferation of classes, in which a new class is usually needed for each possible superposition of processes in order to properly identify all situations. It is common to use a representation in the transform domain, where

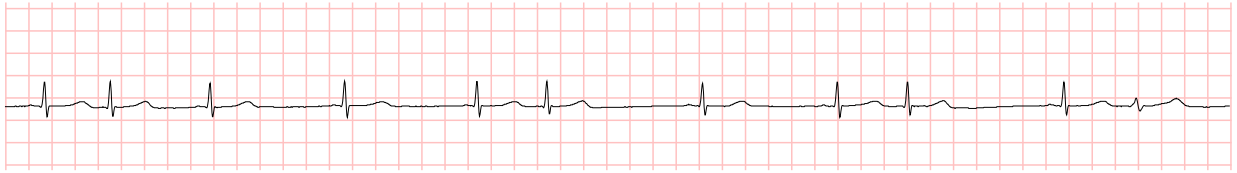


Fig. 7. False atrial fibrillation episode. [Source: Mobiguide Project [38], private recording].

certain regular processes are easily separable, although at the expense of a cumbersome representation of the temporal information [30]. In contrast, in the proposed framework, the hypothesize-and-test cycle aims to conjecture those hypotheses that best explain the available evidence, including simultaneous hypotheses in a natural way as long as these are not mutually exclusive.

ECG interpretation provides some interesting examples of this type of problem. Atrial fibrillation, a common heart arrhythmia caused by the independent and erratic contractions of the atrial muscle fibers, is characterized by an irregularly irregular heart rhythm [46]. Most of the classification techniques for the identification of atrial fibrillation are based on the analysis of the time interval between consecutive beats, and attempt to detect this irregularity [34]. These techniques offer good results in those situations in which atrial fibrillation is the only anomaly, but they fail to properly identify complex scenarios which go beyond the distinction between atrial fibrillation and normal rhythm. In the strip shown in Fig. 7, obtained during a pilot study for the home follow-up of patients with cardiac diseases [38], such a classifier would wrongly identify this segment as an atrial fibrillation episode, since the observed rhythm variability is consistent with the description of this arrhythmia. In contrast, the present interpretation framework correctly explains the first five beats as a sinus bradycardia, compatible with the presence of a premature ectopic beat in the second position, followed by a trigeminy pattern during six beats, and finally another ectopic beat with a morphology change. The reason to choose this interpretation, despite being more complex than the atrial fibrillation explanation, is that it is able to abstract some of the small P waves before the QRS complexes, increasing the interpretation coverage.

6.2. Coping with ignorance

Most of the classifiers solve a separability problem among classes, either by learning from a training set or by eliciting prior knowledge, and these are implicitly based on the closed-world assumption, i.e., every new instance to be classified is assigned to one of the predefined classes. Such classifiers may additionally include a 'reject' option for all those instances that could be misclassified since they appear too close to the classification boundaries [7,17]. This reject option is added as another possible answer expressing doubt. However, such classifiers fail to classify new instances of unknown classes, since they cannot express ignorance. An approach to this problem can be found in novelty detection proposals [35], which can detect when a new instance does not fit any of the predefined classes as it substantially differs from those instances available during training. Still, these are limited to a common feature representation for every instance, hindering the identification of what is unintelligible from the available knowledge.

The proposed framework provides an expression of ignorance as a common result of the interpretation problem. As long as the abstraction model is incomplete, the non-coverage of some piece of evidence by any interpretation is an expression of partial ignorance. In the extreme case, the trivial interpretation I_0 may be a correct solution of an interpretation problem, expressing total ignorance. Furthermore, abduction naturally includes the notion of ignorance in the reasoning process, since any single piece of evidence can be sufficient to guess an interpretation, and the hypothesize-and-test cycle can be understood as a process of incremental addition of evidence against an initial state of ignorance, while being able to provide an interpretation at any time based on the available evidence.

As an example, consider the interpretation problem illustrated in Fig. 8. Let the initial evidence be the set of QRS annotations obtained by a state-of-the-art detection algorithm [47]. In this short strip, the eighth and ninth annotations correspond to false positives caused by noise. A classification-based strategy processes these two annotations as true QRS complexes, and the monotone nature of the reasoning prevents their possible refutation, probably leading to beat misclassification and false arrhythmia detection, with errors propagating onwards to the end of the processing. In contrast, the present framework provides a single normal rhythm as the best interpretation, which explains all but the two aforementioned annotations, which are ignored and considered unintelligible in the available model. It is also worth noting the ability of this framework to integrate the results of an available classifier as a type of constraint specification in the interpretation cycle.

6.3. Looking for missing evidence

The application of the classification paradigm to pattern detection also entails the potential risk of providing false negative results. In the worst case, a false negative result may be interpreted by a decision maker as evidence of absence, leading to interpretation errors with their subsequent costs, or in the best case as an absence of evidence caused by the lack of a proper detection instrument.

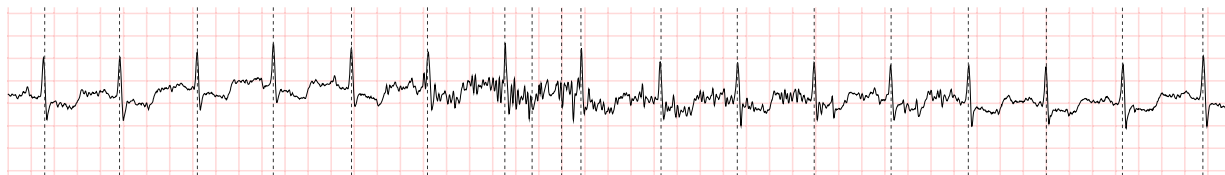


Fig. 8. Unintelligible evidence due to noise. [Source: MIT-BIH arrhythmia DB, recording: 112, between 13:46.200 and 13:56.700].

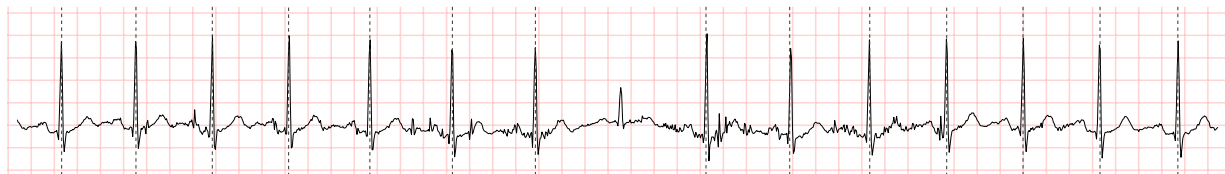


Fig. 9. Missing evidence that may be discovered by prediction. [Source: MIT-BIH normal sinus rhythm DB, recording: 18184, between 09:12:45.000 and 09:12:55.500].

Even though abduction is fallible, and false negative results persist, the hypothesize-and-test cycle involves a prediction mechanism that points to missing evidence that is expected and, moreover, estimates when it should appear. Both the bottom-up and top-down processing performed in this cycle reinforces confidence in the interpretation, since the semantics of any conclusion is widened according to its explanatory power.

As an example, consider the interpretation problem illustrated in Fig. 9. The initial evidence is again a set of QRS annotations obtained by a state-of-the-art detection algorithm [47]. Note that the eighth beat has not been annotated, due to a sudden decrease in the signal amplitude. This error can be amended in the hypothesize-and-test cycle, since the normal rhythm hypothesis that abstracts the first seven QRS annotations predicts the following QRS to be in the position of the missing annotation, and the PREDICT() procedure can look for this (e.g., checking an alternative set of constraints).

The capability of abduction to ignore or look for new evidence has been tested with a simplified version of the present framework in the QRS detection problem [43], leading to a statistically significant improvement over a state-of-the-art algorithm.

6.4. Interpretability of the reasoning process and the results

The interpretability of a reasoning formalism, defined as the ability to understand and evaluate its conclusions, is an essential feature for achieving an adequate confidence in decision making [31]. In this sense, there are a number of classification methods with good interpretability; however, the methods that typically offer the best performance belong to the so-called *black box* approaches.

The present interpretation framework is able to provide a justification of any result in relation to the available model. Given any solution or partial solution of an interpretation problem, the searching path up to I_0 gives full details of all the reasoning steps taken to this end, and any abstraction hypothesis can be traced back to the information supporting it.

This interpretation framework is also able to answer the question of why a certain hypothesis has been rejected or neglected at any reasoning step. This is done by exploring the branches outside the path between I_0 and the solution. Since the K exploration parameter within the CONSTRUE() algorithm has been chosen as the maximum number of hypotheses that may explain a given observable, it is possible to reproduce the reasoning steps taken in the conjecture of any abstraction hypothesis, and to check why this did not succeed (non-satisfaction of pattern constraints, lower coverage, etc.). This can be useful in building and refining the knowledge base.

7. Experimental evaluation: beat labeling and arrhythmia detection

The interpretation of electrocardiograms has served both as a challenge and as an inspiration for the AI community due to a number of factors that can be summarized as: (1) the complexity of the physiological processes underlying what is observed; and (2) the absence of an accurate model of the heart and the hardly formalizable knowledge that constitutes the experience of the cardiologist. There are numerous problems falling within the scope of ECG interpretation, the most relevant being heartbeat labeling [29]. We have tested the present framework by abductively identifying and measuring a set of qualitative morphological and rhythm attributes for each heartbeat, and using a rule-based classifier to assign a label to clusters of similar heartbeats [44]. It is noteworthy that an explicit representation of knowledge has been adopted, namely the kind of knowledge that can be found in an ECG handbook. Table 1 reproduces the performance comparison between this approach and the most relevant automatic and assisted approaches of the state-of-the-art, using sensitivity and positive predictivity of ventricular and supraventricular ectopic beat classes.

As it can be seen, this method significantly outperforms any other automatic approaches in the state-of-the-art, and even improves most of the assisted approaches that require expert aid. The most remarkable improvement concerns the

Table 1

VEB and SVEB classification performance of the abductive approach and comparison with the most relevant automatic and assisted methods of the state-of-the-art.

Dataset	Method	VEB		SVEB	
		Se	P+	Se	P+
MIT-BIH Arrhythmia DS1+DS2	Teixeiro et al. – Automatic	92.82	92.23	85.10	84.51
	Llamedo et al. – Assisted	90±1	97±0	89±2	88±3
	Kiranyaz et al. – Assisted	93.9	90.6	60.3	63.5
	Ince et al. – Assisted	84.6	87.4	63.5	53.7
	Llamedo et al. – Automatic	80±2	82±3	76±2	43±2
MIT-BIH Arrhythmia DS2	Teixeiro et al. – Automatic	94.63	96.79	87.17	83.98
	Llamedo et al. – Assisted	93±1	97±1	92±1	90±3
	Kiranyaz et al. – Assisted	95.0	89.5	64.6	62.1
	Chazal et al. – Assisted	93.4	97.0	94.0	62.5
	Zhang et al. – Automatic	85.48	92.75	79.06	35.98
	Llamedo et al. – Automatic	89±1	87±1	79±2	46±2
	Chazal et al. – Automatic	77.7	81.9	75.9	38.5

classification of supraventricular ectopic beats, which are usually hard to distinguish using only morphological features. The abductive interpretation in multiple abstraction levels, including a rhythm description of signal, is what enables a more precise classification of each individual heartbeat.

Furthermore, the abductive interpretation approach has been used for arrhythmia detection in short single-lead ECG records, focusing on atrial fibrillation [45]. The interpretation results are combined with machine learning techniques to obtain an arrhythmia classifier, achieving the best score in the 2017 Physionet/CinC Challenge dataset and outperforming some of the most popular techniques such as deep learning and random forests [8].

8. Discussion

A new model-based framework for time series interpretation is proposed. This framework relies on some basic assumptions: (i) interpretation of the behavior of a system from the set of available observations is a sort of conjecturing, and as such follows the logic of abduction; (ii) the interpretation task involves both bottom-up and top-down processing of information along a set of abstraction levels; (iii) at the lower levels of abstraction, the interpretation task is a form of precompiled knowledge-based pattern recognition; (iv) the interpretation task involves both the representation of time and reasoning about time and along time.

Model-based representation in the present framework is based on the notion of abstraction pattern, which defines an abstraction relation between observables and provides the knowledge and methods to conjecture new observations from previous ones. Let us deepen in both the backward and forward logical meaning of an abstraction pattern, following a reasoning similar to that of [4]:

- *Backward meaning.* From the backward reading of an abstraction pattern P , a hypothesis h is a possible abstraction of m_1, \dots, m_n , provided that the constraints in C_P hold. An abstraction pattern satisfies the *compositionality principle* of abductive reasoning, and hence an abstraction hypothesis can be conjectured from a single piece of evidence, and new pieces of evidence can be added later [16]. On the other hand, if there are multiple ways of observing h by means of multiple patterns, and their respective constraints are inconsistent with the evidence, we do not conclude $\neg h$, interpreted as failure to prove h ; we will only conclude $\neg h$ in all those interpretations conjecturing an observation of a different h' , where h and h' are mutually exclusive.
- *Forward meaning.* An abductive observation is built upon an archetypical representation of a hypothesis h , creating an observation as an instance of h by estimating, from the available evidence, its attribute values \mathbf{A} and its temporal location T^b and T^e by means of an observation procedure Θ_P . From a forward reading, assuming h is true, there is an observation for each observable of the set m_1, \dots, m_n such that the constraints in C_P hold. However, the estimated nature of abstraction does not usually allow us to infer, from the observation of h , the same observations of m_1, \dots, m_n that have been abstracted into h . We must presume instead that assuming h is true entails the occurrence of an observation for each observable of m_1, \dots, m_n , without necessarily entailing its attribute values and its temporal location.

Both the forward and the backward meanings of an abstraction pattern support the incremental building of an interpretation in the present framework. Thus, what initially was defined as a set covering problem of a time series fragment – a completely intractable problem as it moves away from a toy example – can be feasibly solved if it is properly structured in a set of abstraction levels, on which four reasoning modes (abduction, deduction, subsumption and prediction) can make a more efficient search of the best explanation under a parsimony criterion. Moreover, this incremental reasoning primarily follows the time direction, since the available knowledge is usually compiled in the form of a set of processes that can be expected to be found in a certain sequence, which underscores the anticipatory information contained in the evidence.

An abstraction model, built on a set of abstraction patterns, establishes a causal responsibility for the behavior observed in a complex system [24]. This responsibility is expressed in the language of processes: a process is said to be observable if it is assumed that it causes a recognizable trace in the physical quantity to be interpreted. This notion of causality is behind perception, i.e., concerned with the explanation of sensory data, in contrast with the notion of causality in diagnosis, concerned with the explanation of abnormality [10].

Representing and reasoning about context is a relevant issue in model-based diagnosis [4,10,33,40]. A contextual observation is nothing more than another observation that need not be explained by a diagnosis. In most of the bibliography, the distinction between these two roles must be defined beforehand. Several other works enable the same observation to play different roles in different causal patterns, thus providing some general operations for expressing common changes made by the context in a diagnostic pattern [25,32]. In the present interpretation framework, an observation can either be part of the evidence to be explained in a certain abstraction pattern, or can be part of the environment in another abstraction pattern. Both types of observation play a part in the hypothesize-and-test cycle, with the only difference that observations of the environment of an abstraction pattern are not expected to be abstracted by this pattern. Hence, observations of the environment are naturally included in the deduction, subsumption and prediction modes of reasoning.

An important limitation of the present framework is its knowledge-intensive nature, requiring a non-trivial elicitation of expert knowledge. It is worth exploring different possibilities for the inclusion of machine learning strategies, both for the adaption and the definition of the knowledge base. A first approach may address the automatic adjustment of the initial constraints among recurrent findings in abstraction grammars. In this manner, for example, temporal constraints between consecutive heartbeats in a normal rhythm abstraction grammar could be adapted to the characteristics of the subject whose ECG is being interpreted, allowing the identification of possible deviations from normality with greater sensitivity. On the other hand, the discovery of new abstraction patterns and abstraction grammars by data mining methods appears as a key challenge. In this regard, the CONSTRUCT() algorithm should be extended by designing an INDUCE() procedure aimed at conjecturing new observables after an inductive process. To this end, new default assumptions should be made in order to define those grammar structures that should rule the inductive process. These grammar structures may lead to discovery new morphologies or rhythms not previously included in the knowledge base.

The proposed framework formulates an interpretation problem as an abduction problem with constraints, targeted at finding a set of hypotheses covering all the observations while satisfying a set of constraints on their attribute and temporal values. Thus, consistency is the only criterion to evaluate the plausibility of a hypothesis, resulting in a true or false value, and any evoked hypothesis (no matter how unusual it is) for which inconsistent evidence cannot be found is considered as plausible and, consequently, it will be explored in the interpretation cycle. Even though this simple approach has provided remarkable results, it can be expected that the inclusion of a hypothesis evaluation scheme, typically based on probability [33,37] or possibility [13,32] theories, will allow us to better discriminate between plausible and implausible hypotheses, leading to better explanations with fewer computational requirements.

The expressiveness of the present framework should also be enhanced to support the representation of the *absence* of some piece of evidence, in the form of negation, so that $\neg q$ represents the absence of q . The *exclusion relation* is a first approach to manage with the notion of absence in the hypothesize-and-test cycle, since the occurrence of a process is negated by the concurrent occurrence of any of the processes related to it by the exclusion relation. On the other hand, an *inhibitory relation* can enable us to represent a certain process preventing another from occurring under some temporal constraints, providing a method to insert the prediction of the absence of some observable in the hypothesize-and-test cycle. Furthermore, other forms of interaction between processes, possibly modifying the respective initial patterns of evidence, should be modeled.

Further efforts should be made to improve the efficiency of the interpretation process. To this end, two main strategies are currently being explored. In the first strategy, the model structure is exploited to identify necessary and sufficient conditions for every hypothesis to be conjectured; the necessary conditions avoid the expansion of the hypotheses that can be ruled out because they are inconsistent with observations, while sufficient conditions avoid the construction of redundant interpretations [9]. Another strategy entails additional restrictions in the amount of computer memory and time needed to run the algorithm, resulting in a selective pruning of the node expansion while sacrificing optimality; this strategy is similar to the one used in the K-Beam algorithm [14].

The CONSTRUCT() algorithm is based on the assumption that all the evidence to be explained is available at the beginning of the interpretation task. A new version of the algorithm should be provided to cope with a wide range of problems, where the interpretation must be updated as new evidence becomes available over time. Examples of such problems are continuous biosignal monitoring or plan execution monitoring [3]. At the emergence of a new piece of evidence, two reasoning modes may come into play triggered by the CONSTRUCT() algorithm: a new explanatory hypothesis can be conjectured by means of the ABDUCE() procedure, or the evidence can be incorporated in an existing hypothesis by means of the SUBSUME() procedure. In this way, the incorporation of new evidence over time is seamlessly integrated into the hypothesize-and-test cycle. Furthermore, to properly address these interpretation scenarios, the heuristics used to guide the search must be updated to account for the timing of the interpretation process, which will lead to the definition of a covering ratio until time t , and a complexity until time t .

Implementation

With the aim of supporting reproducible research, the full source code of the algorithms presented in this paper has been published under an Open Source License,¹ along with a knowledge base for the interpretation of the ECG signal strips of all examples in this paper.

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¹ <https://github.com/citiususc/construe>.

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