Artificial Intelligence 175 (2011) 791–813

Contents lists available at ScienceDirect

Artificial Intelligence

www.elsevier.com/locate/artint

On the resolution-based family of abstract argumentation semantics and its grounded instance

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ARTICLE INFO

Article history:
Received 14 January 2010
Received in revised form 11 October 2010
Accepted 16 November 2010
Available online 19 November 2010

Keywords:
Abstract argumentation semantics
Argumentation framework
Grounded semantics
Semantics evaluation criteria
Computational complexity

ABSTRACT

This paper introduces a novel parametric family of semantics for abstract argumentation called *resolution-based* and analyzes in particular the resolution-based version of the traditional *grounded semantics*, showing that it features the unique property of satisfying a set of general desirable properties recently introduced in the literature. Additionally, an investigation of its computational complexity properties reveals that resolution-based grounded semantics is satisfactory also from this perspective.

1. Introduction

In the context of Dung's theory of abstract argumentation frameworks [18] a variety of argumentation semantics have been proposed including the four "traditional" semantics considered in the original Dung's paper (namely stable, complete, grounded and preferred semantics) and several more recent approaches like ideal [19], semi-stable [13], *CF2* [10], and prudent [15] semantics.

As discussed in [6], the motivations supporting the introduction and investigation of a new semantics range from the desire to formalize some high-level intuition, not captured by other proposals, to the need to achieve the "correct" treatment of a particular example (or family of examples), regarded as particularly significant. Heterogeneous and often scarcely formalized motivations are probably the main reason for the lack of systematic principle-based semantics evaluation and comparison until recent years. Two major efforts to fill this gap can be identified in the literature. On one hand, in [6] a comprehensive set of evaluation and comparison criteria has been introduced and their satisfaction by several abstract argumentation semantics verified. On the other hand in [14] general *rationality postulates* have been introduced for argumentation systems at a different abstraction level, where argument structure and construction are explicitly dealt with.

The present work, which is focused on abstract argumentation semantics, stems from an essentially negative result provided in [6]: none of the traditional or more recent literature proposals listed above is able to satisfy all the desirable properties for an abstract argumentation semantics. This raises two "natural" questions:

- are the desirable properties identified in [6] actually achievable altogether?

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0004-3702/$ – see front matter
doi:10.1016/j.artint.2010.11.005

- is it feasible and practically useful to drive the definition of abstract argumentation semantics by formal criteria rather than basic intuitions?

The paper provides a positive answer to both questions in a constructive way by introducing a novel family of abstract argumentation semantics called *resolution-based* and then focusing on its instance *resolution-based grounded semantics* (GR^* in the following), whose properties are investigated. It turns out that GR^* satisfies all the general desiderata of [6] while being at the same time computationally more tractable than any non-trivial multiple-status semantics analyzed up to now in the literature.

The paper is organized as follows. Section 2 provides the necessary background concepts, while Section 3 reviews the desirable properties for abstract argumentation semantics introduced in [6]. In Section 4 the general class of resolution-based argumentation semantics is introduced, whose definition is purposely "principle-driven". Section 5 introduces GR^* as an instance of the resolution-based family of semantics and shows that it satisfies all the desirable properties altogether, while computational properties of GR^* are investigated in Section 6 in the general case and in Section 7 considering restricted classes of argumentation frameworks. Discussion and conclusions are provided in Section 8.

2. Background concepts and notation

2.1. Abstract argumentation frameworks

This work lies in the frame of Dung's theory of abstract argumentation frameworks [18].

Definition 1. An *argumentation framework* (AF for short) is a pair $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, written also $\mathcal{G}(\mathcal{A}, \mathcal{R})$, where $\mathcal{A}$ is a finite set of *arguments* and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary *attack relation* on $\mathcal{A}$. The pair $\langle \emptyset, \emptyset \rangle$ is called the *empty* argumentation framework.

The meaning and possible internal structure of the elements of $\mathcal{A}$ and of the attack relation $\mathcal{R}$ are abstracted away and therefore not specified. An argumentation framework has an obvious representation as a directed graph, often called *defeat graph*.

Some additional notation[1] and terminology concerning attacks will be helpful.

Definition 2. A pair $\langle x, y \rangle \in \mathcal{R}$ is also denoted as $x \to y$ and referred to as '*x attacks y*' or '*y is attacked by x*'. On the other hand $\langle x, y \rangle \notin \mathcal{R}$ is also denoted as $x \not\to y$. An argument x is *self-defeating* if $x \to x$. Two distinct arguments x and y are involved in a *mutual attack* if $x \to y$ and $y \to x$. The set of mutual attacks of $\mathcal{G}$ will be denoted as $M_{\mathcal{G}} = \{\langle x, y \rangle \in \mathcal{R} \mid x \neq y \wedge \langle y, x \rangle \in \mathcal{R}\}$ (note that self-attacking arguments are *not* considered to define mutual attacks).

Since we will frequently deal with sets of arguments, it is also useful to define suitable notations for them.

Definition 3. Given a set of arguments $S \subseteq \mathcal{A}$ and an argument $x \in \mathcal{A}$ we say that '*S attacks x*' (denoted as $S \to x$) if $\exists y \in S : y \to x$ and that '*x attacks S*' (denoted as $x \to S$) if $\exists y \in S : x \to y$. Given a set of arguments $S \subseteq \mathcal{A}$ we denote the set of *attackers* (or *defeaters*) of S as $S^- = \{x \in \mathcal{A} \mid x \to S\}$ and the set of arguments attacked by S as $S^+ = \{x \in \mathcal{A} \mid S \to x\}$. When necessary, these sets will be referred to a specific AF $\mathcal{G}$ using the notations $S^-_{\mathcal{G}}$ and $S^+_{\mathcal{G}}$. The set $S \cup S^+$ is called the *range* of S [32] and denoted as $\rho(S)$.

An argument x is *unattacked* (or *initial*) if $\{x\}^- = \emptyset$. The set of unattacked arguments in $\mathcal{G}$ will be denoted as $IN(\mathcal{G})$. A set of arguments S is *conflict-free* if $\nexists x, y \in S : x \to y$, denoted in the following as $cf(S)$. The set of maximal (with respect to set inclusion) conflict-free sets of $\mathcal{G}$ will be denoted as $\mathcal{MCF}(\mathcal{G})$.

We need to define also the *complement* of a set of arguments and the *restriction* of an argumentation framework.

Definition 4. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and a set of arguments $S \subseteq \mathcal{A}$:

- the *complement* of S is defined as $S^C = \mathcal{A} \setminus S$;
- the *restriction* of $\mathcal{G}$ to S is the AF $\mathcal{G}\downarrow_S = \langle S, \mathcal{R} \cap (S \times S) \rangle$.

The notion of *resolution* of an argumentation framework (introduced in [28]) arises from the idea that each mutual attack represents a sort of "undecided" situation, which might be resolved in favour of one of the arguments involved by suppressing the attack it receives, i.e. transforming the mutual attack into a unidirectional one.

[1] A summary of the notation adopted in the paper is provided in Appendix A.

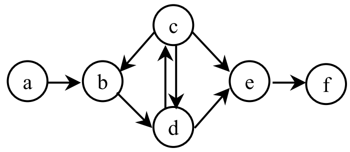

Fig. 1. $\mathcal{G}_1$: an argumentation framework with two resolutions.

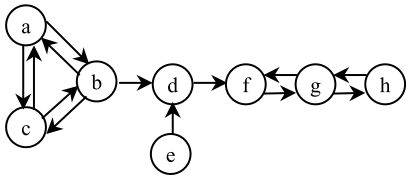

Fig. 2. $\mathcal{G}_2$: an argumentation framework with many resolutions.

Definition 5. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, a (partial) resolution of $\mathcal{G}$ is defined as any subset $\beta \subset M_{\mathcal{G}}$ such that if $\langle x, y \rangle \in \beta$ then $\langle y, x \rangle \notin \beta$, i.e. such that *at most one element* of each of the pairs $\langle x, y \rangle$, $\langle y, x \rangle$ of $M_{\mathcal{G}}$ is in β. The AF arising from applying the partial resolution β to $\mathcal{G}$ is denoted as $\mathcal{G}_\beta = \langle \mathcal{A}, \mathcal{R} \setminus \beta \rangle$. A *full* resolution γ is any partial resolution in which *exactly* one element of each mutual attack occurs, i.e. γ is a full resolution if $M_{\mathcal{G}_\gamma} = \emptyset$. The set of full resolutions of $\mathcal{G}$ is denoted as $\mathcal{FR}(\mathcal{G})$, while the set of argumentation frameworks arising from them is denoted as $\mathcal{FRAF}(\mathcal{G}) = \{\mathcal{G}_\beta \mid \beta \in \mathcal{FR}(\mathcal{G})\}$.

To exemplify this notion let us introduce two examples we will use throughout the paper.

The AF $\mathcal{G}_1$ shown in Fig. 1 includes only one mutual attack, involving arguments c and d, and has just two non-empty resolutions (both full). Formally, $\mathcal{FR}(\mathcal{G}_1) = \{\{\langle c, d \rangle\}, \{\langle d, c \rangle\}\}$.

The AF $\mathcal{G}_2$ shown in Fig. 2 includes five mutual attacks. Given that to define any resolution there are three choices for each mutual attack (selecting one of the attacks or neither) and excluding the empty resolution it follows that $\mathcal{G}_2$ admits 242 resolutions. On the other hand, to define a full resolution there are two choices for each mutual attack entailing that $\mathcal{G}_2$ has 32 full resolutions, including for instance $\{\langle a, b \rangle, \langle c, a \rangle, \langle c, b \rangle, \langle g, f \rangle, \langle g, h \rangle\}$.

2.2. Abstract argumentation semantics

The notions of acceptable argument and characteristic function provide the basis for the definition of some of the traditional Dung's semantics.

Definition 6. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, an argument $x \in \mathcal{A}$ is *acceptable* with respect to (or, equivalently, is *defended* by) a set $S \subseteq \mathcal{A}$ if $\forall y \in \{x\}^- \; S \rightarrow y$ (denoted in the following as $acc(x, S)$). The function $\mathcal{F}_{\mathcal{G}} : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ defined as $\mathcal{F}_{\mathcal{G}}(S) = \{x \in \mathcal{A} \mid acc(x, S)\}$ is called the *characteristic function* of $\mathcal{G}$. We will use the notation $\mathcal{F}^1_{\mathcal{G}}(S) \triangleq \mathcal{F}_{\mathcal{G}}(S)$ and for $i > 1$, $\mathcal{F}^i_{\mathcal{G}}(S) \triangleq \mathcal{F}_{\mathcal{G}}(\mathcal{F}^{i-1}_{\mathcal{G}}(S))$.

In Dung's theory, an (extension-based) argumentation semantics is defined by specifying the criteria for deriving, given a generic argumentation framework, the set of all possible extensions, each one representing a set of arguments able to "survive together" the conflict represented by the attack relation. For a detailed account of traditional and more recent semantics proposals in abstract argumentation the reader may refer to [18,6,7]. Here we recall only the notions that will be used in the sequel of the paper.

Given a generic argumentation semantics $\mathcal{S}$, the set of extensions prescribed by $\mathcal{S}$ for a given argumentation framework $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ is denoted as $\mathcal{E}_{\mathcal{S}}(\mathcal{G}) \subseteq 2^{\mathcal{A}}$. A relevant question concerns the existence of extensions. For a given semantics $\mathcal{S}$, we define $\mathcal{D}_{\mathcal{S}} = \{\mathcal{G} \mid \mathcal{E}_{\mathcal{S}}(\mathcal{G}) \neq \emptyset\}$, namely the set of argumentation frameworks where $\mathcal{S}$ admits at least one extension. If no argumentation framework is outside $\mathcal{D}_{\mathcal{S}}$ we will say that $\mathcal{S}$ is *universally defined*. Most literature semantics are universally defined for finite argumentation frameworks, with the notable exception of stable semantics [18]. As a further terminological note, if it holds that $\forall \mathcal{G} \in \mathcal{D}_{\mathcal{S}} \; |\mathcal{E}_{\mathcal{S}}(\mathcal{G})| = 1$ then the semantics $\mathcal{S}$ is said to belong to the *unique-status approach*, otherwise it is said to belong to the *multiple-status approach*.

The following semantics definitions[2] need to be recalled.

[2] For the sake of conciseness, in some cases we use as definition what is actually an equivalent characterization with respect to the definition given in the relevant original paper.

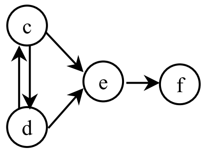

Fig. 3. The argumentation framework $\text{CUT}(\mathcal{G}_1)$.

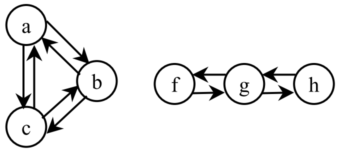

Fig. 4. The argumentation framework $\text{CUT}(\mathcal{G}_2)$.

Definition 7. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, a set $S \subseteq \mathcal{A}$ is:

- a *complete* extension if $cf(S)$ and $S = \mathcal{F}_{\mathcal{G}}(S)$;
- the (unique) *grounded* extension if S is the minimal (with respect to set inclusion) complete extension;
- a *preferred* extension if it is a maximal (with respect to set inclusion) complete extension;
- a *stable* extension if $cf(S)$ and $S^+ = S^C$;
- a *semi-stable* extension if it is a preferred extension such that $\rho(S)$ is maximal (with respect to set inclusion);
- the (unique) *ideal* extension if it is the maximal (with respect to set inclusion) admissible set included in all preferred extensions.

Commenting on semantics definitions, a complete extension is a conflict-free set of arguments which includes all the arguments it defends (i.e., it is a fixed point of $\mathcal{F}_{\mathcal{G}}$), the (provably unique) grounded extension is the smallest such set, while the preferred extensions are the maximal ones, and a stable extension is a conflict-free set which attacks all arguments not belonging to it. A semi-stable extension is a preferred extension where the maximization requirement is extended to attacked arguments too (this implies that semi-stable extensions coincide with stable extensions when the latter exist). The definition of ideal extension is self-explanatory and implies that it includes the grounded extension. Other recent semantics proposals include *CF2* [10] and *prudent* versions of traditional semantics [15]. Reviewing their articulate definitions is beyond the scope of the present paper: the reader may consult the original references for further details.

Complete, grounded, preferred, stable, semi-stable, and ideal semantics are denoted as $\mathcal{CO}$, *GR*, $\mathcal{PR}$, $\mathcal{ST}$, $\mathcal{SST}$, and $\mathcal{ID}$, respectively. The grounded and ideal extension of an AF $\mathcal{G}$ are denoted respectively as $GE(\mathcal{G})$ and $ID(\mathcal{G})$. Due to the key role played by *GR* in this paper we need to introduce some additional notations and recall an important property.

Definition 8. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, $\text{CUT}(\mathcal{G})$ is the AF obtained by suppressing the arguments in the grounded extension and those attacked by them, i.e. $\text{CUT}(\mathcal{G}) = \mathcal{G}{\downarrow}_{(\mathcal{A} \setminus \rho(GE(\mathcal{G})))}$. Letting β be a resolution of $\mathcal{G}$, we will use the shorthand notation $\text{CUT}(\mathcal{G})_\beta$ to denote $\text{CUT}(\mathcal{G})_{\beta \cap ((\mathcal{A} \setminus \rho(GE(\mathcal{G}))) \times (\mathcal{A} \setminus \rho(GE(\mathcal{G}))))}$.

It is proved in [18] that $GE(\mathcal{G})$ results from the iterated application of the characteristic function starting from the empty set when $\mathcal{G}$ is *finitary*.

Definition 9. An AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ is *finitary* if for any $x \in \mathcal{A}$ $\{x\}^-$ is finite.

Proposition 1. *For any finitary* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ $GE(\mathcal{G}) = \bigcup_{i=1}^{\infty} \mathcal{F}^i_{\mathcal{G}}(\emptyset)$.

To exemplify these notions we refer again to Figs. 1 and 2. Given that $GE(\mathcal{G}_1) = \{a\}$ and $GE(\mathcal{G}_2) = \{e\}$, the resulting $\text{CUT}(\mathcal{G}_1)$ and $\text{CUT}(\mathcal{G}_2)$ are as shown in Figs. 3 and 4 respectively. Note also that if $\rho(GE(\mathcal{G})) = \mathcal{A}$ then $\text{CUT}(\mathcal{G})$ is the empty argumentation framework. Later in the paper we will need to consider the grounded extension of an empty argumentation framework: by definition $GE(\langle \emptyset, \emptyset \rangle) = \emptyset$.

Finally, we also need to introduce a notion of "stability" concerning sets of arguments.

Definition 10. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and two sets $S, T \subseteq \mathcal{A}$, S is *stable in* T with respect to $\mathcal{G}$, denoted as $st_{\mathcal{G}}(S, T)$, if $\forall x \in (T \setminus S)$ $x \in (S \cap T)^+$.

3. Evaluation criteria for abstract argumentation semantics

We recall in this section the definition of the main principles and criteria (corresponding to desirable semantics properties) discussed in [6], to which the reader is referred for all details and a more extensive analysis.

3.1. Extension evaluation criteria

Given that an extension can be intuitively conceived as a set of arguments that can be accepted together according to some semantics-specific requirements, one may consider as an additional constraint that no extension can be a proper subset of another one. This is in particular advantageous with respect to the issue of formally defining the justification states of arguments and has a straightforward formal counterpart.

Definition 11. A set of extensions $\mathcal{E}$ is I-maximal iff $\forall E_1, E_2 \in \mathcal{E}$ it holds that $(E_1 \subseteq E_2) \Rightarrow (E_1 = E_2)$. A semantics $\mathcal{S}$ satisfies the *I-maximality criterion* if and only if for any AF $\mathcal{G} \in \mathcal{D}_{\mathcal{S}}$, $\mathcal{E}_{\mathcal{S}}(\mathcal{G})$ is I-maximal.

The requirement of *admissibility*, which actually lies at the heart of all semantics discussed in [18], is based on the notion of admissible set and corresponds to the idea that an extension should be able to defend itself against attacks.

Definition 12. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, a set $E \subseteq \mathcal{A}$ is *admissible* if and only if $cf(E)$ and any argument $x \in E$ is acceptable with respect to E. The set of the admissible sets of $\mathcal{G}$ is denoted as $\mathcal{AS}(\mathcal{G})$.

A semantics $\mathcal{S}$ satisfies the *admissibility criterion* if for any AF $\mathcal{G} \in \mathcal{D}_{\mathcal{S}}$, it holds that $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathcal{G})\ E \in \mathcal{AS}(\mathcal{G})$, namely:

$$x \in E \quad \Rightarrow \quad \forall y \in \{x\}^- \quad E \to y \tag{1}$$

The property of *reinstatement* corresponds to the converse of the implication (1) prescribed by the admissibility criterion. Intuitively, an argument x is *reinstated* if its attackers are in turn attacked and, as a consequence, one may assume that they should have no effect on the extension membership of x. Under this assumption, if an extension E reinstates x then x should belong to E. Formally this leads to the following *reinstatement criterion*:

Definition 13. A semantics $\mathcal{S}$ satisfies the *reinstatement criterion* if $\forall \mathcal{G} \in \mathcal{D}_{\mathcal{S}}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathcal{G})$ it holds that:

$$\bigl(\forall y \in \{x\}^-\ E \to y\bigr) \quad \Rightarrow \quad x \in E \tag{2}$$

The notion of *directionality* is based on the idea that the extension membership of an argument x should be affected only by the attackers of x (which in turn are affected by their attackers and so on), while any argument y such that there is no path from y to x should not have any effect on x. The directionality criterion can be specified by requiring that a set of arguments not receiving attacks from outside is not affected by the remaining parts of the argumentation framework, as far as extensions are concerned.

Definition 14. Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, a set $S \subseteq \mathcal{A}$ is *externally unattacked* if and only if $\nexists x \in S^C : x \to S$. The set of externally unattacked sets of $\mathcal{G}$ will be denoted as $\mathcal{US}(\mathcal{G})$.

Definition 15. A semantics $\mathcal{S}$ satisfies the *directionality criterion* if and only if for any AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle \in \mathcal{D}_{\mathcal{S}}$, $\forall T \in \mathcal{US}(\mathcal{G})$ it holds that $\mathcal{AE}_{\mathcal{S}}(\mathcal{G}, T) = \mathcal{E}_{\mathcal{S}}(\mathcal{G}{\downarrow}_T)$ where $\mathcal{AE}_{\mathcal{S}}(\mathcal{G}, T) = \{(R \cap T) \mid R \in \mathcal{E}_{\mathcal{S}}(\mathcal{G})\} \subseteq 2^{\mathcal{A}}$.

In words, directionality prescribes that one obtains the same result (i.e. the same set of sets of arguments) by either projecting the extensions of the whole framework to an externally unattacked set or computing the extensions of the framework restricted to the same externally unattacked set. More precisely, the intersection of any extension prescribed by $\mathcal{S}$ for $\mathcal{G}$ with an externally unattacked set T is equal to one of the extensions prescribed by $\mathcal{S}$ for the restriction of $\mathcal{G}$ to T, and vice versa.

3.2. Skepticism related criteria

Semantics *adequacy* criteria introduced in [6] are based on comparisons of sets of extensions which in turn exploit some recently introduced notions concerning the formalization of *skepticism*.

3.2.1. Skepticism relations

The notion of skepticism has often been used in the literature in informal or semi-formal ways to discuss semantics behavior, e.g. by observing that a semantics $\mathcal{S}_1$ is "more skeptical" than another semantics $\mathcal{S}_2$, which intuitively means that $\mathcal{S}_1$ makes less committed choices than $\mathcal{S}_2$ about the justification state of the arguments. A more formal and general analysis has been carried out in [6,8] and is partly recalled here. First, we consider as a basic concept a generic relation of skepticism $\preccurlyeq^E$ between sets of extensions: given two sets of extensions $\mathcal{E}_1, \mathcal{E}_2$ of an argumentation framework $\mathcal{G}$, $\mathcal{E}_1 \preccurlyeq^E \mathcal{E}_2$ will simply denote that $\mathcal{E}_1$ is at least as skeptical as $\mathcal{E}_2$ in some sense. In the approach to semantics evaluation we are recalling, a skepticism relation is used to compare the sets of extensions prescribed by a particular semantics on different but related argumentation frameworks. To this purpose, one first needs to define a skepticism relation between argumentation frameworks: given two argumentation frameworks based on the same set of arguments $\mathcal{G}_1 = \langle \mathcal{A}, \mathcal{R}_1 \rangle$ and $\mathcal{G}_2 = \langle \mathcal{A}, \mathcal{R}_2 \rangle$, $\mathcal{G}_1 \preccurlyeq^A \mathcal{G}_2$ denotes that $\mathcal{G}_1$ (actually its attack relation) is inherently less committed (to be precise, not more committed) than $\mathcal{G}_2$. Then, one may reasonably require that any semantics reflects in its extensions the skepticism relations between argumentation frameworks. Requirements of this kind for a generic semantics $\mathcal{S}$ will be called *adequacy criteria*. Having laid out the general framework for the definition of adequacy criteria, we first recall the actual skepticism relations used in the sequel.

Let us start, at a basic level, by noting that defining a relation of skepticism between two extensions is intuitively straightforward: an extension E_1 is more skeptical than an extension E_2 if and only if $E_1 \subseteq E_2$. In fact, a more skeptical attitude corresponds to a smaller set of selected arguments. Directly extending the above intuition to the comparison of sets of extensions leads to define the following skepticism relation $\preccurlyeq^E_\cap$.

Definition 16. Given two non-empty sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework $\mathcal{G}$, $\mathcal{E}_1 \preccurlyeq^E_\cap \mathcal{E}_2$ iff $\bigcap_{E_1 \in \mathcal{E}_1} E_1 \subseteq \bigcap_{E_2 \in \mathcal{E}_2} E_2$.

Finer (and actually stronger) skepticism relations can then be defined by considering relations of pairwise inclusion between extensions. We recall that to compare a single extension E_1 with a set of extensions $\mathcal{E}_2$, the relation $\forall E_2 \in \mathcal{E}_2$, $E_1 \subseteq E_2$ has often been used in the literature. A direct generalization to the comparison of two sets of extensions is represented by the following weak skepticism relation $\preccurlyeq^E_W$.

Definition 17. Given two non-empty sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an AF $\mathcal{G}$, $\mathcal{E}_1 \preccurlyeq^E_W \mathcal{E}_2$ iff

$$\forall E_2 \in \mathcal{E}_2 \quad \exists E_1 \in \mathcal{E}_1\colon\ E_1 \subseteq E_2 \tag{3}$$

It is worth noting that (as it is easy to see), given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework $\mathcal{G}$, $\mathcal{E}_1 \preccurlyeq^E_W \mathcal{E}_2 \Rightarrow \mathcal{E}_1 \preccurlyeq^E_\cap \mathcal{E}_2$.

In a sense, relation $\preccurlyeq^E_W$ is unidirectional, since it only constrains the extensions of $\mathcal{E}_2$, while $\mathcal{E}_1$ may contain additional extensions unrelated to those of $\mathcal{E}_2$. One may then consider also a more symmetric (and stronger) relationship $\preccurlyeq^E_S$, where it is also required that any extension of $\mathcal{E}_1$ is included in one extension of $\mathcal{E}_2$. However, as discussed in [9,5,6] this relationship is definitely too strong since it actually prevents comparability of any pair of multiple-status semantics. For this reason, it will not be considered here. It is also worth mentioning that analogous relations (called *generality* relations, as they are defined in the opposite direction of comparison) have been considered for comparing extensions of default logic in [27] and that, at a more general level, this kind of relations can be put in correspondence with Smyth- and Hoare-orderings, considered in the context of nondeterministic computations [30,1].

Turning to skepticism relations between argumentation frameworks, a relation $\preccurlyeq^A$ has been proposed in [6], generalizing some more specific but related notions introduced in [5] and [28]. The underlying idea, as already mentioned in Section 2 when introducing the notion of resolution, is that a mutual attack corresponds to a less committed (or, equivalently, more undecided) situation than a unidirectional attack. Therefore if an argumentation framework $\mathcal{G}'$ is obtained from another argumentation framework $\mathcal{G}$ by transforming some mutual attacks into unidirectional ones, it is reasonable to assume that $\mathcal{G}'$ is more committed than $\mathcal{G}$. On this basis, we define a skepticism relation $\preccurlyeq^A$ between argumentation frameworks based on the same set of arguments.

Definition 18. Given two argumentation frameworks $\mathcal{G}_1 = \langle \mathcal{A}, \mathcal{R}_1 \rangle$ and $\mathcal{G}_2 = \langle \mathcal{A}, \mathcal{R}_2 \rangle$, $\mathcal{G}_1 \preccurlyeq^A \mathcal{G}_2$ if and only if there is a resolution β of $\mathcal{G}_1$ such that $\mathcal{G}_2 = \mathcal{G}_{1\beta}$.

It is easy to see that the above definition covers all cases where some (possibly none) mutual attacks of $\mathcal{G}_1$ correspond to unidirectional attacks in $\mathcal{G}_2$, while unidirectional attacks of $\mathcal{G}_1$ are the same in $\mathcal{G}_2$.

Comparable argumentation frameworks are characterized by having the same set of arguments and the same set of conflicting pairs of arguments. It is immediate to see that $\preccurlyeq^A$ is a partial order, as it is equivalent to requiring set inclusion between attack relations, under the constraint that at least an attack is preserved for any pair of conflicting arguments in $\mathcal{G}_1$. It is also worth noting that within a set of comparable argumentation frameworks there are, in general, several maximal elements with respect to $\preccurlyeq^A$, namely all argumentation frameworks where no mutual attack is present (corresponding to all full resolutions).

3.2.2. Skepticism adequacy

Given that an argumentation framework is considered inherently more skeptical than another one, it is reasonable to require that when applying the same semantics to both, the skepticism relation between them is preserved by the relevant sets of extensions. This kind of criterion, called *skepticism adequacy*, has first been proposed in [5] and is formulated here in a generalized version.

Definition 19. Given a skepticism relation $\preccurlyeq^E$ between sets of extensions, a semantics $\mathcal{S}$ is $\preccurlyeq^E$-skepticism-adequate, denoted $\mathcal{SA}_{\preccurlyeq^E}(\mathcal{S})$, if and only if for any pair of argumentation frameworks $\mathcal{G}, \mathcal{G}' \in \mathcal{D}_\mathcal{S}$ such that $\mathcal{G} \preccurlyeq^A \mathcal{G}'$ it holds that $\mathcal{E}_\mathcal{S}(\mathcal{G}) \preccurlyeq^E \mathcal{E}_\mathcal{S}(\mathcal{G}')$.

According to the definitions provided in Section 3.2.1 we have two skepticism adequacy properties, which are clearly related by the same order of implication: $\mathcal{SA}_{\preccurlyeq^E_W}(\mathcal{S}) \Rightarrow \mathcal{SA}_{\preccurlyeq^E_\cap}(\mathcal{S})$.

3.2.3. Resolution adequacy

Resolution adequacy generalizes a criterion first proposed in [28] and relies on the intuition that if an argument is included in any extensions of any element of $\mathcal{FRAF}(\mathcal{G})$ then it should be included in all extensions of $\mathcal{G}$ too. This criterion is called *resolution adequacy* in [6] where a generalization of its original formulation is provided, in order to make it parametric with respect to skepticism relations between sets of extensions.

Definition 20. Given a skepticism relation $\preccurlyeq^E$ between sets of extensions, a semantics $\mathcal{S}$ is $\preccurlyeq^E$-resolution-adequate, denoted $\mathcal{RA}_{\preccurlyeq^E}(\mathcal{S})$, if and only if for any argumentation framework $\mathcal{G} \in \mathcal{D}_\mathcal{S}$ such that $\forall \mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$ $\mathcal{G}' \in \mathcal{D}_\mathcal{S}$ it holds that $\mathcal{UR}(\mathcal{G}, \mathcal{S}) \preccurlyeq^E \mathcal{E}_\mathcal{S}(\mathcal{G})$, where $\mathcal{UR}(\mathcal{G}, \mathcal{S}) = \bigcup_{\mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})} \mathcal{E}_\mathcal{S}(\mathcal{G}')$.

Again, we have two resolution adequacy properties, related by the usual order of implication: $\mathcal{RA}_{\preccurlyeq^E_W}(\mathcal{S}) \Rightarrow \mathcal{RA}_{\preccurlyeq^E_\cap}(\mathcal{S})$.

4. The family of resolution-based semantics

As already mentioned, it is shown in [6] that none among the traditional grounded, complete, stable, and preferred semantics nor among the more recent ideal, semi-stable, *CF2*, and (several flavors[3] of) prudent semantics satisfies the properties of I-maximality, admissibility, reinstatement, directionality, $\preccurlyeq^E_\cap$- and $\preccurlyeq^E_W$-skepticism adequacy, $\preccurlyeq^E_\cap$- and $\preccurlyeq^E_W$-resolution adequacy altogether. In the search for a semantics able to satisfy all of them, we introduce a family of semantics which is parametric with respect to another semantics $\mathcal{S}$ and whose definition is purposely oriented to the direct satisfaction (or preservation, if they already hold for $\mathcal{S}$) of some of the properties listed above. This family is called *resolution-based* since its definition is based on the notion of resolution of an argumentation framework introduced in Definition 5.

Definition 21. Given an argumentation semantics $\mathcal{S}$ which is universally defined, its resolution-based version is the semantics $\mathcal{S}^\star$ such that for any argumentation framework $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, \mathcal{S}))$, where given a set $\mathcal{E}$ of subsets of $\mathcal{A}$, $\mathcal{MIN}(\mathcal{E})$ denotes the set of the minimal (with respect to set inclusion) elements of $\mathcal{E}$ and $\mathcal{UR}(\mathcal{G}, \mathcal{S})$ is as in Definition 20.

Operationally, the idea underlying Definition 21 is as follows: given an argumentation framework $\mathcal{G}$ the set $\mathcal{FRAF}(\mathcal{G})$ of all argumentation frameworks derivable from $\mathcal{G}$ by transforming all mutual attacks into unidirectional ones is determined. Then semantics $\mathcal{S}$ is applied to each $\mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$ to obtain $\mathcal{E}_\mathcal{S}(\mathcal{G}')$. The union $\mathcal{UR}(\mathcal{G}, \mathcal{S})$ of these sets of extensions is then considered and its minimal elements (with respect to set inclusion) selected as the extensions prescribed for $\mathcal{G}$ by $\mathcal{S}^\star$. Note that this definition directly enforces the property of I-maximality.

To exemplify, let us refer again to the examples of Figs. 1 and 2 and consider the resolution-based version of grounded semantics. It is quite easy to see that $\mathcal{UR}(\mathcal{G}_1, GR) = \{\{a, c, f\}, \{a, d, f\}\}$ from which it follows that $\mathcal{E}_{GR^*}(\mathcal{G}_1) = \{\{a, c, f\}, \{a, d, f\}\}$. Turning to $\mathcal{G}_2$, it is a bit more laborious to see that $\mathcal{UR}(\mathcal{G}_2, GR) = \{\{e, f, h\}, \{e, g\}, \{e, f, h, a\}, \{e, f, h, b\}, \{e, f, h, c\}, \{e, g, a\}, \{e, g, b\}, \{e, g, c\}\}$. Selecting its minimal elements yields $\mathcal{E}_{GR^*}(\mathcal{G}_2) = \{\{e, f, h\}, \{e, g\}\}$.

Let us now show that for any semantics $\mathcal{S}$, $\mathcal{S}^\star$ satisfies $\preccurlyeq^E_W$-skepticism adequacy (and therefore also $\preccurlyeq^E_\cap$-skepticism adequacy).

Proposition 2. *For every argumentation semantics $\mathcal{S}$, its resolution-based version $\mathcal{S}^\star$ satisfies $\preccurlyeq^E_W$-skepticism adequacy.*

Proof. On the basis of Definition 19 we have to show that for any pair of argumentation frameworks $\mathcal{G}, \mathcal{G}'$ such that $\mathcal{G} \preccurlyeq^A \mathcal{G}'$ it holds that $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \preccurlyeq^E_W \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}')$. First, it is easy to see that for any such a pair of argumentation frameworks $\mathcal{FRAF}(\mathcal{G}') \subseteq \mathcal{FRAF}(\mathcal{G})$, and, therefore, by definition of $\mathcal{UR}$, $\mathcal{UR}(\mathcal{G}', \mathcal{S}) \subseteq \mathcal{UR}(\mathcal{G}, \mathcal{S})$. Recalling that $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, \mathcal{S}))$ and $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}') = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}', \mathcal{S}))$, it follows that $\forall E' \in \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}')$ $E' \in \mathcal{UR}(\mathcal{G}, \mathcal{S})$ and hence $\exists E \in \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, \mathcal{S})) = \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G})$ such that $E \subseteq E'$, namely $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \preccurlyeq^E_W \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}')$. $\square$

[3] A prudent version has been defined for each of the four traditional Dung's semantics.

Adding the (not particularly restrictive) hypothesis that $\mathcal{S}$ is I-maximal, $\mathcal{S}^\star$ achieves also the property of $\preccurlyeq^E_W$- (and therefore $\preccurlyeq^E_\cap$-) resolution adequacy, as shown by the following proposition.

Proposition 3. *For every I-maximal argumentation semantics $\mathcal{S}$, its resolution-based version $\mathcal{S}^\star$ satisfies $\preccurlyeq^E_W$-resolution adequacy.*

Proof. According to Definition 20 we have to show that $\mathcal{UR}(\mathcal{G},\mathcal{S}^\star) \preccurlyeq^E_W \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G})$. By definition of $\mathcal{UR}$ we have $\mathcal{UR}(\mathcal{G},\mathcal{S}^\star) = \bigcup_{\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})}\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}')$. In turn, by Definition 21, $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}') = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}',\mathcal{S})) = \mathcal{MIN}(\bigcup_{\mathcal{G}''\in\mathcal{FRAF}(\mathcal{G}')}\mathcal{E}_{\mathcal{S}}(\mathcal{G}''))$. Since any $\mathcal{G}'$ belongs to $\mathcal{FRAF}(\mathcal{G})$, it is immediate to see that $\mathcal{FRAF}(\mathcal{G}') = \{\mathcal{G}'\}$ therefore $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}') = \mathcal{MIN}(\mathcal{E}_{\mathcal{S}}(\mathcal{G}')) = \mathcal{E}_{\mathcal{S}}(\mathcal{G}')$, where the last equality holds by the hypothesis of I-maximality of $\mathcal{S}$. It follows that $\mathcal{UR}(\mathcal{G},\mathcal{S}^\star) = \bigcup_{\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})}\mathcal{E}_{\mathcal{S}}(\mathcal{G}') = \mathcal{UR}(\mathcal{G},\mathcal{S})$. On the other hand, by definition $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) = \mathcal{MIN}(\mathcal{UR}(\mathcal{G},\mathcal{S}))$. It follows that $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \subseteq \mathcal{UR}(\mathcal{G},\mathcal{S}) = \mathcal{UR}(\mathcal{G},\mathcal{S}^\star)$ which directly implies $\mathcal{UR}(\mathcal{G},\mathcal{S}^\star) \preccurlyeq^E_W \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G})$. □

Turning to defense related criteria, a significant result can be obtained using as a basis the following lemma concerning the relationship between the complete extensions of the elements of $\mathcal{FRAF}(\mathcal{G})$ and those of $\mathcal{G}$ itself.

Lemma 1. *For any* AF $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$, $\forall\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})\ \mathcal{E}_{\mathcal{CO}}(\mathcal{G}') \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G})$.

Proof. It is shown in Lemma 1 of [28] that for any AF $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$, $\mathcal{AS}(\mathcal{G}) = \bigcup_{\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})}\mathcal{AS}(\mathcal{G}')$. Let $E \in \mathcal{E}_{\mathcal{CO}}(\mathcal{G}')$, since any complete extension is an admissible set ($E \in \mathcal{AS}(\mathcal{G}')$), we also have that $E \in \mathcal{AS}(\mathcal{G})$. Then, to prove that E is a complete extension in $\mathcal{G}$ we only need to show that $\forall x\in\mathcal{A}$, $(\forall y \in \{x\}^-_{\mathcal{G}}\ y \in E^+_{\mathcal{G}}) \Rightarrow x \in E$. To this purpose we show that any x defended by E in $\mathcal{G}$ is also defended by E in $\mathcal{G}'$. First note that according to Definition 18, it must be the case that $\forall y \in E^+_{\mathcal{G}}$, $\exists z \in E$ such that either $z \in \{y\}^-_{\mathcal{G}'}$ or $y \in \{z\}^-_{\mathcal{G}'}$. Since E is admissible in $\mathcal{G}'$, if $y \in \{z\}^-_{\mathcal{G}'}$ then also $y \in E^+_{\mathcal{G}'}$. In sum, we have that $\forall y \in \{x\}^-_{\mathcal{G}}$, $y \in E^+_{\mathcal{G}'}$ (any attacker of x in $\mathcal{G}$ is attacked by E in $\mathcal{G}'$). Now, since no additional attack edges are present in $\mathcal{G}'$ with respect to those of $\mathcal{G}$ it must be the case that $\{x\}^-_{\mathcal{G}'} \subseteq \{x\}^-_{\mathcal{G}}$ and we can derive that $\forall z \in \{x\}^-_{\mathcal{G}'}$, $z \in E^+_{\mathcal{G}'}$ (any attacker of x in $\mathcal{G}'$ is attacked by E in $\mathcal{G}'$). Since E is a complete extension in $\mathcal{G}'$ it follows that $x \in E$. □

Proposition 4. *If a semantics $\mathcal{S}$ is such that for any argumentation framework $\mathcal{G}$ $\mathcal{E}_{\mathcal{S}}(\mathcal{G}) \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G})$, then also $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G})$.*

Proof. $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \subseteq \mathcal{UR}(\mathcal{G},\mathcal{S}) = \bigcup_{\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})}\mathcal{E}_{\mathcal{S}}(\mathcal{G}')$. By the hypothesis, for any $\mathcal{G}'\in\mathcal{FRAF}(\mathcal{G})$ $\mathcal{E}_{\mathcal{S}}(\mathcal{G}') \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G}')$ and, by Lemma 1, $\mathcal{E}_{\mathcal{CO}}(\mathcal{G}') \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G})$. It follows that $\mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}) \subseteq \mathcal{E}_{\mathcal{CO}}(\mathcal{G})$. □

Since it is known [6] that complete extensions satisfy the property of admissibility (1) and reinstatement (2) any resolution-based version $\mathcal{S}^\star$ of a semantics $\mathcal{S}$ satisfying the hypothesis of Proposition 4 also satisfies these properties.

5. Resolution-based grounded semantics

The results provided in Section 4 show that given a universally defined and I-maximal semantics satisfying the hypothesis of Proposition 4, its resolution-based version satisfies the properties of I-maximality, admissibility, reinstatement, $\preccurlyeq^E_\cap$- and $\preccurlyeq^E_W$-skepticism adequacy, $\preccurlyeq^E_\cap$- and $\preccurlyeq^E_W$-resolution adequacy, i.e. all the desirable properties but directionality.

Following these lines, let us look among the semantics considered in this paper for a candidate to which to apply the resolution-based scheme. The requirement of universal definition leaves out stable semantics (and its prudent version), the one of I-maximality excludes complete semantics,[4] while the requirement that all extensions are complete excludes *CF2* and all other versions of prudent semantics, as shown in [6].

The four remaining semantics satisfy the above requirements, but directionality of the resolution-based versions of preferred, ideal, and semi-stable semantics is ruled out by a single example, illustrated in Fig. 5.

The argumentation framework $\mathcal{G}$ contains just one mutual attack between arguments a and c, it holds therefore that $\mathcal{FRAF}(\mathcal{G}) = \{\mathcal{G}',\mathcal{G}''\}$. By applying the relevant definition, it is relatively easy to identify the preferred extensions (i.e. the maximal admissible sets) of $\mathcal{G}'$ and $\mathcal{G}''$: $\mathcal{E}_{\mathcal{PR}}(\mathcal{G}') = \{\{a,e,h\},\{a,f,g,i\}\}$, $\mathcal{E}_{\mathcal{PR}}(\mathcal{G}'') = \{\{f,g,i\}\}$. It is also easy to see that $\mathcal{E}_{\mathcal{SST}}(\mathcal{G}') = \mathcal{E}_{\mathcal{PR}}(\mathcal{G}')$ and $\mathcal{E}_{\mathcal{SST}}(\mathcal{G}'') = \mathcal{E}_{\mathcal{PR}}(\mathcal{G}'')$, namely semi-stable extensions coincide with preferred extensions in both $\mathcal{G}'$ and $\mathcal{G}''$. In fact, in the case of $\mathcal{G}'$ both preferred extensions are also stable, hence necessarily semi-stable, while in the case of $\mathcal{G}''$ the coincidence is immediate given that there is just one preferred extension. As to ideal semantics, in the case of $\mathcal{G}'$ we note that the intersection of the preferred extensions, namely $\{a\}$, is admissible, hence $\mathcal{E}_{\mathcal{ID}}(\mathcal{G}') = \{\{a\}\}$, while, by definition, when there is just one preferred extension it is also the only ideal extension, hence $\mathcal{E}_{\mathcal{ID}}(\mathcal{G}'') = \{\{f,g,i\}\}$. Applying Definition 21 it follows that $\mathcal{E}_{\mathcal{SST}^\star}(\mathcal{G}) = \mathcal{E}_{\mathcal{PR}^\star}(\mathcal{G}) = \{\{a,e,h\},\{f,g,i\}\}$ and $\mathcal{E}_{\mathcal{ID}^\star}(\mathcal{G}) = \{\{a\},\{f,g,i\}\}$. Consider now the subset $S = \{a,b,c\}$ which is clearly externally unattacked in $\mathcal{G}$ (formally $S \in \mathcal{US}(\mathcal{G})$). The restricted AF $\mathcal{G}{\downarrow}_S$ has

[4] Note that stable extensions may not exist for argumentation frameworks with simple topologies not involving mutual attacks (e.g. a three-length unidirectional attack cycle). The same holds for complete semantics failing to satisfy I-maximality (e.g. a four-length unidirectional attack cycle).

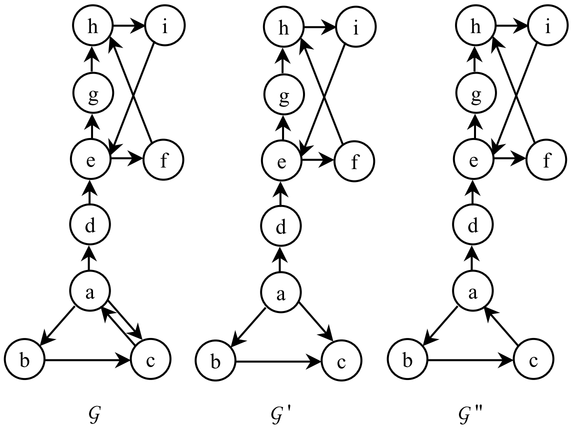

Fig. 5. An example showing that $\mathcal{PR}^\star$, $\mathcal{SST}^\star$, and $\mathcal{ID}^\star$ are not directional.

exactly two full resolutions, actually giving rise to the corresponding restrictions of $\mathcal{G}'$ and $\mathcal{G}''$ (formally $\mathcal{FRAF}(\mathcal{G}\downarrow_S) = \{\mathcal{G}'\downarrow_S, \mathcal{G}''\downarrow_S\}$). It is easy to see that $\mathcal{E}_{\mathcal{PR}}(\mathcal{G}'\downarrow_S) = \mathcal{E}_{\mathcal{SST}}(\mathcal{G}'\downarrow_S) = \mathcal{E}_{\mathcal{ID}}(\mathcal{G}'\downarrow_S) = \{\{a\}\}$ and $\mathcal{E}_{\mathcal{PR}}(\mathcal{G}''\downarrow_S) = \mathcal{E}_{\mathcal{SST}}(\mathcal{G}''\downarrow_S) = \mathcal{E}_{\mathcal{ID}}(\mathcal{G}''\downarrow_S) = \{\emptyset\}$ from which it follows $\mathcal{E}_{\mathcal{SST}^\star}(\mathcal{G}\downarrow_S) = \mathcal{E}_{\mathcal{PR}^\star}(\mathcal{G}\downarrow_S) = \mathcal{E}_{\mathcal{ID}^\star}(\mathcal{G}\downarrow_S) = \{\emptyset\}$. We can now see that $\mathcal{PR}^\star$, $\mathcal{SST}^\star$, and $\mathcal{ID}^\star$ fail to satisfy Definition 15 with respect to the externally unattacked set S. In fact, for any semantics $\mathcal{S}^\star \in \{\mathcal{PR}^\star, \mathcal{SST}^\star, \mathcal{ID}^\star\}$ it turns out that $\{(E \cap S) \mid E \in \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G})\} = \{\{a\}, \emptyset\} \neq \mathcal{E}_{\mathcal{S}^\star}(\mathcal{G}\downarrow_S) = \{\emptyset\}$.

Consider instead the behavior in this case of the only remaining candidate, namely the resolution-based version of grounded semantics. It is easy to see that $GE(\mathcal{G}') = \{a\}$ while $GE(\mathcal{G}'') = \emptyset$. Applying Definition 21 it follows that $\mathcal{E}_{GR^\star}(\mathcal{G}) = \mathcal{MIN}(\{\{a\}, \emptyset\}) = \{\emptyset\}$. It is also easily seen that $\mathcal{E}_{GR^\star}(\mathcal{G}\downarrow_S) = \{\emptyset\}$, thus satisfying the directionality property in this case.[5]

Happily, compliance of $GR^\star$ with the directionality property turns out to hold in general, as shown by Proposition 5, which requires a preliminary lemma.

Lemma 2. *For any* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and for any set* $S \subseteq \mathcal{A}$, $\{\mathcal{G}'\downarrow_S \mid \mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})\} = \mathcal{FRAF}(\mathcal{G}\downarrow_S)$.

Proof. Let us first prove that, given $\mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$, $\mathcal{G}'\downarrow_S \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$. By definition, $\exists \beta \in \mathcal{FR}(\mathcal{G})$ such that $\mathcal{G}' = \mathcal{G}_\beta$. Then considering $\beta^* = \beta \cap (S \times S)$ it is easy to see that $\beta^* \in \mathcal{FR}(\mathcal{G}\downarrow_S)$ and $\mathcal{G}'\downarrow_S = (\mathcal{G}\downarrow_S)_{\beta^*}$, hence $\mathcal{G}'\downarrow_S \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$.

Turning to the other direction of the proof, given a generic $\mathcal{G}'' \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$ we have to prove that $\exists \mathcal{G}' \in \mathcal{FR}(\mathcal{G})$ such that $\mathcal{G}'\downarrow_S = \mathcal{G}''$. $\mathcal{G}'$ can be constructed from $\mathcal{G}$ by selecting a unidirectional attack for each mutual attack of $\mathcal{G}$, with the constraint that $\forall x, y \in S$ the chosen attack between x and y is the one included in $\mathcal{G}''$ (this is possible since $\mathcal{G}'' \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$). It is then easy to see that $\mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$ and that $\mathcal{G}'\downarrow_S = \mathcal{G}''$. $\square$

Proposition 5. $GR^\star$ *satisfies the directionality property.*

Proof. According to Definition 15, we have to prove that for any AF $\mathcal{G}$, $\forall S \in \mathcal{US}(\mathcal{G})$ $\{(E \cap S) \mid E \in \mathcal{E}_{GR^\star}(\mathcal{G})\} = \mathcal{E}_{GR^\star}(\mathcal{G}\downarrow_S)$.

Let $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ as usual. First, we prove that given a set $E \in \mathcal{E}_{GR^\star}(\mathcal{G})$ $(E \cap S) \in \mathcal{E}_{GR^\star}(\mathcal{G}\downarrow_S)$. Since $E \in \mathcal{E}_{GR^\star}(\mathcal{G})$, according to Definition 21 $\exists \mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$ such that $E \in \mathcal{E}_{GR}(\mathcal{G}')$. Since grounded semantics belongs to the unique status approach, this is equivalent to $\exists \mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$ such that $E = GE(\mathcal{G}')$. It is easy to see that the set S, being externally unattacked in $\mathcal{G}$, also belongs to $\mathcal{US}(\mathcal{G}')$. Since grounded semantics satisfies the directionality property [6], we can derive $(E \cap S) = GE(\mathcal{G}'\downarrow_S)$. By Lemma 2, we have $\mathcal{G}'\downarrow_S \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$, which implies that $(E \cap S) \in \mathcal{UR}(\mathcal{G}\downarrow_S, GR)$. To see that $(E \cap S) \in \mathcal{MIN}(\mathcal{UR}(\mathcal{G}\downarrow_S, GR))$ suppose by contradiction that $\exists \mathcal{G}^\sim \in \mathcal{FRAF}(\mathcal{G}\downarrow_S)$: $GE(\mathcal{G}^\sim) \subsetneq (E \cap S)$. To see that this is impossible, consider now an argumentation framework $\mathcal{G}'' = \langle \mathcal{A}, \mathcal{R}'' \rangle$ such that $\mathcal{G}''\downarrow_S = \mathcal{G}^\sim$, $\mathcal{G}''\downarrow_{\mathcal{A}\setminus S} = \mathcal{G}'\downarrow_{\mathcal{A}\setminus S}$, and $\forall \langle x, y \rangle \in \mathcal{R}$: $x \in S, y \notin S$ $\langle x, y \rangle \in \mathcal{R}''$. In words, $\mathcal{G}''$ is obtained from $\mathcal{G}$ by applying within S the same substitutions of mutual into unidirectional attacks as in $\mathcal{G}^\sim$, and the same substitutions as in $\mathcal{G}'$ outside S. Note in particular that since S is externally unattacked, any attack involving an element $x \in S$ and an element $y \notin S$ is unidirectional and has the form $x \to y$, therefore the same attack is necessarily present in $\mathcal{G}'$ (and in any argumentation framework arising from a resolution of $\mathcal{G}$). We will show that $GE(\mathcal{G}'') \subsetneq GE(\mathcal{G}') = E$ which contradicts the hypothesis that $E \in \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, GR))$. First note that, by directionality of grounded semantics, $GE(\mathcal{G}'') \cap S = GE(\mathcal{G}''\downarrow_S) = GE(\mathcal{G}^\sim) \subsetneq (E \cap S) = GE(\mathcal{G}'\downarrow_S) = GE(\mathcal{G}') \cap S$, yielding in particular

$$GE(\mathcal{G}'') \cap S \subsetneq GE(\mathcal{G}') \cap S \tag{4}$$

[5] $GR^\star$ admits a unique extension here. It is however easy to see that $GR^\star$, differently from GR, belongs to the multiple-status approach. For instance, in the AF $\mathcal{G}_1$ shown in Fig. 1 there are two $GR^\star$ extensions, namely $\{a, c, f\}$ and $\{a, d, f\}$.

It is then sufficient to show that $GE(\mathcal{G}'') \subseteq GE(\mathcal{G}')$. To this purpose, we exploit Proposition 1. It is easy to see that $\mathcal{F}^1_{\mathcal{G}''}(\emptyset) \subseteq \mathcal{F}^1_{\mathcal{G}'}(\emptyset)$. In fact for any $\mathcal{G}$, $\mathcal{F}^1_{\mathcal{G}}(\emptyset) = IN(\mathcal{G})$ (see Definition 3). By construction, $IN(\mathcal{G}'') \cap (\mathcal{A} \setminus S) = IN(\mathcal{G}') \cap (\mathcal{A} \setminus S)$ while (4) entails that $IN(\mathcal{G}'') \cap S \subseteq IN(\mathcal{G}') \cap S$. Having shown that $\mathcal{F}^1_{\mathcal{G}''}(\emptyset) \subseteq \mathcal{F}^1_{\mathcal{G}'}(\emptyset) \subseteq GE(\mathcal{G}')$ let us prove by induction that if $\mathcal{F}^i_{\mathcal{G}''}(\emptyset) \subseteq GE(\mathcal{G}')$ then $\mathcal{F}^{i+1}_{\mathcal{G}''}(\emptyset) \subseteq GE(\mathcal{G}')$. Consider any argument x in $\mathcal{F}^{i+1}_{\mathcal{G}''}(\emptyset) \setminus \mathcal{F}^i_{\mathcal{G}''}(\emptyset)$. If $x \in S$ then from (4) we directly have that $x \in GE(\mathcal{G}')$. If $x \notin S$ then by definition of $\mathcal{F}^i_{\mathcal{G}''}(\emptyset)$ it holds that $\forall y : y \to x$ in $\mathcal{G}''$ $\exists z \in \mathcal{F}^i_{\mathcal{G}''}(\emptyset) : z \to y$ in $\mathcal{G}''$, and by construction of $\mathcal{G}''$ the defeaters of x in $\mathcal{G}''$ are precisely those in $\mathcal{G}'$, therefore $\forall y : y \to x$ in $\mathcal{G}'$ $\exists z \in \mathcal{F}^i_{\mathcal{G}''}(\emptyset) : z \to y$ in $\mathcal{G}''$. Now, by the inductive hypothesis $z \in GE(\mathcal{G}')$, and since both $\mathcal{G}'$ and $\mathcal{G}''$ belong to $\mathcal{FRAF}(\mathcal{G})$ either z attacks y or y attacks z in $\mathcal{G}'$, where in the latter case, by the admissibility of $GE(\mathcal{G}')$ there must be another element of $GE(\mathcal{G}')$ attacking y in $\mathcal{G}'$. Summing up, when $x \notin S$ in any case all its defeaters in $\mathcal{G}'$ are in turn attacked in $\mathcal{G}'$ by $GE(\mathcal{G}')$: since the latter is a complete extension it must be the case that $x \in GE(\mathcal{G}')$.

Turning to the second part of the proof and letting $S \in \mathcal{US}(\mathcal{G})$ and $T \in \mathcal{E}_{GR^*}(\mathcal{G}{\downarrow_S}) = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}{\downarrow_S}, GR))$, we have to show that $\exists E \in \mathcal{E}_{GR^*}(\mathcal{G}) = \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, GR))$ such that $E \cap S = T$. First consider the set $\mathcal{H} = \{E \in \mathcal{UR}(\mathcal{G}, GR) \mid E \cap S = T\}$. We know that $\mathcal{H} \neq \emptyset$. In fact $T = GE(\mathcal{G}^{\sim})$ for some $\mathcal{G}^{\sim} \in \mathcal{FRAF}(\mathcal{G}{\downarrow_S})$. By Lemma 2, $\exists \mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})$: $\mathcal{G}'{\downarrow_S} = \mathcal{G}^{\sim}$. Then $T = GE(\mathcal{G}'{\downarrow_S})$ and, by directionality of GR, $GE(\mathcal{G}') \cap S = T$, where $GE(\mathcal{G}') \in \mathcal{UR}(\mathcal{G}, GR)$. Therefore $GE(\mathcal{G}') \in \mathcal{H}$. Now, we have to show that $\mathcal{H} \cap \mathcal{MIN}(\mathcal{UR}(\mathcal{G}, GR)) \neq \emptyset$, namely that at least an element of $\mathcal{H}$ is also a minimal element of $\mathcal{UR}(\mathcal{G}, GR)$. Suppose by contradiction that this is not the case. Since $\mathcal{H} \subseteq \mathcal{UR}(\mathcal{G}, GR)$ this entails that $\forall E \in \mathcal{H}$ $\exists E^* \in (\mathcal{UR}(\mathcal{G}, GR) \setminus \mathcal{H})$ such that $E^* \subsetneq E$. Now consider $E^* \cap S$: since $E^* \subsetneq E$, it must be the case that $(E^* \cap S) \subseteq (E \cap S) = T$, but since $E^* \in (\mathcal{UR}(\mathcal{G}, GR) \setminus \mathcal{H})$ it cannot be the case that $(E^* \cap S) = T$, therefore $(E^* \cap S) \subsetneq T$. Summing up, $\exists E^* \in \mathcal{UR}(\mathcal{G}, GR) \mid (E^* \cap S) \subsetneq T$. Since $E^* \in \mathcal{UR}(\mathcal{G}, GR)$ there exists $\mathcal{G}'' \in \mathcal{FRAF}(\mathcal{G}) \mid E^* = GE(\mathcal{G}'')$. Since $S \in \mathcal{US}(\mathcal{G})$ it also holds that $S \in \mathcal{US}(\mathcal{G}'')$ and, by directionality of GR, $GE(\mathcal{G}'') \cap S = GE(\mathcal{G}''{\downarrow_S})$. By Lemma 2, $\mathcal{G}''{\downarrow_S} \in \mathcal{FRAF}(\mathcal{G}{\downarrow_S})$, and therefore $GE(\mathcal{G}'') \cap S \in \mathcal{UR}(\mathcal{G}{\downarrow_S}, GR)$ but $GE(\mathcal{G}'') \cap S = E^* \cap S \subsetneq T$ and this contradicts the hypothesis that $T \in \mathcal{MIN}(\mathcal{UR}(\mathcal{G}{\downarrow_S}, GR))$. □

6. Computational properties of *GR**

While GR^* turns out satisfactory from a principle-based point of view, its definition may seem, at a first glance, prohibitive from a computational perspective since a straightforward implementation would involve the enumeration of the (possibly) exponentially many full resolutions of an argumentation framework. Actually this explicit enumeration is not necessary to solve several of the standard computational problems for argumentation semantics, which indeed turn out to be tractable in the case of GR^*. In particular we will analyze computational properties of GR^* with respect to the problems listed below (see [26] for a comprehensive treatment of computational complexity issues in abstract argumentation):

(a) Given $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $S \subseteq \mathcal{A}$ decide if S is an extension of GR^*. Formally this relates to the decision problem $\textsc{ver}_{GR^*}$ with instances $\langle \mathcal{G}, S \rangle$ $(S \subseteq \mathcal{A})$ accepted if and only if $S \in \mathcal{E}_{GR^*}(\mathcal{G})$.
(b) Given $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ determine whether its resolution-based grounded extensions are exactly the grounded extension of $\mathcal{G}$. Formally this relates to the decision problem $\textsc{coin}_{GR,GR^*}$ with instances $\mathcal{G}$ accepted if and only if $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$.
(c) Given $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ determine whether there is at least one non-empty resolution-based grounded extension of $\mathcal{G}$. Formally this relates to the decision problem $\textsc{ne}_{GR^*}$ with instances $\mathcal{G}$ accepted if and only if $\mathcal{E}_{GR^*}(\mathcal{G}) \neq \{\emptyset\}$.
(d) Given $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x \in \mathcal{A}$ determine whether x is *credulously* accepted with respect to $\mathcal{E}_{GR^*}(\mathcal{G})$. Formally this relates to the decision problem $\textsc{ca}_{GR^*}$ with instances $\langle \mathcal{G}, x \rangle$ accepted if and only if $\exists S \in \mathcal{E}_{GR^*}(\mathcal{G})$ s.t. $x \in S$.
(e) Given $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x \in \mathcal{A}$ determine whether x is *skeptically* accepted with respect to $\mathcal{E}_{GR^*}(\mathcal{G})$. Formally this relates to the decision problem $\textsc{sa}_{GR^*}$ with instances $\langle \mathcal{G}, x \rangle$ accepted if and only if $\forall S \in \mathcal{E}_{GR^*}(\mathcal{G})$ $x \in S$.

We show that the problems (a), (b) and (c) admit polynomial time decision processes, while (d) is NP-complete and (e) coNP-complete.

6.1. Polynomial time decidable problems

We need several preliminary lemmata concerning properties of GR. First, any argument attacked by the grounded extension receives at least one non-mutual attack from it.

Lemma 3. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and its grounded extension* $GE(\mathcal{G})$, $\forall x \in (GE(\mathcal{G}))^+$ $\exists y \in GE(\mathcal{G}) : y \to x \wedge x \not\to y$.

Proof. The proof is based on the equality $GE(\mathcal{G}) = \bigcup_{i=1}^{\infty} \mathcal{F}^i_{\mathcal{G}}(\emptyset)$ (Proposition 1) and proceeds by induction on i, showing that the thesis holds for any $x \in (\mathcal{F}^i_{\mathcal{G}}(\emptyset))^+$. As to the basis step, note that the thesis holds trivially for $x \in (\mathcal{F}^1_{\mathcal{G}}(\emptyset))^+$ since $\forall y \in \mathcal{F}^1_{\mathcal{G}}(\emptyset)$ $\{y\}^- = \emptyset$. As to the inductive step, consider $x \in (\mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset))^+$ with $i \geqslant 1$. Then $\exists y \in \mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset) : y \to x$. If $x \not\to y$ the thesis obviously holds, otherwise since $y \in \mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset)$ it holds that $x \in (\mathcal{F}^i_{\mathcal{G}}(\emptyset))^+$ and, by the inductive hypothesis, $\exists y' \in \mathcal{F}^i_{\mathcal{G}}(\emptyset) : y' \to x \wedge x \not\to y'$. □

The following quite technical lemma states that if $GE(\mathcal{G})$ is stable in a set S then the part of $GE(\mathcal{G})$ outside S coincides with the grounded extension of the AF obtained from $\mathcal{G}$ by suppressing the arguments in S and those attacked by $GE(\mathcal{G}) \cap S$.

Lemma 4. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and a set* $S \subseteq \mathcal{A}$ *such that* $st_{\mathcal{G}}(GE(\mathcal{G}), S)$, *it holds that* $GE(\mathcal{G}) \cap S^C = GE(\mathcal{G}\downarrow_{S^C \setminus (GE(\mathcal{G}) \cap S)^+})$.

Proof. First note that the conclusion easily follows if $S^C = \emptyset$, therefore in the following we assume $S^C \neq \emptyset$. To shorten notation let $\bar{S} \triangleq S^C \setminus (GE(\mathcal{G}) \cap S)^+$. We will exploit again the property $GE(\mathcal{G}) = \bigcup_{i=1}^{\infty} \mathcal{F}^i_{\mathcal{G}}(\emptyset)$. Let us first show by induction on i that for any i $(\mathcal{F}^i_{\mathcal{G}}(\emptyset) \cap S^C) \subseteq GE(\mathcal{G}\downarrow_{\bar{S}})$. As to the base case, for any argument x in $(\mathcal{F}^1_{\mathcal{G}}(\emptyset) \cap S^C)$ it holds that $\{x\}^- = \emptyset$ in $\mathcal{G}$ and hence in any restriction of $\mathcal{G}$ including x. It follows that $x \in \bar{S}$ and $x \in GE(\mathcal{G}\downarrow_{\bar{S}})$. Let us now assume inductively that $(\mathcal{F}^i_{\mathcal{G}}(\emptyset) \cap S^C) \subseteq GE(\mathcal{G}\downarrow_{\bar{S}})$ and show that $(\mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset) \cap S^C) \subseteq GE(\mathcal{G}\downarrow_{\bar{S}})$. Letting $x \in (\mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset) \cap S^C)$, we show that x is acceptable with respect to $GE(\mathcal{G}\downarrow_{\bar{S}})$ in $\mathcal{G}\downarrow_{\bar{S}}$, namely $\forall y \in (\{x\}^- \cap \bar{S})$ $y \in (GE(\mathcal{G}\downarrow_{\bar{S}}))^+$. In fact, since $x \in \mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset)$, it holds that $y \in (\mathcal{F}^i_{\mathcal{G}}(\emptyset))^+$. Now, $\mathcal{F}^i_{\mathcal{G}}(\emptyset) \subseteq GE(\mathcal{G})$ and since $y \notin (GE(\mathcal{G}) \cap S)^+$ it follows that $y \in (\mathcal{F}^i_{\mathcal{G}}(\emptyset) \cap S^C)^+$, which, by the inductive hypothesis, entails $y \in (GE(\mathcal{G}\downarrow_{\bar{S}}))^+$. It follows that $x \in GE(\mathcal{G}\downarrow_{\bar{S}})$ given the fact that the grounded extension satisfies the reinstatement principle.

We have now to prove that $GE(\mathcal{G}\downarrow_{\bar{S}}) \subseteq (GE(\mathcal{G}) \cap S^C)$, by showing (again by induction on i) that for any i $\mathcal{F}^i_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset) \subseteq (GE(\mathcal{G}) \cap S^C)$, which, given the definition of $\bar{S}$, is obviously equivalent to $\mathcal{F}^i_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset) \subseteq GE(\mathcal{G})$. Preliminarily, we show that $\forall x \in \bar{S}$, $\forall y \in \{x\}^-$ such that $y \notin \bar{S}$, $y \in (GE(\mathcal{G}))^+$. In fact $y \notin \bar{S}$ implies $y \in (S^C)^C \cup (S^C \cap (GE(\mathcal{G}) \cap S)^+) = S \cup (S^C \cap (GE(\mathcal{G}) \cap S)^+)$. We have two possible cases. If $y \in S$ then, in particular, $y \in (S \setminus GE(\mathcal{G}))$ since $x \notin (GE(\mathcal{G}) \cap S)^+$. Then, since $st_{\mathcal{G}}(GE(\mathcal{G}), S)$ it follows $y \in (GE(\mathcal{G}))^+$. If otherwise $y \in (S^C \cap (GE(\mathcal{G}) \cap S)^+)$ it follows in particular $y \in (GE(\mathcal{G}) \cap S)^+$ hence $y \in (GE(\mathcal{G}))^+$. Turning to the inductive proof, consider for the base case any $x \in \mathcal{F}^1_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset)$: we have that for any $y \in \{x\}^-$ in $\mathcal{G}$ it must be the case that $y \notin \bar{S}$, which, as shown above, entails $y \in (GE(\mathcal{G}))^+$. It follows that x is acceptable with respect to $GE(\mathcal{G})$, hence $x \in GE(\mathcal{G})$. Now assume inductively that $\mathcal{F}^i_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset) \subseteq GE(\mathcal{G})$ and consider any $x \in \mathcal{F}^{i+1}_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset)$. For any $y \in \{x\}^-$ in $\mathcal{G}$ we can consider two cases: if $y \in \bar{S}$ it must be the case that $y \in (\mathcal{F}^i_{\mathcal{G}\downarrow_{\bar{S}}}(\emptyset))^+$, which, by the inductive hypothesis, entails $y \in (GE(\mathcal{G}))^+$. Otherwise, $y \notin \bar{S}$ which again entails $y \in (GE(\mathcal{G}))^+$, as shown above. Summing up, it turns out that x is acceptable with respect to $GE(\mathcal{G})$, hence $x \in GE(\mathcal{G})$, which completes the proof. $\square$

An equality involving the operations of restriction and resolution of an argumentation framework will also be useful.

Lemma 5. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and a set* $S \subseteq \mathcal{A}$, *for any resolution* β *of* $\mathcal{G}$ *it holds that* $\mathcal{G}_\beta\downarrow_S = (\mathcal{G}\downarrow_S)_\beta$.

Proof. $\mathcal{G}_\beta\downarrow_S = (\mathcal{A}, \mathcal{R} \setminus \beta)\downarrow_S = (\mathcal{A} \cap S, (\mathcal{R} \setminus \beta) \cap (S \times S)) = (\mathcal{A} \cap S, (\mathcal{R} \cap (S \times S)) \setminus \beta) = (\mathcal{G}\downarrow_S)_\beta$. $\square$

We can now show that the grounded extension of a "resolved" AF $\mathcal{G}_\beta$ can be "decomposed" into the grounded extension of the original AF $\mathcal{G}$ and the grounded extension of the AF resulting from applying the same resolution to $\text{CUT}(\mathcal{G})$ (see Definition 8).

Lemma 6. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, *for any resolution* β *of* $\mathcal{G}$ *it holds that* $GE(\mathcal{G}_\beta) = GE(\mathcal{G}) \cup GE(\text{CUT}(\mathcal{G})_\beta)$.

Proof. The fact that for any resolution β $GE(\mathcal{G}) \subseteq GE(\mathcal{G}_\beta)$ is proved in [6] in relation with the property of resolution adequacy of GR. Given that $GE(\mathcal{G}_\beta)$ is conflict-free, it follows that

$$GE(\mathcal{G}_\beta) \cap \rho\big(GE(\mathcal{G})\big) = GE(\mathcal{G}) \tag{5}$$

In the particular case where $\rho(GE(\mathcal{G})) = \mathcal{A}$ condition (5) reduces to $GE(\mathcal{G}_\beta) = GE(\mathcal{G})$ which is equivalent to the desired conclusion since in this case $\text{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset \rangle$. Therefore we can assume in the following $\text{CUT}(\mathcal{G}) \neq \langle \emptyset, \emptyset \rangle$. Clearly $GE(\mathcal{G})$ is stable in $\rho(GE(\mathcal{G}))$ with respect to $\mathcal{G}$, i.e. $st_{\mathcal{G}}(GE(\mathcal{G}), \rho(GE(\mathcal{G})))$. By Lemma 3 any argument attacked by $GE(\mathcal{G})$ receives at least one non-mutual attack from $GE(\mathcal{G})$. Since non-mutual attacks are obviously preserved when applying β, it follows that $st_{\mathcal{G}_\beta}(GE(\mathcal{G}), \rho(GE(\mathcal{G})))$, which by (5) is equivalent to $st_{\mathcal{G}_\beta}(GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G})), \rho(GE(\mathcal{G})))$, and by definition this holds if and only if $st_{\mathcal{G}_\beta}(GE(\mathcal{G}_\beta), \rho(GE(\mathcal{G})))$. Then Lemma 4 can be applied, yielding $GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G}))^C = GE(\mathcal{G}_\beta\downarrow_{\rho(GE(\mathcal{G}))^C \setminus (GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G})))^+})$. Considering (5) in the second member, it is easy to see that $\rho(GE(\mathcal{G}))^C \setminus (GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G})))^+ = \rho(GE(\mathcal{G}))^C$, yielding $GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G}))^C = GE(\mathcal{G}_\beta\downarrow_{\rho(GE(\mathcal{G}))^C})$, which by Lemma 5 is equivalent to $GE(\mathcal{G}_\beta) \cap \rho(GE(\mathcal{G}))^C = GE(\text{CUT}(\mathcal{G})_\beta)$. Putting together (5) and this (disjoint) piece we obtain $GE(\mathcal{G}_\beta) = GE(\mathcal{G}) \cup GE(\text{CUT}(\mathcal{G})_\beta)$. $\square$

The decomposition identified in Lemma 6 can then be applied to the extensions of GR^*.

Corollary 1. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$, $\mathcal{E}_{GR^*}(\mathcal{G}) = \{GE(\mathcal{G}) \cup S \mid S \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))\}$.

Proof. Using definitions and exploiting Lemma 6 at the second equality we have: $\mathcal{E}_{GR^*}(\mathcal{G}) = \mathcal{MIN}(\{GE(\mathcal{G}_\beta) \mid \beta \in \mathcal{FR}(\mathcal{G})\}) = \mathcal{MIN}(\{GE(\mathcal{G}) \cup GE(\text{CUT}(\mathcal{G})_\beta), \beta \in \mathcal{FR}(\mathcal{G})\}) = \mathcal{MIN}(\{GE(\mathcal{G}) \cup GE(\text{CUT}(\mathcal{G})_\beta), \beta \in \mathcal{FR}(\text{CUT}(\mathcal{G}))\}) = \{GE(\mathcal{G}) \cup S\colon S \in \mathcal{MIN}(\{GE(\text{CUT}(\mathcal{G})_\beta), \beta \in \mathcal{FR}(\text{CUT}(\mathcal{G}))\})\} = \{GE(\mathcal{G}) \cup S \mid S \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))\}$. □

Note in particular that in the case where $\text{CUT}(\mathcal{G})$ is empty $\mathcal{E}_{GR^*}(\mathcal{G}) = \{GE(\mathcal{G})\}$ since $\mathcal{E}_{GR^*}(\langle\emptyset,\emptyset\rangle) = \{\emptyset\}$ by definition.

Corollary 2. *For any* AF $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$, $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$ *if and only if* $\text{CUT}(\mathcal{G}) = \langle\emptyset,\emptyset\rangle$ *or there is a full resolution* β *of* $\text{CUT}(\mathcal{G})$ *such that in* $\text{CUT}(\mathcal{G})_\beta$ *every argument has at least one attacker.*

Proof. From Corollary 1 we have $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G}) \Leftrightarrow \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G})) = \{\emptyset\}$, i.e. if and only if either $\text{CUT}(\mathcal{G}) = \langle\emptyset,\emptyset\rangle$ or there is a full resolution β of $\text{CUT}(\mathcal{G})$ such that $GE(\text{CUT}(\mathcal{G})_\beta) = \emptyset$. This entails the conclusion by recalling that $GE(\mathcal{G}) = \emptyset$ if and only if $\mathcal{F}^1_{\mathcal{G}}(\emptyset) = \emptyset$, i.e. if and only if $\forall x \in \mathcal{A}\ \{x\}^- \neq \emptyset$. □

Corollary 2 provides a condition for COIN_{GR,GR^*} involving the existence of unattacked arguments in all resolutions of $\text{CUT}(\mathcal{G})$. While checking the existence of unattacked arguments is easy, using this condition would impose considering the whole set $\mathcal{FR}(\text{CUT}(\mathcal{G}))$, whose enumeration would give rise to a combinatorial explosion. Next, we will first derive a simpler to check condition, concerning the case of argumentation frameworks consisting of a single strongly-connected component, and then exploit this result in the general case. We recall that the *strongly-connected component* (SCC) decomposition of $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$ partitions $\mathcal{A}$ according to the equivalence classes induced by the relation $\rho(x,y)$ defined over $\mathcal{A} \times \mathcal{A}$ so that $\rho(x,y)$ holds if and only if $x = y$ or there are directed paths from x to y *and* from y to x in $\mathcal{G}$. We will denote the set of strongly-connected components of $\mathcal{G}$ as $\text{SCCS}(\mathcal{G})$. It is well known that the graph obtained by considering strongly-connected components as single nodes is acyclic. As a consequence, a partial order $\prec$ over the SCC decomposition $\{\mathcal{A}_1,\dots,\mathcal{A}_k\}$ is defined as $(\mathcal{A}_i \prec \mathcal{A}_j) \Leftrightarrow (i \neq j)$ and $\exists x \in \mathcal{A}_i$, $y \in \mathcal{A}_j$ such that there is a directed path from x to y.

To exemplify, in the case of Fig. 1 we have $\text{SCCS}(\mathcal{G}_1) = \{\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3,\mathcal{A}_4\} = \{\{a\},\{b,c,d\},\{e\},\{f\}\}$, with $\mathcal{A}_1 \prec \mathcal{A}_2 \prec \mathcal{A}_3 \prec \mathcal{A}_4$, while in the case of Fig. 2 we have $\text{SCCS}(\mathcal{G}_2) = \{\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3,\mathcal{A}_4\} = \{\{a,b,c\},\{e\},\{d\},\{f,g,h\}\}$ with $\mathcal{A}_1 \prec \mathcal{A}_3 \prec \mathcal{A}_4$ and $\mathcal{A}_2 \prec \mathcal{A}_3 \prec \mathcal{A}_4$. Considering then Figs. 3 and 4, we have $\text{SCCS}(\text{CUT}(\mathcal{G}_1)) = \{\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3\} = \{\{c,d\},\{e\},\{f\}\}$, with $\mathcal{A}_1 \prec \mathcal{A}_2 \prec \mathcal{A}_3$, and $\text{SCCS}(\text{CUT}(\mathcal{G}_2)) = \{\mathcal{A}_1,\mathcal{A}_2\} = \{\{a,b,c\},\{f,g,h\}\}$, with no precedence relation between $\mathcal{A}_1$ and $\mathcal{A}_2$.

The following lemma states that given an SCC S and an argument $x \in S$ it is possible to find a resolution β of $\mathcal{G}$ which resolves all mutual attacks involving elements of S such that in $\mathcal{G}_\beta$ all elements of S, with the only possible exception of x, receive an attack from an element of S itself.

Lemma 7. *Given an* AF $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$ *and an* SCC $S \in \text{SCCS}(\mathcal{G})$, *for any* $x \in S$ *there is a full resolution* β *of* $\mathcal{G}{\downarrow}_S$ *such that* $\forall y \in (S \setminus \{x\})$ $y \in S^+_{\mathcal{G}_\beta}$.

Proof. For a generic $x \in S$, define inductively the following sequence of sets: $L_0 = \{x\}$, $L_{i+1} = L_i^+ \setminus (\bigcup_{j=0}^{i} L_j)$ for $i \geqslant 0$. Observe that for any $y \in S$ $\exists i$: $y \in L_i$. In fact, $x \in L_0$ and for any $y \neq x$ there is a path from x to y, S being an SCC. Letting d be the minimal path length from x to y it is evident that $y \in L_d$. We can now build the full resolution β as follows: for any mutual attack involving consecutive sets in the sequence insert in β the attack coming from the set with higher index, namely for any $\{\langle y',y''\rangle,\langle y'',y'\rangle\} \subseteq \mathcal{R}$ such that $y' \in L_i$, $y'' \in L_{i+1}$ for some i, let $\langle y'',y'\rangle \in \beta$. Then, resolve arbitrarily any other mutual attack. It is evident that for any $y \neq x$ any path with minimal length from x to y within S is preserved in $\mathcal{G}_\beta$, hence $y \in S^+_{\mathcal{G}_\beta}$. □

We can now obtain the important result concerning argumentation frameworks consisting of a single SCC anticipated above.

Lemma 8. *Given a non-empty* AF $\mathcal{G} = \langle\mathcal{A},\mathcal{R}\rangle$ *such that* $|\text{SCCS}(\mathcal{G})| = 1$, *the condition* (i) *for any full resolution* β *of* $\mathcal{G}$ $\exists x$ *such that* $\{x\}^-_{\mathcal{G}_\beta} = \emptyset$ *is equivalent to the conjunction* (ii) *of the following three conditions*:

(a) $\forall x \in \mathcal{A}$, $\langle x,x\rangle \notin \mathcal{R}$;
(b) $\mathcal{R}$ *is* symmetric, *i.e.* $\langle x,y\rangle \in \mathcal{R} \Leftrightarrow \langle y,x\rangle \in \mathcal{R}$;
(c) *the* undirected *graph* $\overline{\mathcal{G}}$ *formed by replacing each* (*directed*) *pair* $\{\langle x,y\rangle,\langle y,x\rangle\}$ *with a single undirected edge* $\{x,y\}$ *is acyclic.*

Proof. We first prove that (i) implies (ii) by showing that if any of the conditions (a)–(c) is violated then (i) is violated too. If (a) does not hold we can apply Lemma 7 to x such that $\langle x,x\rangle \in \mathcal{R}$ and derive the existence of a full resolution β such that $\forall y \neq x\ \{y\}^-_{\mathcal{G}_\beta} \neq \emptyset$ while $\{x\}^-_{\mathcal{G}_\beta} \supseteq \{x\}$, thus denying (i). If either (b) or (c) is violated there is a cycle consisting of at least three distinct elements in $\mathcal{G}$. In fact, if $\mathcal{R}$ is not symmetric, for any $\langle x,y\rangle \in \mathcal{R}$ such that $\langle y,x\rangle \notin \mathcal{R}$ there must be a path from y to x involving at least another distinct argument z, while if the undirected graph contains a cycle it involves necessarily at least three elements. Now it is easy to build a (possibly empty) resolution β' resolving only the mutual attacks, if any, involving elements of the cycle and preserving the existence of such a cycle in $\mathcal{G}_{\beta'}$. Therefore $\mathcal{G}_{\beta'}$ still consists of exactly one scc. Consider now any argument x in the cycle: clearly $\{x\}^-_{\mathcal{G}_{\beta'}} \neq \emptyset$ and this condition will still hold for any argumentation framework arising from a full resolution of $\mathcal{G}_{\beta'}$. But we can apply now Lemma 7 to x and derive the existence of a full resolution β'' of $\mathcal{G}_{\beta'}$ such that $\forall y \neq x\ \{y\}^- \neq \emptyset$. Summing up we have obtained a full resolution $\beta = \beta' \cup \beta''$ of $\mathcal{G}$ such that $\forall x\ \{x\}^-_{\mathcal{G}_\beta} \neq \emptyset$.

Turning to the other direction of the proof, it is easy to see that the conclusion is verified in case $|\mathcal{A}| = 1$. Otherwise, we will now show that the conjunction of (a), (b) and (c) implies (i), by trying to build a full resolution β such that $\forall x \in \mathcal{A}\ \{x\}^- \neq \emptyset$ in $\mathcal{G}_\beta$ and showing that this is impossible. Since (c) holds, the undirected graph $\overline{\mathcal{G}}$ obtained from $\mathcal{G}$ is a tree. Let r be the tree root and for any $y \neq r$ denote as $d(y)$ the length of the unique (simple) path from r to y. Let $m = \max_{y \in \mathcal{A} \setminus \{r\}} d(y)$: for any y such that $d(y) = m$ it is clearly the case that y is directly connected in $\overline{\mathcal{G}}$ with exactly one element z such that $d(z) = m - 1$. This entails that y can only attack or be attacked by z in $\mathcal{G}$, and, by (b), actually both cases hold, i.e. $\{\langle y, z\rangle, \langle z, y\rangle\} \subseteq \mathcal{R}$. Then necessarily $\langle y, z\rangle \in \beta$, otherwise y, not being self-defeating by (a), would be unattacked in $\mathcal{G}_\beta$. This entails that for any z such that $d(z) = m - 1$, z does not receive attacks in $\mathcal{G}_\beta$ from any argument y such that $d(y) = m$. But now we can iterate the same reasoning on any argument z such that $d(z) = m - 1$ showing that there is exactly one w such that $d(w) = m - 2$, $\{\langle z, w\rangle, \langle w, z\rangle\} \subseteq \mathcal{R}$ and necessarily $\langle z, w\rangle \in \beta$. Iterating the same reasoning we reach the arguments x such that $d(x) = 1$ and $\{\langle x, r\rangle, \langle r, x\rangle\} \subseteq \mathcal{R}$. For any such argument x it must be the case that $\langle x, r\rangle \in \beta$ (otherwise x would be unattacked in $\mathcal{G}_\beta$) but then r is unattacked in $\mathcal{G}_\beta$, showing that the construction of the desired β is impossible. □

Lemma 8 has provided three simple topological conditions which, on the basis of Corollary 2, allow to check the condition $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$ (while avoiding the enumeration of full resolutions) when $|\mathrm{SCCS}(\mathcal{G})| = 1$. To extend this result to a generic $\mathcal{G}$ we need to focus our attention on the strongly-connected components which are minimal with respect to $\prec$ (i.e. do not receive attacks from other strongly-connected components) and satisfy conditions (a)–(c) of Lemma 8.

Definition 22. Given a non-empty AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R}\rangle$, $S \in \mathrm{SCCS}(\mathcal{G})$ is *minimal relevant* if S is a minimal element of $\prec$ and $\mathcal{G}{\downarrow_S}$ satisfies conditions (a)–(c) stated in Lemma 8. The set of the minimal relevant sccs of $\mathcal{G}$ is denoted as $\mathcal{MR}(\mathcal{G})$.

Let us exemplify this notion on the usual examples of Figs. 1–4. It is relatively easy to see that $\mathcal{MR}(\mathcal{G}_1) = \{\{a\}\}$, $\mathcal{MR}(\mathcal{G}_2) = \{\{e\}\}$, $\mathcal{MR}(\mathrm{CUT}(\mathcal{G}_1)) = \{\{c, d\}\}$, and $\mathcal{MR}(\mathrm{CUT}(\mathcal{G}_2)) = \{\{f, g, h\}\}$.

The following theorem achieves the desired generalization by showing that verifying the coincidence between *GR* and *GR** for an AF $\mathcal{G}$ is equivalent to checking whether $\mathrm{CUT}(\mathcal{G})$ has some minimal relevant component.

Theorem 1. *Given an* AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R}\rangle$, $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$ *if and only if* $\mathrm{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset\rangle$ *or* $\mathcal{MR}(\mathrm{CUT}(\mathcal{G})) = \emptyset$.

Proof. Suppose first $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$. By Corollary 2, either $\mathrm{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset\rangle$ or there is a full resolution β of $\mathrm{CUT}(\mathcal{G})$ such that in $\mathrm{CUT}(\mathcal{G})_\beta$ every argument has at least one attacker. Let S be any scc of $\mathrm{CUT}(\mathcal{G})$ minimal with respect to $\prec$. Clearly there is a full resolution β_S of $\mathrm{CUT}(\mathcal{G}){\downarrow_S}$ ($\beta_S = \beta \cap (S \times S)$) such that every element of S has at least one attacker in $(\mathrm{CUT}(\mathcal{G}){\downarrow_S})_{\beta_S}$ and therefore $\mathrm{CUT}(\mathcal{G}){\downarrow_S}$ does not satisfy conditions (a)–(c) of Lemma 8. It follows $\mathcal{MR}(\mathrm{CUT}(\mathcal{G})) = \emptyset$.

Turning to the other direction of the proof, by Corollary 2 if $\mathrm{CUT}(\mathcal{G})$ is empty the conclusion follows directly, otherwise it is sufficient to show that there is a full resolution β of $\mathrm{CUT}(\mathcal{G})$ such that in $\mathrm{CUT}(\mathcal{G})_\beta$ every argument has at least one attacker. To build such a β, consider first any scc of $\mathrm{CUT}(\mathcal{G})$ minimal with respect to $\prec$. Given the hypothesis $\mathcal{MR}(\mathrm{CUT}(\mathcal{G})) = \emptyset$, by Lemma 8 there is a full resolution β_S of $\mathrm{CUT}(\mathcal{G}){\downarrow_S}$ such that every element of S has at least one attacker in $(\mathrm{CUT}(\mathcal{G}){\downarrow_S})_{\beta_S}$. Turning now to the sccs of $\mathrm{CUT}(\mathcal{G})$ which are not minimal with respect to $\prec$, we can proceed following the (partial) order induced by $\prec$. In fact, for any such scc S' we can assume inductively that there is a full resolution β such that for every scc $S \prec S'$ every element of S has at least one attacker in $\mathrm{CUT}(\mathcal{G})_\beta$ and we need to show that the same holds also for S'. Note first that there must be an element x of S' which receives at least an attack from an element y of a scc S such that $S \prec S'$ and that any such attack must be non-mutual (otherwise S and S' would not be distinct sccs), hence for any resolution β $\langle y, x\rangle \notin \beta$. Now, by Lemma 7 we can define a resolution $\beta' \subseteq (S' \times S')$ such that any argument $z \neq x$ in S' has at least an attacker, while, as shown above, x has at least an attacker in any resolution. Summing up, we have shown a procedure to incrementally build a full resolution β of $\mathrm{CUT}(\mathcal{G})$ such that any element of $\mathrm{CUT}(\mathcal{G})$ has at least an attacker in $\mathrm{CUT}(\mathcal{G})_\beta$, as desired. □

Polynomial complexity results for COIN_{GR,GR^*} and NE_{GR^*} follow directly from Theorem 1.

Corollary 3. $\mathrm{COIN}_{GR,GR^*} \in \mathrm{P}$.

Proof. By Theorem 1, to check $\mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$ do the following steps: (i) compute $GE(\mathcal{G})$; (ii) compute $\mathrm{CUT}(\mathcal{G})$ and if it is empty return true; (iii) compute the scc decomposition of $\mathrm{CUT}(\mathcal{G})$; (iv) identify those sccs of $\mathrm{CUT}(\mathcal{G})$ which are minimal with respect to $\prec$; (v) on each of them check conditions (a)–(c) of Lemma 8. Each of these steps is known (or easily seen) to belong to P. □

Corollary 4. $\text{NE}_{GR^*} \in \text{P}$.

Proof. By Corollary 1, $\mathcal{E}_{GR^*}(\mathcal{G}) = \{\emptyset\} \Leftrightarrow GE(\mathcal{G}) = \emptyset \wedge \mathcal{E}_{GR}(\mathcal{G}) = \mathcal{E}_{GR^*}(\mathcal{G})$, thus NE_{GR^*} reduces to checking first NE_{GR}, which is known to belong to P [26], and then (possibly) COIN_{GR,GR^*}, which belongs to P by Corollary 3. □

We now turn to the problem VER_{GR^*}. Preliminarily, we have to identify some quite technical but useful properties of GR and GR^* in relation with minimal relevant components.

Lemma 9. *Given an argumentation framework $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\text{CUT}(\mathcal{G}) \neq \langle \emptyset, \emptyset \rangle$ and $\mathcal{MR}(\text{CUT}(\mathcal{G})) \neq \emptyset$, letting $\Pi_\mathcal{G} = \bigcup_{V \in \mathcal{MR}(\text{CUT}(\mathcal{G}))} V$ it holds that* (i) *for any full resolution β of $\text{CUT}(\mathcal{G})$ $GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}}) \in \mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}})$ (i.e. $GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}})$ is a stable extension of $\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}}$) and* (ii) *$\mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}) = \mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$.*

Proof. Recall first that a stable extension of an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ is a conflict-free set $T \subseteq \mathcal{A}$ such that $\forall x \in (\mathcal{A} \setminus T)\ T \to x$. As for (ii), we show that $\{GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma) \mid \gamma \in \mathcal{FR}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})\} = \mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$, and, since no stable extension can be a proper subset of another one, this set turns out to be equal to $\mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$. Condition (i) will arise as an intermediate result.

To show $\{GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma) \mid \gamma \in \mathcal{FR}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})\} \subseteq \mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$ we observe first that, by the definition of minimal relevant components, $(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma$ is acyclic. In fact, it does not contain self-defeating arguments, any cycle of length 2 in $\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}$ is resolved by γ and no cycles of length > 2 can be present. It is well known [18] that in an acyclic argumentation framework the grounded extension is also a stable extension, thus $GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma)$ is a stable extension of $(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma$ ((i) can thus be proved taking into account Lemma 5 and the fact that for any $\beta \in \mathcal{FR}(\text{CUT}(\mathcal{G}))$ $\beta \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G}) \in \mathcal{FR}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$) and we can observe further that $GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma)$ is also a stable extension of $\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}$ since in $\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}$ it clearly preserves both the properties of being conflict-free and of attacking all other arguments.

To show that $\mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}) \subseteq \{GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma) \mid \gamma \in \mathcal{FR}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})\}$, for any stable extension T of $\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}$ we have to build a full resolution γ of $\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}}$ such that $T = GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma)$. To obtain such a γ, note that, by the symmetry condition of minimal relevant components, for any x in T either x is unattacked or is involved in mutual attacks with some other elements y of $\Pi_\mathcal{G}$ and we can include in γ all the pairs of the form $\langle y, x \rangle$. It turns out that any element of T is unattacked in $(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma$, and, T being a stable extension, that any element $y \notin T$ is attacked by T. If follows that $T = \mathcal{F}^1_{(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma}(\emptyset) = \mathcal{F}^i_{(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma}(\emptyset)$ for any $i \geqslant 1$ and hence $T = GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})_\gamma)$. □

The following theorem provides a characterization of the extensions of GR^* in terms of three (still quite technical) conditions.

Theorem 2. *Given an argumentation framework $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\text{CUT}(\mathcal{G}) \neq \langle \emptyset, \emptyset \rangle$ (i.e. $\rho(GE(\mathcal{G})) \subsetneq \mathcal{A}$) and $\mathcal{MR}(\text{CUT}(\mathcal{G})) \neq \emptyset$, letting $\Pi_\mathcal{G} = \bigcup_{V \in \mathcal{MR}(\text{CUT}(\mathcal{G}))} V$ and $T \triangleq U \setminus \rho(GE(\mathcal{G}))$, $U \in \mathcal{E}_{GR^*}(\mathcal{G})$ if and only if the following conditions hold:*

(a) $U \cap \rho(GE(\mathcal{G})) = GE(\mathcal{G})$;
(b) $st_{\text{CUT}(\mathcal{G})}(T, \Pi_\mathcal{G})$;
(c) $(T \cap \Pi^C_\mathcal{G}) \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+})$.

Proof. As to the first direction of the proof, assume $U \in \mathcal{E}_{GR^*}(\mathcal{G})$. Condition (a) follows directly from Corollary 1. As to condition (b), note first that (again from Corollary 1) $T \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))$ and observe that $\Pi_\mathcal{G}$ is, by definition of minimal relevant components, an externally unattacked set of $\text{CUT}(\mathcal{G})$. By directionality of GR^* it follows that $(T \cap \Pi_\mathcal{G}) \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$, and condition (b) follows from Lemma 9, which entails $(T \cap \Pi_\mathcal{G}) \in \mathcal{E}_{\mathcal{ST}}(\text{CUT}(\mathcal{G})\downarrow_{\Pi_\mathcal{G}})$.

Turning to condition (c), note first that since $T \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))$, there exists a full resolution β of $\text{CUT}(\mathcal{G})$ such that $T = GE(\text{CUT}(\mathcal{G})_\beta)$. By directionality of GR we have that $T \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}})$, while from Lemma 9 $GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}})$ is a stable extension of $\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}}$, hence $st_{\text{CUT}(\mathcal{G})_\beta}((T \cap \Pi_\mathcal{G}), \Pi_\mathcal{G})$. Since this entails by definition $st_{\text{CUT}(\mathcal{G})_\beta}(T, \Pi_\mathcal{G})$, recalling $T = GE(\text{CUT}(\mathcal{G})_\beta)$ we can then apply Lemma 4 to obtain $T \cap \Pi^C_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+}) = GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+})_\beta)$ where Lemma 5 is exploited for the second equality. Letting $\beta^* = \beta \cap ((\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+) \times (\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+))$ it turns out that $T \cap \Pi^C_\mathcal{G} = GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+})_{\beta^*})$, β^* being a full resolution of $\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+}$, and to prove condition (c) we need to show that there is no full resolution γ of $\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+}$ such that $V \triangleq GE((\text{CUT}(\mathcal{G})\downarrow_{\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+})_\gamma) \subsetneq (T \cap \Pi^C_\mathcal{G})$. We will show that assuming that such a full resolution γ exists leads to a contradiction. Let us start by observing that since $\Pi_\mathcal{G} \in \mathcal{US}(\text{CUT}(\mathcal{G}))$, it holds also that $\Pi_\mathcal{G} \in \mathcal{US}(\text{CUT}(\mathcal{G})_\beta)$ and then, by directionality of GR, $T \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_\beta \downarrow_{\Pi_\mathcal{G}})$. Consider now a full resolution β' of $\text{CUT}(\mathcal{G})$ such that $\beta' \supseteq (\beta \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G})) \cup \gamma$: β' can be easily constructed since $(\beta \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G}))$ resolves all mutual attacks within $\Pi_\mathcal{G}$, γ resolves all mutual attacks within $\Pi^C_\mathcal{G} \setminus (T \cap \Pi_\mathcal{G})^+$, and any other mutual attack can be resolved arbitrarily without affecting the line of the proof. Since $\beta' \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G}) = \beta \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G})$, we have then $T \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_{\beta'} \downarrow_{\Pi_\mathcal{G}})$ and since $\Pi_\mathcal{G} \in \mathcal{US}(\text{CUT}(\mathcal{G})_{\beta'})$, by directionality of GR we have $GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}}) = GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G}$, obtaining (i) $T \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G}$. Moreover, by definition of β', $V = GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})_{\beta'})$, which is equal to $GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})$ by Lemma 5. Thus, we obtain (ii) $V = GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})$. We now apply the directionality of GR to $\Pi_\mathcal{G}$, obtaining $GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}})$. By Lemma 9 $GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G}$ is a stable extension of $\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}}$. From Lemma 4 we then have $GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G}^C = GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}^C \setminus (GE(\text{CUT}(\mathcal{G})_{\beta'})\cap\Pi_\mathcal{G})^+})$, which by (i) is equal to $GE(\text{CUT}(\mathcal{G})_{\beta'}{\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})$. Taking into account (ii) we then obtain $V = GE(\text{CUT}(\mathcal{G})_{\beta'}) \cap \Pi_\mathcal{G}^C$. Summing up the last equality and (i) we have $V \cup (T \cap \Pi_\mathcal{G}) = GE(\text{CUT}(\mathcal{G})_{\beta'})$. Now $V \subseteq \Pi_\mathcal{G}^C$ and, by the absurd hypothesis, $V \subsetneq (T \cap \Pi_\mathcal{G}^C)$, therefore it must be the case that $(V \cup (T \cap \Pi_\mathcal{G})) \subsetneq T$. But then we have $GE(\text{CUT}(\mathcal{G})_{\beta'}) \subsetneq T$, which contradicts the hypothesis $T \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))$.

We can now turn to the other direction of the proof, by assuming that conditions (a)–(c) hold. By Corollary 1 and condition (a), to prove the thesis $U \in \mathcal{E}_{GR^*}(\mathcal{G})$ it is sufficient to show that $T \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}))$. To this aim, we first build a full resolution β of $\text{CUT}(\mathcal{G})$ such that $T = GE(\text{CUT}(\mathcal{G})_\beta)$, then we prove that $\nexists \gamma \in \mathcal{FR}(\text{CUT}(\mathcal{G}))$ such that $GE(\text{CUT}(\mathcal{G})_\gamma) \subsetneq T$.

Considering the first part, from condition (b) and Lemma 9 it follows that $(T \cap \Pi_\mathcal{G}) \in \mathcal{E}_{GR^*}(\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}})$ which implies that (i) $\exists \beta'$ full resolution of $\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}}$ such that $(T \cap \Pi_\mathcal{G}) = GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}})_{\beta'})$ and (ii) $\nexists \beta'_*$ full resolution of $\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}}$ such that $GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}})_{\beta'_*}) \subsetneq (T \cap \Pi_\mathcal{G})$. Letting for conciseness $\underline{\mathcal{G}} = \text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+}$, from condition (c) we have (iii) $\exists \beta''$ full resolution of $\underline{\mathcal{G}}$ such that $(T \cap \Pi_\mathcal{G}^C) = GE(\underline{\mathcal{G}}_{\beta''})$ and (iv) $\nexists \beta''_*$ full resolution of $\underline{\mathcal{G}}$ such that $GE(\underline{\mathcal{G}}_{\beta''_*}) \subsetneq (T \cap \Pi_\mathcal{G}^C)$. We can now build a full resolution β of $\text{CUT}(\mathcal{G})$ such that: $\beta \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G}) = \beta'$, $\beta \cap ((\Pi_\mathcal{G}^C \setminus (T \cap \Pi_\mathcal{G})^+) \times (\Pi_\mathcal{G}^C \setminus (T \cap \Pi_\mathcal{G})^+)) = \beta''$ and any mutual attack not resolved by $\beta' \cup \beta''$ is resolved arbitrarily. From (i) and (iii), applying also Lemma 5 we then obtain respectively (v) $(T \cap \Pi_\mathcal{G}) = GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}})$ and (vi) $(T \cap \Pi_\mathcal{G}^C) = GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})$. Now from Lemma 9 $GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}})$ is a stable extension of $\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}}$, and by the directionality of the grounded semantics $GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}}) = GE(\text{CUT}(\mathcal{G})_\beta) \cap \Pi_\mathcal{G}$. Therefore, it holds that $st_{\text{CUT}(\mathcal{G})_\beta}(GE(\text{CUT}(\mathcal{G})_\beta), \Pi_\mathcal{G})$, and by Lemma 4 we have $GE(\text{CUT}(\mathcal{G})_\beta) \cap \Pi_\mathcal{G}^C = GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}^C \setminus (GE(\text{CUT}(\mathcal{G})_\beta)\cap\Pi_\mathcal{G})^+})$. Using (v) and directionality of GR the latter can be expressed as $GE(\text{CUT}(\mathcal{G})_\beta{\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})$, which by (vi) is equal to $T \cap \Pi_\mathcal{G}^C$. From $(T \cap \Pi_\mathcal{G}^C) = GE(\text{CUT}(\mathcal{G})_\beta) \cap \Pi_\mathcal{G}^C$ (just obtained) and $(T \cap \Pi_\mathcal{G}) = GE(\text{CUT}(\mathcal{G})_\beta) \cap \Pi_\mathcal{G}$ (obtained from (v) by the directionality of GR) we finally obtain $T = GE(\text{CUT}(\mathcal{G})_\beta)$.

Let us turn now to the second part: we have to show that $\nexists \gamma \in \mathcal{FR}(\text{CUT}(\mathcal{G}))$ such that $GE(\text{CUT}(\mathcal{G})_\gamma) \subsetneq T$. Suppose by contradiction that such a full resolution γ exists. From directionality of GR and Lemma 5 we have (vii) $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G} = GE(\text{CUT}(\mathcal{G})_\gamma{\downarrow}_{\Pi_\mathcal{G}}) = GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}})_\gamma)$. By Lemma 9 and the directionality of GR it holds that $GE(\text{CUT}(\mathcal{G})_\gamma)$ is stable in $\Pi_\mathcal{G}$, hence by Lemma 4 we obtain (viii) $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G}^C = GE(\text{CUT}(\mathcal{G})_\gamma{\downarrow}_{\Pi_\mathcal{G}^C \setminus (GE(\text{CUT}(\mathcal{G})_\gamma)\cap\Pi_\mathcal{G})^+}) = GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}^C \setminus (GE(\text{CUT}(\mathcal{G})_\gamma)\cap\Pi_\mathcal{G})^+})_\gamma)$, using Lemma 5 for the second equality. Referring now to the absurd hypothesis $GE(\text{CUT}(\mathcal{G})_\gamma) \subsetneq T$ it must be the case that $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G} \subsetneq T \cap \Pi_\mathcal{G}$ or $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G}^C \subsetneq (T \cap \Pi_\mathcal{G}^C)$. We show that both are impossible. If $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G} \subsetneq T \cap \Pi_\mathcal{G}$, from (vii) we obtain $GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}})_\gamma) \subsetneq (T \cap \Pi_\mathcal{G})$ which contradicts (ii) putting $\beta'_* = \gamma \cap (\Pi_\mathcal{G} \times \Pi_\mathcal{G})$. Therefore we can assume $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G} = T \cap \Pi_\mathcal{G}$, and from (viii) we get $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G}^C = GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})_\gamma)$. Then, if $GE(\text{CUT}(\mathcal{G})_\gamma) \cap \Pi_\mathcal{G}^C \subsetneq (T \cap \Pi_\mathcal{G}^C)$ we have $GE((\text{CUT}(\mathcal{G}){\downarrow}_{\Pi_\mathcal{G}^C \setminus (T\cap\Pi_\mathcal{G})^+})_\gamma) \subsetneq (T \cap \Pi_\mathcal{G}^C)$, which analogously contradicts (iv). □

We are now in a position to prove the polynomial complexity of VER_{GR^*}.

Theorem 3. $\text{VER}_{GR^*} \in \text{P}$.

Proof. The proof refers to the recursive Algorithm 1. As to its correctness, we can distinguish several cases. If $\mathcal{G}$ is empty then we get $S = \emptyset$ at line 2 (and therefore $T = U$ at line 6), moreover the condition at line 7 is obviously satisfied: then Algorithm 1 terminates returning true if U is empty too (line 9) or returning false if U is non-empty (line 11), which is the correct behavior in either case. If $\mathcal{G}$ is not empty but $\text{CUT}(\mathcal{G})$ is empty or $\mathcal{MR}(\text{CUT}(\mathcal{G})) = \emptyset$ then we know that $\mathcal{E}_{GR^*}(\mathcal{G}) = \mathcal{E}_{GR}(\mathcal{G}) = \{GE(\mathcal{G})\}$ from Theorem 1, therefore in this case we have to check that Algorithm 1 returns true if and only if $U = GE(\mathcal{G})$. Note first that the algorithm returns false at line 4 if $U \cap \rho(GE(\mathcal{G})) \neq GE(\mathcal{G})$, otherwise the algorithm proceeds by letting T be the set difference between U and $GE(\mathcal{G})$ (line 6). By assumption the condition at line 7 is satisfied and Algorithm 1 terminates returning true if the above mentioned set difference is empty or returning false if it is not. Summing up, in the considered case Algorithm 1 returns true if and only if $U \cap \rho(GE(\mathcal{G})) = GE(\mathcal{G})$ and $U \setminus GE(\mathcal{G}) = \emptyset$ which is clearly equivalent to the desired condition $U = GE(\mathcal{G})$. In the other cases Algorithm 1 is easily seen to correspond to checking whether conditions of Theorem 2 hold: lines 3–4 for condition (a), lines 16–17 for condition (b), line 19 for condition (c).

As to complexity, first note that every step of Algorithm 1 is in P. In particular, it is well known that computing $GE(\mathcal{G})$ is in P, as it clearly is also computing $\rho(S)$. Computing the minimal relevant components of $\mathcal{G}$ is a task we have already commented to be in P in Corollary 3. Verifying whether a set is stable in another one and identifying the arguments attacked by a set is linear in the number of attack relations, while all other operations (e.g. those involved in computing $\text{CUT}(\mathcal{G})$, $\text{CUT}(\mathcal{G}){\downarrow}_{W^C \setminus (T\cap W)^+}$ and $(T \cap W^C)$) only require basic set manipulations. It remains to be seen that the recursion is well-founded and terminates after a polynomial number of calls. To this purpose it is sufficient to observe that at each recursive

Algorithm 1 Verifying that $U \in \mathcal{E}_{GR^*}(\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle)$

1: **procedure** GR^*-VER$(\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle, U)$ **returns boolean**
2: $S := GE(\mathcal{G})$
3: **if** $(U \cap \rho(S) \neq S)$ **then**
4: **return false**
5: **end if**
6: $T := U \setminus S$
7: **if** $\text{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset \rangle$ **or** $\mathcal{MR}(\text{CUT}(\mathcal{G})) = \emptyset$ **then**
8: **if** $T = \emptyset$ **then**
9: **return true**
10: **else**
11: **return false**
12: **end if**
13: **else**
14: $W := \Pi_{\mathcal{G}}$
15: **end if**
16: **if** $\neg st_{\text{CUT}(\mathcal{G})}(T, W)$ **then**
17: **return false**
18: **else**
19: **return** GR^*-VER$(\text{CUT}(\mathcal{G}){\downarrow_{W^C \setminus (T \cap W)^+}}, (T \cap W^C))$
20: **end if**
21: **end**

Algorithm 2 Producing $\mathcal{E}_{GR^*}(\mathcal{G})$

1: **procedure** GR^*-COMP-EXT$(\mathcal{G})$ **returns** $\mathcal{E} \subseteq 2^{\mathcal{A}}$
2: $S := GE(\mathcal{G})$
3: **if** $\text{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset \rangle$ **or** $\mathcal{MR}(\text{CUT}(\mathcal{G})) = \emptyset$ **then**
4: $W := \emptyset$
5: **else**
6: $W := \Pi_{\mathcal{G}}$
7: **end if**
8: **if** $W = \emptyset$ **then**
9: **return** $\{S\}$
10: **end if**
11: $\mathcal{E} := \emptyset$
12: **for** each $T \in \mathcal{MCF}(\mathcal{G}{\downarrow_W})$ **do**
13: $\mathcal{H} := GR^*$-COMP-EXT$(\text{CUT}(\mathcal{G}){\downarrow_{W^C \setminus (T \cap W)^+}})$
14: **for** each $R \in \mathcal{H}$ **do**
15: $\mathcal{E} = \mathcal{E} \cup \{S \cup T \cup R\}$
16: **end for**
17: **end for**
18: **return** $\mathcal{E}$

call an argumentation framework with a strictly lesser number of arguments is considered, since at least the elements of W (which is not empty, otherwise the condition at line 7 would be satisfied and the procedure would terminate) are suppressed. Moreover the procedure clearly terminates without further recursive calls if invoked on $\langle \emptyset, \emptyset \rangle$. It follows that the procedure terminates after a number of calls which is linear in the number of arguments. $\square$

Let us exemplify an execution of Algorithm 1 with arguments $\mathcal{G} = \mathcal{G}_1$ (Fig. 1) and $U = \{a, d, f\}$. Line 2 gives $S = \{a\}$, and it follows $U \cap \rho(S) = \{a, d, f\} \cap \{a, b\} = \{a\} = S$. Then the condition of the if statement of line 3 is false, we get $T = \{d, f\}$ at line 6, and, recalling $\text{CUT}(\mathcal{G}_1)$ shown in Fig. 3, we note that the condition of the if statement of line 7 is false too. We then obtain $W = \{\{c, d\}\}$ at line 14 and since T is stable in W the condition at line 16 is false and we enter the else branch at line 19. Here we note that $W^C = \{e, f\}$ and $(T \cap W)^+ = \{c, e\}$ yielding arguments $\langle \{f\}, \emptyset \rangle$ and $\{f\}$ for the recursive invocation of the procedure. At line 3 we have then $S = \{f\}$ and, since it follows that $\text{CUT}(\mathcal{G}) = \langle \emptyset, \emptyset \rangle$ and $T = \emptyset$ the algorithm returns true at line 9. In fact, $\{a, d, f\} \in \mathcal{E}_{GR^*}(\mathcal{G}_1)$.

Consider instead an execution of Algorithm 1 with arguments $\mathcal{G} = \mathcal{G}_2$ (Fig. 2) and $U = \{a, e, g\}$. Line 2 gives $S = \{e\}$, and, skipping easy observations, we obtain $W = \{\{f, g, h\}\}$ at line 14 and we are led to a recursive invocation with arguments $\langle \{a, b, c\}, \{(a, b), (b, a), (b, c), (c, b), (a, c), (c, a)\} \rangle$ and $\{a\}$. Now we obtain $S = \emptyset$ at line 2, $T = \{a\}$ at line 6, and since $\mathcal{MR}(\text{CUT}(\mathcal{G})) = \emptyset$ the condition of the if statement of line 7 is satisfied and the algorithm returns false at line 11. In fact, $\{a, e, g\} \notin \mathcal{E}_{GR^*}(\mathcal{G}_2)$.

6.2. A side result: an algorithm to compute $\mathcal{E}_{GR^*}(\mathcal{G})$

Algorithm 1 provides the basis to devise the non-naive Algorithm 2 to compute the set of GR^* extensions of an AF $\mathcal{G}$. Algorithm 2 relies on the property that the attack relation of the restricted argumentation framework $\mathcal{G}{\downarrow_W}$ consisting of minimal relevant components is symmetric by definition and on the well-known fact [16] that in a symmetric argumentation framework $\mathcal{G}$ stable extensions coincide with maximal conflict-free sets, namely $\mathcal{E}_{\mathcal{ST}}(\mathcal{G}) = \mathcal{MCF}(\mathcal{G})$.

Given this observation, it is easy to see that any set produced by Algorithm 2 produces **true** as return value if given in input to Algorithm 1 and conversely that any set yielding **true** in Algorithm 1 is included in the result produced by Algorithm 2.

6.3. Intractable decision problems

Turning to the credulous and skeptical acceptance problems, in contrast to the polynomial time methods identified in Section 6.1 we have the following result.

Theorem 4.

(a) $\textsc{ca}_{GR^*}$ *is* NP-*complete.*
(b) $\textsc{sa}_{GR^*}$ *is* CONP-*complete.*

Proof. For part (a), that $\textsc{ca}_{GR^*} \in \textsc{np}$ follows by observing that any instance $\langle \mathcal{G}(\mathcal{A}, \mathcal{R}), x\rangle$ can be decided by checking $\exists T \subseteq \mathcal{A}$: $(x \in T) \wedge \textsc{ver}_{GR^*}(\mathcal{G}, T)$. By virtue of Theorem 3 this yields an NP algorithm.

For NP-hardness we use a reduction (analogous to the one introduced in [17]) from 3-SAT, the problem of deciding if a propositional formula, $\varphi(Z_n)$, in conjunctive normal form (CNF) with at most three literals in each clause, has an assignment to its variables, Z_n, for which at least one literal in every clause takes the value $\top$, i.e. a *satisfying assignment*.

Given an instance $\varphi(Z_n)$ of 3-SAT with clauses $\{C_1, C_2, \ldots, C_m\}$ form the instance $\langle \mathcal{G}(\mathcal{A}_\varphi, \mathcal{R}_\varphi), \varphi\rangle$ of $\textsc{ca}_{GR^*}$ in which,

$$\begin{aligned}
\mathcal{A}_\varphi &= \{\varphi\} \cup \{C_j\colon 1 \leqslant j \leqslant m\} \cup \{z_i, \neg z_i\colon 1 \leqslant i \leqslant n\} \\
\mathcal{R}_\varphi &= \bigl\{\langle C_j, \varphi\rangle\colon 1 \leqslant j \leqslant m\bigr\} \cup \bigl\{\langle z_i, C_j\rangle\colon z_i \text{ occurs in } C_j\bigr\} \cup \bigl\{\langle \neg z_i, C_j\rangle\colon \neg z_i \text{ occurs in } C_j\bigr\} \\
&\quad \cup \bigl\{\langle z_i, \neg z_i\rangle, \langle \neg z_i, z_i\rangle\colon 1 \leqslant i \leqslant n\bigr\}
\end{aligned}$$

We claim that there is some $T \in \mathcal{E}_{GR^*}(\mathcal{G})$ for which $\varphi \in T$ if and only if there is a satisfying instantiation of $\varphi(Z_n)$.

It is easily seen that $\mathcal{E}_{GR}(\mathcal{G}) = \{\emptyset\}$. Furthermore, noting that $M_{\mathcal{G}}$ contains exactly the set of pairs $\{\langle z_i, \neg z_i\rangle, \langle \neg z_i, z_i\rangle\colon 1 \leqslant i \leqslant n\}$ *every* full resolution of these yields a distinct set in $\mathcal{E}_{GR^*}(\mathcal{G})$.

Suppose first that $\underline{\alpha} = \langle \alpha_1, \alpha_2, \ldots, \alpha_n\rangle$ describes a satisfying assignment for $\varphi(Z_n)$ and consider the full resolution $\gamma(\underline{\alpha})$ given by

$$\langle z_i, \neg z_i\rangle \in \gamma(\underline{\alpha}) \quad \Leftrightarrow \quad \alpha_i = \bot$$

The grounded extension of the AF $\mathcal{G}_{\gamma(\underline{\alpha})} = \langle \mathcal{A}_\varphi, \mathcal{R}_\varphi \setminus \gamma(\underline{\alpha})\rangle$ contains exactly the arguments $\{z_i\colon \alpha_i = \top\} \cup \{\neg z_i\colon \alpha_i = \bot\} \cup \{\varphi\}$: each of the literal arguments (i.e. the $y_i \in \{z_i, \neg z_i\}$ selected) has $\{y_i\}^- = \emptyset$. Furthermore, since $\underline{\alpha}$ satisfies φ, each clause argument C_j attacking φ must be attacked by at least one of these literal arguments. It remains only to note that the resulting subset is minimal with respect to the grounded extensions resulting from full resolutions of $\mathcal{G}$.

On the other hand suppose that $\gamma \subset M_{\mathcal{G}}$ defines a full resolution for which φ is in the grounded extension, T, of $\mathcal{G}_\gamma = \langle \mathcal{A}_\varphi, \mathcal{R}_\varphi \setminus \gamma\rangle$. From $\varphi \in T$, it follows that $C_j \notin T$ for any $1 \leqslant j \leqslant m$, and thus (at least) one literal $y_j \in \{z_i, \neg z_i\}$ among the literals defining C_j must belong to T. It follows that for each clause, $C_j = y_{j,1} \vee y_{j,2} \vee y_{j,3}$, γ must contain at least one of the attacks $\langle \neg y_{j,k}, y_{j,k}\rangle$ in order for $y_{j,k} \in T$ to hold. Now defining the instantiation $\langle \alpha_1^\gamma, \alpha_2^\gamma, \ldots, \alpha_n^\gamma\rangle$ of Z_n via $\alpha_i^\gamma = \top \Leftrightarrow \langle \neg z_i, z_i\rangle \in \gamma$ yields a satisfying assignment of $\varphi(Z_n)$ as required.

For part (b), to decide $\textsc{sa}_{GR^*}(\mathcal{G}(\mathcal{A}, \mathcal{R}), x)$ by a CONP method, simply involves verifying for every $T \subseteq \mathcal{A}$ that

$$T \in \mathcal{E}_{GR^*}(\mathcal{G}) \quad \Rightarrow \quad x \in T$$

Again, by virtue of Theorem 3 the required test (for $T \in \mathcal{E}_{GR^*}(\mathcal{G})$) can be performed in polynomial time.

For CONP-hardness, we use a similar construction to that of (a) applied to deciding *unsatisfiability* of a 3-CNF $\varphi(Z_n)$. Given $\varphi(Z_n)$, let us form the AF, $\mathcal{G}(\mathcal{A}_\varphi \cup \{\psi\}, \mathcal{R}_\varphi \cup \{\langle \varphi, \psi\rangle\})$, where $\langle \mathcal{A}_\varphi, \mathcal{R}_\varphi\rangle$ is the AF described in (a) and ψ is a new argument (whose sole attacker is φ). The instance of $\textsc{sa}_{GR^*}$ is given by $\langle \mathcal{G}, \psi\rangle$. Suppose that $\varphi(Z_n)$ is unsatisfiable. By similar arguments to those given above it follows that *every* full resolution γ is such that, T_γ (the grounded extension of $\langle \mathcal{A}_\varphi \cup \{\psi\}, \mathcal{R}_\varphi \setminus \gamma \cup \{\langle \varphi, \psi\rangle\}\rangle$) satisfies $T_\gamma \cap \{C_1, C_2, \ldots, C_m\} \neq \emptyset$ and, hence, $\psi \in T_\gamma$ as required. Similarly, if it is the case that $\textsc{sa}_{GR^*}(\mathcal{G}, \psi)$, then it must hold that $\neg\textsc{ca}_{GR^*}(\mathcal{G}, \varphi)$, and now by an identical argument to that used in (a) we deduce that $\varphi(Z_n)$ must be unsatisfiable. □

That the credulous and skeptical acceptance problems with respect to *GR** turn out to be intractable is unsurprising: all of the multiple-status extension semantics that have been proposed within Dung's frameworks are known be NP-hard (or worse) for credulous acceptance and CONP-hard (or worse) for skeptical acceptance [17,22,24]. Of course, one could object to this complexity comparison on the grounds that resolution-based semantics is defined w.r.t. the *set* of Dung-style systems generated from a *single* framework, i.e. those formed as a result of applying full resolutions of mutual attacks, rather than just a single given framework. In this regard, a "more suitable" comparative basis would be the semantics defined through different translations, $\tau(\langle\mathcal{A},\mathcal{R}\rangle)$ of $\langle\mathcal{A},\mathcal{R}\rangle$ to such sets of frameworks. A number of proposals of this nature have been made in the literature to date: for example, Amgoud and Cayrol's *preference-based* argumentation frameworks PAFS [2], Bench–Capon's *value-based* frameworks VAFS [11], and, recently, the *weighted* frameworks and *inconsistency budget* approach of Dunne et al. [25]. In the first of these, $\tau(\langle\mathcal{A},\mathcal{R}\rangle)$ is defined as the set of acyclic frameworks resulting by discarding attacks that fail under a preference ordering of the arguments involved in an attack, i.e. if (the argument) p "is preferred to" (the argument) q in the attack $\langle q,p\rangle$ then the attack $\langle q,p\rangle$ is removed from $\mathcal{R}$.[6] Defining $\mathcal{E}_{PAF}(\langle\mathcal{A},\mathcal{R}\rangle)$ as

$$\bigcup_{\mathcal{G}\in\tau(\langle\mathcal{A},\mathcal{R}\rangle)} \mathcal{E}_{GR}(\mathcal{G})$$

the associated "credulous" and "skeptical" decision questions are trivial: every $p\in\mathcal{A}$ is credulously accepted (simply prefer p to any argument that attacks it); $p\in\mathcal{A}$ is skeptically accepted if and only if $\{p\}^-=\emptyset$ (if $q\in\{p\}^-$ then p is *not* preferred to q in the attack $\langle q,p\rangle$ whereas q is preferred to each of its attackers, leading to q being credulously accepted in the resulting framework – $\{q\}^-=\emptyset$ – but with the attack $\langle q,p\rangle$ still present).

Bench–Capon's model, defines the translation $\tau(\langle\mathcal{A},\mathcal{R}\rangle)$ via (qualitative) "values" associated with arguments so that failing attacks $\langle q,p\rangle$ are rationalized in terms of "the *value* endorsed by (the argument) p is preferred to the *value* endorsed by (the argument) q".[7] Given a finite value set $\mathcal{V}$ so that each $x\in\mathcal{A}$ is associated with exactly one value, $\eta(x)$ in $\mathcal{V}$,[8] the set $\tau(\langle\mathcal{A},\mathcal{R},\mathcal{V},\eta\rangle)$ contains

$$\bigcup_{\alpha:\ \alpha \text{ is a total ordering of } \mathcal{V}} \Big\langle \mathcal{A}, \mathcal{R}\setminus\big\{\langle p,q\rangle:\ \eta(q) \text{ is preferred to } \eta(p) \text{ under } \alpha\big\}\Big\rangle$$

In [11] it is shown that every framework in $\tau(\langle\mathcal{A},\mathcal{R},\mathcal{V},\eta\rangle)$ is acyclic so that the extension semantics for VAFS are simply

$$\mathcal{E}_{VAF}\big(\langle\mathcal{A},\mathcal{R},\mathcal{V},\eta\rangle\big)=\bigcup_{\mathcal{G}\in\tau(\langle\mathcal{A},\mathcal{R},\mathcal{V},\eta\rangle)} \mathcal{E}_{GR}(\mathcal{G})$$

In contrast to PAFS, the credulous and skeptical problems – referred to as subjective and objective acceptance (SBA and OBA) – are NP-complete and CONP-complete [23,12].

Finally the *weighted* frameworks proposed in [25], augment $\langle\mathcal{A},\mathcal{R}\rangle$ using a positive real-valued weighting function $w:\mathcal{R}\to\mathbf{R}^+$, with $w(\langle p,q\rangle)$ interpreted as how "strong" the attack is. Each $\beta\in\mathbf{R}^+$ gives rise to a set of systems $\tau(\langle\mathcal{A},\mathcal{R},w\rangle,\beta)$ defined via

$$\tau\big(\langle\mathcal{A},\mathcal{R},w\rangle,\beta\big)=\bigcup_{\{T\subseteq\mathcal{R}:\ \sum_{\langle p,q\rangle\in T} w(\langle p,q\rangle)\leqslant\beta\}} \big\{\langle\mathcal{A},\mathcal{R}\setminus T\rangle\big\}$$

The structures considered in [25] are the so-called *weighted grounded extensions* with *inconsistency budget* β, i.e.

$$\mathcal{E}_{\beta\text{-}GR}\big(\langle\mathcal{A},\mathcal{R},w\rangle\big)=\bigcup_{\mathcal{G}\in\tau(\langle\mathcal{A},\mathcal{R},w\rangle,\beta)} \big\{\mathcal{E}_{GR}(\mathcal{G})\big\}$$

As with VAFS and GR^* credulous acceptance w.r.t. $\mathcal{E}_{\beta\text{-}GR}$ is NP-complete and skeptical acceptance CONP-complete [25].

In summary, credulous and skeptical acceptance with respect to $\mathcal{E}_{GR^*}$ exhibits similar complexity to multiple-status semantics in both Dung-style argumentation frameworks and the variants of these represented by VAFS and weighted systems.

7. Computational properties of *GR** in restricted frameworks

In the previous section it was seen that, in general, CA_{GR^*} and SA_{GR^*} are computationally intractable (under the standard assumption $\text{P}\neq\text{NP}$). There are, however, a number of restricted cases in which CA_s and SA_s are known to be polynomial time decidable for particular semantics s. In particular, CA_s and SA_s are polynomial time decidable if $\langle\mathcal{A},\mathcal{R}\rangle$ is acyclic or symmetric or bipartite (see Definition 23) for all of the standard multiple-status semantics that have been proposed in Dung style argument systems [18,16,20]. If, however, we consider the corresponding problems within the "augmented" systems reviewed at the conclusion of the previous subsection, i.e. VAFS and weighted frameworks, these reductions in complexity do not always result, as shown in Table 1 for credulous acceptance.

So for VAFS two of the restricted topologies do not yield efficient algorithms[9] and the remaining case (symmetric frameworks) is uninteresting: every argument is subjectively accepted and only those p for which $\{p\}^-=\emptyset$ are objectively

[6] [2] imposes the condition that preferences between arguments expressed in discarding attacks must be consistent with a preorder of $\mathcal{A}$.

[7] Focusing on values endorsed by arguments rather than the arguments themselves means that if p and q are associated with identical values then attacks between these cannot be discounted.

[8] $|\mathcal{V}|$ may be much smaller than $|\mathcal{A}|$ so the mapping is not, in general, bijective.

[9] In fact decision problems remain intractable even if $\langle\mathcal{A},\mathcal{R}\rangle$ is a *binary tree* [20].

Table 1
Complexity of credulous acceptance in restricted systems.

Formalism	Acyclic framework	Symmetric framework	Bipartite framework
VAF	NP-complete	Trivial	NP-complete
Weighted	NP-complete	NP-complete	Open

accepted.[10] Similarly, in weighted frameworks no improvement in complexity results with either acyclic or symmetric systems. While the case of general bipartite frameworks is open, there are efficient methods known if $\langle \mathcal{A}, \mathcal{R} \rangle$ is both bipartite *and* acyclic.

Obviously a question arises about the behavior of GR^* with respect to these restricted frameworks. For acyclic frameworks it is clear that $\mathcal{E}_{GR^*}(\mathcal{G}) = \mathcal{E}_{GR}(\mathcal{G})$ and thus this case is polynomial time decidable for both $\textsc{ca}_{GR^*}$ and $\textsc{sa}_{GR^*}$.

For a symmetric framework $\mathcal{G}$, one may first note that any strongly-connected component of $\mathcal{G}$ cannot receive attacks from another one (i.e. it is minimal with respect to the $\prec$ relation). Then, exploiting the results of the previous section, we can consider the following cases:

- arguments not receiving any attack: they are skeptically (and hence credulously) accepted;
- arguments in strongly-connected components which are not minimal relevant: they are not included in any extension of GR^*;
- arguments receiving attacks in strongly-connected components which are minimal relevant: they are credulously but not skeptically accepted.

It follows that both $\textsc{ca}_{GR^*}$ and $\textsc{sa}_{GR^*}$ are polynomial time decidable for symmetric frameworks.

In the following we focus on the case of bipartite frameworks and show that $\textsc{sa}_{GR^*}$ is polynomial time decidable also in this case.

Definition 23. An AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ is *bipartite* if $\exists \mathcal{X}, \mathcal{Y} \subseteq \mathcal{A}$ such that $\mathcal{A} = \mathcal{X} \cup \mathcal{Y}$, $\mathcal{X} \cap \mathcal{Y} = \emptyset$ and both $\mathcal{X}$ and $\mathcal{Y}$ are conflict-free.

Sometimes a bipartite AF will be indicated as $\langle \mathcal{X}, \mathcal{Y}, \mathcal{R} \rangle$, denoting the argumentation framework $\mathcal{G} = \langle \mathcal{X} \cup \mathcal{Y}, \mathcal{R} \rangle$ where $\mathcal{X}$ and $\mathcal{Y}$ satisfy the previous definition.

A sequence of preliminary lemmata is needed.

Lemma 10. *Let $\langle \mathcal{X}, \mathcal{Y}, \mathcal{R} \rangle$ be a bipartite AF and let x be an argument such that $x \in \mathcal{X}$. Given an argument z, if there is an even length path between x and z then $z \in \mathcal{X}$, if there is an odd length path between x and z then $z \in \mathcal{Y}$.*

Proof. Since $\mathcal{X}$ is conflict-free, any argument a_1 which attacks x or is attacked by it belongs to $\mathcal{Y}$. In the same way, any argument a_2 which attacks a_1 or is attacked by it belongs to $\mathcal{X}$. Iterating the same step of reasoning, it is easy to see that any a_i with i even belongs to $\mathcal{X}$, any a_i with i odd belongs to $\mathcal{Y}$. $\square$

Lemma 11. *Given an AF $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x \in GE(\mathcal{G})$, either x is initial or there is an even length path $u \to a_0 \to a_1 \to \cdots \to a_{2i} \to x$ with u initial.*

Proof. The proof is based on the equality $GE(\mathcal{G}) = \bigcup_{i=1}^{\infty} \mathcal{F}^i_{\mathcal{G}}(\emptyset)$ (Proposition 1) and proceeds by induction on i, showing that the thesis holds for all arguments of $\mathcal{F}^i_{\mathcal{G}}(\emptyset)$. As to the basis step, it is easy to see that all arguments of $\mathcal{F}^1_{\mathcal{G}}(\emptyset)$ are initial. As to the inductive step, any argument $x \in \mathcal{F}^{i+1}_{\mathcal{G}}(\emptyset)$ either is initial or $\exists y : y \to x$. In the latter case it holds that $\mathcal{F}^i_{\mathcal{G}}(\emptyset) \to y$, and the thesis follows from the inductive hypothesis on the arguments of $\mathcal{F}^i_{\mathcal{G}}(\emptyset)$. $\square$

Lemma 12. *Given an argumentation framework $\mathcal{G} = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\textsc{cut}(\mathcal{G}) \neq \langle \emptyset, \emptyset \rangle$, there are no initial arguments in $\textsc{cut}(\mathcal{G})$.*

Proof. We reason by contradiction. If an argument v is initial in $\textsc{cut}(\mathcal{G})$, then $\forall y : y \to v \ \ y \in \rho(GE(\mathcal{G}))$, and in particular $y \in GE(\mathcal{G})^+$ otherwise v would belong to $\rho(GE(\mathcal{G}))$ and would not be an argument of $\textsc{cut}(\mathcal{G})$. Therefore, v is acceptable with respect to $GE(\mathcal{G})$ entailing $v \in GE(\mathcal{G})$, which contradicts the fact that v is an argument of $\textsc{cut}(\mathcal{G})$. $\square$

Lemma 13. *Given a bipartite AF $\mathcal{G} = \langle \mathcal{X}, \mathcal{Y}, \mathcal{R} \rangle$ such that $\textsc{cut}(\mathcal{G}) \neq \langle \emptyset, \emptyset \rangle$ and $\mathcal{MR}(\textsc{cut}(\mathcal{G})) \neq \emptyset$, letting $\Pi_{\mathcal{G}} = \bigcup_{V \in \mathcal{MR}(\textsc{cut}(\mathcal{G}))} V$ it holds that $\exists T_1, T_2 \in \mathcal{E}_{\mathcal{ST}}(\mathcal{G}{\downarrow}_{\Pi_{\mathcal{G}}})$ such that $T_1 \cap \Pi_{\mathcal{G}} = \Pi_{\mathcal{G}} \cap \mathcal{X}$ and $T_2 \cap \Pi_{\mathcal{G}} = \Pi_{\mathcal{G}} \cap \mathcal{Y}$.*

Proof. We prove the claim by showing how to construct T_1 and T_2. According to the definition of minimal relevant components, for any $V \in \mathcal{MR}(\textsc{cut}(\mathcal{G}))$ replacing the direct attacks with undirected edges gives rise to a tree. We can then

[10] The notions of subjective and objective acceptance are specific to VAFs.

select an argument r in $V \cap \mathcal{X}$ as the root: by Lemma 12 such an argument exists, since $|V| \geqslant 2$ and any two conflicting arguments belong to $\mathcal{X}$ and $\mathcal{Y}$ respectively. We include r into T_1, then we consider all of its children in the rooted tree, i.e. the nodes with depth 1: since $\mathcal{X}$ is conflict-free, all of them belong to $\mathcal{Y}$, and we include them into T_2. We can proceed in this way along the tree, including the nodes of even depth into T_1 and the nodes of odd depth into T_2. It is easy to see that each of the nodes of V is included in either T_1 or T_2. Moreover, given the symmetry of minimal relevant components all nodes of even depth are attacked by nodes of odd depth, and vice versa. Finally, notice that this process can be applied to all minimal relevant components, obtaining $T_1, T_2 \in \mathcal{E}_{\mathcal{ST}}(\mathcal{G}{\downarrow}_{\Pi_\mathcal{G}})$. □

The following proposition provides the main result for skeptical acceptance.

Proposition 6. *Given a bipartite* AF $\mathcal{G} = \langle \mathcal{X}, \mathcal{Y}, \mathcal{R}\rangle$, *an argument z is skeptically accepted if and only if $z \in GE(\mathcal{G})$.*

Proof. The fact that if $z \in GE(\mathcal{G})$ then z is skeptically accepted follows from Corollary 1, therefore we have to prove that if $z \notin GE(\mathcal{G})$ then z is not skeptically accepted. The conclusion follows from Theorem 1 in case $\textsc{cut}(\mathcal{G}) = \langle \emptyset, \emptyset\rangle$ or $\mathcal{MR}(\textsc{cut}(\mathcal{G})) = \emptyset$, therefore in the following we consider the case $\textsc{cut}(\mathcal{G}) \neq \langle \emptyset, \emptyset\rangle$ and $\mathcal{MR}(\textsc{cut}(\mathcal{G})) \neq \emptyset$. By Lemma 13, $\exists T_1, T_2 \in \mathcal{E}_{\mathcal{ST}}(\mathcal{G}{\downarrow}_{\Pi_\mathcal{G}})$ such that $T_1 \cap \Pi_\mathcal{G} = \Pi_\mathcal{G} \cap \mathcal{X}$ and $T_2 \cap \Pi_\mathcal{G} = \Pi_\mathcal{G} \cap \mathcal{Y}$. On the basis of Theorem 2 it is easy to see that, to be skeptically accepted, z must be skeptically accepted both in $\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+}$ and in $\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_2 \cap \Pi_\mathcal{G})^+}$. In fact, $z \notin GE(\mathcal{G})$ by the hypothesis and $z \notin \Pi_\mathcal{G}$ since in this case by Theorem 2 it should belong both to T_1 and to T_2 which are instead disjoint. We can then assume inductively that $z \in GE(\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+})$ and $z \in GE(\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_2 \cap \Pi_\mathcal{G})^+})$. We show that the first condition entails that there is an even-length path from an argument of T_1 to z, and analogously that the second condition entails that an even-length path exists from an argument of T_2 to z: since $T_1 \subseteq \mathcal{X}$ and $T_2 \subseteq \mathcal{Y}$, according to Lemma 10 it should be the case that $z \in \mathcal{X}$ and $z \in \mathcal{Y}$, leading to a contradiction.

On the basis of Lemma 11, the first condition $z \in GE(\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+})$ entails that there is an even-length path $u \rightarrow a_0 \rightarrow a_1 \rightarrow \cdots \rightarrow a_{2i} \rightarrow z$ with u initial in $\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+}$ (if z is initial we can assume $z = u$ and a path of length 0). Now, by Lemma 12 u is not initial in $\textsc{cut}(\mathcal{G})$, therefore $\exists y \in \rho(GE(\mathcal{G}))^C$ such that $y \rightarrow u$. If $y \in \Pi_\mathcal{G}$ then $y \notin T_1$, otherwise $u \in (T_1 \cap \Pi_\mathcal{G})^+$ while $u \in \Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+$, therefore since T_1 is stable in $\Pi_\mathcal{G}$ it must be the case that $y \in (T_1 \cap \Pi_\mathcal{G})^+$. If, on the other hand, $y \in \Pi^C_\mathcal{G}$ then $y \in (T_1 \cap \Pi_\mathcal{G})^+$ directly follows from the fact that u initial in $\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_1 \cap \Pi_\mathcal{G})^+}$. Summing up, $(T_1 \cap \Pi_\mathcal{G}) \rightarrow y \rightarrow u$, and there is an even-length path from u to z, clearly entailing the desired conclusion.

Finally, the fact that there is an even-length path from $(T_2 \cap \Pi_\mathcal{G})$ to z can be proved in the same way using the second condition $z \in GE(\textsc{cut}(\mathcal{G}){\downarrow}_{\Pi^C_\mathcal{G} \setminus (T_2 \cap \Pi_\mathcal{G})^+})$. □

Theorem 5. *Let* $\textsc{sa}^{(2)}_{GR^*}$ *denote the problem* $\textsc{sa}_{GR^*}$ *restricted to instances which describe bipartite* AFs. *Then* $\textsc{sa}^{(2)}_{GR^*}$ *is decidable in polynomial time.*

Proof. It directly follows from Proposition 6. □

We now turn to credulous acceptance. Although – as with the weighted systems of [25] – the exact status of credulous acceptance in bipartite frameworks w.r.t. *single* arguments is open, in the case of *sets* of arguments Theorem 6 suggests this variant to be intractable.

Theorem 6. *Let* $\textsc{ca}^{(2)^{\{\}}}_{GR^*}$ *denote the problem whose instances are bipartite* AFs $\langle \mathcal{X}, \mathcal{Y}, \mathcal{R}\rangle$ *together with a subset S of $\mathcal{X} \cup \mathcal{Y}$, such instances being accepted if there is some $T \in \mathcal{E}_{GR^*}(\langle \mathcal{X}, \mathcal{Y}, \mathcal{R}\rangle)$ for which $S \subseteq T$. The decision problem* $\textsc{ca}^{(2)^{\{\}}}_{GR^*}$ *is* NP-*complete even for instances with $|S| = 2$.*

Proof. Membership in NP is immediate via Theorem 3. To establish NP-hardness we use a reduction from *Monotone* 3-SAT, the restricted version of 3-SAT in which clauses are constrained to consist entirely of positive literals or entirely of negated literals, e.g. $(x \vee y \vee z)(\neg x \vee \neg y \vee \neg z)$ defines a valid instance of Monotone 3-SAT while $(\neg x \vee y \vee z)(x \vee y \vee z)$ fails to do so.

Given $\varphi(Z_n)$ an instance of Monotone 3-SAT let $C^+ = \{C^+_1, \ldots, C^+_k\}$ denote the set of clauses comprising only positive literals and $C^\neg = \{C^\neg_1, \ldots, C^\neg_r\}$ be the remaining clauses (those defined using only negated literals). Using a construction similar to Theorem 4, we define the following bipartite AF, $\langle \mathcal{X}_\varphi, \mathcal{Y}_\varphi, \mathcal{R}_\varphi\rangle$:

$$
\begin{aligned}
\mathcal{X}_\varphi &= \{\varphi^+\} \cup \{C^\neg_j\colon 1 \leqslant j \leqslant r\} \cup \{z_i\colon 1 \leqslant i \leqslant n\} \\
\mathcal{Y}_\varphi &= \{\varphi^\neg\} \cup \{C^+_j\colon 1 \leqslant j \leqslant k\} \cup \{\neg z_i\colon 1 \leqslant i \leqslant n\} \\
\mathcal{R}_\varphi &= \{\langle C^+_j, \varphi^+\rangle\colon 1 \leqslant j \leqslant k\} \cup \{\langle C^\neg_j, \varphi^\neg\rangle\colon 1 \leqslant j \leqslant r\} \cup \{\langle z_i, C^+_j\rangle\colon z_i \text{ occurs in } C^+_j\} \\
&\quad \cup \{\langle \neg z_i, C^\neg_j\rangle\colon \neg z_i \text{ occurs in } C^\neg_j\} \cup \{\langle z_i, \neg z_i\rangle, \langle \neg z_i, z_i\rangle\colon 1 \leqslant i \leqslant n\}
\end{aligned}
$$

Reasoning in an analogous way as in Theorem 4, it is easy to see that $\langle\langle \mathcal{X}_\varphi, \mathcal{Y}_\varphi, \mathcal{R}_\varphi\rangle, \{\varphi^{\neg}, \varphi^{+}\}\rangle$ is accepted as an instance of $\mathrm{CA}_{GR^*}^{(2),\{\}}$ if and only if $\varphi(Z_n)$ is satisfiable. □

On the other hand, in the case of single arguments we have a positive result for a special class of bipartite frameworks.

Definition 24. An AF $\langle \mathcal{X}, \mathcal{R}\rangle$ is *tree-like* if the *undirected* graph $\langle \mathcal{X}, \mathcal{F}\rangle$ in which $\mathcal{F} = \{\{x, y\}: \langle x, y\rangle \in \mathcal{R}$ or $\langle y, x\rangle \in \mathcal{R}\}$ is acyclic.

In case of tree-like argumentation frameworks all of the extensions prescribed by GR^* are stable, and their definition can be simplified by dropping the minimality condition.

Lemma 14. *Let $\mathcal{G} = \langle \mathcal{X}, \mathcal{R}\rangle$ be a tree-like argumentation framework. Then, $\mathcal{E}_{GR^*}(\mathcal{G}) = \{GE(\mathcal{G}_\gamma) \mid \gamma \in \mathcal{FR}(\mathcal{G})\} \subseteq \mathcal{E}_{\mathcal{ST}}(\mathcal{G})$.*

Proof. Since $\mathcal{G}$ is tree-like, $\forall \gamma \in \mathcal{FR}(\mathcal{G})$ $\mathcal{G}_\gamma$ is acyclic, and it is known from [18] that in an acyclic graph the grounded extension is also stable. Therefore $\{GE(\mathcal{G}_\gamma) \mid \gamma \in \mathcal{FR}(\mathcal{G})\} \subseteq \mathcal{E}_{\mathcal{ST}}(\mathcal{G})$, and since a stable extension cannot be a proper subset of another one all of the elements of $\{GE(\mathcal{G}_\gamma) \mid \gamma \in \mathcal{FR}(\mathcal{G})\}$ are minimal, i.e. this set is equal to $\mathcal{E}_{GR^*}(\mathcal{G})$. □

It is immediate from the following theorem that deciding credulous acceptance in the case of tree-like frameworks can be done in polynomial time.

Theorem 7. *Let $\mathcal{G} = \langle \mathcal{X}, \mathcal{R}\rangle$ be a tree-like argumentation framework with grounded extension $GE(\mathcal{G})$. For all $z \in \mathcal{X}$*

$$\neg \mathrm{CA}_{GR^*}(\mathcal{G}, z) \quad \Leftrightarrow \quad z \in GE(\mathcal{G})^{+}$$

Proof. It is obvious from Corollary 1 that the arguments of $GE(\mathcal{G})$ are skeptically (and thus credulously) accepted, while those attacked by $GE(\mathcal{G})$ are not credulously accepted. The remaining arguments to consider, namely those in $\mathrm{CUT}(\mathcal{G})$, are not attacked by $GE(\mathcal{G})$ by definition, therefore we have to prove that any argument z of $\mathrm{CUT}(\mathcal{G})$ is credulously accepted.

First, notice that $\mathrm{CUT}(\mathcal{G})$ is tree-like, therefore the undirected graph obtained as in Definition 24 is acyclic: without loss of generality, we assume that this graph is connected, i.e. is a tree (otherwise, by directionality of GR^*, it is sufficient to consider the connected subgraph z belongs to). Let us denote as T the corresponding rooted tree where z is assumed as its root: any argument of $\mathrm{CUT}(\mathcal{G})$ is then characterized by its depth, i.e. the length of the (undirected) path in T from z to it (the depth of z is 0). Notice in particular that, given two arguments y and x of $\mathrm{CUT}(\mathcal{G})$ such that either $y \to x$ or $x \to y$, if the depth of y is even then the depth of x is odd, and vice versa. To prove the desired conclusion, we consider the full resolution γ of $\mathrm{CUT}(\mathcal{G})$ where all mutual attacks between two arguments x and y are resolved in favour of the one with even depth, i.e. if x has even depth then $\langle y, x\rangle \in \gamma$, and we show that $GE(\mathcal{G}_\gamma)$ is an extension of GR^* including z. The fact that $GE(\mathcal{G}_\gamma) \in \mathcal{E}_{GR^*}(\mathcal{G})$ derives from Lemma 14, from which we also know that it is a stable extension. As to the inclusion of z, note that, since $\mathrm{CUT}(\mathcal{G})$ has no initial arguments by Lemma 12, the only initial arguments in $\mathcal{G}_\gamma$ are obtained by attack suppression, then by construction they have even depth w.r.t. T. As a consequence, by Lemma 11 all elements of $GE(\mathcal{G}_\gamma)$ have even depth w.r.t. T. Suppose by contradiction that $z \notin GE(\mathcal{G}_\gamma)$: since $GE(\mathcal{G}_\gamma)$ is stable there is $y \in GE(\mathcal{G}_\gamma)$ attacking z, but then y has odd depth (actually 1) w.r.t. T, which is impossible. □

8. Discussion and conclusions

The results provided in this paper can be assessed from several perspectives.

From a principle-oriented perspective, it has been shown that the semantics evaluation criteria introduced in [6] are not incompatible altogether since a novel semantics satisfying all of them, namely GR^*, has been introduced. This can be regarded as a sort of confirmation of their global coherence, dispelling any doubt possibly raised by the fact that none of the previous literature semantics was able to comply with all criteria.

From the perspective of investigation on argumentation semantics definition, the new resolution-based family of semantics (which is parametric with respect to the selection of a "base" argumentation semantics) represents an interesting tool for further investigation, as it has been shown that all desirable properties except directionality are directly satisfied by any resolution-based semantics, given that the base argumentation semantics satisfies very mild conditions. This has provided a solid starting point to identify a member of this family satisfying directionality too, which turned out to be the one based on the traditional grounded semantics. Devising general conditions on the base semantics to guarantee the satisfaction of the directionality property by its resolution-based version appears indeed an interesting open problem.

From a more practical perspective, it has been shown that the principle driven approach to semantics definition is satisfactory also from the viewpoint of computational complexity. In fact, we have investigated computational properties of GR^* with reference to a standard set of decision problems for abstract argumentation semantics, proving that some of them (VER_{GR^*}, COIN_{GR,GR^*}, NE_{GR^*}) are tractable while others (CA_{GR^*}, SA_{GR^*}) are in general not, but are shown to be tractable in some restricted frameworks. Leaving apart the (unique-status) grounded semantics which is known to be computable with a polynomial algorithm, it is known that the same decision problems are generally intractable for both stable and preferred semantics (see in particular [17,22,26]) with the only exception of $\text{VER}_{\mathcal{ST}} \in \text{P}$. In particular $\text{CA}_{\mathcal{PR}}$, $\text{NE}_{\mathcal{PR}}$ and $\text{CA}_{\mathcal{ST}}$ are NP-complete, $\text{VER}_{\mathcal{PR}}$ is coNP-complete, $\text{SA}_{\mathcal{PR}}$ is Π_2^p-complete, and $\text{SA}_{\mathcal{ST}}$ is D^p-complete. We can state therefore that GR^* has better complexity properties than the traditional multiple-status semantics $\mathcal{ST}$ and $\mathcal{PR}$. With the exception of ideal semantics in [21], complexity properties of recently proposed semantics, e.g. semi-stable or prudent, have not been fully analyzed yet but preliminary non-tractability results exist [26]. Actually, as to our knowledge, no other non-trivial multiple-status semantics in the literature has been shown, up to now, to admit polynomial time decision processes (in the general case) for any of the standard decision problems considered here. Taking also into account the comparisons drawn in Section 6 with extended frameworks, these results qualify GR^* as the non-trivial multiple-status semantics with best computational properties considered in the literature up to now. This suggests that, leaving apart its merits with respect to principled conceptual requirements, GR^* may turn out to be advantageous also from the viewpoint of its practical application, an issue which deserves further investigation.

Concerning relationships with other approaches, it can be noted that the definition of resolution-based semantics stands on two basic points: the resolution of mutual attacks and the minimality requirement. The idea of “resolving” or “suppressing” attacks according to some criterion has been considered in several works in the literature. As already mentioned, the notion of resolution has been introduced in [28] in the context of hierarchical argumentation, which aims at capturing argumentation over preference information to resolve indecisions corresponding to mutual attacks. Somehow similarly, preferences [2] and values [11] have been considered as additional information extending the basic Dung’s framework and giving implicitly rise to the suppression of some attacks (possibly including non-mutual ones). In a less abstract framework encompassing structured arguments, an ordering over arguments is used in an analogous way to determine which attacks are successful [31]. Other extensions of Dung’s framework encompass attacks to attacks [29,3,4], which can be regarded as an explicit form of attack suppression (again, possibly including non-mutual ones) and provide an alternative way to represent preferences, values, and any other entity affecting attacks, in a formal argumentation setting. While (mutual) attack suppression is actually present in the definition of resolution-based semantics, it must be remarked that this semantics is not conceived to be specifically applied to contexts where attack suppression (either preference-based, value-based, or of any other kind) plays a central role. In fact, due to the minimality requirement, the results produced by resolution-based semantics may be very different from those arising, for instance, from all possible ordering of values in a value-based framework. A deeper exploration of the relationships between resolution-based semantics and the above mentioned works represents an interesting direction of future research.

Finally, as a further issue for future work we mention the investigation of other significant instances of the resolution-based definition scheme and the study of skepticism and agreement properties of GR^* with respect to other literature semantics.

Appendix A. Notation summary

Notation	Comment	Reference
$\mathcal{G}(\mathcal{A},\mathcal{R})$	An argumentation framework (AF)	Def. 1, Sec. 2.1
$M_{\mathcal{G}}$	The set of mutual attacks of $\mathcal{G}$	Def. 2, Sec. 2.1
S^-	The set of arguments attacking set S	Def. 3, Sec. 2.1
S^+	The set of arguments attacked by set S	Def. 3, Sec. 2.1
$\rho(S)$	$S \cup S^+$: the range of set S	Def. 3, Sec. 2.1
$IN(\mathcal{G})$	The unattacked arguments in $\mathcal{G}$	Def. 3, Sec. 2.1
$cf(S)$	The set S is conflict-free	Def. 3, Sec. 2.1
$\mathcal{MCF}(\mathcal{G})$	The set of maximal conflict-free sets of $\mathcal{G}$	Def. 3, Sec. 2.1
S^C	The complement of set S	Def. 4, Sec. 2.1
$\mathcal{G}{\downarrow}_S$	The *restriction* of $\mathcal{G}$ to S	Def. 4, Sec. 2.1
$\mathcal{G}_\beta$	The AF arising from applying the partial resolution β to $\mathcal{G}$	Def. 5, Sec. 2.1
$\mathcal{FR}(\mathcal{G})$	The set of full resolutions of $\mathcal{G}$	Def. 5, Sec. 2.1
$\mathcal{FRAF}(\mathcal{G})$	The set of AFs arising from applying the full resolutions of $\mathcal{G}$	Def. 5, Sec. 2.1
$\mathcal{F}_{\mathcal{G}}$	The characteristic function of $\mathcal{G}$	Def. 6, Sec. 2.2
$\mathcal{E}_{\mathcal{S}}(\mathcal{G})$	The set of extensions prescribed by the semantics $\mathcal{S}$ for $\mathcal{G}$	Sec. 2.2
$\mathcal{D}_{\mathcal{S}}$	The set of AFs where $\mathcal{S}$ admits at least one extension	Sec. 2.2
$GE(\mathcal{G})$	The grounded extension of $\mathcal{G}$	Sec. 2.2
$ID(\mathcal{G})$	The ideal extension of $\mathcal{G}$	Sec. 2.2
$\text{CUT}(\mathcal{G})$	The AF obtained by suppressing $\rho(GE(\mathcal{G}))$ from $\mathcal{G}$	Def. 8, Sec. 2.2
$\mathcal{AS}(\mathcal{G})$	The set of the admissible sets of $\mathcal{G}$	Def. 12, Sec. 3.1
$\mathcal{US}(\mathcal{G})$	The set of externally unattacked sets of $\mathcal{G}$	Def. 14, Sec. 3.1
$\mathcal{AE}_{\mathcal{S}}(\mathcal{G},T)$	$\{(R \cap T) \mid R \in \mathcal{E}_{\mathcal{S}}(\mathcal{G})\}$	Def. 15, Sec., 3.1
$\mathcal{E}_1 \preccurlyeq^E_\cap \mathcal{E}_2$	$\bigcap_{E_1 \in \mathcal{E}_1} E_1 \subseteq \bigcap_{E_2 \in \mathcal{E}_2} E_2$	Def. 16, Sec. 3.2.1
$\mathcal{E}_1 \preccurlyeq^E_W \mathcal{E}_2$	$\forall E_2 \in \mathcal{E}_2 \ \ \exists E_1 \in \mathcal{E}_1\colon\ E_1 \subseteq E_2$	Def. 17, Sec. 3.2.1
$\mathcal{G}_1 \preccurlyeq^A \mathcal{G}_2$	$\mathcal{G}_2 = \mathcal{G}_{1\beta}$ for some β resolution of $\mathcal{G}_1$	Def. 18, Sec. 3.2.1
$\mathcal{UR}(\mathcal{G},\mathcal{S})$	$\bigcup_{\mathcal{G}' \in \mathcal{FRAF}(\mathcal{G})} \mathcal{E}_{\mathcal{S}}(\mathcal{G}')$	Def. 20, Sec. 3.2.3
$\text{SCCS}(\mathcal{G})$	The set of strongly-connected components of $\mathcal{G}$	Sec. 6.1
$\mathcal{MR}(\mathcal{G})$	The set of the minimal relevant SCCS of $\mathcal{G}$	Def. 22, Sec. 6.1
$\Pi_{\mathcal{G}}$	$\bigcup_{V \in \mathcal{MR}(\text{CUT}(\mathcal{G}))} V$	Sec. 6

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