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McCarthy variations in a modal key

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ABSTRACT

We take a fresh look at some major strands in John McCarthy's work from a logician's perspective. First, we re-analyze circumscription in dynamic logics of belief change under hard and soft information. Next, we re-analyze the regression method in the Situation Calculus in terms of update axioms for dynamic–epistemic temporal logics. Finally, we draw some general methodological comparisons between 'Logical AI' and practices in modal logic, pointing at some creative tensions.

1. Introduction

John McCarthy is a colleague whom I have long respected and admired. He is a truly free spirit who always communicates about issues in an open-minded way, away from beaten academic tracks and entrenched academic ranks. Each encounter has been a pleasure, from being involved with his creative Ph.D. students in Logical AI (each a remarkable character) to his lively presence at our CSLI workshop series on Logic, Language and Computation – and most recently, his participation in the Handbook of the Philosophy of Information [1]. Interacting with John makes me see my own work and my field in new ways, even in places where I eventually disagree. This brief note presents three illustrations. As a logician, I find John's work intriguing because it is clearly about logic, but not in the 'internal' professional mode that I am used to. True, many ideas of his have provided grist to our technical mills, and that is good. But more importantly, John's work reminds us of broader issues: what logic should be about, and also, what methodology best suits the resulting agenda. All these themes play in my illustrations. In doing so, I merely give new perspectives on what exists, pointing out some new directions: there are no new formal results.

2. Circumscription, logical consequence, and logical dynamics

My first encounter with the classic [18] introducing circumscription was when my student Wilfried Meyer Viol rushed into my office, and said he had just seen the first truly new logical idea in years, and that: not coming from a logician! We quickly read the paper, and I was struck at once by the liberating effect of other consequence relations that retain basic structure that makes them 'logical', while breaking new ground in terms of new varieties of reasoning. Ever since, I have

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been intrigued by what circumscription means, and I will tell a bit of the story up to my current somewhat iconoclastic reinterpretation.

2.1. Classical consequence, circumscription, and beyond

Minimal models. Classical consequence from premises $\boldsymbol{P}$ to a conclusion C says that all models of $\boldsymbol{P}$ are models for C. The models for $\boldsymbol{P}$ are the current range of options, encoding what we know: a logical conclusion does not add to that knowledge, but elucidates it. McCarthy [18] taught us that problem solving and planning go beyond this format, getting more out of the stated premises in special models that are most congenial to the scenario at hand. A *circumscriptive consequence* from $\boldsymbol{P}$ to C says that

$$C \text{ is true in all minimal models for } \boldsymbol{P}.$$

Here, minimality refers to some relevant comparison order for models: inclusion of object domains, inclusion of denotations for specified predicates, and so on. The general idea is minimizing over any reflexive–transitive order of 'relative plausibility' [28].[1] This is not just a trick for problem solving: circumscription seems a natural phenomenon in common sense reasoning broadly construed.

Circumscription and classical logics. Circumscription quickly caught the imagination of logicians, since it raises new questions about familiar systems. An example is the fine structure of second-order logic (cf. [39]). Letting the comparison order be inclusion of predicates over the same domain, and switching to standard notation, truth of a circumscriptive sequent $\varphi \Rightarrow_P C$ with φ a conjunction of premises (where the predicate P may occur in both φ and C) in a first-order model $\boldsymbol{M}$ is truth of a following second-order implication:

$$\bigl(\varphi(P) \wedge \neg\exists P' \subset P : \varphi\bigl(P'\bigr)\bigr) \to C(P).$$

Here the antecedent defines the 'predicate minimal' models of $\varphi(P)$.

Thus, circumscription involves second-order logic, and high complexity is lurking. Still, basic results in [15] found special syntactic forms of $\varphi(P)$ that reduce circumscription to first-order reasoning.[2] This line of analysis can be extended. [34] analyzes just when there exists a unique minimal predicate $\mathit{MIN}\,P \bullet \varphi(P)$ satisfying a given first-order description $\varphi(P)$ (φ may also contain predicates Q that are not minimized):

Definition 1. A first-order formula $\varphi(P, \boldsymbol{Q})$ has the *Intersection Property* for a predicate letter P if, in every model $\boldsymbol{M}$, whenever $\boldsymbol{M}, \boldsymbol{P}_i \models \varphi(P, \boldsymbol{Q})$ holds for all predicates in some family $\{\boldsymbol{P}_i \mid i \in I\}$ (here the predicate letter P gets interpreted as the predicate $\boldsymbol{P}_i$), then it also holds for the intersection of all these predicates, that is: $\boldsymbol{M}, \cap\boldsymbol{P}_i \models \varphi(P, \boldsymbol{Q})$.

If the formula $\varphi(P, \boldsymbol{Q})$ is satisfied in a model $\boldsymbol{M}$ by any predicate P at all, this ensures there is a unique smallest predicate in $\boldsymbol{M}$ that does: and that is the earlier $\mathit{MIN}\,P \bullet \varphi(P)$. A simple formula with the Intersection Property is $\forall x(Qx \to Px)$: the minimal predicate for P is just Q. A more complex example is $\forall x(Qx \to Px) \wedge \forall xy((Px \wedge Rxy) \to Py)$: the minimal predicate is the reflexive–transitive R-closure of Q in the model. These two cases suggest a syntactic format matching the semantic Intersection Property:

Definition 2. A first-order formula is a *PIA* form ('positive antecedent implies atom') if it has the syntactic format (with $\boldsymbol{x}$ a finite tuple of variables)

$$\forall \boldsymbol{x}\bigl(\varphi(P, \boldsymbol{Q}, \boldsymbol{x}) \to P\boldsymbol{x}\bigr) \quad \text{with } P \text{ occurring only positively in } \varphi(P, \boldsymbol{Q}, \boldsymbol{x}).$$

Here is a model-theoretic preservation result connecting the two notions:

Theorem 1. *(See van Benthem [34].) The following assertions are equivalent for all first-order formulas $\psi(P, \boldsymbol{Q})$:*

(a) $\psi(P, \boldsymbol{Q})$ *has the Intersection Property w.r.t. predicate P,*
(b) $\psi(P, \boldsymbol{Q})$ *is definable by a conjunction of PIA formulas.*

This analysis relates circumscription to well-known languages in the study of computation, with operators for smallest and greatest fixed-points:

Theorem 2. *(See van Benthem [34].) First-order logic with added predicate minimization over PIA-conditions has the same expressive power as $\mathit{LFP}(\mathit{FO})$: first-order logic with added least fixed-point operators.*

[1] This is much as in the Lewis semantics for conditional logic since around 1970 – an analogy which has been often noted (cf. [30]). This analogy will return below.

[2] The latter are reminiscent of 'correspondence theory' in modal logic [39] where we ask when modal logics whose axioms express second-order conditions on binary relations are already completely captured by matching first-order properties.

The connection between circumscription and fixed-point logic seems worth developing, but this technical strand is not my main theme in this paper.

Further travels of the idea. The next noticeable phenomenon is that circumscription, like all good ideas, has crossed over to other areas, in maybe unintended ways. Non-monotonic default reasoning is important in philosophy. [30] notes how philosophers of science in the 1950s were quite close to it in their accounts of scientific explanation as what follows from a theory 'under normal circumstances'.[3] A more detailed account is found in [35].

Circumscription made its way into linguistics as well. [30] noted also how it accounts for 'exhaustive readings' of answers to questions. An answer "John and Mary" to a question "Who are walking?" is usually read minimally: *only* John and Mary are walking. Such predicate minimizations are elaborated in [43]: the art is then finding the right model order to minimize over for concrete semantic purposes. There is also work on linguistic formulations for McCarthy's common sense puzzles. The innovative [12] analyzed these as a source of cues directing reasoning toward a solution.

The complete travels of circumscription and related ideas remain to be chronicled, and they would also include cognitive science and mathematics. Instead, we turn to the more general impact of circumscription in views of logic itself.

The Bolzano Program: logic as a study of the varieties of consequence. By now, many styles of non-classical consequence have been found, with their structural properties [6,8,24], and it has been suggested that logic itself should be viewed as a study of a plurality of consequence relations. Indeed, there is a historical precedent, if we go back to the agenda before Frege and Boole. Logic was defined as the study of different natural styles of reasoning in the work of the great pioneer Bernard Bolzano, who already provided a highly original theory of sub-structural properties [29,32].

But this diversity of reasoning styles also raises quite a few problems of its own. What is the nature of this diversity: do we really 'infer' in many different ways, and why? Can we safely combine different styles in useful ways? And if these styles are to reflect cognitive practice or common sense, what about their computational complexity? Over a first-order language, circumscription has a much more complex notion of validity than classical consequence, so what total package of benefits are we buying at this price?

I have no answers to such questions of reasoning architecture. But in what follows, I suggest that a shift in perspective may be helpful – from a steaming jungle of non-classical 'consequence relations' to the world of modal logics for belief revision and other *informational processes*.

Rethinking the issues: problem solving involves knowledge and belief. Recall the scenarios that motivated non-monotonic logic in the first place. We are given some initial information – and solving a puzzle means finding out what the true situation is. We may also get further information on the way. Perhaps the central phenomenon here is *not inference at all*, but our processing of that initial information, and its subsequent updates:

Say we are playing a card game, with the table in public view. At each stage, I know things about the outcome of the game, while I believe more than what I strictly know, based on expectations about cards that other players hold, or their temperaments: timid, bluffing, ... Now, new information comes in, say you play a card on the table. This changes my current state: I know more now, and my observation may also induce further beliefs about cards you are still holding. Of course, these beliefs may be refuted by your further moves, unlike the hard knowledge that I got about what is on the board. Solving puzzles, playing games, or planning in action seems all about such events.

Now, I claim it is illuminating to think about classical versus circumscriptive 'inference' in the same way. The very motivation for non-monotonic reasoning seems epistemic, having to do with managing our knowledge and beliefs – but this key feature is *left implicit*. To me, even classical consequence is really about the *knowledge update* that takes place when new information comes in. And in tandem with this, I would say that

circumscriptive inference is about belief formation

that takes place on the basis of incoming new information.[4] Clearly, knowledge update and belief revision are intertwined, and they provide mutual support. I think it is this diversity of responses to information that truly explains the modern galaxies of notions of consequence. But if so, what should be our preferred logical analysis for these phenomena?

Logical Dynamics. Informational events like the above are the arena of current dynamic logics of information update and belief revision. The program behind such systems is 'Logical Dynamics' [31] that makes cognitive actions and events key themes in logic. This makes *systematic changes* in what agents know, believe, or prefer the engine of 'intelligent interaction'.

This new focus leads to a re-analysis of what we studied earlier. We start with existing 'static' logics of knowledge and belief, and then create 'dynamified' versions. Then we discuss what they tell us about circumscription.

From static to dynamic epistemic logic. The epistemic base language has operators $K_i\varphi$ for 'agent i knows that φ', interpreted over models $M = (W, \{R_i\}_{i\in I}, V)$ where W is a set of relevant situations containing the actual world, the R_i are epistemic accessibility relations among the worlds for agents, and V is a valuation assigning (possibly varying) truth values to proposition letters in each world. The key semantic clause is then this:

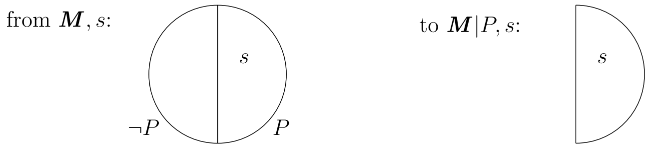
$$\boldsymbol{M}, s \models K_i\varphi \quad \text{iff} \quad \boldsymbol{M}, t \models \varphi \quad \text{for all } t \text{ with } R_i st.$$

[3] Domain-minimizing circumscription occurs explicitly as early as [10].

[4] 'Belief' should not be taken in any deep sense here: I just mean the temporary views we entertain as most plausible given our information so far.

In what follows, we will write $\langle K\rangle\varphi$ for the existential dual of this notion. Epistemic models $\boldsymbol{M}$ define agents' current range of uncertainty, i.e., they represent group information states in an obvious folklore sense. An agent 'knows' in this informational sense whatever is true at all worlds in her current range. We refer to the abundant literature for details of these systems (cf. [7]).

These ranges come with another commonsense idea: new information decreases the current range, while full information is just the singleton set $\{w\}$ with w the actual world. The paradigmatic events that make this happen are *public announcements* $!P$ of new 'hard information' that P is the case, which change irrevocably what agents know by elimination of all worlds that fail to satisfy P. Formally, a public announcement (or equally well, a public observation) $!P$ triggers a change in the current epistemic model $(\boldsymbol{M}, s)$ with actual world s: it eliminates all worlds in $\boldsymbol{M}$ that are incompatible with P, zooming in on the actual situation. Thus $(\boldsymbol{M}, s)$ changes into its definable sub-model $(\boldsymbol{M}|P, s)$, whose domain is the set $\{t \in \boldsymbol{M} \mid M, t \models P\}$. In a picture, one goes



In line with the informational interpretation, we assume that public announcements are made only of facts that are true.[5] Truth values of formulas may change in this update process: in particular, with factual assertions, agents who did not know P do after the event $!P$. This switching can be complex, but the following logic keeps track of it:

Definition 3. The language of public announcement logic *PAL* extends epistemic logic with action expressions denoting the preceding update steps:

Formulas	P:	$p \mid \neg\varphi \mid \varphi \vee \psi \mid K_i\varphi \mid C_G\varphi \mid [A]\varphi$.
Action expressions	A:	$!P$.

Here the basic new action modality $[A]\varphi$ comes from dynamic logic of programs, and it says that 'after action A has been performed at the current world, φ has become the case'. The precise semantic clause for this dynamic action modality is as follows, with the antecedent reflecting the partiality of the model change operation (its precondition is that P be true):

$$\boldsymbol{M}, s \models [!P]\varphi \quad \text{iff} \quad \text{if } \boldsymbol{M}, s \models P, \quad \text{then } \boldsymbol{M}|P, s \models \varphi.$$

We refer to [42] for a textbook exposition of *PAL* and much richer systems of 'dynamic epistemic logic'. Here is the complete logical calculus of 'hard information flow' in the above sense [22,9,41]:

Theorem 3. *PAL is axiomatized completely by the usual complete laws of epistemic logic for the chosen model class*[6] *plus the following recursion axioms*:

$$\begin{aligned}
&[!P]q && \leftrightarrow && P \to q \quad \textit{for atomic facts } q,\\
&[!P]\neg\varphi && \leftrightarrow && P \to \neg[!P]\varphi,\\
&[!P](\varphi \wedge \psi) && \leftrightarrow && [!P]\varphi \wedge [!P]\psi,\\
&[!P]K_i\varphi && \leftrightarrow && P \to K_i\big(P \to [!P]\varphi\big),\\
&[!P]C_G\varphi && \leftrightarrow && P \to C_G^P[!P]\varphi.
\end{aligned}$$

We give some informal explanation. The antecedents P express the earlier-mentioned partiality of the operation $!P$. The first axiom says that announcements do not change atomic facts. The second axiom interchanges the modality and negation, something that holds for a modal operator when its accessibility relation is a partial function: and indeed, $!P$ is. The knowledge recursion axiom replaces knowledge after announcement by conditional knowledge before, taking proper care of the right recursion. Finally, for completeness, we added the recursion axiom for $C_G\varphi$: common knowledge of φ in group G, whose semantics we omit here. $C_G^P\varphi$ is then the matching 'conditionalized version' of common knowledge (cf. [41]).[7]

These axioms are the 'recursion equations' of public information flow, performing step-by-step analysis of epistemic effects of incoming hard information. In particular, they relate knowledge that agents get after getting new information to

[5] We can easily deal with more general announcement scenarios, all the way up to unreliability, lying and cheating, but they would lead us too far here.

[6] This logic can be $S4$, $S5$ or $KD45$: it does not matter for the dynamic analysis.

[7] [41] define *P-conditional common knowledge* as common knowledge with accessibilities restricted to finite paths consisting entirely of P-worlds, and provide the full-blown complete recursion axiom for it: $[!P]C_G^{\psi}\varphi \leftrightarrow (P \to C_G^{P\wedge[!P]\psi}\varphi[!P]\varphi)$.

conditional knowledge they already had before. There is much more to this system, but for the purposes of this paper, we know enough.

Dynamic consequence relations. One more thing is essential to our discussion of McCarthy's work: the system *PAL* supports new notions of consequence!

Definition 4. A sequent $P_1, \ldots, P_k \Rightarrow \varphi$ is *dynamically valid* if, starting with any epistemic model $(\boldsymbol{M}, s)$ whatsoever, successive announcements of the premises result in a final model where announcement of φ effects no further change: i.e., in the model $((\ldots(\boldsymbol{M}|P_1)\ldots)|P_k, s)$ the formula φ was already true everywhere, even before it is announced.

Dynamic validity amounts to *PAL* validity of the following dynamic–epistemic formula, which says that the conclusion becomes common knowledge:

$$[!P_1]\ldots[!P_k]C_G\varphi.$$ [8]

On the surface, this seems close to classical consequence. Indeed, we have:

Fact 1. *For purely factual (non-epistemic) formulas $P_1, \ldots, P_k, \varphi$, dynamic valid consequence holds if and only if φ follows classically from $P_1, \ldots, P_k$.*

The special reason behind the fact is this: purely factual formulas do not change their truth values at worlds when passing from a model $\boldsymbol{M}$ to an updated model $\boldsymbol{M}|P$. Things change when we admit announcements of epistemic formulas, because then, truth values can, and will, typically change, like when ignorance changes into knowledge.

Structural rules. Now consider the famous *structural rules* governing classical logical consequence in a sequent format $\boldsymbol{P} \Rightarrow C$ from premises to conclusion. These are *Permutation* of formulas in $\boldsymbol{P}$, *Contraction* of occurrences of the same formula in $\boldsymbol{P}$ to just one, *Monotonicity* (adding antecedents to $\boldsymbol{P}$), *Reflexivity* ($C \Rightarrow C$), and *Cut* ($\boldsymbol{P} \Rightarrow C$ and $\boldsymbol{Q}, C \Rightarrow D$ imply that $\boldsymbol{P}, \boldsymbol{Q} \Rightarrow D$). Prima facie, dynamic consequence loses all of these, since it treats premises as update instructions, whose order and multiplicity matter (cf. [31]):

Fact 2. *All classical structural rules fail for valid dynamic consequence.*

But once can find modified structural rules that do remain valid in this setting:

Fact 3. *Dynamic consequence satisfies the following structural rules*:

$$\begin{array}{ll}
\textit{if } \boldsymbol{P} \Rightarrow C, \quad \textit{then } A, \boldsymbol{P} \Rightarrow C & (\textit{Left-Monotonicity}),\\
\textit{if } \boldsymbol{P} \Rightarrow A \textit{ and } \boldsymbol{P}, A, \boldsymbol{Q} \Rightarrow C, \quad \textit{then } \boldsymbol{P}, \boldsymbol{Q} \Rightarrow C & (\textit{Left-Cut}),\\
\textit{if } \boldsymbol{P} \Rightarrow A \textit{ and } \boldsymbol{P}, \boldsymbol{Q} \Rightarrow C, \quad \textit{then } \boldsymbol{P}, A, \boldsymbol{Q} \Rightarrow C & (\textit{Cautious Monotonicity}).
\end{array}$$

Theorem 4. *(See van Benthem [33].) A sequent σ is derivable from a set of sequents X by means of the preceding three structural rules iff σ is true in all models where all sequents in X are true.*

Altogether, this shows how dynamic logics of knowledge suggest new consequence relations. We now move to analogous phenomena for agents' beliefs:

From static to dynamic doxastic logic. Real agents jump to conclusions, form and drop beliefs, *correcting themselves* under the influence of new information. Logics of belief analyze assertions $B_i\varphi$ for *'agent i believes that φ'*. Their semantics adds gradations to the information ranges in epistemic models, using a ternary *plausibility ordering* of worlds x, y as seen from some vantage point s:

$$\leqslant_{i,s} xy \quad \text{in world } s \text{, agent } i \text{ considers } x \text{ at least as plausible as } y.$$

We then define belief semantically as *'truth in the most plausible options'*:

$$\boldsymbol{M}, s \models B_i\varphi \quad \text{iff} \quad \boldsymbol{M}, t \models \varphi \quad \text{for all minimal worlds } t \text{ in the order } \lambda xy.\leqslant_{i,s} xy.$$

[8] For a single $S5$-agent, we can replace $C_G\varphi$ by just knowledge: $K\varphi$.

Example 1. Consider the following model with two mutually epistemically accessible worlds, but the one with $\neg P$ is more plausible than the other:

$$\boxed{\neg P \;\overset{\leq}{\rule{3em}{0.4pt}}\; \boldsymbol{P}}$$

At the actual world to the right, with $\boldsymbol{P}$, the agent does not know whether P, but she does (mistakenly!) believe that $\neg P$. Crucially, beliefs can be false.

Now one soon finds that absolute beliefs are not sufficient for explaining agents' behavior. We want to know what they would believe were they to receive new information. This pre-encoding, in our earlier sense, requires conditional belief:

$$\boldsymbol{M}, s \models B_i^{\psi}\varphi \quad \text{iff} \quad \boldsymbol{M}, t \models \varphi \text{ for all worlds } t \text{ that are } \textit{maximal} \text{ for } \lambda xy.\leqslant_{i,s} xy \text{ in the set } \{u \mid \boldsymbol{M}, u \models \psi\}.$$

Conditional beliefs $B_i^{\psi}\varphi$ say that i believes φ, given that ψ. This is like conditionals [14]: they express what might happen under different circumstances.

Now, the earlier events of public hard information also affect agents' *beliefs*:

Theorem 5. *(See van Benthem [37].) The complete logic of conditional belief under public announcements is axiomatized by* (a) *any complete static logic for knowledge and belief,* (b) *PAL reduction axioms for atomic facts and Boolean operations, and* (c) *a new recursion axiom for conditional belief*:

$$[!P]B_i^{\psi}\varphi \leftrightarrow \big(P \rightarrow B_i^{P\wedge[!P]\psi}[!P]\varphi\big).$$

A special case is formation of absolute beliefs: $[!P]B_i\varphi \leftrightarrow (P \rightarrow B_i^P[!P]\varphi)$.

Dynamic consequence subsumes circumscriptive inference. Now, just as with information update and knowledge, the dynamic setting of belief change suggests a new notion of consequence:

$$[!P_1]\ldots[!P_k]B\varphi.$$

That is, when updated with the information in the successive premises, the agent will come to believe the conclusion.[9] Now the point to observe is that this notion generalizes the earlier-mentioned most general version of circumscription minimizing over arbitrary pre-orders, as truth of the conclusion in all minimal models of the premises[10]:

$$P_1, \ldots, P_k \Rightarrow \varphi.$$

Given our semantics of belief, with the plausibility order now the obvious candidate for minimization, and working with factual propositions only, that do not change their truth values across model updates from $\boldsymbol{M}$ to $\boldsymbol{M}|P$, $[!P_1]\ldots[!P_k]B\varphi$ says exactly the same as truth in all minimal models of the premises. Thus, we have the following informal

Claim 1. *Our dynamic logic specialized to belief formation under incoming factual propositions describes everything circumscription was designed to do!*

This is of course a conceptual claim, not a mathematical theorem. But it is a serious proposal: if we analyze problem solving in terms of belief formation and belief change, then circumscription becomes a special case of the appropriate logical notion of dynamic consequence.

In fact, the latter notion even does more. Our full dynamic logic also admits complex *non-factual* propositions for premises and conclusions, that express agents knowledge and beliefs. The usual accounts of structural rules and the intuitions associated with circumscription do not seem to take such more sophisticated information into account at all. But the above calculus does, and it will keep all this extra structure absolutely straight.

Belief change under soft information. But there is more. In using circumscription, one often fixes one comparison relation between models, that does not change in the process of inference. In abstract non-monotonic logics, this choice is left implicit in context. But since comparing worlds by relative plausibility determines agents' beliefs, and hence their 'conclusions', it is important to have explicit control over how we choose, and *change that ordering*. Indeed, triggers for changing beliefs need not be 'hard information' of the public announcement type, ruling out certain worlds for good. They can rather be 'soft information' affecting just our plausibility ordering of the worlds.

[9] With groups of agents, we would require *common belief*.

[10] We forego the usual technical complications with suitably modified definitions on non-well-founded orderings here, as these will be very similar for both dynamic consequence and circumscription.

An event that makes us believe that P need only rearrange worlds making the most plausible ones P: it works by 'promotion' rather than elimination of worlds. Thus, on the earlier models $\boldsymbol{M} = (W, \sim_i, \leqslant_i, V)$, we change the relations $\leqslant_i$, rather than the domain of worlds W or epistemic accessibilities $\sim_i$.

Here is a well-known soft trigger from the area of belief revision, sometimes called 'radical revision'. A *lexicographic upgrade* $\Uparrow P$ is an instruction for changing the current ordering relation $\leqslant$ between worlds as follows:

all P-worlds in the current model become better than all $\neg P$-worlds,
while, within those two zones, the old plausibility ordering remains.

We have the following corresponding dynamic modality

$$M, s \models [\Uparrow P]\varphi \quad \text{iff} \quad M \Uparrow P, s \models \varphi$$

with $\boldsymbol{M} \Uparrow P$ the model $\boldsymbol{M}$ with its order $\leqslant$ changed as above. This dynamic doxastic language describes how beliefs change under soft information:

Theorem 6. *(See van Benthem [37].) The dynamic logic of lexicographic upgrade is axiomatized completely by* (a) *any complete axiom system for conditional belief on the static models, plus* (b) *the following recursion axioms*:

$$\begin{array}{lll} [\Uparrow P]q & \leftrightarrow & P \to q, \quad \textit{for atomic facts } q, \\ [\Uparrow P]\neg\varphi & \leftrightarrow & P \to \neg[\Uparrow P]\varphi, \\ [\Uparrow P](\varphi \wedge \psi) & \leftrightarrow & [\Uparrow P]\varphi \wedge [\Uparrow P]\psi, \\ [\Uparrow P]B^{\psi}\varphi & \leftrightarrow & \big(\Diamond\big(P \wedge [\Uparrow P]\psi\big) \wedge B^{P \wedge [\Uparrow P]\psi}[\Uparrow P]\varphi\big) \\ & & \quad \vee \big(\neg\Diamond\big(P \wedge [\Uparrow P]\psi\big) \wedge B^{[\Uparrow P]\psi}[\Uparrow P]\varphi\big). \end{array}$$

The crucial final equivalence states which conditional beliefs agents form after soft upgrade. (Here $\Diamond$ is the earlier epistemic existential modality $\langle K \rangle$.) This formula may look daunting, but try to read the principles of some non-monotonic default logics existing today! And there is a reward. We now see *explicitly* how new triggers affect the plausibility order $\leqslant$ among worlds, and hence our beliefs at any given stage, and thus, the 'non-monotonic inferences' available to us on the basis of the ambient order $\leqslant$.[11]

New circumscriptive consequence relations. Given these options in belief revision and plausibility change, which is the true analogue of a circumscriptive inference $\boldsymbol{P} \Rightarrow \varphi$ in this dynamic setting? In particular, are the premises 'fed in' as hard or soft information? We cannot tell once and for all, because we now live in a richer universe of informational events that may determine how we solve our problem, make our plan, or play our game. Indeed, here is what seems a new notion of circumscriptive consequence:

In addition to 'hard circumscription' $[!P_1] \ldots [!P_k]B\varphi$
consider also '*soft circumscription*' $[\Uparrow P_1] \ldots [\Uparrow P_k]B\varphi$.

Definition 5. A *soft circumscriptive inference* $\boldsymbol{P} \Rightarrow_{circ\text{-}soft} C$ is valid when, in every model, after performing radical upgrades for all successive premises in $\boldsymbol{P}$, the conclusion C holds in the model obtained in this way.

The two options are not the same [38]:

Fact 4. *For factual assertions* P, Q: (i) $P, Q \Rightarrow_{circ\text{-}hard} P$, (ii) *not* $P, Q \Rightarrow_{circ\text{-}soft} P$.

The reason for (i) is that successive hard updates always yield subsets of the P-worlds. The reason for (ii) is that the last upgrade with Q may have demoted all P-worlds from their former top positions.

These generalized notions of circumscription are axiomatized by our dynamic logics of belief change. Still, an open problem of detail remains:

What are the complete structural rules for both hard and soft circumscription?

[11] There are many further ways of taking soft information. Conservative belief revision puts not *all* P-worlds on top qua plausibility, but just *the most plausible P-worlds*. 'After the revolution', this policy co-opts just the leaders of the under-class, not all of them. Complete dynamic logics for these and other policies exist [37,4].

2.2. Conclusion

The intuitions behind circumscriptive inference are epistemic, involving knowledge and belief. They are also dynamic, involving agents' responses to incoming information. Circumscription and other styles of non-monotonic reasoning are at heart about cognitive attitudes and responses to information. This was made explicit in dynamic logics that make processes of *inference* and *self-correction* go hand in hand. Moreover, the events driving such processes can be quite diverse, from hard information update eliminating worlds to soft mechanisms of plausibility change in world order. Thus, the circumscriptive reasoner, or her theorist, gets a framework describing problem solving in a much richer way than just choosing between a few reasoning styles.

Here are some other noteworthy features. Our analysis does away with 'deviant' notions of consequence. The above systems implement an 'equation':

non-standard consequence is *classical consequence plus dynamics*,

with the latter being an explicit account of knowledge and belief-changing events. And the mechanisms for the dynamics have an independent interest. They perform what may be called 'model management', keeping our current models attuned to information received, and in particular, dynamically adjusting the minimality ordering that was so crucial to circumscription. Thus, we can also formulate intuitions concerning that ordering at a dynamic level. For instance, on an Ockhamist stance, we would only add individual objects or worlds when really forced by the incoming informational events.

Our perspective raises new open problems, such as capturing the structural properties of softer variants of circumscription. Also, it raises issues of over-all architecture of reasoning. Dynamic notions of consequence care for details of presentation of premises and conclusions, reflecting the history of matching informational events. There may be different 'scales' here, since these seem local effects, forgotten in the longer term. Indeed, circumscription itself already seems more local than classical consequence, since we use the current context of presentation for premises ('just these right now, no more'). Our dynamic notions go one step further in this local direction. Thus, there may be a place for different notions of consequence, and short-term and long-term dynamics.

Viewed in this way, the study of circumscription and similar non-monotonic reasoning styles and the study of dynamic modal logics of knowledge and belief are after the same things. They may compete, but also, help each other.

3. From Situation Calculus to epistemic temporal logic

Classical logic and circumscription describe single steps of inference. The next step in studying agency is the longer term, where many such steps combine into problem solving procedures and behavior over time. In this realm, many frameworks compete: Dynamic Logic, Temporal Logic, and so on – often without communication. All work with more or less the same structures, viz. branching trees of events, possibly endowed with epistemic structure.[12]

This temporal realm is where John McCarthy has made another fundamental contribution, viz. his Situation Calculus, developed in [17,19].[13]

My initial plan for this paper was a crisp comparison between Situation Calculus and my own habitat of Modal Logic (see [5] for the current state of the art).[14] But two things made me change my mind. One was a convergence of approaches that had already happened without my knowing it. When opening Reiter's *Knowledge in Action* from 2001 [23], I was struck by great similarities. The very title sounds like 'Language in Action' and 'Logic in Action', fashionable book titles in my world since the early 1990s. More concretely, models of the Situation Calculus are branching trees of event sequences, the standard semantics of all process theories of rational action [40], major methods like 'Regression' are standard compositional techniques for computing preconditions of events, knowledge is treated in a standard philosophical epistemic style (cf. [7]), and the logic programming underpinning of specific scenarios is a general resource that we all use.[15] Thus the modern Situation Calculus seems to be part of the air that we all breathe.

Next I wanted to look at fine-structure, comparing Situation Calculus techniques in detail with those used in modal logic. In particular, striking analogies exist between the 'regression calculus' and the dynamic logics of Section 2. But I was just too late. Many points in my first draft were found independently in [11]. I will therefore just give a brief discussion here, starting from a generalization of the dynamic logics in Section 2.

[12] [40] show how this structure occurs across AI, computer science, linguistics, and philosophy, while the temporal logics and technical results about them are in close harmony, even when developed independently. [37] provide comparisons and merges between the frameworks of dynamic epistemic logic and epistemic temporal logic.

[13] In what follows, we will refer to the formulations given in [23] – while related temporal frameworks are found in [28,25,27].

[14] One special reason was that John himself sometimes grumbles about modal logicians persisting in their evil ways, ignoring what he sees as his knock-down arguments against their approach.

[15] To avoid misunderstanding here, I am *not* saying that Reiter's book contains nothing new. To the contrary, I was deeply impressed by its content, and I stand by what I said on its cover!

3.1. Dynamic epistemic logics of time and change

Events and general dynamic epistemic logic. The dynamic logics of information change in Section 2 have general event-based versions where update is more sophisticated than mere elimination of worlds, or rearranging plausibility order [3,36]. General observation of events involves two ingredients:

(a) a current epistemic information model $\boldsymbol{M}$ as above, plus,
(b) an epistemic event model $\boldsymbol{E}$ of all relevant possible events,

with accessibility relations in $\boldsymbol{E}$ representing what agents know about these events, their 'epistemic access'. Say, in a card game, playing cards on the table is a publicly observable event, but in drawing a card from the stack, I see the card I am drawing, but you only know it is one of a certain set. Also, you and I may have different beliefs. Perhaps my smile means that I am drawing the Ace of Hearts, but you cannot know this.

Formally, an *event model* $\boldsymbol{E}$ is a triple $(E, \{\sim_i\}_{i\in I}, PRE)$ with E the set of relevant events, indistinguishability relations $\sim_i$ describing which events agent i cannot (observationally) distinguish in the scenario described by $\boldsymbol{E}$, and a *precondition function PRE* assigning to each event e a precondition PRE_e for its occurrence, expressed as a formula of epistemic logic.[16]

In this setting, information flows through observation since events e come with preconditions PRE_e on their successful execution. A truthful public announcement $P!$ can only happen in P-worlds, I can only draw the Ace of Hearts if it is actually on the stack, etc. This information flow gets implemented in the following update mechanism:

Definition 6 *(Product update).* Dynamic epistemic update takes an epistemic model $\boldsymbol{M}$ and an event model $\boldsymbol{E}$ to a

$$\text{product model } \boldsymbol{M} \times \boldsymbol{E} \text{ with domain } \{(s,e) \mid \boldsymbol{M}, s \models PRE(e)\}.$$

Here a pair (s,e) records a new event e occurring at the old world s, provided that the preconditions of e held at $\boldsymbol{M}, s$ (the latter restriction may change the total space of options). The basic epistemic idea of information flow is now as follows. Uncertainty among new worlds can only come from existing uncertainty via indistinguishable events:

$$(s,e) \sim_i (t,f) \quad \text{iff} \quad \text{both } s \sim_i t \text{ and } e \sim_i f.$$ [17]

The valuation for proposition letters at (s,e) just remains that of s.

This framework for knowledge update can also be extended to deal with beliefs and belief change, providing a generalization for all of Section 2 above (cf. [4,37]). It can also deal with actual world change through physical events (cf. [41]). In the latter set-up, events e in an event model $\boldsymbol{E}$ are specified using both 'pre-' and 'post-conditions', similar to those used in the Situation Calculus.[18]

Dynamic epistemic logics. Again there are complete dynamic languages and matching logics governing this style of update (cf. [41]). As in Section 2, their core consists of recursion axioms analyzing epistemic, doxastic, and physical effects of events, with the earlier operators $[!P]\varphi$ generalized to forms $[\boldsymbol{E},e]K_i\varphi$, $[\boldsymbol{E},e]B_i\varphi$, etc. These are somewhat adventurous, as they put epistemic event models inside the language, but this can be done consistently.

Again, the crucial recursion uses the ability of our static logical languages to pre-encode what would become true after certain events. For instance, here is the central recursion axiom for the case of knowledge (note the role of the precondition for the event e):

$$[\boldsymbol{E},e]K_i\varphi \leftrightarrow PRE_e \to \bigwedge\{K_i(PRE_f \to [\boldsymbol{E},f]\varphi) \mid f \sim_i e \text{ in } \boldsymbol{E}\}.$$

Dynamic epistemic logic in this style may be viewed as a well-chosen fragment of the richer *epistemic temporal logics* of [7,21]. But the latter also describe longer-term environments of events, into the future and the past.

3.2. A striking encounter: regression principles are recursion axioms

We will just sketch one striking convergence with the Situation Calculus. Consider the central *Regression Method* in [23] for 'pre-computing' the effects of a plan, or a structured program, at the initial stage of a process. Here are some basic notions in this setting.

Possibility predicates. First, when describing events e, we often know explicitly when they can occur, and the 'possibility predicates' $Poss(e)$ in the Situation Calculus are just the above preconditions. In modal terms, this uses formulas

$$\langle e\rangle\top \leftrightarrow \varphi(e), \quad \text{where } \varphi \text{ is a formula referring to the current state only.}$$ [19]

[16] This format of definition can be extended, but it will do for an illustration.

[17] Actually, this clause amounts to an assumption of agents having 'Perfect Recall' [41] similar to those discussed in [7,23], or in game theory [20].

[18] It is tempting to identify events with the same pre- and postconditions, but in the presence of different epistemic agents, this is impossible – just as we cannot identify worlds with the same factual content in multi-agent settings.

Successor state axioms. Next, we often specify effects of events via conditions on the current state. My lighting the match will make the cigarette burn (a post-condition) iff certain preconditions hold. 'Successor state axioms' in the Situation Calculus describe when an event produces a state satisfying a certain atomic predicate in terms of some local condition on the current state. Again, the pattern is familiar from modal logic. Successor state axioms are like *preconditions* $\langle\pi\rangle\varphi$ of dynamic logics of programs, where the existential modality says that φ will hold after some successful execution of the program π. For specific events, the reduction principle takes the following form

$$\langle e\rangle\psi \leftrightarrow \Psi(e), \quad \text{where } \Psi \text{ is a formula referring to the current state only.}$$ [20]

Program structure. On top of these basic events, plans have complex structure, that can be reduced via the usual principles of dynamic logic, such as the axiom for action composition:

$$\langle\pi_1;\pi_2\rangle\varphi \leftrightarrow \langle\pi_1\rangle\langle\pi_2\rangle\varphi.$$

Object recursion axioms. Finally, and this goes beyond *propositional* dynamic logic, the Situation Calculus employs reduction axioms for existential quantifiers over domains of objects. E.g., the following 'Barcan Axiom' from modal predicate logic will hold, provided that the event does not change the domain by actions of destruction or creation:

$$\langle e\rangle\exists x\varphi \leftrightarrow \exists x\langle e\rangle\varphi.$$

The Regression Theorem. Now, arguably the central result in [23] (where it occurs in both factual and epistemic versions) is the *Regression Theorem*. It says essentially that one can *pre-compute in the current state* when some given complex event or plan has certain specified effects following its execution. The key point is this, when stated in the above modal terms. Given preconditions for events and their atomic effects, all further constructions for complex effects *can be pushed recursively through the event modality*. This is even an effective method for analyzing plans.

But all this was exactly the point of our method of recursion axioms and 'pre-encoding' in Sections 2 and 3.1. Thus, Situation Calculus and Dynamic Epistemic Logic exploit essentially the same natural features of events, and of how we understand them in terms of information flow and physical changes. Of course, there are differences in style – but perhaps unbeknownst to each other, they have found similar solutions to similar problems.

Adding knowledge. Further analogies reinforce this point. The original Situation Calculus has no explicit account of knowledge. But [23] does have one, based on the extension of regression in the Situation Calculus to knowledge first given in [26]. Once this epistemic structure is added to the temporal universe, the discussions of knowledge and informative events in [23] are virtually identical to those found in [7], or the dynamic–epistemic literature. Indeed, the very treatment of knowledge is that of standard epistemic logic, be it in a rich language that also describes accessibility relations directly.[21]

In particular, Reiter's thoughtful discussion of regression axioms for information after events makes a point that is also central to dynamic–epistemic logic. Recursion axioms for knowledge after events like the ones stated in Section 2 are not neutral: they presuppose a special kind of agency. In particular, they encode two major epistemic presuppositions about agents [7,40]. One is *Perfect Recall*: future uncertainties only arise out of past uncertainties plus uncertainty introduced by partial observation – there are no memory lapses. The other major presupposition is *No Miracles*: current uncertainty between worlds persists, unless it is resolved by some newly observed event.[22]

3.3. Conclusion

The Situation Calculus seems to have evolved to a pretty standard epistemic-temporal framework, of which many variants exist in many areas. In particular, it uses recursive machinery that has been proposed independently in dynamic epistemic logics. Thus, we can profitably compare, and perhaps even merge frameworks. A concrete example is computational complexity. We know a lot about the complexity of dynamic–epistemic logics over their static base logics (cf. [16,11]), and accordingly, we can learn a lot about the design of well-chosen fragments of the full Situation Calculus.

4. General methodological issues

We have found striking similarities between McCarthy's World and the modus operandi of (modal) logicians. In this section, we list a few methodological differences, at least in emphasis. These are worth-while, too, as 'intensional differences' are often instructive.

'Logical systems' versus doing the actual work. One difficulty in making comparisons is that 'Logical AI' is about systems that really describe some reasoning practice in detail, while many logicians live in a world of formal systems whose uses

[19] In Reiter's language, these predicates can still involve individual terms, and lush higher-order languages, far beyond the usual austere modal formalisms – but we will discuss *that* difference in Section 4 below.

[20] For the Situation Calculus, where events are usually deterministic, the difference with the universal modal $[\pi]\varphi$ in dynamic logic is slight.

[21] This itself is the familiar 'Tandem Approach' beloved by working modal logicians: cf. the chapter 'Modal Logic: A Semantic Perspective' in [5].

[22] For further productive analogies between the two traditions, cf. [13].

are largely fictional, or at least, confined to scoring points in internal discussions with other designers of formal systems. But Circumscription or Situation Calculus are not just *core theories* in the logician's sense, as a set of formal definitions plus mathematical meta-theorems. They are also *bodies of practice*, including sophisticated exemplars of reasoning with hierarchies of classificatory 'abnormality predicates', temporal 'inertia predicates', and the like. By contrast, dynamic epistemic logics so far have just a modest 'modeling practice': a historically growing fund of successful applications that serve as exemplars for new researchers. Another practical aspect is computational implementation. For instance, whether you are bothered by computational complexity is like sensitivity to sin: some people have it, some do not. This paper has provided no framework comparison at these broader practical levels.

Modal logic and fine-structure. A second difference is the role of formalism. In logic, formal languages make concepts explicit and computationally manipulable, trying to model major aspects of some reasoning practice. A crucial aspect in this perspective is their expressive poverty. First-order logic itself is a small fragment of Frege's higher-order logic, proposed by Hilbert & Ackermann, which strikes a special *balance* between two desiderata: *expressive* power, and low *complexity* of validity – the valid first-order inferences are effectively axiomatizable. The same concern with fine-structure drives modal logic, which weakens the expressive power of first-order logic still further, in return for even decidability of valid inference. Given this stance, a logician might say that the complexity in Logical AI comes from mixing two strands. One are *core principles* of non-monotonic or temporal reasoning, the other are effects of working with *particular classes of models* ordered by concrete relations of comparison and temporal succession. That this mixture of logic and 'mathematics' is highly complex goes without saying.

Bringing these perspectives together seems worthwhile. Logicians keep finding new fragments that strike a good balance between expressive power for key patterns of reasoning and complexity of validity. One new decidable system in between modal and first-order logic is the *Guarded Fragment* [2], and it would be of interest to see how much of existing Logical AI can work with guarded (re-)formulations.[23] Experience from modal logic might help 'deconstruct' highly complex systems and provide illuminating fine-structure. And if you find this motivation too internal-logical, think of this: less expressive languages represent higher levels of *abstraction*, and we all know the advantages of that!

First-order and the proper level of generality. John McCarthy has sometimes engaged in polemics against modal logic. I will not be drawn into this arena, partly because my own work on modal correspondence theory has always advocated a tandem approach of studying modal languages jointly with translations into classical ones, first- or higher-order. But there is one interesting issue where John's criticisms resonate with me. It can be described in several ways. One is that propositional modal logics may be missing generality. Even though one can encode information flow in given modal models, such solutions often miss the genericity of first-order formulations, that work across many models at once. Thus, it is always of interest to also look upward in expressive power, toward languages that bring out the generic nature of solutions in a most perspicuous manner. This issue is related to what I have called the 'system imprisonment' of modern logic. We are bound by our methodology to defining specific formal systems, and these may not be the most appropriate vehicle for stating our general insights. Consider the recursion axioms of dynamic–epistemic logic, and compare them to Reiter's regression calculus. Clearly, the recursion method is not tied at all to the modal language, but one might forget this generality when staying 'down below'.[24]

Richer Ontology. Of course, there is also a less methodological and more practical reason for moving up in expressive power. The Situation Calculus has structured objects all around: events have agents and objects, agents know people and their telephone numbers, etc. In line with this, formal languages in Logical AI are usually based on at least predicate logic with atomic predicates, variables, terms that cross between objects and predicates, and other ways of describing individual objects and their properties and interrelations. This, too, should appeal to modal logicians. Dynamic epistemic logics and epistemic temporal logics extol the importance of events, but they are silent on the structure of these crucial entities. In contrast, theories of events in formal semantics endow them with a lot of structure, including agents, objects, patients, locations, modes, etc. Agents knowing objects and persons, in addition to propositions, were crucial to epistemic logic as originally developed in philosophy. The agenda shift in modal logic taking this richer object structure to the margin of research can be questioned. Well, nothing was lost for good. Each preceding item presents a challenge to people in my world, viz. the design of richer and more generic versions of dynamic epistemic logic.[25]

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[23] In constructive mathematics, much attention has been paid to the actual minimal expressive power needed to formalize bodies of mathematical proof. Why not do the same for common sense reasoning?

[24] Still, the real issue with our methodology for information and belief may not be total system complexity, but relative complexity of a dynamic logic over its static base.

[25] Occasionally, McCarthy's insistence on first-order logic also has a 'downward-looking' flavor. In his view of common knowledge, groups are just new first-order objects, where modal logicians move to non-first-order fixed-point logics for group knowledge and related phenomena. I find McCarthy's alternative interesting, and indeed, one can analyze fixed-point axioms for epistemic logic with common knowledge in first-order terms by modal frame correspondence techniques.

5. Conclusion

John McCarthy's world and that of modal logic seem highly congenial, though not quite the same. I hope to have shown with concrete examples that this allows for meaningful communication, and through that, fruitful forms of agreement and disagreement. And even if our life-styles do not merge completely in this encounter, creative friction is the engine of scientific progress.

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