

Rough intervals—enhancing intervals for qualitative modeling of technical systems

M. Rebolledo

Universität Stuttgart, Institute of Industrial Automation and Software Engineering, Pfaffenwaldring 47, 70569 Stuttgart, Germany

Received 8 October 2004; received in revised form 29 December 2005; accepted 15 February 2006

Abstract

The success of model-based industrial applications generally depends on how exactly models reproduce the behavior of the real system that they represent. However, the complexity of industrial systems makes the construction of accurate models difficult. An alternative is the qualitative description of process states, for example by means of the discretization of continuous variable spaces in intervals. In order to reach the required precision in the modeling of complex dynamic systems, interval-based representations usually produce qualitative models, which are sometimes too large for practical use. The approach introduced in this paper incorporates vague and uncertain information based on principles of the Rough Set Theory as a way of enhancing the information contents in interval-based qualitative models. The resulting models are more compact and precise than ordinary qualitative models.

© 2006 Elsevier B.V. All rights reserved.

Keywords: Rough set; Qualitative modeling; Qualitative reasoning; Interval-based knowledge representation; Vagueness and uncertainty management

1. Introduction

Many industrial systems rely on models to support applications such as process simulation, advanced control, fault diagnosis and online monitoring. Working on a correct and precise system model is always cheaper, faster, easier and safer than working with the real system. But modeling real systems is not always straightforward. Models can only be employed if they mimic the original system precisely. However, this model precision, i.e. how well this model reproduces the system behavior or characteristics, is not determined by the model itself, but by the intended application, and is therefore often more a requirement than a feature. A further problem is that industrial systems are usually too complex to be described, meaningfully and wholly, with precise deterministic models. To make matters worse, system complexity and precision demand in model-based applications increase with each new technological development.

Humans are able of abstracting complex processes and describing them qualitatively using expert knowledge and common sense. Therefore, a way of dealing with the described problems is focusing only upon the system aspects to be analyzed, enhanced or supervised, and representing these aspects in qualitative models. Such qualitative models usually describe continuous variables by dividing the spaces where they are defined in intervals [3,15]. Yet, using in-

E-mail address: mrrw2002@yahoo.com (M. Rebolledo).

Table 1
Representing qualities as binary messages

(a)	A	AB	B	BC	C
	0	1	0	0	0

(b)	A	B	C
	1	1	0

tervals to represent continuous physical dimensions implies losing information about the system behavior during state transitions, because an interval-based partitioned variable can change its qualitative value instantaneously. Defining additional transitional intervals reduces this information loss. Important transitions can be well described, for instance, using one-point intervals, but it is impossible to describe the behavior of the system as it nears or leaves these points. Besides, defining one-point intervals may result in models that are too large to describe entire complex dynamic systems.

Interval-based qualitative models are analogous to using just one bit at a time in a N -bits message (e.g. 10000, 01000, 00100). This is underutilizing the 2^N representation capability of these N -bits. However, because of continuity, it is not always the case that 2^N different messages are physically possible. Using a message such as 01001 to describe a particular system state may be nonsense. For instance, a given continuous variable may be represented using three intervals (representing qualities A, B and C respectively) and two one-point transitional intervals. The equivalent binary representation is a 5-bits word, which is only allowed to get one of five possible values: 10000, 01000, 00100, 00010 or 00001. Table 1(a) shows the equivalent binary representation of a system in the transition point between quality A and quality B.

This paper introduces a concept to enhance intervals as information representation framework, by allowing the overlapping of qualities in intermediate states. This eliminates, for example, the need of defining one-point transitional intervals and makes easier modeling the vagueness and uncertainty in transitions. Following the previous example, this would correspond to using only three bits to code the five possible states of the variable (Table 1(b)). Of course, binary values 000 and 101 are still not allowed.

The proposed concept takes advantage of principles of the Rough Set Theory. The resulting representation, the Rough Interval, is more suited for dealing with partially unknown or ill-defined parameters and variables. Additionally, Rough Intervals are capable of coding more information than ordinary intervals, as it is suggested by the example of the analog binary representation. The resulting qualitative models are, therefore, more compact for a required precision, or the same, more precise for a given model size. This improvement in the relation precision/size in qualitative models expands the applicability of qualitative modeling methods in complex systems. The new concept was integrated in the qualitative modeling method SQMA (Situation-based Qualitative Modeling and Analysis) [33], which allowed the verification of the described enhancements. This concept is, however, independent from SQMA and can therefore be integrated into other qualitative reasoning techniques as well.

2. Theoretic framework

The following sections describe briefly the techniques and methods that served as basis for the development and further application of the proposed concept. In the first place, the most cited qualitative reasoning techniques are introduced, followed for a detailed analysis of interval-based methods. This gradual approach, from general to particular, closes with a brief description of SQMA, technique used as case study. The Rough Set Theory, the theory upon which the interval-based representation of qualitative information is enhanced, considering vagueness and uncertainty, is introduced in this section as well.

2.1. Qualitative reasoning

A way of dealing with system complexity is describing system behavior using rules or tables, based on experience and common sense. One approach that implements this idea is *qualitative reasoning*, which approximates human thinking and reasoning by capturing fundamental system behavior in a computer model, while suppressing much of

the detail. Qualitative reasoning is an area of Artificial Intelligence that supports reasoning with very little information, providing promising tools for research and engineering activities.

The first of two processes normally included in qualitative reasoning is Qualitative modeling. It abstracts continuous aspects of real processes, such as space, time and other physical quantities to model the fundamental behavior of a system. The second process is *qualitative simulation*, which uses previously determined qualitative models to predict possible behaviors of the modeled system. In order to achieve this, a qualitative simulator generates an expected output from the real system's inputs based on the qualitative model and compares simulated and actual behavior. Important representatives of qualitative reasoning are the methods introduced by de Kleer and Brown [15], Kuipers [3], Forbus [18,19], Lunze [16] and Laufenberg and Fröhlich [33]:

- De Kleer's approach employs a special *Physics based on Confluences* for qualitative system modeling. Confluences are the qualitative depiction of differential equations, where only the symbols $[-]$, $[0]$, $[+]$ and $[?]$ (for negative, zero, positive and undetermined) are accepted as arguments. Confluences are managed with sign arithmetic represented in a tabular form. This arithmetic includes the derivative function $\delta[x] = \text{Sign}(dx/dt)$ to represent system dynamics.
- Kuipers' *Qualitative Simulation* (QSim) is one of the most cited qualitative reasoning methods. QSim's qualitative model uses an interval-based representation of physical variables and qualitative differential equations. Intervals and qualitative differential equations are operated based on the sign $(+/-)$ of the variables and threshold values associated with qualitative system changes.
- The *Qualitative Process Theory* introduced by Forbus regards the modeled system as a collection of *Objects*, *Quantities* and *Processes*. *Objects* are entities in the universe of the problem that are capable of interacting with each other. *Quantities* describe the continuous properties of *Objects* in terms of ordinal relations, while *Processes* modify the *Objects*, their relationships and their properties. The Qualitative Process Theory also models the possibility of entities being created and destroyed, as part of a dynamic universe.
- Lunze introduced an approach based on the qualitative representation of the process dynamics with nondeterministic automata. This approach considers process states and events as sets of qualities, which results in a *Qualitative Automaton*. If the probability of each particular transition is represented in the model, the resulting state machine is known as a *Qualitative Stochastic Automaton*.
- Laufenberg and Fröhlich's *Situation-based Qualitative Modeling and Analysis* approach (SQMA) models a system using interval-based qualitative descriptions of situations that can take place. These situations, gathered in a table of valid situations, correspond in technical systems to a qualitative description of the finite set of states where the process can reside. SQMA serves as case study for the evaluation of the proposed representation concept and is, for this reason, described in detail in Section 2.3.

The above described approaches have been employed, for example, to monitor and analyze the correct functioning of industrial processes. Many successful process monitoring and diagnosis applications, such as MIMIC [5,6], SQUID [13] and SQMD [30], have been developed based on these methods. However, qualitative modeling has been also applied to the treatment of problems in other areas, such as biological [11] and socio-economical [10] systems, and to support academic activities as in the case of CyclePad [20]. Each technique has strengths and weaknesses, according to its application. In common, they have their capability of handling incomplete information by system modeling.

2.2. Interval based qualitative representation

In mathematics, a container that includes or excludes any given element of a completely defined universe of discourse is called a *set*. According to the Classic Set Theory, an object (element) can either be part of a given set or be excluded; i.e. each element of the universe of discourse is either in the set or out of it. A classic set can represent any idea or concept, as shown in the example in Fig. 1, where the set *C* should contain all the big objects of the depicted universe of six elements.

Even though concept borders in most real problems are not crisp, classic sets are usually employed to approximate them, as it is shown in Fig. 1. But objects 3 and 4 have almost the same size, with object 3 just a little bit bigger. It can be considered, for instance, that object 4 is also big. On the other hand, it is perhaps not big enough. Similar

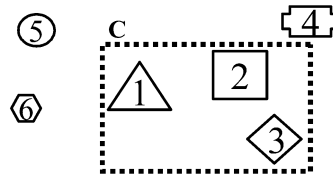
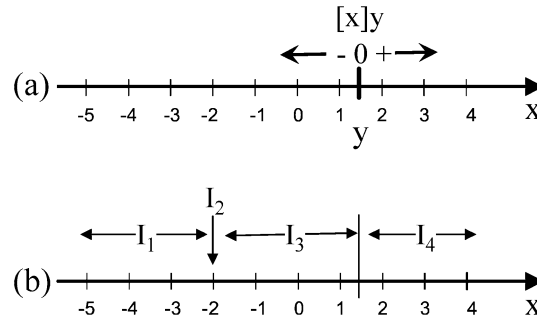
Fig. 1. Classic set C of the big objects.

Fig. 2. Prefix and interval representation.

questions can be formulated about object 3. A classic set resolves only approximately this problem by adopting a trade-off position, such as that of the set C .

This classic set notion can be adapted to continuous variables (e.g. the object's cross area) instead of objects, just by delimiting value ranges in the dimension where these variables are defined. Value ranges are frequently used for the depiction of vague concepts and imprecise values, such as in the case of “big” in the previous example. They are usually associated with specific system characteristics or behaviors enabling the qualitative modeling of a system. Value ranges can be used to represent discrete and logic values as well, which offers a common framework for the qualitative modeling of real process variables and parameters.

A form of value range notation is the prefix sign representation relative to a reference point [15]. In this notation system, a qualitative value is derived from the prefix of the quantitative value in the reference system determined by the referential point. Fig. 2(a) shows this kind of qualitative representation. $[x]y$ is a qualitative variable with the values “+”, “0” or “−”; where x is a variable and y is a parameter that serves as reference point for the qualitative variable $[x]y$. The qualitative variable $[x]y$ obtains respectively, for $x < y$, $x = y$ and $x > y$, the qualitative values “−”, “0” and “+”. It is valid therefore that $[x]0 = \text{Sign}(x)$. De Kleer and Brown's Physics based on Confluences [15] relies on prefix sign representation. Additionally, this notational principle is generally used to represent process dynamics in qualitative models.

Based on the principle above-described, many reference points may be defined at will, in order to generalize the interval bordering. The result of the superposition of these qualitative variables is an interval representation. The interval $[a b]$, with $a < b$, describes the value range between the reference points a and b . For example, $I_3 = [-2 \ 1.4]$ in Fig. 2(b) includes the values 0, 1, and 0.5 among others. Following this notation, intervals of the form $[a a]$ describe the reference point exactly as the corresponding real number a (e.g. $I_2 = [-2 \ -2] = -2$ in Fig. 2(b)). Closed and open interval limits (square brackets and parentheses respectively) define whether a given reference point must be included in the interval or not. In any case, the borders of these intervals are crisp (from here the name “crisp interval”); there is no continuous transition or common point between adjacent intervals. Finally, there are also unbounded intervals in the form $(-\infty a]$ and $[b +\infty)$. They describe the area from $-\infty$ to point a ($I_1 = (-\infty -2]$ in Fig. 2(b)) and the region between the reference point b and $+\infty$ ($I_4 = [1.4 \ \infty)$ in Fig. 2(b)). It is not necessary to portray the entire dimension; it often suffices representing excerpts of it.

For working with intervals, Moore [29] developed the interval arithmetic as an extension of the basic arithmetic operations with real numbers (addition, subtraction, etc.). This arithmetic also redefines the relational operators. A relationship, such as “greater than”, “lower than” or “equal to” is satisfied, as soon as individual points of the compared intervals satisfy it. Equality, for example, is determined based on the existence of intersection (i.e. common points) be-

tween intervals. Relational operators in interval arithmetic are therefore nondeterministic, i.e. for any two overlapping intervals A and B it can be possible to satisfy the three relationships $A = B$, $A > B$ and $A < B$ at the same time. This is the case, for example, of the intervals $(1\ 5)$ and $(3\ 6)$. This nondeterminism reproduces the natural nondeterminism in human qualitative appraisals.

Interval arithmetic enabled the realization of complex calculations with intervals, and thus, the development of the interval-based qualitative methods. Situation-based Qualitative Modeling and Qualitative Automata are two methods based on the representation of physical variables using previously defined intervals. One of the most cited qualitative modeling techniques, Kuipers's Qualitative Simulation, defines intervals whose reference values are dynamically determined during the simulation process.

2.3. Situation-based qualitative modeling and analysis, SQMA

Situation-based Qualitative Modeling and Analysis (SQMA) is a modeling method developed at the Institute of Industrial Automation and Software Engineering of the Universität Stuttgart. SQMA was conceived for safety-related applications, such as hazard analysis [31], fault detection [25,30], diagnosis [2] and reliability assurance [12], which require the comprehensive modeling of complex systems. SQMA models a system using qualitative descriptions of the situations that can take place. These situations, gathered in a table of valid situations, correspond in technical systems to a qualitative description of the finite set of states where the process can reside. Each situation considers for each system variable a particular qualitative value, which is represented by an interval. SQMA modeling works as shown in Fig. 3.

The system modeler structures the whole system hierarchically and decomposes the innermost level (#1 in Fig. 3) into components. After that, component variables are modeled using intervals and characteristic values represented as one-value-intervals (#2 in Fig. 3). Physical rules that will be used for the situation verification are formulated using interval arithmetic to complete the description of each component. The computer-aided modeling of a component checks every possible interval combination (component situation). A given component situation is valid if it complies with all the rules declared for the corresponding component. This makes it possible to detect and to remove impossible

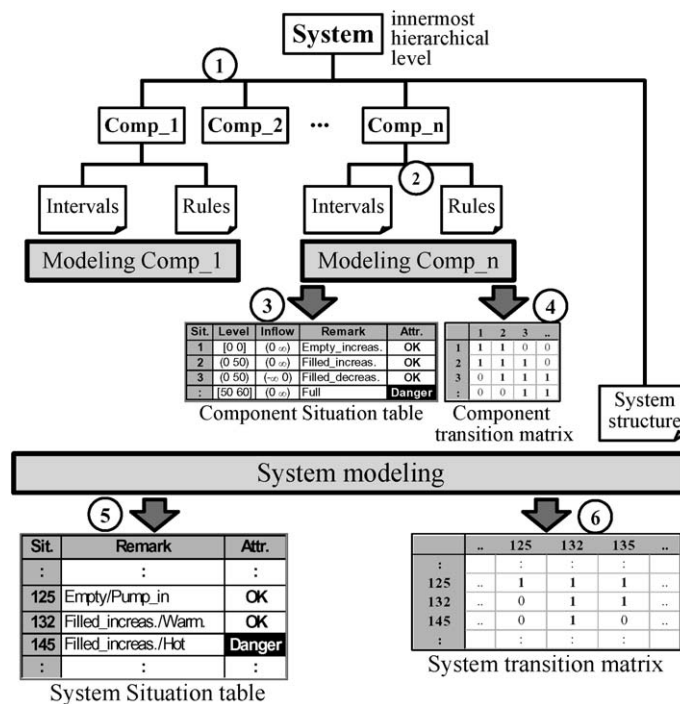


Fig. 3. Situation based qualitative modeling, SQMA.

interval combinations. An SQMA component model is defined upon these valid interval combinations collected in a component situation table (#3 in Fig. 3), which describes the normal behavior of the component.

Envisioning in SQMA components relies on a transition rule testing procedure. This procedure is similar to the procedure implemented for situation validation. It uses the relationships determining the system dynamics, which are described with interval arithmetic as well, to validate the transitions between component situations. The modeling tool checks each possible situation transition and marks (“1”) the valid ones, while invalid transitions remain unmarked (“0”). A quadratic matrix with situations in rows and columns, the component transition matrix (#4 in Fig. 3), collects the result of this validation and completes the component model. This matrix, representing all states the system can take, with all of the transitions between them, is formally known as its envisionment in qualitative reasoning literature. SQMA component models can be reused (in case of similar components in the system) or further combined at system level. Moreover, complete component models can be stored in a component library for future use.

To model the current hierarchical system level, the situation tables of all the components at this level are combined. An automated procedure assembles the rules that describe the relationships between these components and between components and system terminals following the way they are connected with each other. These system rules express material and energy balances, similar to Kirchhof’s laws in electric circuits, and are used to validate coincidences of component situations collected in an initial system situation table. The system rule validation follows the same rule verification primitives employed for situation and transition validation in the components. That results in a situation table, which lists all the possible behaviors of the entire system (#5 in Fig. 3).

The corresponding system transition matrix (#6 in Fig. 3) is built by verifying the component transition matrices after the following rule: *A transition between two system situations (in the system transition matrix) is only possible (and therefore marked), if the corresponding transition (in the component transition matrix) is also possible for each single component.* The envisioning at system level completes the situation based qualitative model of the current hierarchical level. The structure of this model is identical to the structure of component models, and can be similarly reused, further combined or stored in an SQMA model library.

In very complex systems, it can be necessary to resolve several hierarchical levels to model the whole system. For example, pumps, valves, heaters and vessels can be modeled as single components and combined to model evaporators, reactors or distillation columns. These process units can be combined, following different configurations, to form a number of bigger and more complex process units. These process units can then be combined to model first process plants and then entire petrochemical or refinery complexes with a group of plants, tens of processing units and hundreds of single-component types. In these cases, the described procedure is repeated for each hierarchical level until the completion of the system model.

A common problem of qualitative modeling techniques is the size of envisionments that result from modeling (for instance) entire process plants. In the same way, SQMA transition matrices grow very fast with the complexity and size of the modeled system. Even if building up the envisionment hierarchically is faster and easier, the resulting table can be too large for practical use. An advantage of SQMA, however, is the possibility of generating and managing models at multiple levels of abstraction. This allows a convenient definition of the different hierarchical levels to avoid or at least to mitigate this problem.

2.4. Rough Set Theory

Pawlak proposed the Rough Set Theory in 1982 [34] as a method for the joint management of vagueness and uncertainty. It is based on the Classic Set Theory, but is inspired by Zadeh’s fuzzy sets [23]. Reasoning in the Rough Set Theory is a matter of classification, not a matter of degrees of truth. A basic assumption in Rough Set Theory is that any concept can be defined through the collection of all the objects that exhibit the properties associated with it. Therefore, a vague concept, expressed as a vague classification criterion, is always susceptible of being decomposed in two well-delimited concepts (here, two classification criteria) that can be further handled independently. For a vague concept R , it can be formulated:

- a lower approximation R_* containing all the elements that *are surely included* in R , and
- an upper approximation R^* containing those elements that *cannot be excluded*, beyond doubt, from R .

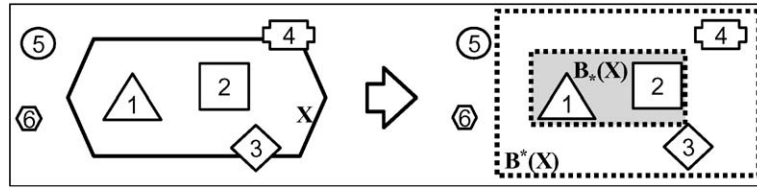


Fig. 4. Vague information with rough sets.

Table 2

Properties of the rough membership

$\rho_X^B(z) = 1$	if $z \in B_*(X)$
$\rho_X^B(z) = 0$	if $z \notin B^*(X)$
$0 < \rho_X^B(z) < 1$	if $z \in BN_B(X)$
$\rho_{U-X}^B(z) = 1 - \rho_X^B(z)$	for any $z \in U$
$\rho_{X \cup Y}^B(z) \geq \max(\rho_X^B(z), \rho_Y^B(z))$	for any $z \in U$
$\rho_{X \cap Y}^B(z) \leq \min(\rho_X^B(z), \rho_Y^B(z))$	for any $z \in U$

Another important premise is that some information about the elements of the universe of discourse is available. If the same information can be associated with several elements of this universe, these elements are indiscernible; i.e. they cannot be separated from each other. Hence, the lower and the upper approximation of a set X , representing the original vague concept, can be defined based on this indiscernibility regarding the available information B :

$$\text{Lower approximation: } B_*(X) = \{z \in U: B(z) \subseteq X\}, \quad (1)$$

$$\text{Upper approximation: } B^*(X) = \{z \in U: B(z) \cap X \neq \emptyset\}, \quad (2)$$

$B(z)$ represents in these definitions the set of all the elements of U , which are indiscernible from z based on the information in B .

Fig. 4 illustrates the classification-based notion of rough set. The left side shows the elements $z \in U = \{1, 2, 3, 4, 5, 6\}$ to be classified according to their membership to a vague concept X (e.g. representing the feature “big”) and the available information B . The right side of the figure represents the vague concept X following rough set’s principles. The inner set B_* (lower approximation) comprises elements $\{1, 2\}$ that are unequivocally included in X . The outer set consists of elements $\{5, 6\}$ that definitively are not part of X (e.g. they are small objects). The classification of $\{3, 4\}$ is uncertain, e.g. it is not possible to decide whether they are big or small. Elements $\{1, 2, 3, 4\}$ define together the upper approximation of X .

The vagueness problem is so transformed in an uncertainty problem, which is concentrated in the set $B^*(X) - B_*(X)$, known as the Boundary region ($BN_B(X)$) of the concept X based on the information B . This boundary set represents the uncertainty in the concept, where no solid conclusion can be reached about whether objects are big or small. A probability-based rough membership function is then necessary to address the uncertainty problem. This rough membership can be defined using the indiscernibility relationship:

$$\rho_X^B(z) = \frac{|X \cap B(z)|}{|B(z)|}. \quad (3)$$

The symbol ρ will be used to represent rough membership instead of the generally used μ , with the intention of explicitly marking the difference with fuzzy membership, where the symbol μ is also employed. The function $|\dots|$ represents the size of the set, i.e. the number of elements in it. Table 2 shows the most important properties of the rough membership function (see [35] for a complete list). There is no further restriction about the form of a rough membership function beyond these properties.

3. Enhancing intervals considering vagueness and uncertainty

Most variables in industrial processes are defined in continuous numerical spaces, and most of these continuous variables can only change their values gradually, because their behavior is bound with the energy flow in the system.

Under these conditions, representing dynamic systems based on crisp intervals is very imprecise. Crisp intervals provide static views of the continuous dynamic variables; therefore, interval-based models can only provide static views of reality.

This section describes a new approach for partitioning continuous variables, which is based on Rough Intervals. The approach takes advantage of the representation and management of vagueness and uncertainty in interval-based qualitative models, applying principles of the Rough Set Theory to improve the model precision and compactness.

3.1. Interval-based management of uncertainty and vagueness

The term “incomplete information” is usually cited in association with interval-based qualitative modeling. However, this term does not necessarily mean that the system information is incomplete in the sense of its availability (partial or total ignorance) or in any way insufficient. For instance, even though intervals can be arbitrarily chosen, a deep knowledge about the system may support a better partition of the variable spaces. Incomplete information in qualitative modeling can be associated with uncertainty or ignorance about the process, but also with the deliberate simplification (abstraction) of system information before using it to build a model. Crisp intervals are successfully employed to approximate this kind of “incomplete” information. They allow representing the most probable parameter values or scenarios in qualitative relationships. Another problem, however, regards the uncertainty arising from the nondeterministic nature of qualitative models [24], in the form of the typically huge envisionments delivered by qualitative reasoning methods. Interval-based qualitative models do not offer indexes or measures such as probabilities or possibilities [4], which could support the analysis of a solution space containing multiple alternative behaviors.

A different aspect is the modeling based on approximate or inaccurate descriptions of a system. Modeling vague level concepts such as “too high” or “almost empty” would require defining new intervals for each one of these cases, but the definition of these additional intervals would mean enlarging the model drastically. The space of a qualitative model grows in a geometrical progression with the number of intervals defined on its variables. Thus, it is necessary to achieve a fair qualitative representation of vague information without increasing the model size.

An alternative way of representing vagueness is using weighting values to express the confidence on the available information. Qualitative modeling methods, such as FuSim [28], QuaSi [1] and FRenSi [32], follow this approach. They introduce the management of vague information using fuzzy sets and fuzzy numbers. FRenSi and QuaSi, for instance, preprocess vagueness using fuzzy numbers and then transform the intermediate results into equivalent intervals to continue operating with interval arithmetic, while FuSim adapts QSim and Moore’s interval arithmetic to work with fuzzy sets, which results in a complex hybrid fuzzy/interval notation and an extended fuzzy arithmetic. However, none of these methods take uncertainty into account.

A new method is required, which offers a consistent solution for the compact modeling and further handling of vagueness and uncertainty in interval-based qualitative models. Hence, a method based on rough sets is proposed. Rough Set Theory unifies vagueness and uncertainty as different aspects of the same problem. Additionally, information representation with rough sets is based on crisp sets, which is less demanding than representing and processing continuous fuzzy membership functions.

3.2. Rough intervals

Rough sets are defined over sets and elements and therefore cannot be used for the representation of continuous variables [9,22] as it is the case with crisp intervals and fuzzy sets. Rough Intervals are introduced to adapt the rough set principles to the modeling of continuous variables. A Rough Interval (RI) is a particular case of a rough set. They fulfill all the rough set’s properties and core concepts, including the upper and lower approximation definitions. A Rough Interval comprises two parts: an Upper Approximation Interval (UAI) and a Lower Approximation Interval (LAI). The variable could assume inside of the UAI the represented qualitative value (a vague concept in rough sets), or what is the same, it is clear that the variable cannot get this qualitative value outside of this interval. The second element of a RI, the LAI, can be also redefined on this basis; In the LAI it is sure that the variable takes the represented qualitative value.

The Lower and Upper Approximation Interval concepts satisfy the mathematical definition of rough set’s upper and lower approximation in (1) and (2), adapted to intervals. If a particular qualitative value C must be represented over a variable, the two enveloping intervals, A^* and A_* , can be defined. The implications (4) and (5) represent the

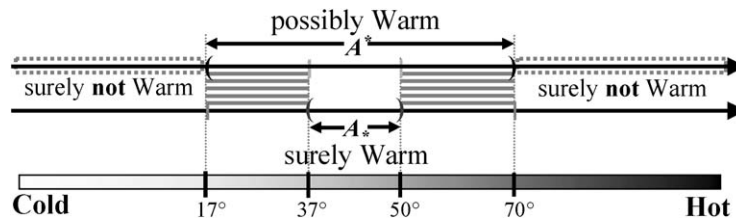


Fig. 5. “Warm” represented as a Rough Interval.

relationship between the variable $x \in U$ (where the universe U can be any complete set of continuous or discrete values), the qualitative value A defined on x , and the intervals A^* and A .

$$x \in A^* \Rightarrow x \in A, \quad (4)$$

$$x \notin A^* \Rightarrow x \notin A. \quad (5)$$

No special knowledge is required to use these simple definitions as design rules. They do not complement each other, in fact, the second one is a subset of the first one; the Lower Approximation Interval can be and must be defined inside of the Upper Approximation Interval. For instance, the definition of “Warm” in a temperature variable may be considered. Common sense and general knowledge can help defining its limits, for example:

- The temperature of the human body is about 37 °C. Nothing warmer will be considered “Cold”.
- In average, a human hand cannot hold an object, whose temperature is over 70°C, because it is “Hot”.
- If something is colder than the environment (let us say 17 °C), it cannot be considered “Warm” anymore.

A few statistical considerations, together with some sense of symmetry may assist an expert completing the definition of this Rough Interval and its neighbors. A set of two crisp intervals can represent the resulting RIs. The Rough Interval shown in Fig. 5 represents the qualitative value “Warm”. The key fact in this example was the use of precise concepts to define an imprecise one. They may be supported by verifiable knowledge, statistics or physical laws, which are in general measurable, trustworthy and easier to model than the original vague concept.

The Rough Set Theory reduces the vagueness of a concept to uncertainty areas at their borders. Within these uncertainty areas, in Rough Intervals as in rough sets, no definitive conclusion about the problem is possible. Hence, rough membership values (ρ) must be defined for Rough Intervals as well. These rough memberships membership values, expressed in rough membership functions, must satisfy all the conditions listed in Table 2 and [35], but also the following properties:

Complementarity. RI overlaps are transient areas between two clearly typified regions (LAIs, where the membership of any value to the RI is 1). In an overlap, the variable changes gradually from one quality to another, like from “warm” to “cold” or “short” to “long”. Thus, the rough membership of a given real value inside of this transition area to the one RI decreases, as the rough membership of the same real value to the adjacent one grows. Consequently, totalizing the rough membership of any value to its corresponding RIs will always result in 1.

Monotonicity. In a numeric space, there is a relation of precedence and consequence, an order among its elements, which determines the necessity of using monotonic growing and decreasing functions in the boundary regions of a Rough Interval to represent these rough membership values.

Border Conditions. Rough membership is an expression of the probability of being part of a clearly defined concept, which is in RI delimited by the LAI. Thus, and considering the properties above, a given number in the boundary region is part of the represented concept with a probability that increases with the proximity of this number to the LAI, where this probability, and consequently the rough membership function, reaches its maximum (1). For the same reason, a rough membership function has its minimum (0) at the borders of the UAI.

Vague intermediate qualitative values such as “between warm and cold”, “almost cold” or “not warm enough” can describe the uncertainty in the boundary regions, or what is the same, vague qualitative values can be represented as the

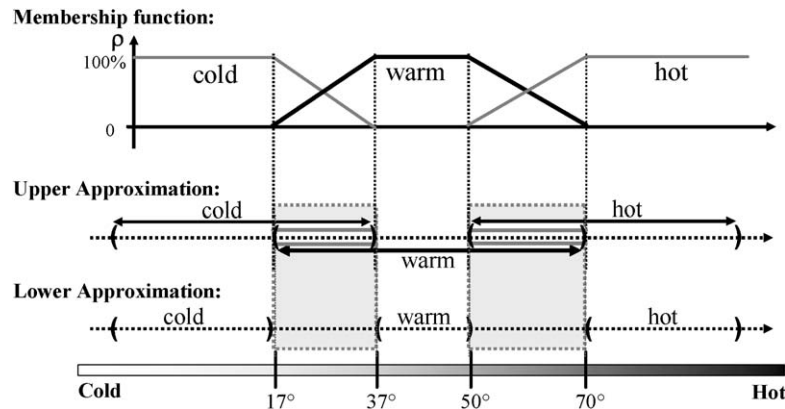


Fig. 6. The Rough Interval notation.

superposition of two clearly defined intervals. By analyzing a particular value inside of these overlaps, the qualitative values corresponding to both intervals must be considered together with their complementary rough memberships. This enhances the precision and descriptive power of the model in the transition areas, where a higher degree of detail is usually required.

For practical reasons a straight linear function is adopted to describe rough interval membership (Fig. 6, top). It not only satisfies all the conditions described above offering a trade-off among the infinite possibilities, but also implies the lowest calculation and representation effort, which at the end results in a more efficient model processing. This linear representation of the probabilistic rough membership function corresponds to the assumption of a uniform probability distribution, which is adopted, as a rule, as worse case scenario for the representation of systems under uncertainty, i.e. when no information about the probabilistic distribution of the modeled events is available.

The separate representation of UAIs and LAIs makes evident the redundancy in this notation (shaded area in Fig. 6). First, the empty spaces in the Lower Approximation Intervals coincide exactly with the overlapping of Upper Approximation Intervals. Second, the Rough Interval membership function can be totally reproduced from any of both segments, without further parameters. Hence, just the overlapping representation of the UAIs will be taken as notation for the Rough Intervals, which is not different from the interval-based notation employed for qualitative modeling.

The described representation and interpretation framework, based on the UAIs, could have been developed using the LAIs instead. However, the superposition of qualitative features (overlaps) is a notion that is closer to the human understanding (e.g. “almost cold” can be understood as cold and warm at the same time) than representing featureless empty areas with qualitative descriptors assigned ad hoc (“almost cold” \neq “neither cold, nor warm”). The chosen approach, representing UAIs, allows handling Rough Intervals independently from their overlaps during the modeling. Overlaps emerge from the evaluation of a real situation based on the qualitative model.

Indiscernibility [34], a property defined inside of the Rough Set Theory, allows the extraction of precise information from vague concepts, by comparing these vague concepts with the available information about the problem. Based on these properties Rough Intervals associate qualitative values to the different value ranges, as crisp intervals do. Furthermore, RI can represent the indiscernibility in the region between contiguous qualitative concepts, providing a new framework for the interpretation of interval-based qualitative models based on indiscernibility.

The resulting crisp-interval-like notation is crucial for the integration of Rough Intervals in interval-based qualitative methods. So represented, there is no difference between Rough Intervals and overlapping crisp intervals. This representation assures a minimal impact with the introduction of the Rough Interval concept in interval-based modeling procedures. Qualitative models with crisp intervals and RIs are identical. Only the interval overlaps, which are not permitted in the conventional interval-based qualitative methods, indicate the use of RIs.

4. Qualitative computing with Rough Intervals

The question: “how do a qualitative model profit from using RIs instead of common crisp intervals?” must be answered for each particular qualitative modeling and reasoning approach separately. Section 5 makes a detailed analysis of the use of RI in SQMA as case study. Similar analysis can be performed for other qualitative and hybrid

techniques such as QSim, SQMD or Lunze's Qualitative Automata. On the other hand, the utility of using RIs in qualitative models and reasoning based on these models must take into account the complexity of computing with RI and how difficult is their determination for a real problem. These two aspects (computing with RI and RI modeling) are addressed in the following sections in detail, taking the modeling and the operations with crisp intervals and fuzzy sets as reference.

4.1. Rough Interval arithmetic

As it was explained in Section 3.2, RIs can be completely represented in a qualitative model using only the UAIs because of the probabilistic complementarity of contiguous RIs. Nevertheless, computing with RI requires using every single RI isolated from other RIs in the same process variable. For this reason, each RI in a single-interval notation (as it is found in the model) must be changed to its equivalent double-interval notation before being used in arithmetical or logical operations. This double-interval notation declares UAI and LAI of the Rough Interval explicitly, separated by a colon (UAI : LAI).

This extended RI notation made possible the definition of arithmetic RI operations based on Moore's Interval Arithmetic [29]. The resulting Rough Interval Arithmetic (RIA) separates an arithmetic operation, such as addition or subtraction, into two identical interval operations. The first one operating over the UAIs and the second using the corresponding LAI as operands. No attention must be paid to membership functions during these operations. For example, with the Rough Interval $A = [4\ 9] : [5\ 8]$ and $B = [2\ 8] : [4\ 6]$, the following operations can be performed:

$$\text{Addition: } A + B = [4\ 9] : [5\ 8] + [2\ 8] : [4\ 6] = [4 + 2\ 9 + 8] : [5 + 4\ 8 + 6] = [6\ 17] : [9\ 14];$$

$$\text{Subtraction: } B - A = [2\ 8] : [4\ 6] - [4\ 9] : [5\ 8] = [2\ 8] : [4\ 6] + [-9\ -4] : [-8\ -5] = [-7\ 4] : [-4\ 1];$$

$$\text{Negation: } -A = -[4\ 9] : [5\ 8] = [-9\ -4] : [-8\ -5].$$

Set and logic operations exploit the "set" nature of the Rough Intervals. The set operations *unification*, *intersection* and *complementation* are defined for the RIA. The same analogies between set and logic operations while working with crisp intervals and fuzzy sets (AND \Leftrightarrow intersection; OR \Leftrightarrow unification; NOT \Leftrightarrow complement) can be translated and adapted for RIs. For the unification of Rough Intervals no changes were introduced, since this operation coincides with that defined for rough sets (see shaded area in Fig. 7(a) for $A \cup B$).

The complement of a Rough Interval must be clearly disjointed from the sign inversion operation (Negation), which regards the interval (not the set) nature of the Rough Intervals. To avoid notational ambiguity, as symbol for RI complement, its logical equivalent " \neg " (NOT) is used instead of the traditional " $-$ ", which remains consequent with Moore's interval arithmetic. The complement operation results in the set of all the elements in the universe of discourse that are not contained in the RI introduced as argument for this operation. The shaded area in Fig. 7(c) shows the set $\neg A$.

By the application of the intersection (compare with shaded area in Fig. 7(b) for $A \cap B$), however, particular conditions may arise that require further definition. An arithmetic operation over one or two RIs must always result in a RI. However, the result of a RI intersection may not fulfill the defined notation conventions, as in the intersection of A with the RI $C = [3\ 5] : [4\ 4]$ in Fig. 8. $A \cap B$ produces a new RI $[4\ 5] : [5\ 4]$ (see Fig. 8) with a maximum membership of 0.5, and where the end of the LAI precedes its starting point. In this case, a maximum rough membership lower than one would denote that there is no certainty region about this concept. This kind of RI is called "Minor". Minor

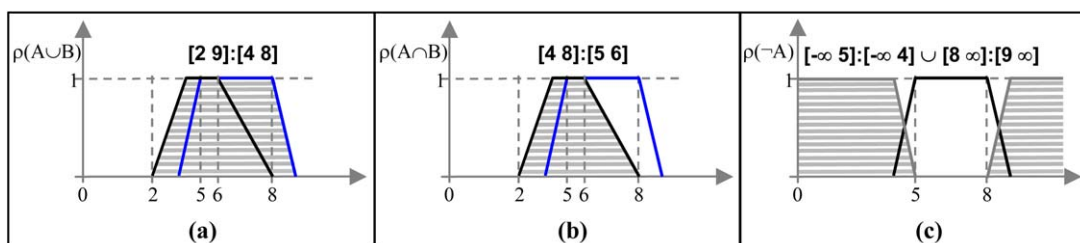


Fig. 7. Set operations in RAI.

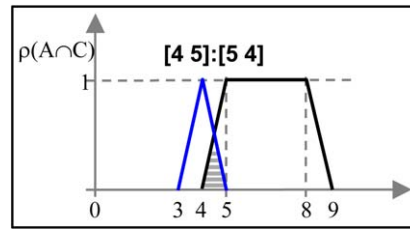


Fig. 8. RI intersection.

RIs represent qualitative values or logic statements without 100% of certainty area, but that cannot be completely discarded.

In general, a logic statement is TRUE (logic 1), if a non-empty RI can be found as result of the logic operations that describe this statement. An empty or non-existing RI would indicate that the logic expression is FALSE (logic 0). The effect of a minor RI resulting from an AND operation can be interpreted as a confidence between 0 and 1 for a TRUE result instead of a clear TRUE or FALSE. “Rough Interval confidence” is introduced here as the maximum membership value that can be reached within a minor RI. According to this definition, RI confidence is always 1 for normal RIs. This value represents the “degree of truth” of the evaluated logic statement.

Even though each arithmetic operation with RI must be performed twice (once with the UAIs and then with the LAIs), the single operations are not more complex than computing with crisp intervals. The same applies for logic and set operations. Only the final interpretation of the RI intersection was enhanced to accommodate the management of vagueness and uncertainty through the definition of a confidence factor. However, as comparison, set and logic operations with RI are simpler than the same operations with fuzzy sets. Fuzzy sets must handle continuous membership functions and cannot take advantage of Moore’s interval calculus; in the way RIs do to reduce computational complexity.

4.2. Modeling of Rough Intervals

Determining an adequate partitioning of the process variables is one of the most challenging tasks in building interval-based qualitative models. This is also the case when Rough Intervals are to be used. Nonetheless, techniques adapted from other areas such as Artificial Intelligence, probabilities and curve analysis can support the determination of a basic set of Rough Intervals that can be then adapted to fit the problem requirements. Some approaches to RI modeling [26] are:

- (a) *Heuristic definition of Rough Intervals.* Determination of RI based on the interpretation of the variable according to the definitions of Upper and Lower Approximation Intervals. First, the different qualitative descriptors are separately defined based on the available knowledge about the variable, as in the example of “warm” in Section 3.2. The resulting RIs, put together, define a sub optimal partitioning of the space of the variable (compare Fig. 6).
- (b) *Experimental definition of Rough Intervals.* In many applications, the knowledge about the process and process variables is insufficient to allow the complete definition of RIs based on the previously described heuristic definition. Assuming that some structural and behavioral information is available, the still-required knowledge about the process parameters may be gained through experimentation. This approach looks for “interesting points (points associated with alarms, events or important changes in the process) using an approximate model of the system. An alternative modeling procedure is based on the concurrent simulation of the system behavior and ad-hoc defined monitoring application. Initial values for RIs can be arbitrarily chosen and gradually tuned by trial and error [8, 24].
- (c) *Episodic interval identification.* The Interval Identification method [7] can be adapted to support the identification of Rough Intervals as well. It decomposes an experimental run of the system, where the input variable (the one to be modeled) monotonically increases or decreases, into qualitative “episodes”. Episodes are defined through the combination of the sign of the input variable, the sign of the system response and the first and second derivatives of this response. Those value areas of the variable where there is no change of episode would correspond to the

Table 3
Comparison of qualitative representation principles

	Fuzzy subset	Rough Interval	Crisp interval
Representation of	Vagueness	Vagueness and uncertainty	Computational/ measurement imprecision
Membership functions	Trapezoid, Gaussian, sigmoid, ...	Crisp In/Out discrimination with straight membership in boundary reg.	Crisp In/Out discriminations
Overlaps	No restrictions or conditions	Mandatory total overlapping of boundary regions, not allowed in LAI	Not allowed
Cross-dependency	Each set is defined independently	Probabilistic complementarity	No overlaps and no empty spaces allowed
Total membership Interpretation	$\sum \mu > 1$ allowed (Fuzziness) Membership or degrees of truth, possibility	$\sum \rho = 1$ at any point (Probability) Probability of being member of the interval	$\sum m = 1$ at any point ($m \in \{1, 0\}$) Decision: inside or outside of the interval

same interval. The undefined region between two stable sequences of episodes would correspond to the overlap of contiguous RIs.

In general, Rough Intervals may be represented as two superposed intervals with straight lines in the boundary regions. So results a trapezoid, with the first interval, i.e. the UAI, as base. The second interval delimits the “lower approximation”, the region where the concept certainty is total. The terms “upper” and “lower” in Rough Intervals (and rough sets) are not used regarding their graphic representation. This trapezoid representation of RIs, however, is very similar to the one frequently used for fuzzy membership functions. An UAI in a Rough Interval would correspond to the “base” of the Fuzzy Interval, whereas the LAI corresponds to its “top” or “core”. A fourth approach for the determination of Rough Intervals takes advantage of these similarities between fuzzy membership functions and Rough Intervals.

(d) *Rough Intervals definition based on fuzzy set identification.* Tools and methods, such as clustering and neuro-fuzzy approaches, have been developed for the identification of sub-optimal variable space partitions in fuzzy set applications, using sampled process data. Most of these concepts can be adapted for the determination of RIs as well. Adaptation is required because the identification process must fulfill the particular features of RIs: First, fuzzy sets and membership functions are identified. Second, the membership functions of the fuzzy sets resulting from this first step are converted into trapezoid functions. Third, these trapezoid membership functions are transformed into Rough Intervals, which assures that the resulting trapezoid covers the entire variable space and complies with characteristics of the RI overlapping [17] such as complementarity.

Nonetheless, the described similarities of fuzzy sets and RI must not lead to confusion between both techniques. Table 3 summarizes some important characteristics and properties of Rough Intervals and compares them with that of crisp intervals and the equivalent application of the membership of continuous variables to fuzzy subsets for their qualitative representation.

5. Case study: Situation-based qualitative modeling with Rough Intervals

The previous sections made a general description of Rough Intervals and their application to the qualitative representation of continuous variables, using vague concepts such as “warm”, “far” or “low”. Now it is described the actual integration of RIs in the interval-based qualitative modeling approach SQMA. This will support the evaluation of the effect of using RIs, instead of crisp intervals, for the qualitative modeling of dynamic systems.

5.1. Enhancing SQMA with Rough Intervals

The Rough Interval concept is compatible with the interval-based information modeling and representation scheme used in SQMA. By modeling a component variable using RIs, i.e. declaring the RIs’ Upper Approximation Intervals, characterizes the complete variable space. SQMA processes UAIs, exactly as if they were crisp intervals, in order to

produce component and system situation tables, as well as component and system transition matrices. The resulting model differs physically from ordinary SQMA models only in the overlaps between their declared intervals.

Interval overlapping and membership functions enhance the information contents in SQMA models. A variable defined over n Rough Intervals (which defines the size of the model) contains information distributed in n LAIs plus $(n - 1)$ overlapping areas, i.e. in $(2 * n - 1)$ sections. Because each one of these $(2 * n - 1)$ sections can be independently perceived and interpreted, the resulting model delivers almost twice as much information as an SQMA model using crisp intervals, which only can identify n different qualitative values. For instance, “Hot” (with $\rho = 1$), “Cold” (with $\rho = 1$) or both qualitative values simultaneously, with different but complementary ρ values, correspond to three different states. The third case could be described as “moderately Hot”, “between Hot and Warm” or “almost Hot”, depending on the relationship between the membership values and given linguistic conventions. Vague linguistic expressions can be used and generated from the model, which are closer to the human way of description.

Another effect is the superposition of system situations in the model. As a result of the interval overlapping, multiple situations with different, but complementary, membership values (ρ) appear, which describe transient system conditions that otherwise would remain invisible in the qualitative model [25]. Eq. (6) shows the calculation of the global rough membership of a particular system situation (ρ_{SIT}) for a given transient system condition. ρ_{SIT} is computed as the multiplication of the rough membership of each process value (x_i) in the described system condition to its corresponding RI ($\rho(x_i)$, compare Fig. 6, top). Based on the probabilistic interpretation of the rough membership, ρ_{SIT} would correspond to the compound probability of all the qualitative values that define the situation, taking place simultaneously.

$$\rho_{\text{SIT}} = \prod_i \rho(x_i). \quad (6)$$

The following example illustrates the improvement in the descriptive power of SQMA models through the identification of multiple situations for each process state. Fig. 9 shows two possible SQMA descriptions of a given process state. A single situation (top) results of using an SQMA model, where only crisp intervals are defined. At most, a plant operator would interpret this information as: “The system is in a normal situation. The tank is filled with warm liquid and has nominal inflow and outflow”.

The second table (Fig. 9, bottom) corresponds to an SQMA model with Rough Intervals. The membership values for each situation are calculated using (6) for the following process condition:

Inflow = 85 ml/sec → Nominal ($\rho = 0.6$); High ($\rho = 0.4$).

Level = 50 cm → Filled ($\rho = 1$).

Outflow = 70 ml/sec → Nominal ($\rho = 1$).

Temperature = 56 °C → Warm ($\rho = 0.7$); Hot ($\rho = 0.3$).

So results for Sit 134, for example, a membership value $\rho_{\text{SIT}} = 0.6 * 1.0 * 1.0 * 0.7 = 0.42$. Now the same process state could be described, based on the several identified situations, as: “The liquid filling the tank is moderately hot (between “Warm” and “Hot”). The tank inflow is a little high (between “High” and “Nominal”) while the outflow is nominal; the inflow is in any case greater than the outflow, therefore, the level is increasing. With that, the system

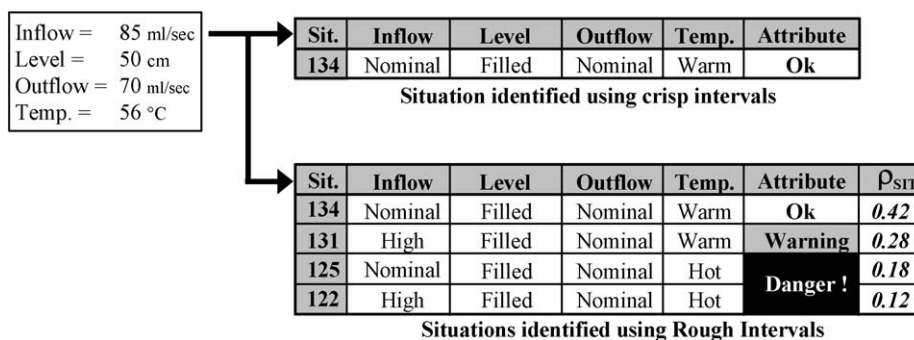


Fig. 9. SQMA description of a process state with and without Rough Intervals.

transits towards a dangerous state". The descriptive power of the SQMA model using RIs is visibly superior to that using crisp intervals, even though the number of situations in both models (and consequently their sizes) is the same.

The above-described property of qualitative models with RI also implies that it is possible to reduce the size of the model by keeping the original resolution. Each overlapping area can be precisely identified and handled in the SQMA model with RIs, which allows their interpretation as independent intervals. In consequence, the number of intervals defined in a variable may be reduced, following the considerations presented at the beginning of this section, in a relation $0.5 * (n + 1) : n$ without losing information in the model. That is, using RIs to describe a given process variable can reduce the initial number of situations in a SQMA model to e.g. 2/3, 3/5 or 5/9 of the amount of situations it would have had, if the same variable would have been described with 3, 5 or 9 crisp intervals instead.

This is an important size reduction for SQMA models, whose situation tables grow geometrically with the amount of intervals defined on the system's variables. For the same case described in the previous paragraph, the initial SQMA model space (before validating against the situation rules) can be compressed to 4/9, 9/25 or 25/81 of its original size. The final ratio of model size reduction is a geometric function of how many variables are described with R and how many intervals are defined on them. In general, assuming that the n crisp intervals in m process variables (for simplification, assume also that these process variables are equally partitioned) are replaced with $(n + 1)/2$ RIs, it can be verified that:

$$\lim_{n \rightarrow \infty} \{\text{new size}\} = \frac{1}{2^m} \{\text{original size}\}. \quad (7)$$

An analogous analysis can be made for the resolution increase by keeping the number of intervals, i.e. when a RI substitutes each crisp interval in a process variable. The model resolution can grow up to 2^m (when $n \rightarrow \infty$) the original one, if m variables are expressed using RI. These two competing tendencies (size reduction and resolution enhancement) originate two opposing ways of considering the employment of Rough Intervals. At the end, as in most engineering solutions, the problem is reduced to a trade-off between both criteria.

The situation membership (ρ_{SIT}) provides an important criterion to order, rank, classify or filter the delivered information. However, ρ_{SIT} must not be understood only as a post-processing criterion. Using multiple situations with their respective ρ -values to represent a process state (as in Fig. 9, bottom) allow for a better simulation, monitoring and analysis of dynamic systems. The different situation membership values would continuously change in a time-varying system, in a direction, sense and magnitude, which corresponds to the system dynamics. Qualitative models based on crisp intervals (e.g. Fig. 9, top) cannot deliver this kind of information.

5.2. Spurious situations in the SQMA model

A consequence of using RI in SQMA is that some impossible situations may remain in the situation table after completing the model. These spurious situations are component or system situations that are reachable according to the model, but actually cannot take place in real systems. The adjective spurious denotes, in this case, situations that accredit false descriptions to the real system state. Because of the RI notation, a situation is accepted if the Upper Approximation Intervals defined in this situation satisfy all the component or system rules. Since UAIs are bigger than the equivalent crisp intervals, i.e. crisp intervals with limits in the midpoints of the overlaps, using RI results in the acceptance of situations that the original SQMA method would reject.

Analyzing the example of the system rule $\{T1.h = \text{Valve.dp} + T2.h\}$ corresponding to the system Tank-Valve-Tank shown in Fig. 10, may help to the comprehension of this problem. T1.h and T2.h are the pressure heads in the output of tanks T1 and T2 respectively, while Valve.dp corresponds to the pressure loss across Valve. The combinations of the intervals defined in these variables (Fig. 10, right) are validated against this rule, which results in the situations shown in Table 4 (the last column, "Confidence" is described in the next section). No invalid interval combination resulted from the rule evaluation (compare the sequence in the situation numbers).

A variable partitioning using the equivalent crisp intervals results in the elimination of situations 4, 6, 7 and 9 (italic in Table 4). If their respective descriptions are considered, these four situations are impossible, for example situation 6 claims the existence of water flowing from a tank with low level to a filled one, while situations 4 and 9 indicate that there is no flow between two tanks with different levels. Yet these four situations are not eliminated from the system situation table, because they satisfy the rule $\{T1.h = \text{Valve.dp} + T2.h\}$ when evaluated with the UAIs of the defined Rough Intervals.

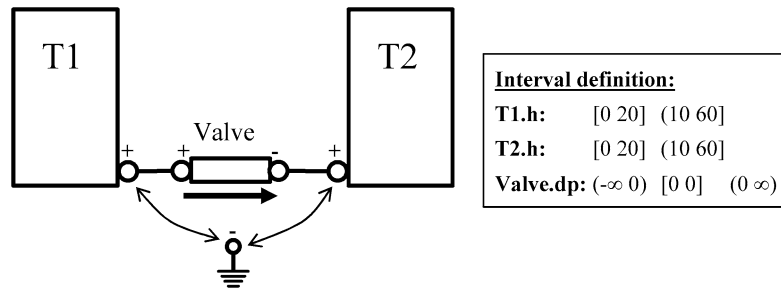


Fig. 10. The Tank-Valve-Tank system.

Table 4
Situation table for the system Tank-Valve-Tank

Sit	T1.h	Valve.dp	T2.h	Description	Confidence
1	[0 20]	$(-\infty 0)$	[0 20]	low \leftarrow low	1
2	[0 20]	$(-\infty 0)$	(10 60]	low \leftarrow filled	1
3	[0 20]	[0 0]	[0 20]	low–low	1
4	[0 20]	[0 0]	(10 60]	low–filled	0.5
5	[0 20]	(0 ∞)	[0 20]	low \rightarrow low	1
6	[0 20]	(0 ∞)	(10 60]	low \rightarrow filled	0.5
7	(10 60]	$(-\infty 0)$	[0 20]	filled \leftarrow low	0.5
8	(10 60]	$(-\infty 0)$	(10 60]	filled \leftarrow filled	1
9	(10 60]	[0 0]	[0 20]	filled–low	0.5
10	(10 60]	[0 0]	(10 60]	filled–filled	1
11	(10 60]	(0 ∞)	[0 20]	filled \rightarrow low	1
12	(10 60]	(0 ∞)	(10 60]	filled \rightarrow filled	1

Spurious situations appear because of the uncertainty inside of RI overlaps and the no determinism in the interval equality operator (see end of Section 2.2) that is also present in Rough Interval Arithmetic. The combination of these two factors causes the SQMA modeling procedure to accept impossible situations during the rule verification. Setting intervals and reference values conveniently to avoid evaluating component rules in RI overlaps can resolve this problem at component level. However, this is impossible with the system rules, because they are determined automatically during the modeling process. This demands enhancing the SQMA situation analysis at system level for the reduction of spurious situations.

5.3. Confident-based reduction of spurious situations

Using Upper Approximation Intervals to represent RIs causes spurious situations to appear in the system situation table. On the other hand, using Lower Approximation Intervals instead may cause the rejection of situations that are actually possible. A rule evaluation against LAIs should result in less than or, at most, the same number of situations resulting from evaluating the equivalent crisp intervals. A trade-off between these two extreme cases is developed based on the RI intersection (see Section 4.1), which is always in SQMA the operation closing the evaluation of a rule. Fig. 11 shows the three possible outcomes for this operation:

- The RI intersection does not exist (top-left).
- The intersection exists and delivers a conventional RI (top-right).
- The intersection exists, but delivers a minor RI (bottom).

The maximum value reachable by the corresponding membership functions is therefore: exactly zero, exactly one or a value in between (one and zero excluded) respectively. This value can express the confidence in the satisfaction of the rule evaluated with the RI intersection:

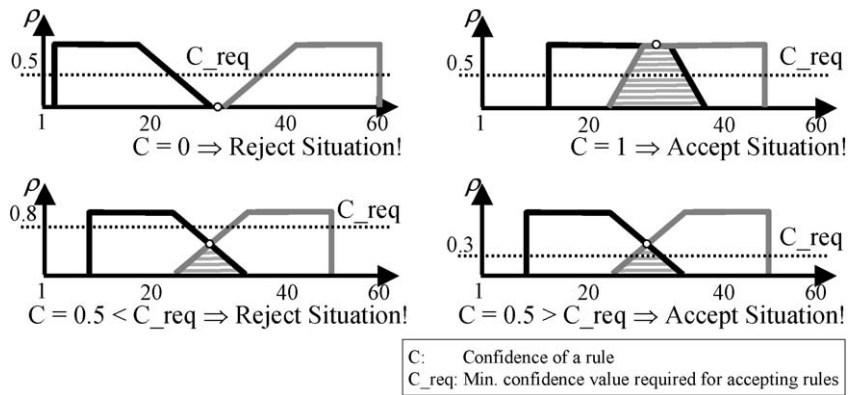


Fig. 11. Confidence-based rule evaluation with RI.

- Maximum membership value is zero \Rightarrow the rule is not fulfilled.
- Maximum membership value is one \Rightarrow the rule is fulfilled.
- Maximum membership value in between \Rightarrow no direct decision is possible.

The third case results in a number between zero and one, which is proportional to the confidence in the fulfillment of the rule by the corresponding situation. This value is called rule confidence (C), and is defined as *the maximum membership value in the Rough Interval intersection that determines the verification of this rule for the current situation*. Rule confidence is a value between zero and one, where $C = 0$ denotes an empty RI (no intersection), $C = 1$ a trapezoidal RI and $C \in (0,1)$ a minor RI.

By using rule confidence, it is possible to define thresholds to decide, during the situation validation process, whether a given rule is fulfilled or not. This enables the reduction of spurious situations in the SQMA model. Resuming the example of the Tank-Valve-Tank system, a confidence $C = 0.5$ results from the rule evaluation with situation 9 (s. Table 4, last column). This is a triangular RI with a maximum value of 0.5 (compare the intersections in Fig. 11, bottom). Establishing $C > 0.3$ as acceptance criterion for the rule validation would determine its acceptance. This minimum confidence value required for accepting a rule is called C_{req} . The example in Fig. 11, bottom-right corresponds in consequence to defining $C_{req} = 0.3$. On the contrary, this situation must be eliminated from the model, if a confidence higher than 0.5, e.g. $C > 0.8$, is demanded (compare Fig. 11, bottom-left, $C_{req} = 0.8$). The same analysis is also applicable to situations 4, 6 and 7. Establishing the rejection of any situation with a confidence under 0.5 ($C_{req} = 0.5$) would result in a system situation table similar to the one resulting from using equivalent crisp intervals for the tank levels, i.e. without situations 4, 6, 7 and 9.

Eliminating situations with a small confidence value reduces the spurious situations in the situation table. This reduces the size of the SQMA model and supports a better interpretation of process states by avoiding conceptual inconsistencies in situation remarks such as “liquid flowing from a tank with low level to a filled one” in situations 6 and 7 of the Tank-Valve-Tank system. But how can an adequate value be determined for C_{req} ? The answer to this question can be supported on the fact that the total membership of any particular value of a variable to the RIs defined for this variable must be equal to one. This rule arises from the total probability condition for complementary events and must be fulfilled even inside of the overlaps between two RIs. Hence, there exists a complementarity condition (see Section 3.2) between the membership values of a given point to the two RIs in the overlap: the probability of being part of one RI or the other must be one. So, if situations resulting with a confidence lower than 0.5 (or any C_{req} under 0.5) are eliminated during the situation validation, there must always exist at least one complementary situation that represents the current process state better than the one eliminated.

Another approach to explain the effect of C_{req} on the validation process is based on the α -cut operation. Nguyen ([14] cited by [1]) proved that by computing the value of a fuzzy mapping, a generic h -level α -cut of the resulting set only depends on the h -level α -cuts of the arguments:

$$y_h = F(x_1, x_2, \dots, x_n)_h = F(x_{1h}, x_{2h}, \dots, x_{nh}) \quad (8)$$

where a h -level α -cut (with $0 \leq h \leq 1$) of a fuzzy set A is the crisp set (the interval) A_h of all $x \in A$ with a fuzzy membership value greater than, or equal to, h . Consequently, any algebraic computation may be decomposed into several interval computations, each one having as arguments the α -cut intervals of the fuzzy arguments at the same level. The result is the α -cut of the original fuzzy result. The fuzzy calculus problem is then simplified to interval calculus.

Since RIs and trapezoidal fuzzy sets share the same shape and basic operations, this principle can be applied to RIs operations. Consequently, eliminating situations with a confidence $C < C_{\text{req}}$ during the situation validation is the same as eliminating those situations where the C_{req} -level α -cut of the RI resulting from the last intersection operation is an empty interval. Considering the SQMA situation validation as function F in Eq. (8), and the RIs composing the evaluated situation as its arguments x_1 to x_n , the same spurious situations can be filtered based on the C_{req} -level α -cut of the RIs and interval arithmetic. For this reason, the equivalent crisp intervals introduced in last section deliver the same result as eliminating spurious situations with $C_{\text{req}} = 0.5$.

Choosing $C_{\text{req}} > 0.5$ (e.g. $C_{\text{req}} = 0.8$) would deliver non-intersecting intervals after applying the α -cut. There are in fact gaps between these intervals. These gaps cause possible situations not being considered at all. Thus, choosing a C_{req} value greater than 0.5 violates the completeness of SQMA models. On the contrary, a conservative C_{req} between 0.5 and zero produces overlapping crisp intervals. The solution provided by these overlapping intervals is not optimal, but preserves the completeness of the SQMA model. In general, a reasonable threshold value for the rule confidence (C_{req}) would vary between 0.3 and 0.5.

6. Conclusions

This article introduced a new concept for the qualitative modeling of complex systems, which is based on the representation and handling of vague and uncertain information in interval-based methods. The management of vague and uncertain information enhances the relation size/precision and the descriptive power of the resulting qualitative models, improving their applicability. Principles taken from Pawlak's Rough Set Theory are employed for the representation of vague and uncertain knowledge in the original interval-based notation. The resulting Rough Interval concept enhances the resolution of the representation of dynamic systems without increasing the size of the models, or what is equivalent, allows compressing qualitative models without losing resolution.

A further pay-off of Rough Intervals is that thanks to the introduction of a rough membership function, it is now possible to identify and characterize process transitions precisely. This enables at least rough probability judgments. The uniformity assumption by the selection of the straight membership function may or may not be realistic for particular process plants, but it is still a better starting point than the probability-free estimates provided in interval-based qualitative models, particularly when no probabilistic information about the system behavior is available.

The new concept was successfully integrated in the Situation-based Qualitative Modeling and Analysis method. It allowed the integration of vague and uncertain process knowledge into SQMA models, taking advantage of the rich information hidden in the human way of perceiving and describing nature. This improved the precision and compactness of SQMA models, compared to the conventional technique based on crisp intervals. However, the introduction of vague and uncertain information causes the appearance of spurious situations in the model, which must be conveniently filtered out to assure the targeted model compactness. SQMA models using Rough Intervals have been successfully employed in online process monitoring and fault detection applications [21,27].

References

- [1] A. Bonarini, G. Bontempi, A qualitative simulation approach for fuzzy dynamical models, *ACM Transactions on Modeling and Computer Simulation* 4 (4) (1994) 285–313.
- [2] A. Grzesiak, Analysis of the applicability of SQMA to fault analysis, Diplomarbeit, IAS, Universität Stuttgart, Germany, 2003.
- [3] B. Kuipers, Qualitative simulation, *Artificial Intelligence* 29 (1986) 289–338.
- [4] D. Dubois, H. Prade, J. Lang, Possibilistic logic, in: D. Gabbay, et al. (Eds.), *Handbook of Logic in Artificial Intelligence and Logic Programming*, vol. 3, Oxford University Press, USA, 1994, pp. 439–514.
- [5] D. Dvorak, B. Kuipers, Process monitoring and diagnosis: A model-based approach, *IEEE Expert* 6 (3) (1991) 67–74.
- [6] D. Dvorak, Monitoring and diagnosis of continuous dynamic systems using semiquantitative simulation, PhD dissertation, University of Texas, Austin, TX, USA, 1992.
- [7] D. Schaich, R. King, U. Keller, M. Chantler, Interval identification—a modelling and design technique for dynamic systems, in: 13th International Workshop of Qualitative Reasoning, Loch Awe Scotland, June, 1999.

- [8] F. Berlin, Integration von Fuzzy-Intervallen in die qualitative Modellierungssprache SQMA, Diplomarbeit, IAS, Universität Stuttgart, Germany, 2002.
- [9] F. Tay, L. Shen, Fault diagnosis based on rough set theory, *Engineering Applications of Artificial Intelligence* 16 (1) (2003) 39–43.
- [10] G. Brajnik, M. Lines, Qualitative modeling and simulation of socio-economic phenomena, *Journal of Artificial Societies and Social Simulation* 1 (1) (1998).
- [11] G. Coghill, S. Garrett, R. King, Learning qualitative metabolic models, in: *Proceedings of the European Conference on Artificial Intelligence—ECAI'04*, Valencia, Spain, 2004.
- [12] G. Mayer, Development of a method concept for increasing the reliability of software-intensive systems in early development phases, Diplomarbeit, IAS, Universität Stuttgart, Stuttgart, Germany, 2004.
- [13] H. Kay, B. Rinner, B. Kuipers, Semi-quantitative system identification, *Artificial Intelligence* 119 (1–2) (2000) 103–140.
- [14] H. Nguyen, A note on the extension principle for fuzzy sets, *Journal of Mathematical Analysis Applications* 64 (1978) 369–380.
- [15] J. de Kleer, J. Brown, A qualitative physics based on confluences, *Artificial Intelligence* 24 (1–3) (1984) 7–83.
- [16] J. Lunze, Qualitative modelling of dynamical systems: Motivation, methods, and prospective applications, *Mathematics and Computers in Simulation* 46 (5–6) (1998) 465–483.
- [17] J. Wille, Development of a concept for the experimental determination of Rough Intervals, Diplomarbeit, IAS, Universität Stuttgart, Germany, 2003.
- [18] K. Forbus, Qualitative process theory, *Artificial Intelligence* 24 (1–3) (1984) 85–168.
- [19] K. Forbus, Qualitative process theory: twelve years after, *Artificial Intelligence* 59 (1–2) (1993) 115–123.
- [20] K. Forbus, P. Whalley, J. Everett, L. Ureel, M. Brokowski, J. Baher, S. Kuehne, CyclePad: An articulate virtual laboratory for engineering thermodynamics, *Artificial Intelligence* 114 (1–2) (1999) 297–347.
- [21] L. Cachero, Online probabilistic forecasting in a three tanks system, Master Thesis, IAS, Universität Stuttgart, Germany, 2003.
- [22] L. Shen, F. Tay, L. Qu, Y. Shen, Fault diagnosis using rough sets theory, *Computers in Industry* 43 (2000) 61–72.
- [23] L. Zadeh, Fuzzy sets, *Information and Control* 8 (1965) 338–353.
- [24] M. Rebolledo, Development of a concept for the handling of vagueness in the SQMA modeling approach, Special Project, IAS, Universität Stuttgart, Germany, 2002.
- [25] M. Rebolledo, A combined rough-stochastic approach to process monitoring based on qualitative models, in: *IASTED International Conference on Intelligent Systems and Control*, Salzburg, Austria, 2003, pp. 69–74.
- [26] M. Rebolledo, Situation-based process monitoring in complex systems considering vagueness and uncertainty, PhD Thesis, Universität Stuttgart, Germany, 2004.
- [27] P. Katamaneni, Online process supervision of a three tank system, Master Thesis, IAS, Universität Stuttgart, Germany, 2003.
- [28] Q. Shen, R. Leitch, Fuzzy qualitative simulation, *IEEE Transactions on Systems, Man, and Cybernetics* 23 (4) (1993) 1038–1060.
- [29] R. Moore, *Interval Analysis*, Prentice-Hall, Englewood-Cliffs, NJ, 1966.
- [30] S. Manz, Online fault detection and diagnosis of complex systems based on hybrid component models, in: *Proceedings of the 14th International Congress on Condition Monitoring and Diagnostic Engineering Management—COMADEM 2001*, Manchester, UK, 2001, pp. 865–872.
- [31] U. Biegert, Computer-aided safety analysis of computer-controlled systems: A case example, in: *Proceedings of the 6th IEEE International Conference on Methods and Models in Automation and Robotics (MMAR)*, Miedzyzdroje, Poland, 2000, pp. 777–782.
- [32] U. Keller, T. Wyatt, R. Leitch, FrenSi—a fuzzy qualitative simulation method, in: *Proceedings of the MISC Workshop on Application of Interval Analysis to System and Control*, Girona, Spain, 1999.
- [33] X. Laufenberg, P. Fröhlich, A qualitative model based approach to integrate system and hazard analysis, in: *Proceedings of the IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes—SAFEPROCESS 94*, Helsinki, Finland, 1994.
- [34] Z. Pawlak, Rough sets: A new approach to vagueness, in: L. Zadeh, J. Kacprzyk (Eds.), *Fuzzy Logic for the Management of Uncertainty*, Wiley Professional Computing, USA, 1982, pp. 105–118.
- [35] Z. Pawlak, Rough sets, rough relations and rough functions, ICS Research Report 24/94, Warsaw University of Technology, Warsaw, 1994.