Lista 10 – Cálculo 2

1) Verifique se as funções são LI ou LD no intervalo I.

a) $y_1 = senax,\ y_2 = \cos ax, \quad I = \mathbb{R}$; Resp. LI

b) $y_1 = \ln x,\ y_2 = -\ln x^3, \quad y_3 = g(x) \quad I = (0,+\infty)$; Resp. LD

c) $y_1 = x+1,\ y_2 = x-1, \quad y_3 = x^2 - a \quad I = \mathbb{R}$. Resp. LI

2) Determine a solução geral para a EDO homogênea.

a) $y''-2y'+10y=0$; b) $4y''+17y'+4y=0$; c) $y''+4y'=0$;

d) $y''+4y'+5y=0$; e) $y''+6y'+9y=0$; f) $4y''-4y'+y=0$.

Resp. a) $y = e^x(c_1\cos 3x + c_2 sen3x)$, b) $y = c_1 e^{\frac{-x}{4}} + c_2 e^{-4x}$, c) $y = c_1 + c_2 e^{-4x}$

d) $y = e^{-2x}(c_1 \cos x + c_2 senx)$, e) $y = e^{-3x}(c_1 + c_2 x)$, f) $y = e^{\frac{x}{2}}(c_1 + c_2 x)$

3) Apresente um conjunto fundamental de soluções para $y''-2\sqrt{3}y'+3y=0$;

Resp. $\{e^{\sqrt{3}x}, xe^{\sqrt{3}x}\}$

4) Apresente a EDO linear homogênea cuja solução geral é:

a) $y = e^{-x}(A\cos x + Bsenx)$; b) $y = e^{4x}(c_1 + c_2 x)$;

c) $y = c_1 + e^{4x}c_2$; d) $y = c_1 e^x + c_2 x e^x + c_3 e^{-x}$.

Resp. a) $y''+2y'+2y=0$, b) $y''-8y'+16y=0$, c) $y''-4y'=0$, d) $y'''-y''-y'+y=0$

5) Resolver as EDO de Cauchy-Euler.

a) $x^2y''-xy'+y=0$; Resp. $y=(c_1+c_2\ln x)x$

b) $x^2y''-3xy'+3y=0$; Resp. $y=c_1x+c_2x^3$

6) Resolva pelo Método dos Coeficientes a Determinar.

a) $y''+3y'+2y=x^2$; Resp. $y = c_1e^{-2x}+c_2e^{-x}+\dfrac{x^2}{2}-\dfrac{3x}{2}+\dfrac{7}{4}$

b) $y''-2y'=sen4x$; Resp. $y = c_1+c_2e^{2x}+\dfrac{1}{40}\cos 4x-\dfrac{1}{20}sen4x$

c) $y''+y'-2y=x^2+e^x$. Resp. $y = c_1e^x+c_2e^{-2x}-\dfrac{x^2}{2}-\dfrac{x}{2}-\dfrac{3}{4}+\dfrac{xe^x}{3}$

7) Resolva pelo Método da Variação dos Parâmetros.

a) $y''+y=\sec x$; Resp. $y=(c_1+x)senx+(c_2+\ln(\cos x))\cos x$

b) $y''-y=\dfrac{1}{x}$; Resp. $y=e^{-x}(c_1-\dfrac{1}{2}\int\dfrac{e^x}{x}dx)+e^x(c_2+\dfrac{1}{2}\int\dfrac{e^{-x}}{x}dx)$

c) $y''-4y'+4y=\dfrac{e^{2x}}{x}$; Resp. $y=(c_1+c_2x+x\ln x-x)e^{2x}$

d) $y''+y=\mathrm{cosec}\, x$. Resp. $y=c_1senx+c_2\cos x+senx.\ln(senx)-x\cos x$.