Lista 14 – Cálculo 2

1) Calcule as convoluções:

a) $1 * t^n$; b) $e^{-t} * \cos t$.

Resp. a) $\dfrac{t^{n+1}}{n+1}$, b) $\dfrac{1}{2}\left[sent + \cos t - e^{-t} \right]$

2) Calcule $\mathcal{L}(e^{-t} * e^t \cos t)$. Resp. $\left(\dfrac{1}{s+1}\right)\dfrac{s-1}{(s-1)^2+1}$

3) Calcule a transformada inversa de Laplace, usando o Teorema da Convolução.

a) $H(s) = \dfrac{s^2}{(s^2+k^2)^2}$; Resp. $h(t) = \dfrac{sen(kt)}{2k} + \dfrac{t\cos(kt)}{2}$

b) $H(s) = \dfrac{1}{s(s^2+1)}$. Resp. $h(t) = 1 - \cos t$

4) Use o Teorema da Convolução para resolver a equação integral

$y(t) = sen2t + \int_0^t y(\sigma) sen(2(t-\sigma))d\sigma$. Resp. $y(t) = \sqrt{2} sen(\sqrt{2}\,t)$

5) Use transformada de Laplace para resolver os PVI.

a) $y'' + y = sen(3t), \quad y(0) = 0, \quad y'(0) = 0$; Resp. $y(t) = \dfrac{3}{8} sent - \dfrac{1}{8} sen(3t)$

b) $y'' + 3y' + 2y = r(t), \quad y(0) = 0, \quad y'(0) = 0$, quando $r(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$;

Resp. $y(t) = \begin{cases} \dfrac{1}{2} - e^{-t} + \dfrac{1}{2}e^{-2t}, & 0 \le t < 1 \\ e^{-t}(-1+e) + \dfrac{e^{-2t}}{2}(1-e^2), & t \ge 1 \end{cases}$

c) $y'' + y = r(t), \quad y(0) = 0, \quad y'(0) = 1$, quando $r(t) = \begin{cases} 0, & 0 \le t < \pi \\ 1, & \pi \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$;

Resp. $y(t) = sent + [1 - \cos(t-\pi)]\mathcal{U}_\pi(t) - [1 - \cos(t-2\pi)]\mathcal{U}_{2\pi}(t)$

d) $y'' + y = \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 1$; Resp $y(t) = sent + [sent]\mathcal{U}_{2\pi}(t)$

e) $y'' + y = \delta(t - \dfrac{\pi}{2}) + \delta(t - \dfrac{3\pi}{2}), \quad y(0) = 0, \quad y'(0) = 0$; Resp. $y(t) = -[\cos t]\mathcal{U}_{\frac{\pi}{2}}(t) + [\cos t]\mathcal{U}_{\frac{3\pi}{2}}(t)$

f) $y'' + 2y' = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 1$. Resp. $y(t) = \begin{cases} \dfrac{1}{2} - \dfrac{1}{2}e^{-2t}, & 0 \le t < 1 \\ 1 - \dfrac{e^{-2t}}{2}(1+e^2), & t \ge 1 \end{cases}$