Lista 13 – Cálculo 2

1) Calcule a transformada de Laplace.

a) $f(t) = t\cos(kt)$; b) $f(t) = te^{-at}\cos(kt)$; c) $f(t) = t^2\,\mathrm{s}en(kt)$.

Resp. a) $F(s) = \dfrac{s^2 - k^2}{(s^2 + k^2)^2}$; b) $F(s) = \dfrac{(s+a)^2 - k^2}{((s+a)^2 + k^2)^2}$; c) $F(s) = \dfrac{6ks^2 - 2k^3}{(s^2 + k^2)^3}$.

2) Calcule a transformada inversa de Laplace.

a) $F(s) = \dfrac{1}{s(s-a)}$, $a \neq 0$; b) $F(s) = \dfrac{1}{s(s^2 - 1)}$;

c) $F(s) = \ln\left(\dfrac{s}{s-1}\right)$; d) $F(s) = \ln\left(\dfrac{s+a}{s+b}\right)$.

Resp. a) $f(t) = \dfrac{e^{at} - 1}{a}$; b) $f(t) = \cosh(t) - 1$, c) $f(t) = \dfrac{e^t - 1}{t}$, d) $f(t) = \dfrac{e^{-bt} - e^{-at}}{t}$.

3) Resolva o PVI, usando transformada de Laplace.

a) $y'' + 2y' - 8y = 0, \quad y(0) = 1, \quad y'(0) = 8$; Resp. $y(t) = 2e^{2t} - e^{-4t}$

b) $y'' + 2y' + 10y = 0, \quad y(0) = 0, \quad y'(0) = 3$. Resp. $y(t) = e^{-t} sen 3t$

4) Esboce o gráfico de $f(t)$, escreva $f(t)$ em termos da função degrau unitário e calcule a transformada de Laplace de $f(t)$.

a) $f(t) = \begin{cases} 0, & 0 \le t < a \\ k, & a \le t < b, \quad a < b \\ 0, & t \ge b \end{cases}$; b) $f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t-1, & 1 \le t < 2 \\ 1, & t \ge 2 \end{cases}$.

Resp. a) $F(s) = \dfrac{k}{s}(e^{-as} - e^{-bs})$, b) $F(s) = \dfrac{e^{-s} - e^{-2s}}{s^2}$

5) Calcule a transformada inversa de Laplace.

a) $F(s) = \dfrac{e^{-as}}{s^n}$; b) $F(s) = \dfrac{e^{-2s}}{s-2}$; c) $F(s) = \dfrac{e^{-\pi s}}{s^2 + 2s + 2}$.

Resp. a) $f(t) = \dfrac{(t-a)^{n-1}}{(n-1)!}\mathcal{U}_a(t)$, b) $f(t) = e^{2(t-2)}\mathcal{U}_2(t)$, c) $f(t) = -e^{\pi - t} sen t\, \mathcal{U}_\pi(t)$

6) Esboce o gráfico e calcule a transformada de Laplace das funções periódicas.

a) $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}, \quad T = 2$; Resp. $F(s) = \dfrac{1}{1 - e^{-2s}}\left(\dfrac{-e^{-s}}{s} + \dfrac{1 - e^{-s}}{s^2}\right)$

b) $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}, \quad T = 2$; Resp. $F(s) = \dfrac{1}{s(1 + e^{-s})}$

c) $f(t) = e^t, \quad T = 2\pi$. Resp. $F(s) = \dfrac{e^{2\pi(1-s)} - 1}{(1-s)(1 - e^{-2\pi s})}$