

Practical_maximum_likelihood_estimation_ Oscar_Contreras_Rafael_Castilla

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R Markdown

Resolve the following exercise in groups of two students. Write your solution in a Word, Latex or Markdown document and generate a pdf file with your solution. Upload the pdf file with your solution to the corresponding task at the Moodle environment of the course, no later than the hand-in date.

1.(16p) ML estimation of a one-parameter distribution. Let X be a random variable with probability density $f(x|\beta) = \beta x^{b-1}$ with $x \geq 1$, $\beta > 0$ we consider a random sample of n observation of this distribution.

a) (2p) Write down the likelihood function for a sample of n observations of this distribution.

Answer: $L(\beta|x) = \prod_{i=1}^n f(x_i|\beta) = \prod_{i=1}^n \beta x_i^{b-1} = \beta^n \prod_{i=1}^n x_i^{b-1}$

```
likeli<-function(b,x){  
  n<-row(x)  
  l<-b^n*prod(x^(b-1))  
  return(l)  
}
```

b) (1p) Obtain the log-likelihood function $\log(L(\beta|x_i)) = n\log(\beta) + \sum_{i=1}^n (\beta - 1)\log(x_i) = n\log(\beta) + n(\beta - 1) \sum_{i=1}^n \log(x_i)$

```
loglike<-function(b,x){  
  n<-nrow(x)  
  l<-n*log(b)+(n*(b-1))*sum(log(x))  
  return(l)  
}
```

c) (2p) Find the stationary point(s) of the log-likelihood function analytically. $\frac{d\log(L(\beta|x))}{d\beta} = \frac{n}{\beta} + n \sum_{i=1}^n \log(x_i)$ for find the stationary point the derivate equal to 0 $0 = \frac{n}{\beta} + n \sum_{i=1}^n \log(x_i) \Rightarrow \beta = \frac{1}{\sum_{i=1}^n \log(x_i)}$

d)(1p) Determine whether the stationary point(s) are maxima or minima.

for know if the stationary point is maxima or minima the need the 2 derivate $\frac{d^2 l}{d\beta^2} = -\frac{n}{\beta^2}$ and if the result is positive is the minimum and if is negative is a maximum

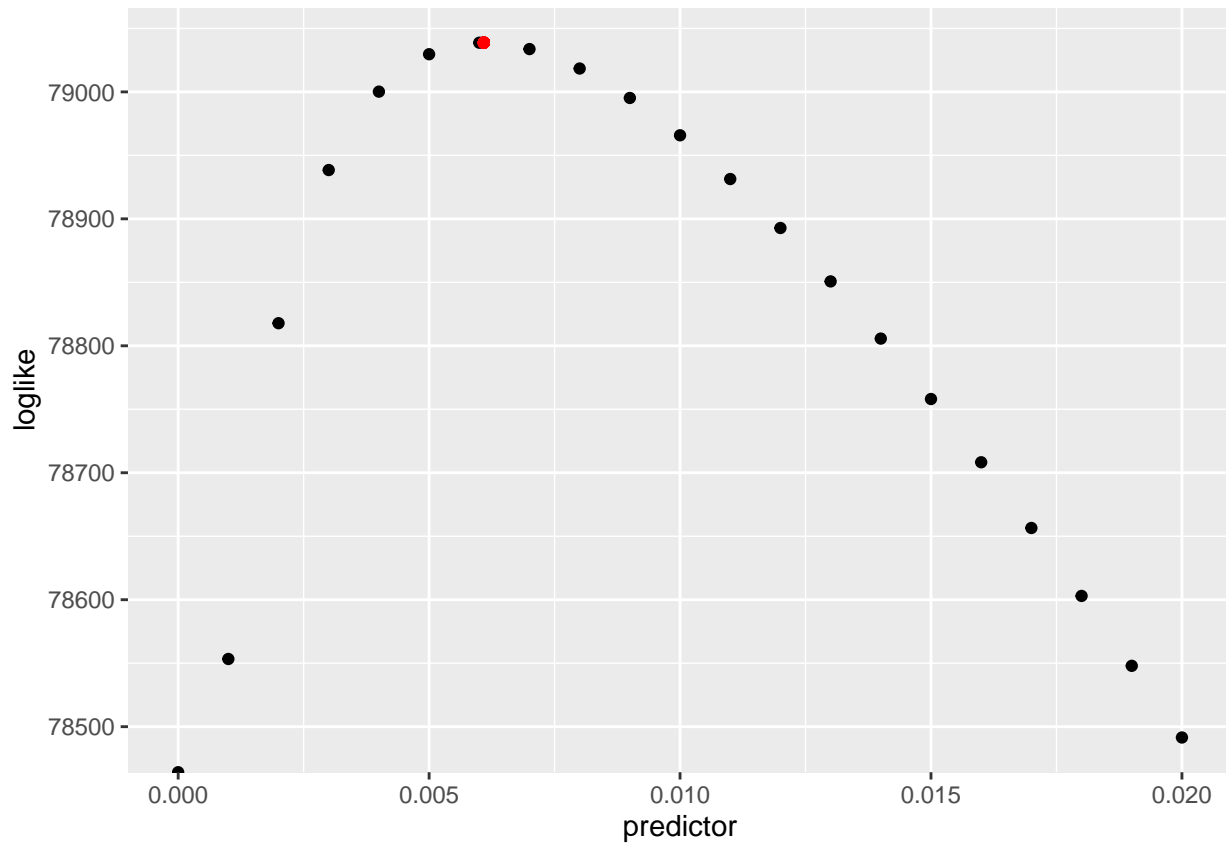
e) 1p) Download the file Sample.dat, which contains sample of observations from this probability distribution. Determine the sample size and calculate the value of the ML estimator for this sample.

```
x<-read.table('~/.Statistic/Sample.dat')  
print(paste0("the sample size is: ", nrow(x)))  
  
## [1] "the sample size is: 500"  
  
#if use the derivate of loglikelihood  
ml<--1/sum(log(x))  
print(paste0("the ML estimator is: ",ml))
```

```
## [1] "the ML estimator is: 0.00609092952042536"
```

f) 2p) Plot the log-likelihood function, and assess graphically if your ML estimate coincides with the maximum of this function.

```
library(ggplot2)
b<-seq(0.0,0.02,by=0.001)
d<-data.frame("predictor"=b,"loglike"=loglike(b,x))
ggplot(d,aes(y=loglike,x=predictor))+
  geom_point()+geom_point(aes(ml,loglike(ml,x)),colour="red")
```



the maximum correspond with the graph