Practical_maximum_likelihood_estimation_ Oscar_Contreras_Rafael_Castilla

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R Markdown

Resolve the following exercise in groups of two students. Write your solution in a Word, Latex or Markdown document and generate a pdf file with your solution. Upload the pdf file with your solution to the corresponding task at the Moodle environment of the course, no later than the hand-in date.

1.(16p) ML estimation of a one-parameter distribution. Let X be a random variable with probability density $f(x|\beta) = \beta x^{b-1}$ with $\geq x \geq 1$, $\beta > 0$ we consider a random sample of n observation of this distribution.

a) (2p) Write down the likelihood function for a sample of n observations of this distribution.
   Answer: $L(\beta|x) = \prod_{i=1}^{n} f(x_i|\beta) = \prod_{i=1}^{n} \beta x_i^{b-1} = \beta^n \prod_{i=1}^{n} x_i^{b-1}$

```
likeli<-function(b,x){
  n<-row(x)
  l<-b^n*prod(x^(b-1))
  return(l)
}
```

b) (1p) Obtain the log-likelihood function $log(L(\beta|x_i) = nlog(\beta) + \sum_{i=1}^{n}(\beta-1)log(x_i) = nlog(\beta) + n(\beta-1)\sum_{i-1}^{n} log(x_i)$

```
loglike<-function(b,x){
n<-nrow(x)
l<-n*log(b)+(n*(b-1))*sum(log(x))
return(l)
}
```

c) (2p) Find the stationary point(s) of the log-likelihood function analytically. $\frac{dlog(L(\beta|x))}{d\beta} = \frac{n}{\beta} + n\sum_{i=1}^{n} x_i$ for find the stationary poitt the derivate qual to 0 $0 = \frac{n}{\beta} + n\sum_{i=1}^{n} x_i \backslash\ \beta = \frac{1}{sum_{i=1}^{n} log(x_i)}$

d)(1p) Determine whether the stationary point(s) are maxima or minima.

for know if the stationary point is maxima or minima the need the 2 derivate $\frac{d^2 l}{d^2 \beta} = -\frac{n}{\beta^2}$ and if the result is positive is the minimum and if is negative is a maximum

e) 1p) Download the file Sample.dat, which contains sample of obser- vations from this probability distribution. Determine the sample size and calculate the value of the ML estimator for this sample.

```
x<-read.table('~/Statistic/Sample.dat')
print(paste0("the sample size is: ", nrow(x)))
```

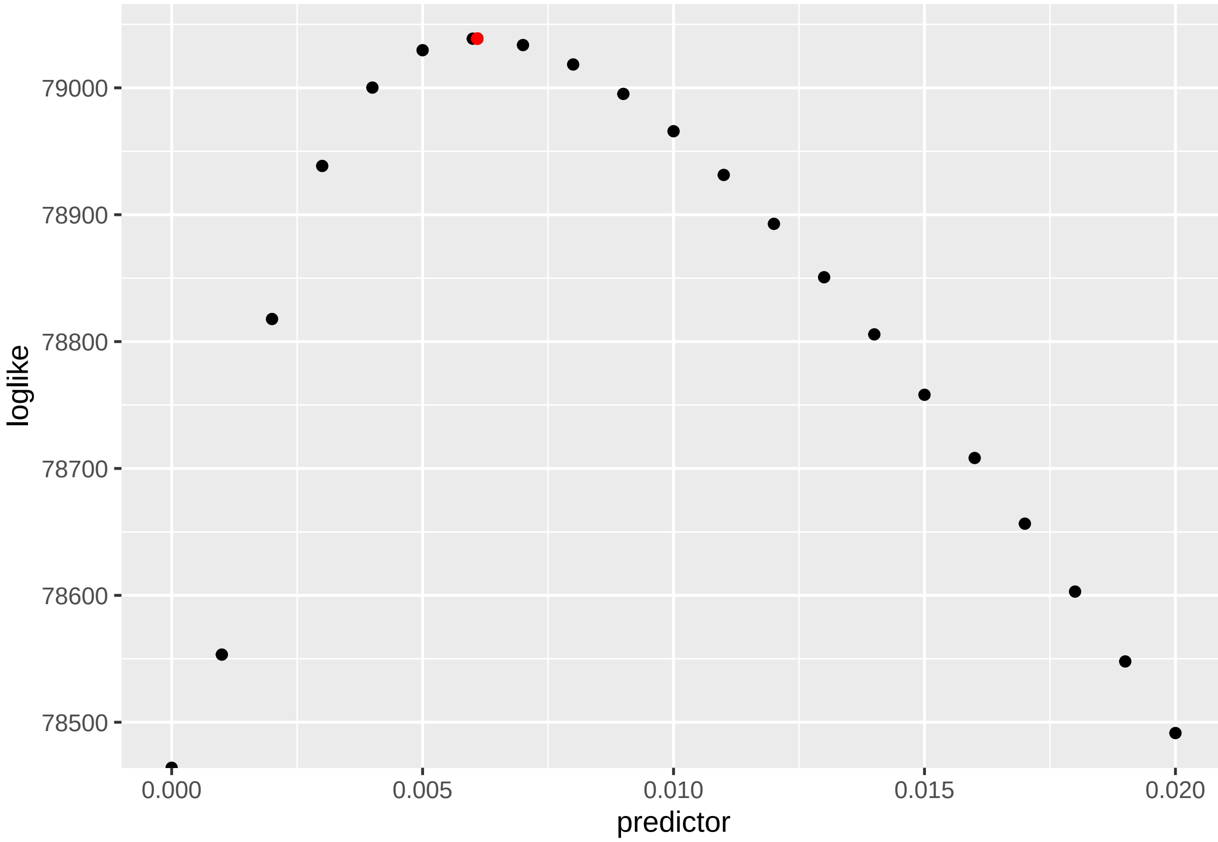
```
## [1] "the sample size is: 500"
```

```
#if use the derivate of loglikelihood
ml<--1/sum(log(x))
print(paste0("the ML estimator is: ",ml))
```

```
## [1] "the ML estimator is: 0.00609092952042536"
```

f) 2p) Plot the log-likelihood function, and assess graphically if your ML estimate coincides with the maximum of this function.

```
library(ggplot2)
b<-seq(0.0,0.02,by=0.001)
d<-data.frame("predictor"=b,"loglike"=loglike(b,x))
ggplot(d,aes(y=loglike,x=predictor))+
  geom_point()+geom_point(aes(ml,loglike(ml,x)),colour="red")
```



the maximum correspond with the graph

g) (1p) Determine an expression for the Fisher information by calculating $-E(\frac{d^2l}{d\beta^2})$ $-E(\frac{n}{\beta^2})$

h) (1p) Use the Fisher information for obtaining an expression for the variance of the maximum likelihood estimator βML. $(\sigma, \frac{1}{l(\beta)})$ $(\sigma, \frac{\beta^2}{n})$ the variance is $\frac{\beta^2}{n}$

i) (1p) Using the asymptotic normality of the ML estimator, give an expression of a confidence interval for .

a clasic example of asymptomatic normality is $N \sim (\mu, \sigma^2)|CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{V(\hat{\beta})}$

$CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\beta^2}{n}}$

j) (1p) Calculate a 95% confidence interval for parameter , using the dataset that you have downloaded $CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\beta^2}{n}}$

```
v=sqrt(ml^2/nrow(x))
CIp=ml+1.96*v
CIn=ml-1.96*v
print(paste0("the confidence interval is ",paste0(round(CIn,5),paste0(" : ",round(CIp,5)))))
```

```
## [1] "the confidence interval is 0.00556 : 0.00662"
```

k) (1p) Do you think it is tenable that $\beta = 1$? Argue your answer.

have 95% chance of 0.00556 and 00662 and $\beta = 1$ is not between into the coefficient

l) (2p) Make a histogram of the data, using function hist, using the argument freq=FALSE. Overplot the histogram with the estimated probability density $f(x|\beta)$, using the maximum likelihood estimate.What do you observe? $f(x|\beta) = \beta x^{b-1}$

```
f=ml*x^{ml-1}
hist(f[,1])
```