# Practical_maximum_likelihood_estimation_Oscar_Contreras_Rafael_Castilla

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## R Markdown

Resolve the following exercise in groups of two students. Write your solution in a Word, Latex or Markdown document and generate a pdf file with your solution. Upload the pdf file with your solution to the corresponding task at the Moodle environment of the course, no later than the hand-in date.

1.(16p) ML estimation of a one-parameter distribution.
Let X be a random variable with probability density $f(x|\beta) = \beta x^{b-1}$ with $\geq x \geq 1$, $\beta > 0$ we consider a random sample of n observation of this distribution.

a) (2p) Write down the likelihood function for a sample of n observations of this distribution.

Answer: $L(\beta|x) = \prod_{i=1}^{n} f(x_i|\beta) = \prod_{i=1}^{n} \beta x_i^{b-1} = \beta^n \prod_{i=1}^{n} x_i^{b-1}$

```
likeli<-function(b,x){
  n<-row(x)
  l<-b^n*prod(x^(b-1))
  return(l)
}
```

b) (1p) Obtain the log-likelihood function

Answer: $log(L(\beta|x_i) = nlog(\beta) + \sum_{i=1}^{n}(\beta-1)log(x_i) = nlog(\beta) + n(\beta-1)\sum_{i-1}^{n} log(x_i)$

```
loglike<-function(b,x){
n<-nrow(x)
l<-n*log(b)+(n*(b-1))*sum(log(x))
return(l)
}
```

c) (2p) Find the stationary point(s) of the log-likelihood function analytically.

Answer: $\frac{dlog(L(\beta|x))}{d\beta} = \frac{n}{\beta} + n\sum_{i=1}^{n} x_i$
for find the stationary poitt the derivate qual to 0
$0 = \frac{n}{\beta} + n\sum_{i=1}^{n} x_i$
$\beta = \frac{1}{sum_{i=1}^{n} log(x_i)}$

d)(1p) Determine whether the stationary point(s) are maxima or minima.

Answer: for know if the stationary point is maxims or minima the need the 2 derivate
$\frac{d^2l}{d^2\beta} = -\frac{n}{\beta^2}$
and if the result is positive is the minimum and if is negative is a maximum

e) 1p) Download the file Sample.dat, which contains sample of observations from this probability distribution. Determine the sample size and calculate the value of the ML estimator for this sample.

```
x<-read.table('~/Statistic/Sample.dat')
print(paste0("the sample size is: ", nrow(x)))
```

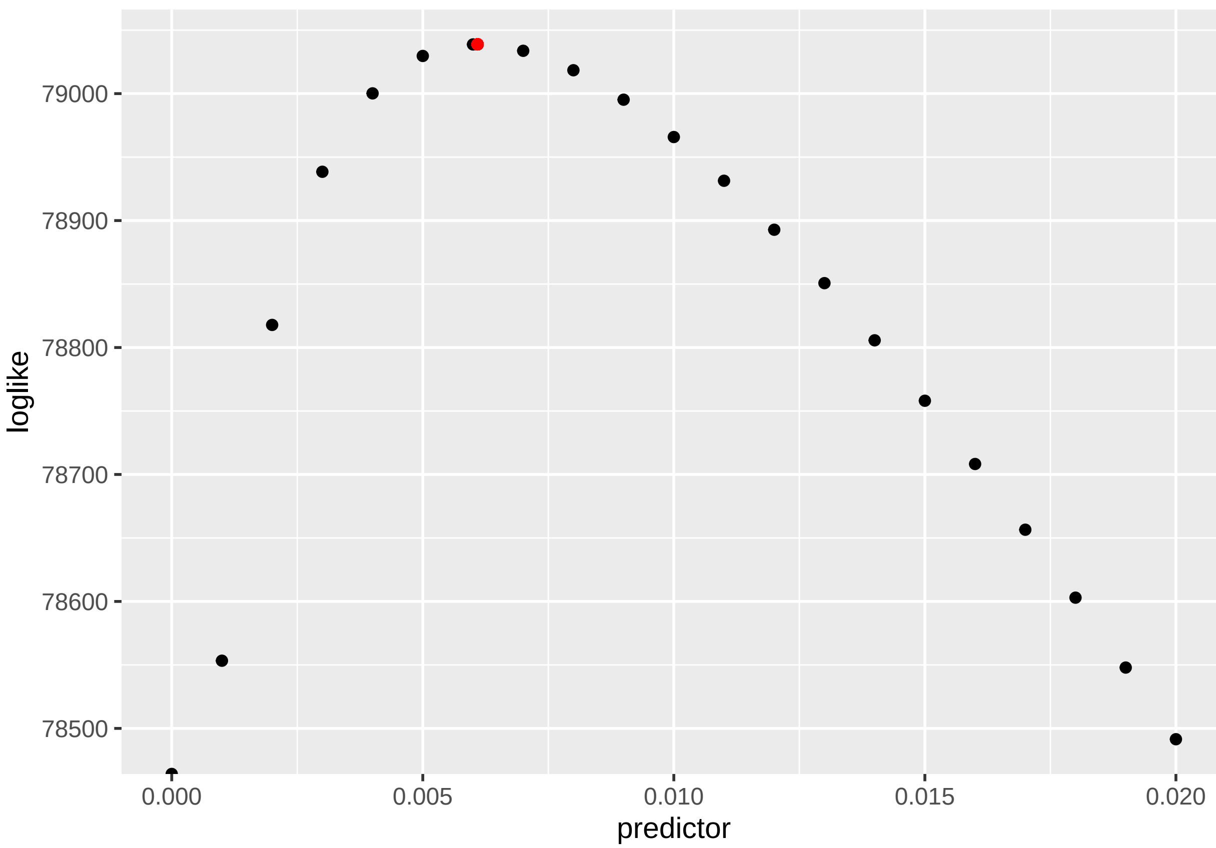
```
## [1] "the sample size is: 500"
```

```
#if use the derivate of loglikelihood
ml<--1/sum(log(x))
print(paste0("the ML estimator is: ",ml))
```

```
## [1] "the ML estimator is: 0.00609092952042536"
```

f) 2p) Plot the log-likelihood function, and assess graphically if your ML estimate coincides with the maximum of this function.

```
library(ggplot2)
b<-seq(0.0,0.02,by=0.001)
d<-data.frame("predictor"=b,"loglike"=loglike(b,x))
ggplot(d,aes(y=loglike,x=predictor))+
  geom_point()+geom_point(aes(ml,loglike(ml,x)),colour="red")
```




Answer: the maximum correspond with the graph

g) (1p) Determine an expression for the Fisher information by calculating

Answer:$-E(\frac{d^2l}{d\beta^2})$
$-E(\frac{n}{\beta^2})$

h) (1p) Use the Fisher information for obtaining an expression for the variance of the maximum likelihood estimator $\beta$ML.

Answer:$(\sigma, \frac{1}{I(\beta)})$
$(\sigma, \frac{\beta^2}{n})$
the variance is $\frac{\beta^2}{n}$

i) (1p) Using the asymptotic normality of the ML estimator, give an expression of a confidence interval for .

Answer:A clasic example of asymptomatic normality is $N \sim (\mu, \sigma^2) | CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{V(\hat{\beta})}$
$CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\beta^2}{n}}$

j) (1p) Calculate a 95% confidence interval for parameter , using the dataset that you have downloaded

Answer:$CI(\mu)_{1-\alpha} = \bar{x} \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\beta^2}{n}}$

```
v=sqrt(ml^2/nrow(x))
CIp=ml+1.96*v
CIn=ml-1.96*v
print(paste0("the confidence interval is ",paste0(round(CIn,5),paste0(" : ",round(CIp,5)))))
```

```
## [1] "the confidence interval is 0.00556 : 0.00662"
```

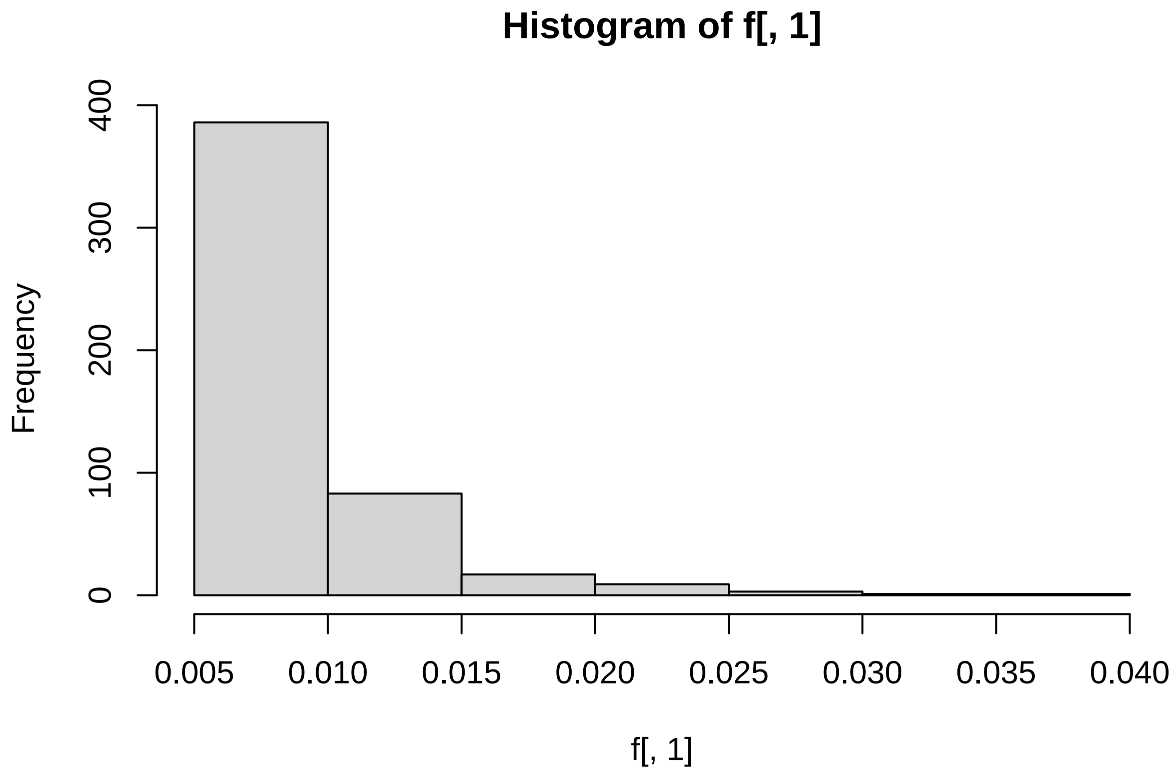
k) (1p) Do you think it is tenable that $\beta = 1$? Argue your answer.

Answer: Have 95% chance of 0.00556 and 00662 and $\beta = 1$ is not between into the coefficient

l) (2p) Make a histogram of the data, using function hist, using the argument freq=FALSE. Overplot the histogram with the estimated probability density $f(x|\beta)$, using the maximum likelihood estimate.What do you observe?

Answer:$f(x|\beta) = \beta x^{b-1}$

```
f=ml*x^{ml-1}
hist(f[,1])
```


Histogram of f[, 1]
Frequency
0
100
200
300
400
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
f[, 1]