Practical_maximum_likelihood_estimation_ Oscar_Contreras_Rafael_Castilla

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R Markdown

Resolve the following exercise in groups of two students. Write your solution in a Word, Latex or Markdown document and generate a pdf file with your solution. Upload the pdf file with your solution to the corresponding task at the Moodle environment of the course, no later than the hand-in date.

1.(16p) ML estimation of a one-parameter distribution. Let X be a random variable with probability density $f(x|\beta) = \beta x^{b-1}$ with $\geq x \geq 1$, $\beta > 0$ we consider a random sample of n observation of this distribution.

a) (2p) Write down the likelihood function for a sample of n observations of this distribution.
Answer: $L(\beta|x) = \prod_{i=1}^{n} f(x_i|\beta) = \prod_{i=1}^{n} \beta x_i^{b-1} = \beta^n \prod_{i=1}^{n} x_i^{b-1}$

```
like<-function(x,b){
  prod<-1
  n<-nrow(x)
  for (i in 1:n){
   prod<-prod*(x[i]^{b-1})
  }
  l<-b^n*prod
  return(l)
}
```

b) (1p) Obtain the log-likelihood function $log(L(\beta|x_i) = nlog(\beta) + \sum_{i=1}^{n}(\beta - 1)log(x_i) = nlog(\beta) + n(\beta - 1)\sum_{i-1}^{n} log(x_i)$

```
loglike<-function(x,b){
  n=nrow(x)
  sum<-0
  for(i in 1:n){
    sum<-sum+log(x[i])
  }
  l<-n*log(b)+n(b-1)*sum
  return(l)
}
```

c) (2p) Find the stationary point(s) of the log-likelihood function analytically. $\frac{dlog(L(\beta|x))}{d\beta} = \frac{n}{\beta}$ for find the stationary poitt the derivate qual to 0 $0 = \frac{n}{\beta} \setminus n = \beta$

d)(1p) Determine whether the stationary point(s) are maxima or minima.

for know if the stationary point is maxima or minima the need the 2 derivate $\frac{d^2l}{d^2\beta} = -\frac{n}{\beta^2}$ and if the result is positive is the minimum and if is negative is a maximum

e) 1p) Download the file Sample.dat, which contains sample of obser- vations from this probability distribution. Determine the sample size and calculate the value of the ML estimator for this sample.

```
x<-read.table('~/Statistic/Sample.dat')
print(paste0("the sample size is: ", nrow(x)))
```

```
## [1] "the sample size is: 500"
```

```
#if use the derivate of loglikelihood
b<-nrow(x)
print(paste0("the ML estimator is: ",b))
```

```
## [1] "the ML estimator is: 500"
```