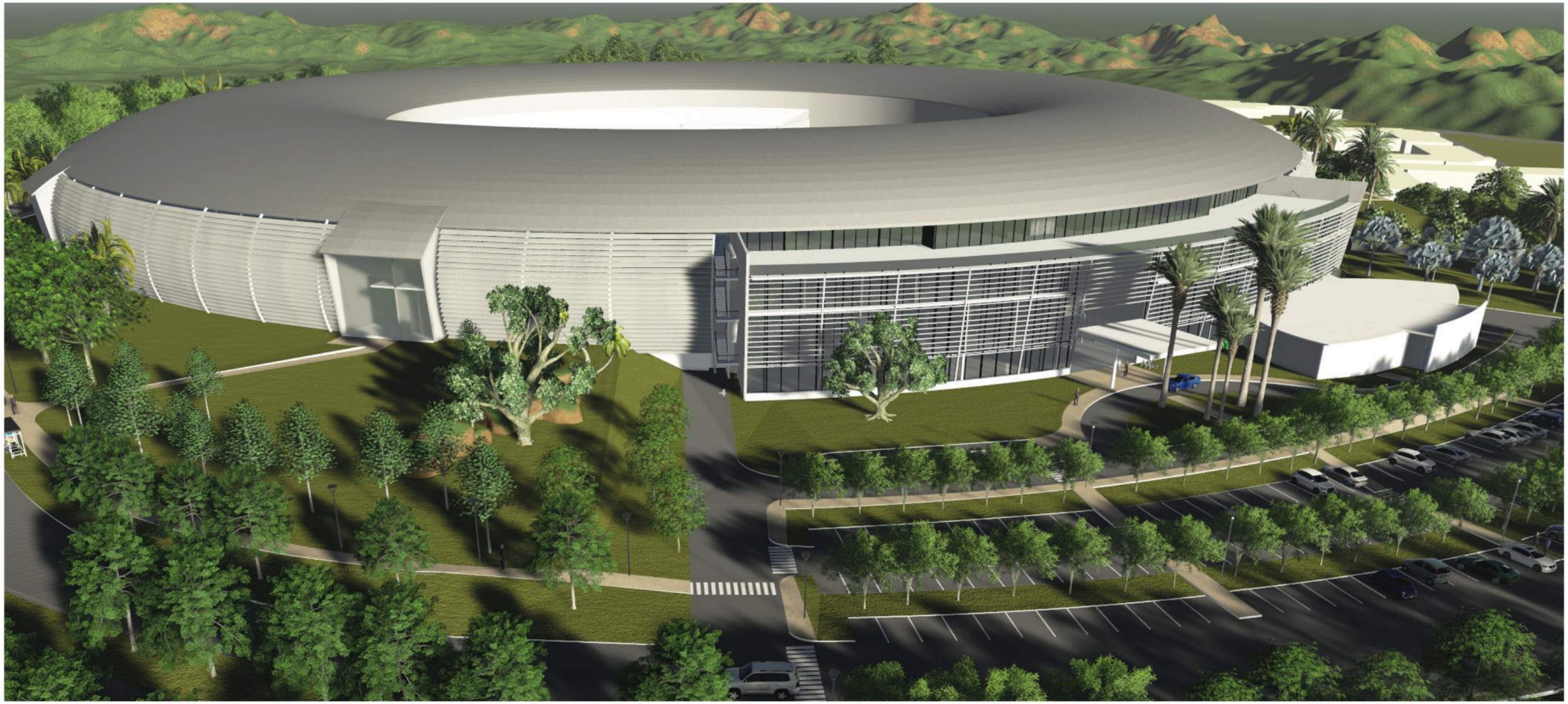
Undulator Radiation



Undulator Radiation – General Introduction

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Outline

- The Sirius project: a 4th generation light source
- Undulator equations
  - The energy spread effect
  - Non-Gaussian beams
- The SR_LNLS code
- Conclusion

LNLS: UVX and Sirius storage rings

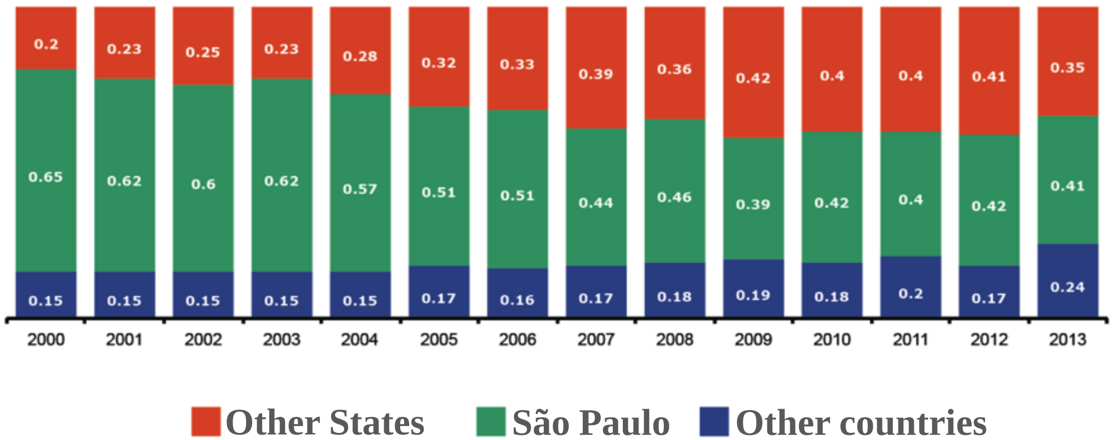
Operation started in 1997

- 1.37 GeV, 100nm.rad emitance;
- 18 beamlines;
  - 1 EPU, 1W (2T), 1 SCW (4T), 15BM;
- Today ~1200 users/year.

Operation to start in 2018

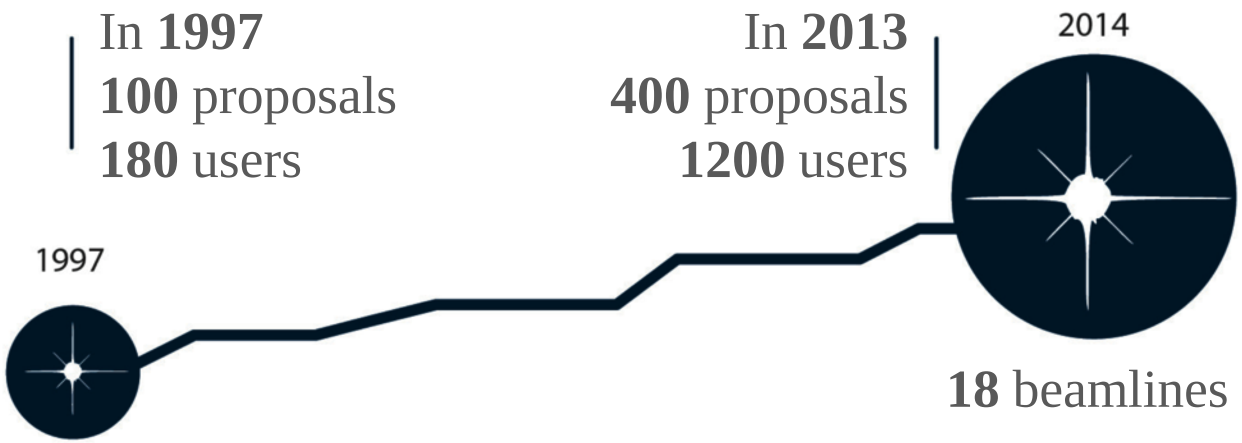
- 3 GeV, 0.28nm.rad emittance;
- Initial phase (2018-2020): 13 beamlines;
  - 5 IVU, 2 EPU, 1 SCW, 5BM.

The UVX user community



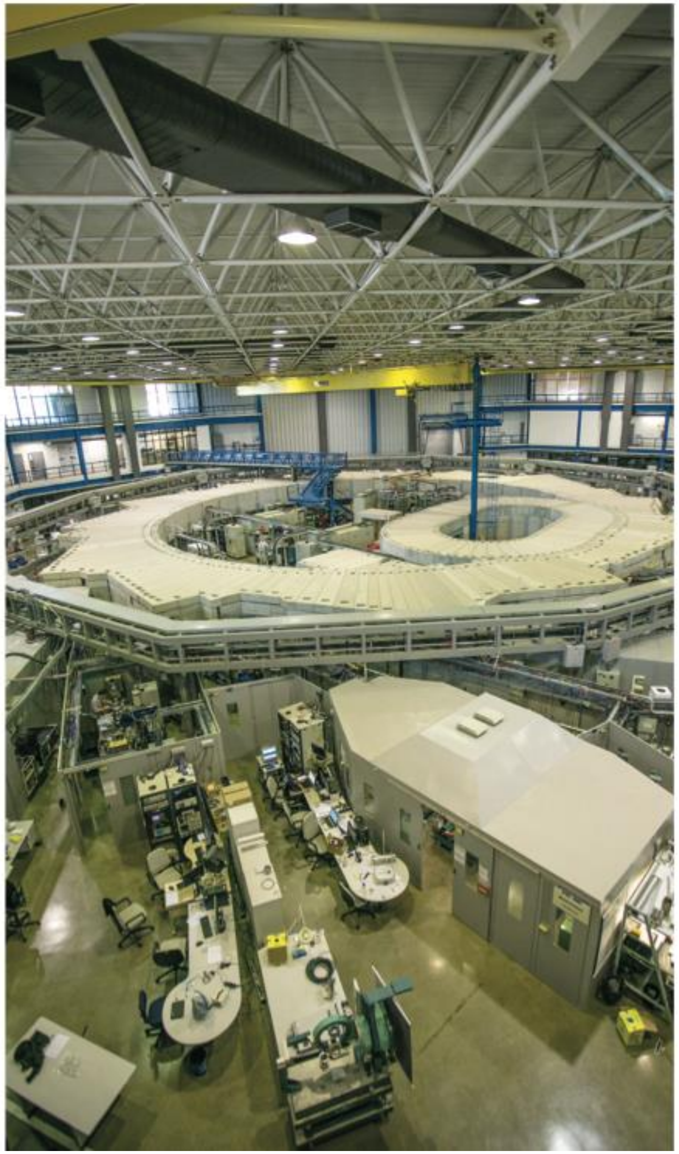
In **1997**
100 proposals
180 users

In **2013**
400 proposals
1200 users



18 beamlines

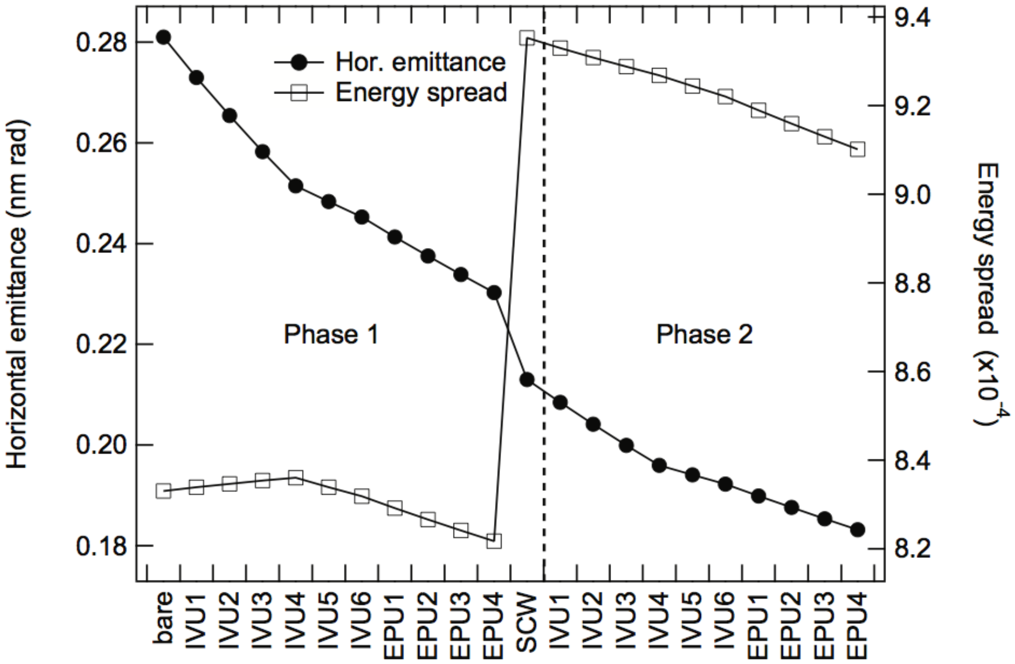
7 beamlines



Sirius storage ring

Sirius storage ring parameters		
Beam energy	3	[GeV]
Horizontal emittance	280	[pm. rad]
Vertical emittance	2.8	[pm. rad]
Beam current (top up)	100	[mA]
Circumference	518	[m]
Number of bunches	864	[ ]
Bunch length	10	[ps]
Energy spread	0.1	[%]

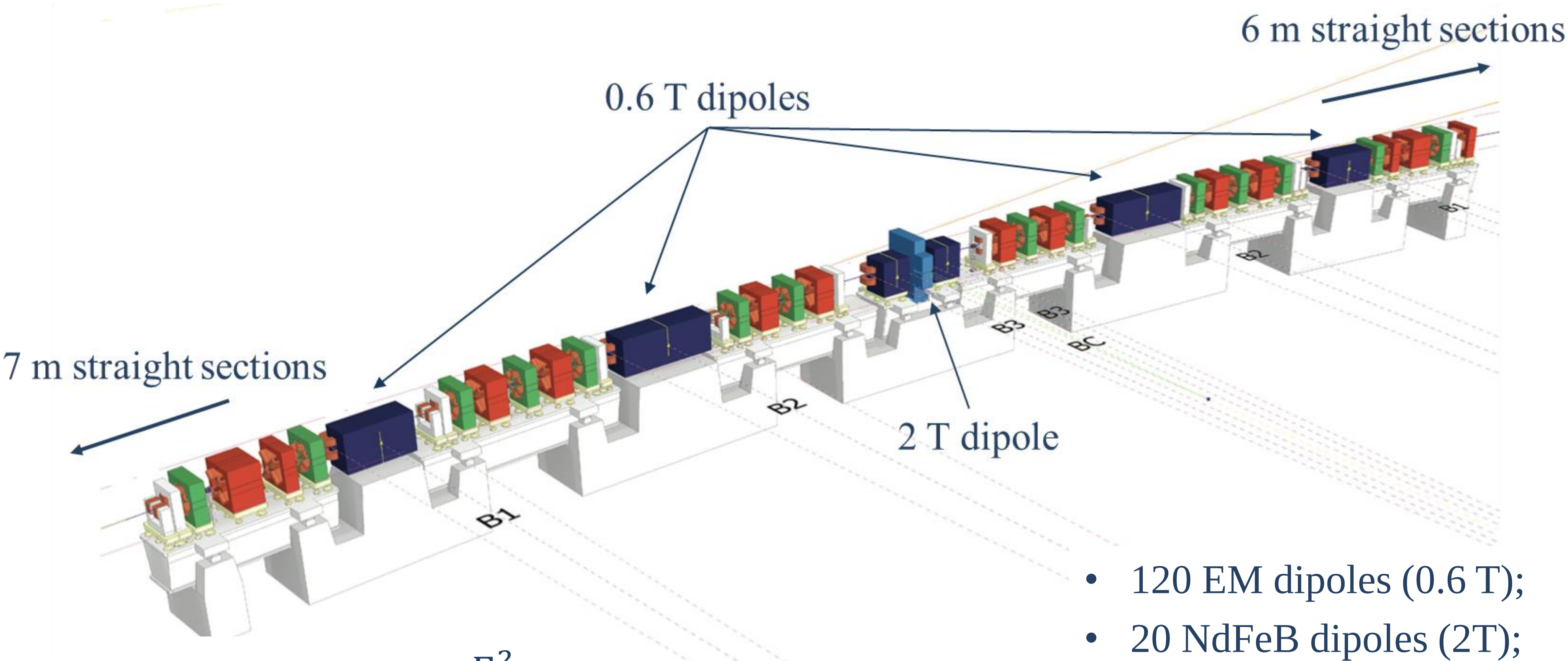
Effect of insertion devices



Insertion device type

(Liu et al., 2014)

Sirius 5BA magnetic lattice

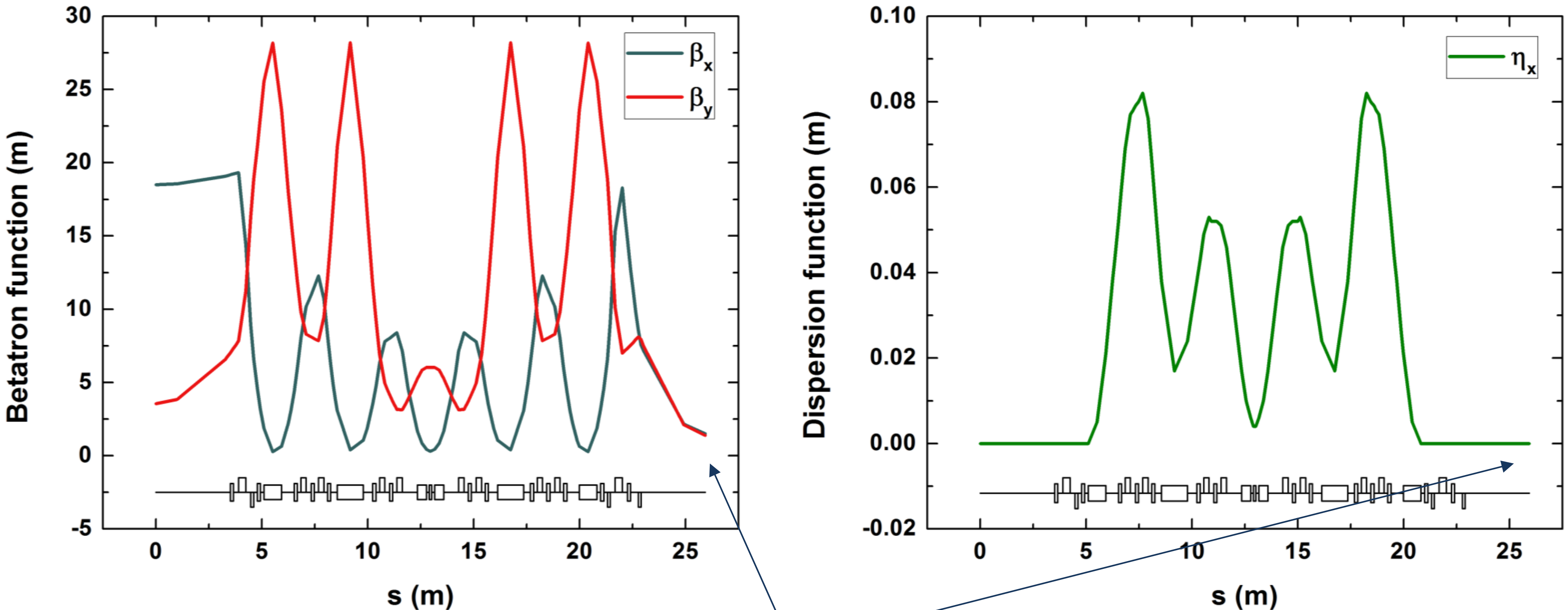


$$\varepsilon = \sigma_e \sigma'_e \propto \frac{E^2}{N_{\mathrm{dipoles}}^3}$$

- 120 EM dipoles (0.6 T);
- 20 NdFeB dipoles (2T);
- 260 quadrupoles;
- 280 sextupoles.

(Liu et al., 2014)

Sirius lattice functions



Parameters at the low β straight section:

$\beta_x = 1.5$ [m] $\beta_y = 1.4$ [m] $\eta = 0$ [m]

(Liu et al., 2014)

Sirius lattice functions

In each plane, the relative energy spread σ_E and the emittance ε of the electron beam are related to the following quantities:

$$\boxed{\sigma_{x,y}^2 = \beta_{x,y}\varepsilon_{x,y} + \left(\eta_{x,y}\sigma_E\right)^2}$$

$$\boxed{\sigma_{x',y'}^2 = \gamma_{x,y}\varepsilon_{x,y} + \left(\eta'_{x,y}\sigma_E\right)^2}$$

With:

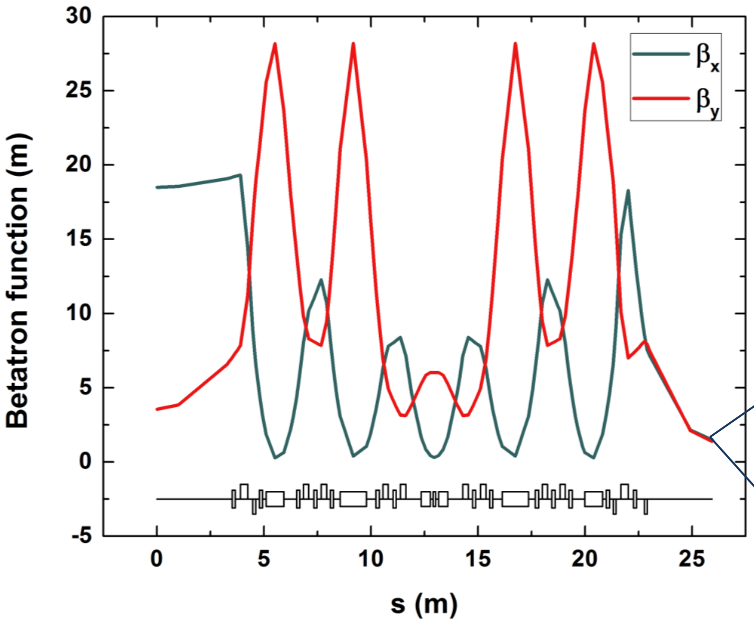
$$\boxed{\gamma_{x,y} = \frac{1+\alpha_{x,y}^2}{\beta_{x,y}}}$$

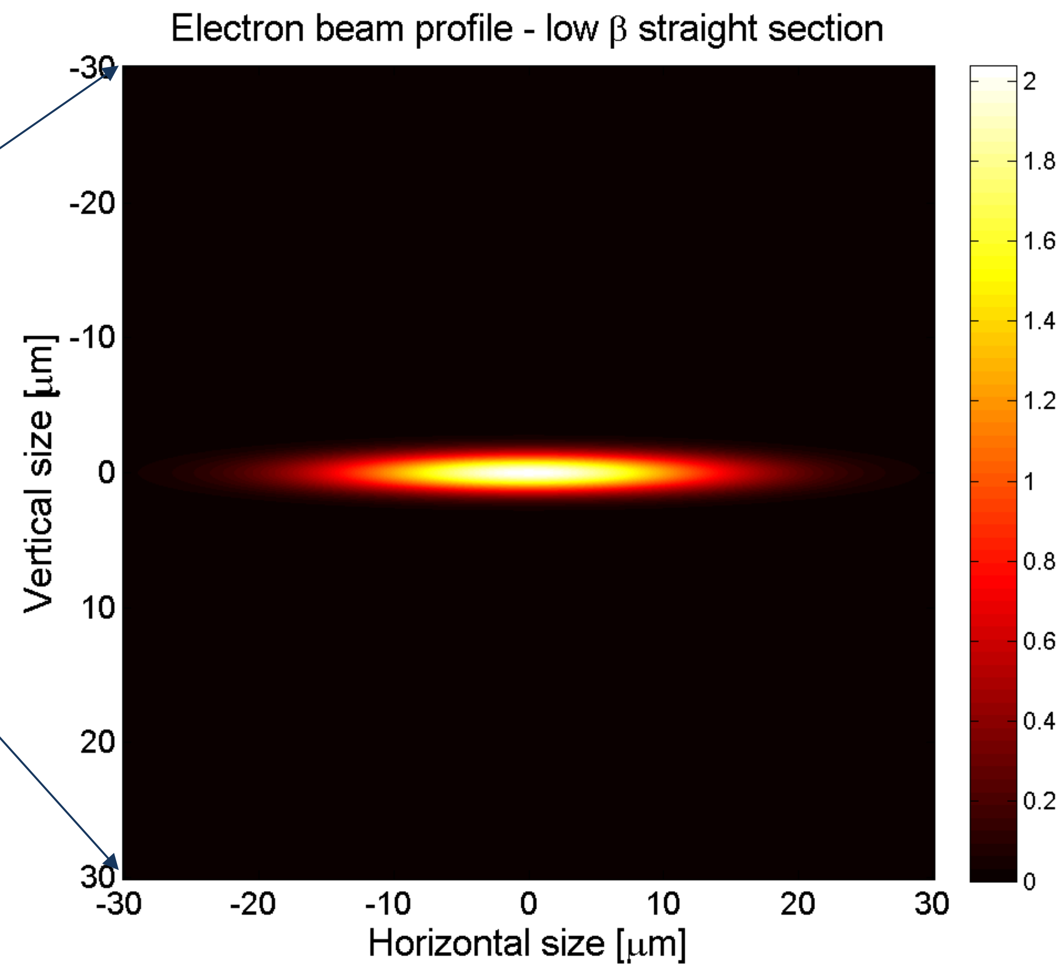
$$\boxed{\alpha_{x,y} = \frac{-\beta'_{x,y}}{2}}$$

Where $\beta_{x,y}$, $\eta_{x,y}$, $\beta'_{x,y}$, $\eta'_{x,y}$ are the betatron and dispertion functions and their derivatives.

(Courant & Snyder, 1957)

Sirius lattice functions

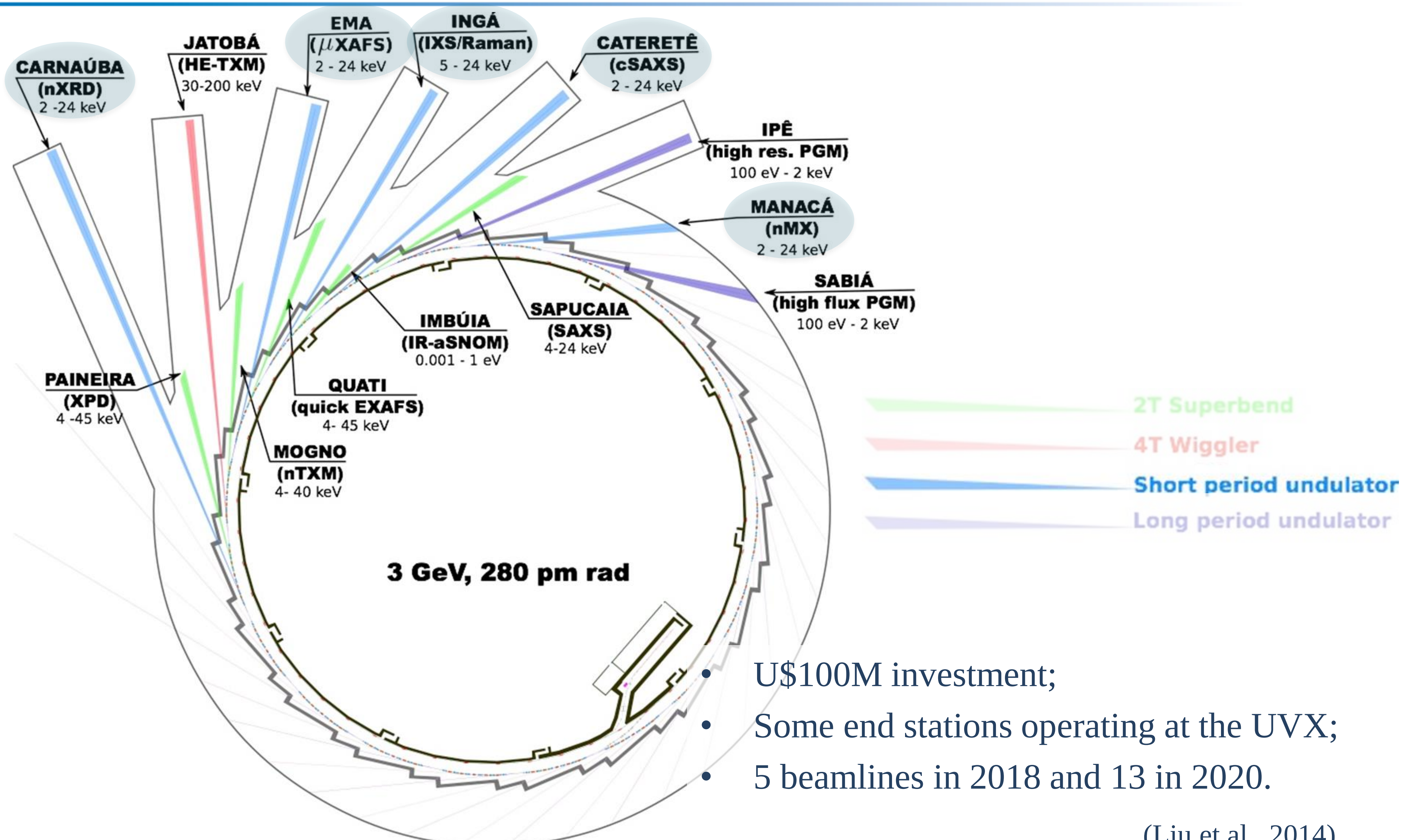




Electron Beam Parameters		
σ_x	20.01	[μm]
$\sigma_{x'}$	13.40	[μrad]
σ_y	1.94	[μm]
$\sigma_{y'}$	1.39	[μrad]

(Liu et al., 2014)

Sirius beamlines



- U$100M investment;
- Some end stations operating at the UVX;
- 5 beamlines in 2018 and 13 in 2020.

(Liu et al., 2014)

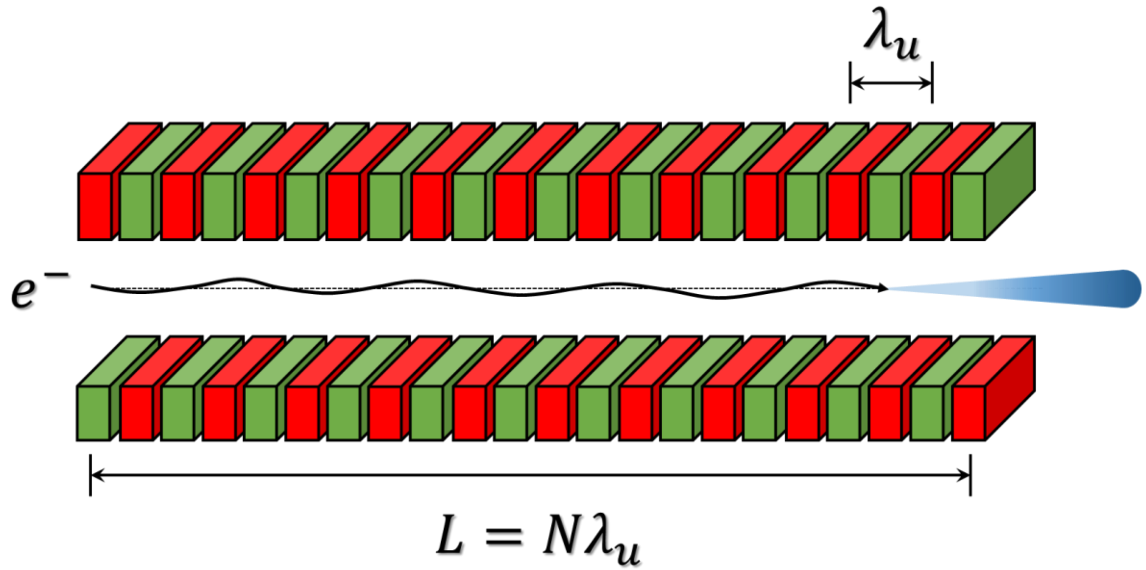
Sirius IVU19

- In vacuum hybrid design;
  - Dy doped (better coercivity) NdFeB with B_r=1.3 T ;
- 5 mm minimum (magnetic) gap
  - Important tender edges (P, S, Cl, K, Ca) in the 1st harmonic;
- λ_u=19 mm for no energy gap between 1st and 3rd harmonics;
- L = 2 m is enough for highest brilliance (not much gain for L > 2m);
- Flux demanding applications may need 3-4 m undulators (inelastic scattering).

IUV19 Undulator Parameters		
Magnetic field	1.15	[T]
λ_u	19	[mm]
N	105	[periods]
Undulator length	1.995	[m]
K_{max}	2.04	[ ]
K_{min}	0.40	[ ]
σ_E	0.083	[%]
# of harmonics*	6	[ ]

*odd harmonics

Undulator equations



$$\boxed{K = \frac{eB_0\lambda_u}{2\pi mc}}$$

$$\boxed{\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

Relativistic contracted wavelenght

Magnetic tunning

Off axis wavelenght increase

$$\gamma^2\theta^2 \backsimeq \frac{\Delta\lambda}{\lambda} = \frac{1}{nN}$$

(Clarke, 2004)

Where $\boldsymbol{\lambda_u}$ is the magnetic period, **N** is the number of periods, **K** is the undulator deflection parameter, $\boldsymbol{B_0}$ is the peak magnetic field, $\boldsymbol{n}$ is the harmonic number and $\boldsymbol{\theta}$ is the observation angle.

Undulator equations

The undulator natural source size (σ_{r0}) and divergence ($\sigma_{r'0}$) are given by the following equations:

$$\boxed{\sigma_{r0} = \frac{\sqrt{2L\lambda}}{4\pi}} \quad \boxed{\sigma_{r'0} = \sqrt{\frac{\lambda}{2L}}}$$

The source size ($\Sigma_{x,y}$) and angular divergence ($\Sigma_{x',y'}$) of the photon beam are given by:

$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_{r0}^2} \quad \Sigma_{x',y'} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'0}^2}$$

where $\sigma_{x,y}$ and $\sigma_{x',y'}$ are the beam size and angular divergence of the electron beam in the horizontal (x) and vertical (y) direction, respectively.

(Kim, 1995)

The energy spread effect

Let us define the growth fator introduced by the energy spread to the source size (Q_s) and divergence (Q_a).

$$\boxed{Q_s(x) = 2[Q_a(x/4)]^{2/3}}$$

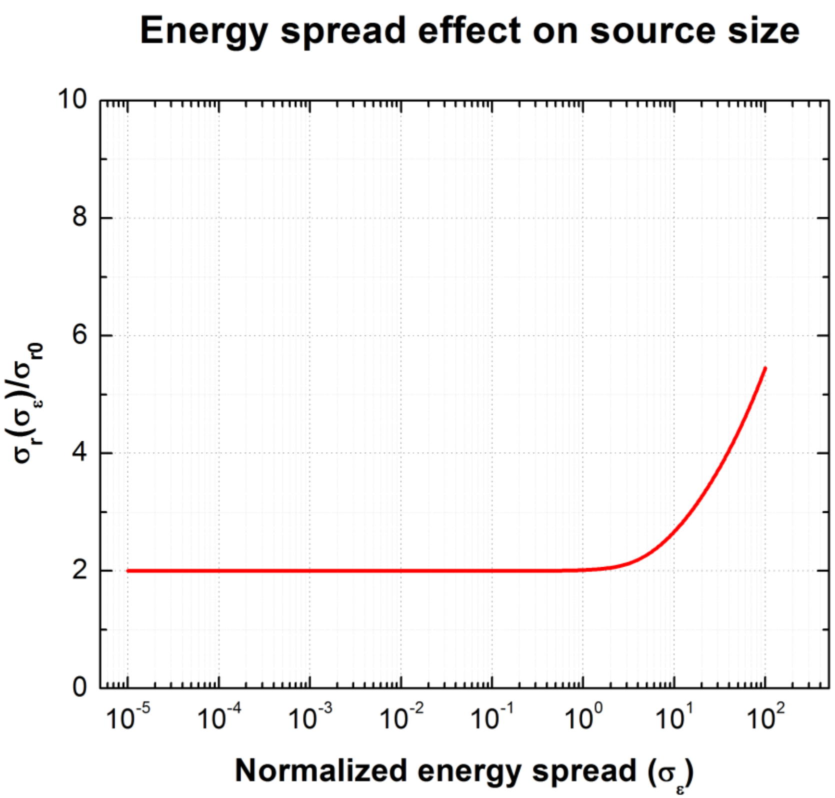
Where Q_a is given by:

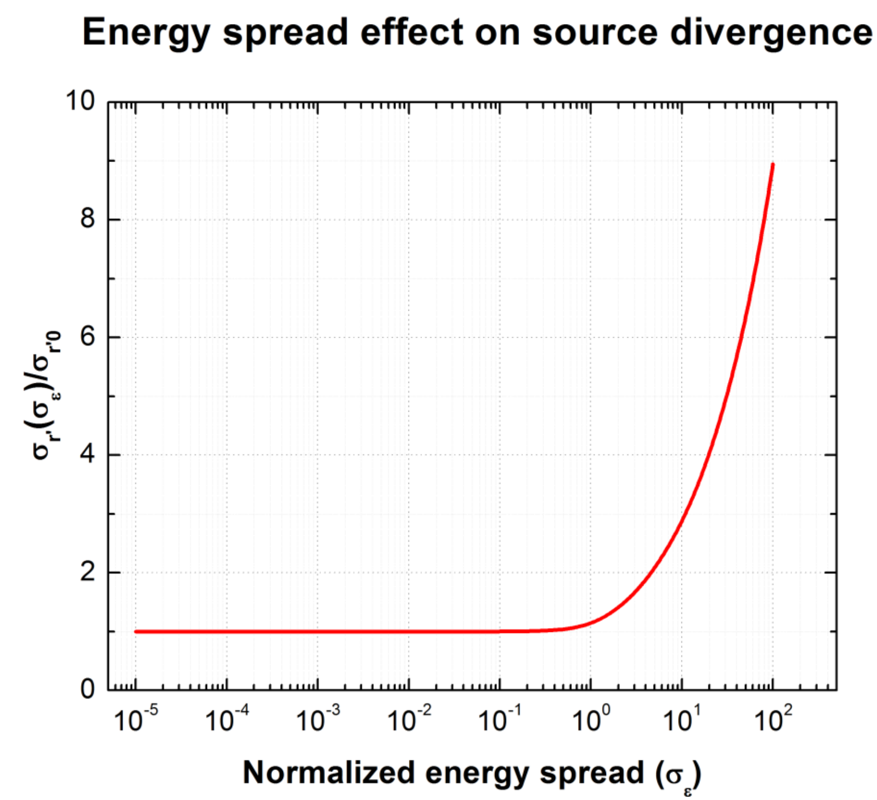
$$\boxed{Q_a(x) = \left[\frac{x^2}{-1 + \exp(-2x^2) + (2\pi)^{1/2}\, x\, \mathrm{erf}(2^{1/2}x)}\right]^{1/2}}$$

with $\mathrm{erf}(x)$ being the Gaussian error function.

(Tanaka & Kitamura, 2009)

The energy spread effect





Effects owing to the energy spread of the electron beam on the source size (left image) and on the beam divergence (right image).

Images adapted from (Tanaka & Kitamura, 2009)

The energy spread effect

Assuming the energy distribution to be a Gaussian function with the standard deviation σ_E, we introduce a normalized energy spread σ_ϵ defined as:

$$\boxed{\sigma_\epsilon = 2\pi n N \sigma_E}$$

Beams with a high energy spread produce in a conventional undulator photon beams with a high wavelength spread.

(Tanaka & Kitamura, 2009)

Corrected equations

Applying the broadning factors to the spatial and angular profiles of the undulator radiation, one obtains:

$$\boxed{\sigma_r(\sigma_\epsilon) = \sigma_{r0} Q_s(\sigma_\epsilon)}$$

$$\boxed{\sigma_{r'}(\sigma_\epsilon) = \sigma_{r'0} Q_a(\sigma_\epsilon)}$$

Which leads to the revised equations for the spatial and angular profiles of the undulator.

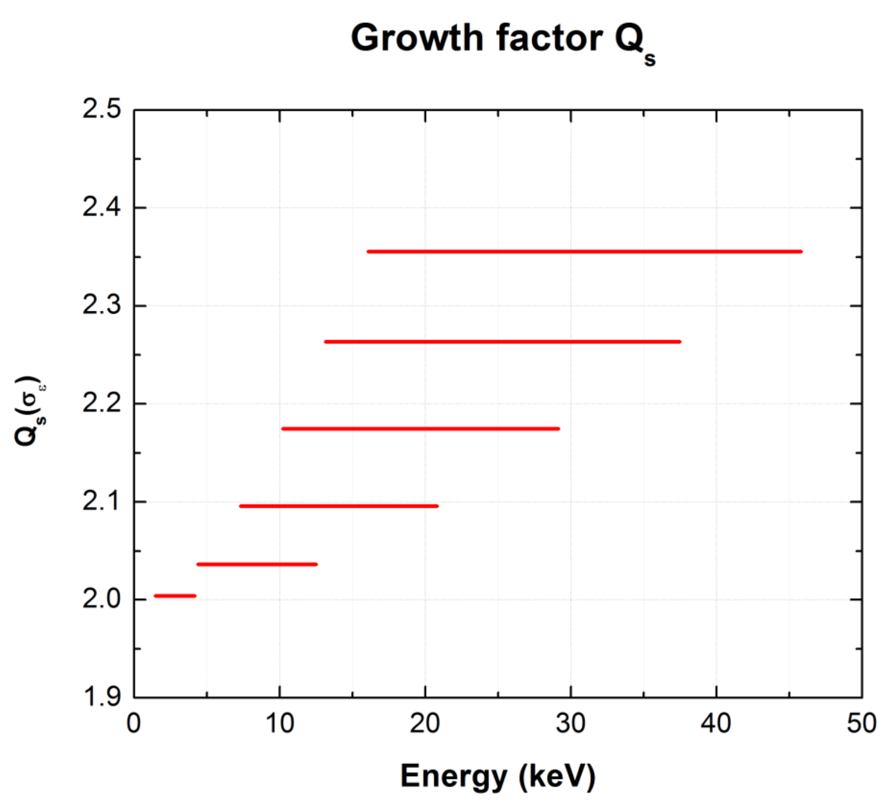
$$\boxed{\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_r{}^2}}$$

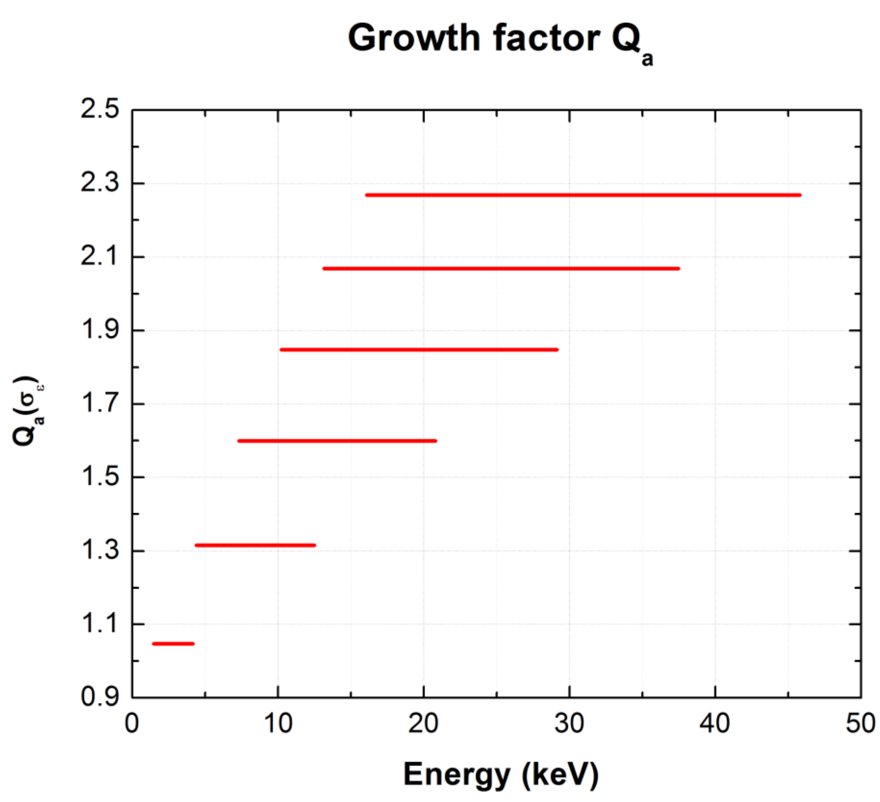
$$\boxed{\Sigma_{x',y'} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'}{}^2}}$$

Note that the argument σ_ϵ was omitted for simplicity.

(Tanaka & Kitamura, 2009)

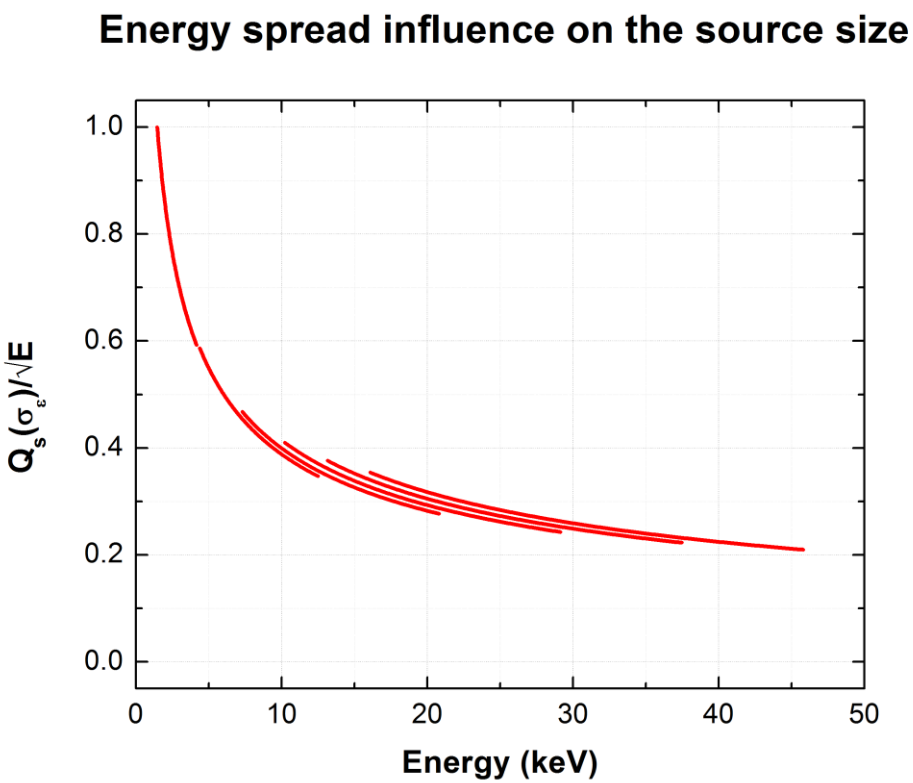
Growth factors

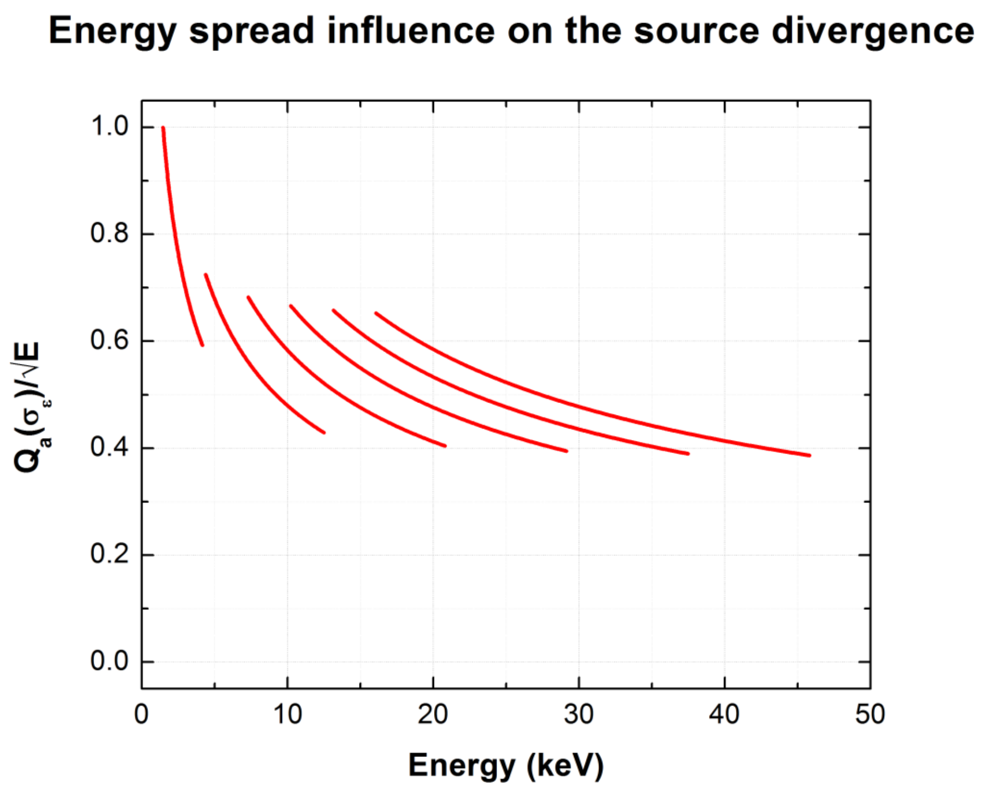




Growth factors calculated for the source size (left image) and on the beam divergence (right image). Calculations were held for the IVU19.

The energy spread influence





Effects owing to the energy spread of the electron beam on the source size (left image) and on the beam divergence (right image). Calculations were held for the IVU19.

Non-Gaussian beam

When the e^- beam and undulator natural emittances are comparable, the brilliance of the source is better represented by the Wigner distribution:

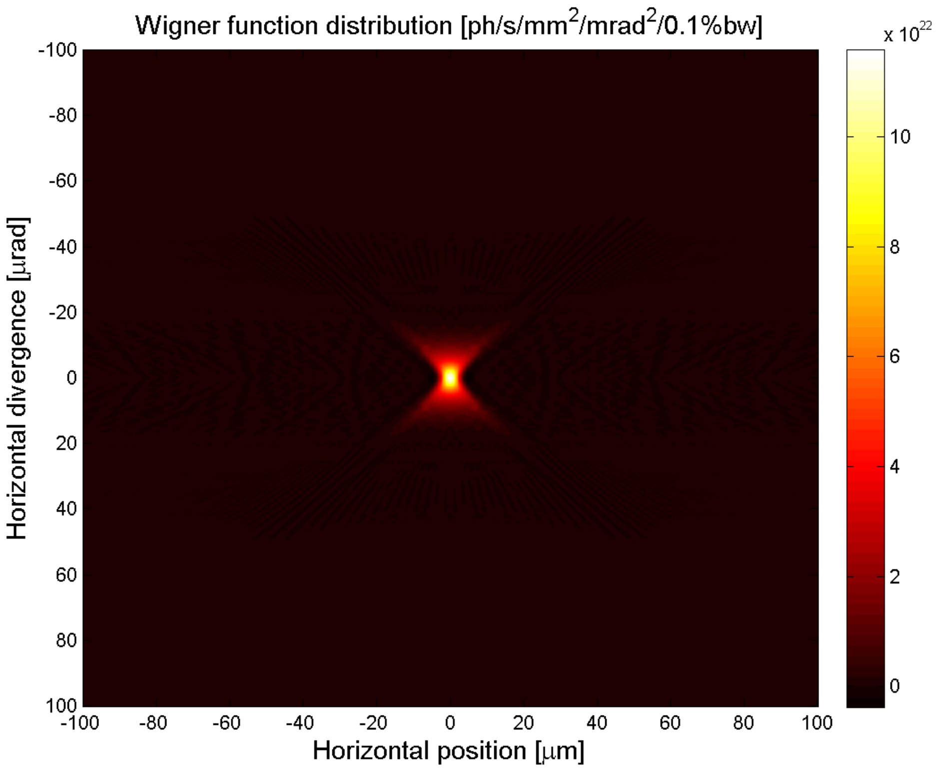
$$\mathbf{W}(\mathbf{r},\boldsymbol{\theta}) = \mathrm{N_e} \iint \mathrm{W_0}(\mathbf{r}-\mathbf{r_e},\boldsymbol{\theta}-\boldsymbol{\theta}_\mathbf{e};\delta_\mathrm{e})$$

$$\times\, \mathrm{P}(\mathbf{r_e},\boldsymbol{\theta}_\mathbf{e},\delta_\mathrm{e})d^2\mathbf{r_e}d^2\boldsymbol{\theta}_\mathbf{e}d\delta_\mathrm{e}$$

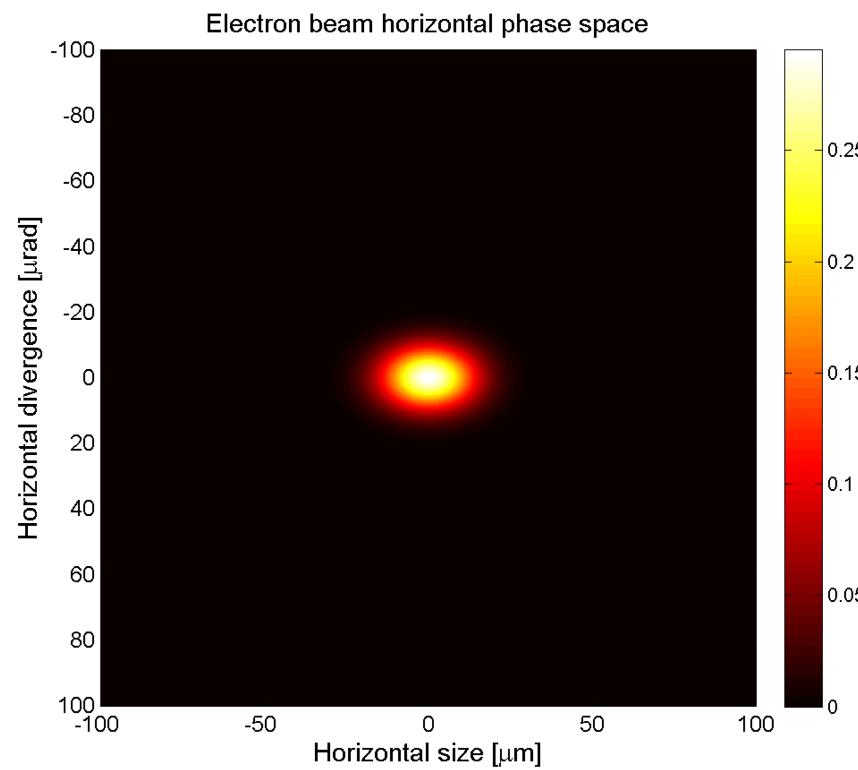
Which is the convolution between pure Wigner distribution function of the photons (first term of the integral) and classical Wigner distribution function of the electrons (second term of the integral).

(Bazarov, 2012) and (Tanaka, 2014)

Non-Gaussian beam

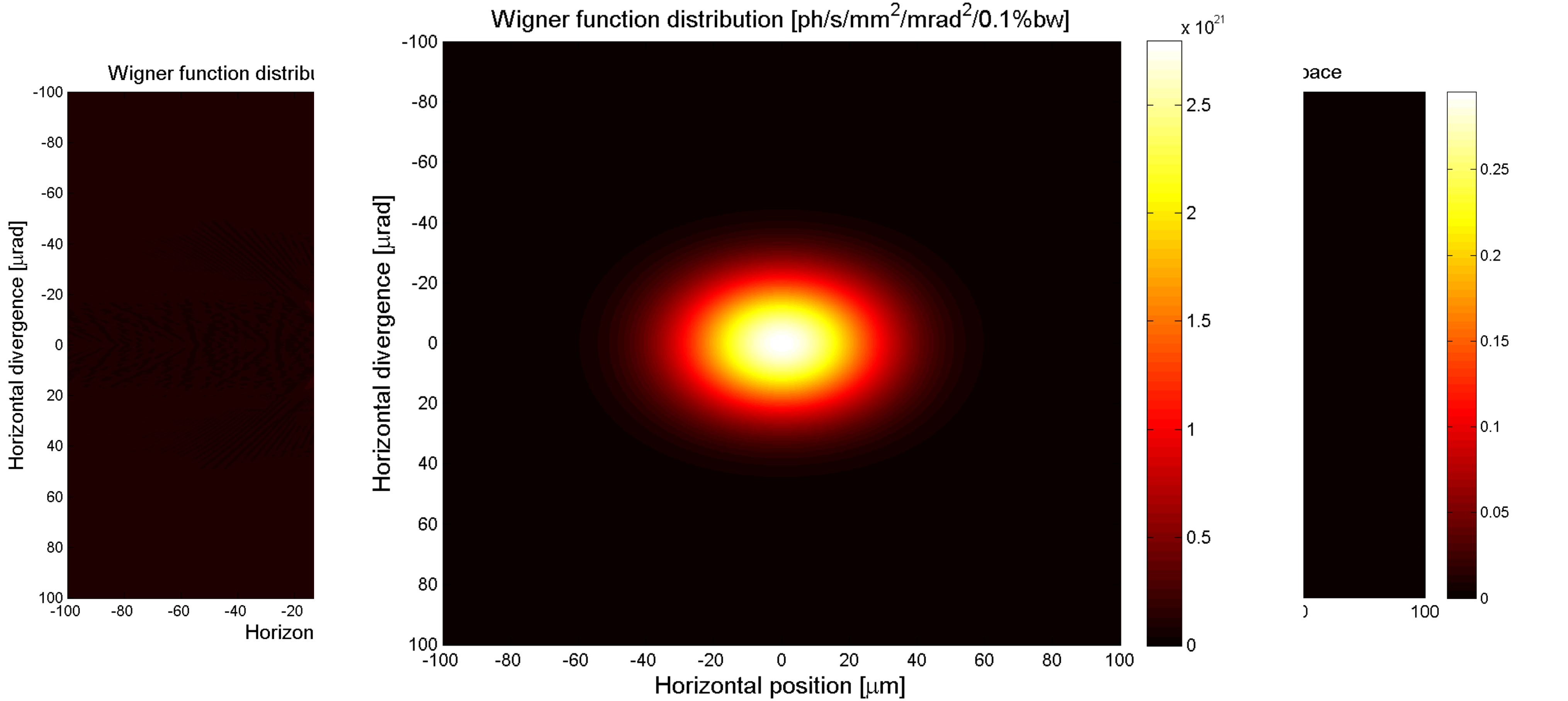


*



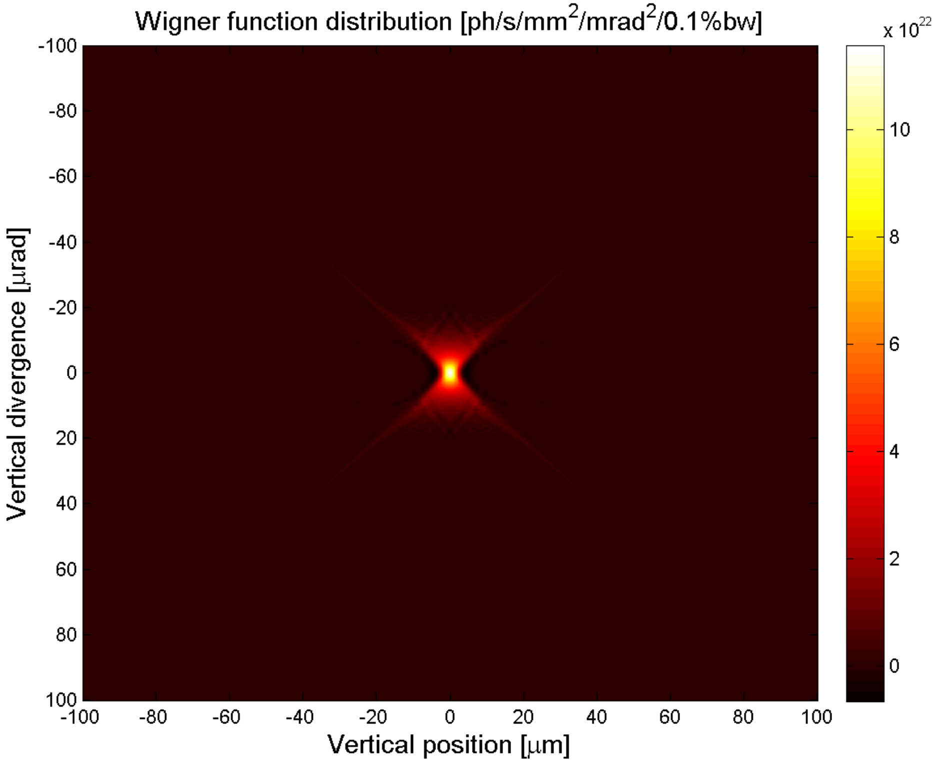
Horizontal Wigner distribution of the pure ondulator IVU19 (left image) and the horizontal electron beam phase space (right image). Calculations were held using SPECTRA 10.1 for the 5^{th} harmonic at $K = 2.04$ ($E_{5th} = 7.3$ keV).

Non-Gaussian beam



Convolution between the horizontal Wigner distribution and the horizontal electron phase space for the IVU19. Calculations were held using SPECTRA 10.1 for the 5^{th} harmonic at $K = 2.04$ ($E_{5th} = 7.3$ KeV).

Non-Gaussian beam



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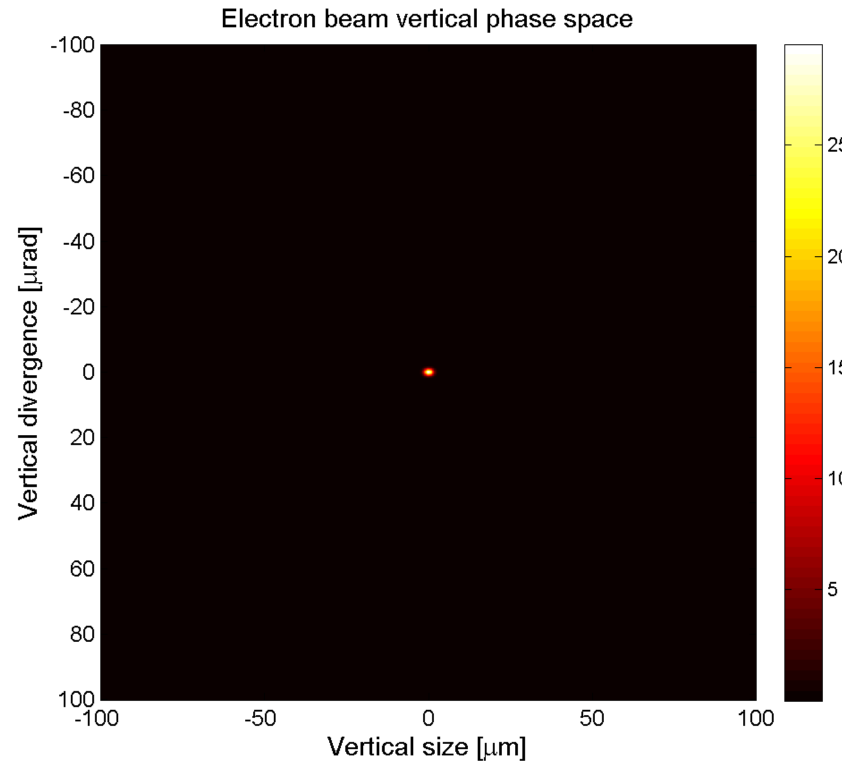
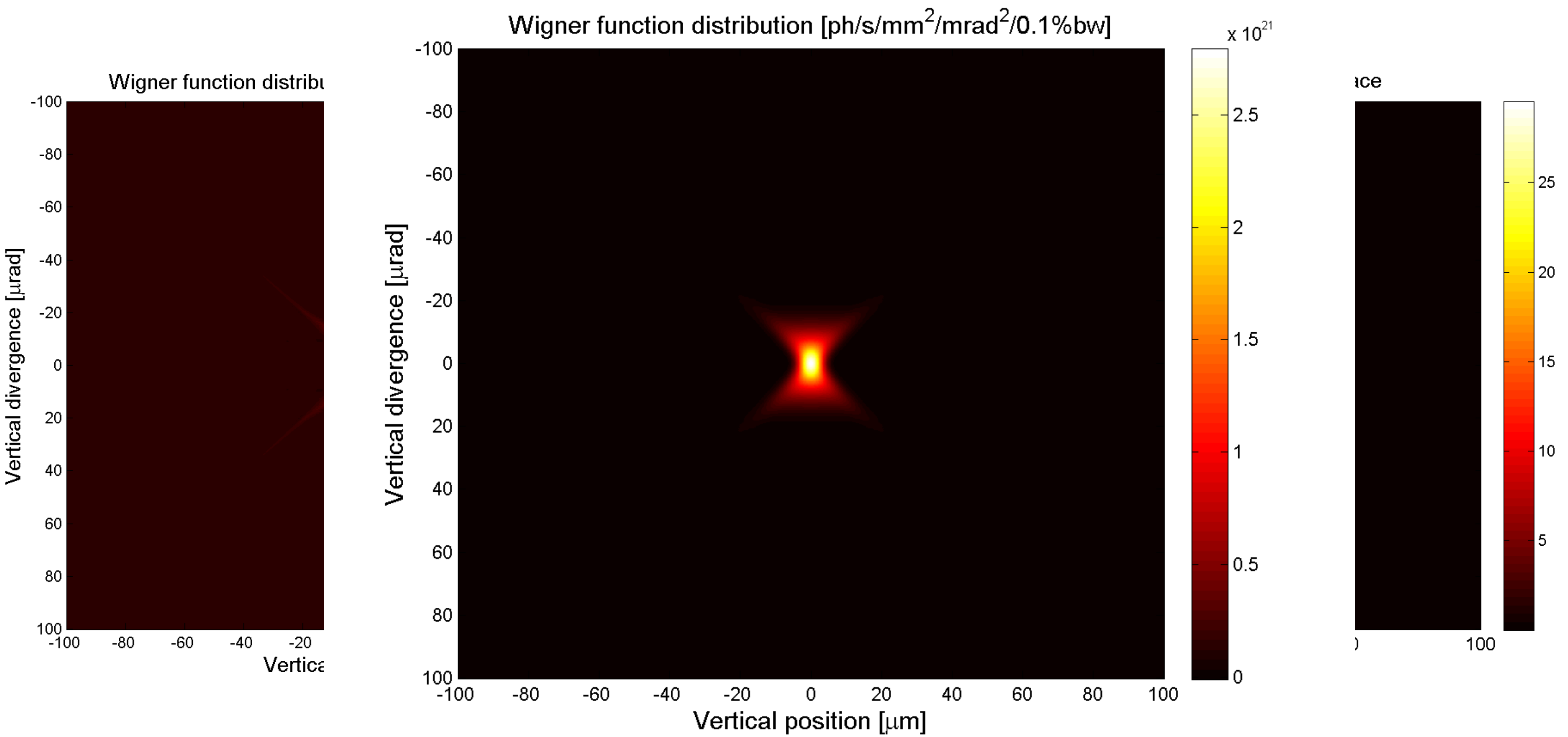


Vertical Wigner distribution of the pure ondulator IVU19 (left image) and the vertical electron beam phase space (right image). Calculations were held using SPECTRA 10.1 for the 5^{th} harmonic at $K = 2.04$ ($E_{5th} = 7.3$ keV).

Non-Gaussian beam



Convolution between the vertical Wigner distribution and the vertical electron phase space for the IVU19. Calculations were held using SPECTRA 10.1 for the 5^{th} harmonic at $K = 2.04$ ($E_{5th} = 7.3$ KeV).

Computer codes

Synchrotron Radiation Workshop (SRW) – ESRF – France

O. Chubar, P. Elleaume, "Accurate And Efficient Computation Of Synchrotron Radiation In The Near Field Region", proc. of the EPAC98 Conference, 22-26 June 1998, p.1177-1179.

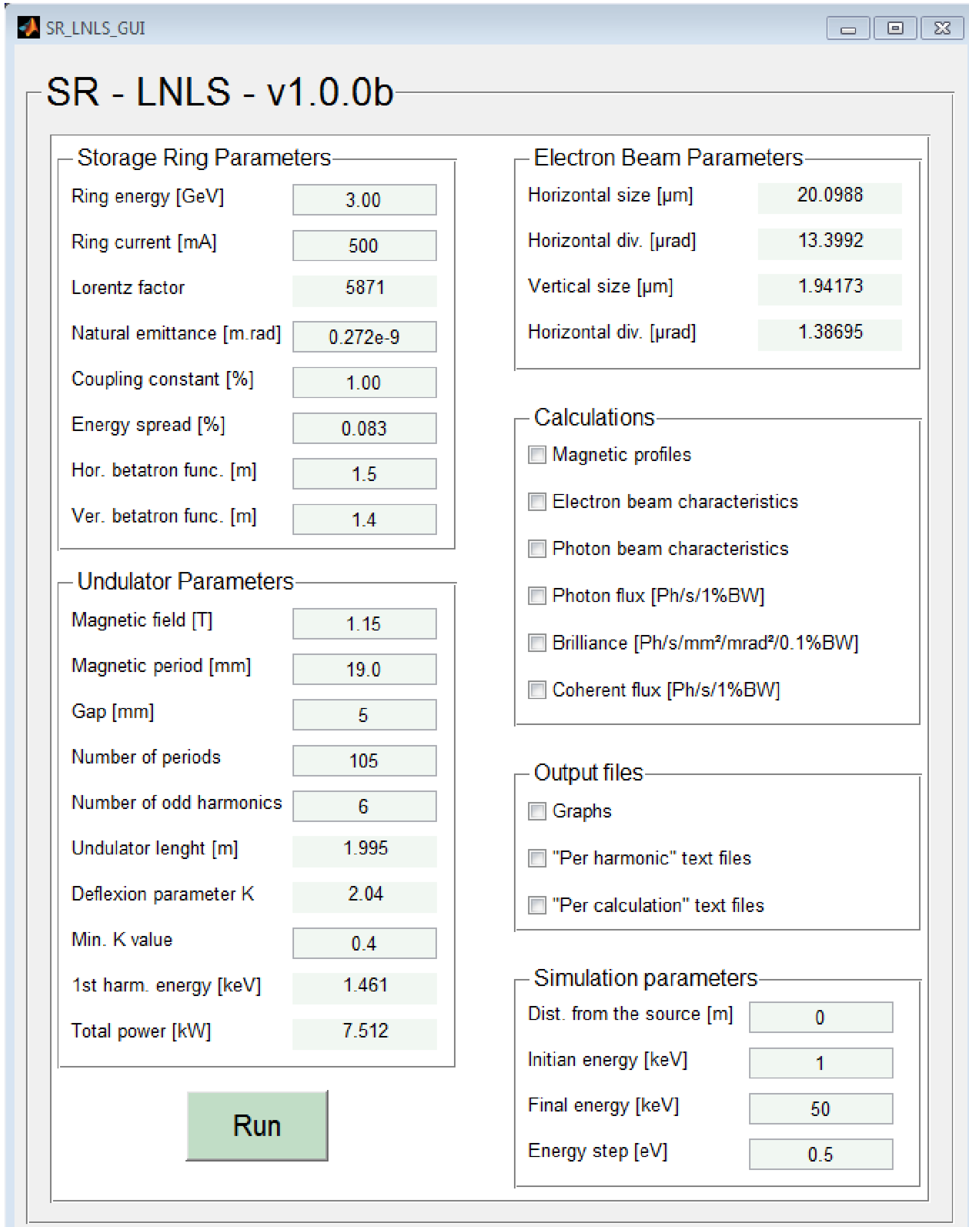
SPECTRA – SPring-8 – Japan

T. Tanaka, H. Kitamura, "SPECTRA: a synchrotron radiation calculation code", J. Synchrotron Rad., 2001, vol. 8, p.1221-1228.

SR_LNLS – LNLS – Brazil

R. Celestre, B. Meyer. Homemade MATLAB code developed by the X-Ray Optics Group based on the revised equations presented before.

The SR_LNLS



User interface from the SR_LNLS. Simulation parameters are given through text boxes and are grouped into 6 categories:

- Storage ring parameters;
- Electron beam parameters;
- Undulator parameters;
- Calculations:
  - Magnetic profiles;
  - Electron beam;
  - Photon beam;
- Output files:
  - Graphs;
  - Text files;
- Simulation parameters.

The SR_LNLS

```
9-May-2015 11:23:13

Planar Undulator Calculations

Storage Ring Parameters:
     Ring Energy:           3.00             [GeV]
     Ring Current:          500              [mA]
     Gamma:                 5870.8543
     Natural emittance:     2.720000e-010    [m.rad]
     Coupling factor:       1                [%]
     Energy spread:         8.300000e-002    [%]
     Hor_beta:              1.500000e+000    [m]
     Ver_beta:              1.400000e+000    [m]

E-beam parameters (sigma):
     Hor_size:              2.009877e+001    [um]
     Hor_div:               1.339918e+001    [urad]
     Ver_size:              1.941725e+000    [um]
     Ver_div:               1.386947e+000    [urad]
```

```
Undulator parameters:
     Magnetic field:        1.150000e+000    [T]
     Magnetic period:       19               [mm]
     Number of periods:     105              [ ]
     Undulator Lenght:      1.995000e+000    [m]
     1st harmonic energy:   1.460468e+000    [keV]
     K max:                 2.040198e+000    [ ]
     K min:                 4.000000e-001    [ ]
     Total Power:           7.511786e+000    [kW]

Simulation parameters:
     Dist. to the source:   0.00             [m]
     Initial energy:        1.00             [keV]
     Final Energy:          50.00            [keV]
     Energy step:           10.00            [ev]
```

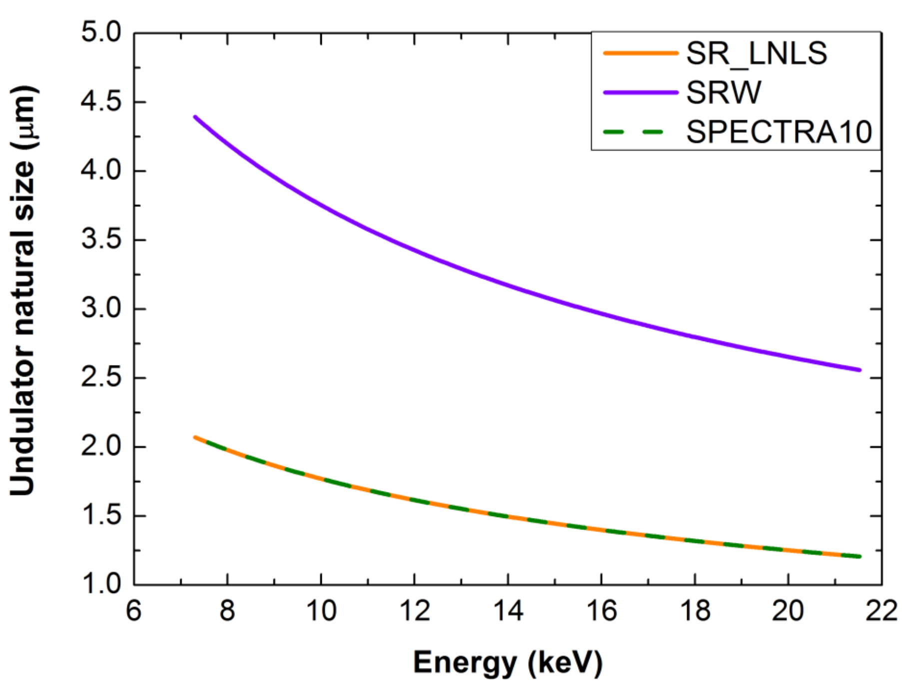
Benchmarking: direct comparisons

Were the first means of benchmarking, since they offer direct and clear results. Each program has an option for generating optical parameters given a number of odd harmonics and a K range.

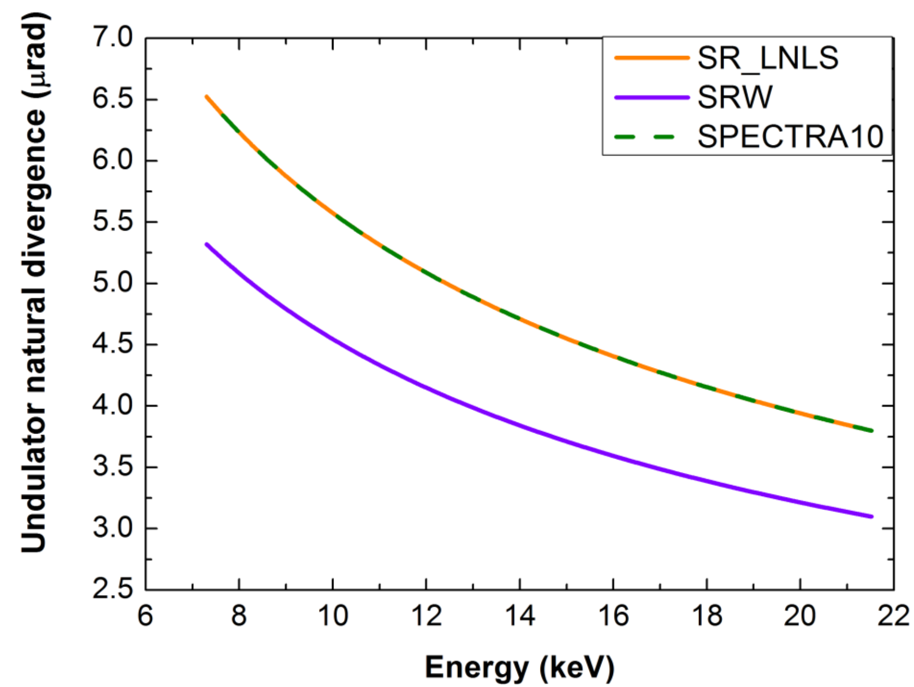
The following calculations (tunning curves) were held for the 5^{th} harmonic of the IVU19, that ranges from $E_{5\mathrm{th}}|_{\mathrm{K}=2.04} = 7.30\,\mathrm{keV}$ to $E_{5\mathrm{th}}|_{\mathrm{K}=0.3} = 21.523\,\mathrm{keV}$.

Benchmarking: direct comparisons

IVU19 - σ_{r0}

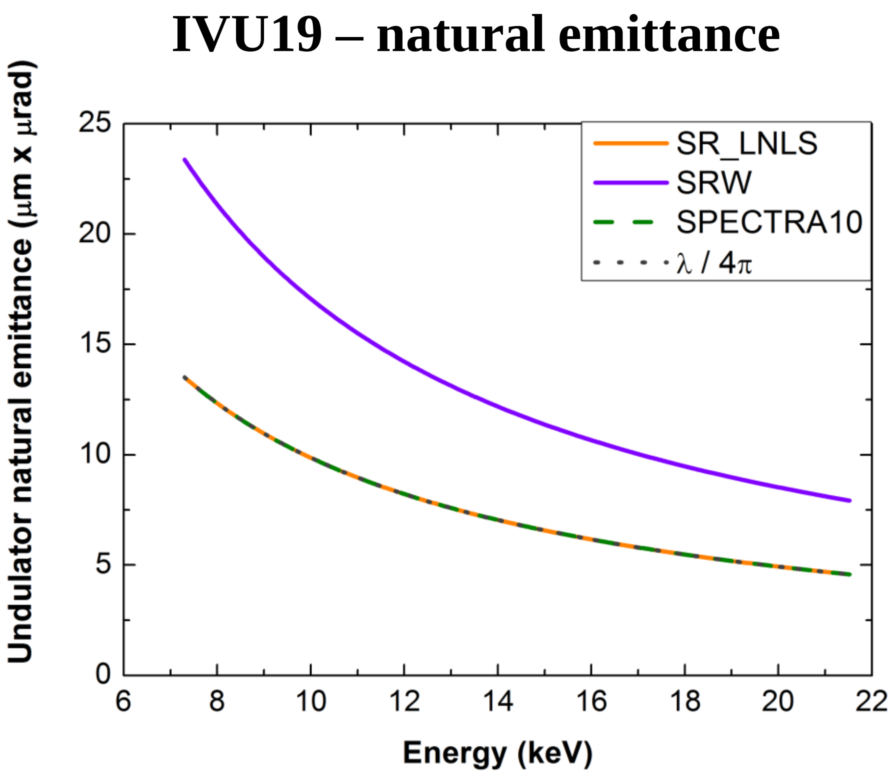


IVU19 - $\sigma_{r'0}$



Direct comparisons between the undulator natural source size (left image) and the undulator natural source divergence (right image).

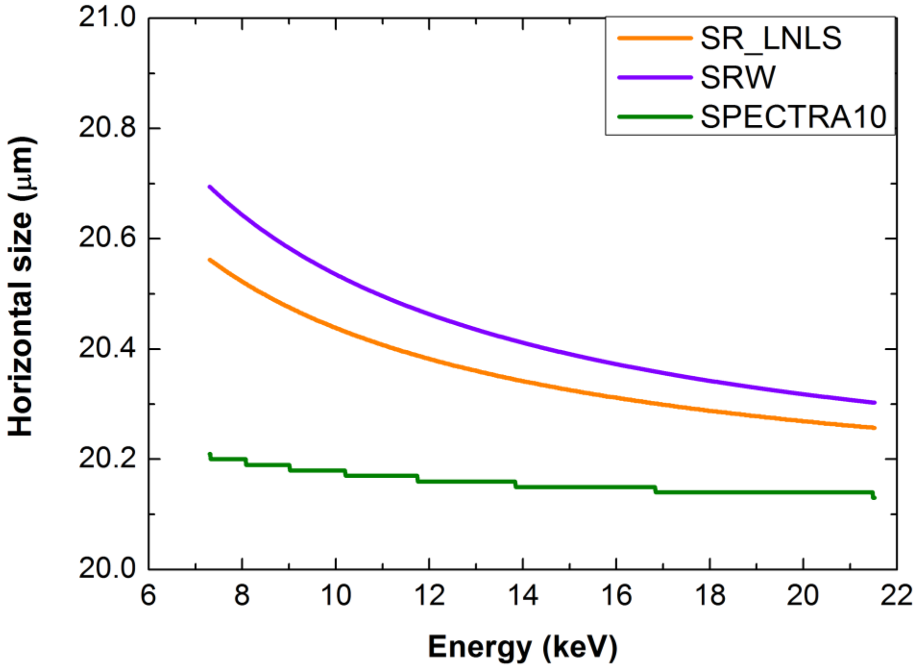
Benchmarking: direct comparisons



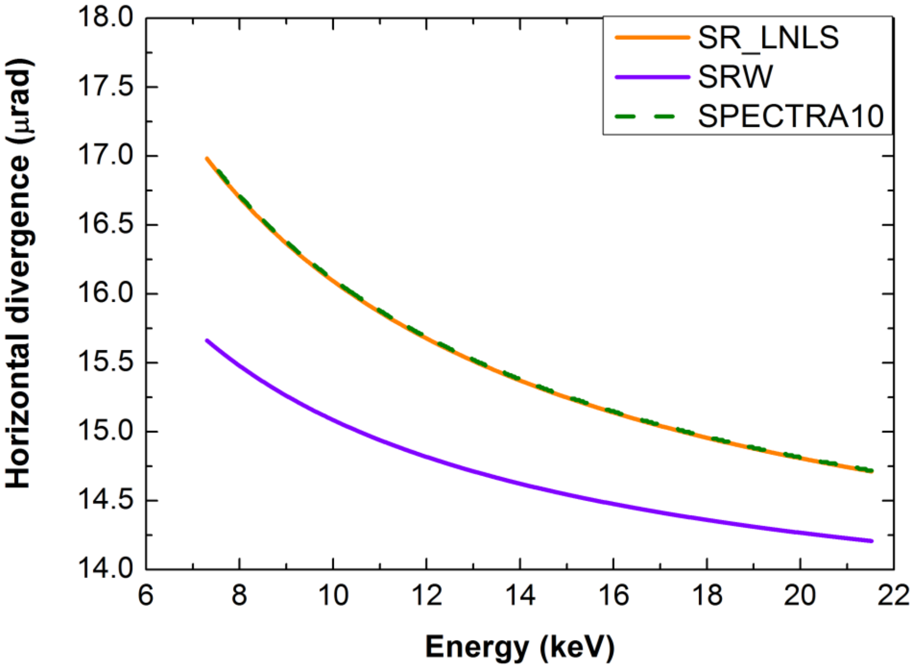
Comparisons between the ondulator natural emittance and the theorethical diffraction limited source value.

Benchmarking: direct comparisons

IVU19 - Σ_x



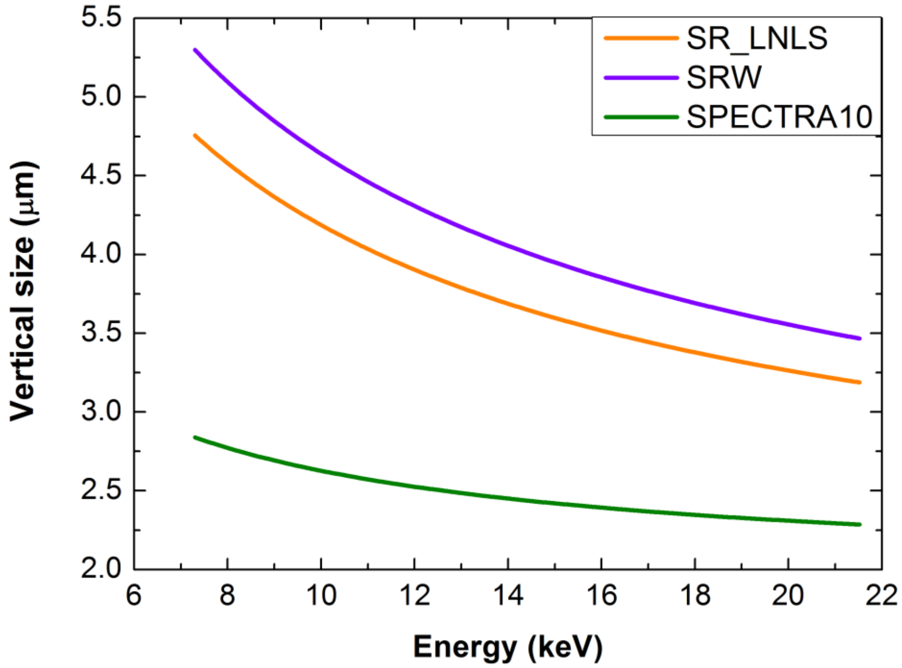
IVU19 - $\Sigma_{x'}$



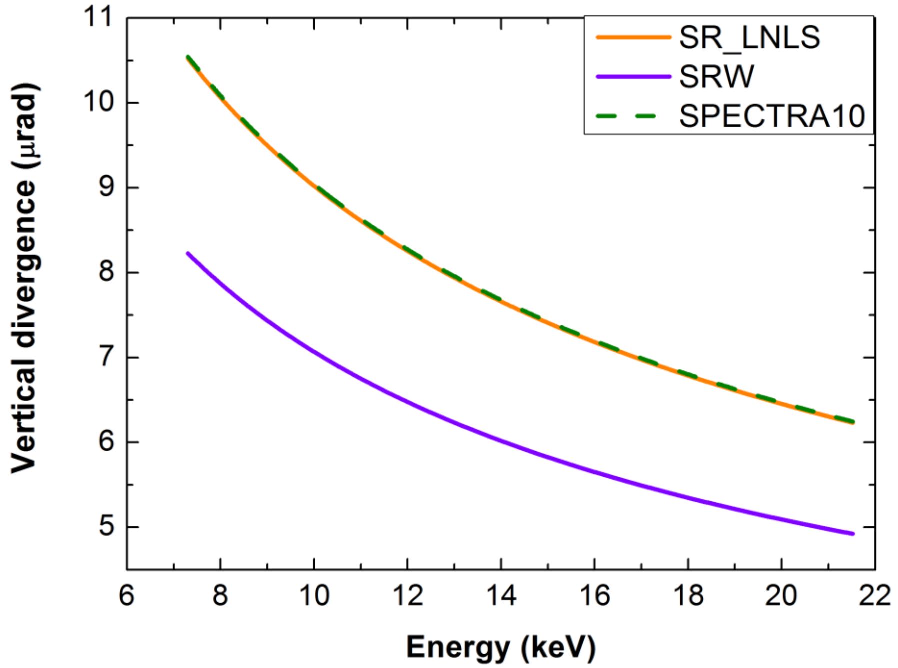
Direct comparisons between the undulator horizontal source size (left image) and the undulator horizontal source divergence (right image).

Benchmarking: direct comparisons

IVU19 - Σ_y



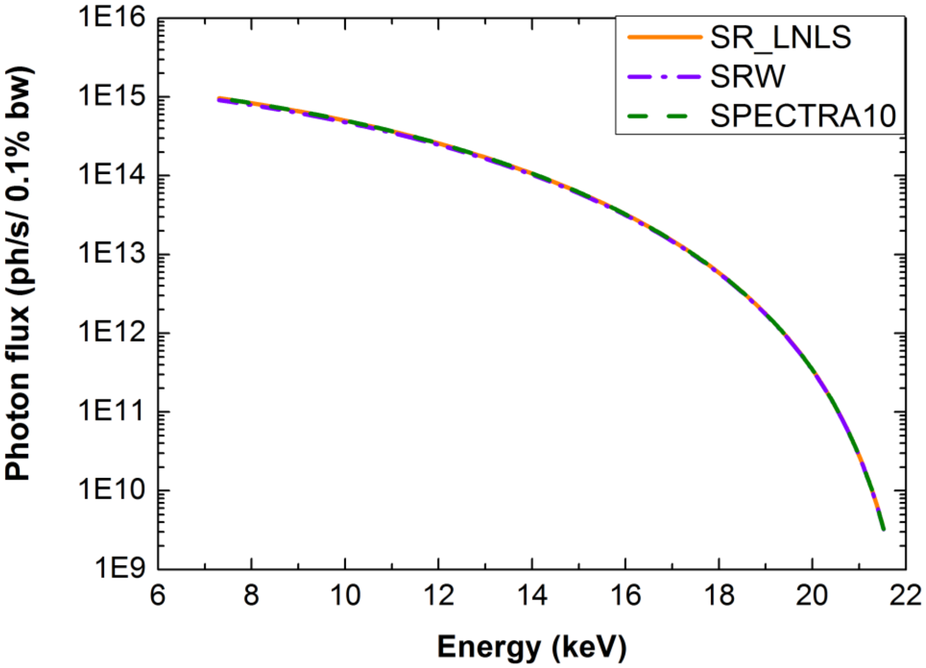
IVU19 - $\Sigma_{y'}$



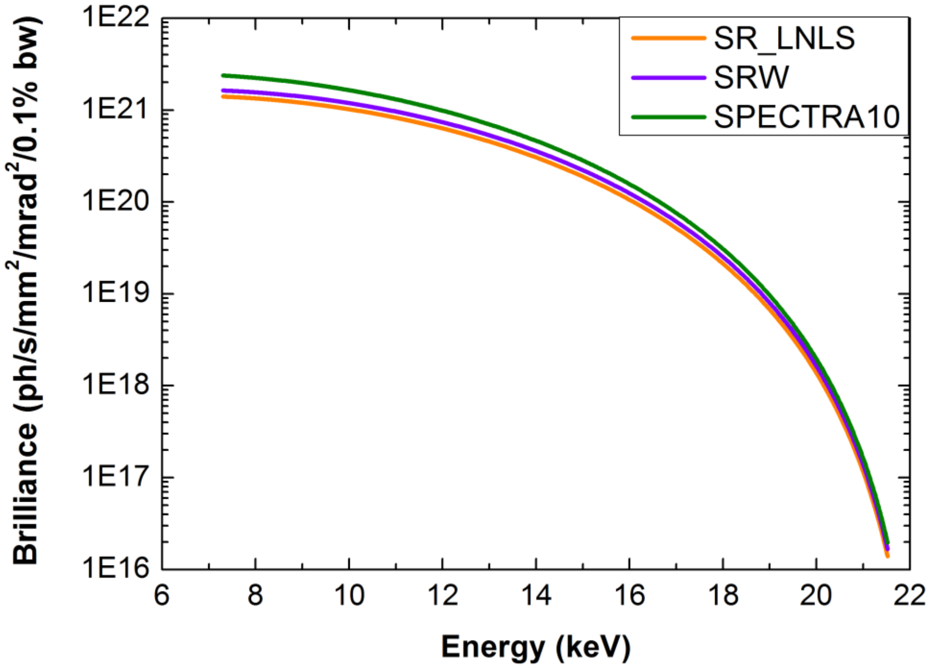
Direct comparisons between the undulator vertical source size (left image) and the undulator vertical source divergence (right image).

Benchmarking: direct comparisons

IVU19 – photon flux



IVU19 – brilliance



Direct comparisons between the photon flux (left image) and the undulator brilliance (right image).

Benchmarking: Wigner function

The Wigner function is the quasi-probability distribution of the photons at the phase space, which is the generic definition of brilliance:

$$\mathfrak{B}(x, y, x', y') \equiv \mathrm{W}(x, y, x', y')$$

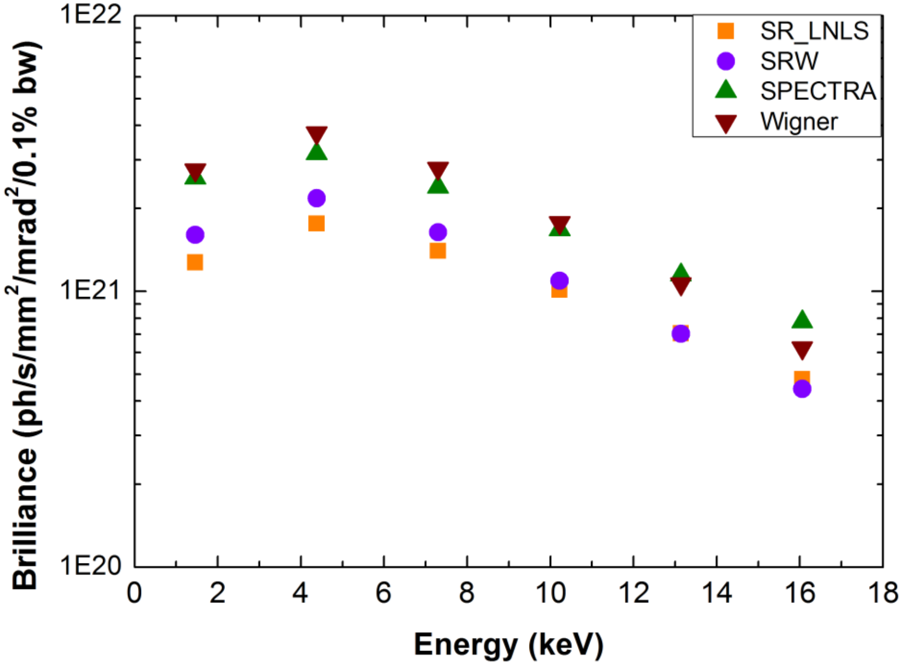
Thus, by calculation of the Wigner function one obtains not only brilliance values, but also optical parameters such as source size and divergence.

The following calculations were held for the first six odd harmonics of the IVU19 at a $\mathrm{K} = 2.04$ with the SPECTRA10.1.

(Bazarov, 2012) and (Tanaka, 2014)

Benchmarking: Wigner function

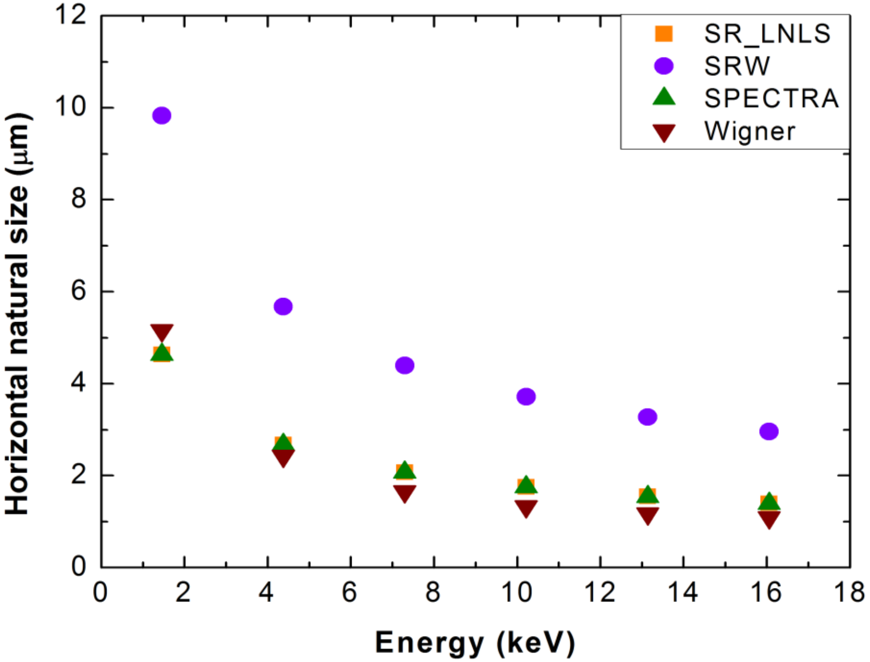
IVU19 – brilliance



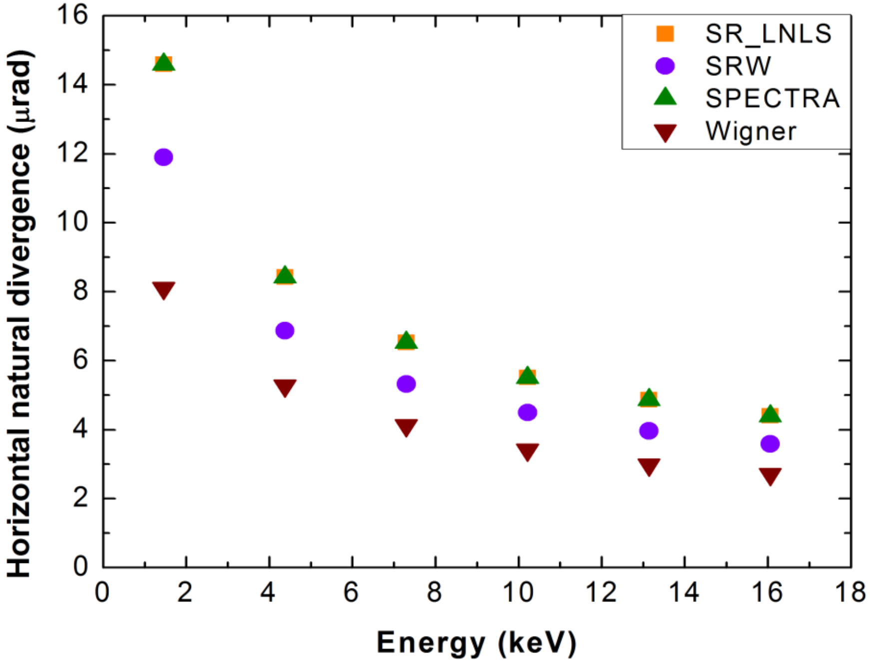
Comparisons between the brilliance values found with the benchmarking codes and the one generated with the calculation of the Wigner function.

Benchmarking: Wigner function

IVU19 - σ_{r0}



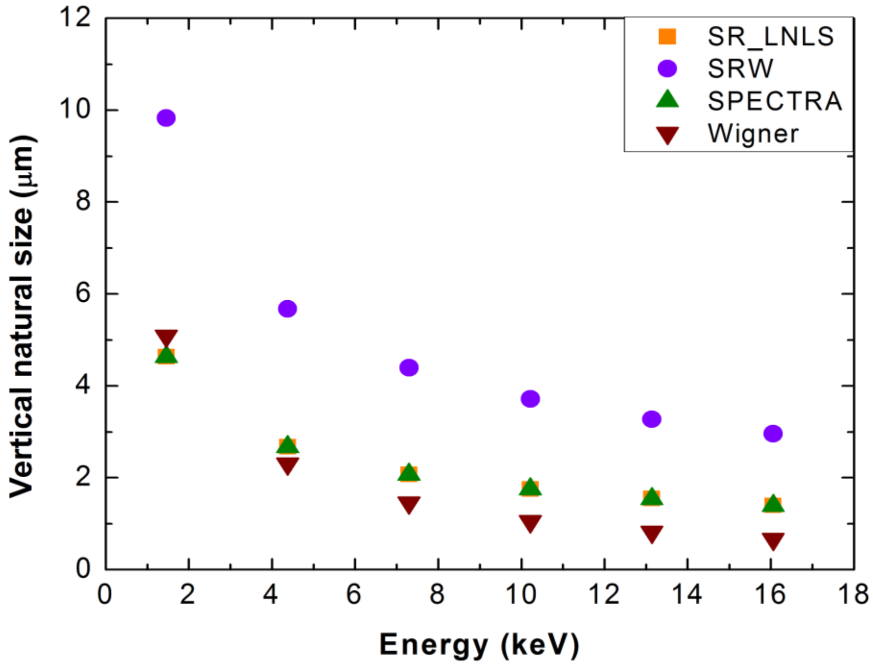
IVU19 - $\sigma_{r'0}$



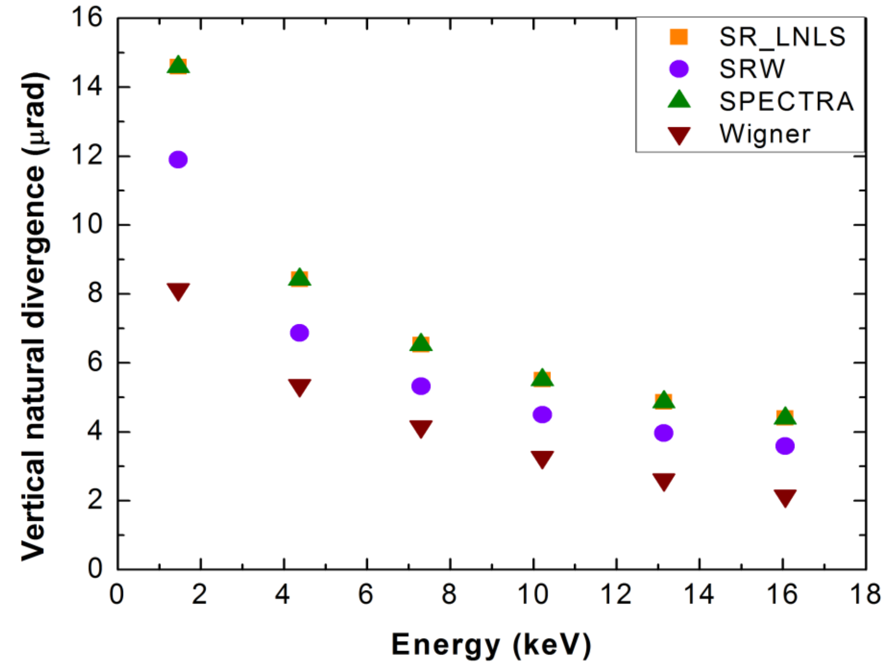
Comparisons between the undulator natural source size (left image) and the undulator natural source divergence (right image).

Benchmarking: Wigner function

IVU19 - σ_{r0}



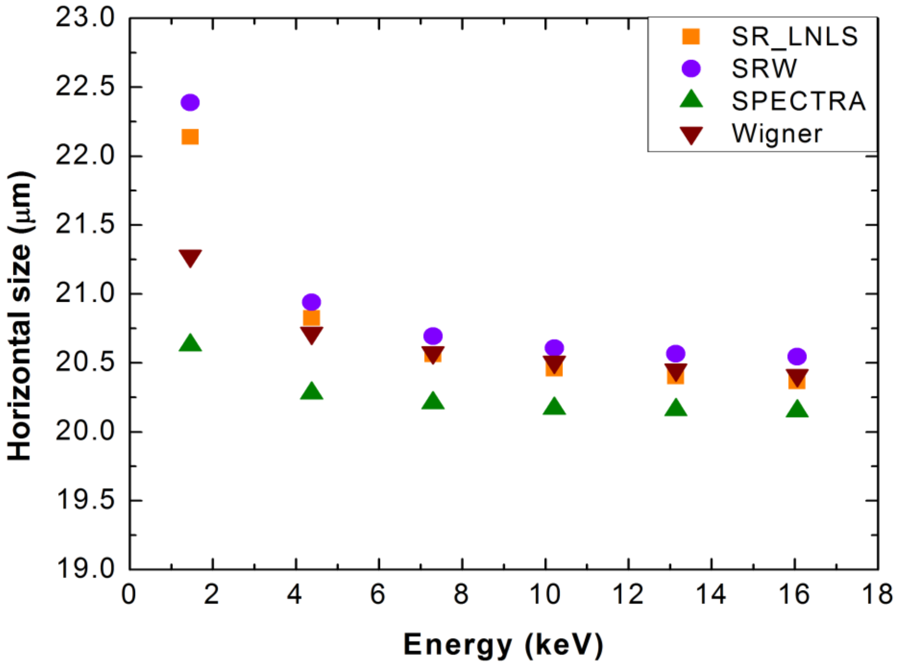
IVU19 - $\sigma_{r'0}$



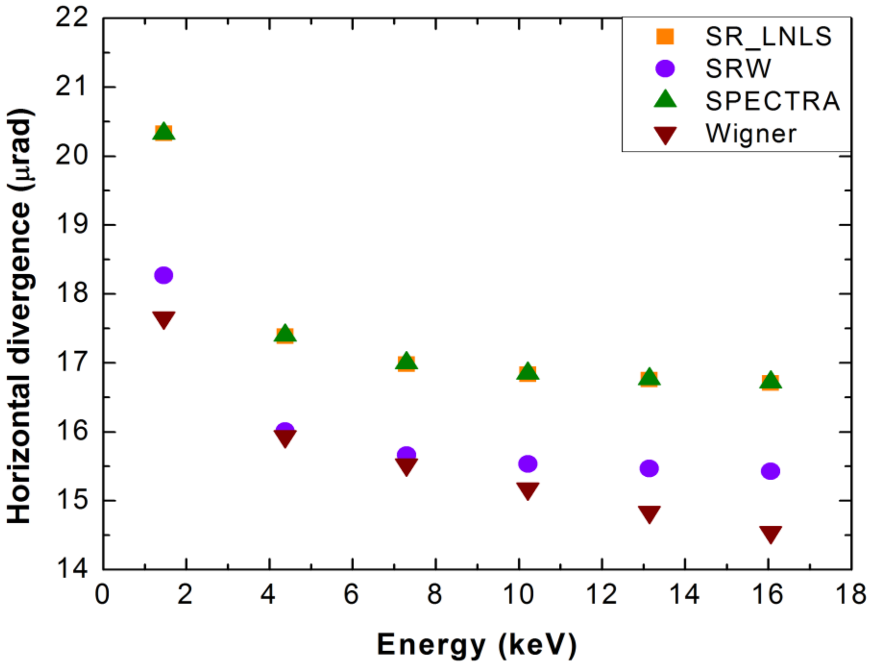
Comparisons between the undulator natural source size (left image) and the undulator natural source divergence (right image).

Benchmarking: Wigner function

IVU19 - Σ_x



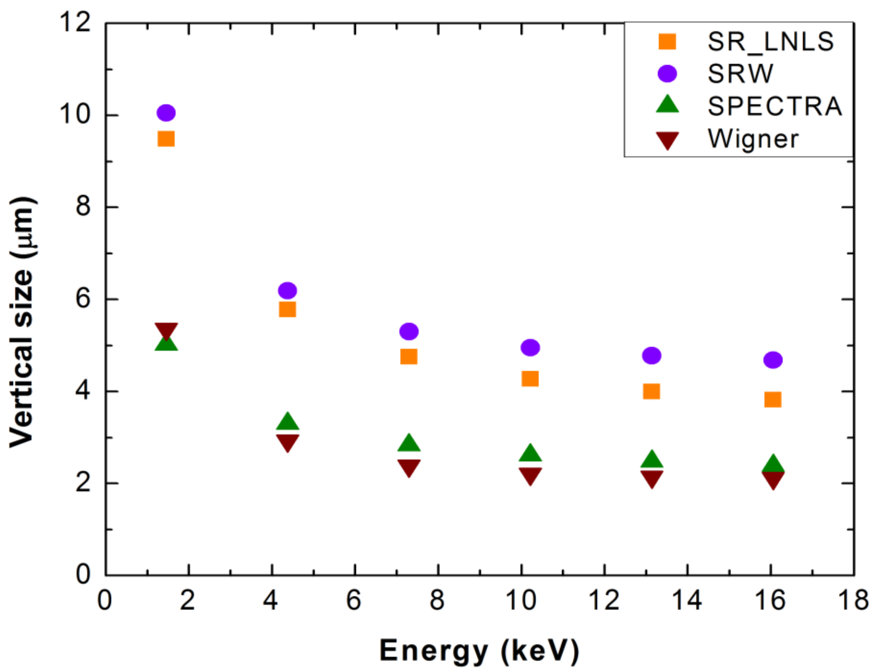
IVU19 - $\Sigma_{x'}$



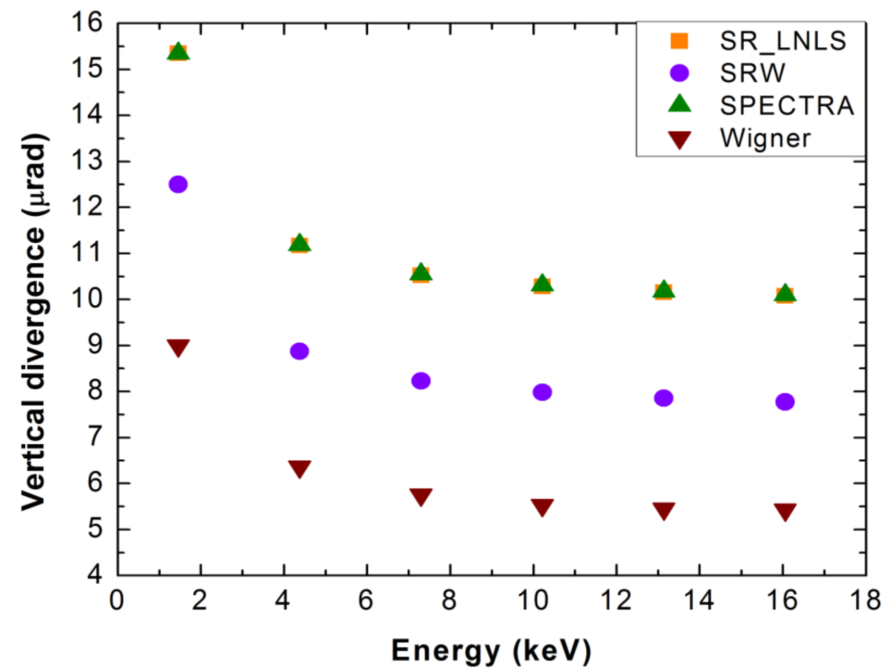
Comparisons between the undulator horizontal source size (left image) and the undulator horizontal source divergence (right image).

Benchmarking: Wigner function

IVU19 - Σ_y



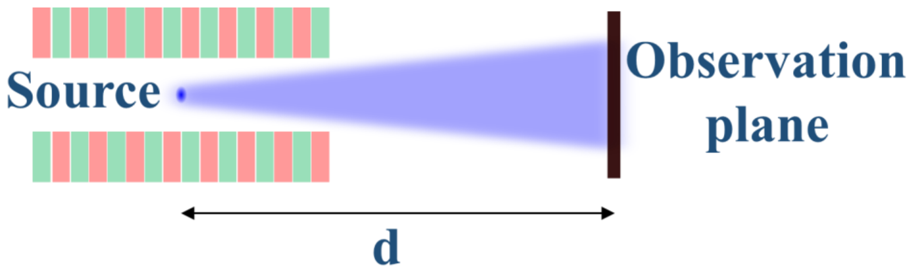
IVU19 - $\Sigma_{y'}$



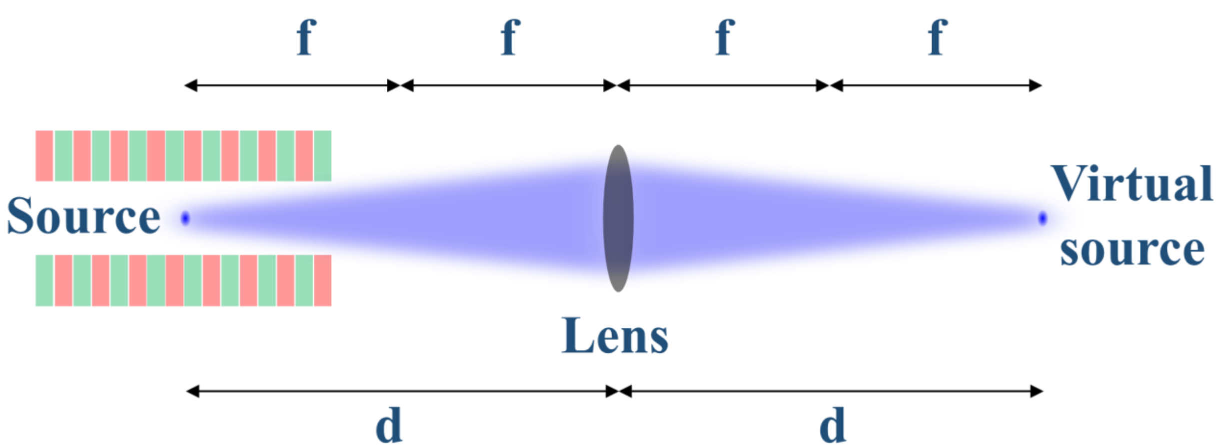
Comparisons between the undulator vertical source size (left image) and the undulator vertical source divergence (right image).

Benchmarking: 1:1 imaging system

a) Far field observation

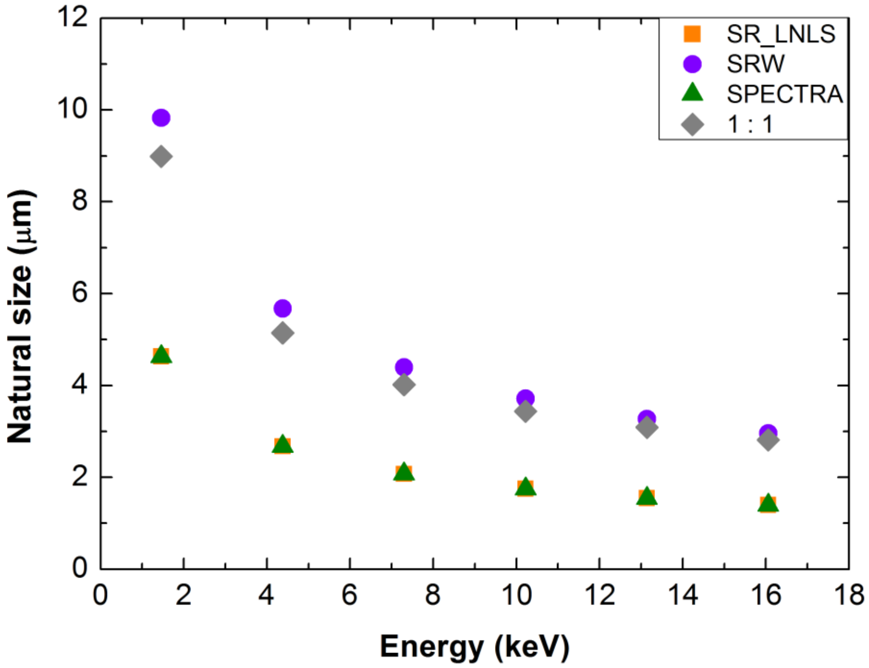


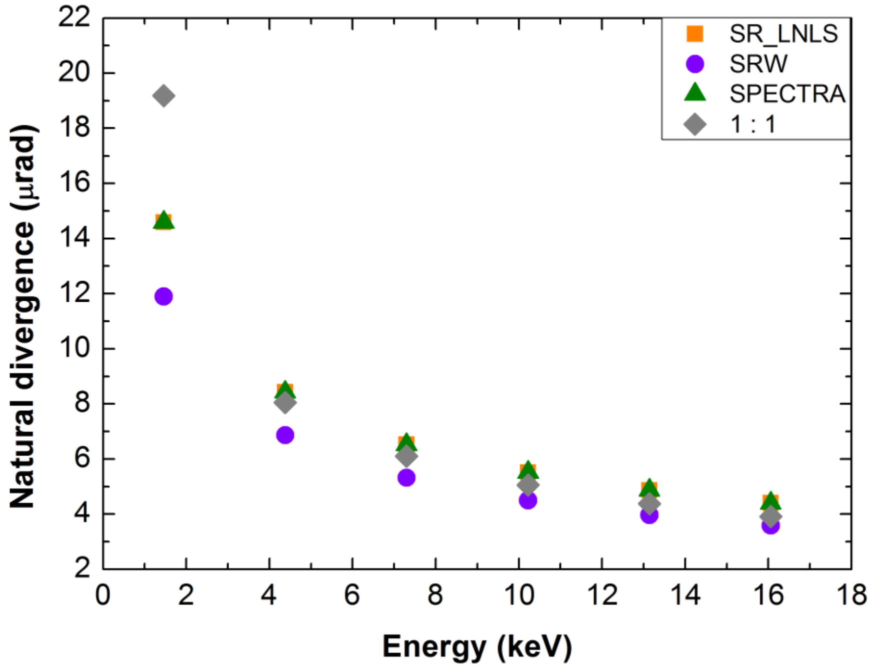
b) Virtual source observation



Setup for simulating a) the beam dimensions after a drift space of 30 m and b) a imaging system of 1:1 magnification, where the source properties are preserved. Wave propagation was done with SRW, that does not take into account energy spread.

Benchmarking: 1:1 imaging system





Comparisons between the undulator natural source size (left image) and the undulator natural source divergence (right image).

Conclusion

Since it only uses equations found at the most relevant literature (see References), the SR_LNLS code has shown satisfactory results and can be used to calculate and generate optical parameters from planar undulators for ray-tracing simulation of hard and tender X-ray beamlines.

Although minor bugs still exist, it is possible to exclude implementation and codification errors, since the code has been through extensive revision.

SR_LNLS was written in MATLAB and could be implemented in Python to provide input parameters to be used by the ray-tracing program Shadow (del Rio et al., 2011).

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References

The complete work can be found at:

Celestre, R. (2014) *SR_LNLS: A computer code for calculating optical parameters from insertion devices for ray-tracing simulation of hard and tender X-ray beamlines.* Bs.Eng Thesis. University of Campinas, Brazil.