Efficient and Verified Non-Terminating Programs with Isabelle-LLVM

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23/11/2023

Introduction

Context

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
      - Time-Aware Computations

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
    - Time-Aware Computations
- Formal Methods
  - Verification using proof assistants
    - Isabelle proofs
    - Verified + executable + efficient code
- Formalization of Time-Aware Stream Processing

Stream Processing

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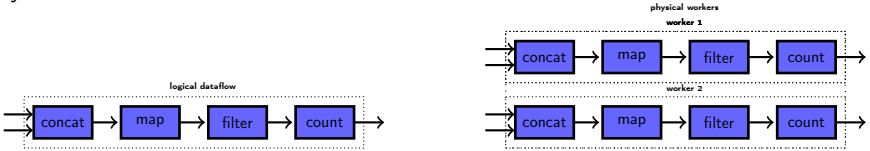
- Dataflow Model:
  - Directed graph of interconnected operators that perform event-wise transformations
  - E.g.: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow

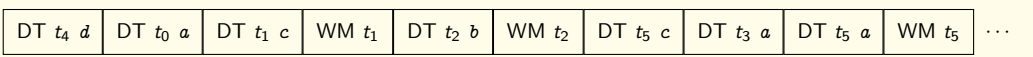
Cloud DataFlow Flink

  - Highly Parallel



Time-Aware Stream Processing

- Time-Aware Computations:
  - Timestamps: Metadata associating the data with some data collection
    - An unix timestamp
    - Version of the data
    - Logical grouping
  - Watermarks: Metadata indicating the completion of a data collection
    - e.g.: A watermark 5 says that there is no data associated with timestamp 5 or bellow arriving
    - Are increasingly monotonic (they don't go backwards in time)
  - e.g.:



Preliminaries

Isabelle/HOL

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Isabelle/HOL

- Classical higher-order logic (HOL): Simple Typed Lambda Calculus + (Hilbert) axiom of choice + axiom of infinity + rank-1 polymorphism
- Isabelle: A generic proof assistant

- Isabelle/HOL: Isabelle's flavor of HOL

Isabelle/HOL: (Co)datatypes

- Datatypes and Codatatypes

```
codatatype (lset: 'a) llist = lnull: LNil | LCons (lhd: 'a) (ltl: 'a llist)
  for map: lmap where ltl LNil = LNil
```

- Examples:
  - LNil
  - LCons *1* (LCons *2* (LCons *3* LNil))
  - LCons *0* (LCons *0* (LCons *0* (...)))
- Proofs by induction
- Proofs by coinduction

State of this work

What have I formalized so far? (part 1)

- Formalization stream processing (model)
  - Using Isabelle/HOL: (co)datatypes, (co)recursion, and (co)induction
  - Streams are lazy lists, and operators as a codatatype
  - Semantics: a `produce` function that runs an operator throughout a lazy lists
    - Mix of recursion and corecursion: inductive and coinductive principles
  - Sequential composition
    - Correctness!

What have I formalized so far? (part 2)

- Time-Aware computations
  - Coinductive properties of streams: monotonicity and productivity
  - Building blocks operators:
    - Convenience operators: batching and incremental computations
      - Incremental computing: only update results that are affected by the new input
      - With verified properties: Soundness, Completeness, preservation of productivity, and preservation of monotonicity
  - Compositional reasoning
- Case studies with the building blocks:
  - Incremental histogram operator
  - Relational join

Next Steps

Efficient Stream Processing

- It is executable! But slow!
  - Code generator: functional languages (OCaml, Haskell, SML...)
  - Functional data-structures: (many algorithms are slow with linked lists)

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  - Functional data-structures: (many algorithms are slow with linked lists)
- How do we make efficient and verified programs in Isabelle/HOL?
- Isabelle-LLVM!
- This is a non-terminating program

Isabelle-LLVM

Isabelle Refinement Framework and Isabelle-LLVM

- Isabelle Refinement Framework
  - Framework for step-wise refinement verification (refinement calculus): Specification $\rightarrow$ Abstract Algorithm $\rightarrow$ Less Abstract Algorithm $\rightarrow$ Executable Code
  - Imperative HOL as backend (lowest layer in the refinement)
    - Shallow Embedding of Monadic programs in HOL
  - Separation Logic (heap memory reasoning)
- Isabelle-LLVM is a new backend for the Isabelle Refinement Framework
  - Generates LLVM code (efficient imperative code)

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- Can we write and verify non-terminating programs in this framework?
  - Yes and No!

Isabelle-LLVM's Recursion Model

- Isabelle-LLVM's monad:

$$\textbf{datatype}\ 'a\ neM = SPEC\ (the\_spec{:}\ 'a \Rightarrow bool) \mid FAIL$$

What the Heck is a CCPO?

- Complete Chain Partial Order

The First Steps

Our CCPO Attempt

-
- Exit argument
- Connect to the Isabelle-LLVM refinement framework

Our Monad Attempt

- Feedback loop
- Exit argument
- Connect to the Isabelle-LLVM refinement framework

Questions, comments and suggestions