Nondeterministic Asynchronous Dataflow in Isabelle/HOL

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Motivation

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Context:

- Stream Processing: programs that compute (possibly) unbounded sequences of data (streams)
- A common problem in the industry
- Frameworks: Google Cloud Dataflow, Apache Flink, and Timely Dataflow

- Why use frameworks?
  - Highly Parallel
  - Low latency (output as soon as possible)
  - Incremental computing (re-uses previous computations)

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Our long term goal:

Mechanically Verify Timely Dataflow algorithms

A Good Foundation

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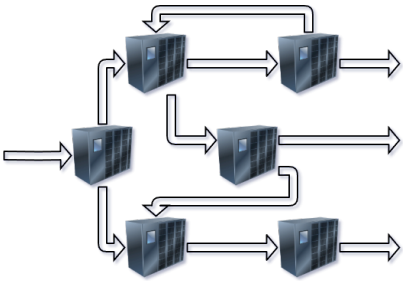

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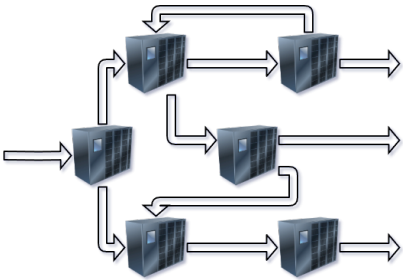

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  - Dataflow: Directed graph of interconnected operators

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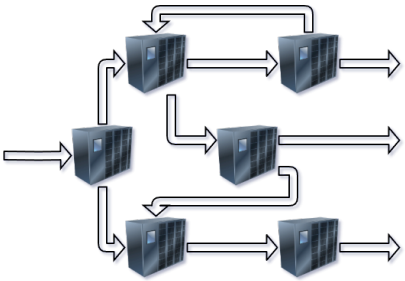

- Nondeterministic Asynchronous Dataflow
  - Dataflow: Directed graph of interconnected operators
  - Asynchronous:
    - Operators execute independently: processes without an orchestrator
    - Operators can freely communicate with the network (read/write); do silent computation steps
    - Networks are unbounded FIFO queues

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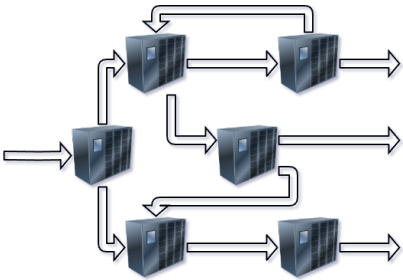

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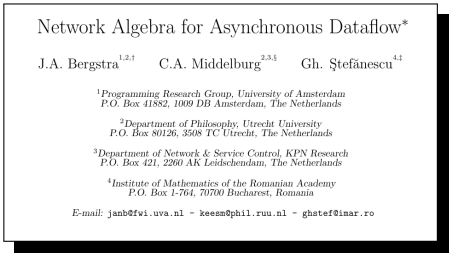
    - Networks are unbounded FIFO queues
  - Nondeterministic:
    - Operators can make nondeterministic choices

The Algebra for Nondeterministic Asynchronous Dataflow

- Bergstra et al. presents an algebra for Nondeterministic Asynchronous Dataflow
- Primitives:
  sequential and parallel composition; feedback loop...
- 52 axioms
- A process calculus instance



Network Algebra for Asynchronous Dataflow*

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Main Contributions

- An Isabelle/HOL instance of Nondeterministic Asynchronous Dataflow
  - Operators as a shallow embedding as codatatypes
  - 51 axioms proved
- Executable via code extraction to Haskell

Isabelle/HOL Preliminaries

Isabelle/HOL

- Classical higher-order logic (HOL):
  Simple Typed Lambda Calculus + axiom of choice + axiom of infinity + rank-1 polymorphism

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- Isabelle: A generic proof assistant

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- Isabelle/HOL: Isabelle's flavor of HOL

Why Isabelle/HOL?

- Codatatypes: (possibly) infinite data structures (e.g., lazy lists, streams)
- Corecursion: always eventually produces some codatatype constructor
- Coinductive predicate: infinite number of introduction rule applications
- Coinduction: reason about coinductive predicates

Operators as a Codatatype

Operators

Operators in Isabelle/HOL

codatatype (inputs: $'i$, outputs: $'o$, $'d$) op =
Read $'i$ ($'d \Rightarrow$ ($'i$, $'o$, $'d$) op) | Write (($'i$, $'o$, $'d$) op) $'o$ $'d$
Silent ($'i$, $'o$, $'d$) op | Choice (($'i$, $'o$, $'d$) op) $cset$

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- Type parameters:
  inputs/output ports; data
- Operator's actions
- inputs/outputs:
  Sets of used ports
- Possibly infinite trees

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Uncommunicative operators

$\oslash$ = Choice $\{\}_c$

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Uncommunicative operators

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$\otimes$ = Choice $\{\otimes\}_c$

More examples

ex1 = Choice $\{$Write ex1 1 42, $\oslash\}_c$

ex2 = Choice $\{$Write ex2 1 42, ex2$\}_c$

ex3 = Choice $\{$Write ex3 1 42, Silent ex3$\}_c$

Operators Equivalences: Motivation

An ideal equivalence relation

- Only equate operators that **behave** the same
- Useful reasoning principle

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An ideal equivalence relation

- Only equate operators that **behave** the same
- Useful reasoning principle

- Milner's approach!
  The classic chapter on **weak bisimilarity**

- Based on labeled transition systems (LTS)
- Labels (actions): Read, Write, Silent (τ)

Operators Equivalences: Label Transition System

Label Transition System

datatype ('i,'o,'d) IO = Inp 'i 'd | Out 'o 'd | Tau

inductive step **where**

step (Inp p x) (Read p f) (f x)

| step (Out q x) (Write op q x) op

| step Tau (Silent op) op

| $op \in_c ops \implies$ step io op $op' \implies$ step io (Choice ops) op'

Operators Equivalences: Weak Bisimilarity

Weakly Simulates

$$\text{estep Tau} = (\text{step Tau})^{==}$$

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$$\text{wstep } io = (\text{step Tau})^{**}\ \text{OO}\ (\text{estep } io)\ \text{OO}\ (\text{step Tau})^{**}$$

$$\text{wsim } R\ op_1\ op_2 = (\forall io\ op_1'.\ \text{step } io\ op_1\ op_1' \longrightarrow (\exists op_2'.\ \text{wstep } io\ op_2\ op_2' \wedge R\ op_1'\ op_2'))$$

- A Relation R is a *weak simulation* if $\forall op_1\ op_2.\ R\ op_1\ op_2 \longrightarrow \text{wsim } R\ op_1\ op_2$.

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Weak Bisimilarity

coinductive wbisim (**infix** $\approx$ 40) **where**

$$\text{wsim } (\approx)\ op_1\ op_2 \implies \text{wsim } (\approx)\ op_2\ op_1 \implies op_1 \approx op_2$$

- R is a *weak bisimulation* when both R and its converse R^{-1} are weak simulations. Weak bisimilarity is the largest weak bisimulation.

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- R is a *weak bisimulation* when both R and its converse R^{-1} are weak simulations. Weak bisimilarity is the largest weak bisimulation.
- $\approx$ has a useful coinduction principle
- $\oslash \approx \otimes \approx \odot$ and ex1 $\approx$ ex2 $\approx$ ex3

Asynchronous Dataflow Operators

Auxiliary Definitions

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Buffer functions

$\mathsf{BHD}\ p\ buf = \mathsf{hd}\ (buf\ p)$

$\mathsf{BTL}\ p\ buf = buf(p := \mathsf{tl}\ (buf\ p))$

$\mathsf{BENQ}\ p\ x\ buf = buf(p := buf\ p\ @\ [x])$

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choices function

$\mathsf{choices} \otimes = \{\}$

$\mathsf{Silent}\ op' \in_c \mathsf{choices}\ op \longleftrightarrow \mathsf{step}\ \mathsf{Tau}\ op\ op'$

$\mathsf{choices}\ (\mathsf{Choice}\ ops) = \bigcup_c\ (\mathsf{choices}\ `_c\ ops)$

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Silent $op' \in_c$ choices $op \longleftrightarrow$ step Tau $op\ op'$

choices (Choice ops) $= \bigcup_c$ (choices `$_c$ ops)

Mapping ports functions

map_op :: $('i_1 \Rightarrow 'i_2) \Rightarrow ('o_1 \Rightarrow 'o_2) \Rightarrow ('i_1, 'o_1, 'd)\ op \Rightarrow ('i_2, 'o_2, 'd)\ op$

projl :: $('a + 'b) \Rightarrow 'a$

projr :: $('a + 'b) \Rightarrow 'b$

$\curvearrowright$ :: $('a + 'b) + 'c \Rightarrow 'a + 'b + 'c$

$\curvearrowleft$ :: $'a + 'b + 'c \Rightarrow ('a + 'b) + 'c$

Identity

Equation of the identity operator

id_op buf = Choice

$\bigcup_c$

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$$\begin{aligned}&\text{id\_op } buf = \text{Choice}\\&(((\lambda p.\ \text{Read } p\ (\lambda x.\ \text{id\_op } (\text{BENQ } p\ x\ buf))) \grave{}_c\ \mathfrak{U}_c)\\&\cup_c\end{aligned}$$

- $\mathfrak{U}_c$ is the set of usable ports provide by its type

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- Stream delayer: always can read, and it may eventually output

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- $\mathfrak{U}_c$ is the set of usable ports provide by its type
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Identity operator with an empty buffer

$$\mathcal{I} = \text{id\_op } (\lambda\_.\ [])$$

Composition: Preliminaries

Composition operator type

comp_op :: $('o_1 \Rightarrow 'i_2\ option) \Rightarrow ('i_2 \Rightarrow 'd\ list) \Rightarrow ('i_1, 'o_1, 'd)\ op \Rightarrow ('i_2, 'o_2, 'd)\ op \Rightarrow$
$('i_1 + 'i_2, 'o_1 + 'o_2, 'd)\ op$

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Sequential and parallel composition

$op_1 \parallel op_2 =$ comp_op $(\lambda\_.\ \text{None})\ (\lambda\_.\ [])\ op_1\ op_2$

$op_1 \bullet op_2 =$ map_op projl projr (comp_op Some $(\lambda\_.\ [])\ op_1\ op_2$)

Composition: Equation

Equation of the composition operator

comp_op *wire buf* op_1 op_2 =

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Read *p f* ⇒ Read (Inl *p*) (λ*x*. comp_op *wire buf* (*f x*) op_2)

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&\ \mid \text{Write } \mathit{op}\ p\ x \Rightarrow (\underline{\text{case}}\ \mathit{wire}\ p\ \underline{\text{of}}\\
&\qquad \text{None} \Rightarrow \text{Write } (\text{comp\_op } \mathit{wire}\ \mathit{buf}\ \mathit{op}\ \mathit{op}_2)\ (\text{Inl } p)\ x\\
&\qquad \mid \text{Some } q \Rightarrow \text{Silent } (\text{comp\_op } \mathit{wire}\ (\text{BENQ } q\ x\ \mathit{buf})\ \mathit{op}\ \mathit{op}_2))\\
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&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \grave{}_c\ \text{sound\_reads } \mathit{wire}\ \mathit{buf}\ (\text{choices } \mathit{op}_2)))
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&\quad\ |\ \text{Some } q \Rightarrow \text{Silent } (\text{comp\_op } wire\ (\text{BENQ } q\ x\ buf)\ op\ op_2))\\
&\ |\ \text{Silent } op \Rightarrow \text{Silent } (\text{comp\_op } wire\ buf\ op\ op_2))\ `_c\ \text{choices } op_1)\ \cup_c\\
&((\lambda op.\ \underline{\text{case}}\ op\ \underline{\text{of}}\\
&\quad \text{Read } p\ f \Rightarrow \underline{\text{if}}\ p \in \text{ran } wire\\
&\qquad \underline{\text{then}}\ \text{Silent } (\text{comp\_op } wire\ (\text{BTL } p\ buf)\ op_1\ (f\ (\text{BHD } p\ buf)))\\
&\qquad \underline{\text{else}}\ \text{Read } (\text{Inr } p)\ (\lambda x.\ \text{comp\_op } wire\ buf\ op_1\ (f\ x))\\
&\ |\ \text{Write } op\ p\ x \Rightarrow \text{Write } (\text{comp\_op } wire\ buf\ op_1\ op)\ (\text{Inr } p)\ x\\
&\ |\ \text{Silent } op \Rightarrow \text{Silent } (\text{comp\_op } wire\ buf\ op_1\ op))\ `_c\ \text{sound\_reads } wire\ buf\ (\text{choices } op_2)))
\end{aligned}
$$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
- Sequential composition: $op_1 \bullet op_2$

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$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

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Transpose operator type

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Dummy source operator type

i $:: ('i, 'o, 'd)\ op$

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$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Asynchronous Dataflow Operators

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Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

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- Sequential composition: $op_1 \bullet op_2$

Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
- Sequential composition: $op_1 \bullet op_2$

Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Copy operator type

$\mathcal{C} :: ('n, 'n + 'n, 'd)\ op$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
- Sequential composition: $op_1 \bullet op_2$

Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Copy operator type

$\mathcal{C} :: ('n, 'n + 'n, 'd)\ op$

Equality test operator type

$\mathcal{Q} :: ('n + 'n, 'n, 'd\ option)\ op$

Asynchronous Dataflow Properties

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op } \curvearrowright \curvearrowright \ (op_1 \parallel op_2) \parallel op_3$
B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl } op$
B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr } op$
B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$
B4_1: $op \bullet \mathcal{I} \approx op$
B4_2: $\mathcal{I} \bullet op \approx op$
B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$
B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$
B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$
B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl) } \mathcal{I}$
B9: $\mathcal{X} \approx \text{map\_op } \curvearrowright \curvearrowright \ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id } \curvearrowleft \ (\mathcal{I} \parallel \mathcal{X})$
B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$
F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$
F2: $\mathcal{X}\uparrow \approx \mathcal{I}$
R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$
R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$
R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op } \curvearrowleft \curvearrowleft \ (op_1 \parallel op_2))\uparrow$
R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$
R5: $\text{map\_op Inl Inl } ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$
R6: $(op\uparrow)\uparrow \approx (\text{map\_op } \curvearrowright \curvearrowright \ op)\uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op}\ \curvearrowright\curvearrowright\ (op_1 \parallel op_2) \parallel op_3$

B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl}\ op$

B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr}\ op$

B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$

B4_1: $op \bullet \mathcal{I} \approx op$

B4_2: $\mathcal{I} \bullet op \approx op$

B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$

B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$

B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$

B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl)}\ \mathcal{I}$

B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

F2: $\mathcal{X}\uparrow \approx \mathcal{I}$

R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$

R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$

R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

R6: $(op\uparrow)\uparrow \approx (\text{map\_op}\ \curvearrowright\curvearrowright\ op)\uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (op_1 \parallel op_2) \parallel op_3$
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B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$
B4_1: $op \bullet \mathcal{I} \approx op$
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B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$
B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl)}\ \mathcal{I}$
B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$
B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$
F1: $\mathcal{I} \uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$
F2: $\mathcal{X} \uparrow \approx \mathcal{I}$
R1: $op_2 \bullet (op_1 \uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1) \uparrow$
R2: $(op_1 \uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I})) \uparrow$
R3: $op_1 \parallel (op_2 \uparrow) \approx (\text{map\_op}\ \curvearrowleft\ \curvearrowleft\ (op_1 \parallel op_2)) \uparrow$
R4: $(op_1 \bullet (\mathcal{I} \parallel op_2)) \uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1) \uparrow$
R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op) \uparrow) \approx op$
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B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

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R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$

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R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\ \curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

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R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

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B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

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R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\ \curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

R6: $(op\uparrow)\uparrow \approx (\text{map\_op}\ \curvearrowright\ \curvearrowright\ op)\uparrow$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op} \curvearrowleft \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet \bigtriangleright \approx \bigtriangleright$ for $\bigtriangleright \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $\bigtriangleleft \bullet \mathcal{X} \approx \bigtriangleleft$ for $\bigtriangleleft \in \{\mathcal{C}, \Lambda\}$

A3$_\mathcal{Q}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

A3$_\mathcal{V}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id}\ \mathcal{I}$

A4: $\bigtriangleright \bullet ! \approx ! \parallel !$ for $\bigtriangleright \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet \bigtriangleleft \approx \text{i} \parallel \text{i}$ for $\bigtriangleleft \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id}\ \curvearrowleft\ (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

A7: $\mathcal{C} \bullet (! \parallel \mathcal{I}) \approx \text{map\_op id Inr}\ \mathcal{I}$

A9: $\text{i} \bullet ! \approx \mathcal{I}$

A10: $\mathcal{Q} \bullet \mathcal{C} \approx (\mathcal{C} \parallel \mathcal{C}) \bullet (\text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})) \bullet (\mathcal{Q} \parallel \mathcal{Q})$

A11: $\mathcal{C} \bullet \mathcal{Q} \approx \mathcal{I}$

A12: $\text{i} \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A16: $! \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A13: $\text{i} \approx \text{i} \parallel \text{i}$

A17: $! \approx ! \parallel !$

A14: $\text{map\_op id Inl}\ (\bigtriangleright :: (0 + 0, 0, \text{'d})\ op) \approx \mathcal{I}$ for $\bigtriangleright \in \{\mathcal{Q}, \mathcal{V}\}$

A15: $\bigtriangleright \approx \text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I}) \bullet (\bigtriangleright \parallel \bigtriangleright)$ for $\bigtriangleright \in \{\mathcal{Q}, \mathcal{V}\}$

A18: $\text{map\_op Inl id}\ (\bigtriangleleft :: (0, 0 + 0, \text{'d})\ op) \approx \mathcal{I}$ for $\bigtriangleleft \in \{\mathcal{C}, \Lambda\}$

A19: $\bigtriangleleft \approx (\bigtriangleleft \parallel \bigtriangleleft) \bullet \text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})$ for $\bigtriangleleft \in \{\mathcal{C}, \Lambda\}$

F3: $(\text{map\_op id Inr}\ \bigtriangleright)\uparrow \approx !$ for $\bigtriangleright \in \{\mathcal{Q}, \mathcal{V}\}$

F4: $(\text{map\_op Inr id}\ \bigtriangleleft)\uparrow \approx \text{i}$ for $\bigtriangleleft \in \{\mathcal{C}, \Lambda\}$

F5: $((\mathcal{I} \parallel \mathcal{C}) \bullet \text{map\_op}\ \curvearrowright\curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet (\mathcal{I} \parallel \mathcal{Q}))\uparrow \approx ! \bullet \text{i}$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op} \curvearrowleft \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet (\!\!\gg) \approx (\!\!\gg)$ for $(\!\!\gg) \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $(\!\!\ll) \bullet \mathcal{X} \approx (\!\!\ll)$ for $(\!\!\ll) \in \{\mathcal{C}, \Lambda\}$

$A3_{\mathcal{Q}}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

$A3_{\mathcal{V}}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id}\ \mathcal{I}$

A4: $(\!\!\gg) \bullet ! \approx ! \parallel !$ for $(\!\!\gg) \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet (\!\!\ll) \approx \text{i} \parallel \text{i}$ for $(\!\!\ll) \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id}\ \curvearrowleft\ (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

A7: $\mathcal{C} \bullet (! \parallel \mathcal{I}) \approx \text{map\_op id Inr}\ \mathcal{I}$

A9: $\text{i} \bullet ! \approx \mathcal{I}$

A10: $\mathcal{Q} \bullet \mathcal{C} \approx (\mathcal{C} \parallel \mathcal{C}) \bullet (\text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})) \bullet (\mathcal{Q} \parallel \mathcal{Q})$

A11: $\mathcal{C} \bullet \mathcal{Q} \approx \mathcal{I}$

A12: $\text{i} \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A16: $! \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A13: $\text{i} \approx \text{i} \parallel \text{i}$

A17: $! \approx ! \parallel !$

A14: $\text{map\_op id Inl}\ ((\!\!\gg) :: (0 + 0, 0, \text{'d})\ op) \approx \mathcal{I}$ for $(\!\!\gg) \in \{\mathcal{Q}, \mathcal{V}\}$

A15: $(\!\!\gg) \approx \text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I}) \bullet ((\!\!\gg) \parallel (\!\!\gg))$ for $(\!\!\gg) \in \{\mathcal{Q}, \mathcal{V}\}$

A18: $\text{map\_op Inl id}\ ((\!\!\ll) :: (0, 0 + 0, \text{'d})\ op) \approx \mathcal{I}$ for $(\!\!\ll) \in \{\mathcal{C}, \Lambda\}$

A19: $(\!\!\ll) \approx ((\!\!\ll) \parallel (\!\!\ll)) \bullet \text{map\_op}\ \curvearrowright\curvearrowright\ (\text{map\_op}\ \curvearrowleft\curvearrowleft\ (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})$ for $(\!\!\ll) \in \{\mathcal{C}, \Lambda\}$

F3: $(\text{map\_op id Inr}\ (\!\!\gg))\uparrow \approx !$ for $(\!\!\gg) \in \{\mathcal{Q}, \mathcal{V}\}$

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F5: $((\mathcal{I} \parallel \mathcal{C}) \bullet \text{map\_op}\ \curvearrowright\curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet (\mathcal{I} \parallel \mathcal{Q}))\uparrow \approx ! \bullet \text{i}$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op}\ \curvearrowleft\ \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet \circledS \approx \circledS \quad \text{for } \circledS \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $\circledM \bullet \mathcal{X} \approx \circledM \quad \text{for } \circledM \in \{\mathcal{C}, \Lambda\}$

A3$_\mathcal{Q}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

A3$_\mathcal{V}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id } \mathcal{I}$

A4: $\circledS \bullet ! \approx ! \parallel ! \quad \text{for } \circledS \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet \circledM \approx \text{i} \parallel \text{i} \quad \text{for } \circledM \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id}\ \curvearrowleft\ (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

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A6: $(\!\!<\!\!) \bullet \mathcal{X} \approx (\!\!<\!\!)$ for $(\!\!<\!\!) \in \{\mathcal{C}, \Lambda\}$

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A4: $(\gtrdot) \bullet ! \approx ! \parallel !$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

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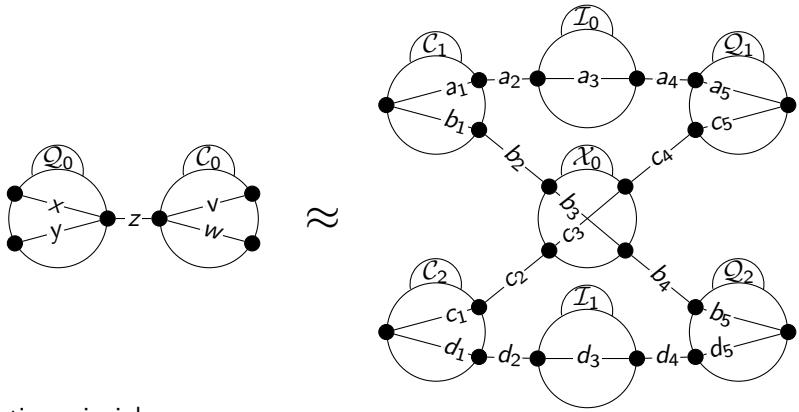
A10



- $\approx$ coinduction principle
- Buffers generalization
- Buffers invariant

Related Work

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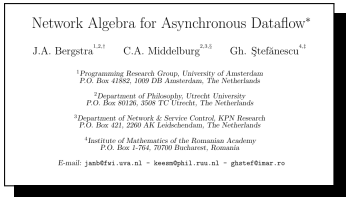
- Instance in Algebra of Communicating Processes (ACP)
- Our operators closely follow their ACP counterparts



Network Algebra for Asynchronous Dataflow*

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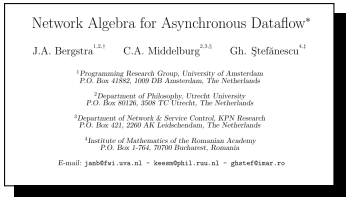

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- Our operators can be seen as a domain specific variant of choice trees
- Monadic composition

Interaction Trees

Representing Recursive and Impure Programs in Coq

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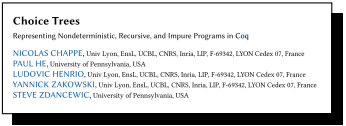

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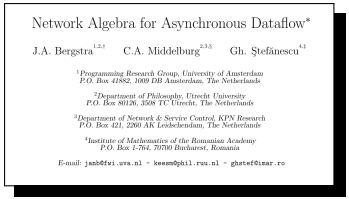

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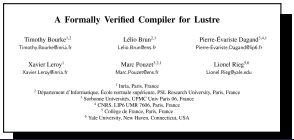
- **Synchronous** dataflow languages have some proof assistant mechanization



A Formally Verified Compiler for Lustre

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[3] Sorbonne Universités, UPMC Univ Paris 06, France
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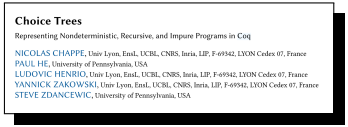

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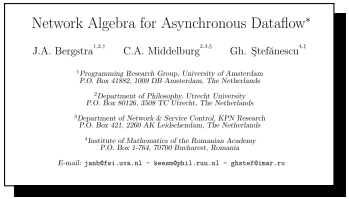
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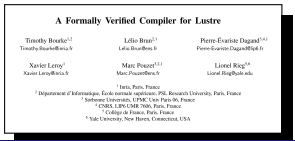
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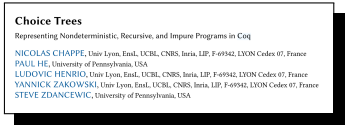
Choice Trees

Representing Nondeterministic, Recursive, and Impure Programs in Coq

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- Flo: a framework for representing streaming computations originating from different systems like Flink and DBSP


Flo: a Semantic Foundation for Progressive Stream Processing

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Conclusion

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  - Axiom A1 for the merge operator does not hold in our instance when equivalence is $\approx$. We conjecture it also does not hold in the process algebra from Bergstra et al.

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- Future work:
  - Brock–Andersen Time anomaly
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  - Verify Timely Dataflow algorithms

Questions, comments and suggestions