Efficient and Verified Non-Terminating Programs with Isabelle-LLVM

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Introduction

Context

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
      - Time-Aware Computations

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
      - Time-Aware Computations
- Formal Methods
  - Verification using proof assistants
    - Isabelle proofs
      - Verified + executable + efficient code
- Formalization of Time-Aware Stream Processing

Stream Processing

- Stream Processing: Abstraction for processing data when the input is not completely presented in the begging of the computation

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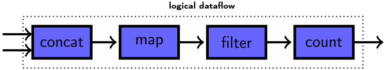
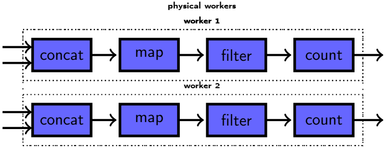
- Stream Processing: Abstraction for processing data when the input is not completely presented in the begging of the computation
- Dataflow Model:
  - Directed graph of interconnected operators that perform event-wise transformations
  - E.g.: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow
  - Highly Parallel





Time-Aware Stream Processing (part 1)

- Time-Aware Computations:
  - Timestamps: Metadata associating the data with some data collection
    - An unix timestamp
    - Version of the data
    - Logical grouping
  - Watermarks: Metadata indicating the completion of a data collection
    - e.g.: A watermark 5 says that there is no data associated with timestamp 5 or bellow arriving
    - Are increasingly monotonic (they don't go backwards in time)

Exameple of Time-Aware Stream Processing

DT 2 b	DT 1 a	WM 1	DT 2 c	DT 3 a	WM 4

buf = []
H = {}

```
λ e. case e of
DT t d => insert (t, d) buf
WM wm =>
 if ∃ t ∈ (map fst buf).  t  ≤ wm
 then
   out <- filter (λ (t, _) . t ≤ wm) buf
    ts <- remdups (map fst out)
    buf <- filter (λ (t, _) . t > wm) buf
    H <- H + (mset (map snd out))
    push (map (λ t . DT t (H + (mset (map snd (filter (λ (t', _) . t' ≤ t) out))))) ts) @ [WM wm]
 else
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Example Time-Aware Stream Processing

DT 1 a	WM 1	DT 2 c	DT 3 a	WM 4

buf = [(2,b)]
H = {}

```
$\lambda case e of
DT t d => insert (t, d) buf
WM wm =>
 if $\exists$ t $\in$ (map fst buf)\math-container{}.  t  $\le$ wm
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Example Time-Aware Stream Processing

WM 1	DT 2 c	DT 3 a	WM 4

buf = [(2,b), (1,a)]
H = {}

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$\lambda case e of
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Example Time-Aware Stream Processing

DT 2 c	DT 3 a	WM 4

buf = [(2,b)]
H = {a}

```
$\lambda case e of
DT t d => insert (t, d) buf
WM wm =>
 if $\exists$ t $\in$ (map fst buf)\math-container{}.  t  $\le$ wm
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```

DT 1 {a}	WM 1

Example Time-Aware Stream Processing

WM 4

buf = [(2,b), (2,c), (3,a)]
H = {a}

```
$\lambda case e of
DT t d => insert (t, d) buf
WM wm =>
 if $\exists$ t $\in$ (map fst buf)\math-container{}.  t  $\le$ wm
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   out <- filter ($\lambda$ (t, _) . t $\le$ wm) buf
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DT 1 {a}	WM 1

Example Time-Aware Stream Processing

buf = []
H = {a,a,b,c}

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DT 1 {a}	WM 1	DT 2 {a,b,c}	DT 3 {a,a,b,c}	WM 4

Preliminaries

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- Isabelle: A generic proof assistant

- Isabelle/HOL: Isabelle's flavor of HOL

Isabelle/HOL: (Co)datatypes

- Datatypes and Codatatypes

  codatatype (lset: $'a$) *llist* = lnull: LNil | LCons (lhd: $'a$) (ltl: $'a$ *llist*)
  for map: lmap **where** ltl LNil = LNil

- Examples:
  - LNil
  - LCons *1* (LCons *2* (LCons *3* LNil))
  - LCons *0* (LCons *0* (LCons *0* (...)))
- Proofs by induction
- Proofs by coinduction

State of this work

What have I formalized so far? (part 1)

- Formalization stream processing (model)
    - Using Isabelle/HOL: (co)datatypes, (co)recursion, and (co)induction
    - Streams are lazy lists, and operators as a codatatype
    - Semantics: a $\textsf{produce} :: {'i}\ \mathit{llist} \Rightarrow ({'i}, {'o})\ \mathit{op} \Rightarrow {'o}\ \mathit{llist}$ function that runs an operator throughout a lazy lists
        - Mix of recursion and corecursion: inductive and coinductive principles
    - Sequential composition
        - Correctness!

What have I formalized so far? (part 2)

- Time-Aware computations
  - Coinductive properties of streams: monotonicity and productivity
  - Building blocks operators:
    - Convenience operators: batching and incremental computations
      - Incremental computing: only update results that are affected by the new input
      - With verified properties: Soundness, Completeness, preservation of monotonicity, and preservation of productivity
  - Compositional reasoning
- Case studies with the building blocks:
  - Incremental histogram operator
  - Relational join

Next Steps

Efficient Stream Processing

- It is executable! But slow!
  - Code generator: functional languages (OCaml, Haskell, SML...)
  - Functional data-structures (often not ideal)

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Efficient Stream Processing

- It is executable! But slow!
  - Code generator: functional languages (OCaml, Haskell, SML...)
  - Functional data-structures (often not ideal)
- How do we make efficient and verified programs in Isabelle/HOL?
- Isabelle-LLVM!
- Let's port this formalization to Isabelle-LLVM then!
- This is a non-terminating program

Isabelle-LLVM

Isabelle Refinement Framework and Isabelle-LLVM

- Isabelle Refinement Framework
  - Framework for step-wise refinement verification (refinement calculus): Specification $\rightarrow$ Abstract Algorithm $\rightarrow$ Less Abstract Algorithm $\rightarrow$ Executable Code
  - Imperative HOL as backend (lowest layer in the refinement)
    - Shallow Embedding of Monadic programs in HOL
  - Separation Logic (heap memory reasoning)
- Isabelle-LLVM is a new backend for the Isabelle Refinement Framework
  - Generates LLVM code (efficient imperative code)

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- Can we write and verify non-terminating programs in this framework?
  - Yes and No!

Isabelle-LLVM's Recursion Model

- Knaster–Tarski theorem
  - Standard way to define the semantics of recursive definitions
    - Isabelle/HOL: Partial Function Package
  - Every monotonic function on Complete Chain Partial Order (CCPO) has a fixed point
  - Induction principle
- No need for well-foundness

What the Heck is a CCPO?

- Chain: A set in which all elements are comparable
- Complete Chain Partial Order:
  1. A partial order: $'a::order$
  2. A function that returns the suprimium (least upper bound) from a chain $'a::order\ \mathrm{set} \Rightarrow 'a$

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- Isabelle-LLVM's monad:

$$\textbf{datatype}\ 'a\ \mathit{neM} = \mathit{SPEC}\ (\mathit{the\_spec}{:}\ 'a \Rightarrow \mathit{bool}) \mid \mathit{FAIL}$$

  - Order: flat (every SPEC is greater than FAIL, SPECs are only comparable when they are equal)
  - Suprimium from a chain: The SPEC, or the only FAIL
  - Bottom: FAIL
    - Non-termination

The First Steps

Our CCPO Attempt

- Let's look in Isabelle!

The Other Steps

More Changes to Isabelle-LLVM?

- Separation Logic?
    - Express properties about the trace of the program
- Refinement Calculus?
- LLVM code generator?

Questions, comments and suggestions