

# Verified Time-Aware Stream Processing

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02/11/2023

# What is this PhD/Status seminar about?

- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
    - Time-Aware Computations
- Formal Methods
  - Verification using proof assistants
    - Isabelle proofs
    - Verified + executable + efficient code
- Formalization of Time-Aware Stream Processing

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- Introduction
- Preliminaries
- Lazy Lists Processors
- Time-Aware Operators
- Case Study
- Next Steps

# Introduction

# Stream Processing

- Stream Processing: Abstraction for processing data when the input is not completely presented in the beginning of the computation
- Dataflow Model:
  - Directed graph of interconnected operators that perform event-wise transformations
  - Examples: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow



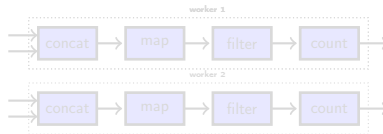
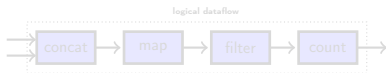
Cloud  
DataFlow



Flink



- Highly Parallel



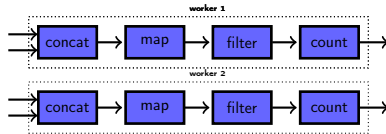
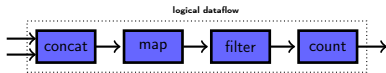
- Time-Aware Computations
  - Explain watermarks
- Bugs in Stream Processing

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- Time-Aware Computations
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# Preliminaries

- Classical higher-order logic (HOL): Simple Typed Lambda Calculus + (Hilbert) axiom of choice + axiom of infinity + rank-1 polymorphism
- Isabelle: A generic proof assistant



- Isabelle/HOL: Isabelle's flavor of HOL
- All functions in Isabelle/HOL must be total

# Isabelle/HOL: (Co)datatypes

- Datatypes and Codatatypes

```
codatatype (lset: 'a) llist = lnull: LNil | LCons (lhd: 'a) (ltl: 'a llist)  
for map: lmap where ltl LNil = LNil
```

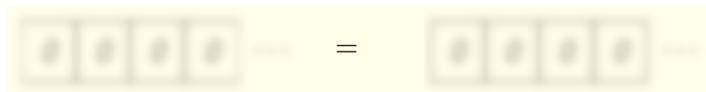
- Examples:

- LNil
- LCons 1 (LCons 2 (LCons 3 LNil))
- LCons 0 (LCons 0 (LCons 0 (...)))

- Induction principle assuming membership in the lazy list

- Coinductive principle for lazy list equality:

- Show that there is a pair of goggles that makes them to look the same, which implies that:
  - The first lazy list is empty iff second is
  - They have the same head
  - Their tail looks the same



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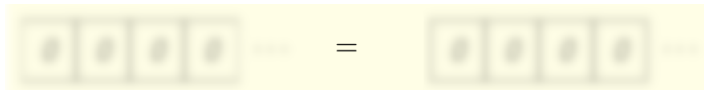
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$$\boxed{0} \dots = \boxed{0} \dots$$

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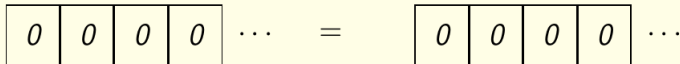
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- Recursion

```
fun lshift :: 'a list  $\Rightarrow$  'a llist  $\Rightarrow$  'a llist (infixr @@ 65) where  
  lshift [] lxs = lxs  
| lshift (x # xs) lxs = LCons x (lshift xs lxs)
```

- While Combinator

```
definition while_option :: ('a  $\Rightarrow$  bool)  $\Rightarrow$  ('a  $\Rightarrow$  'a)  $\Rightarrow$  'a  $\Rightarrow$  'a option where  
  while_option b c s = ...
```

- While rule for invariant reasoning (hoare-style):
  - There is something that holds before a step; that thing still holds after the step

# Isabelle/HOL: Corecursion and Friends

- Corecursion is like recursion, but instead of always eventually reducing an argument it always eventually produces something

- Corec:

```
corec lapp :: 'a llist  $\Rightarrow$  'a llist  $\Rightarrow$  'a llist where  
  lapp lxs lys = case lxs of LNil  $\Rightarrow$  lys | LCons x lxs'  $\Rightarrow$  LCons x (lapp lxs' lys)
```

- Friendly function

- Preserves productivity: it may consume at most one constructor to produce one constructor.

```
friend_of_corec lshift where  
  xs @@ lxs = (case xs of  
    []  $\Rightarrow$  (case lxs of LNil  $\Rightarrow$  LNil | LCons x lxs'  $\Rightarrow$  LCons x lxs')  
  | x#xs'  $\Rightarrow$  LCons x (xs' @@ lxs))  
  by (auto split: list.splits llist.splits) (transfer_prover)
```

```
lconcat lxs = case lxs of LNil  $\Rightarrow$  LNil | LCons xs lxs'  $\Rightarrow$  lshift xs (lconcat lxs')
```

- Coinduction up to congruence: Coinduction for Lazy list equality can be extended to compare an entire finite prefix through a congruence relation



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# Isabelle/HOL: (Co)inductive Predicates

- Inductive predicate
  - Finite number of introduction rule applications

```
inductive in_llist :: 'a  $\Rightarrow$  'a llist  $\Rightarrow$  bool where  
  In_llist: in_llist x (LCons x lxs)  
  | Next_llist: in_llist x lxs  $\Rightarrow$  in_llist x (LCons y lxs)  
  
in_llist 2 (LCons 1 (LCons (2 (...))))
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# Lazy Lists Processors

# Operator formalization

- Operator as a codatatype
  - Taking `'i` as the input type, and `'o` as the output type:  
`codatatype ('o, 'i) op = Logic (apply: ('i  $\Rightarrow$  ('o, 'i) op  $\times$  'o list))`
  - Infinite trees: applying the selector `apply` “walks” a branch of the tree



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  - Infinite trees: applying the selector `apply` “walks” a branch of the tree

- Produce function: applies the logic (co)recursively throughout a lazy list

**definition**  $\text{produce}_1' \text{ op } lxs = \text{while\_option}$

$(\lambda(\text{op}, lxs). \neg \text{Inull } lxs \wedge \text{snd } (\text{apply } \text{op } (\text{lhs } lxs)) = [])$

$(\lambda(\text{op}, lxs). (\text{fst } (\text{apply } \text{op } (\text{lhs } lxs)), \text{tl } lxs)) (\text{op}, lxs)$

**definition**  $\text{produce}_1 \text{ op } lxs =$

$(\text{case } \text{produce}_1' \text{ op } lxs \text{ of } \text{None} \Rightarrow \text{None}$

$| \text{Some } (\text{op}', lxs') \Rightarrow \text{if } \text{Inull } lxs' \text{ then } \text{None} \text{ else}$

$\text{let } (\text{op}'', \text{out}) = \text{apply } \text{op}' (\text{lhs } lxs') \text{ in } \text{Some } (\text{op}'', \text{hd } \text{out}, \text{tl } \text{out}, \text{tl } lxs'))$

**corec** **produce** **where**

$\text{produce } \text{op } lxs = (\text{case } \text{produce}_1 \text{ op } lxs \text{ of } \text{None} \Rightarrow \text{LNil}$

$| \text{Some } (\text{op}', x, xs, lxs') \Rightarrow \text{LCons } x \text{ (xs @@ produce } \text{op}' \text{ lxs')})$

- $\text{produce}_1$  has an induction principle based on the while invariant rule

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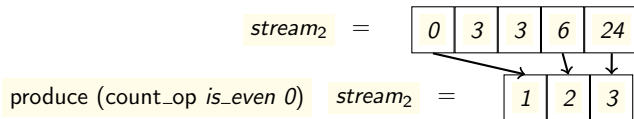
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# Operators: Count

- Example:

```
corec count_op where count_op P n =  
  Logic (λe. if P e then (count_op P (n + 1), [n+1]) else (count_op P n, []))
```



# Sequential Composition

- Sequential composition: take the output of the first operator and give it as input to the second operator.

```
definition fproduce  $op\ xs = \text{fold } (\lambda e\ (op,\ out)).$   
  let  $(op',\ out') = \text{apply } op\ e\ \text{in } (op',\ out\ @\ out')$  xs (op, [])  
corec comp_op where  
  comp_op  $op_1\ op_2 = \text{Logic } (\lambda ev.$   
    let  $(op_1',\ out) = \text{apply } op_1\ ev;$   $(op_2',\ out') = \text{fproduce } op_2\ out$   
    in  $(\text{comp\_op } op_1'\ op_2',\ out')$ 
```

# Sequential Composition: Correctness

- Correctness:

$\text{produce } (\text{comp\_op } op_1 \ op_2) \ xs = \text{produce } op_2 \ (\text{produce } op_1 \ xs)$

- Proof: coinduction principle for lazy list equality and  $\text{produce}_1$  induction principle
  - Generalization: we must be able to reason about elements in arbitrary positions

$\text{corec skip\_op}$  where

$\text{skip\_op } op \ n = \text{Logic } (\lambda ev. \text{let } (op', out) = \text{apply } op \ ev \text{ in}$   
if  $\text{length } out < n$  then  $(\text{skip\_op } op' \ (n - \text{length } out), [])$   
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# Time-Aware Operators

- Time-Aware lazy lists

```
datatype ('t::order, 'd) event = DT (tmp: 't) (data: 'd) | WM (wmk: 't)
```

- Generalization to partial orders
  - Cycles
  - Operators with multiple inputs

# Time-Aware Streams

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# Monotone Time-Aware Streams

- Monotone: watermarks do not go back in time

**coinductive** monotone  $:: ('t::\text{order}, 'd) \text{ event llist} \Rightarrow 't \text{ set} \Rightarrow \text{bool}$  where

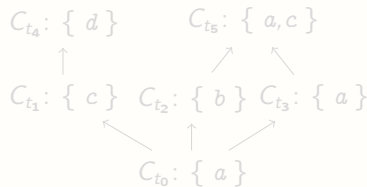
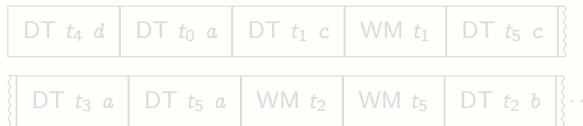
LNil: monotone LNil  $W$

| LConsR:  $(\forall wm' \in W. \neg wm' \geq wm) \longrightarrow \text{monotone } lxs (\{wm\} \cup W) \longrightarrow$   
monotone (LCons (WM  $wm$ )  $lxs$ )  $W$

| LConsL:  $(\forall wm \in W. \neg wm \geq t) \longrightarrow \text{monotone } lxs W \longrightarrow$   
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- Up to congruence coinduction principle
- Example:

$stream_3 =$



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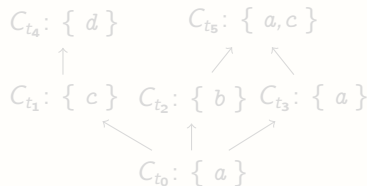
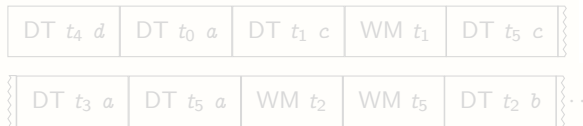
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# Monotone Time-Aware Streams

- Monotone: watermarks do not go back in time

**coinductive** monotone  $:: ('t::\text{order}, 'd) \text{ event llist} \Rightarrow 't \text{ set} \Rightarrow \text{bool}$  where

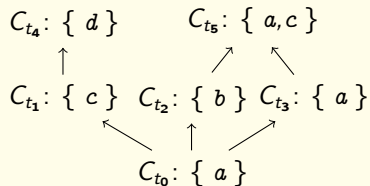
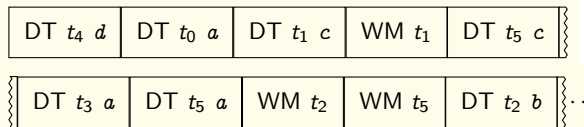
LNil: monotone LNil  $W$

| LConsR:  $(\forall wm' \in W. \neg wm' \geq wm) \longrightarrow \text{monotone } lxs (\{wm\} \cup W) \longrightarrow$   
 monotone (LCons (WM  $wm$ )  $lxs$ )  $W$

| LConsL:  $(\forall wm \in W. \neg wm \geq t) \longrightarrow \text{monotone } lxs W \longrightarrow$   
 monotone (LCons (DT  $t d$ )  $lxs$ )  $W$

- Up to congruence coinduction principle
- Example:

$stream_3 =$



# Productive Time-Aware Streams

- Productive: always eventually allows the production

- Batching operators: accumulate data until its completion
- Data is always eventually completed by some watermark

coinductive productive where

LFinite: lfinite  $lxs \longrightarrow$  productive  $lxs$

| EnvWM:  $\neg$  lfinite  $lxs \longrightarrow (\exists u \in \text{vimage WM } (\text{lset } lxs). u \geq t) \longrightarrow$   
productive  $lxs \longrightarrow$  productive (LCons (DT  $t$   $d$ )  $lxs$ )

| SkipWM:  $\neg$  lfinite  $lxs \longrightarrow$  productive  $lxs \longrightarrow$   
productive (LCons (WM  $t$ )  $lxs$ )

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 $\text{productive } lxs \longrightarrow \text{productive } (\text{LCons } (\text{DT } t \ d) \ lxs)$

|  $\text{SkipWM}: \neg \text{lfinite } lxs \longrightarrow \text{productive } lxs \longrightarrow$   
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# Building Blocks: Batch Operator

# Batch Operator: Soundness

# Batch Operator: Completeness

- Uses soundness of `batch_op`
- Proof by induction over `n`

$$\begin{aligned} \text{mono\_prod } lxs \ W \longrightarrow & (\exists i \ d. \text{enat } i < \text{llength } lxs \wedge \text{Inth } lxs \ i = \text{DT } t \ d \wedge n = \text{Suc } i) \vee \\ n = 0 \wedge t \in \text{set\_t } buf \longrightarrow & (\forall t' \in \text{set\_t } buf. \text{lfinite } lxs \vee \exists wm \geq t'. \text{WM } wm \in \text{lset } lxs) \longrightarrow \\ \exists wm \ batch. \text{DT } wm \ batch \in \text{lset } & (\text{produce } (\text{batch\_op } buf) \ lxs) \wedge t \in \text{set\_t } batch \vee \\ (\forall k \in \{n .. < \text{the\_enat } & (\text{llength } lxs)\} . \neg (\exists t' \geq t. \text{Inth } lxs \ k = \text{WM } t')) \wedge \text{lfinite } lxs \end{aligned} \quad (1)$$

# Batch Operator: Monotone

# Batch Operator: Productive



# Building Blocks: Incremental Operator

# Batch Operator: Soundness

# Batch Operator: Completeness

# Batch Operator: Monotone

# Batch Operator: Productive



## Case Study

# Histogram



# Histogram: Soundness

# Histogram: Completeness

# Histogram: Monotone

# Histogram: Productive

# Efficient Histogram

- Foo





# Join: Completeness



# Join: Monotone

## Next Steps

# Next Steps

Questions, comments and suggestions