Nondeterministic Asynchronous Dataflow in Isabelle/HOL

Rafael Castro G. Silva, Laouen Fernet and Dmitriy Traytel

rasi@di.ku.dk

Department of Computer Science
University of Copenhagen

14/05/2025

Introduction

Introduction

- Nondeterministic Asynchronous Dataflow: Directed graph of interconnected operators that perform event-wise transformations
  - Dataflow: Directed graph of interconnected operators that perform event-wise transformations
  - Asynchronous:
    - Operators execute independently (processes without an orchestrator)
    - Operators can communicate with the network (read/write); do silent computation steps
    - Networks are unbounded FIFO queues
  - Nondeterministic: Operators can make nondeterministic choices

The Process Algebra Model

- Nondeterministic Asynchronous Dataflow defined as an algebra by Bergstra et al.:

Network Algebra for Asynchronous Dataflow[*]

J.A. Bergstra[1,2,†]   C.A. Middelburg[2,3,§]   Gh. Ştefănescu[4,†]

[1]Programming Research Group, University of Amsterdam
P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

[2]Department of Philosophy, Utrecht University
P.O. Box 80126, 3508 TC Utrecht, The Netherlands

[3]Department of Network & Service Control, KPN Research
P.O. Box 421, 2260 AK Leidschendam, The Netherlands

[4]Institute of Mathematics of the Romanian Academy
P.O. Box 1-764, 70700 Bucharest, Romania

E-mail: janb@fwi.uva.nl - keesm@phil.ruu.nl - ghstef@imar.ro

- Provides a process algebra model (a process calculi) of Nondeterministic Asynchronous Dataflow
- Defined basic primitives: sequential and parallel composition; feedback loop
- Defined basic primitives: sequential and parallel composition; feedback loop

- Stream Processing: programs that compute (possibly) unbounded sequences of data (streams)
- E.g.: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow

- Why?
  - Highly Parallel
  - Low latency (output as soon as possible)
  - Incremental computing (re-uses previous computations)

Isabelle/HOL Preliminaries

- Classical higher-order logic (HOL): Simple Typed Lambda Calculus + (Hilbert) axiom of choice + axiom of infinity + rank-1 polymorphism

Isabelle/HOL

- Classical higher-order logic (HOL): Simple Typed Lambda Calculus + (Hilbert) axiom of choice + axiom of infinity + rank-1 polymorphism
- Isabelle: A generic proof assistant

- Isabelle/HOL: Isabelle's flavor of HOL

- Codatatypes

- Recursion: always eventually reduces an argument
- Corecursion: always eventually produces something

- Recursion: always eventually reduces an argument
- Corecursion: always eventually produces something
- Corec:

Isabelle/HOL: (Co)inductive Predicates

- Inductive predicate
  - Finite number of introduction rule applications

Isabelle/HOL: (Co)inductive Predicates

- Inductive predicate
  - Finite number of introduction rule applications
- Coinductive predicate
  - Infinite number of introduction rule applications

Isabelle/HOL: (Co)inductive Predicates

- Inductive predicate
  - Finite number of introduction rule applications
- Coinductive predicate
  - Infinite number of introduction rule applications
- Coinduction principle

Operators as a Codatatype

- Codatatypes

- Codatatypes

- foo

- foo

- foo

Operators Equivalences: Trace Equivalence

- foo

Numeral Types for Ports

- foo

Asynchronous Dataflow Operators

Buffer Infrastructure

- foo

Asynchronous Dataflow Properties

Conclusion

Conclusion

- Isabelle/HOL has a good tool set to formalize and reason about stream processing:
  - Codatatypes, coinductive predicates, corecursion with friends, reasoning up to friends (congruence),
    - Coinduction up to congruence principle is automatically derived for codatatypes (but not for coinductive principles)
- Next step: Feedback loop

Questions, comments and suggestions