# Verified Time-Aware Stream Processing

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04/06/2024

# Introduction

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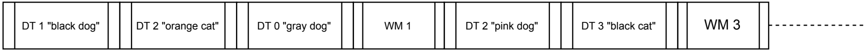
    - Watermark: A value of the same type of the timestamp. Represents data-completeness.



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| | | | | | | |
|---|---|---|---|---|---|---|
| DT 1 "black dog" | DT 2 "orange cat" | DT 0 "gray dog" | WM 1 | DT 2 "pink dog" | DT 3 "black cat" | WM 3 |

- Asynchronous Dataflow Programming: Directed graph of interconnected operators that perform event-wise transformations
- E.g.: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow

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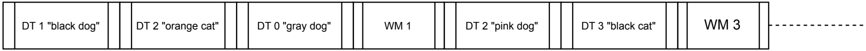


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Cloud DataFlow Flink

- Why?
  - Highly Parallel
  - Low latency (output as soon as possible)
  - Incremental computing

# Time-Aware Stream Processing Example

Example:
Incrementally count the occurrences of the words “dog” and “cat”

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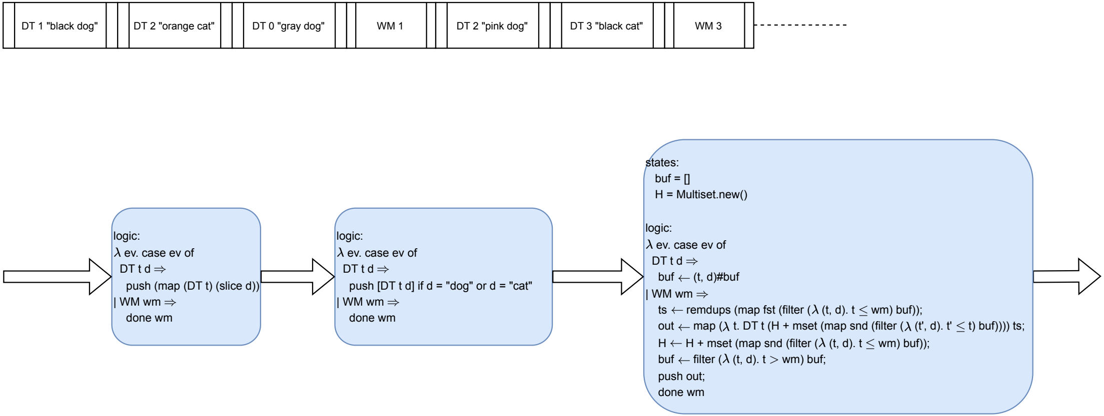

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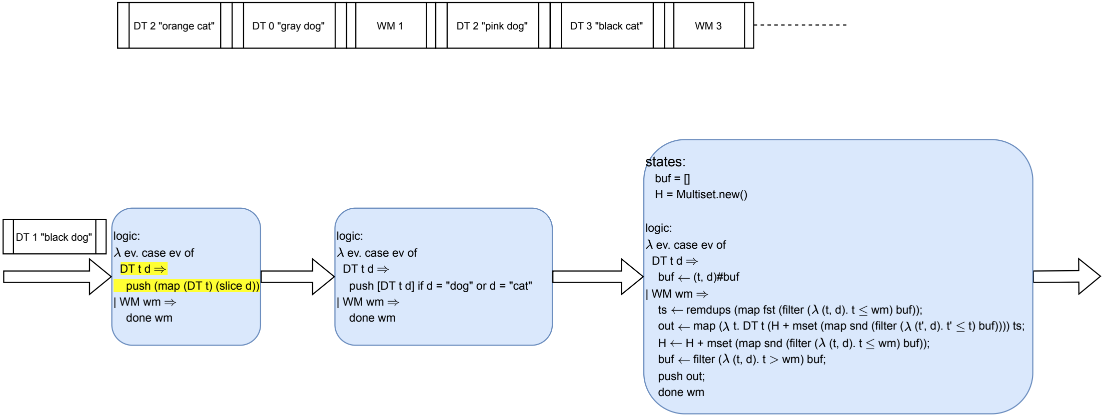

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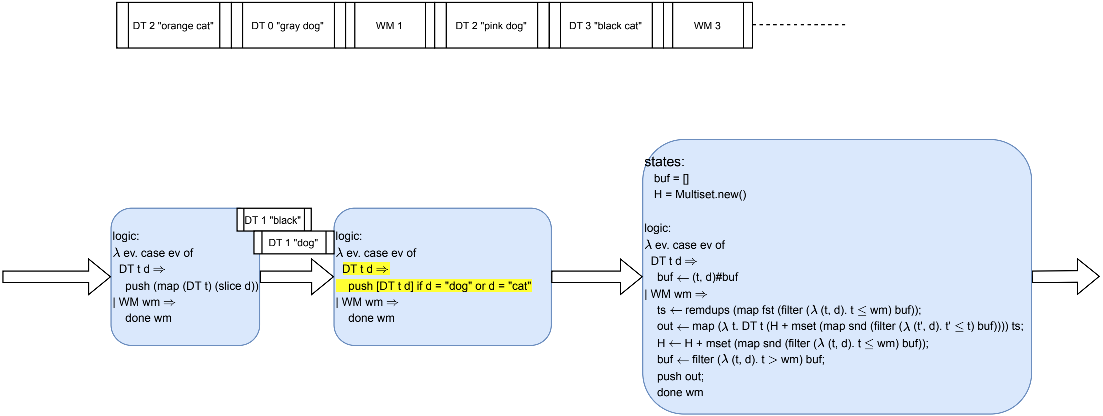

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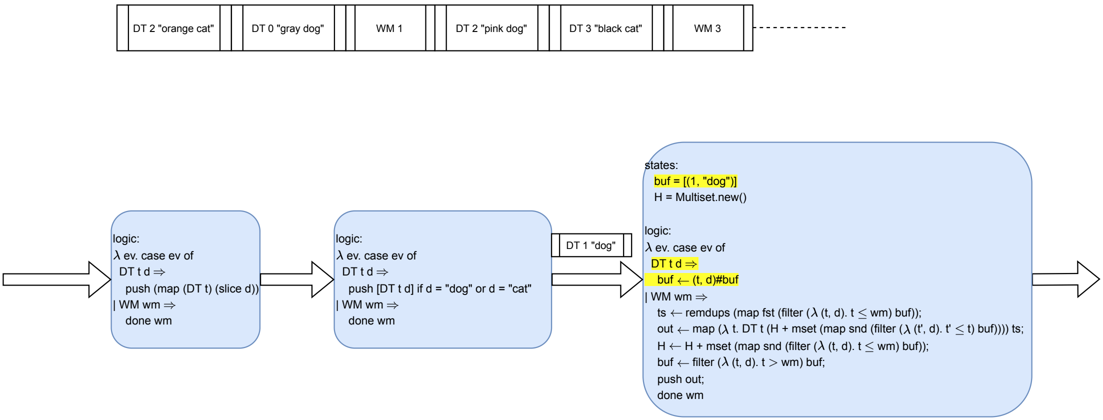

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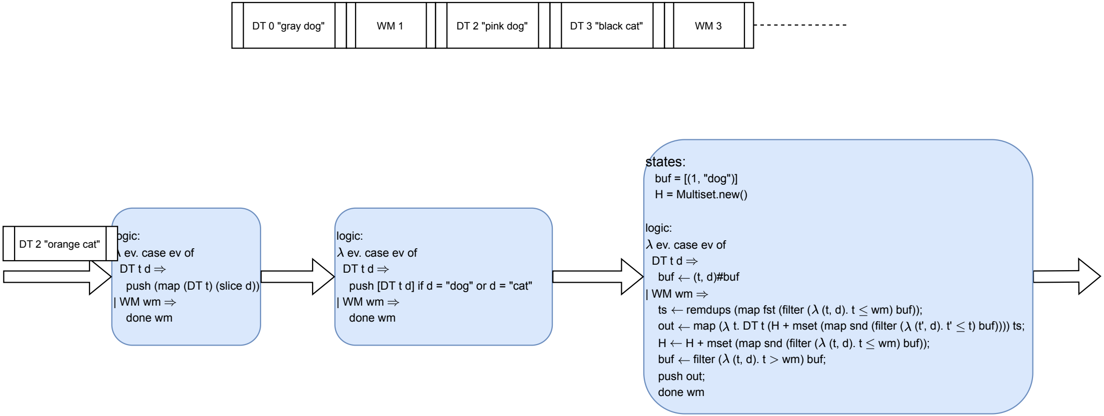

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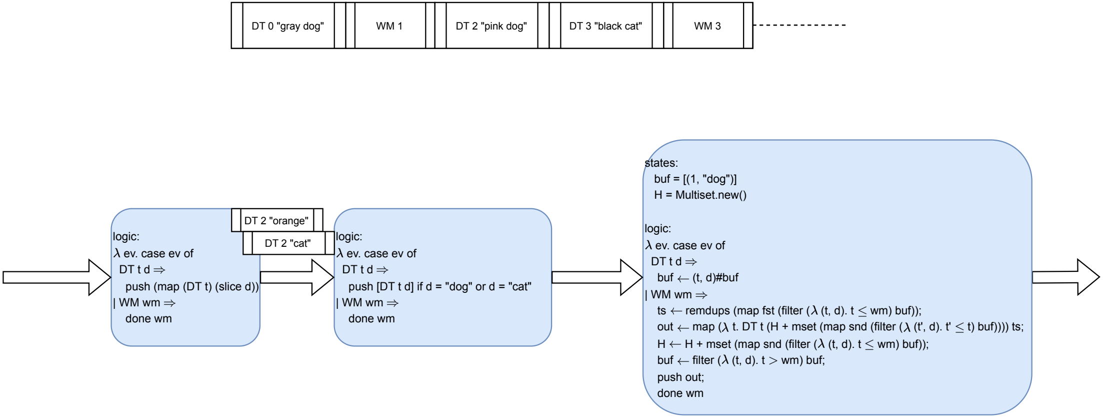

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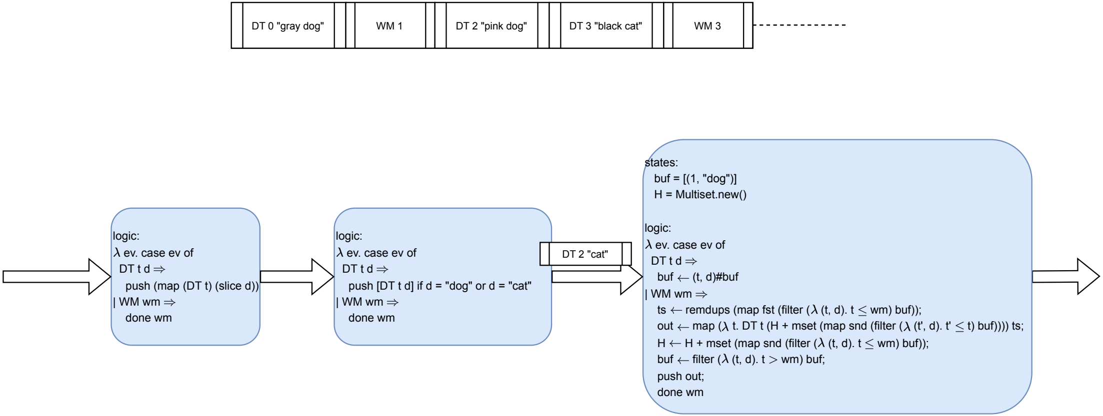

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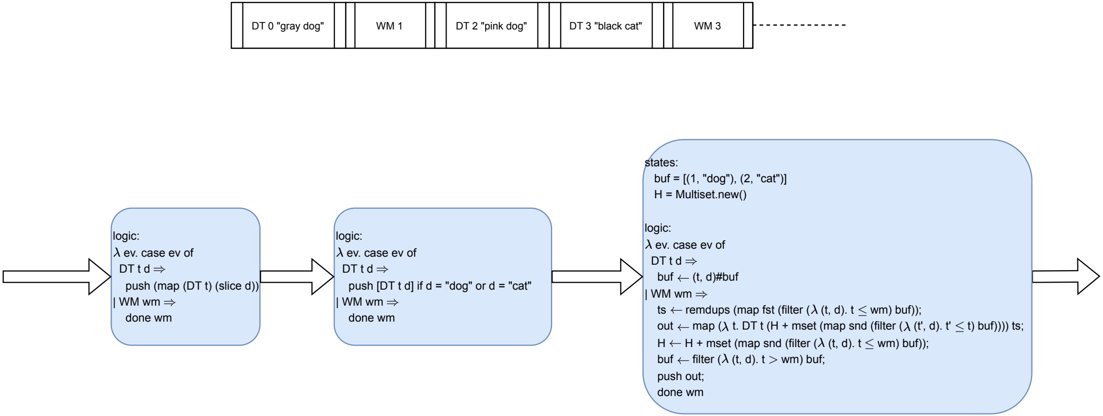

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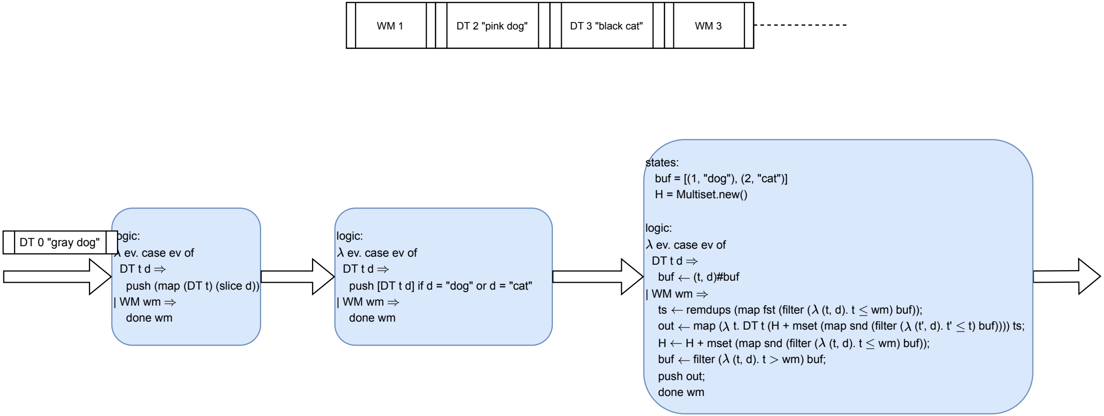

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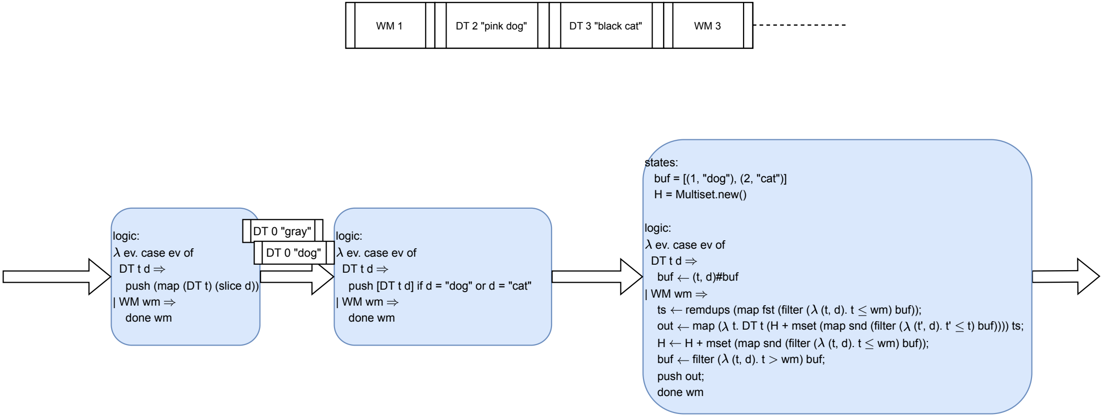

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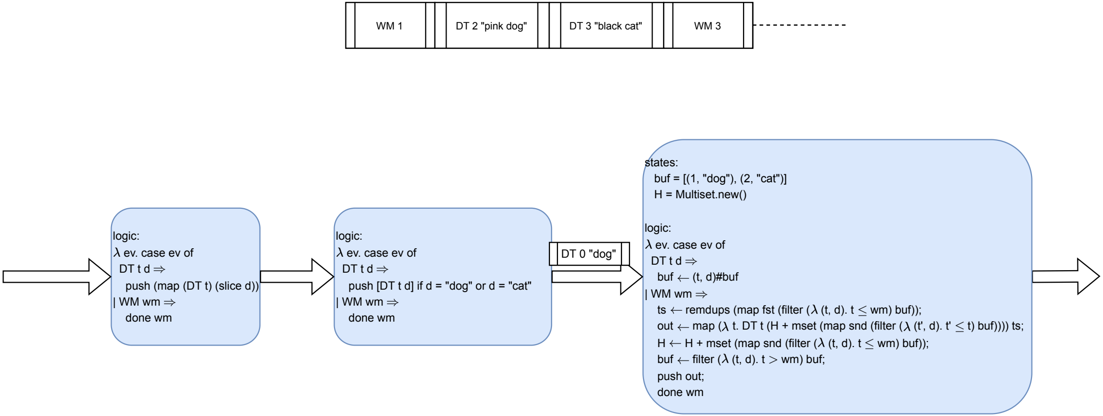

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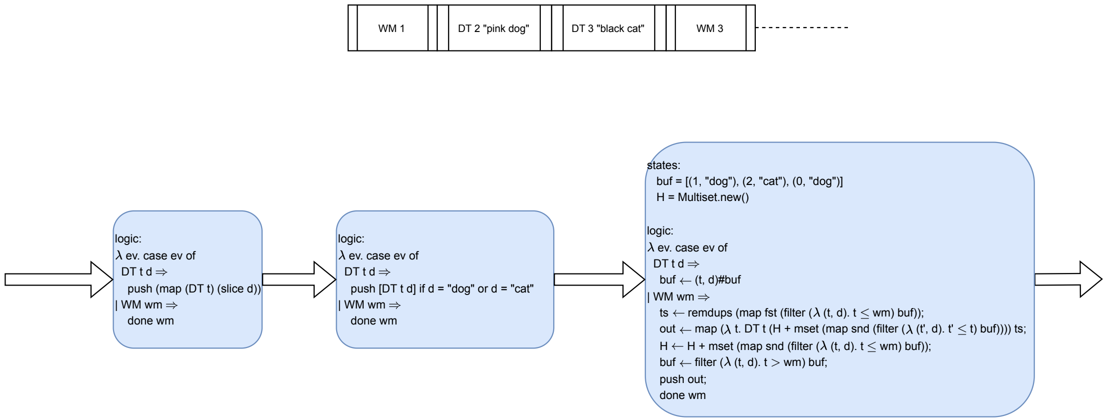

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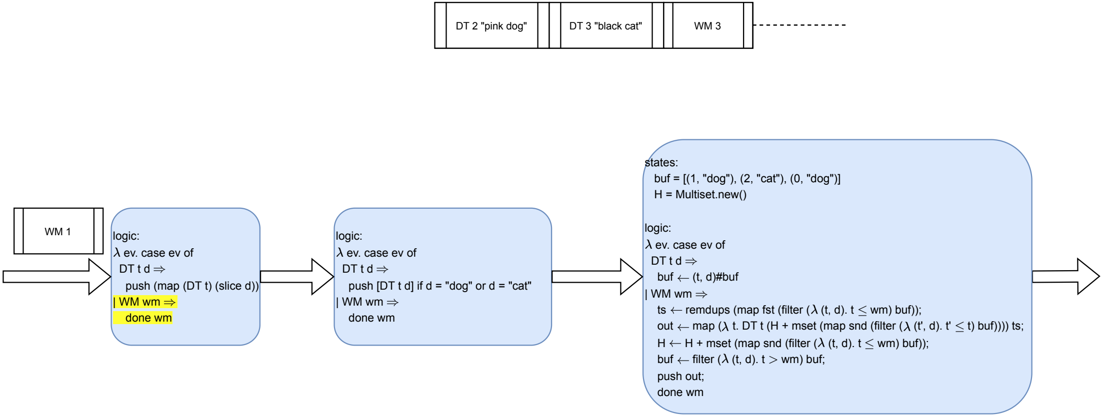

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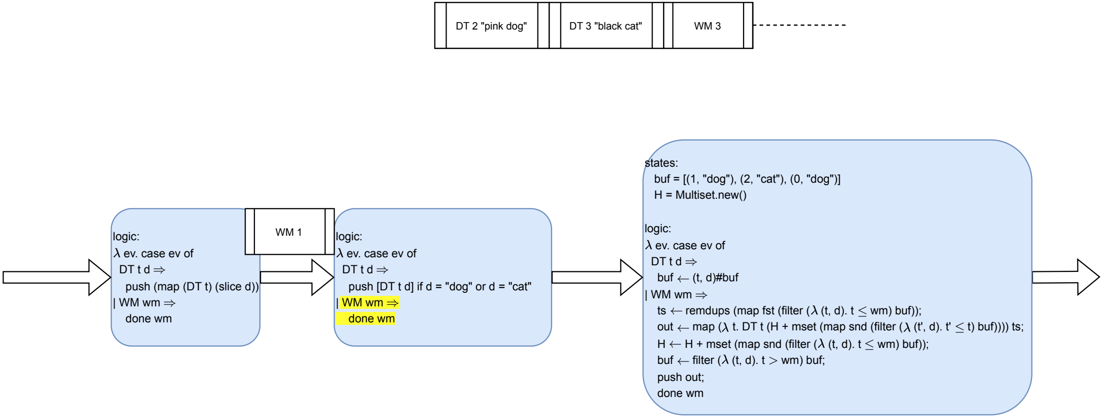

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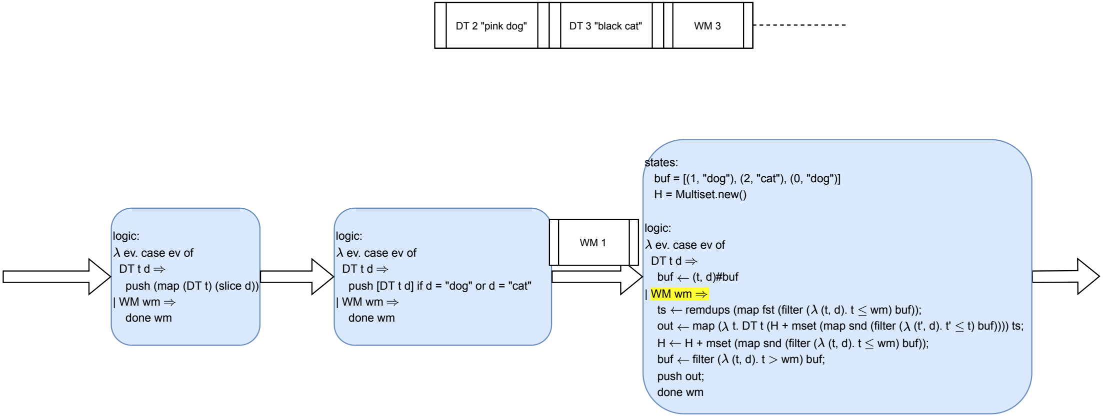

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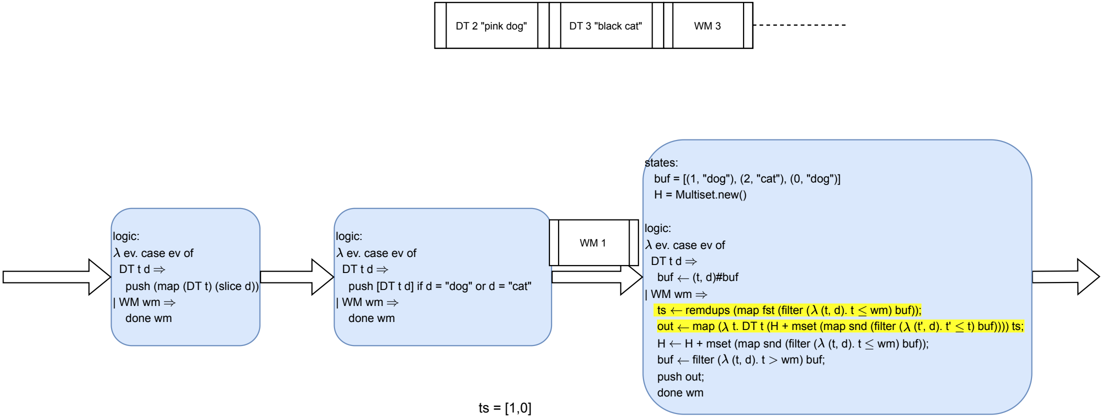


ts = [1,0]

out = [DT 1 {"dog": 2}, DT 0 {"dog": 1}]

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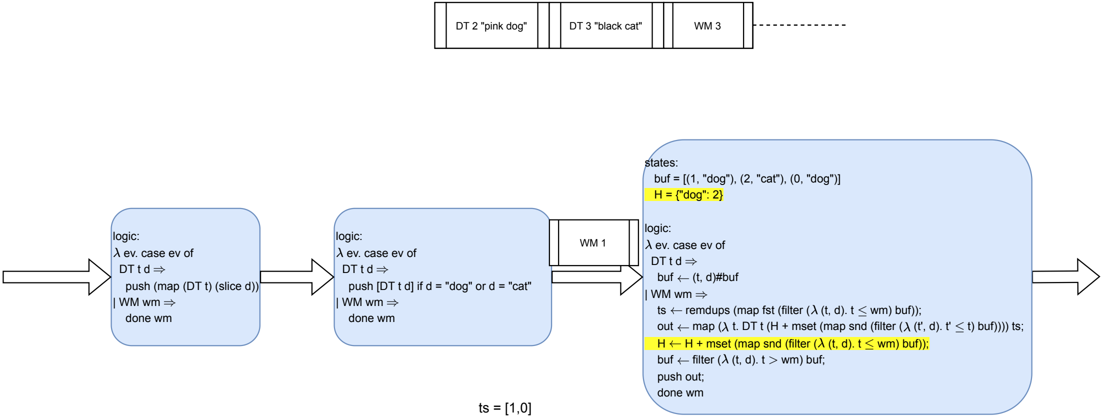


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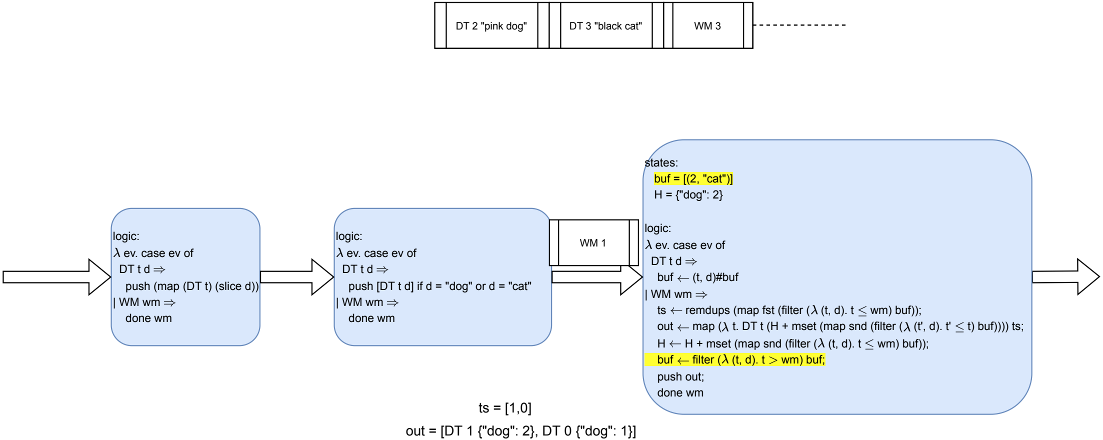

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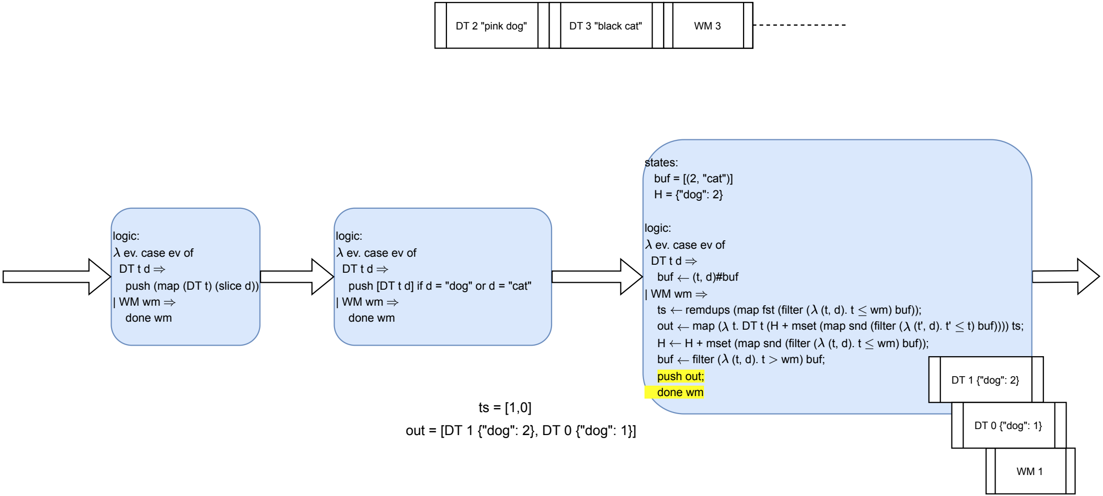

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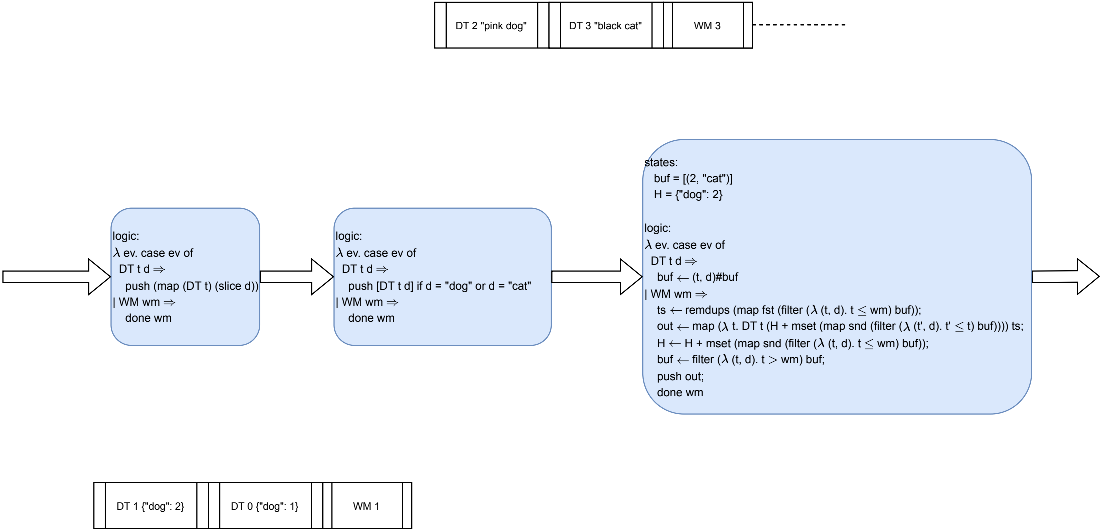

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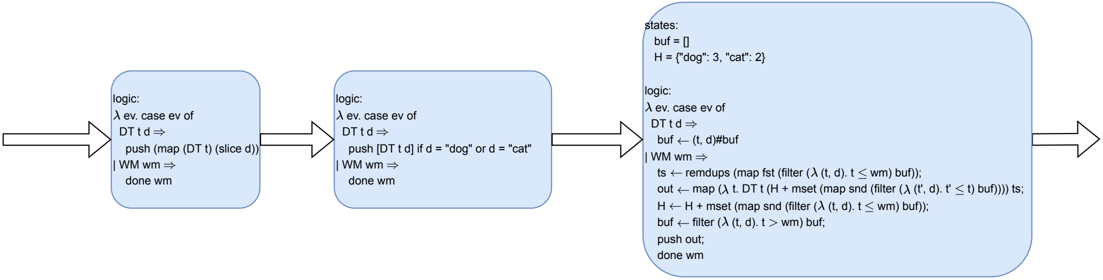


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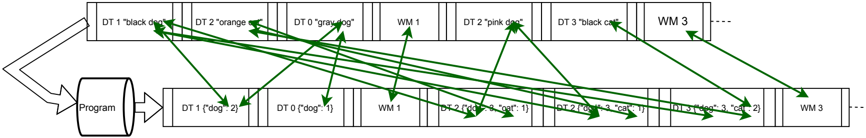
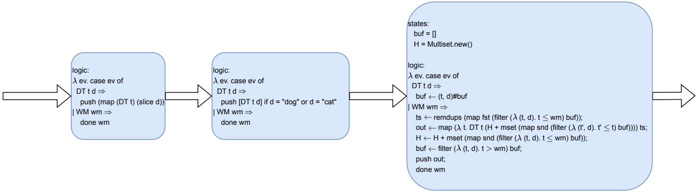
- We need a correctness specification
- **Intuition of the specification**:
    - Soundness: for every output $DT\ t\ H$, the "dog" count in $H$ is the count of events with timestamp $(\leq)t$ which contains the string "dog"; similarly for "cat". The count for any other word is always 0.
    - Completeness: The other way around.

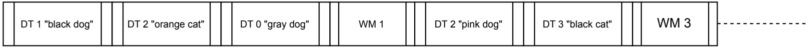


# How to prove it

- **lets break down the problem!**:
  - The correctness of the entire Dataflow emerges from the correctness of each part (operator)
    - Operator 1: Slicer
    - Operator 2: Filter
    - Operator 3: Incremental histogram
    - Assumptions about the incoming stream:
    1. Monotone: after WM wm no *DT t d* such that $t \leq wm$.
    2. Productive: after *DT t d* eventually WM wm such that $t \leq wm$




- The original incoming stream must respect monotonicity and productivity




- Each operator must preserve monotonicity and productivity!

## Writing it down in Isabelle/HOL!

# Isabelle/HOL: (Co)datatypes

- Datatypes and Codatatypes

  **codatatype** (lset: $'a$) *llist* = lnull: LNil | LCons (lhd: $'a$) (ltl: $'a$ *llist*)
  **for** map: lmap **where** ltl LNil = LNil

- Examples:
  - LNil
  - LCons *1* (LCons *2* (LCons *3* LNil))
  - LCons *0* (LCons *0* (LCons *0* (...)))

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- Coinductive principle for lazy list equality

# Isabelle/HOL: Recursion and While Combinator

- Recursion

$$\textbf{fun}\ \mathsf{lshift} :: {}'a\ list \Rightarrow {}'a\ llist \Rightarrow {}'a\ llist\ (infixr\ @@\ 65)\ \textbf{where}$$
$$\mathsf{lshift}\ []\ lxs = lxs$$
$$|\ \mathsf{lshift}\ (x \# xs)\ lxs = \mathsf{LCons}\ x\ (\mathsf{lshift}\ xs\ lxs)$$

- While Combinator

$$\textbf{definition}\ \mathsf{while\_option} :: ({}'a \Rightarrow bool) \Rightarrow ({}'a \Rightarrow {}'a) \Rightarrow {}'a \Rightarrow {}'a\ option\ \textbf{where}$$
$$\mathsf{while\_option}\ b\ c\ s = \ldots$$

- While rule for invariant reasoning (Hoare-style):
  - There is something that holds before a step; that thing still holds after the step

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- Corec:

  **corec** lapp $:: 'a\ llist \Rightarrow 'a\ llist \Rightarrow 'a\ llist$ **where**
  lapp $lxs\ lys$ = case $lxs$ of LNil $\Rightarrow lys$ | LCons $x\ lxs' \Rightarrow$ LCons $x$ (lapp $lxs'\ lys$)

# Isabelle/HOL: (Co)inductive Predicates

- Inductive predicate
  - Finite number of introduction rule applications

**inductive** in_llist $:: {'a} \Rightarrow {'a}\ \mathit{llist} \Rightarrow \mathit{bool}$ **where**
  In_llist: in_llist $x$ (LCons $x$ $\mathit{lxs}$)
| Next_llist: in_llist $x$ $\mathit{lxs}$ $\Rightarrow$ in_llist $x$ (LCons $y$ $\mathit{lxs}$)

in_llist $2$ (LCons $1$ (LCons ($2$ (...))))

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- Coinductive predicate
  - Infinite number of introduction rule applications

**coinductive** lprefix $:: 'a\ llist \Rightarrow 'a\ llist \Rightarrow bool$ **where**
  LNil_lprefix: lprefix LNil $lys$
| LCons_lprefix: lprefix $lxs$ $lys$ $\Rightarrow$ lprefix (LCons $x$ $lxs$) (LCons $x$ $lys$)

lprefix (LCons $1$ (LCons ($2$ (...)))) (LCons $1$ (LCons ($2$ (...))))

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- Coinductive principle

# Lazy Lists Processors

# Operator formalization

- Operator as a codatatype
  - Taking $'i$ as the input type, and $'o$ as the output type:
    $\textbf{codatatype}\ ('o, 'i)\ \mathit{op} = \mathrm{Logic}\ (\mathrm{apply}{:}\ ('i \Rightarrow ('o, 'i)\ \mathit{op} \times 'o\ \mathit{list}))$

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    $\textbf{codatatype}$ $('o, 'i)\ op = \text{Logic}\ (\text{apply}: ('i \Rightarrow ('o, 'i)\ op \times 'o\ list))$
  - Infinite trees: applying the selector apply "walks" a branch of the tree

# Execution formalization

- Produce function: applies the logic (co)recursively throughout a lazy list

**definition** $\mathsf{produce}_1$ $op$ $lxs$ $=$ $\mathsf{while\_option}$ $\ldots$

**corec** produce **where**
produce $op$ $lxs$ $=$ (case $\mathsf{produce}_1$ $op$ $lxs$ of
  None $\Rightarrow$ LNil
| Some $(op', x, xs, lxs')$ $\Rightarrow$ LCons $x$ $(xs$ @@ produce $op'$ $lxs'))$

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- Produce function: applies the logic (co)recursively throughout a lazy list

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| Some (*op*′, *x*, *xs*, *lxs*′) ⇒ LCons *x* (*xs* @@ produce *op*′ *lxs*′))

- produce$_1$ has an induction principle based on the while invariant rule

- Example:

**corec** count_op **where** count_op $P$ $n$ =
  Logic ($\lambda e.$ `if` $P$ $e$ `then` (count_op $P$ ($n$ + $1$), [$n$+$1$]) `else` (count_op $P$ $n$, []))

$$\mathit{stream}_1 = \boxed{0}\,\boxed{3}\,\boxed{3}\,\boxed{6}\,\boxed{24}$$

$$\text{produce (count\_op } \mathit{is\_even}\ 0) \quad \mathit{stream}_1 = \boxed{1}\,\boxed{2}\,\boxed{3}$$

# Sequential Composition operator

- Sequential composition: take the output of the first operator and give it as input to the second operator.
- Correctness:

  $\text{produce } (\text{comp\_op } op_1 \ op_2) \ lxs = \text{produce } op_2 \ (\text{produce } op_1 \ lxs)$

- Proof: coinductive principle for lazy list equality and $\text{produce}_1$ induction principle

# Time-Aware Operators

# Time-Aware Streams

- Time-Aware lazy lists

  $\mathbf{datatype}\ ('t{::}order,\ 'd)\ \mathit{event} = \mathsf{DT}\ (\mathsf{tmp}{:}\ 't)\ (\mathsf{data}{:}\ 'd) \mid \mathsf{WM}\ (\mathsf{wmk}{:}\ 't)$

# Time-Aware Streams

- Time-Aware lazy lists

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- Generalization to partial orders
  - Cycles
  - Operators with multiple inputs
- Productive and monotone streams: Coinductive predicates over lazy lists of events.

# Proving histogram correct: building Blocks

- Histogram operator: batching and incremental computing
- Building Blocks: reusable operators
  - Batching: batch_op
  - Incremental computing: incr_op
  - Soundness, completeness, preservation of monotonicity and productivity

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- Histogram operator: batching and incremental computing
- Building Blocks: reusable operators
  - Batching: batch_op
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  - Soundness, completeness, preservation of monotonicity and productivity
- Define histogram using the building blocks
- Compositional Reasoning: correctness follows from the correctness of the building blocks

# Next Steps

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- Feedback loop

Questions, comments and suggestions