Nondeterministic Asynchronous Dataflow in Isabelle/HOL

Rafael Castro G. Silva, Laouen Fernet and Dmitriy Traytel

rasi@di.ku.dk

Department of Computer Science
University of Copenhagen

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Motivation

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Context:

- Stream Processing: programs that compute (possibly) unbounded sequences of data (streams)
- A common problem in the industry
- Frameworks:
  Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow

- Why use frameworks?
  - Highly Parallel
  - Low latency (output as soon as possible)
  - Incremental computing (re-uses previous computations)

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Our goal:

Mechanically Verify Timely Dataflow algorithms

A Good Foundation

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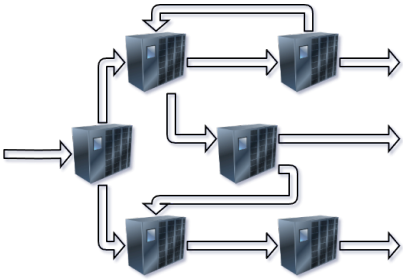

- Nondeterministic Asynchronous Dataflow

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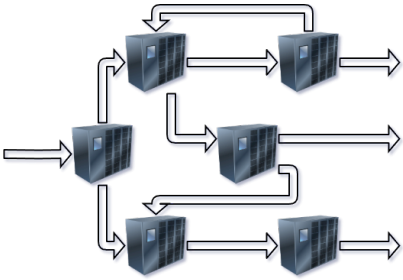

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  - Dataflow: Directed graph of interconnected operators

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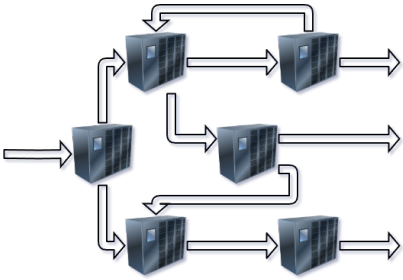

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  - Asynchronous:
    - Operators execute independently: processes without an orchestrator
    - Operators can freely communicate with the network (read/write); do silent computation steps
    - Networks are unbounded FIFO queues

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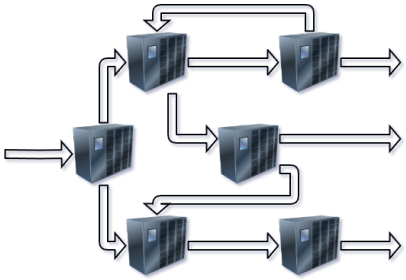

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    - Operators can freely communicate with the network (read/write); do silent computation steps
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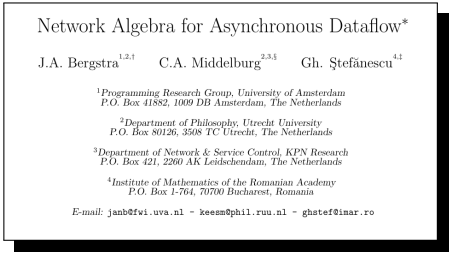
  - Nondeterministic:
    - Operators can make nondeterministic choices
    - Operators are relations between inputs and outputs sequences

The Algebra for Nondeterministic Asynchronous Dataflow

- Bergstra et al. presents an algebra for Nondeterministic Asynchronous Dataflow
- Primitives:
  sequential and parallel composition; feedback loop...
- The 52 axioms
- An process calculus instance



Network Algebra for Asynchronous Dataflow*

J.A. Bergstra[1,2,†] C.A. Middelburg[2,3,§] Gh. Ştefănescu[4,‡]

[1]Programming Research Group, University of Amsterdam
P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

[2]Department of Philosophy, Utrecht University
P.O. Box 80126, 3508 TC Utrecht, The Netherlands

[3]Department of Network & Service Control, KPN Research
P.O. Box 421, 2260 AK Leidschendam, The Netherlands

[4]Institute of Mathematics of the Romanian Academy
P.O. Box 1-764, 70700 Bucharest, Romania

E-mail: janb@fwi.uva.nl - keesm@phil.ruu.nl - ghstef@imar.ro

Isabelle/HOL Preliminaries

Isabelle/HOL

- Classical higher-order logic (HOL):
  Simple Typed Lambda Calculus + axiom of choice + axiom of infinity + rank-1 polymorphism

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- Isabelle/HOL: Isabelle's flavor of HOL

Why Isabelle/HOL?

- Codatatypes: (possibly) infinite data structures (e.g., lazy lists, streams)
- Corecursion: always eventually produces some codatatype constructor
- Coinductive predicate: infinite number of introduction rule applications
- Coinduction: reason about coinductive predicates

Operators as a Codatatype

Operators

Operators in Isabelle/HOL

codatatype (inputs: $'i$, outputs: $'o$, $'d$) op =
Read $'i$ ($'d \Rightarrow$ ($'i$, $'o$, $'d$) op) | Write (($'i$, $'o$, $'d$) op) $'o$ $'d$
Silent ($'i$, $'o$, $'d$) op | Choice (($'i$, $'o$, $'d$) op) $cset$

- Type parameters: inputs/output ports; data
- Operator's actions
- Possibly infinite trees
- inputs/outputs: Sets of used ports

Examples 1

Operators in Isabelle/HOL

corec silent_op $(\odot)$ **where**
$\odot$ = Silent $\odot$

corec spin_op $(\otimes)$ **where**
$\otimes$ = Choice $((\lambda\_.\ \otimes), \{()\}_c)$

lemma spin_op_code:
$\otimes$ = Choice $\{\otimes\}_c$

- Codatatypes

Operators Equivalences: Motivation

- foo

Operators Equivalences: Strong Bisimilarity

- foo

Operators Equivalences: Weak Bisimilarity

- foo

Asynchronous Dataflow Operators

Buffer Infrastructure

- foo

Asynchronous Dataflow Properties

Conclusion

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- Isabelle/HOL has a good tool set to formalize and reason about stream processing:
  - Codatatypes, coinductive predicates, corecursion with friends, reasoning up to friends (congruence),
    - Coinduction up to congruence principle is automatically derived for codatatypes (but not for coinductive principles)
- Next step: Feedback loop

Questions, comments and suggestions