Nondeterministic Asynchronous Dataflow in Isabelle/HOL

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Motivation

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Context:

- Stream Processing: programs that compute (possibly) unbounded sequences of data (streams)
- Frameworks: Google Cloud Dataflow, Apache Flink, and Timely Dataflow

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Our long term goal:

Mechanically Verify Timely Dataflow algorithms

A Good Foundation

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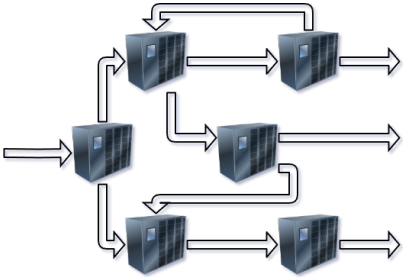

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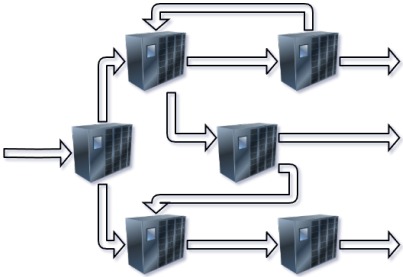

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  - Dataflow: Directed graph of interconnected operators

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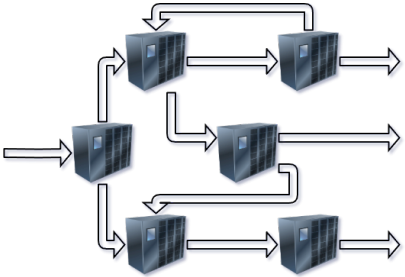

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  - Asynchronous:
    - Operators execute independently: processes without an orchestrator
    - Operators can freely communicate with the network (read/write); do silent computation steps
    - Networks are unbounded FIFO queues

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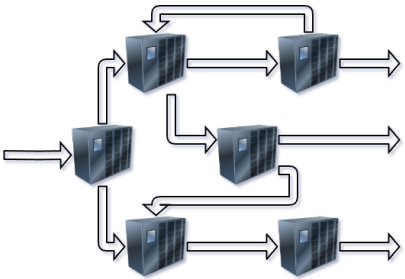

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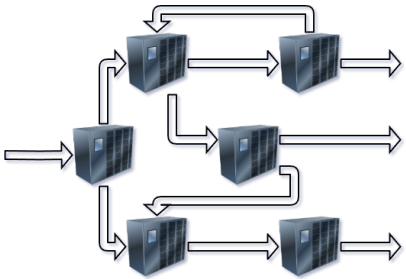

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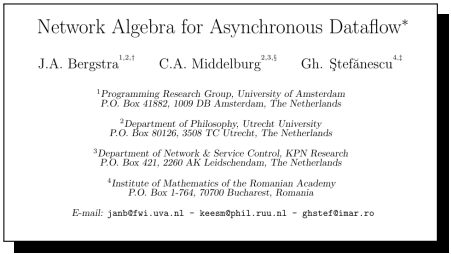
Question

How do we know something is Nondeterministic Asynchronous Dataflow?

The Algebra for Nondeterministic Asynchronous Dataflow

- Bergstra et al. presents an algebra for Nondeterministic Asynchronous Dataflow
- Primitives:
  sequential and parallel composition; feedback loop...
- 52 axioms
- A process calculus instance



Network Algebra for Asynchronous Dataflow*

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Main Contributions

- An Isabelle/HOL instance of Nondeterministic Asynchronous Dataflow
  - Operators as a shallow embedding as codatatypes
  - 51 axioms proved

Isabelle/HOL Preliminaries

What is Isabelle/HOL?

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- Isabelle: A generic proof assistant

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Why Isabelle/HOL?

- Codatatypes: (possibly) infinite data structures (e.g., lazy lists, streams)
- Corecursion: always eventually produces some codatatype constructor
- Coinductive predicates: infinite number of introduction rule applications
- Coinduction: reason about coinductive predicates

Operators as a Codatatype

Operators

Operators in Isabelle/HOL

codatatype $('i, 'o, 'd)\ op =$

Read $'i\ ('d \Rightarrow ('i, 'o, 'd)\ op)$ | Write $(('i, 'o, 'd)\ op)\ 'o\ 'd$

Silent $('i, 'o, 'd)\ op$ | Choice $(('i, 'o, 'd)\ op)\ cset$

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  inputs/output ports; data
- Operator's actions
- Possibly infinite trees

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Uncommunicative operators

$\oslash$ = Choice $\{\}_c$

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Uncommunicative operators

$\oslash$ = Choice $\{\}_c$

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$\otimes$ = Choice $\{\otimes\}_c$

More examples

ex1 = Choice $\{$Write ex1 1 42, $\oslash\}_c$

ex2 = Choice $\{$Write ex2 1 42, ex2$\}_c$

ex3 = Choice $\{$Write ex3 1 42, Silent ex3$\}_c$

Operators Equivalences: Weak Bisimilarity

An ideal equivalence relation

- Only equate operators that **behave** the same
- Useful reasoning principle

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  The classic chapter on **weak bisimilarity**

- Based on labeled transition systems (LTS)
- **Weak**: silent computations are abstracted away

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Weak bisimilarity of operators

- Labels (actions): read, write, silent (τ)
- Weak bisimilarity of operators ($\approx$): LTS + weak simulation + greatest weak bisimulation
- $\approx$ has a useful coinduction principle
- $\oslash \approx \otimes \approx \otimes$ and ex1 $\approx$ ex2 $\approx$ ex3

Asynchronous Dataflow Operators

Auxiliary Definitions

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- Buffers: $'p \Rightarrow 'd\ list$

Buffer functions

$\mathsf{BHD}\ p\ buf = \mathsf{hd}\ (buf\ p)$

$\mathsf{BTL}\ p\ buf = buf(p := \mathsf{tl}\ (buf\ p))$

$\mathsf{BENQ}\ p\ x\ buf = buf(p := buf\ p\ @\ [x])$

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Mapping ports functions

$map\_op :: ('i_1 \Rightarrow 'i_2) \Rightarrow ('o_1 \Rightarrow 'o_2) \Rightarrow ('i_1, 'o_1, 'd)\ op \Rightarrow ('i_2, 'o_2, 'd)\ op$

$\curvearrowright :: ('a + 'b) + 'c \Rightarrow 'a + 'b + 'c$

$\curvearrowleft :: 'a + 'b + 'c \Rightarrow ('a + 'b) + 'c$

Identity

Equation of the identity operator

$\mathsf{id\_op}\ buf = \mathsf{Choice}$

$\bigcup_c$

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Equation of the identity operator

$$\begin{aligned}&\text{id\_op } buf = \text{Choice}\\&(((\lambda p.\ \text{Read } p\ (\lambda x.\ \text{id\_op } (\text{BENQ } p\ x\ buf))) \grave{}_c\ \mathfrak{U}_c)\\&\cup_c\end{aligned}$$

- $\mathfrak{U}_c$ is the set of usable ports provide by its type

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- Stream delayer: always can read, and it may eventually output

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Identity operator with an empty buffer

$$\mathcal{I} = \text{id\_op } (\lambda\_.\ [])$$

Composition: Preliminaries

Composition operator type

$$\text{comp\_op} :: ('o_1 \Rightarrow 'i_2\ option) \Rightarrow ('i_2 \Rightarrow 'd\ list) \Rightarrow ('i_1, 'o_1, 'd)\ op \Rightarrow ('i_2, 'o_2, 'd)\ op \Rightarrow ('i_1 + 'i_2, 'o_1 + 'o_2, 'd)\ op$$

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Sequential and parallel composition

$op_1 \parallel op_2 =$ comp_op $(\lambda\_.\ \text{None})\ (\lambda\_.\ [])\ op_1\ op_2$

$op_1 \bullet op_2 =$ map_op projl projr (comp_op Some $(\lambda\_.\ [])\ op_1\ op_2$)

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&\ \mid \text{Write } \mathit{op}\ p\ x \Rightarrow (\underline{\text{case}}\ \mathit{wire}\ p\ \underline{\text{of}}\\
&\qquad \text{None} \Rightarrow \text{Write } (\text{comp\_op } \mathit{wire}\ \mathit{buf}\ \mathit{op}\ \mathit{op}_2)\ (\text{Inl } p)\ x\\
&\qquad \mid \text{Some } q \Rightarrow \text{Silent } (\text{comp\_op } \mathit{wire}\ (\text{BENQ } q\ x\ \mathit{buf})\ \mathit{op}\ \mathit{op}_2))\\
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| Write *op p x* ⇒ (case *wire p* of
None ⇒ Write (comp_op *wire buf op* op_2) (Inl *p*) *x*
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- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
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Sink operator type

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$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
- Sequential composition: $op_1 \bullet op_2$

Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
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Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Copy operator type

$\mathcal{C} :: ('n, 'n + 'n, 'd)\ op$

Asynchronous Dataflow Operators

- Identity: $\mathcal{I}$
- Parallel composition: $op_1 \parallel op_2$
- Sequential composition: $op_1 \bullet op_2$

Feedback operator type ($op \uparrow$)

$\uparrow :: ('i + 'm, 'o + 'm, 'd)\ op \Rightarrow ('i, 'o, 'd)\ op$

Transpose operator type

$\mathcal{X} :: ('n + 'm, 'm + 'n, 'd)\ op$

Dummy source operator type

$\mathsf{i} :: ('i, 'o, 'd)\ op$

Sink operator type

$! :: ('i, 'o, 'd)\ op$

Split operator type

$\Lambda :: ('n, 'n + 'n, 'd)\ op$

Merge operator type

$\mathcal{V} :: ('n + 'n, 'n, 'd)\ op$

Copy operator type

$\mathcal{C} :: ('n, 'n + 'n, 'd)\ op$

Equality test operator type

$\mathcal{Q} :: ('n + 'n, 'n, 'd\ option)\ op$

Asynchronous Dataflow Properties

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op } \curvearrowright \curvearrowright \ (op_1 \parallel op_2) \parallel op_3$
B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl } op$
B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr } op$
B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$
B4_1: $op \bullet \mathcal{I} \approx op$
B4_2: $\mathcal{I} \bullet op \approx op$
B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$
B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$
B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$
B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl) } \mathcal{I}$
B9: $\mathcal{X} \approx \text{map\_op } \curvearrowright \curvearrowright \ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id } \curvearrowleft \ (\mathcal{I} \parallel \mathcal{X})$
B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$
F1: $\mathcal{I} \uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$
F2: $\mathcal{X} \uparrow \approx \mathcal{I}$
R1: $op_2 \bullet (op_1 \uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1) \uparrow$
R2: $(op_1 \uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I})) \uparrow$
R3: $op_1 \parallel (op_2 \uparrow) \approx (\text{map\_op } \curvearrowleft \curvearrowleft \ (op_1 \parallel op_2)) \uparrow$
R4: $(op_1 \bullet (\mathcal{I} \parallel op_2)) \uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1) \uparrow$
R5: $\text{map\_op Inl Inl } ((op :: ('i + 0, 'o + 0, 'd)\ op) \uparrow) \approx op$
R6: $(op \uparrow) \uparrow \approx (\text{map\_op } \curvearrowright \curvearrowright \ op) \uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op } \curvearrowright \curvearrowright\ (op_1 \parallel op_2) \parallel op_3$

B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl } op$

B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr } op$

B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$

B4_1: $op \bullet \mathcal{I} \approx op$

B4_2: $\mathcal{I} \bullet op \approx op$

B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$

B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$

B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$

B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl) } \mathcal{I}$

B9: $\mathcal{X} \approx \text{map\_op } \curvearrowright \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id } \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I} \uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

F2: $\mathcal{X} \uparrow \approx \mathcal{I}$

R1: $op_2 \bullet (op_1 \uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1) \uparrow$

R2: $(op_1 \uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I})) \uparrow$

R3: $op_1 \parallel (op_2 \uparrow) \approx (\text{map\_op } \curvearrowleft \curvearrowleft\ (op_1 \parallel op_2)) \uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2)) \uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1) \uparrow$

R5: $\text{map\_op Inl Inl } ((op :: ('i + 0, 'o + 0, 'd)\ op) \uparrow) \approx op$

R6: $(op \uparrow) \uparrow \approx (\text{map\_op } \curvearrowright \curvearrowright\ op) \uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (op_1 \parallel op_2) \parallel op_3$

B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl}\ op$

B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr}\ op$

B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$

B4_1: $op \bullet \mathcal{I} \approx op$

B4_2: $\mathcal{I} \bullet op \approx op$

B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$

B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$

B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$

B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl)}\ \mathcal{I}$

B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

F2: $\mathcal{X}\uparrow \approx \mathcal{I}$

R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$

R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$

R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\ \curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

R6: $(op\uparrow)\uparrow \approx (\text{map\_op}\ \curvearrowright\ \curvearrowright\ op)\uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op } \curvearrowright \curvearrowright \ (op_1 \parallel op_2) \parallel op_3$

B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl } op$

B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr } op$

B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$

B4_1: $op \bullet \mathcal{I} \approx op$

B4_2: $\mathcal{I} \bullet op \approx op$

B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$

B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$

B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$

B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl) } \mathcal{I}$

B9: $\mathcal{X} \approx \text{map\_op } \curvearrowright \curvearrowright \ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id } \curvearrowleft \ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

F2: $\mathcal{X}\uparrow \approx \mathcal{I}$

R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$

R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$

R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op } \curvearrowleft \curvearrowleft \ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl } ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

R6: $(op\uparrow)\uparrow \approx (\text{map\_op } \curvearrowright \curvearrowright \ op)\uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op } \curvearrowright \curvearrowright (op_1 \parallel op_2) \parallel op_3$
B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl } op$
B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr } op$
B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$
B4_1: $op \bullet \mathcal{I} \approx op$
B4_2: $\mathcal{I} \bullet op \approx op$
B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$
B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$
B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$
B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl) } \mathcal{I}$
B9: $\mathcal{X} \approx \text{map\_op } \curvearrowright \curvearrowright (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id } \curvearrowleft (\mathcal{I} \parallel \mathcal{X})$
B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$
F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$
F2: $\mathcal{X}\uparrow \approx \mathcal{I}$
R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$
R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$
R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op } \curvearrowleft \curvearrowleft (op_1 \parallel op_2))\uparrow$
R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$
R5: $\text{map\_op Inl Inl } ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$
R6: $(op\uparrow)\uparrow \approx (\text{map\_op } \curvearrowright \curvearrowright op)\uparrow$

Basic Network Algebra Properties

B1: $op_1 \parallel (op_2 \parallel op_3) \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (op_1 \parallel op_2) \parallel op_3$

B2_1: $op \parallel (\mathcal{I} :: (0, 0, 'd)\ op) \approx \text{map\_op Inl Inl}\ op$

B2_2: $(\mathcal{I} :: (0, 0, 'd)\ op) \parallel op \approx \text{map\_op Inr Inr}\ op$

B3: $(op_1 \bullet op_2) \bullet op_3 \approx op_1 \bullet (op_2 \bullet op_3)$

B4_1: $op \bullet \mathcal{I} \approx op$

B4_2: $\mathcal{I} \bullet op \approx op$

B5: $(op_1 \parallel op_2) \bullet (op_3 \parallel op_4) \approx (op_1 \bullet op_3) \parallel (op_2 \bullet op_4)$

B6: $\mathcal{I} \parallel \mathcal{I} \approx \mathcal{I}$

B7: $\mathcal{X} \bullet \mathcal{X} \approx \mathcal{I}$

B8: $(\mathcal{X} :: ('i + 0, 0 + 'i, 'd)\ op) \approx \text{map\_op id (case\_sum Inr Inl)}\ \mathcal{I}$

B9: $\mathcal{X} \approx \text{map\_op}\ \curvearrowright\ \curvearrowright\ (\mathcal{X} \parallel \mathcal{I}) \bullet \text{map\_op id}\ \curvearrowleft\ (\mathcal{I} \parallel \mathcal{X})$

B10: $(op_1 \parallel op_2) \bullet \mathcal{X} \approx \mathcal{X} \bullet (op_2 \parallel op_1)$

F1: $\mathcal{I}\uparrow \approx (\mathcal{I} :: (0, 0, 'd)\ op)$

F2: $\mathcal{X}\uparrow \approx \mathcal{I}$

R1: $op_2 \bullet (op_1\uparrow) \approx ((op_2 \parallel \mathcal{I}) \bullet op_1)\uparrow$

R2: $(op_1\uparrow) \bullet op_2 \approx (op_1 \bullet (op_2 \parallel \mathcal{I}))\uparrow$

R3: $op_1 \parallel (op_2\uparrow) \approx (\text{map\_op}\ \curvearrowleft\ \curvearrowleft\ (op_1 \parallel op_2))\uparrow$

R4: $(op_1 \bullet (\mathcal{I} \parallel op_2))\uparrow \approx ((\mathcal{I} \parallel op_2) \bullet op_1)\uparrow$

R5: $\text{map\_op Inl Inl}\ ((op :: ('i + 0, 'o + 0, 'd)\ op)\uparrow) \approx op$

R6: $(op\uparrow)\uparrow \approx (\text{map\_op}\ \curvearrowright\ \curvearrowright\ op)\uparrow$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op} \curvearrowleft \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet \bigtriangledown \approx \bigtriangledown$ for $\bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $\bigtriangleup \bullet \mathcal{X} \approx \bigtriangleup$ for $\bigtriangleup \in \{\mathcal{C}, \Lambda\}$

A3$_\mathcal{Q}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

A3$_\mathcal{V}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id}\ \mathcal{I}$

A4: $\bigtriangledown \bullet ! \approx ! \parallel !$ for $\bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet \bigtriangleup \approx \text{i} \parallel \text{i}$ for $\bigtriangleup \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id} \curvearrowleft (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

A7: $\mathcal{C} \bullet (! \parallel \mathcal{I}) \approx \text{map\_op id Inr}\ \mathcal{I}$

A9: $\text{i} \bullet ! \approx \mathcal{I}$

A10: $\mathcal{Q} \bullet \mathcal{C} \approx (\mathcal{C} \parallel \mathcal{C}) \bullet (\text{map\_op} \curvearrowright \curvearrowright (\text{map\_op} \curvearrowleft \curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})) \bullet (\mathcal{Q} \parallel \mathcal{Q})$

A11: $\mathcal{C} \bullet \mathcal{Q} \approx \mathcal{I}$

A12: $\text{i} \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A16: $! \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A13: $\text{i} \approx \text{i} \parallel \text{i}$

A17: $! \approx ! \parallel !$

A14: $\text{map\_op id Inl}\ (\bigtriangledown :: (0 + 0, 0, \text{'d})\ op) \approx \mathcal{I}$ for $\bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A15: $\bigtriangledown \approx \text{map\_op} \curvearrowright \curvearrowright (\text{map\_op} \curvearrowleft \curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I}) \bullet (\bigtriangledown \parallel \bigtriangledown)$ for $\bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A18: $\text{map\_op Inl id}\ (\bigtriangleup :: (0, 0 + 0, \text{'d})\ op) \approx \mathcal{I}$ for $\bigtriangleup \in \{\mathcal{C}, \Lambda\}$

A19: $\bigtriangleup \approx (\bigtriangleup \parallel \bigtriangleup) \bullet \text{map\_op} \curvearrowright \curvearrowright (\text{map\_op} \curvearrowleft \curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})$ for $\bigtriangleup \in \{\mathcal{C}, \Lambda\}$

F3: $(\text{map\_op id Inr}\ \bigtriangledown) \uparrow \approx !$ for $\bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

F4: $(\text{map\_op Inr id}\ \bigtriangleup) \uparrow \approx \text{i}$ for $\bigtriangleup \in \{\mathcal{C}, \Lambda\}$

F5: $((\mathcal{I} \parallel \mathcal{C}) \bullet \text{map\_op} \curvearrowright \curvearrowright (\mathcal{X} \parallel \mathcal{I}) \bullet (\mathcal{I} \parallel \mathcal{Q})) \uparrow \approx ! \bullet \text{i}$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op} \curvearrowleft \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet (\gtrdot) \approx (\gtrdot)$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $(\lessdot) \bullet \mathcal{X} \approx (\lessdot)$ for $(\lessdot) \in \{\mathcal{C}, \Lambda\}$

A3$_\mathcal{Q}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

A3$_\mathcal{V}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id}\ \mathcal{I}$

A4: $(\gtrdot) \bullet ! \approx ! \parallel !$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet (\lessdot) \approx \text{i} \parallel \text{i}$ for $(\lessdot) \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id} \curvearrowleft (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

A7: $\mathcal{C} \bullet (! \parallel \mathcal{I}) \approx \text{map\_op id Inr}\ \mathcal{I}$

A9: $\text{i} \bullet ! \approx \mathcal{I}$

A10: $\mathcal{Q} \bullet \mathcal{C} \approx (\mathcal{C} \parallel \mathcal{C}) \bullet (\text{map\_op} \curvearrowright\curvearrowright (\text{map\_op} \curvearrowleft\curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})) \bullet (\mathcal{Q} \parallel \mathcal{Q})$

A11: $\mathcal{C} \bullet \mathcal{Q} \approx \mathcal{I}$

A12: $\text{i} \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A16: $! \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A13: $\text{i} \approx \text{i} \parallel \text{i}$

A17: $! \approx ! \parallel !$

A14: $\text{map\_op id Inl}\ ((\gtrdot) :: (0 + 0, 0, \text{'d})\ op) \approx \mathcal{I}$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

A15: $(\gtrdot) \approx \text{map\_op} \curvearrowright\curvearrowright (\text{map\_op} \curvearrowleft\curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I}) \bullet ((\gtrdot) \parallel (\gtrdot))$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

A18: $\text{map\_op Inl id}\ ((\lessdot) :: (0, 0 + 0, \text{'d})\ op) \approx \mathcal{I}$ for $(\lessdot) \in \{\mathcal{C}, \Lambda\}$

A19: $(\lessdot) \approx ((\lessdot) \parallel (\lessdot)) \bullet \text{map\_op} \curvearrowright\curvearrowright (\text{map\_op} \curvearrowleft\curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})$ for $(\lessdot) \in \{\mathcal{C}, \Lambda\}$

F3: $(\text{map\_op id Inr} (\gtrdot)) \uparrow \approx !$ for $(\gtrdot) \in \{\mathcal{Q}, \mathcal{V}\}$

F4: $(\text{map\_op Inr id} (\lessdot)) \uparrow \approx \text{i}$ for $(\lessdot) \in \{\mathcal{C}, \Lambda\}$

F5: $((\mathcal{I} \parallel \mathcal{C}) \bullet \text{map\_op} \curvearrowright\curvearrowright (\mathcal{X} \parallel \mathcal{I}) \bullet (\mathcal{I} \parallel \mathcal{Q})) \uparrow \approx ! \bullet \text{i}$

Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

A1: $(\mathcal{Q} \parallel \mathcal{I}) \bullet \mathcal{Q} \approx \text{map\_op} \curvearrowleft \text{id}\ ((\mathcal{I} \parallel \mathcal{Q}) \bullet \mathcal{Q})$

A2: $\mathcal{X} \bullet \bigtriangledown \approx \bigtriangledown \quad \text{for } \bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A6: $\bigtriangleup \bullet \mathcal{X} \approx \bigtriangleup \quad \text{for } \bigtriangleup \in \{\mathcal{C}, \Lambda\}$

A3$_\mathcal{Q}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{Q} \approx (! :: (0 + \text{'a}, 0, \text{'d})\ op) \bullet \text{i}$

A3$_\mathcal{V}$: $((\text{i} :: (0, \text{'a}, \text{'d})\ op) \parallel \mathcal{I}) \bullet \mathcal{V} \approx \text{map\_op Inr id } \mathcal{I}$

A4: $\bigtriangledown \bullet ! \approx ! \parallel ! \quad \text{for } \bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A8: $\text{i} \bullet \bigtriangleup \approx \text{i} \parallel \text{i} \quad \text{for } \bigtriangleup \in \{\mathcal{C}, \Lambda\}$

A5: $\mathcal{C} \bullet (\mathcal{C} \parallel \mathcal{I}) \approx \text{map\_op id} \curvearrowleft (\mathcal{C} \bullet (\mathcal{I} \parallel \mathcal{C}))$

A7: $\mathcal{C} \bullet (! \parallel \mathcal{I}) \approx \text{map\_op id Inr } \mathcal{I}$

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A10: $\mathcal{Q} \bullet \mathcal{C} \approx (\mathcal{C} \parallel \mathcal{C}) \bullet (\text{map\_op} \curvearrowright \curvearrowright (\text{map\_op} \curvearrowleft \curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I})) \bullet (\mathcal{Q} \parallel \mathcal{Q})$

A11: $\mathcal{C} \bullet \mathcal{Q} \approx \mathcal{I}$

A12: $\text{i} \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A16: $! \approx (\mathcal{I} :: (0, 0, \text{'d})\ op)$

A13: $\text{i} \approx \text{i} \parallel \text{i}$

A17: $! \approx ! \parallel !$

A14: $\text{map\_op id Inl } (\bigtriangledown :: (0 + 0, 0, \text{'d})\ op) \approx \mathcal{I} \quad \text{for } \bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A15: $\bigtriangledown \approx \text{map\_op} \curvearrowright \curvearrowright (\text{map\_op} \curvearrowleft \curvearrowleft (\mathcal{I} \parallel \mathcal{X}) \parallel \mathcal{I}) \bullet (\bigtriangledown \parallel \bigtriangledown) \quad \text{for } \bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

A18: $\text{map\_op Inl id } (\bigtriangleup :: (0, 0 + 0, \text{'d})\ op) \approx \mathcal{I} \quad \text{for } \bigtriangleup \in \{\mathcal{C}, \Lambda\}$

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F3: $(\text{map\_op id Inr } \bigtriangledown) \uparrow \approx ! \quad \text{for } \bigtriangledown \in \{\mathcal{Q}, \mathcal{V}\}$

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Properties of Equality Test, Merge, Copy, Split, Source and Sink operators

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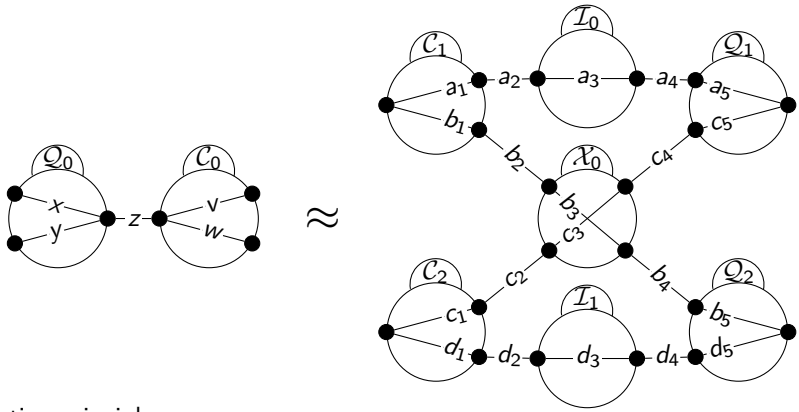
A10



- $\approx$ coinduction principle
- Buffers generalization
- Buffers invariant

Related Work

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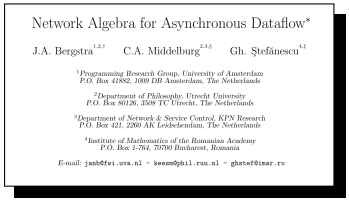
- Instance in Algebra of Communicating Processes (ACP)
- Our operators closely follow theirs



Network Algebra for Asynchronous Dataflow*

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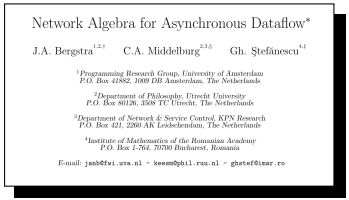

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- Our operators can be seen as a domain specific variant of choice trees

Interaction Trees

Representing Recursive and Impure Programs in Coq

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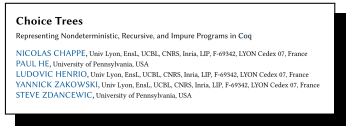

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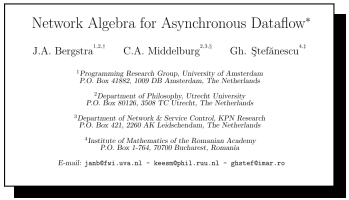

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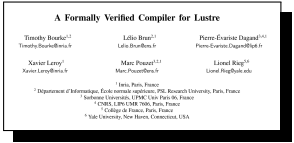
- **Synchronous** dataflow languages have some proof assistant mechanization



A Formally Verified Compiler for Lustre

Timothy Bourke[1,2] Timothy.Bourke@inria.fr

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Pierre-Évariste Dagand[1,4,5] Pierre-Evariste.Dagand@lip6.fr

Xavier Leroy[1] Xavier.Leroy@inria.fr

Marc Pouzet[1,2,3] Marc.Pouzet@ens.fr

Lionel Rieg[5,6] Lionel.Rieg@yale.edu

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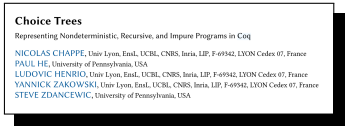

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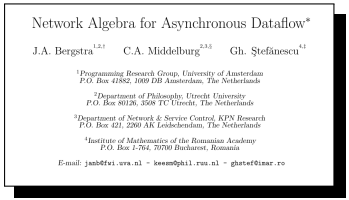

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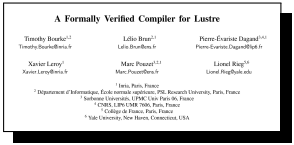

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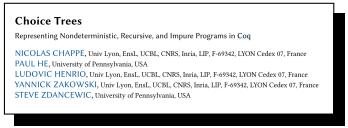

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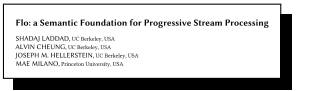
- Flo: a framework for representing streaming computations originating from different systems like Flink and DBSP



Flo: a Semantic Foundation for Progressive Stream Processing

SHADAJ LADDAD, UC Berkeley, USA
ALVIN CHEUNG, UC Berkeley, USA
JOSEPH M. HELLERSTEIN, UC Berkeley, USA
MAE MILANO, Princeton University, USA

Conclusion

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- Future work:
  - Brock–Andersen Time anomaly
  - Formalize Timely Dataflow infrastructure
  - Verify Timely Dataflow algorithms

Questions, comments and suggestions