Efficient and Verified Non-Terminating Programs with Isabelle-LLVM

Rafael Castro G. Silva

rasi@di.ku.dk

Department of Computer Science
University of Copenhagen

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Introduction

Context

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
      - Time-Aware Computations

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- Distributed Systems
  - Stream processing frameworks
    - Dataflow models
      - Time-Aware Computations
- Formal Methods
  - Verification using proof assistants
    - Isabelle proofs
      - Verified + executable + efficient code
- Formalization of Time-Aware Stream Processing

Stream Processing

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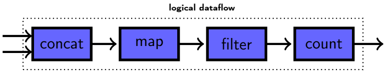
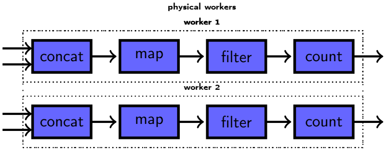
- Dataflow Model:
  - Directed graph of interconnected operators that perform event-wise transformations
  - E.g.: Apache Flink, Apache Samza, Apache Spark, Google Cloud Dataflow, and Timely Dataflow
  - Highly Parallel





Time-Aware Stream Processing

- Time-Aware Computations:
  - Timestamps: Metadata associating the data with some data collection
    - An unix timestamp
    - Version of the data
    - Logical grouping
  - Watermarks: Metadata indicating the completion of a data collection
    - e.g.: A watermark 5 says that there is no data associated with timestamp 5 or bellow arriving
    - Are increasingly monotonic (they don't go backwards in time)
  - e.g.:

DT t_4 d	DT t_0 a	DT t_1 c	WM t_1	DT t_2 b	WM t_2	DT t_5 c	DT t_3 a	DT t_5 a	WM t_5	$\cdots$

Preliminaries

Isabelle/HOL

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- Isabelle: A generic proof assistant

- Isabelle/HOL: Isabelle's flavor of HOL

Isabelle/HOL: (Co)datatypes

- Datatypes and Codatatypes

  codatatype (lset: $'a$) *llist* = lnull: LNil | LCons (lhd: $'a$) (ltl: $'a$ *llist*)
  for map: lmap **where** ltl LNil = LNil

- Examples:
  - LNil
  - LCons *1* (LCons *2* (LCons *3* LNil))
  - LCons *0* (LCons *0* (LCons *0* (...)))
- Proofs by induction
- Proofs by coinduction

State of this work

What have I formalized so far? (part 1)

- Formalization stream processing (model)
  - Using Isabelle/HOL: (co)datatypes, (co)recursion, and (co)induction
  - Streams are lazy lists, and operators as a codatatype
  - Semantics: a produce function that runs an operator throughout a lazy lists
    - Mix of recursion and corecursion: inductive and coinductive principles
  - Sequential composition
    - Correctness!

What have I formalized so far? (part 2)

- Time-Aware computations
  - Coinductive properties of streams: monotonicity and productivity
  - Building blocks operators:
    - Convenience operators: batching and incremental computations
      - Incremental computing: only update results that are affected by the new input
      - With verified properties: Soundness, Completeness, preservation of productivity, and preservation of monotonicity
  - Compositional reasoning
- Case studies with the building blocks:
  - Incremental histogram operator
  - Relational join

Next Steps

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  - Code generator: functional languages (OCaml, Haskell, SML...)
  - Functional data-structures: (many algorithms are slow with linked lists)

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- How do we make efficient and verified programs in Isabelle/HOL?
- Isabelle-LLVM!
- Let's port this formalization to Isabelle-LLVM then!
- This is a non-terminating program

Isabelle-LLVM

Isabelle Refinement Framework and Isabelle-LLVM

- Isabelle Refinement Framework
  - Framework for step-wise refinement verification (refinement calculus): Specification $\rightarrow$ Abstract Algorithm $\rightarrow$ Less Abstract Algorithm $\rightarrow$ Executable Code
  - Imperative HOL as backend (lowest layer in the refinement)
    - Shallow Embedding of Monadic programs in HOL
  - Separation Logic (heap memory reasoning)
- Isabelle-LLVM is a new backend for the Isabelle Refinement Framework
  - Generates LLVM code (efficient imperative code)

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  - Yes and No!

Isabelle-LLVM's Recursion Model

- Knaster–Tarski theorem
  - Standard way to define the semantics of recursive definitions
    - Isabelle/HOL: Partial Function Package
  - Every monotonic function on Complete Chain Partial Order (CCPO) has a fixed point
  - Induction principle
- No need for well-foundness

What the Heck is a CCPO?

- Chain: A set in which all elements are comparable
- Complete Chain Partial Order:
  1. A partial order: $'a::order$
  2. A function that returns the suprimium (least upper bound) from a chain $'a::order \text{ set} \Rightarrow 'a$

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- Isabelle-LLVM's monad:

$$\textbf{datatype}\ 'a\ neM = SPEC\ (the\_spec\text{:}\ 'a \Rightarrow bool) \mid FAIL$$

  - Order: flat (every $SPEC$ is greater than $FAIL$, $SPEC$s are only comparable when they are equal)
  - Suprimium from a chain: The $SPEC$, or the only $FAIL$
  - Bottom: $FAIL$
    - Non-termination

The First Steps

Our CCPO Attempt

- Let's look in Isabelle!

Questions, comments and suggestions