How to Use Mathematics to Write Better Programs

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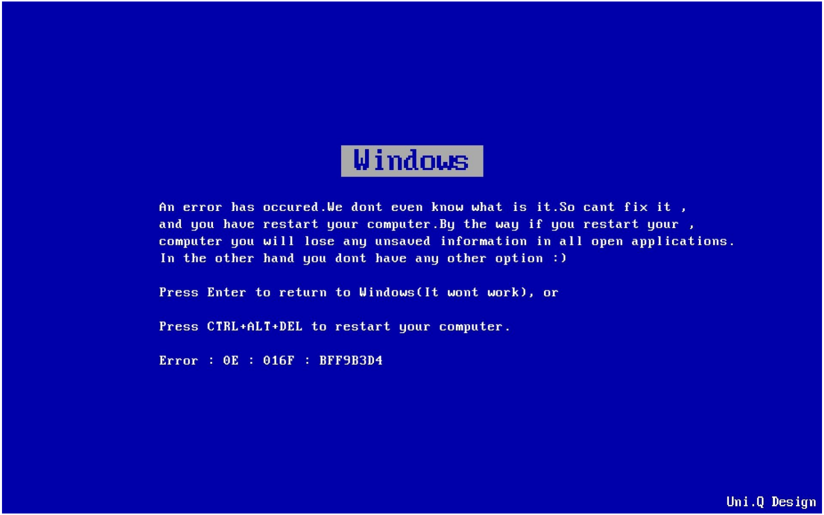
Bugs are annoying



Windows

An error has occured.We dont even know what is it.So cant fix it ,
and you have restart your computer.By the way if you restart your ,
computer you will lose any unsaved information in all open applications.
In the other hand you dont have any other option :)

Press Enter to return to Windows(It wont work), or

Press CTRL+ALT+DEL to restart your computer.

Error : 0E : 016F : BFF9B3D4

Uni.Q Design

- why they happen?

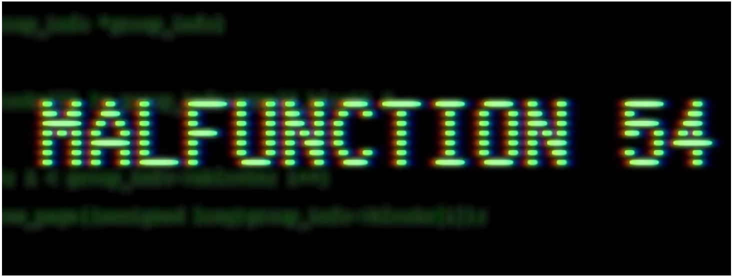
Therac-25: The killing radiation therapy machine

People got hurt

- At least 6 accidents between 1985 and 1987, 3 died later

Malfunction messages

- Malfunction messages from 1 to 64, with no extra information



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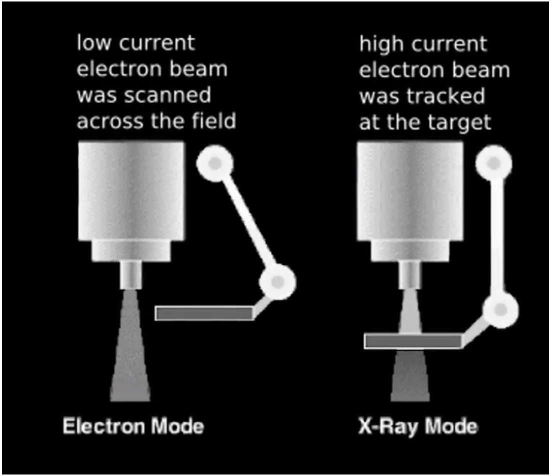
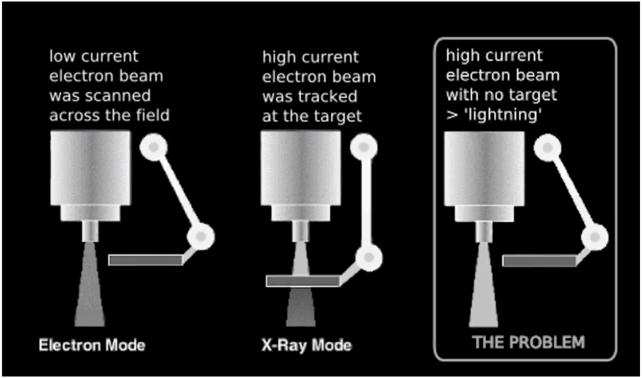
MALFUNCTION 54
```

The machine a bug

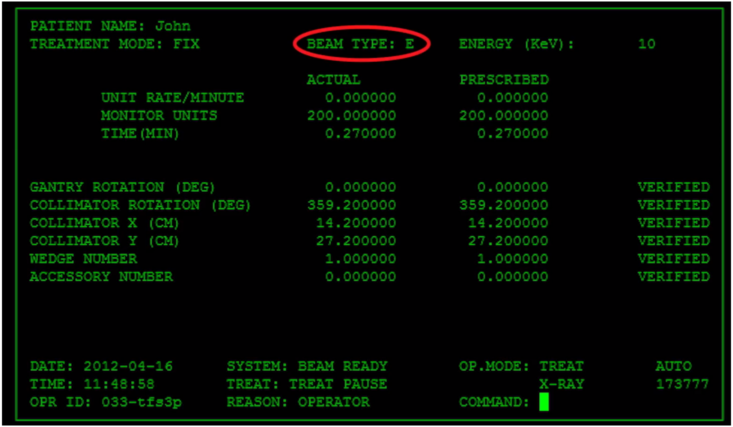
- The machine had three modes: field light, direct electron beam, x-ray



The machine a bug


low current electron beam was scanned across the field
Electron Mode
high current electron beam was tracked at the target
X-Ray Mode
high current electron beam with no target > 'lightning'
THE PROBLEM

How to cause the bug



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PATIENT NAME: John
TREATMENT MODE: FIX            BEAM TYPE: E      ENERGY (KeV):       10

                               ACTUAL            PRESCRIBED
       UNIT RATE/MINUTE          0.000000          0.000000
       MONITOR UNITS           200.000000        200.000000
       TIME(MIN)                 0.270000          0.270000


GANTRY ROTATION (DEG)            0.000000          0.000000         VERIFIED
COLLIMATOR ROTATION (DEG)      359.200000        359.200000         VERIFIED
COLLIMATOR X (CM)               14.200000         14.200000         VERIFIED
COLLIMATOR Y (CM)               27.200000         27.200000         VERIFIED
WEDGE NUMBER                     1.000000          1.000000         VERIFIED
ACCESSORY NUMBER                 0.000000          0.000000         VERIFIED




DATE: 2012-04-16      SYSTEM: BEAM READY       OP.MODE: TREAT      AUTO
TIME: 11:48:58        TREAT: TREAT PAUSE                X-RAY      173777
OPR ID: 033-tfs3p     REASON: OPERATOR         COMMAND:
```
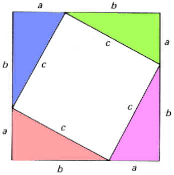

How to write software that don't cause catastrophic failures?

- Formal Methods!
- There are many techniques: static analysis, model checking and formal proving

What is to prove something?

- Establishing mathematical facts that are universally true
- You know some: Pythoragoras theorem: $\forall abc.c^2 = a^2 + b^2$



Two ways to calculate this area:

1. $(a+b)(a+b) = a^2 + ab + ab + b^2$
2. $4(ab)/2 + c^2$
   they should be the same!
   $a^2 + ab + ab + b^2 = 4(ab)/2 + c^2$
   $a^2 + b^2 = c^2$
3. How can you be sure that there is no mistake in the proof?

Proof assistants

- Proof assistants: programs that verify if your proof is correct!
    - We trust the verifier!
- I use a proof assistant called Isabelle/HOL to verify programs
- How my research works:
    1. I write a specification of the program: what it should do, and what it should not do
    2. I write the program
    3. I write a proof that the program respects the specification
        - Usually this requires writing many other auxiliary proofs

Examples!

- Pythagoras
- Sorting

This is a real job!

- TLA+/TLC: Amazon AWS, Intel, Microsoft. . .
- Coq (CompCert): Airbus
- Isabelle/HOL (seL4): Defense Advanced Research Projects Agency (DARPA)
- Formal methods companies: Absint, Trustworthy Systems
- Apple: `https://jobs.apple.com/en-us/details/200343072/formal-verification-engineer`

Thank you!

- Questions?